

# Task 2. Mesh Animation

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## 1 Introduction

Generating a plausible mesh which is a morphing of two given meshes ...

## 2 Implemented techniques

We present two approaches to solve the mesh morphing problem.

### 2.1 Linear interpolation

Linear interpolation is a simple and intuitive first approach. The new position of a vertex  $\mathbf{v}_{it} = [v_{ix}, v_{iy}, v_{iz}]^T$  in an interval  $t = \{0, \dots, 1\}$  is given by

$$\mathbf{v}_{it} = (1 - t)\mathbf{p}_i + t\mathbf{q}_i \quad i \in \{1, \dots, n\},$$

where  $n$  is the number of vertices,  $\mathbf{p}_i = [p_x, p_y, p_z]^T$  and  $\mathbf{q}_i = [q_x, q_y, q_z]^T$  are the corresponding vertices in the source and target meshes respectively. Some meshes generated with this method are shown in Figure 1.

### 2.2 As rigid as possible interpolation

The previous method produces distorted meshes due to its non-rigid nature. One way to produce better results is to use a transform-based technique. Lets define an affine transformation from  $\mathbf{p}_i$  to  $\mathbf{q}_i$  such that:

$$A\mathbf{p}_i + \mathbf{l} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{pmatrix} + \begin{pmatrix} l_x \\ l_y \\ l_z \end{pmatrix} = \mathbf{q}_i.$$

We have a matrix  $A_s$  for each triangle, since there are 12 unknowns and each triangle only gives 9 equations, we add a extra point to be able to compute  $A_i$ . This new point is computed as  $\mathbf{p}_{\text{new}} = \mathbf{c}_{\text{tr}} + w\mathbf{n}_{\text{tr}}$ , where  $\mathbf{c}_{\text{tr}}$  is the triangle centroid,  $w$  is the average triangle

edge length and  $\mathbf{n}_{\text{tr}}$  is the triangle normal. Since in a general mesh vertices are shared among different triangles, let's define  $B_j$  as the actual transformation matrix, where

$$E(t) = \sum_s^S \|A_s(t) - B_s(t)\|_F^2,$$

where  $\|\cdot\|_F$  is the matrix Frobenius norm, and the objective is to minimize  $E(t)$ . If we ignore the  $l$  term, rename  $\mathbf{q}_i$  to treat each vertex in individual vertex in the mesh as  $\mathbf{v}_i$  the mesh, and rearrange  $B$  such that

$$PB_{col} = \begin{pmatrix} \mathbf{p}_i^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_i^T \\ \mathbf{p}_j^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_j^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_j^T \\ \mathbf{p}_k^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_k^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_k^T \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{21} \\ b_{22} \\ b_{23} \\ b_{31} \\ b_{32} \\ b_{33} \end{pmatrix} = \begin{pmatrix} v_{ix} \\ v_{iy} \\ v_{iz} \\ v_{jx} \\ v_{jy} \\ v_{jz} \\ v_{kx} \\ v_{ky} \\ v_{kz} \end{pmatrix} \text{ and } P^{-1} = \begin{pmatrix} \alpha_1 & 0 & 0 & \delta_1 & 0 & 0 & \lambda_1 & 0 & 0 \\ \alpha_2 & 0 & 0 & \delta_2 & 0 & 0 & \lambda_2 & 0 & 0 \\ \alpha_3 & 0 & 0 & \delta_3 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & \epsilon_1 & 0 & 0 & \mu_1 & 0 \\ 0 & \beta_2 & 0 & 0 & \epsilon_2 & 0 & 0 & \mu_2 & 0 \\ 0 & \beta_3 & 0 & 0 & \epsilon_3 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & \gamma_1 & 0 & 0 & \kappa_1 & 0 & 0 & \pi_1 \\ 0 & 0 & \gamma_2 & 0 & 0 & \kappa_2 & 0 & 0 & \pi_2 \\ 0 & 0 & \gamma_3 & 0 & 0 & \kappa_3 & 0 & 0 & \pi_3 \end{pmatrix},$$

