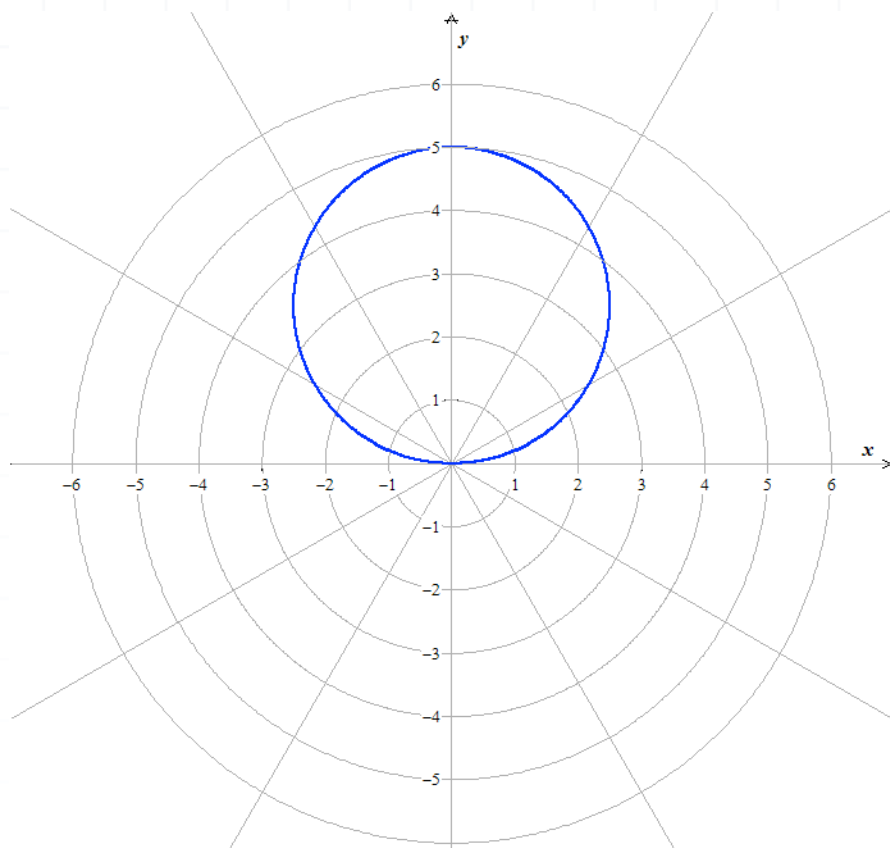


Topic: Graph the polar curve, circle

Question: The following curve is the graph of one of the polar equations given below. Which polar equation is it?

**Answer choices:**

- A $r = -5 \sin \theta$
- B $r = (5/2)\cos \theta$
- C $r = 5 \sin \theta$
- D $r = (5/2)\sin \theta$



Solution: C

The given curve is the circle with center at the point

$$(x, y) = (0, k) = \left(0, \frac{5}{2}\right)$$

and a radius of

$$c = |k| = \frac{5}{2}$$

Since k is positive, one pair of polar coordinates of the center of this circle is

$$(r, \theta) = \left(k, \frac{\pi}{2}\right) = \left(\frac{5}{2}, \frac{\pi}{2}\right)$$

Thus the given curve is the graph of the polar equation

$$r = a \sin \theta$$

for $a = 2k$. Since $k = 5/2$, we have $a = 2k = 2(5/2) = 5$. Therefore, the curve is the graph of the polar equation

$$r = 5 \sin \theta$$

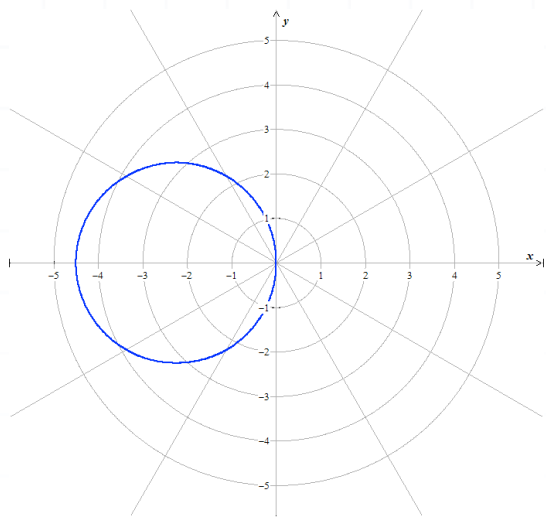


Topic: Graph the polar curve, circle

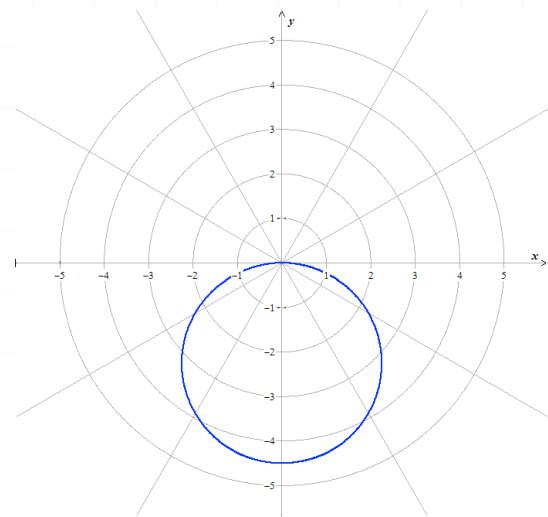
Question: Which of the following is the graph of the polar equation?

