Cosecant, secant, cotangent, and the reciprocal identities

In the last lesson we defined the first three of the six trig functions as the sine, cosine, and tangent functions:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

In this lesson, we'll define the other three of the six trig functions as cosecant, secant, and cotangent. We abbreviate these three functions as $\csc$, $\sec$, and $\cot$. These three functions are the reciprocals of the first three trig functions.

- Sine is the reciprocal of cosecant, and vice versa
- Cosine is the reciprocal of secant, and vice versa
- Tangent is the reciprocal of cotangent, and vice versa

Remember that the reciprocal of a fraction is what we get when we flip the fraction upside down. So the reciprocal of a/b is b/a. Therefore, in terms of side lengths, we can define these three new trig functions as

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Notice how these three are just the reciprocals of $\sin$, $\cos$, and $\tan$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

We can also define cosecant, secant, and cotangent in terms of x, y, and r, and they'll of course still be the reciprocals of sine, cosine, and cotangent.

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

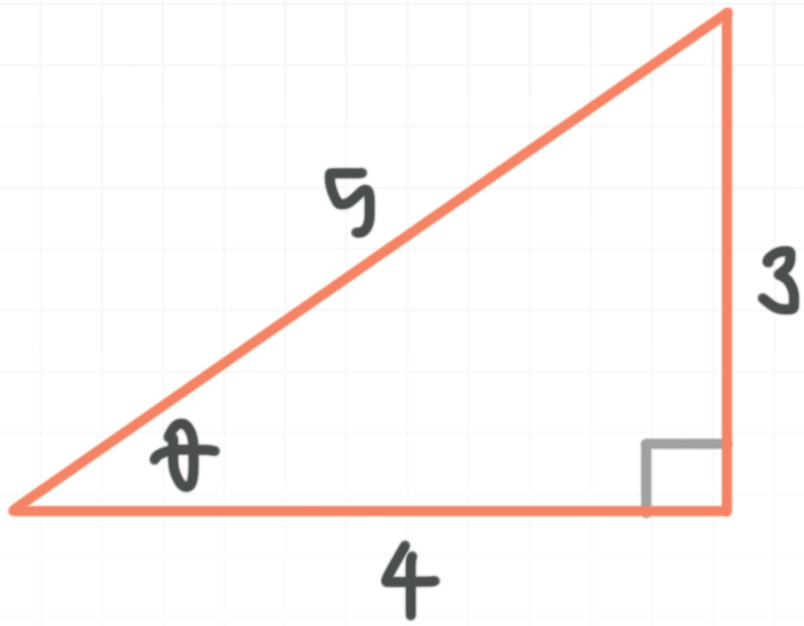
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Let's do an example where we calculate $\csc$, $\sec$, and $\cot$ for a particular triangle.

Example

Find the values of the cosecant, secant, and cotangent functions for θ.



Given the position of the angle θ in the right triangle, the length of the opposite side is 3, the length of the adjacent side is 4, and the length of the hypotenuse is 5.

Then the values of cosecant, secant, and cotangent for the angle are

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{3}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{4}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3}$$

The reciprocal identities

Throughout trigonometry, we'll frequently work with trigonometric identities, which are simply relationships between different trig functions. The identity set we'll talk about here is the set of reciprocal identities.

Of course, these are the reciprocal relationships we've just defined that relate sine, cosine, and tangent to cosecant, secant, and cotangent.

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

With these reciprocal identities, we can start to use the value of one trig function to find the value of another trig function.

Example

Given the value of $\sec\theta$, find the value of $\cos\theta$.

$$\sec\theta = \frac{\sqrt{2}}{2}$$

The cosine and secant functions are related to each other by the reciprocal identities, so we can substitute the value of $\sec\theta$ into the reciprocal identity for $\cos\theta$.

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\cos\theta = \frac{1}{\frac{\sqrt{2}}{2}}$$

Remember that dividing by a fraction is equivalent to multiplying by its reciprocal. So because we're dividing by $\sqrt{2}/2$, we can simplify by instead multiplying by $2/\sqrt{2}$.

$$\cos\theta = 1 \cdot \frac{2}{\sqrt{2}}$$

$$\cos\theta = \frac{2}{\sqrt{2}}$$

We never want to leave an answer with a root in the denominator. It's standard practice to "rationalize the denominator" in order to get roots out of the denominator. So we'll multiply both the numerator and denominator by $\sqrt{2}$. This is equivalent to multiplying by 1, so we're not changing the value of the fraction.

$$\cos\theta = \frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$\cos\theta = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}}$$

$$\cos\theta = \frac{2\sqrt{2}}{2}$$

$$\cos\theta = \sqrt{2}$$