Task 2. Mesh Animation

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1 Introduction

Generating a realistic mesh which is a morphing of two given meshes can be a challenging task. We present two approaches to produce a intermediate 3D mesh from conforming source and target 3D meshes.

2 Linear interpolation

Linear interpolation is a simple and intuitive first approach. The new position of a vertex $\mathbf{v_{it}} = [v_{ix}, v_{iy}, v_{iz}]^T$ in an interval $t = \{0, \dots, 1\}$ is given by

$$\mathbf{v_{it}} = (1-t)\mathbf{p_i} + t\mathbf{q_i} \quad i \in \{1, \dots, n\},$$

where n is the number of vertices, $\mathbf{p_i} = [p_x, p_y, p_z]^T$ and $\mathbf{q_i} = [q_x, q_y, q_z]^T$ are the corresponding vertices in the source and target meshes respectively. Some meshes generated with this method are shown in Figure 1.

3 As rigid as possible interpolation

The previous method produces distorted meshes due to its non-rigid nature. One way to produce better results is to use transform-based techniques [1]. Lets define an affine transformation from $\mathbf{p_i}$ to $\mathbf{q_i}$ such that:

$$A\mathbf{p_i} + \mathbf{l} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{pmatrix} + \begin{pmatrix} l_x \\ l_y \\ l_z \end{pmatrix} = \mathbf{q_i}.$$

We have a matrix A_s for each triangle, since there are 12 unknowns and each triangle only gives 9 equations, we add a extra point to be able to compute A_i. This new point is computed as $\mathbf{p_{new}} = \mathbf{c_{tr}} + w\mathbf{n_{tr}}$, where $\mathbf{c_{tr}}$ is the triangle centroid, w is the average triangle edge length and $\mathbf{n_{tr}}$ is the triangle normal. Since in a general mesh vertices a shared among different triangles, lets define B_j as the actual transformation matrix, where

$$E(t) = \sum_{s}^{S} \|A_s(t) - B_s(t)\|_F^2 ,$$

where $\|\cdot\|_F$ is the matrix Frobenius norm, and the objective is to minimize $E(t)$. If we ignore the l term, rename $\mathbf{q_i}$ to treat each vertex in individual vertex in the mesh as $\mathbf{v}_i$ the mesh, and rearrange B such that

$$PB_{col} = \begin{pmatrix} \mathbf{p}_i^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_i^T \\ \mathbf{p}_j^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_j^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_j^T \\ \mathbf{p}_k^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_k^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_k^T \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{21} \\ b_{22} \\ b_{23} \\ b_{31} \\ b_{32} \\ b_{33} \end{pmatrix} = \begin{pmatrix} v_{ix} \\ v_{iy} \\ v_{iz} \\ v_{jx} \\ v_{jy} \\ v_{jz} \\ v_{kx} \\ v_{ky} \\ v_{kz} \end{pmatrix} \text{ and } P^{-1} = \begin{pmatrix} \alpha_1 & 0 & 0 & \delta_1 & 0 & 0 & \lambda_1 & 0 & 0 \\ \alpha_2 & 0 & 0 & \delta_2 & 0 & 0 & \lambda_2 & 0 & 0 \\ \alpha_3 & 0 & 0 & \delta_3 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & \epsilon_1 & 0 & 0 & \mu_1 & 0 \\ 0 & \beta_2 & 0 & 0 & \epsilon_2 & 0 & 0 & \mu_2 & 0 \\ 0 & \beta_3 & 0 & 0 & \epsilon_3 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & \gamma_1 & 0 & 0 & \kappa_1 & 0 & 0 & \pi_1 \\ 0 & 0 & \gamma_2 & 0 & 0 & \kappa_2 & 0 & 0 & \pi_2 \\ 0 & 0 & \gamma_3 & 0 & 0 & \kappa_3 & 0 & 0 & \pi_3 \end{pmatrix},$$

then we have that $B = P^{-1}V$. We would have the above system for each triangle in the mesh, with its own P and B matrices. Writing explicitly the Frobenius terms in $E(t)$, and expressing B as a linear combination of $\mathbf{v}_i$, we get

$$\begin{aligned} E(t) = \{ & a_{11}^2 + a_{12}^2 + \ldots + a_{33}^2 + \alpha_1^2 v_{1x}^2 + \ldots + \pi^2 v_{3z}^2 - 2a_{11}\alpha_1 v_{1x} \\ & - 2a_{11}\delta_1 v_{2x} - 2a_{11}\lambda_1 v_{3x} - \ldots - 2a_{33}\pi_3 v_{3z} + 2\alpha_1 v_{1x}\delta_1 v_{2x} \\ & + 2\alpha_1 v_{1x}\lambda_1 v_{3x} + \ldots + 2\kappa_3 v_{2z}\pi_3 v_{3z} \}_{j=1} + \ldots + \{\cdots\}_{j=J}, \end{aligned}$$

where we have a $\{\cdots\}_j$ for each triangle. Predetermining the transformation of $\mathbf{v_1}$, and setting $\mathbf{u}(t)^T = \big(1, v_{2x}, v_{2y}, v_{2z}, \ldots, v_{mx}, v_{my}, v_{mz}\big)$, where m is the number of vertices, we can stack the terms in a quadratic matrix form

$$E(t) = \mathbf{u}^T \begin{pmatrix} c & \mathbf{g}(t)^T \\ \mathbf{g}(t) & H \end{pmatrix} \mathbf{u},$$

