Trig functions of real numbers

Up to now, we've been evaluating trig functions at angles. In other words, we've treated the domain of the six trig functions as a set of angles, meaning that we've only plugged angle values in the trig functions.

But sometimes we want to evaluate trig functions at real numbers, not angles. When we use the six trig functions with real-number arguments, we often call them the six **circular functions**. And we usually indicate the real-number argument with a variable like s or t.

So when we see $\sin\theta$, it indicates a trig function being evaluated at an angle. But when we see $\sin t$ or $\sin s$, it indicates a trig function being evaluated at a real number.

Radian interpretation

When we see a trig function with a real-number argument, like $\sin 2$, we should always interpret it as a real number or as an angle in radians, such that $\sin 2$ means "sine of the real number 2" or "sine of 2 radians." We should never interpret $\sin 2$ to mean "sine of 2 degrees." If we'd wanted to specify the argument as an angle in degrees, we'd use the degree symbol to write $\sin 2^\circ$.

Therefore, because we're always interpreting the real-number argument in radians, if we're using a calculator to find values, we have to make sure the calculator is set to radian mode, not degree mode.

Let's do an example.

Example

Evaluate the six circular functions at $t = 1.732$.

We'll use a calculator to evaluate the circular functions at 1.732, making sure the calculator is set to radian mode.

$$\sin 1.732 \approx 0.9870 \qquad \csc 1.732 \approx 1.0131$$

$$\cos 1.732 \approx -0.1605 \qquad \sec 1.732 \approx -6.2303$$

$$\tan 1.732 \approx -6.1495 \qquad \cot 1.732 \approx -0.1626$$

In the same way we evaluated trig functions at angles, realize here that, alternatively, we could have used the calculator to find sine and cosine at 1.732, and then used the quotient identity to find tangent, and then the reciprocal identities to find cosecant, secant, and cotangent.

Let's do one more example with a negative real number.

Example

Find the values of the six circular functions at $s = -0.6428$.

We'll use a calculator to evaluate the circular functions at -0.6428, making sure the calculator is set to radian mode.

$$\sin(-0.6428) \approx -0.5994 \qquad \csc(-0.6428) \approx -1.6682$$

$$\cos(-0.6428) \approx 0.8004 \qquad \sec(-0.6428) \approx 1.2493$$

$$\tan(-0.6428) \approx -0.7489 \qquad \cot(-0.6428) \approx -1.3353$$