# Trigonometry Workbook

## Trig identities

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## SUM-DIFFERENCE IDENTITIES FOR SINE AND COSINE

1. Evaluate the expression.

$$\cos\left(\frac{13\pi}{12}\right)$$

2. Find $\sin 75°$.

3. Simplify the expressions.

$$\cos\left(\frac{\pi}{2}+\theta\right) \text{ and } \cos\left(\frac{\pi}{2}-\theta\right)$$

4. Find the value of $a - 2b$, if $a$ and $b$ are real numbers.

$$\sin(\theta-\alpha) = a\sin\theta\cos\alpha + b\cos\theta\sin\alpha$$

5. Find the exact value of the expression.

$$\cos\left(\sin^{-1}\frac{\sqrt{3}}{2} - \cos^{-1}\frac{4}{5}\right)$$

6. Find the solutions to the equation in the interval $[0,\pi)$.

$$\cos\left(\theta-\frac{\pi}{2}\right)+\sin\left(\theta-\frac{3\pi}{2}\right)=0$$

## COFUNCTION IDENTITIES

1. Find an angle $\theta$ that satisfies the equation.

$$\tan\left(-\frac{3\pi}{4}\right) = \cot\theta$$

2. Find an acute angle that satisfies the equation.

$$\sin\left(2\alpha - \frac{5\pi}{6}\right) = \cos\left(4\alpha - \frac{\pi}{3}\right)$$

3. What is the value of $\theta$?

$$\tan\left(\frac{\pi}{6} - \theta\right) = \cot\left(\frac{\pi}{6}\right)$$

4. Find the value of $\cos\theta$.

$$\sin\left(\frac{\pi}{2} - \theta\right) + \frac{1}{4}\csc\left(\frac{\pi}{2} - \theta\right) = 1$$

5. Rewrite the expression as the cosine of an angle in terms of $\alpha$ and $\beta$.

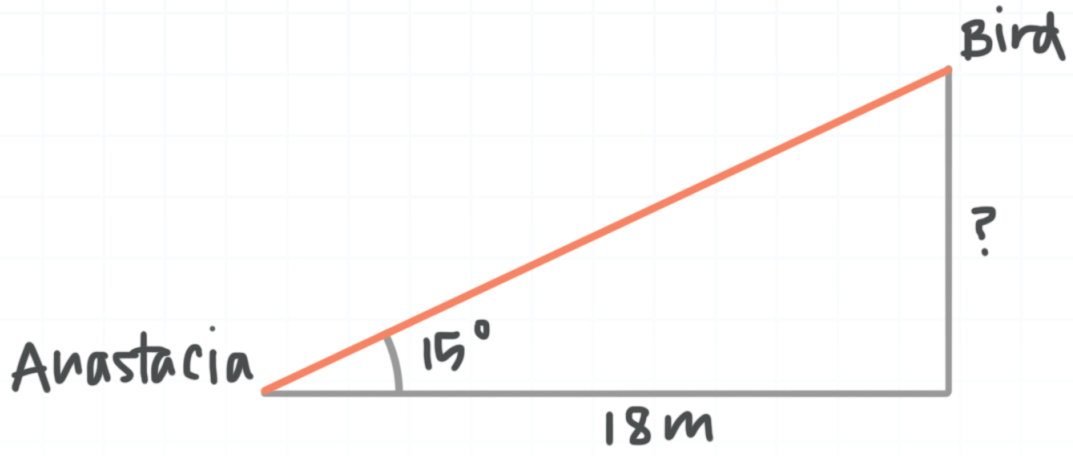
$$\sin\left(\frac{\pi}{2}-\alpha-\beta\right)$$

6. Find an angle $\theta$ that satisfies the equation.

$$\csc\left(\frac{\pi}{5}\right)=\sec\theta$$

## SUM-DIFFERENCE IDENTITIES FOR TANGENT

1. Cara is watching a bird on a tree. She measured the angle of elevation of the bird as $15°$, and the distance to the tree as $18$ meters. Find the exact altitude of the bird above the ground.




2. Find the exact value of $\tan 105°$.

3. Find the exact values of $\tan(\theta - \alpha)$ if $\theta$ is an angle in the first quadrant whose cosine is $3/5$ and $\alpha$ is an angle in the fourth quadrant whose sine is $-5/13$.

4. Simplify the expressions $\tan(\pi + \theta)$ and $\tan(\pi - \theta)$.

5. Find the exact values of $\tan(\theta + \alpha)$ if $\theta$ is an angle in the second quadrant whose cosine is $-4/7$ and $\alpha$ is an angle in the third quadrant whose cosine is $-9/10$.

