



Precalculus Workbook

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MATH

COMPLEX NUMBERS

- 1. Simplify the imaginary number.

$$i^{437}$$

- 2. Simplify the imaginary number.

$$i^{2,314}$$

- 3. Name the real and imaginary parts of the complex number.

$$z = -5 + 17i$$

- 4. Name the real and imaginary parts of the complex number.

$$z = \sqrt{7} - 4\pi i$$

- 5. How can the numbers be classified?

$$z = -3 + 9i$$

$$z = 0 - 15i$$

$$z = 6 + 0i$$



■ 6. How can the numbers be classified?

$$z = 0 - \pi i$$

$$z = -\sqrt{5} + 0i$$

$$z = -11 + \frac{2}{3}i$$



COMPLEX NUMBER OPERATIONS

- 1. Find the sum and difference of the complex numbers.

$$\frac{7}{5} - \frac{2}{3}i$$

$$\frac{7}{2} - \frac{8}{3}i$$

- 2. Find the product of the complex numbers.

$$-7i$$

$$-5 + 9i$$

- 3. Find the product of the complex numbers.

$$5 - 2i$$

$$6 - 11i$$

- 4. Divide the complex number $-4 + 15i$ by the imaginary number $5i$.

- 5. Find the complex conjugate of each complex number.



$$9 - 9i$$

$$-3 + 13i$$

$$11 - 22i$$

■ 6. Express the fraction in the form $a + bi$ where a and b are real numbers.

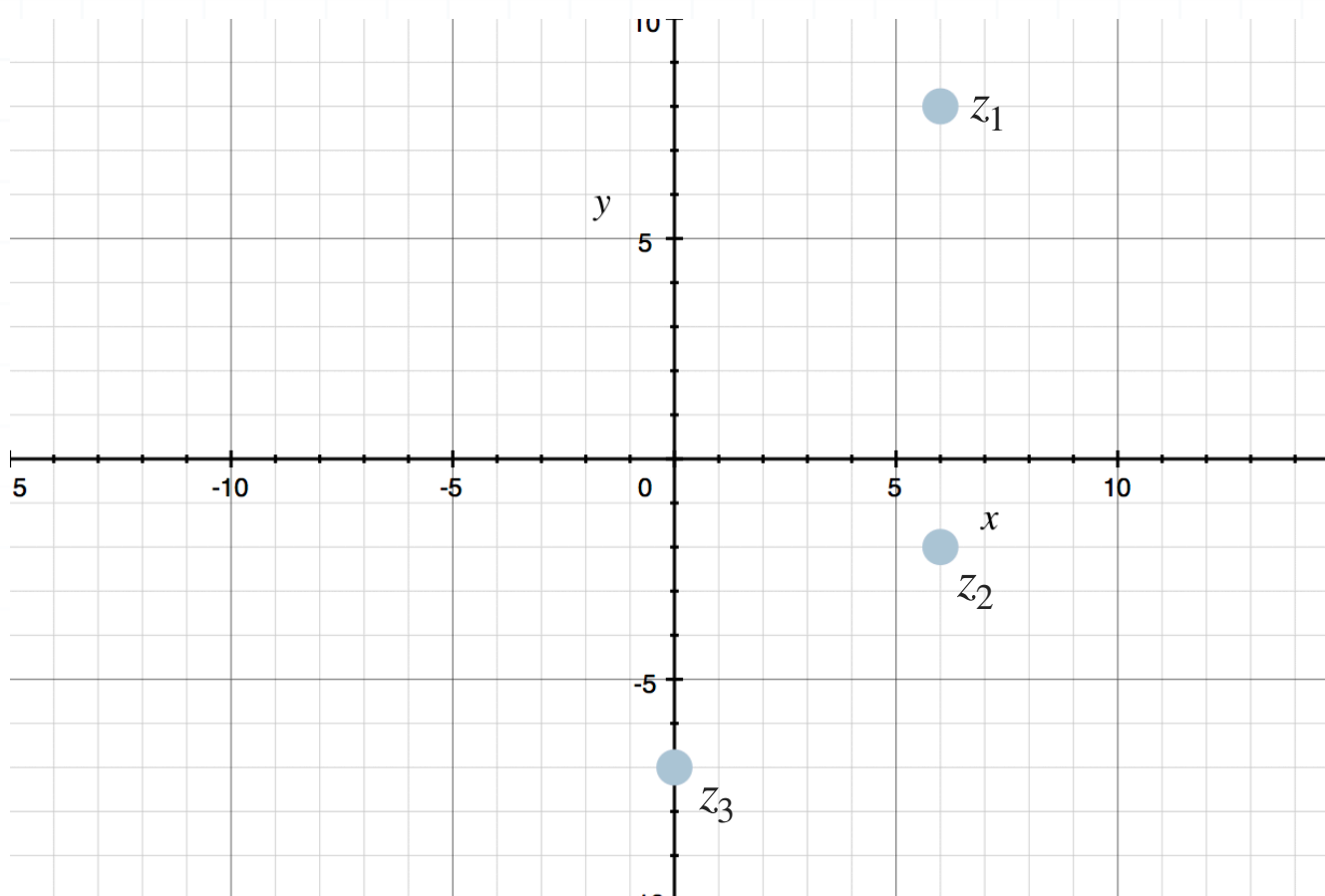
$$\frac{-3 + 7i}{4 - 5i}$$



GRAPHING COMPLEX NUMBERS

■ 1. Graph $-3 + 5i$, $2 - 4i$, and 5 in the complex plane.

