Scalar multiplication and zero matrices

We've learned about matrix addition and subtraction, and in this lesson we want to start looking at matrix multiplication. But before we multiply two matrices together, we'll tackle something a little simpler: scalar multiplication.

Scalar multiplication

You can multiply any matrix by a scalar. If we want to multiply a matrix by 3, then 3 is the **scalar**, we distribute the scalar across every entry in the matrix, and the result of the scalar multiplication looks like this:

$$3\begin{bmatrix}6 & 2\\ -1 & -4\end{bmatrix}=\begin{bmatrix}3(6) & 3(2)\\ 3(-1) & 3(-4)\end{bmatrix}=\begin{bmatrix}18 & 6\\ -3 & -12\end{bmatrix}$$

Notice that this also translates to dividing through a matrix by a scalar. Because, if you want to divide through a matrix by 6, that's the same as multiplying through the matrix by $1/6$, and we're right back to the same kind of scalar multiplication.

$$\frac{1}{6}\begin{bmatrix}6 & 2\\ -1 & -4\end{bmatrix}=\begin{bmatrix}\frac{1}{6}(6) & \frac{1}{6}(2)\\ \frac{1}{6}(-1) & \frac{1}{6}(-4)\end{bmatrix}=\begin{bmatrix}1 & \frac{1}{3}\\ -\frac{1}{6} & -\frac{2}{3}\end{bmatrix}$$

This translates as well to linear equations.

Example

Solve the equation for k.

$$2\begin{bmatrix} 5 & -7 \\ -1 & 0 \end{bmatrix} + k = -3\begin{bmatrix} 2 & 4 \\ 11 & -9 \end{bmatrix}$$

Apply the scalars to the matrices.

$$\begin{bmatrix} 2(5) & 2(-7) \\ 2(-1) & 2(0) \end{bmatrix} + k = \begin{bmatrix} -3(2) & -3(4) \\ -3(11) & -3(-9) \end{bmatrix}$$

$$\begin{bmatrix} 10 & -14 \\ -2 & 0 \end{bmatrix} + k = \begin{bmatrix} -6 & -12 \\ -33 & 27 \end{bmatrix}$$

Subtract the matrix on the left from both sides of the equation in order to isolate k.

$$k = \begin{bmatrix} -6 & -12 \\ -33 & 27 \end{bmatrix} - \begin{bmatrix} 10 & -14 \\ -2 & 0 \end{bmatrix}$$

$$k = \begin{bmatrix} -6 - 10 & -12 - (-14) \\ -33 - (-2) & 27 - 0 \end{bmatrix}$$

$$k = \begin{bmatrix} -16 & 2 \\ -31 & 27 \end{bmatrix}$$

Zero matrices

A **zero matrix** is a matrix with all zero values, like these:

$$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} \qquad \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\end{bmatrix} \qquad \begin{bmatrix}0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

We always name the zero matrix with a capital O. And optionally, you can add a subscript with the dimensions of the zero matrix. Since the values in a zero matrix are all zeros, just having the dimensions of the zero matrix tells you what the entire matrix looks like. So $O_{2\times3}$ is

$$O_{2\times3} = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Adding and subtracting the zero matrix

Adding the zero matrix to any other matrix doesn't change the matrix's value. And subtracting the zero matrix from any other matrix doesn't change that matrix's value.

Just like with non-zero matrices, matrix dimensions have to be the same in order to be able to add or subtract them.

Adding the zero matrix:

$$\begin{bmatrix}9 & -6 & 2\\1 & 0 & -7\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\end{bmatrix} = \begin{bmatrix}9 & -6 & 2\\1 & 0 & -7\end{bmatrix}$$

Subtracting the zero matrix:

$$\begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix}$$

Adding opposite matrices

Matrix K and $-K$ are **opposite matrices**. In other words, to get the opposite of a matrix, multiply it by a scalar of -1. So if

$$K = \begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix},$$

then the opposite of K is

$$-K = (-1)\begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} = \begin{bmatrix} (-1)9 & (-1)(-6) & (-1)2 \\ (-1)1 & (-1)0 & (-1)(-7) \end{bmatrix} = \begin{bmatrix} -9 & 6 & -2 \\ -1 & 0 & 7 \end{bmatrix}$$

The reason we're talking about opposite matrices in this section is because adding opposite matrices results in the zero matrix.

$$K + (-K) = \begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} + \begin{bmatrix} -9 & 6 & -2 \\ -1 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 9 + (-9) & -6 + 6 & 2 + (-2) \\ 1 + (-1) & 0 + 0 & -7 + 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Multiplying by a zero scalar

Multiplying any matrix by a scalar of 0 results in a zero matrix.

$$(0)\begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 9(0) & -6(0) & 2(0) \\ 1(0) & 0(0) & -7(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$