Task 1.2. Supervised Learning: Bayesian Linear Regressor

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1 Introduction

Simple linear regressors are overconfident in their predictions. Bayesian linear regressors are an extension of this model used to solve this particular issue.

2 The problem

In this model we want to compute a posterior distribution given a set of training samples, as shown in Equation 1. Where $\mathbf{w}$ is a one dimensional array with the world state, $\mathbf{X}$ is a matrix with the data points, ϕ are the parameters of a linear function of the data.

$$Pr(\phi|\mathbf{X}, \mathbf{w}) = \frac{Pr(\mathbf{w}|\mathbf{X}, \phi)Pr(\phi)}{Pr(\mathbf{w}|\mathbf{X})} \tag{1}$$

The prior in Equation 1 is a normal distribution with 0 mean and spherical covariance. And the likelihood is a multivariate normal distribution, as shown in Equation 2. Where σ^2 is the covariance, $\mathbf{I}$ is the identity matrix and $\boldsymbol{\theta} = \{\phi, \sigma^2\}$.

$$Pr(\mathbf{w}|\mathbf{X}, \boldsymbol{\theta}) = Norm_{\mathbf{w}}\left[\mathbf{X}^T\phi, \sigma^2\mathbf{I}\right], \tag{2}$$

The posterior distribution is shown in Equation 3.

$$\begin{gathered} Pr(\phi|\mathbf{X}, \mathbf{w}) = Norm_{\phi}\left[\frac{1}{\sigma^2}\mathbf{A}^{-1}\mathbf{X}\mathbf{w}, \mathbf{A}^{-1}\right], \\ \mathbf{A} = \frac{1}{\sigma^2}\mathbf{X}\mathbf{X}^T + \frac{1}{\sigma_p^2}\mathbf{I}, \end{gathered} \tag{3}$$

The probability distribution of a new world state w^* over new test data x^* is a Normal distribution as shown in Equation 4.

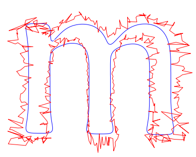
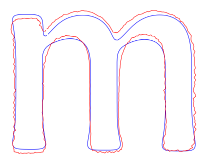
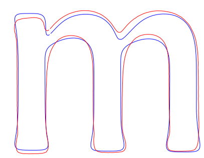
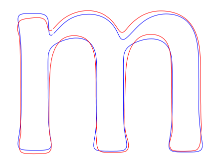
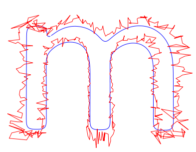
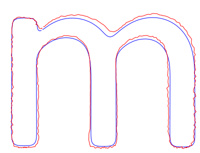
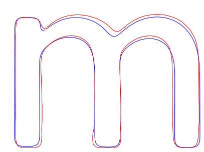
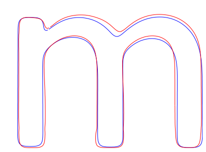
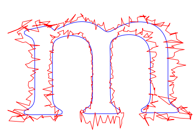
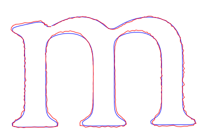
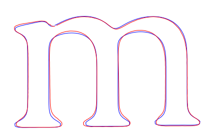
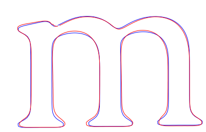
$$
\begin{aligned}
Pr(w^*|\mathbf{x}^*, \mathbf{X}, \mathbf{w}) &= \int Pr(w^*|\mathbf{x}^*, \phi) Pr(\phi|\mathbf{X}, \mathbf{w}) d\phi, \\
&= Norm_{w^*}\left[\frac{1}{\sigma^2}\mathbf{x}^{*T}\mathbf{A}^{-1}\mathbf{X}\mathbf{w}, \mathbf{x}^{*T}\mathbf{A}^{-1}\mathbf{x}^* + \sigma^2\right],
\end{aligned} \tag{4}
$$

Since our world states $\mathbf{w_i}$ are multivariate, a separate regressor is used for each dimension in $\mathbf{w_i}$.

3 Results

In Table 1 we show results for a font data set. The Bayesian linear regressors is trained with pictures of n in different fonts and their corresponding m in the same font. For testing, n with new fonts are used.

Table 1: Table with comparing the reconstrucction of several m with different fonts and variances. In blue is the ground truth and in red is the reconstructed character.

	$\sigma^2 = 10$	$\sigma^2 = 1$	$\sigma^2 = 0.01$	$\sigma^2 = 0.001$
Font1				
Font2				
Font3				

As the Table 1 shows smaller variances σ^2 produce better results. Therefore, we can conclude that ....