■ 2. Which three complex numbers are represented in the graph?

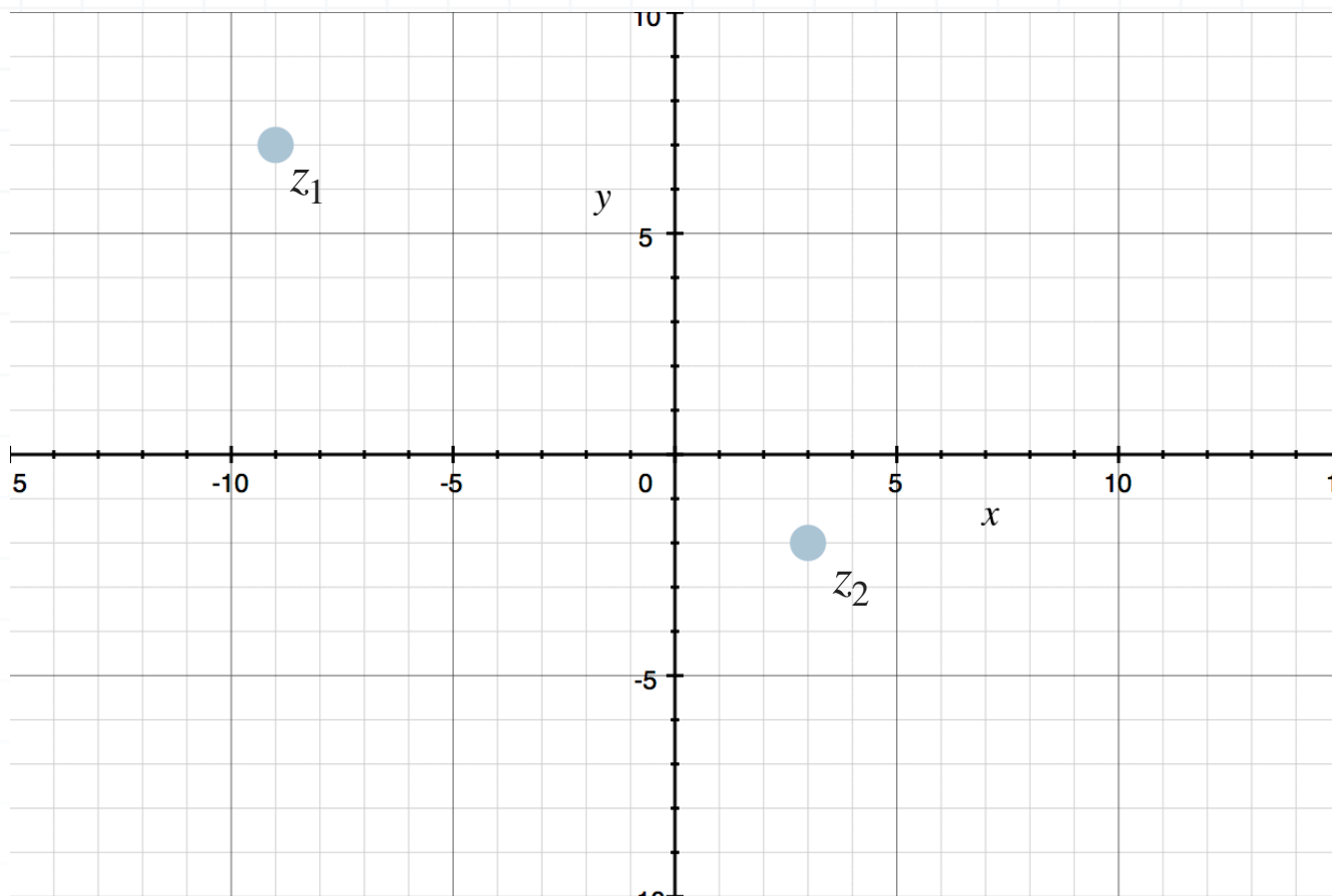


■ 3. Graph the sum of the complex numbers $5 - 4i$ and $-1 + 10i$.

■ 4. Graph the difference of the complex numbers $8 - 7i$ and $13 - 4i$.