$$r = -9 \cos \theta$$

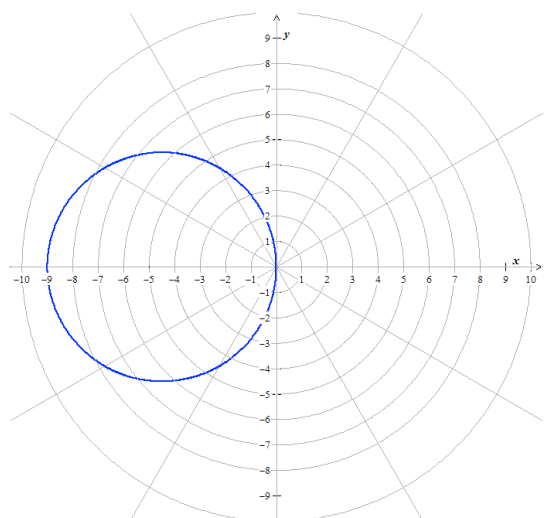
Answer choices:



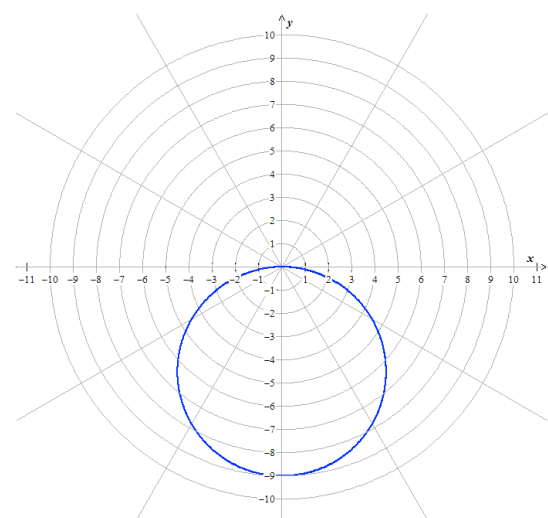
A



C



B



D



Solution: B

Since the coefficient of $\cos \theta$ is $a = -9$, we can set a equal to $2h$ (hence $h = a/2 = -9/2$), so

$$r = -9 \cos \theta$$

is the polar equation of the circle that's centered at the point

$$(x, y) = (h, 0) = \left(\frac{a}{2}, 0\right) = \left(-\frac{9}{2}, 0\right)$$

and has a radius of

$$c = \frac{|a|}{2} = \frac{9}{2}$$

Since a is negative, one pair of polar coordinates of the center of this circle is

$$(r, \theta) = (-h, \pi) = \left(-\frac{a}{2}, \pi\right) = \left(\frac{9}{2}, \pi\right)$$

Inspection of the graphs given in the answer choices tells us that the graph of the polar equation $r = -9 \cos \theta$ is the curve shown in answer choice B.



Topic: Graph the polar curve, circle

Question: Exactly one of the curves described below is the graph of the polar equation. Which curve is it?

$$r = 8 \cos \left(\theta - \frac{\pi}{2} \right)$$

Answer choices:

- A The circle with center $(x, y) = (0, 4)$ and radius 4.
- B The circle with center $(x, y) = (4, 0)$ and radius 8.
- C The circle with center $(x, y) = (-4, 0)$ and radius 4.
- D The circle with center $(x, y) = (0, -4)$ and radius 8.



Solution: A

By the difference identity for cosine,

$$\cos\left(\theta - \frac{\pi}{2}\right) = \cos\theta \cos\left(\frac{\pi}{2}\right) + \sin\theta \sin\left(\frac{\pi}{2}\right) = \cos\theta(0) + \sin\theta(1) = \sin\theta$$

Therefore, the polar equation

$$r = 8 \cos\left(\theta - \frac{\pi}{2}\right)$$

is equivalent to the polar equation

$$r = 8 \sin\theta$$

Since the coefficient of $\sin\theta$ is $a = 8$, we can set a equal to $2k$ (hence $k = a/2 = 4$), so

$$r = 8 \cos\left(\theta - \frac{\pi}{2}\right)$$

is the polar equation of the circle with center at the point

$$(x, y) = (0, k) = \left(0, \frac{a}{2}\right) = (0, 4)$$

and a radius of

$$c = \frac{|a|}{2} = 4$$