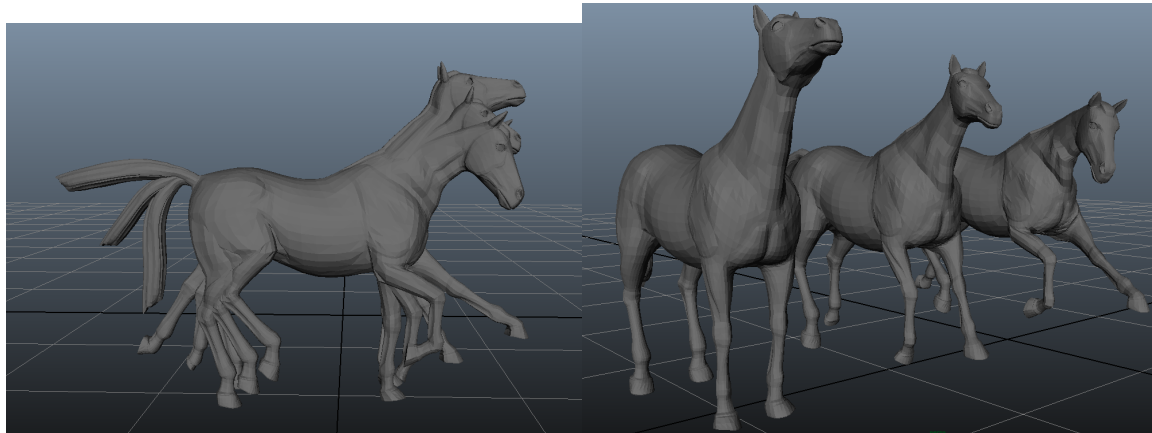
then we have that  $B = P^{-1}V$ . We would have the above system for each triangle in the mesh, with its own  $P$  and  $B$  matrices. Writing explicitly the Frobenius terms in  $E(t)$ , and expressing  $B$  as a linear combination of  $\mathbf{v}_i$ , we get

$$\begin{aligned} E(t) = & \{a_{11}^2 + a_{12}^2 + \dots + a_{33}^2 + \alpha_1^2 v_{1x}^2 + \dots + \pi^2 v_{3z}^2 - 2a_{11}\alpha_1 v_{1x} \\ & - 2a_{11}\delta_1 v_{2x} - 2a_{11}\lambda_1 v_{3x} - \dots - 2a_{33}\pi_3 v_{3z} + 2\alpha_1 v_{1x}\delta_1 v_{2x} \\ & + 2\alpha_1 v_{1x}\lambda_1 v_{3x} + \dots + 2\kappa_3 v_{2z}\pi_3 v_{3z}\}_{j=1} + \dots + \{\dots\}_{j=J}, \end{aligned}$$

where we have a  $\{\dots\}_j$  for each triangle. Predetermining the transformation of  $\mathbf{v}_1$ , and setting  $\mathbf{u}^T = (1, v_{2x}, v_{2y}, v_{2z}, \dots, v_{mx}, v_{my}, v_{mz})$ , where  $m$  is the number of vertices, we can stack the terms in a quadratic matrix form

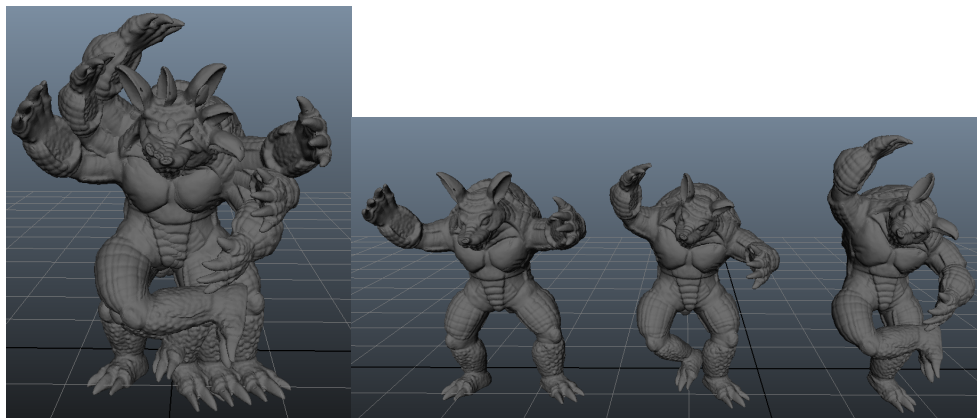
$$E(t) = \mathbf{u}^T \begin{pmatrix} c & \mathbf{g}^T \\ \mathbf{g} & H \end{pmatrix} \mathbf{u},$$

where  $c$  are the constant coefficients,  $\mathbf{g}$  is a vector with the terms linear on  $a_i$  and  $H$  is matrix with the quadratic and mixed terms. Note that the terms involving  $\mathbf{v}_1$  are treated differently since it is not an unknown.



(a) fig 3

(b) fig 4



(c) fig 1

(d) fig 2

Figure 1: Horse and armadillo meshes, morphed mesh is in the middle.