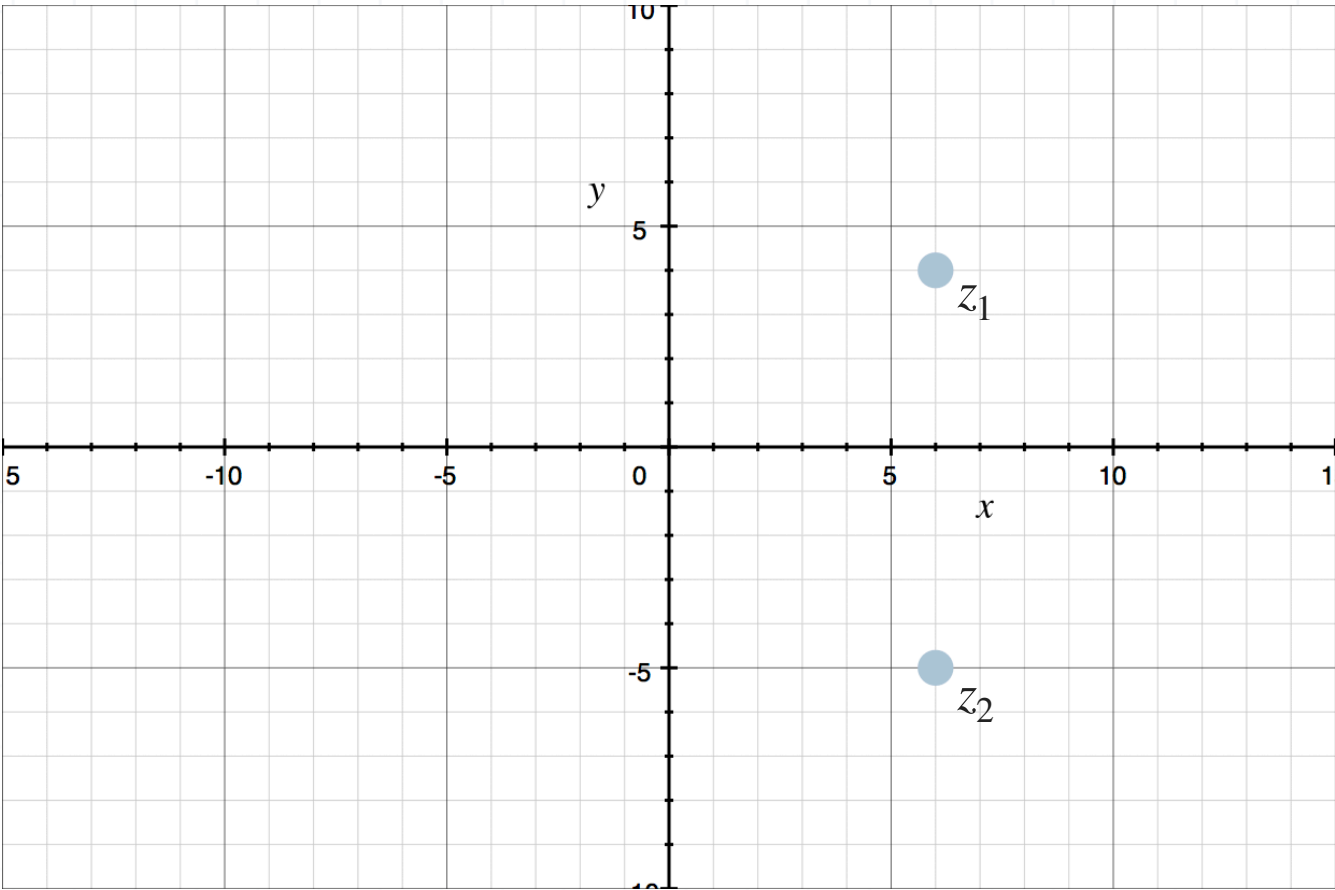


- 5. Graph the sum of the complex numbers z_1 and z_2 .



- 6. Graph the difference of the complex numbers z_1 and z_2 .





DISTANCES AND MIDPOINTS

- 1. Find the distance between $s = 5 + 3i$ and $t = 1 - i$.
- 2. Find the distance between $u = -5 - 3i$ and $v = 4 + 2i$.
- 3. Find the distance between $w = 2 + 6i$ and $z = -2 - 6i$.
- 4. Find the midpoint between $s = 5 + 3i$ and $t = 1 - i$.
- 5. Find the midpoint between $u = -7 - 5i$ and $z = 2 + 2i$.
- 6. Graph the midpoint between $w = 6 + 8i$ and $z = 2 + 4i$.



COMPLEX NUMBERS IN POLAR FORM

■ 1. If the complex number $6 - 2i$ is expressed in polar form, which quadrant contains the angle θ ?

