Task 2. Mesh Animation

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1 Introduction

Generating a plausible mesh which is a morphing of two given meshes ...

2 Implemented techniques

We present two approaches to solve the mesh morphing problem.

2.1 Linear interpolation

Linear interpolation is a simple and intuitive first approach. The new position of a vertex $\mathbf{v_{it}} = [v_{ix}, v_{iy}, v_{iz}]^T$ in an interval $t = \{0, \ldots, 1\}$ is given by

$$\mathbf{v_{it}} = (1-t)\mathbf{p_i} + t\mathbf{q_i} \quad i \in \{1, \ldots, n\},$$

where n is the number of vertices, $\mathbf{p_i} = [p_x, p_y, p_z]^T$ and $\mathbf{q_i} = [q_x, q_y, q_z]^T$ are the corresponding vertices in the source and target meshes respectively. Some meshes generated with this method are shown in Figure 1.

2.2 As rigid as possible interpolation

The previous method produces distorted meshes due to its non-rigid nature. One way to produce more better results is to use a transform-based technique. Lets define an affine transformation from $\mathbf{p_i}$ to $\mathbf{q_i}$ such that:

$$A\mathbf{p_i} + \mathbf{l} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{pmatrix} + \begin{pmatrix} l_x \\ l_y \\ l_z \end{pmatrix} = \mathbf{q_i}.$$

We have a matrix A_s for each triangle. Since in a general mesh vertices a shared among different triangles, lets define B_j as the actual transformation matrix, where:

$$E(t) = \sum_{s}^{S} \left\| A_s(t) - B_s(t) \right\|_F^2 ,$$

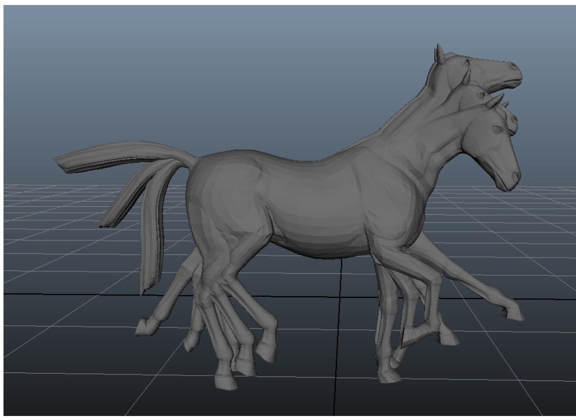
where $\|\cdot\|_F$ is the matrix Frobenius norm, and the objective is to minimize $E(t)$. If we ignore the l term and rearrange B such that:

$$PB_{col} = \begin{pmatrix} \mathbf{p}_i^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_i^T \\ \mathbf{p}_j^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_j^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_j^T \\ \mathbf{p}_k^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_k^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_k^T \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{21} \\ b_{22} \\ b_{23} \\ b_{31} \\ b_{32} \\ b_{33} \end{pmatrix} = \begin{pmatrix} \mathbf{q}_{ix} \\ \mathbf{q}_{iy} \\ \mathbf{q}_{iz} \\ \mathbf{q}_{jx} \\ \mathbf{q}_{jy} \\ \mathbf{q}_{jz} \\ \mathbf{q}_{kx} \\ \mathbf{q}_{ky} \\ \mathbf{q}_{kz} \end{pmatrix} \text{ and } P^{-1} = \begin{pmatrix} \alpha_1 & 0 & 0 & \delta_1 & 0 & 0 & \lambda_1 & 0 & 0 \\ \alpha_2 & 0 & 0 & \delta_2 & 0 & 0 & \lambda_2 & 0 & 0 \\ \alpha_3 & 0 & 0 & \delta_3 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & \epsilon_1 & 0 & 0 & \mu_1 & 0 \\ 0 & \beta_2 & 0 & 0 & \epsilon_2 & 0 & 0 & \mu_2 & 0 \\ 0 & \beta_3 & 0 & 0 & \epsilon_3 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & \gamma_1 & 0 & 0 & \kappa_1 & 0 & 0 & \pi_1 \\ 0 & 0 & \gamma_2 & 0 & 0 & \kappa_2 & 0 & 0 & \pi_2 \\ 0 & 0 & \gamma_3 & 0 & 0 & \kappa_3 & 0 & 0 & \pi_3 \end{pmatrix},$$

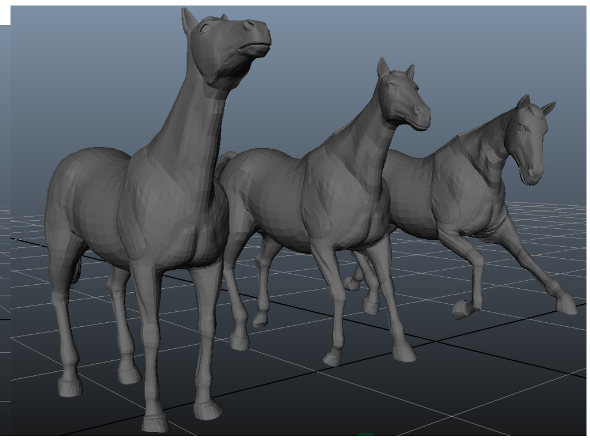
then we have that $B = P^{-1}Q$. Writing out the Frobenius terms in $E(t)$ in and expressing B as a linear combination of $P^{-1}Q$, we get:

$$\begin{aligned} E(t) = \{ & a_{11}^2 + a_{12}^2 + \ldots + a_{33}^2 + \boldsymbol{\alpha}^2 v_{1x}^2 + \ldots + \boldsymbol{\pi}^2 v_{3z}^2 - 2a_{11}\alpha_1 v_{1x} \\ & - 2a_{11}\delta_1 v_{2x} - 2a_{11}\lambda_1 v_{3x} - \ldots - 2a_{33}\pi_3 v_{3z}\}_{j=1} + \ldots + \{\cdots\}_{j=J}, \end{aligned}$$

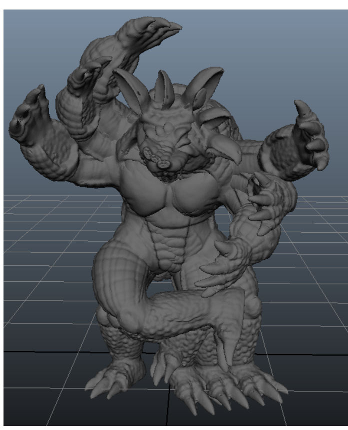
where each {



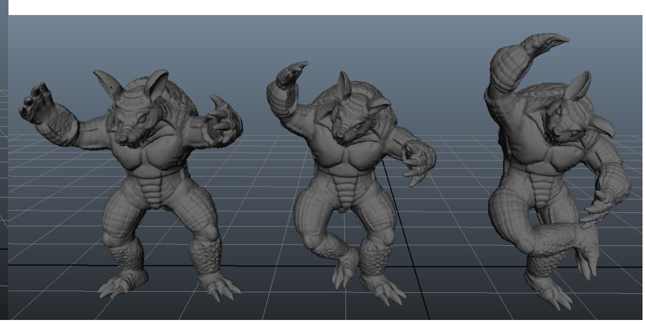
(a) fig 3



(b) fig 4



(c) fig 1



(d) fig 2

Figure 1: Horse and armadillo meshes, morphed mesh is in the middle.