



# Trigonometry Final Exam Solutions

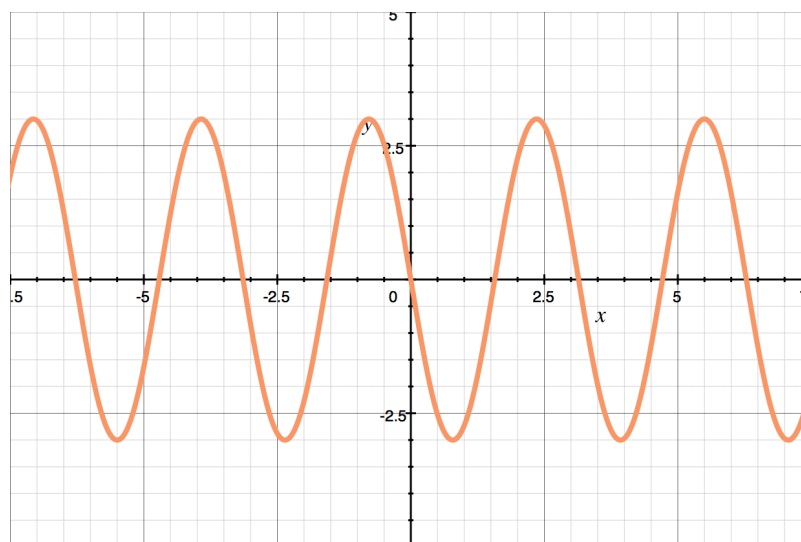
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# Trigonometry Final Exam Answer Key

1. (5 pts)	A	B	C	D	
2. (5 pts)	A	B	C	D	
3. (5 pts)	A	B	C		E
4. (5 pts)	A	B		D	E
5. (5 pts)	A		C	D	E
6. (5 pts)		B	C	D	E
7. (5 pts)		B	C	D	E
8. (5 pts)	A	B		D	E

9. (15 pts) 681 ft

10. (15 pts)  $c = 16.95$ ,  $A = 19.79^\circ$ , and  $B = 52.21^\circ$



11. (15 pts)

12. (15 pts)  $\csc^2 \theta = \csc^2 \theta$



## Trigonometry Final Exam Solutions

1. E. Since the central angle  $\theta$  is in degrees, find the area of the circular sector using the formula.

$$A = \pi r^2 \left( \frac{\theta}{360} \right)$$

Since the diameter is 8 ft, the radius is 4 ft.

$$A = \pi(4)^2 \left( \frac{150}{360} \right)$$

$$A = \pi(16) \left( \frac{5}{12} \right)$$

$$A = \frac{20}{3}\pi$$

2. E. Angular velocity is found using the formula  $\omega = v/r$  where  $\omega$  is angular velocity,  $v$  is linear velocity, and  $r$  is the radius.

$$\omega = \frac{60 \text{ mi}}{10 \text{ hr-in}}$$

Now we'll convert to revolutions per second using unit conversions.

$$\omega = \frac{60 \text{ mi}}{10 \text{ hr-in}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ rev}}{2\pi}$$



$$\omega = \frac{60 \cdot 5,280 \cdot 12 \text{ rev}}{10 \cdot 60 \cdot 60 \cdot 2\pi \text{ sec}}$$

$$\omega = \frac{528 \text{ rev}}{10\pi \text{ sec}}$$

$$\omega \approx 16.81 \text{ revolutions per second}$$

### 3. D. The law of sines is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We know that  $A = 126^\circ$ ,  $a = 12$ , and  $b = x$ . Since the sum of the angles of a triangle must equal  $180^\circ$ ,

$$B = 180^\circ - 126^\circ - 28^\circ$$

$$B = 26^\circ$$

Substitute the known values into the law of sines.

$$\frac{12}{\sin 126^\circ} = \frac{x}{\sin 26^\circ}$$

Cross multiply and divide both sides by  $\sin 126^\circ$ .