■ 2. Find r for the complex number.

$$-9 - 3i$$

■ 3. What is the polar form of the complex number?

$$5 + 12i$$

■ 4. Write the complex number in polar form.

$$11i$$

■ 5. What is the polar form of the complex number?

$$z = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$



■ 6. Write the complex number in polar form.

$$-5$$



MULTIPLYING AND DIVIDING POLAR FORMS

■ 1. What is the product $z_1 z_2$ of the complex numbers in polar form?

$$z_1 = 5 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$z_2 = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

■ 2. What is the product $z_1 z_2$ of the complex numbers in polar form?

$$z_1 = \sqrt{3} \left(\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5} \right)$$

$$z_2 = \frac{\sqrt{5}}{3} \left(\cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8} \right)$$

■ 3. What is the quotient z_1/z_2 of the complex numbers in polar form?

$$z_1 = 12 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$$

$$z_2 = 15 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$



■ 4. What is the quotient z_1/z_2 of the complex numbers in polar form?

$$z_1 = \sqrt{7} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$$

$$z_2 = \frac{1}{\sqrt{2}} \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

■ 5. What is the product $z_1 z_2$ of the complex numbers in polar form?

$$z_1 = \frac{\sqrt{15}}{4} \left(\cos \frac{7\pi}{2} + i \sin \frac{7\pi}{2} \right)$$

$$z_2 = \frac{1}{\sqrt{5}} \left(\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5} \right)$$

■ 6. Suppose that a complex number z is the product $z_1 \cdot z_2$ of the given complex numbers. If z is expressed in polar form, $r(\cos \theta + i \sin \theta)$, where is θ located?

$$z_1 = 3\sqrt{5} \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5} \right)$$

$$z_2 = 6 \left(\cos \frac{7\pi}{10} + i \sin \frac{7\pi}{10} \right)$$



POWERS OF COMPLEX NUMBERS AND DE MOIVRE'S THEOREM

- 1. Find z^5 in polar form.

$$z = 2 \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$$

- 2. Find z^7 in polar form.

$$z = \sqrt{5} \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5} \right)$$

- 3. Find z^6 in rectangular form $a + bi$.

$$z = \frac{\sqrt{2}}{2} \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right)$$

- 4. Find z^3 in rectangular form $a + bi$.

$$z = 2\sqrt{6} \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right)$$

- 5. Find z^5 in polar form.



$$z = -4 - 4i$$

■ 6. Find z^4 in rectangular form $a + bi$.

$$z = \sqrt{6} - \sqrt{2}i$$



COMPLEX NUMBER EQUATIONS

- 1. Find the solutions of the complex equation.

$$z^2 = 49$$

- 2. Find the solution of the complex equation that lies in the third quadrant.

$$z^3 = 216$$

- 3. Find the solutions of the complex equation.

$$z^4 = 256$$

- 4. Find the solutions of the complex equation.

$$z^6 = 729$$

- 5. Find the solutions of the complex equation.

$$z^5 = 32$$



■ 6. How many solutions of the complex equation lie in the second quadrant?

$$z^8 = 256$$



ROOTS OF COMPLEX NUMBERS

- 1. Find the cube roots of the complex number.

$$z = 27 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

- 2. Find the 4th root of the complex number.

$$z = 256 (\cos 60^\circ + i \sin 60^\circ)$$

- 3. Find the 5th roots of the complex number that lies in the first quadrant of the complex plane.

$$z = 25 (\cos 80^\circ + i \sin 80^\circ)$$

- 4. Find the 4th roots of the complex number.

$$z = 34 \left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5} \right)$$

- 5. Find the 6th roots of the complex number that lie in the second quadrant of the complex plane.



$$z = 11 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

■ 6. Find the 7th roots of the complex number.

$$z = 20 (\cos 120^\circ + i \sin 120^\circ)$$



MATRIX DIMENSIONS AND ENTRIES

- 1. Give the dimensions of the matrix.

$$D = \begin{bmatrix} 11 & 9 \\ -4 & 8 \end{bmatrix}$$

- 2. Give the dimensions of the matrix.

$$A = [3 \quad 5 \quad -2 \quad 1 \quad 8]$$

- 3. Given matrix J , find $J_{4,1}$.

$$J = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 1 \end{bmatrix}$$

- 4. Given matrix C , find $C_{1,2}$.

$$C = \begin{bmatrix} 3 & 12 \\ 1 & 4 \\ 9 & 5 \\ -3 & 2 \end{bmatrix}$$



- 5. Given matrix N , state the dimensions and find $N_{1,3}$.

$$N = \begin{bmatrix} 1 & 5 & 9 \\ 14 & -8 & 6 \end{bmatrix}$$

- 6. Given matrix S , state the dimensions and find $S_{3,4}$.

$$S = \begin{bmatrix} 3 & 6 & -7 & 1 & 0 \\ 0 & 9 & 15 & 3 & 4 \\ 4 & 0 & 2 & 11 & 8 \\ -5 & 8 & 7 & 9 & 2 \end{bmatrix}$$



REPRESENTING SYSTEMS WITH MATRICES

- 1. Represent the system with an augmented matrix called A .

