Task 1.1. Supervised Learning: Standard Classifier

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1 Introduction

Given pictures from the world and been asked to classify them in several groups, we are faced with a problem of multi-class classification. One of the options would be to create N one-against-all binary classifiers. Using x to denote our data, ω to for the world state, and *lambda* for the probability of observing the given class. However a better one involves using a categorical distribution to model our world. Where $\boldsymbol{\lambda}$ is a vector that contains a λ for each class.

$$Pr(\omega|\mathbf{x}) = Cat_w[\boldsymbol{\lambda}[\mathbf{x}]] \tag{1}$$

2 Mathematical derivation

As stated in the introduction, we are going to fit a Categorical probability model into our data. Denoting I as the total number of data points that we are given. Then, using Bayes' rule we have:

$$Pr(\theta|x_{1\cdots I}) = \frac{\prod_{i=1}^{I} Pr(\omega = k_n|x, \theta)Pr(\theta)}{Pr(x_{1\cdots I})} \tag{2}$$

We need N activation functions to enforce the constrains. Since we are solving for multi-class classification a logistic sigmoid function as activation will not be valid. Therefore a softmax function is used instead for each activation a_n.

$$a_n = \phi_n^T x \tag{3}$$

$$\lambda_n = softmax_n[a_1, a_2 \cdots a_N] = \frac{exp[a_n]}{\sum_{m=1}^{N} exp[a_m]} \tag{4}$$

For the Prior we are going to use a Normal distribution with zero mean and σ variance. In order to simplify the calculations we are going to minimise the log of the probability:

$$
\begin{aligned}
Ł &= -\sum_{i=1}^{I} log\left[P(\omega = k|\mathbf{x}, \phi)\right] + log\left[Pr(\phi)\right] \\
&= -log\sum_{i=1}^{I} y_{in} + \frac{1}{2\sigma^2}\phi^T\phi
\end{aligned} \tag{5}
$$

With gradient and Hessian updates being:

$$
\begin{aligned}
\frac{\delta L}{\delta \phi_n} &= \sum_{i=1}^{I} \left(y_{in} - \delta\left[\omega_i - n\right]\right)\mathbf{x}_i + \frac{\phi}{\sigma^2} \\
\frac{\delta^2 L}{\delta \phi_m \phi_n} &= \sum_{i=1}^{I} y_{im}\left(\delta\left[m - n\right] - y_{in}\mathbf{x}_i\mathbf{x}_i^T\right) + \frac{\delta\left[m - n\right]}{\sigma^2}
\end{aligned} \tag{6}
$$

To make the predictions we evaluate a new sample doing a Laplace approximation and then a Monte Carlo integration

3 Implementation

4 Results

Digits initial phi 2.5 works, 5 or bigger runs on NaN error, -10 gives NaN too, -1 works Digits dataset, with prior 100, initial phi ones Elapsed time is 414.619122 seconds. Hits: 71.87

5 Conclusion