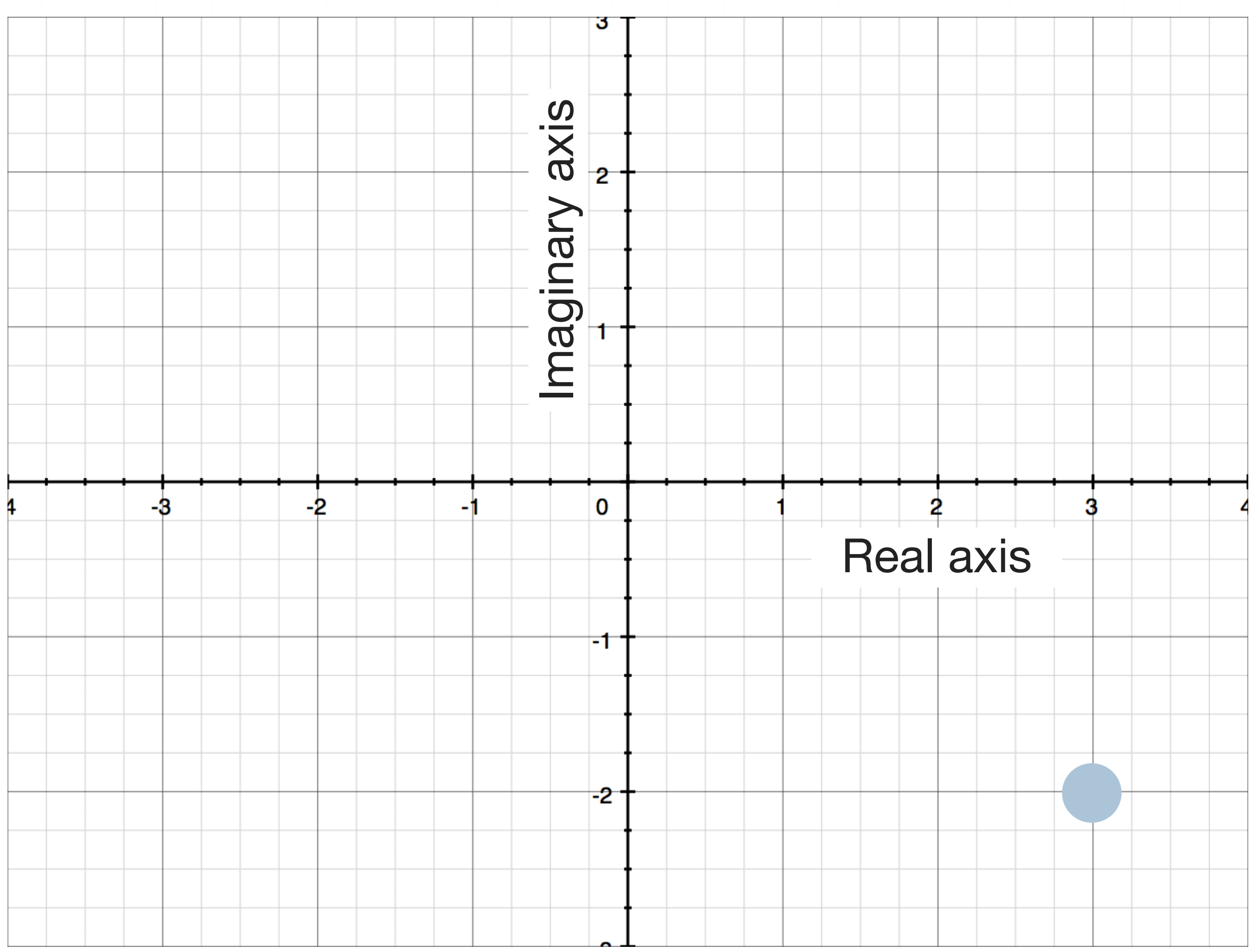
Graphing complex numbers

Let's start with a little review. When you graph a coordinate point (x, y), you graph it in the xy-plane, also called the rectangular coordinate plane, or Cartesian coordinate plane. The x-value is represented along the horizontal axis, and the y-value is represented along the vertical axis.