$$-2x + 5y = 12$$

$$6x - 2y = 4$$

- 2. Represent the system with an augmented matrix called D .

$$9y - 3x + 12 = 0$$

$$8 - 4x = 11y$$

- 3. Represent the system with an augmented matrix called H .

$$4a + 7b - 5c + 13d = 6$$

$$3a - 8b = -2c + 1$$

- 4. Represent the system with an augmented matrix called M .

$$-2x + 4y = 9 - 6z$$

$$7y + 2z - 3 = -3t - 9x$$



■ 5. Represent the system with an augmented matrix called A .

$$3x - 8y + z = 7$$

$$2z = 3y - 2x + 4$$

$$5y = 12 - 9x$$

■ 6. Represent the system with an augmented matrix called K .

$$-4b + 2c = 3 - 7a$$

$$9c = 4 - 2b$$

$$8a - 2c = 5b$$



SIMPLE ROW OPERATIONS

- 1. Write the new matrix after $R_1 \leftrightarrow R_2$.

$$\begin{bmatrix} 2 & 6 & -4 & 1 \\ 8 & 2 & 1 & -5 \end{bmatrix}$$

- 2. Write the new matrix after $R_2 \leftrightarrow R_4$.

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 6 & 1 & 5 & -4 \\ -7 & 7 & 0 & 3 \\ 9 & 2 & 8 & 3 \end{bmatrix}$$

- 3. Write the new matrix after $R_1 \leftrightarrow 3R_2$.

$$\begin{bmatrix} 9 & 2 & -7 \\ 1 & 6 & 4 \end{bmatrix}$$

- 4. Write the new matrix after $3R_2 \leftrightarrow 3R_4$.

$$\begin{bmatrix} 0 & 11 & 6 \\ 7 & -3 & 9 \\ 8 & 8 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$



- 5. Write the new matrix after $R_1 + 2R_2 \rightarrow R_1$.

$$\begin{bmatrix} 6 & 2 & 7 \\ 1 & -5 & 15 \end{bmatrix}$$

- 6. Write the new matrix after $4R_2 + R_3 \rightarrow R_3$.

$$\begin{bmatrix} 13 & 5 & -2 & 9 \\ 8 & 2 & 0 & 6 \\ 4 & 1 & 7 & -3 \end{bmatrix}$$



GAUSS-JORDAN ELIMINATION AND REDUCED ROW-ECHELON FORM

■ 1. Use Gauss-Jordan elimination to solve the system.

$$x + 2y = -2$$

$$3x + 2y = 6$$

■ 2. Use Gauss-Jordan elimination to solve the system.

$$2x + 4y = 22$$

$$3x + 3y = 15$$

■ 3. Use Gauss-Jordan elimination to solve the system.

$$x - 3y - 6z = 4$$

$$y + 2z = -2$$

$$-4x + 12y + 21z = -4$$

■ 4. Use Gauss-Jordan elimination to solve the system.

$$2y + 4z = 4$$

$$x + 3y + 3z = 5$$



$$2x + 7y + 6z = 10$$

■ 5. Use Gauss-Jordan elimination to solve the system.

$$3x + 12y + 42z = -27$$

$$x + 2y + 8z = -5$$

$$2x + 5y + 16z = -6$$

■ 6. Use Gauss-Jordan elimination to solve the system.

$$4x + 8y + 4z = 20$$

$$4x + 6y = 4$$

$$3x + 3y - z = 1$$



MATRIX ADDITION AND SUBTRACTION

■ 1. Add the matrices.

$$\begin{vmatrix} 7 & 6 \\ 17 & 9 \end{vmatrix} + \begin{vmatrix} 0 & 8 \\ -2 & 5 \end{vmatrix}$$

■ 2. Add the matrices.

$$\begin{vmatrix} 8 & 3 \\ -4 & 7 \\ 6 & 0 \\ 1 & 13 \end{vmatrix} + \begin{vmatrix} 6 & 7 \\ 2 & -3 \\ 9 & 11 \\ 7 & -2 \end{vmatrix}$$

■ 3. Subtract the matrices.

$$\begin{vmatrix} 7 & 9 \\ 4 & -1 \end{vmatrix} - \begin{vmatrix} 3 & 8 \\ 12 & -3 \end{vmatrix}$$

■ 4. Subtract the matrices.

$$\begin{vmatrix} 8 & 11 & 2 & 9 \\ 6 & 3 & 16 & 8 \end{vmatrix} - \begin{vmatrix} 6 & 11 & 7 & -4 \\ 5 & 8 & 1 & 15 \end{vmatrix}$$



■ 5. Solve for m .

$$\begin{vmatrix} 6 & 5 \\ 9 & -9 \end{vmatrix} + \begin{vmatrix} 3 & 7 \\ 1 & 6 \end{vmatrix} = m + \begin{vmatrix} 7 & 12 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 8 \\ 4 & -7 \end{vmatrix}$$

■ 6. Solve for n .