6. Find the exact value of the expression.

$$\tan\left(\sin^{-1}\frac{1}{2} - \cos^{-1}\frac{1}{2}\right)$$

## DOUBLE-ANGLE IDENTITIES

■ 1. If $\theta$ is an angle in the fourth quadrant whose sine is $-3/5$, what are the values of $\tan 2\theta$?

■ 2. If $\theta$ is an angle in the third quadrant whose tangent is $3/4$, what are the values of $\cos 2\theta$?

■ 3. Use a double-angle identity to rewrite the expression.

$$(\sin x + \cos x)^2$$

■ 4. If $\theta$ is an angle in the third quadrant whose sine is $-1/\sqrt{5}$, what is the value of $\sin 2\theta$?

■ 5. If $\theta$ is an angle in the third quadrant whose tangent is $7/24$, what is the value of $\tan 2\theta$?

■ 6. Use a double-angle formula to rewrite the expression.

$$12\sin(4x)\cos(4x)$$

## HALF-ANGLE IDENTITIES

■ 1. Use a half-angle identity to find the exact value of the expression.

$$\sin 15^\circ$$

■ 2. If $\theta$ is the angle in Quadrant II with $\sin\theta = 7/25$, what are the values of $\sin(\theta/2)$ and $\cos(\theta/2)$?

■ 3. If $\theta$ is the angle in the interval $(0,\pi/2)$ with $\tan\theta = 2$, what are the values of $\sin(\theta/2)$ and $\cos(\theta/2)$?

■ 4. If $\theta$ is the angle in the interval $(3\pi/2,2\pi)$ with $\sin\theta = -15/17$, what are the values of $\tan(\theta/2)$ and $\cot(\theta/2)$?

■ 5. Use a half-angle identity to find the exact value of the expression.

$$\sec\left(\frac{7\pi}{8}\right)$$

■ 6. Prove the identity.

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

## PRODUCT-TO-SUM IDENTITIES

1. Rewrite $\cos(x-y)\cos(x+y)$ as a sum.

2. Rewrite $\cos(x-15°)\sin(x+15°)$ as a sum.

3. Find a sum equivalent to $\cos^3 x$.

4. Find the exact value of each expression.

$$\left(\sin\frac{3\pi}{8}\right)\left(\cos\frac{3\pi}{8}\right)$$

$$\sin^2\left(\frac{3\pi}{8}\right)$$

$$\cos^2\left(\frac{3\pi}{8}\right)$$

5. Simplify the expression.

$$\sin(x-y)\cos y+\cos(x-y)\sin y$$

6. Find the value of the expression.

$$\sin^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{3\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right)$$

## SUM-TO-PRODUCT IDENTITIES

■ 1. Rewrite the function as a product.

$$f(x) = \sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right)$$

■ 2. Find a product equal to $\sin(x + y) + \sin(x - y)$.

■ 3. Find the exact value of the expression.

$$\frac{\cos 93^\circ + \cos 27^\circ}{\cos 33^\circ}$$

■ 4. Simplify the expression.

$$\frac{\sin(7\theta) + \sin(3\theta)}{\cos(7\theta) + \cos(3\theta)}$$

■ 5. Find a product equal to $\cos(3\theta) + \cos(5\theta) - 2\cos(\theta)\cos(8\theta)$.

■ 6. Find the exact value of the expression.

$$16 \sin 390^\circ + 22 \sin 240^\circ + 16 \sin 150^\circ - 22 \sin 120^\circ$$

## PROVING THE TRIG EQUATION

■ 1. Prove the trig equation.

$$\tan\left(\frac{x}{2}\right) = \frac{1 - \cos x}{\sin x}$$

■ 2. Prove the trigonometric equation.

$$\frac{\sin(5x) - \sin x}{\cos(5x) + \cos x} = \tan(2x)$$

■ 3. Prove the trigonometric equation.

$$\sin(x - \pi)\sin(x + \pi) = \sin^2 x$$

■ 4. Prove the trigonometric equation.

$$\sin(-x)\cos(-x)\tan(-x)\csc(-x) = -\sin x$$

■ 5. Prove the trigonometric equation.

$$(\sin t + \cos t)^2 - 1 = \sin(2t)$$

6. Prove the trigonometric equation.

$$\frac{\cos(270^\circ + x)}{\sin(180^\circ - x)} = 1$$

## COMPLETE SOLUTION SET OF THE EQUATION

■ 1. Find the complete solution set of the equation $\cos^2 x - 3\cos x + 2 = 0$.

