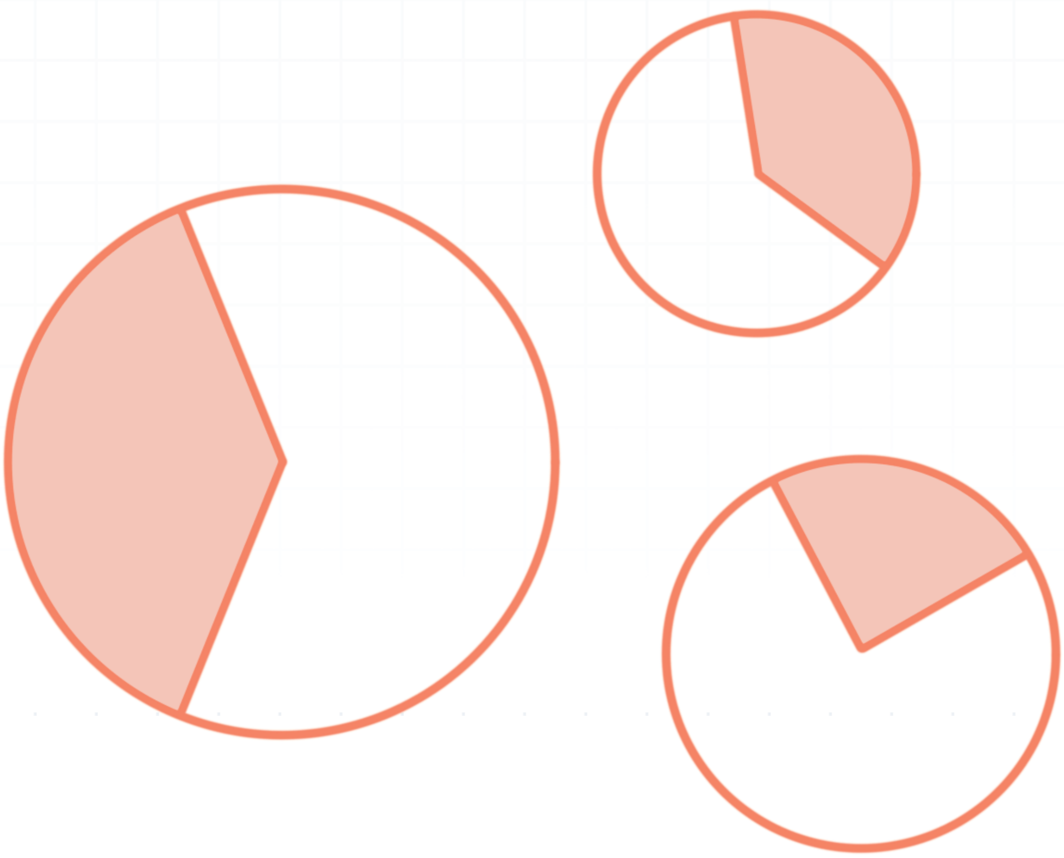
Area of a circular sector

Think of a **circular sector** as a wedge in a circle, like a piece in a pie. Whenever we have one sector in a circle, keep in mind that the rest of the circle also forms another circular sector.

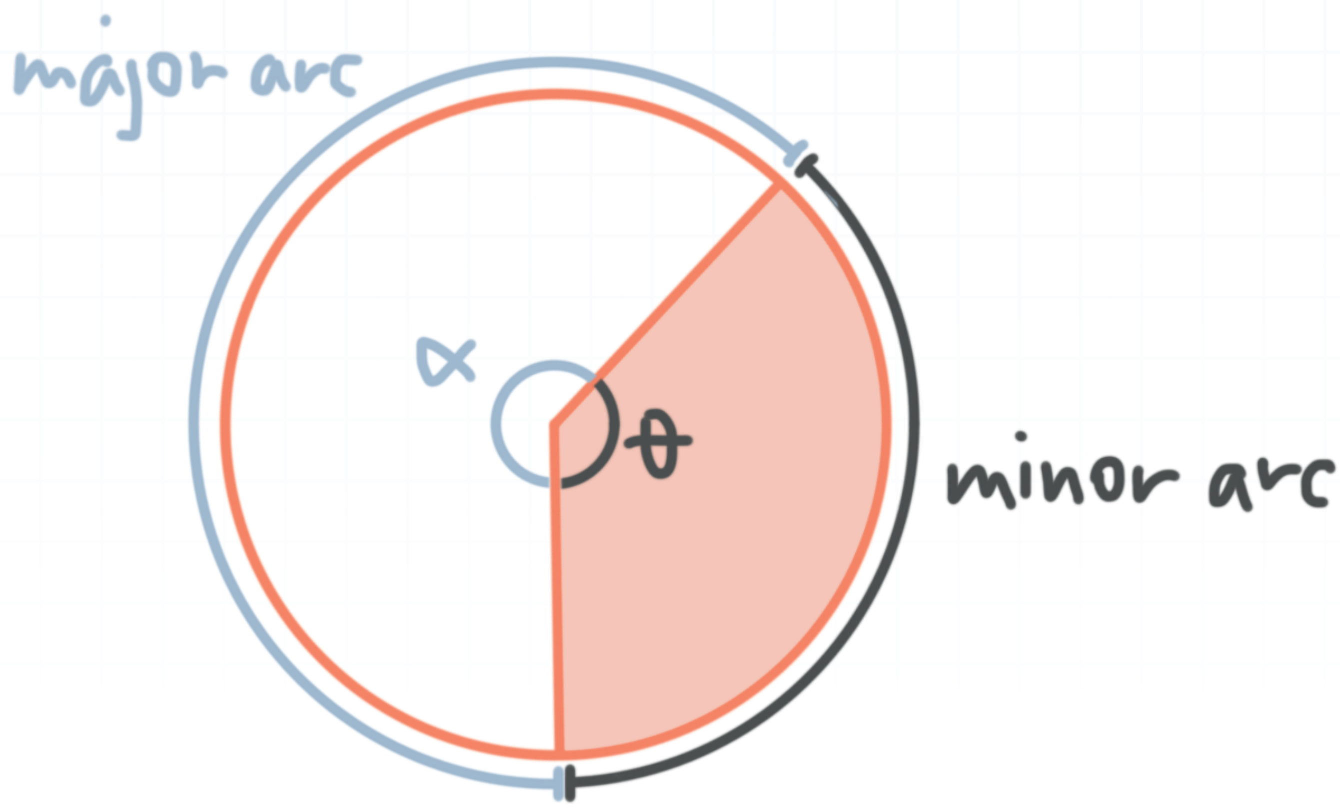
In this figure, each of the red sections are the sectors we've carved out, but then by definition each of the white sections is also a sector.