$$\begin{vmatrix} 4 & 12 \\ 9 & 8 \end{vmatrix} - \begin{vmatrix} 0 & 3 \\ 9 & 9 \end{vmatrix} = n - \begin{vmatrix} 6 & 3 \\ 5 & 11 \end{vmatrix} + \begin{vmatrix} 7 & -4 \\ -18 & 1 \end{vmatrix}$$



SCALAR MULTIPLICATION AND ZERO MATRICES

- 1. Use scalar multiplication to simplify the expression.

$$\frac{1}{4} \begin{vmatrix} 12 & 8 & 3 \\ 2 & -16 & 0 \\ 1 & 5 & 7 \end{vmatrix}$$

- 2. Solve for y .

$$4 \begin{vmatrix} 2 & 9 \\ -5 & 0 \end{vmatrix} + y = 5 \begin{vmatrix} 1 & -3 \\ 6 & 8 \end{vmatrix}$$

- 3. Solve for n .

$$-2 \begin{vmatrix} 6 & 5 \\ 0 & 11 \end{vmatrix} = n - 4 \begin{vmatrix} 2 & 4 \\ -1 & 9 \end{vmatrix}$$

- 4. Add the zero matrix to the given matrix.

$$\begin{vmatrix} 8 & 17 \\ -6 & 0 \end{vmatrix}$$

- 5. Find the opposite matrix.



$$\begin{vmatrix} 6 & 8 & 0 \\ 2 & -3 & 11 \\ 4 & 12 & 9 \end{vmatrix}$$

■ 6. Multiply the matrix by a scalar of 0.

$$\begin{vmatrix} 14 & -1 & 7 & 5 \\ 3 & 7 & 18 & -4 \end{vmatrix}$$



MATRIX MULTIPLICATION

■ 1. If matrix A is 3×3 and matrix B is 3×4 , say whether AB or BA is defined, and give the dimensions of any product that's defined.

■ 2. Find the product of matrices A and B .

$$A = \begin{bmatrix} 2 & 6 \\ -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 0 \\ 5 & -4 \end{bmatrix}$$

■ 3. Find the product of matrices A and B .

$$A = \begin{bmatrix} 5 & -1 \\ 0 & 11 \\ 7 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 1 & 8 \\ -3 & 0 & 4 \end{bmatrix}$$

■ 4. Find the product of matrices A and B .



$$A = \begin{bmatrix} 3 & -2 \\ 1 & 8 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 \\ 4 & 8 \end{bmatrix}$$

■ 5. Use the distributive property to find $A(B + C)$.

$$A = \begin{bmatrix} 2 & 0 \\ 4 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 \\ 5 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & 1 \\ 3 & -1 \end{bmatrix}$$

■ 6. Find the product of matrices A and B .

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -2 & 8 & 1 \\ 7 & 3 & 5 & 2 \end{bmatrix}$$



IDENTITY MATRICES

■ 1. Write the identity matrix I_4 .

■ 2. If we want to find the product IA , where I is the identity matrix and A is a 4×2 , then what are the dimensions of I ?

■ 3. If we want to find the product IA , where I is the identity matrix and A is a 3×4 , then what are the dimensions of I ?

■ 4. If we want to find the product IA , where I is the identity matrix and A is given, then what are the dimensions of I ? What is the product IA ?

$$A = \begin{bmatrix} 2 & 8 \\ -2 & 7 \\ 3 & 5 \end{bmatrix}$$

■ 5. If we want to find the product IA , where I is the identity matrix and A is given, then what are the dimensions of I ? What is the product IA ?

$$A = \begin{bmatrix} 7 & 1 & 3 & -2 \\ 5 & 5 & 2 & 9 \end{bmatrix}$$



■ 6. If A is a 2×4 matrix what are the dimensions of the identity matrix that make the equation true?

$$A \cdot I = A$$



TRANSFORMATIONS

- 1. Find the resulting vector \vec{b} after $\vec{a} = (1,6)$ undergoes a transformation by matrix M .