■ 2. Find all the solutions of the trig equation, then list only the solutions that lie in the interval $[0,2\pi)$.

$$3\csc^2\theta - 2\cot^2\theta - 4 = 0$$

■ 3. Find the complete solution set of the equation.

$$4\cos^3\theta - 2\cos^2\theta - 2\cos\theta + 1 = 0$$

■ 4. Find all the solutions of the trig equation, then list only the solutions that lie in the interval $[0,2\pi)$.

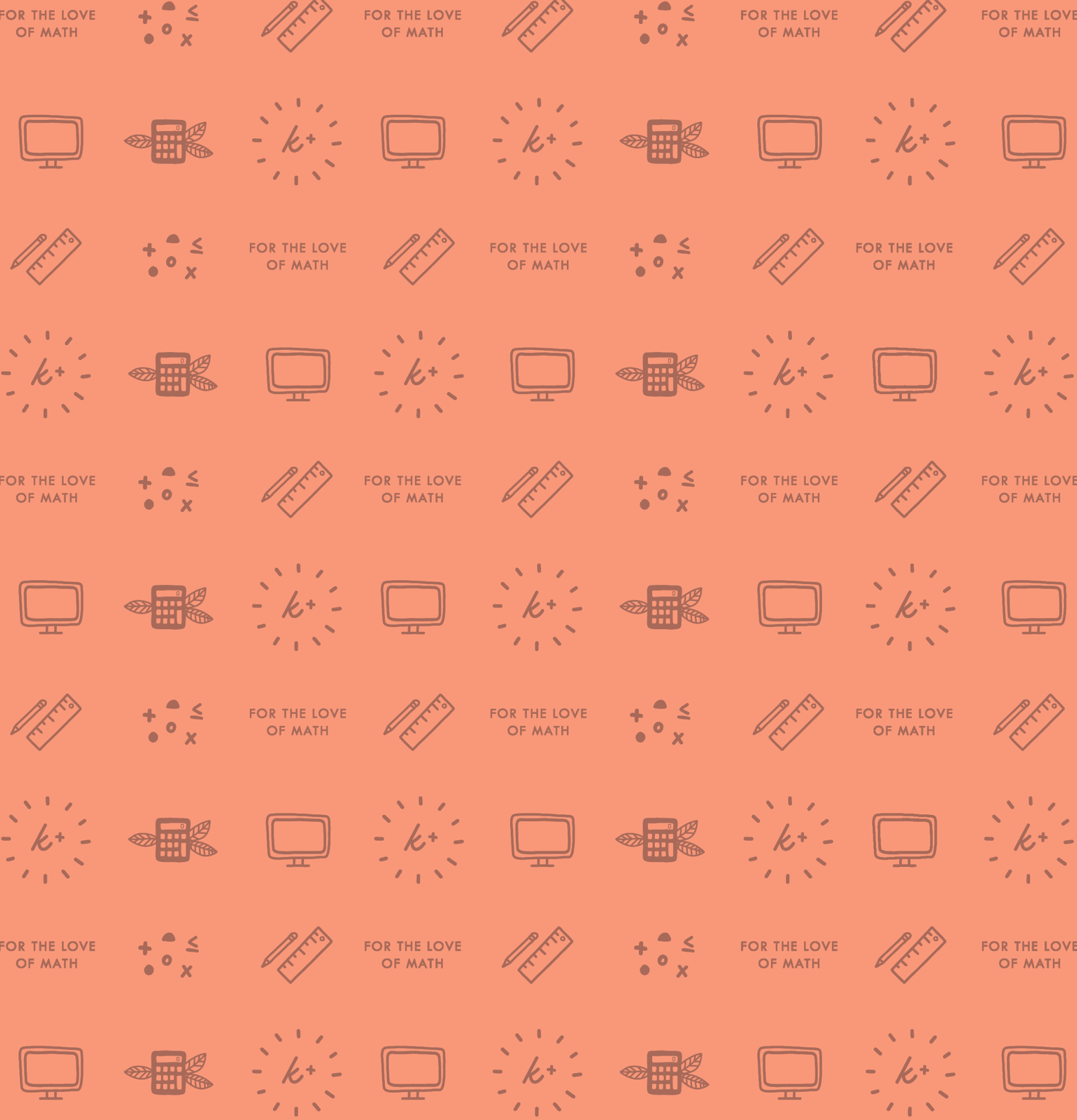
$$\cos\theta + 1 = \sin\theta$$

■ 5. Find all the solutions of the trig equation, then list only the solutions that lie in the interval $[0,2\pi)$.

$$2(\sin^2\theta - \cos^2\theta) = \sqrt{3}$$

6. Find the complete solution set of the equation.

$$4\sin\left(\theta-\frac{\pi}{3}\right)\cos\left(\theta-\frac{\pi}{3}\right)=\sqrt{3}$$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