Topic: Heron's formula

Question: What is the area of a triangle with side lengths 44, 28, and 36?

Answer choices:

A 495

B 503

C 618

D 527

Solution: B

Find s, which is half the perimeter of the triangle.

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(44 + 28 + 36) = \frac{1}{2}(72 + 36) = \frac{1}{2}(108) = 54$$

Then by Heron's formula, the area of the triangle is

$$\text{Area} = \sqrt{54(54 - 44)(54 - 28)(54 - 36)}$$

$$\text{Area} = \sqrt{54(10)(26)(18)}$$

$$\text{Area} = \sqrt{252{,}720}$$

$$\text{Area} \approx 503$$

Topic: Heron's formula

Question: Find the area of the triangle with interior angles $59°$ and $67°$, if the side opposite the $67°$ angle has length 22.

Answer choices:

A 111

B 182

C 216

D 364

Solution: B

Let $A = 59°$ and $B = 67°$. Then $b = 22$, and

$$C = 180° - 59° - 67°$$

$$C = 54°$$

Plug everything we know into the law of sines.

$$\frac{a}{\sin 59°} = \frac{22}{\sin 67°} = \frac{c}{\sin 54°}$$

Find a using the first two parts of this three-part equation.

$$\frac{a}{\sin 59°} = \frac{22}{\sin 67°}$$

$$a = \frac{22 \sin 59°}{\sin 67°} \approx \frac{22(0.857)}{0.921} \approx 20.5$$

Plug what we know into the law of sines for the area of a triangle with $\sin C$.

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} \approx \frac{1}{2}(20.5)(22)\sin 54° \approx \frac{1}{2}(20.5)(22)(0.809) \approx 182$$

Topic: Heron's formula

Question: Find the area of a triangle with side lengths 25 cm and 12 cm and a perimeter of 54 cm.

Answer choices:

A 90

B $1{,}560$

C 515

D 371

Solution: A

The perimeter of the triangle is $p = a + b + c$, so we can find the length of the third side.

$$54 = 25 + 12 + c$$

$$54 = 37 + c$$

$$c = 17$$

Find s, which is half the perimeter of the triangle.

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(p) = \frac{1}{2}(54) = 27$$

Then by Heron's formula, the area of the triangle is

$$\text{Area} = \sqrt{27(27 - 25)(27 - 12)(27 - 17)}$$

$$\text{Area} = \sqrt{27(2)(15)(10)}$$

$$\text{Area} = \sqrt{8100}$$

$$\text{Area} = 90$$