$$x \sin 126^\circ = 12 \sin 26^\circ$$

$$x = \frac{12 \sin 26^\circ}{\sin 126^\circ}$$

$$x = 6.5$$



4. C. Use Heron's formula to find the area of a triangle. If  $a$ ,  $b$ , and  $c$  are the lengths of the sides of the triangle, then part of Heron's formula says

$$s = \frac{1}{2}(a + b + c)$$

$$s = \frac{1}{2}(14 + 18 + 22)$$

$$s = \frac{1}{2}(54)$$

$$s = 27$$

Plugging this value into Heron's formula, we get

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

$$A = \sqrt{27(27 - 14)(27 - 18)(27 - 22)}$$

$$A = \sqrt{27(13)(9)(5)}$$

$$A = \sqrt{15,795}$$

$$A \approx 126$$

5. B. The period of a cosine function is  $2\pi/b$  where  $b$  is the coefficient on  $\theta$ .



$$\frac{2\pi}{\frac{1}{4}}$$

$$2\pi \left( \frac{4}{1} \right)$$

$$8\pi$$

6. A. Remember that a sinusoid written in the form  $y = a \sin(b\theta + c) + d$  has a vertical stretch of  $a$ , horizontal compression of  $b$ , horizontal shift to the left  $c$ , and a vertical shift up  $d$ .

The equation

$$y = 4 \sin \left( 3\theta - \frac{\pi}{4} \right) + 5$$

has an amplitude of 4, horizontal compression of 3, horizontal shift to the right of  $\pi/4$ , and vertical shift up of 5.

7. A. Use the sum of cosines to find the exact value, since  $30^\circ + 45^\circ = 75^\circ$ .

$$\cos(\theta + \alpha) = \cos \theta \cos \alpha - \sin \theta \sin \alpha$$

$$\cos 75^\circ = \cos(30^\circ + 45^\circ)$$

$$\cos(75^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$



$$\cos 75^\circ = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\cos 75^\circ = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos 75^\circ = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos 75^\circ = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\cos 75^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$$

8. C. Use the product identity for  $\cos \theta \sin \alpha$ .

$$\cos \theta \sin \alpha = \frac{1}{2}[\sin(\theta + \alpha) - \sin(\theta - \alpha)]$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2}[\sin(75^\circ + 15^\circ) - \sin(75^\circ - 15^\circ)]$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2}(\sin 90^\circ - \sin 60^\circ)$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2} \left( 1 - \frac{\sqrt{3}}{2} \right)$$