$$M = \begin{bmatrix} -7 & 1 \\ 0 & -2 \end{bmatrix}$$

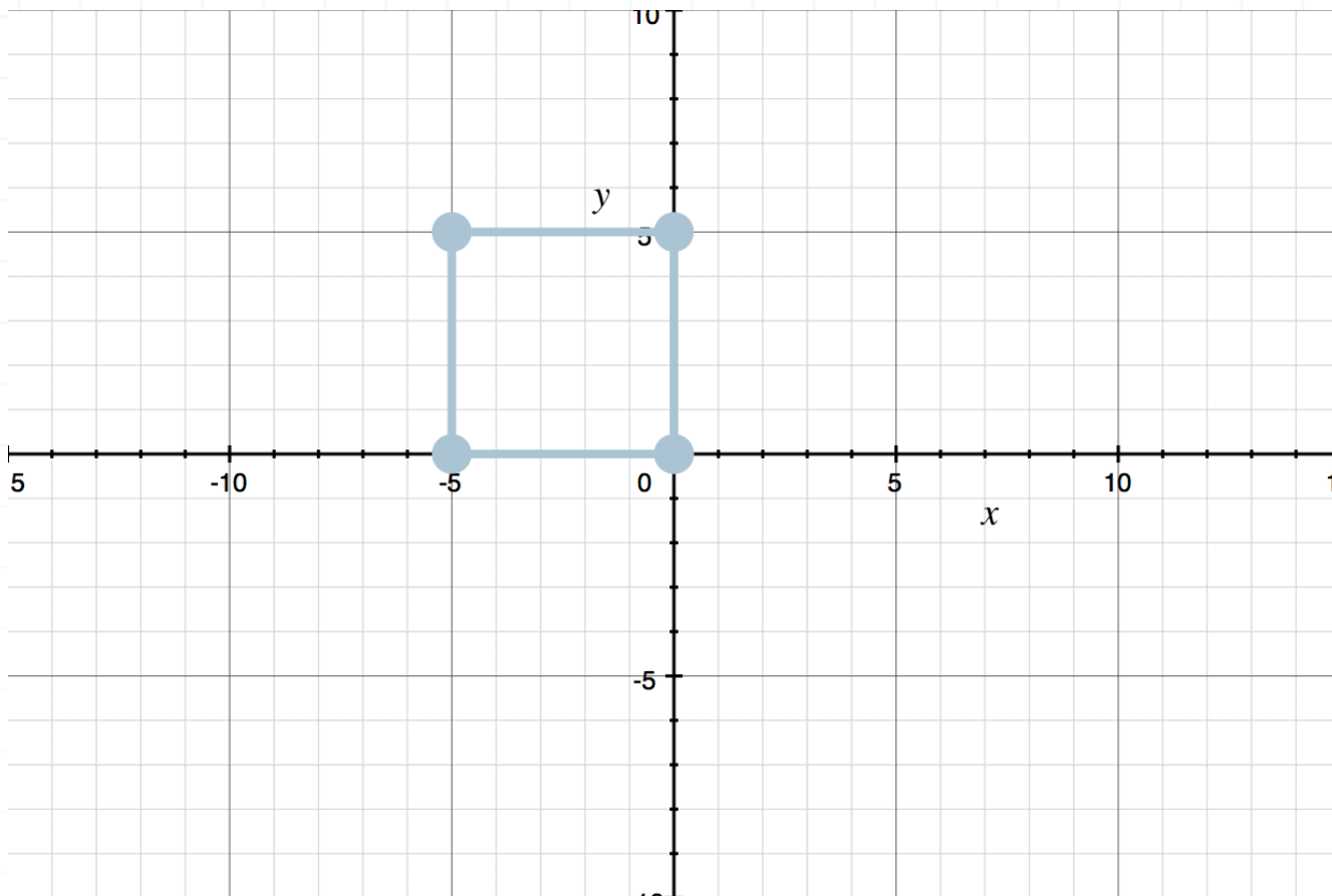
- 2. Sketch triangle $\triangle ABC$ with vertices $(2,3)$, $(-3, -1)$, and $(1, -4)$, and the transformation of $\triangle ABC$ after it's transformed by matrix L .

$$L = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

- 3. Sketch the transformation of the square in the graph after it's transformed by matrix Z .

