Topic: Points not on the unit circle

Question: What is the value of $\sin\theta$ for an angle θ whose terminal side contains the point $(7,-15)$?

Answer choices:

A $\sin\theta = \frac{15}{22}$

B $\sin\theta = -\frac{7}{15}$

C $\sin\theta = -\frac{15\sqrt{274}}{274}$

D $\sin\theta = -\frac{15}{7}$

Solution: C

Substitute $x = 7$ and $y = -15$ into formula for $\sin\theta$ of a point off the unit circle.

$$\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\sin\theta = \frac{-15}{\sqrt{7^2 + (-15)^2}}$$

$$\sin\theta = \frac{-15}{\sqrt{49 + 225}}$$

$$\sin\theta = -\frac{15}{\sqrt{274}}$$

$$\sin\theta = -\frac{15\sqrt{274}}{274}$$

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Question: What is the value of $\cos\theta$ for an angle θ whose terminal side contains the point $(-16,-8)$?

Answer choices:

A $\cos\theta = \frac{1}{2}$

B $\cos\theta = -\frac{2\sqrt{5}}{5}$

C $\cos\theta = -\frac{1}{3}$

D $\cos\theta = \frac{4\sqrt{5}}{5}$

Solution: B

Substitute $x = -16$ and $y = -8$ into formula for $\cos\theta$ of a point off the unit circle.

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos\theta = \frac{-16}{\sqrt{(-16)^2 + (-8)^2}}$$

$$\cos\theta = \frac{-16}{\sqrt{256 + 64}}$$

$$\cos\theta = -\frac{16}{\sqrt{320}}$$

$$\cos\theta = -\frac{16}{8\sqrt{5}}$$

$$\cos\theta = -\frac{2}{\sqrt{5}}$$

$$\cos\theta = -\frac{2\sqrt{5}}{5}$$

Topic: Points not on the unit circle

Question: Let α be an angle whose terminal side contains the point $(12,5)$, and let $\theta = \alpha + \pi$. What are the values of $\sin\theta$ and $\cos\theta$?

Answer choices:

A $\sin\theta = -\frac{5}{13}$ $\cos\theta = -\frac{12}{13}$

B $\sin\theta = -\frac{12}{17}$ $\cos\theta = -\frac{5}{17}$

C $\sin\theta = \frac{5}{17}$ $\cos\theta = -\frac{12}{17}$

D $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$

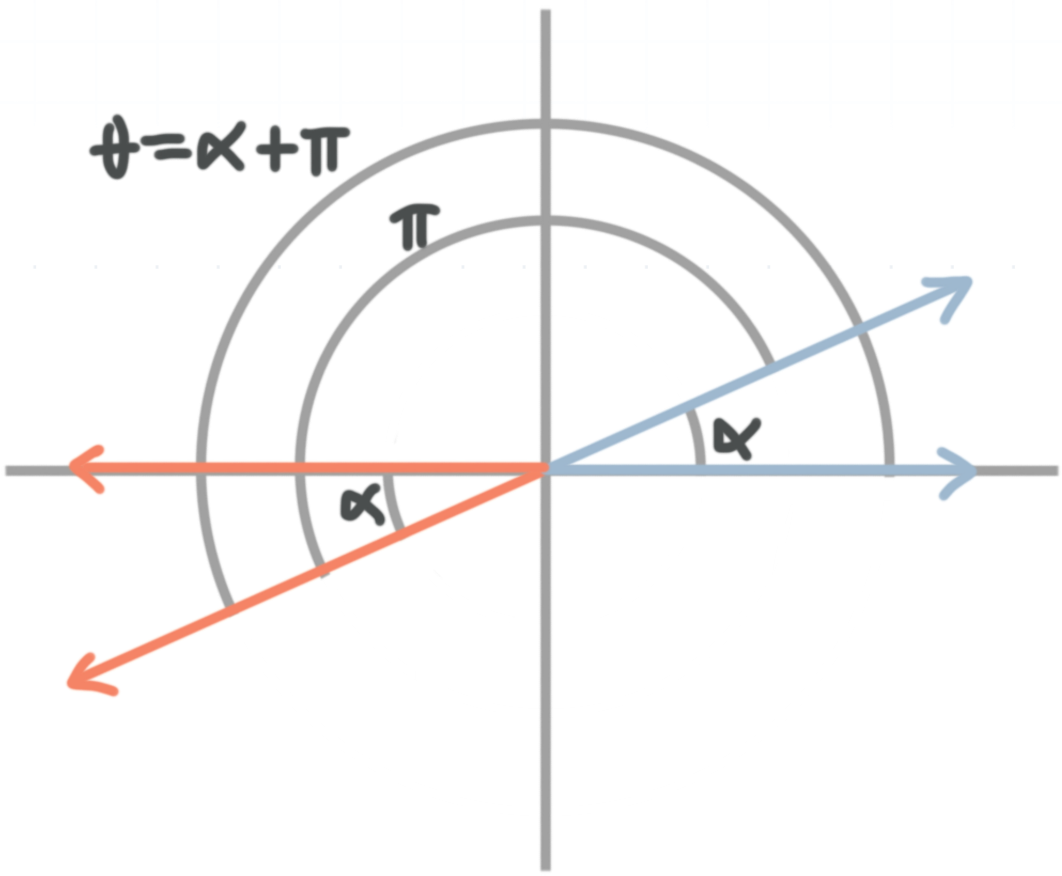
Solution: A

First we'll find $\sin\alpha$ and $\cos\alpha$ by substituting $x = 12$ and $y = 5$ into the following formulas for $\sin\alpha$ and $\cos\alpha$.

$$\sin\alpha = \frac{y}{\sqrt{x^2+y^2}} = \frac{5}{\sqrt{12^2+5^2}} = \frac{5}{\sqrt{144+25}} = \frac{5}{\sqrt{169}} = \frac{5}{13}$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2}} = \frac{12}{\sqrt{12^2+5^2}} = \frac{12}{\sqrt{144+25}} = \frac{12}{\sqrt{169}} = \frac{12}{13}$$

Because $(12,5)$ is on the terminal side of α, we know α is in the first quadrant, because x and y are positive, and therefore that $\alpha+\pi$ is in the third quadrant. Furthermore, the reference angle for both θ and α is α itself.



Then using the reference angle, we get

$$\sin\theta = -\sin\alpha = -\frac{5}{13}$$

$$\cos\theta = -\cos\alpha = -\frac{12}{13}$$