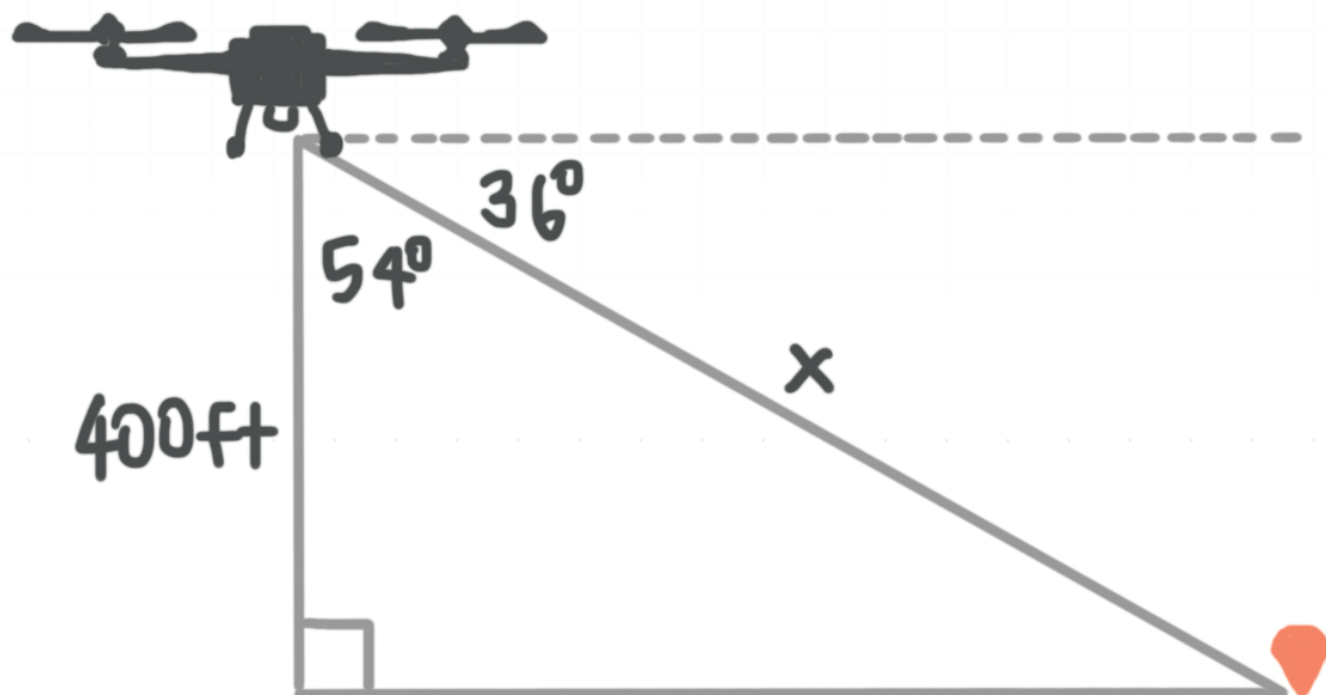


$$\cos 75^\circ \sin 15^\circ = \frac{1}{2} \left( \frac{2}{2} - \frac{\sqrt{3}}{2} \right)$$

$$\cos 75^\circ \sin 15^\circ = \frac{1}{2} \left( \frac{2 - \sqrt{3}}{2} \right)$$

$$\cos 75^\circ \sin 15^\circ = \left( \frac{2 - \sqrt{3}}{4} \right)$$

9. Draw a diagram.



Since the angle of depression is  $36^\circ$ , the angle inside the triangle is  $90^\circ - 36^\circ = 54^\circ$ . To find  $x$ , the slant distance from the drone to the object on the ground, use cosine because

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$





The adjacent is 400 ft and the hypotenuse is  $x$ .

$$\cos 54^\circ = \frac{400}{x}$$

Cross multiply and divide both sides by  $\cos 54^\circ$ .

$$x \cos 54^\circ = 400$$

$$x = \frac{400}{\cos 54^\circ}$$

$$x \approx 680.52 \text{ ft}$$

$$x \approx 681 \text{ ft}$$

10. Use the law of cosines to find side  $c$ .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 6^2 + 14^2 - 2(6)(14)\cos 108^\circ$$

$$c^2 = 36 + 196 - 168 \cos 108^\circ$$

$$c = \sqrt{36 + 196 - 168 \cos 108^\circ}$$

$$c \approx 16.85$$

Use the law of sines to find angle  $A$ .

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$



$$\frac{6}{\sin A} = \frac{16.85}{\sin 108^\circ}$$

Cross multiply and divide both sides by 16.85.

$$16.85 \sin A = 6 \sin 108^\circ$$

$$\sin A = \frac{6 \sin 108^\circ}{16.85}$$

Take the  $\sin^{-1}$  of both sides.

$$A = \sin^{-1} \left( \frac{6 \sin 108^\circ}{16.85} \right)$$

$$A \approx 19.79^\circ$$

Since the sum of all the angles in a triangle is  $180^\circ$ , subtract the measures of angles  $A$  and  $C$  from  $180^\circ$  to find angle  $B$ .

