Topic: Converting rectangular coordinates to polar

Question: Convert the rectangular coordinate point $(0, -14)$ into polar coordinates.

Answer choices:

A $(r, \theta) = \left(-14, -\frac{\pi}{2}\right)$

B $(r, \theta) = \left(14, \frac{3\pi}{2}\right)$

C $(r, \theta) = \left(\sqrt{14}, -\frac{\pi}{2}\right)$

D $(r, \theta) = (14, \pi)$

Solution: B

To find r, we'll use the conversion formula

$$r^2 = x^2 + y^2$$

Plugging $(0, -14)$ into the formula gives

$$r^2 = (0)^2 + (-14)^2$$

$$r^2 = 196$$

$$r = 14$$

To find θ, we realize that since $x = 0$ and y is negative, the point in question is on the negative vertical axis, which must mean that $\theta = 3\pi/2$.

So the given point in polar coordinates is

$$\left(14, \frac{3\pi}{2}\right)$$

Topic: Converting rectangular coordinates to polar

Question: Which choice most closely represents the rectangular coordinate point $(-16, -22)$ in polar coordinates.

Answer choices:

A $(r, \theta) = (27.2, 4.08)$

B $(r, \theta) = (20.7, 54.1)$

C $(r, \theta) = (27.2, -1.20)$

D $(r, \theta) = (13.6, 5.51)$

Solution: A

To find r, we'll use the conversion formula

$$r^2 = x^2 + y^2$$

Plugging $(-16, -22)$ into the formula gives

$$r^2 = (-16)^2 + (-22)^2$$

$$r^2 = 256 + 484$$

$$r^2 = 740$$

$$r \approx 27.2$$

Since both x and y are negative, this point is in the third quadrant, so

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$$

Using a calculator, we find that

$$\frac{y}{x} = \frac{-22}{-16} = \frac{22}{16} = 1.375$$

so

$$\tan^{-1}\left(\frac{y}{x}\right) \approx 0.942 \text{ radians}$$

Therefore,

$$\theta = \left(\tan^{-1}\left(\frac{y}{x}\right)\right) + \pi \approx (0.942 + \pi) \text{ radians}$$

$$\theta \approx 4.08 \text{ radians}$$

So the given point in polar coordinates is

$$(27.2,4.08)$$

Topic: Converting rectangular coordinates to polar

Question: Convert the rectangular coordinate point $\left(17\sqrt{3}, -17\right)$ into polar coordinates.

Answer choices:

A $(r, \theta) = \left(34, \frac{\pi}{6}\right)$

B $(r, \theta) = \left(17, \frac{5\pi}{3}\right)$

C $(r, \theta) = \left(17, \frac{\pi}{3}\right)$

D $(r, \theta) = \left(34, \frac{11\pi}{6}\right)$

Solution: D

To find r, we'll use the conversion formula

$$r^2 = x^2 + y^2$$

Plugging $\left(17\sqrt{3}, -17\right)$ into the formula gives

$$r^2 = \left(17\sqrt{3}\right)^2 + (-17)^2$$

$$r^2 = 289(3) + 289$$

$$r^2 = 1{,}156$$

$$r = 34$$

Since x is positive and y is negative, this point is in the fourth quadrant, so

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) + 2\pi$$

Now

$$\frac{y}{x} = \frac{-17}{17\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

Recall the following:

$$\sin\frac{\pi}{6} = \frac{1}{2} \text{ and } \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

By the odd and even identities for sine and cosine, respectively,

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2} \text{ and } \cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Therefore,

$$\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{y}{x} = \frac{\sin\left(-\frac{\pi}{6}\right)}{\cos\left(-\frac{\pi}{6}\right)} = \tan\left(-\frac{\pi}{6}\right)$$

Note that

$$-\frac{\pi}{2} = -\frac{3\pi}{6} < -\frac{\pi}{6} < 0$$

That is, $-\pi/6$ is in the interval $(-\pi/2,0)$, so $-\pi/6$ is in the range of the inverse tangent function. Thus

$$\tan^{-1}\left(\frac{y}{x}\right) = -\frac{\pi}{6}$$

Using this result, we find that

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) + 2\pi = -\frac{\pi}{6} + 2\pi = \frac{1(-\pi) + 6(2\pi)}{6} = \frac{-\pi + 12\pi}{6} = \frac{11\pi}{6}$$

So the given point in polar coordinates is

$$\left(34, \frac{11\pi}{6}\right)$$