Task 2. Mesh Animation

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1 Introduction

Generating a plausible mesh which is a morphing of two given meshes ...

2 Implemented techniques

We present two approaches to solve the mesh morphing problem.

2.1 Linear interpolation

Linear interpolation is a simple and intuitive first approach. The new position of a vertex $\mathbf{v_{it}} = [v_{ix}, v_{iy}, v_{iz}]^T$ in an interval $t = \{0, \ldots, 1\}$ is given by

$$\mathbf{v_{it}} = (1-t)\mathbf{p_i} + t\mathbf{q_i} \quad i \in \{1, \ldots, n\},$$

where n is the number of vertices, $\mathbf{p_i} = [p_x, p_y, p_z]^T$ and $\mathbf{q_i} = [q_x, q_y, q_z]^T$ are the corresponding vertices in the source and target meshes respectively. Some meshes generated with this method are shown in Figure 1.

2.2 As rigid as possible interpolation

The previous method produces distorted meshes due to its non-rigid nature. One way to produce better results is to use a transform-based technique. Lets define an affine transformation from $\mathbf{p_i}$ to $\mathbf{q_i}$ such that:

$$A\mathbf{p_i} + \mathbf{l} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{pmatrix} + \begin{pmatrix} l_x \\ l_y \\ l_z \end{pmatrix} = \mathbf{q_i}.$$

We have a matrix A_s for each triangle, since there are 12 unknowns and each triangle only gives 9 equations, we add a extra point to be able to compute A_i. This new point is computed as $\mathbf{p_{new}} = \mathbf{c_{tr}} + w\mathbf{n_{tr}}$, where $\mathbf{c_{tr}}$ is the triangle centroid, w is the average triangle edge length and $\mathbf{n_{tr}}$ is the triangle normal. Since in a general mesh vertices a shared among different triangles, lets define B_j as the actual transformation matrix, where

$$E(t) = \sum_{s}^{S} \|A_s(t) - B_s(t)\|_F^2 \,,$$

where $\|\cdot\|_F$ is the matrix Frobenius norm, and the objective is to minimize $E(t)$. If we ignore the l term, rename $\mathbf{q_i}$ to treat each vertex in individual vertex in the mesh as $\mathbf{v}_i$ the mesh, and rearrange B such that

$$PB_{col} = \begin{pmatrix} \mathbf{p}_i^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_i^T \\ \mathbf{p}_j^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_j^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_j^T \\ \mathbf{p}_k^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_k^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_k^T \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{21} \\ b_{22} \\ b_{23} \\ b_{31} \\ b_{32} \\ b_{33} \end{pmatrix} = \begin{pmatrix} v_{ix} \\ v_{iy} \\ v_{iz} \\ v_{jx} \\ v_{jy} \\ v_{jz} \\ v_{kx} \\ v_{ky} \\ v_{kz} \end{pmatrix} \text{ and } P^{-1} = \begin{pmatrix} \alpha_1 & 0 & 0 & \delta_1 & 0 & 0 & \lambda_1 & 0 & 0 \\ \alpha_2 & 0 & 0 & \delta_2 & 0 & 0 & \lambda_2 & 0 & 0 \\ \alpha_3 & 0 & 0 & \delta_3 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & \epsilon_1 & 0 & 0 & \mu_1 & 0 \\ 0 & \beta_2 & 0 & 0 & \epsilon_2 & 0 & 0 & \mu_2 & 0 \\ 0 & \beta_3 & 0 & 0 & \epsilon_3 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & \gamma_1 & 0 & 0 & \kappa_1 & 0 & 0 & \pi_1 \\ 0 & 0 & \gamma_2 & 0 & 0 & \kappa_2 & 0 & 0 & \pi_2 \\ 0 & 0 & \gamma_3 & 0 & 0 & \kappa_3 & 0 & 0 & \pi_3 \end{pmatrix},$$

then we have that $B = P^{-1}V$. We would have the above system for each triangle in the mesh, with its own P and B matrices. Writing explicitly the Frobenius terms in $E(t)$, and expressing B as a linear combination of $\mathbf{v}_i$, we get

$$\begin{aligned} E(t) = \{&a_{11}^2 + a_{12}^2 + \ldots + a_{33}^2 + \alpha_1^2 v_{1x}^2 + \ldots + \pi^2 v_{3z}^2 - 2a_{11}\alpha_1 v_{1x} \\ &- 2a_{11}\delta_1 v_{2x} - 2a_{11}\lambda_1 v_{3x} - \ldots - 2a_{33}\pi_3 v_{3z} + 2\alpha_1 v_{1x}\delta_1 v_{2x} \\ &+ 2\alpha_1 v_{1x}\lambda_1 v_{3x} + \ldots + 2\kappa_3 v_{2z}\pi_3 v_{3z}\}_{j=1} + \ldots + \{\cdots\}_{j=J}, \end{aligned}$$