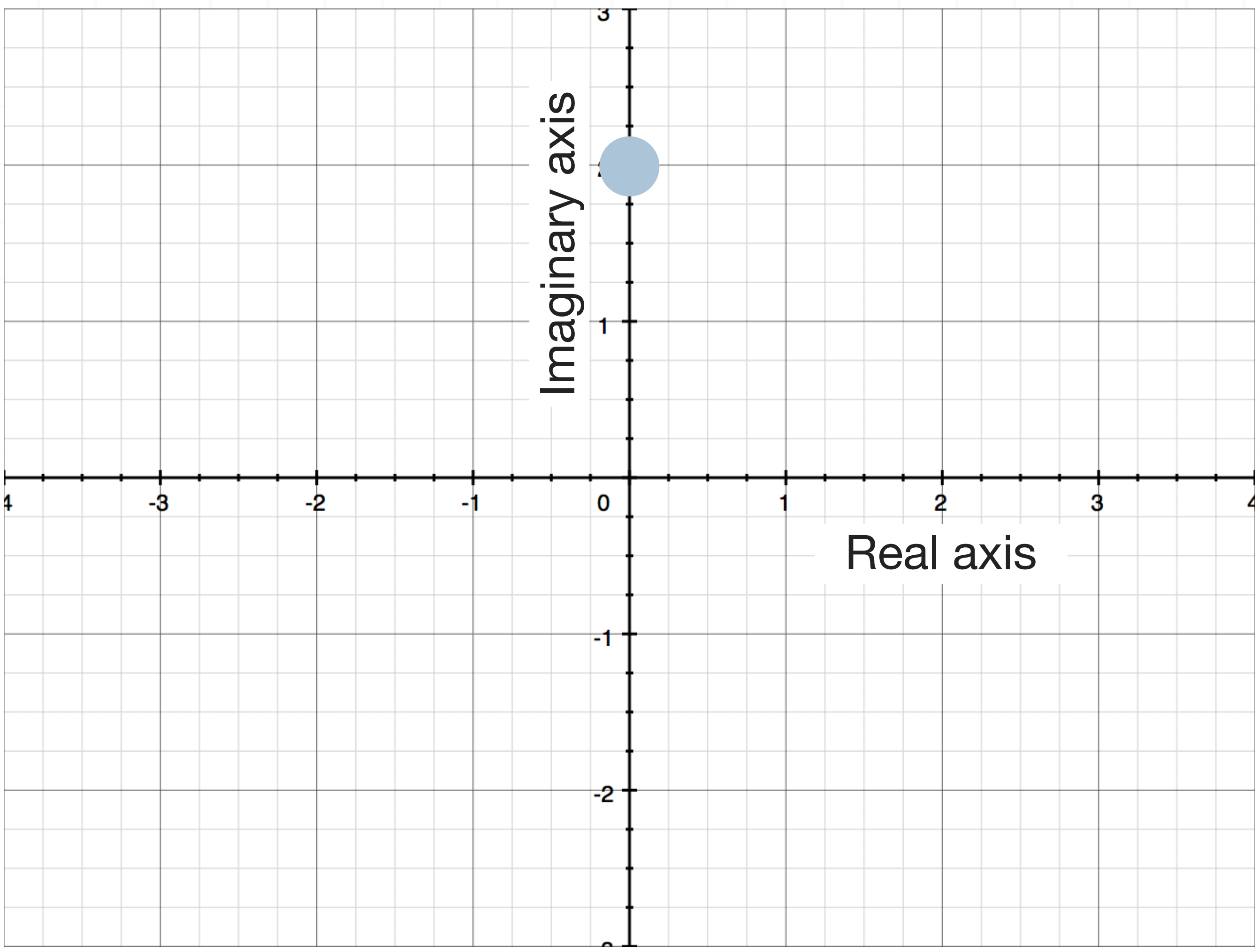
Complex numbers can't be graphed in the same xy-plane. Instead, we graph them in the complex plane. The **complex plane** has horizontal and vertical axes like the xy-plane, but the horizontal axis represents the real part of the complex number, and the vertical axis represents the imaginary part of the complex number.