Every circular sector will have a corresponding arc along the perimeter of the circle. When the two sectors in a circle have equal length arcs, it means each arc is a **semicircle**, or exactly half of the circle. Otherwise, if the arcs aren't the same length, the shorter arc is the **minor arc**, and the longer one is the **major arc**.

The angle at the center of the circle that defines the circular sector is the **central angle**. We say that the arc around the circular sector “subtends” the central angle, or that the central angle “is subtended” by the arc.

So in the figure, θ is the central angle subtended by the **minor arc** (the arc less than half the circle), and α is the central angle subtended by the **major arc** (the arc greater than half the circle).



Area of a circular sector

The area of a circular sector is always proportional to the size of the central angle θ. When θ is defined in radians, the area of a circular sector is

$$A = \frac{1}{2}r^2\theta$$

If the angle we're given is measured in degrees, we can either convert it to radians, or we can use the formula for the area with a degree angle:

$$A = \left(\frac{\pi}{360}\right) r^2\theta$$

Let's use the formula to find the area of a circular sector, given the central angle and the radius of the circle.

Example

Find the area A in square inches of the circular sector with a central angle of $\pi/4$ radians, if the circle has a radius of 9 inches.

Since θ is in radians, the area of this circular sector is

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}(9)^2\left(\frac{\pi}{4}\right)$$

$$A = \frac{81\pi}{8}$$

Let's do another example, but this time with an angle measured in degrees.

Example

In a circle of radius 10 centimeters, calculate the area A in square centimeters of a circular sector with a central angle of 80°.

Since θ is in degrees,

$$A = \left(\frac{\pi}{360}\right) r^2\theta$$

$$A = \left(\frac{\pi}{360}\right)(10)^2(80)$$

$$A = \frac{8{,}000\pi}{360}$$

$$A = \frac{200\pi}{9}$$

Area of the full circle

We may also run into circular sector problems where we're asked to solve for the area of the the entire circle, not just the sector, or where we're given information about the entire circle and asked to solve for the area of a sector of it.

Usually we'll be given the coordinates for the center of the circle and the coordinates of one point on the perimeter of the circle.

The standard form of the equation of a circle is

$$(x-h)^2+(y-k)^2=r^2$$

where (h,k) is the center of the circle, r is its radius, and (x,y) represents the coordinates of any point on the circle.

Let's do an example problem where we use information about the circle to find the area of a sector within it.

Example

A circle passes through the point $(9,-7)$ and has its center at $(4,5)$. Find the area A of a sector of this circle if the sector is defined by the central angle $\theta=\pi/15$ radians.

Since the center of the circle is at $(4,5)$, notice that every point on this circle satisfies the equation

$$(x-4)^2+(y-5)^2=r^2$$

where r is the radius. Moreover, this circle passes through the point $(9,-7)$, so by letting $(x,y)=(9,-7)$, we can find the radius.

$$(9-4)^2+(-7-5)^2=r^2$$

$$5^2+(-12)^2=r^2$$

$$25 + 144 = r^2$$

$$169 = r^2$$

$$r = \pm\sqrt{169}$$

$$r = \pm 13$$

The radius of a circle is a length, so it can't be negative, which means the circle's radius is $r = 13$.

We know the central angle of the circular sector is $\theta = \pi/15$ radians, so we'll plug everything into the formula for the area of a circular sector (with an angle in radians).

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}(13)^2\left(\frac{\pi}{15}\right)$$

$$A = \frac{169\pi}{30}$$