Topic: Gauss-Jordan elimination and reduced row-echelon form

Question: Use Gauss-Jordan elimination to solve the system.

$$x + 3y = 13$$

$$2x + 4y = 16$$

Answer choices:

A $x = 5, y = -2$

B $x = 3, y = -1$

C $x = -1, y = 3$

D $x = -2, y = 5$

Solution: D

The augmented matrix is

$$\begin{bmatrix} 1 & 3 & = & 13 \\ 2 & 4 & = & 16 \end{bmatrix}$$

The first row already has a leading 1. After $2R_1 - R_2 \to R_2$, the matrix is

$$\begin{bmatrix} 1 & 3 & = & 13 \\ 0 & 2 & = & 10 \end{bmatrix}$$

The first column is done. After $(1/2)R_2 \to R_2$, the matrix is

$$\begin{bmatrix} 1 & 3 & = & 13 \\ 0 & 1 & = & 5 \end{bmatrix}$$

After $R_1 - 3R_2 \to R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & = & -2 \\ 0 & 1 & = & 5 \end{bmatrix}$$

The second column is done, and we get the solution set

$$x = -2$$

$$y = 5$$

Topic: Gauss-Jordan elimination and reduced row-echelon form

Question: Use Gauss-Jordan elimination to solve the system.

$$x + 4z = 11$$

$$x - y + 4z = 6$$

$$2x + 9z = 25$$

Answer choices:

A $x = -1, y = 5, z = 3$

B $x = 11, y = 6, z = 25$

C $x = 1, y = 0, z = 12$

D $x = -3, y = 8, z = 3$

Solution: A

The augmented matrix is

$$\begin{bmatrix} 1 & 0 & 4 & = & 11 \\ 1 & -1 & 4 & = & 6 \\ 2 & 0 & 9 & = & 25 \end{bmatrix}$$

The first row already has a leading 1. After $R_1 - R_2 \to R_2$, the matrix is

$$\begin{bmatrix} 1 & 0 & 4 & = & 11 \\ 0 & 1 & 0 & = & 5 \\ 2 & 0 & 9 & = & 25 \end{bmatrix}$$

After $2R_1 - R_3 \to R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & 4 & = & 11 \\ 0 & 1 & 0 & = & 5 \\ 0 & 0 & -1 & = & -3 \end{bmatrix}$$

The first and second columns are done. After $(-1)R_3 \to R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & 4 & = & 11 \\ 0 & 1 & 0 & = & 5 \\ 0 & 0 & 1 & = & 3 \end{bmatrix}$$

After $R_1 - 4R_3 \to R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & -1 \\ 0 & 1 & 0 & = & 5 \\ 0 & 0 & 1 & = & 3 \end{bmatrix}$$

The third column is done, and we get the solution set

$x = -1$

$y = 5$

$z = 3$

Topic: Gauss-Jordan elimination and reduced row-echelon form

Question: Use Gauss-Jordan elimination to solve the system.

$$2x + 4y + 10z = 30$$

$$x + y + 3z = 10$$

$$2x + y + 2z = 9$$

Answer choices:

A $x = 7, y = -3, z = 5$

B $x = -4, y = 1, z = 0$

C $x = 2, y = -1, z = 3$

D $x = 30, y = 10, z = 9$

Solution: C

The augmented matrix is

$$\begin{bmatrix} 2 & 4 & 10 & = & 30 \\ 1 & 1 & 3 & = & 10 \\ 2 & 1 & 2 & = & 9 \end{bmatrix}$$

After $(1/2)R_1 \to R_1$, the matrix is

$$\begin{bmatrix} 1 & 2 & 5 & = & 15 \\ 1 & 1 & 3 & = & 10 \\ 2 & 1 & 2 & = & 9 \end{bmatrix}$$

After $R_1 - R_2 \to R_2$, the matrix is

$$\begin{bmatrix} 1 & 2 & 5 & = & 15 \\ 0 & 1 & 2 & = & 5 \\ 2 & 1 & 2 & = & 9 \end{bmatrix}$$

After $2R_1 - R_3 \to R_3$, the matrix is

$$\begin{bmatrix} 1 & 2 & 5 & = & 15 \\ 0 & 1 & 2 & = & 5 \\ 0 & 3 & 8 & = & 21 \end{bmatrix}$$

The first column is done. After $R_1 - 2R_2 \to R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & 1 & = & 5 \\ 0 & 1 & 2 & = & 5 \\ 0 & 3 & 8 & = & 21 \end{bmatrix}$$

After $R_3 - 3R_2 \to R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & 1 & = & 5 \\ 0 & 1 & 2 & = & 5 \\ 0 & 0 & 2 & = & 6 \end{bmatrix}$$

The second column is done. After $(1/2)R_3 \to R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & 1 & = & 5 \\ 0 & 1 & 2 & = & 5 \\ 0 & 0 & 1 & = & 3 \end{bmatrix}$$

After $R_1 - R_3 \to R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & 2 \\ 0 & 1 & 2 & = & 5 \\ 0 & 0 & 1 & = & 3 \end{bmatrix}$$

After $R_2 - 2R_3 \to R_2$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & 2 \\ 0 & 1 & 0 & = & -1 \\ 0 & 0 & 1 & = & 3 \end{bmatrix}$$

The third column is done, and we get the solution set

$$x = 2$$

$$y = -1$$

$$z = 3$$