Topic: Graph the polar curve, rose

Question: How many petals are there in the graph of the rose?

$$r = 4\sin(6\theta)$$

Answer choices:

A   6 petals

B   4 petals
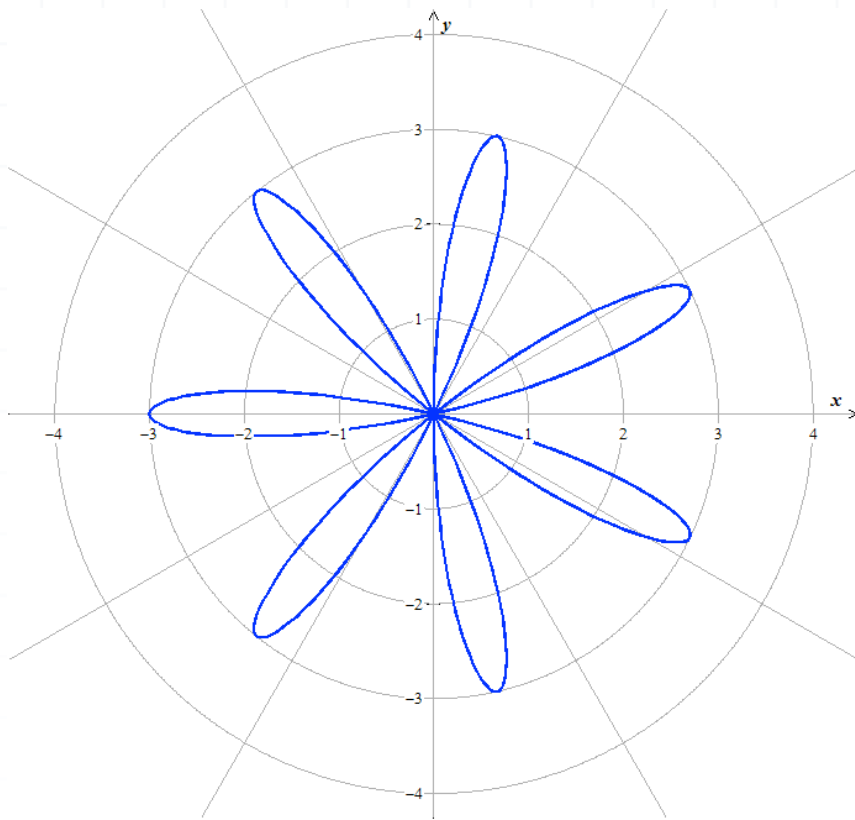
C   8 petals

D   12 petals

Solution: D

The equation $r = 4\sin(6\theta)$ is in the form $r = a\sin(n\theta)$, with $a = 4$ and $n = 6$. Since n is even, this rose has $2n$ petals (i.e., 12 petals).

Topic: Graph the polar curve, rose

Question: The following curve is the graph of one of the polar equations given below. Which polar equation is it?

Answer choices:

A $r = 3\cos(7\theta)$

B $r = 7\sin(3\theta)$

C $r = 3\sin(7\theta)$

D $r = -3\cos(7\theta)$

Solution: D

This rose has 7 petals, which is an odd number, so it's the graph of a polar equation which is either of the form $r = a \cos(7\theta)$ or of the form $r = a \sin(7\theta)$ for some number a. Since the tip of one of the petals is on the horizontal axis, it's the graph of $r = a \cos(7\theta)$ for some number a. And since the tip of that petal is on the negative horizontal axis, and located at a point which is 3 units away from the pole, it's the graph of $r = -3 \cos(7\theta)$.

Topic: Graph the polar curve, rose

Question: What is the measure of the smallest angle θ in the interval $[0,\pi)$ for which there is a petal of the rose $r = 8\sin(5\theta)$ whose tip has polar coordinates (r,θ) with $r = 8$, and what is the measure of the largest angle θ in the interval $[0,\pi)$ for which there is a petal of that rose whose tip has polar coordinates (r,θ) with $r = 8$?

Answer choices:

A   Measure of smallest angle: $\theta = \pi/10$

        Measure of largest angle: $\theta = 7\pi/10$

B   Measure of smallest angle: $\theta = \pi/5$

        Measure of largest angle: $\theta = 4\pi/5$

C   Measure of smallest angle: $\theta = 0$

        Measure of largest angle: $\theta = 4\pi/5$

D   Measure of smallest angle: $\theta = \pi/10$

        Measure of largest angle: $\theta = 9\pi/10$

Solution: D

The equation $r = 8\sin(5\theta)$ is of the form $r = a\sin(n\theta)$, with $a = 8$ and $n = 5$ (hence n is odd), so the tip of every petal of this rose has a pair of polar coordinates (r, θ) with $|r| = 8$ and some angle θ in the set

$$\left\{ \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10} \right\}$$

The angle of smallest measure in this set is $\theta = \pi/10$. Let's see whether $(8, \pi/10)$ is a pair of polar coordinates for one of the petals of this rose.

Well, for $\theta = \pi/10$,

$$r = 8\sin(5\theta) = 8\sin\left(5\left(\frac{\pi}{10} \right) \right) = 8\sin\left(\frac{\pi}{2} \right) = 8(1) = 8$$

so $(8, \pi/10)$ is indeed a pair of polar coordinates for the tip of a petal of this rose, hence $\theta = \pi/10$ is the measure of the smallest angle that we're looking for.

The angle of largest measure in the set given above is $\theta = 9\pi/10$. For the petal whose tip has $\theta = 9\pi/10$,

$$r = 8\sin(5\theta) = 8\sin\left(5\left(\frac{9\pi}{10} \right) \right) = 8\sin\left(\frac{9\pi}{2} \right) = 8\sin\left(4\pi + \frac{\pi}{2} \right)$$

Since 4π is a multiple of 2π, an angle of measure

$$4\pi + \frac{\pi}{2}$$

is coterminal with an angle of measure $\pi/2$, so

$$r = 8 \sin\left(4\pi + \frac{\pi}{2}\right) = 8 \sin\left(\frac{\pi}{2}\right) = 8(1) = 8$$

Therefore, $(8, 9\pi/10)$ is a pair of polar coordinates for the tip of a petal of this rose, so $\theta = 9\pi/10$ is the measure of the largest angle that we're looking for.