So the complex number $z = 3 - 2i$ with a real part of 3 and an imaginary part of -2 is graphed as

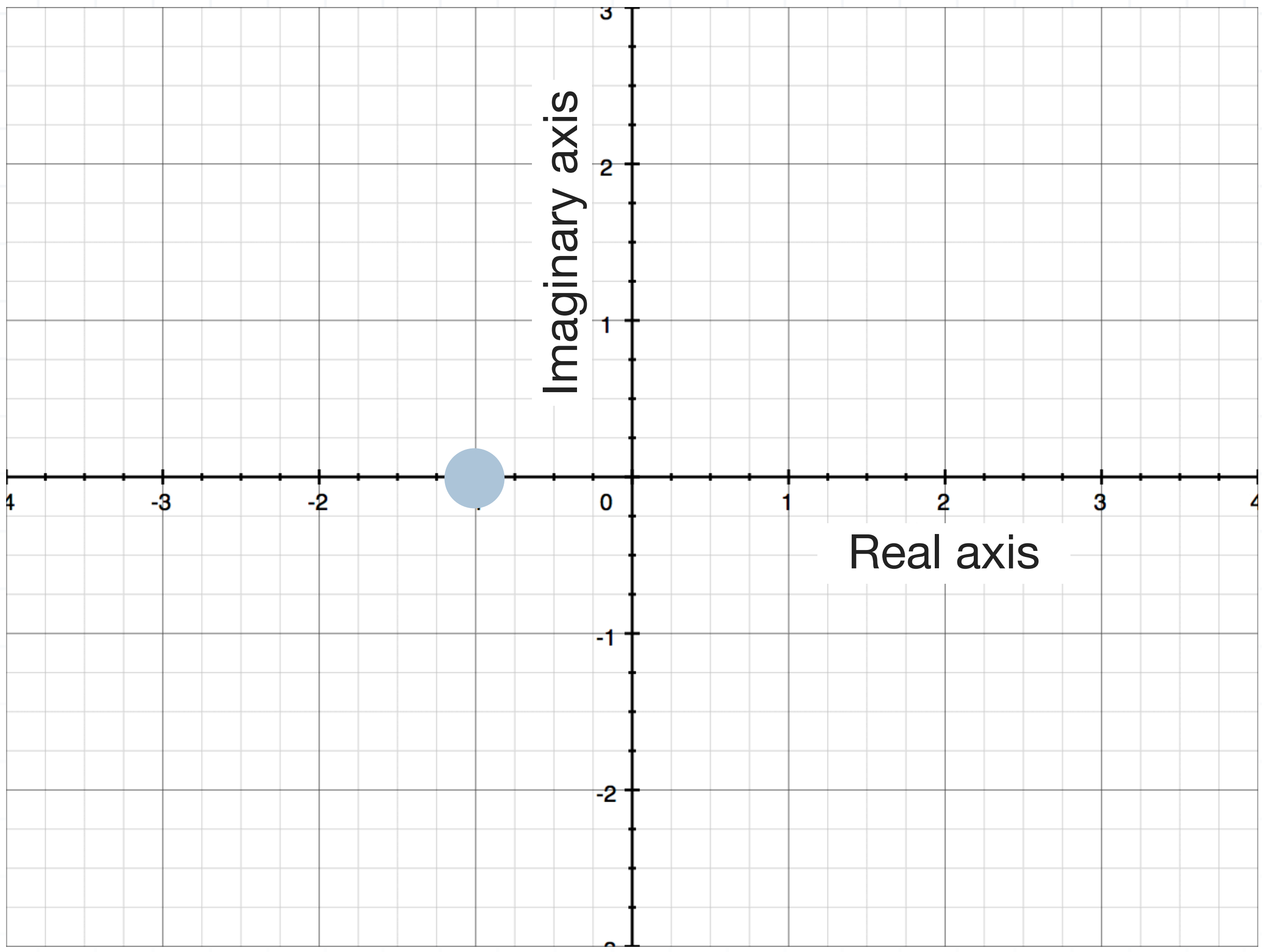


because a real part of $+3$ means we move to the right along the horizontal axis, and because an imaginary part of -2 means we move down along the vertical axis.

If the real part of the complex number is 0, such that $z = 0 + bi = bi$, then the complex number is a pure imaginary number, and it'll lie on the vertical axis in the complex plane. For instance, $z = 2i$ is graphed as



Or if the imaginary part of the complex number is 0, such that $z = a + 0i = a$, then the complex number is a real number, and it'll lie on the real axis in the complex plane. For instance, $z = -1$ is graphed as



Let's do a few more examples.

Example

Graph $3 + 2i$, $-1 - 3i$, and $1.5i$ in the complex plane.

