Topic: Parabolas

Question: Find the vertex, focus, and directrix of the parabola.

$$2y^2 - 5x + 3y - 7 = 0$$

Answer choices:

A $V\left(-\frac{13}{8}, -\frac{3}{4}\right)$ $F\left(-1, -\frac{3}{4}\right)$ Directrix at $x = -\frac{9}{4}$

B $V\left(\frac{13}{8}, \frac{3}{4}\right)$ $F\left(-1, -\frac{3}{4}\right)$ Directrix at $x = \frac{9}{4}$

C $V\left(-\frac{13}{8}, -\frac{3}{4}\right)$ $F\left(-\frac{9}{4}, -\frac{3}{4}\right)$ Directrix at $x = -\frac{9}{4}$

D $V\left(-\frac{3}{4}, -\frac{13}{8}\right)$ $F\left(-1, -\frac{3}{4}\right)$ Directrix at $x = \frac{9}{4}$

Solution: A

First, reduce the equation of the parabola into standard form.

$$2y^2 - 5x + 3y - 7 = 0$$

$$2y^2 + 3y = 5x + 7$$

$$y^2 + \frac{3}{2}y = \frac{5}{2}x + \frac{7}{2}$$

$$y^2 + \frac{3}{2}y + \frac{9}{16} = \frac{5}{2}x + \frac{7}{2} + \frac{9}{16}$$

$$\left(y + \frac{3}{4}\right)^2 = \frac{5}{2}x + \frac{65}{16}$$

$$\left(y + \frac{3}{4}\right)^2 = \frac{5}{2}\left(x + \frac{13}{8}\right)$$

$$\left(y + \frac{3}{4}\right)^2 = 4\frac{5}{8}\left(x + \frac{13}{8}\right)$$

The parabola has a horizontal axis at $y = -3/4$ and opens to the right. Its vertex is at $(-13/8, -3/4)$. Since $a = 5/8$, its focus is at

$$\left(-\frac{13}{8} + \frac{5}{8}, -\frac{3}{4}\right)$$

$$\left(-1, -\frac{3}{4}\right)$$

The equation of the directrix is

$$x = -\frac{13}{8} - \frac{5}{8}$$

$$x = -\frac{9}{4}$$

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Question: Which pair of equations represents a parabola with a directrix at $x = -5$?

Answer choices:

A $x = t^2 + 1$ and $y = 5t - 9$

B $x = t^2 - 1$ and $y = 2t - 3$

C $x = 4t^2 - 1$ and $y = 4t - 1$

D $x = t^2 - 1$ and $y = 4t - 1$

Solution: D

Choosing $x = t^2 - 1$ and $y = 4t - 1$ yields the following equations:

$$y + 1 = 4t$$

$$(y + 1)^2 = (4t)^2$$

$$(y + 1)^2 = 16t^2$$

Solve $x = t^2 - 1$ for t^2.

$$x + 1 = t^2$$

Replace $x + 1 = t^2$ in $(y + 1)^2 = 16t^2$.

$$(y + 1)^2 = 16(x + 1)$$

Therefore, the equation of the directrix is

$$x = -1 - 4$$

$$x = -5$$

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Question: Which pair of parametric equations have their vertices at the same point?

Answer choices:

A $x = t + 1,\ y = 2t^2 - 2$ and $x = t^2 - 4,\ y = 2t + 1$

B $x = t^2 - 4,\ y = 2t + 1$ and $x = t^2 + 4,\ y = t + 1$

C $x = t^2 + 4,\ y = t + 1$ and $x = t^2 + 4,\ y = 2t + 1$

D $x = t + 1,\ y = 2t^2 + 2$ and $x = t^2 - 4,\ y = 2t + 1$

Solution: C

From the parametric equations $x = t^2 + 4$ and $y = t + 1$, we have:

$$y - 1 = t$$

$$(y - 1)^2 = t^2$$

and

$$x = t^2 + 4$$

$$x - 4 = t^2$$

Replace $x - 4 = t^2$ in $(y - 1)^2 = t^2$.

$$(y - 1)^2 = x - 4$$

Thus, the vertex of this parabola is at $(4,1)$.

For parametric equations $x = t^2 + 4$ and $y = 2t + 1$, we have:

$$y - 1 = 2t$$

$$(y - 1)^2 = 4t^2$$

and

$$x = t^2 + 4$$

$$x - 4 = t^2$$

Replace $x - 4 = t^2$ in $(y - 1)^2 = 4t^2$.

$$(y - 1)^2 = 4(x - 4)$$

Thus, the vertex of this parabola is at $(4,1)$.