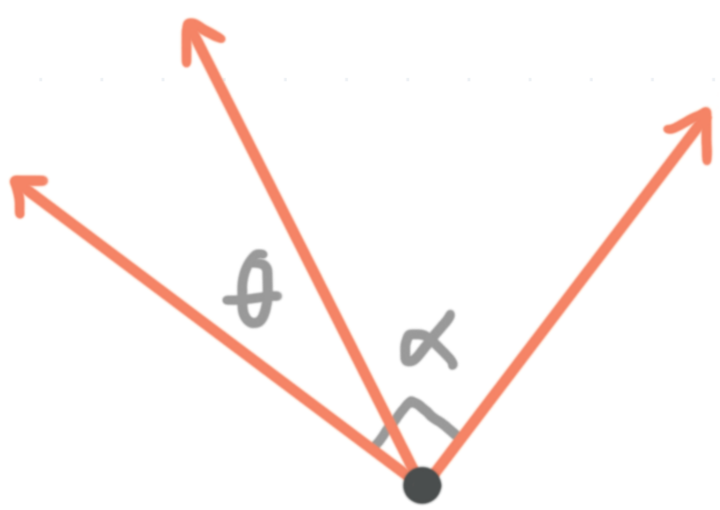
Complementary and supplementary angles

In the last lesson we looked at different types of angles, including right angles, which have measure 90° or $\pi/2$, and straight angles, which have measure 180° or π.

In this lesson, we want to define complementary angle pairs, which have a specific relationship to right angles, and supplementary angle pairs, which have a specific relationship to straight angles.

Complementary angles

Complementary angles are two angles that sum to 90° or $\pi/2$. In other words, if we line up their vertices and match up one ray from each angle, together they form a right angle.



The angles θ and α are complementary because they form a right angle. If we add up their measures, we'll get a total angle of $\theta + \alpha = 90^\circ$ or $\theta + \alpha = \pi/2$.

Let's do an example where we find an angle that's complementary to another angle we've been given. If we're given an angle in degrees, we want to find its complement in degrees, and if we're given an angle in radians, we want to find its complement in radians.

Example

Find the angle θ that's complementary to 37°.

The angle that's complementary to 37° is whatever angle we have to add to 37° in order to get 90°. Therefore, we can set up an equation where 37° and θ sum to 90°.

$$37^\circ + \theta = 90^\circ$$

$$\theta = 90^\circ - 37^\circ$$

$$\theta = 53^\circ$$

So 37° and 53° degrees are complementary angles because they sum to 90° and form a right angle.

We can find complementary angles in radians, too. For example, the angle that's complementary to $\pi/6$ is

$$\frac{\pi}{6} + \theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \frac{\pi}{6}$$

Since 6 is the least common multiple of 2 and 6, we'll multiply the first fraction by $3/3$ to create common denominators.

$$\theta = \frac{\pi}{2}\left(\frac{3}{3}\right) - \frac{\pi}{6}$$

$$\theta = \frac{3\pi}{6} - \frac{\pi}{6}$$

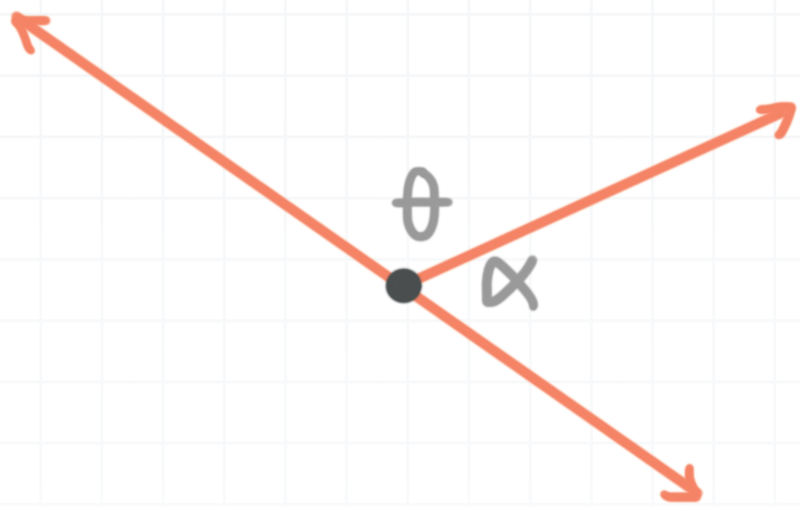
$$\theta = \frac{2\pi}{6}$$

$$\theta = \frac{\pi}{3}$$

So $\pi/6$ and $\pi/3$ are complementary angles because they sum to $\pi/2$ and form a right angle.

Supplementary angles

While complementary angles sum to 90° or $\pi/2$ and form a right angle, **supplementary angles** are two angles that sum to 180° or π and form a straight angle.



The angles θ and α are supplementary because they form a straight angle. If we add up their measures, we'll get a total angle of $\theta + \alpha = 180^\circ$ or $\theta + \alpha = \pi$.

Let's do an example where we find an angle that's supplementary to another angle we've been given. If we're given an angle in degrees, we want to find its supplement in degrees, and if we're given an angle in radians, we want to find its supplement in radians.

Example

Find the angle θ that's supplementary to $\pi/4$.

The angle that's supplementary to $\pi/4$ is whatever angle we have to add to $\pi/4$ in order to get π. Therefore,

$$\frac{\pi}{4} + \theta = \pi$$

$$\theta = \pi - \frac{\pi}{4}$$

Find a common denominator.

$$\theta = \pi\left(\frac{4}{4}\right) - \frac{\pi}{4}$$

$$\theta = \frac{4\pi}{4} - \frac{\pi}{4}$$

$$\theta = \frac{3\pi}{4}$$

So $\pi/4$ and $3\pi/4$ are supplementary angles because they sum to π and form a straight angle.

We can find supplementary angles in degrees, too. For example, the angle that's supplementary to 48° is

$$48^\circ + \theta = 180^\circ$$

$$\theta = 180^\circ - 48^\circ$$

$$\theta = 132^\circ$$

So 48° and 132° are supplementary angles because they sum to 180° and form a straight angle.

Let's do one slightly more complicated example.

Example

Find the angle θ that's twice the supplement to $\pi/3$.

If two angles are supplementary, they sum to 180° or π radians. The angle that's supplementary to $\pi/3$, which we'll call α, is therefore

$$\frac{\pi}{3} + \alpha = \pi$$

$$\alpha = \pi - \frac{\pi}{3}$$

Find a common denominator.

$$\alpha = \pi \left(\frac{3}{3}\right) - \frac{\pi}{3}$$

$$\alpha = \frac{3\pi}{3} - \frac{\pi}{3}$$

$$\alpha = \frac{2\pi}{3}$$

We were asked to find the angle that's twice as big as α, so we'll multiply through this equation by 2 in order to find 2α

$$2\alpha = 2\left(\frac{2\pi}{3}\right)$$

$$2\alpha = \frac{4\pi}{3}$$

The angle $4\pi/3$ is twice the supplement to $\pi/3$.