# Task 2. Mesh Animation

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## 1 Introduction

Generating a plausible mesh which is a morphing of two given meshes ...

## 2 Implemented techniques

We present two approaches to solve the mesh morphing problem.

### 2.1 Linear interpolation

Linear interpolation is a simple and intuitive first approach. The new position of a vertex $\mathbf{v_{it}} = [v_{ix}, v_{iy}, v_{iz}]^T$ in an interval $t = \{0, \dots, 1\}$ is given by

$$\mathbf{v_{it}} = (1-t)\mathbf{p_i} + t\mathbf{q_i} \quad i \in \{1, \dots, n\},$$

where $n$ is the number of vertices, $\mathbf{p_i} = [p_x, p_y, p_z]^T$ and $\mathbf{q_i} = [q_x, q_y, q_z]^T$ are the corresponding vertices in the source and target meshes respectively. Some meshes generated with this method are shown in Figure 1.

### 2.2 As rigid as possible interpolation

The previous method produces distorted meshes due to its non-rigid nature. One way to produce more better results is to use a transform-based technique. Lets define an affine transformation from $\mathbf{p_i}$ to $\mathbf{q_i}$ such that:

$$A\mathbf{p_i} + \mathbf{l} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{pmatrix} + \begin{pmatrix} l_x \\ l_y \\ l_z \end{pmatrix} = \mathbf{q_i}.$$

We have a matrix $A_s$ for each triangle. Since in a general mesh vertices a shared among different triangles, lets define $B_j$ as the actual transformation matrix, where:

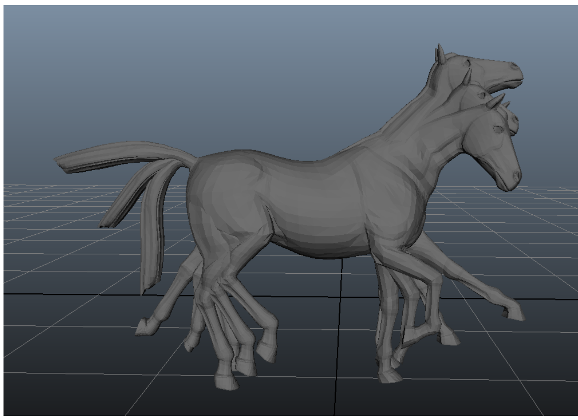
$$E(t) = \sum_{s}^{S} \left\| A_s(t) - B_s(t) \right\|_F^2 ,$$

where $\|\cdot\|_F$ is the matrix Frobenius norm, and the objective is to minimize $E(t)$. If we ignore the $l$ term and rearrange $B$ such that:

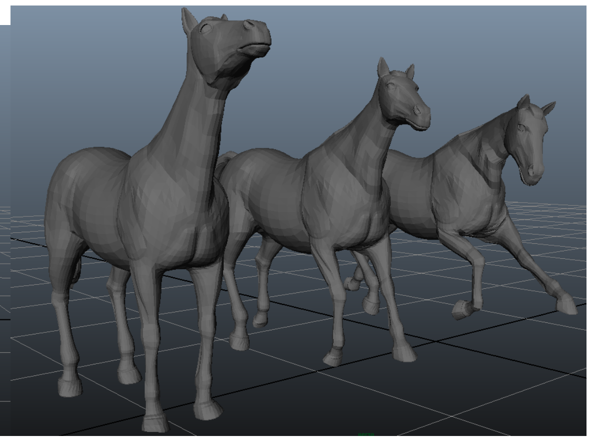
$$PB_{col} = \begin{pmatrix} \mathbf{p}_i^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_i^T \\ \mathbf{p}_j^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_j^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_j^T \\ \mathbf{p}_k^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_k^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_k^T \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{21} \\ b_{22} \\ b_{23} \\ b_{31} \\ b_{32} \\ b_{33} \end{pmatrix} = \begin{pmatrix} \mathbf{q}_{ix} \\ \mathbf{q}_{iy} \\ \mathbf{q}_{iz} \\ \mathbf{q}_{jx} \\ \mathbf{q}_{jy} \\ \mathbf{q}_{jz} \\ \mathbf{q}_{kx} \\ \mathbf{q}_{ky} \\ \mathbf{q}_{kz} \end{pmatrix} \text{ and } P^{-1} = \begin{pmatrix} \alpha_1 & 0 & 0 & \delta_1 & 0 & 0 & \lambda_1 & 0 & 0 \\ \alpha_2 & 0 & 0 & \delta_2 & 0 & 0 & \lambda_2 & 0 & 0 \\ \alpha_3 & 0 & 0 & \delta_3 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & \epsilon_1 & 0 & 0 & \mu_1 & 0 \\ 0 & \beta_2 & 0 & 0 & \epsilon_2 & 0 & 0 & \mu_2 & 0 \\ 0 & \beta_3 & 0 & 0 & \epsilon_3 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & \gamma_1 & 0 & 0 & \kappa_1 & 0 & 0 & \pi_1 \\ 0 & 0 & \gamma_2 & 0 & 0 & \kappa_2 & 0 & 0 & \pi_2 \\ 0 & 0 & \gamma_3 & 0 & 0 & \kappa_3 & 0 & 0 & \pi_3 \end{pmatrix},$$

then we have that $B = P^{-1}Q$. Writing out the Frobenius terms in $E(t)$ in and expressing $B$ as a linear combination of $P^{-1}Q$, we get:

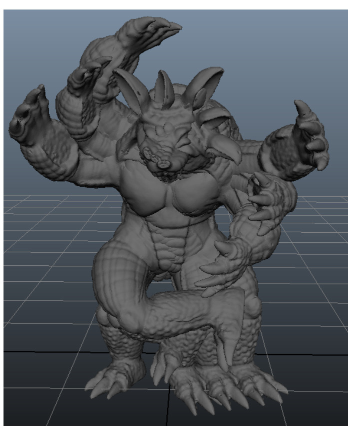
$$\mathbf{v_{it}} = (1-t)\mathbf{p_i} + t\mathbf{q_i} \quad i \in \{1, \ldots, n\},$$



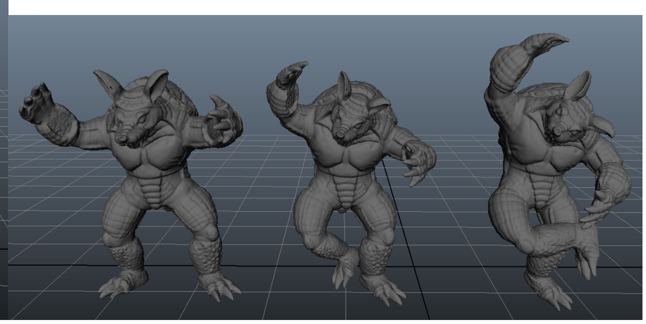
(a) fig 3



(b) fig 4



(c) fig 1



(d) fig 2

Figure 1: Horse and armadillo meshes, morphed mesh is in the middle.