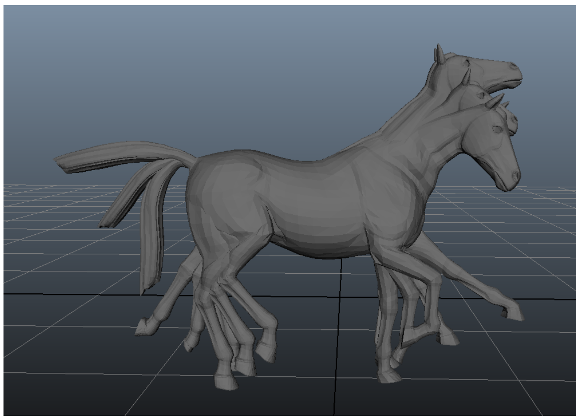
where we have a $\{\cdots\}_j$ for each triangle. Predetermining the transformation of $\mathbf{v_1}$, and setting $\mathbf{u}(t)^T = \big(1, v_{2x}, v_{2y}, v_{2z}, \ldots, v_{mx}, v_{my}, v_{mz}\big)$, where m is the number of vertices, we can stack the terms in a quadratic matrix form

$$E(t) = \mathbf{u}^T \begin{pmatrix} c & \mathbf{g}(t)^T \\ \mathbf{g}(t) & H \end{pmatrix} \mathbf{u},$$

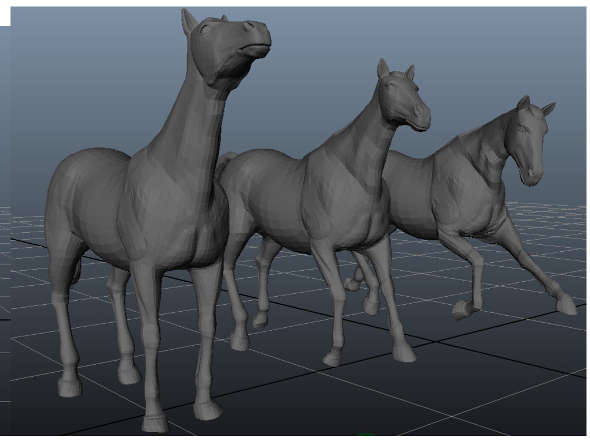
where c are the constant coefficients, $\mathbf{g}$ is a vector with the linear terms and H is matrix with the quadratic and mixed terms. Note that, since $\mathbf{v}_1$ is known, the terms were it appears, become constants or linear in $\mathbf{v}_j$ for $j \neq 1$. Setting the gradient to zero,

$$H\mathbf{u}(t) = -\mathbf{g}(t) \;\rightarrow\; \mathbf{u}(t) = -H^{-1}\mathbf{g}(t).$$

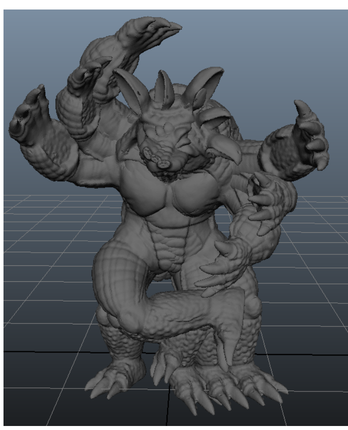
Since H is independent on t, for each iteration we only need to compute the linear interpolation of $\mathbf{v}_1$ and the new $\mathbf{g}(t)$ to solve the system.



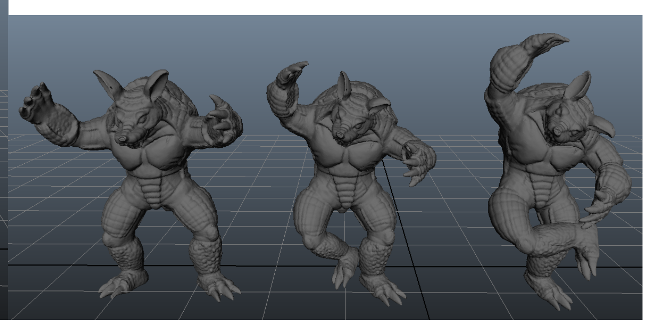
(a) fig 3



(b) fig 4



(c) fig 1



(d) fig 2

Figure 1: Horse and armadillo meshes, morphed mesh is in the middle.