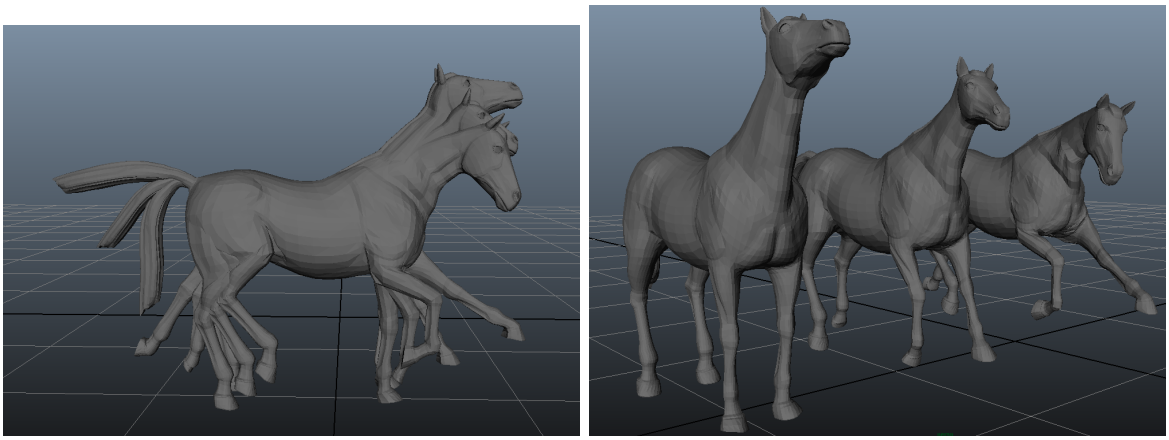
where c are the constant coefficients, $\mathbf{g}$ is a vector with the linear terms and H is matrix with the quadratic and mixed terms. Note that, since $\mathbf{v}_1$ is known, the terms were it appears, become constants or linear in $\mathbf{v}_j$ for $j \neq 1$. Setting the gradient to zero,

$$H\mathbf{u}(t) = -\mathbf{g}(t) \;\rightarrow\; \mathbf{u}(t) = -H^{-1}\mathbf{g}(t).$$

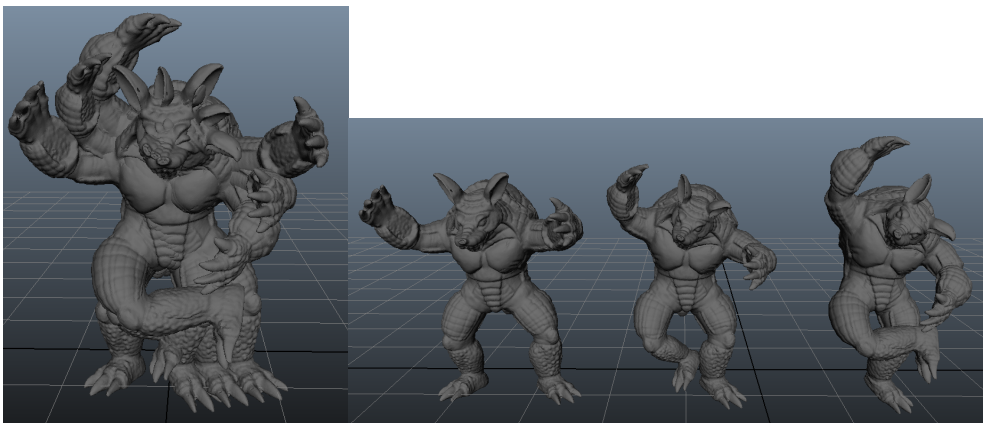
Since H is independent on t, for each iteration we only need to compute the linear interpolation of $\mathbf{v}_1$ and the new $\mathbf{g}(t)$ to solve the system.

4 Results and conclusions

Figure 1 shows that applying a vertex based linear interpolation, rescales the interpolated mesh in unnatural positions. For example, the horse head and tail shapes in Figure 1a



(a) fig 3 (b) fig 4



(c) fig 1 (d) fig 2

Figure 1: Horse and armadillo meshes, morphed mesh is in the middle.

are smaller than their counterparts in the source and target shape. Applying the as-rigid-as-possible interpolation produces better results, because it interpolates the transformation matrices instead of the vertex positions.

References

[1] Marc Alexa, Daniel Cohen-Or, and David Levin. As-rigid-as-possible shape interpolation. In *Proceedings of the 27th annual conference on Computer graphics and interactive techniques - SIGGRAPH '00*, pages 157–164, 2000.