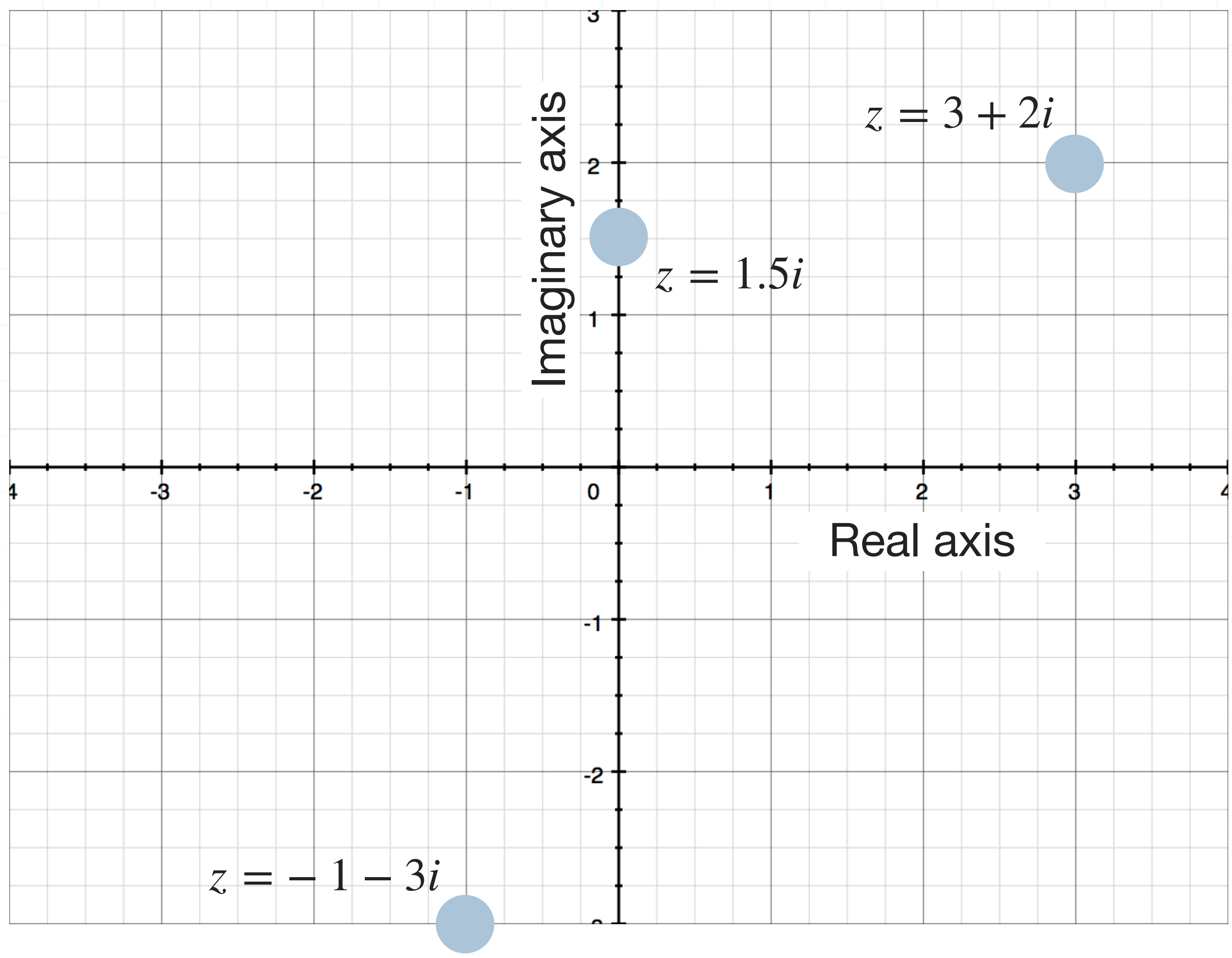
Let's break down each complex number.

The complex number $3 + 2i$ has real part $a = 3$ and imaginary part $b = 2$.

The complex number $-1 - 3i$ has real part $a = -1$ and imaginary part $b = -3$.

The complex number $1.5i$ has real part $a = 0$ and imaginary part $b = 1.5$.

Now we can graph all of them together.



Graphing the sum or difference of complex numbers

Remember that when you add two complex numbers together, you just add the real parts, and then separately add the imaginary parts. So the result is still a complex number.

Therefore, if you're asked to graph the sum of a complex number, find the sum of the complex numbers first, and then graph the resulting sum.

Example

Graph the sum of the complex numbers $-4+3i$ and $2-6i$.

First, find the sum.