$$B = 180^\circ - A - C$$

$$B = 180^\circ - 19.79^\circ - 108^\circ$$

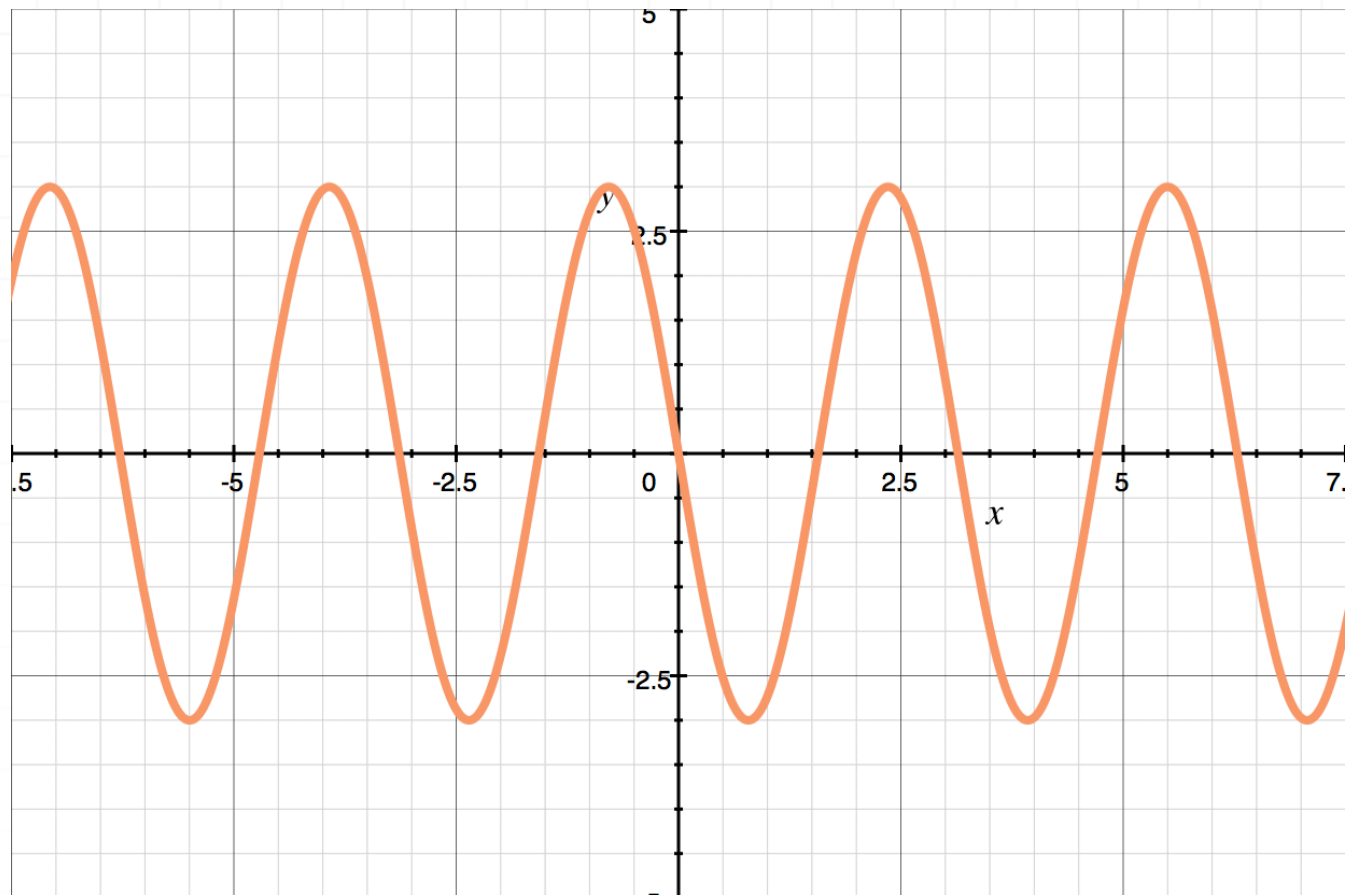
$$B = 52.21^\circ$$

11. Since the graph is the sine function, it will go through the origin. From the origin it will go down to  $-3$  since the amplitude is 3 and the function is negative. The period is

$$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$



so two periods can be graphed from  $-\pi$  to  $\pi$ . There are minimums at  $-3\pi/4$  and  $\pi/4$  and maximums at  $-\pi/4$  and  $3\pi/4$ . The zeros are at  $-\pi$ ,  $-\pi/2$ ,  $0$ ,  $\pi/2$ , and  $\pi$ .



12. To show a trig identity is true, work with one side of the equation until it equals the other side.

$$\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$$

Substitute  $\tan^2 \theta$  in for  $\sec^2 \theta - 1$  using the Pythagorean identity

$$\tan^2 \theta = \sec^2 \theta - 1.$$

$$\frac{\sec^2 \theta}{\tan^2 \theta} = \csc^2 \theta$$



It's often helpful to change everything into terms of sine and cosine when proving trig identities. Remember that  $\sec \theta = 1/\cos \theta$  and  $\tan \theta = \sin \theta / \cos \theta$ .

$$\frac{\frac{1}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta}} = \csc^2 \theta$$

Simplify the fraction by multiplying by a well-chosen 1.

$$\frac{\frac{\cos^2 \theta}{\sin^2 \theta}}{\frac{\cos^2 \theta}{\sin^2 \theta}} \cdot \frac{\frac{1}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta}} = \csc^2 \theta$$

$$\frac{1}{\sin^2 \theta} = \csc^2 \theta$$

The reciprocal identity states that  $\csc \theta = 1/\sin \theta$ .

$$\csc^2 \theta = \csc^2 \theta$$

We've now shown that the equation is a true identity.