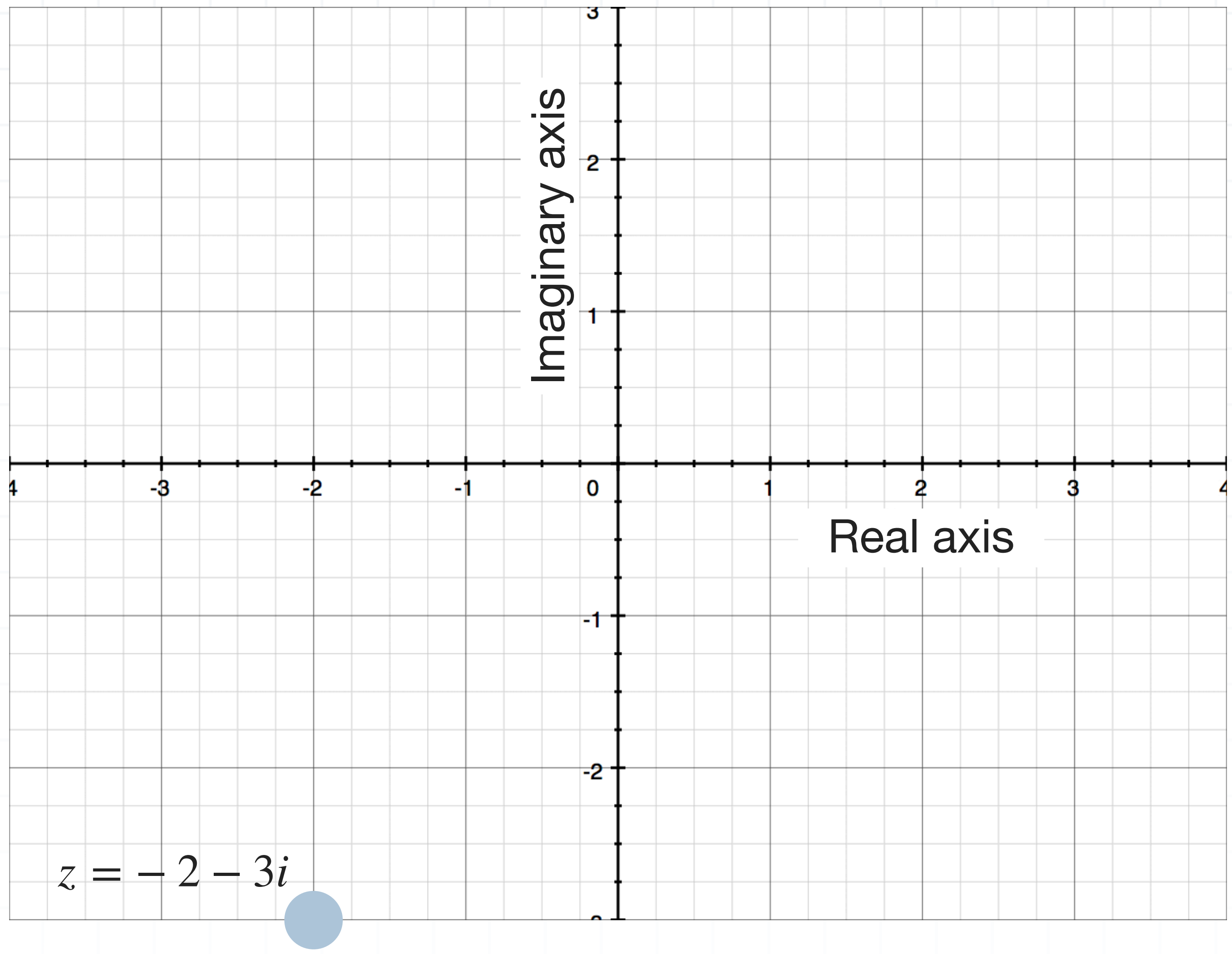
$$(-4+3i)+(2-6i)$$

$$(-4+2)+(3+(-6))i$$

$$(-4+2)+(3-6)i$$

$$-2-3i$$

Now graph the complex number $-2-3i$, which has a real part -2 and an imaginary part -3.



In the same way, to graph the difference of two complex numbers, find the difference first, and then graph the difference.

Example

Graph the difference of the complex numbers 1 and $5 - 3i$.

Write 1 as $1 + 0i$. Then the difference of the complex numbers is

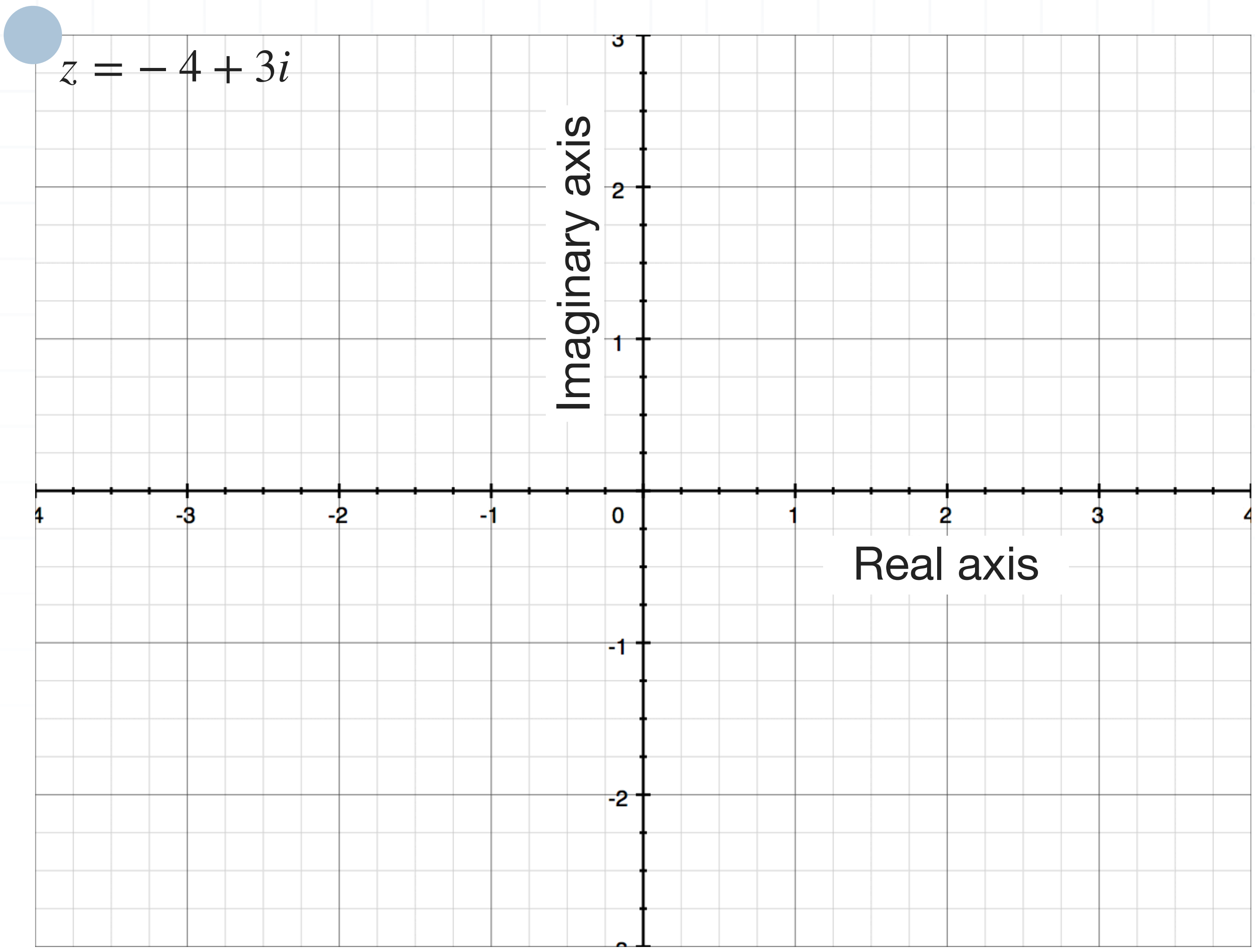
$$(1 + 0i) - (5 - 3i)$$

$$(1 - 5) + (0 - (-3))i$$

$$(-4) + (0 + 3)i$$

$$-4 + 3i$$

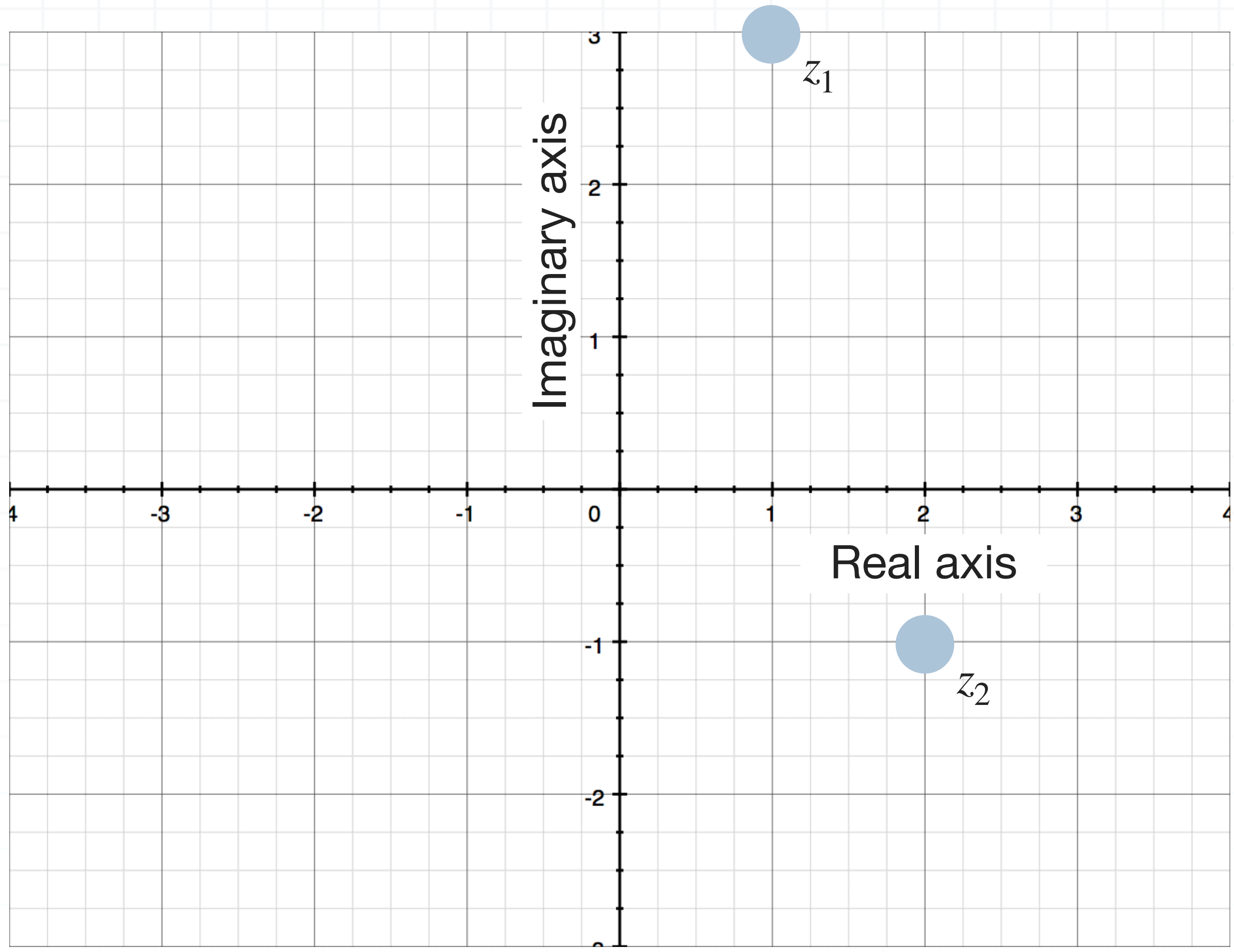
Now graph the complex number $-4 + 3i$, which has a real part of -4 and an imaginary part of 3.



We can also work backwards. If you have imaginary numbers graphed in the complex plane, you can break them down into the real and imaginary parts, then find the sum or difference, and graph those.

Example

Graph the sum of the complex numbers z_1 and z_2.



The point z_1 is 1 unit to the right of the vertical axis and 3 units above the horizontal axis, which means that complex number is $z_1 = 1 + 3i$.

The point z_2 is 2 units to the right of the vertical axis and 1 unit below the horizontal axis, which means that complex number is $z_2 = 2 - 1i = 2 - i$.

The sum of z_1 and z_2 is

$$z_1 + z_2 = (1 + 3i) + (2 - i)$$

$$z_1 + z_2 = (1 + 2) + (3 + (-1))i$$

$$z_1 + z_2 = (1 + 2) + (3 - 1)i$$

$$z_1 + z_2 = 3 + 2i$$

So if we plot the sum on the same set of axes, we get