$$Z = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

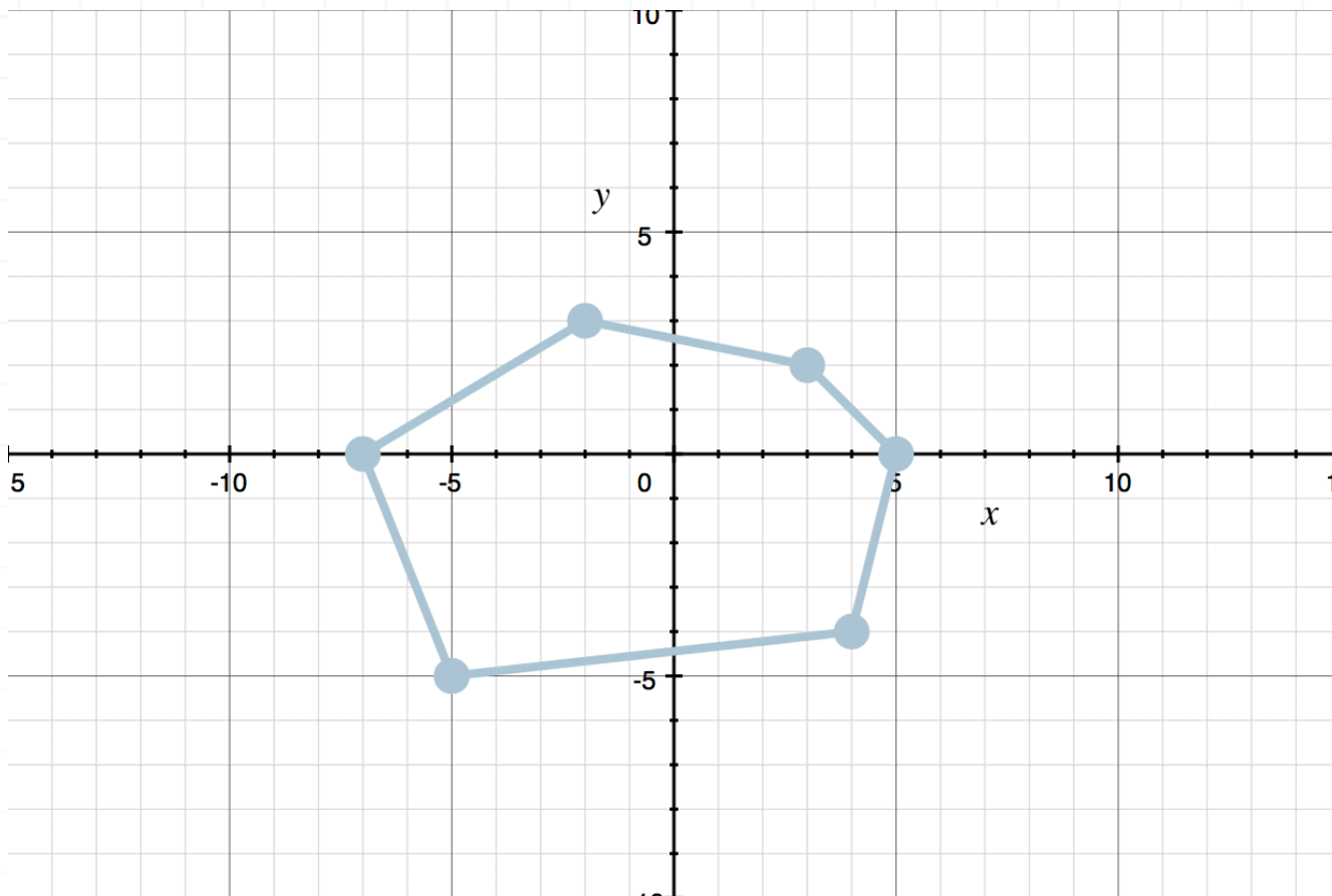




■ 4. Sketch the transformation of the hexagon after it's transformed by matrix Y .

$$Y = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$$





■ 5. What happens to the unit vector $\vec{a} = (1,0)$ after the transformation given by matrix K .

$$K = \begin{bmatrix} 3 & -5 \\ -1 & 0 \end{bmatrix}$$

■ 6. What happens to the unit vector $\vec{b} = (0,1)$ after the transformation given by matrix K .

$$K = \begin{bmatrix} 3 & -5 \\ -1 & 0 \end{bmatrix}$$



MATRIX INVERSES, AND INVERTIBLE AND SINGULAR MATRICES

- 1. Find the determinant of the matrix.

$$B = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

- 2. Find the determinant of the matrix.

$$B = \begin{bmatrix} 1 & -6 \\ 5 & 5 \end{bmatrix}$$

- 3. Find the inverse of matrix G .

$$G = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

- 4. Find the inverse of matrix N .

$$N = \begin{bmatrix} 11 & -4 \\ 5 & -3 \end{bmatrix}$$

- 5. Is the matrix invertible or singular?



$$Z = \begin{bmatrix} 4 & 2 \\ -2 & -1 \end{bmatrix}$$

■ 6. Is the matrix invertible or singular?

$$Y = \begin{bmatrix} 0 & 6 \\ 2 & -1 \end{bmatrix}$$



SOLVING SYSTEMS WITH INVERSE MATRICES

- 1. Use an inverse matrix to solve the system.

$$-4x + 3y = -14$$

$$7x - 4y = 32$$

- 2. Use an inverse matrix to solve the system.

$$6x - 11y = 2$$

$$-10x + 7y = -26$$

- 3. Use an inverse matrix to solve the system.

$$13y - 6x = -81$$

$$7x + 17 = -22y$$

- 4. Sketch a graph of vectors to visually find the solution to the system.

$$3x = 3$$

$$x - y = -2$$



■ 5. Sketch a graph of vectors to visually find the solution to the system.

$$-y = -4$$

$$2x - y = -2$$

■ 6. Sketch a graph of vectors to visually find the solution to the system.

$$x - y = 0$$

$$x + y = 2$$



SOLVING SYSTEMS WITH CRAMER'S RULE

- 1. Use Cramer's rule to find the expression that would give the solution for x . You do not need to solve the system.

$$2x - y = 5$$

$$x + 3y = 15$$

- 2. Use Cramer's rule to find the expression that would give the solution for x . You do not need to solve the system.

$$ax + by = e$$

$$cx + dy = f$$

- 3. Use Cramer's rule to find the expression that would give the solution for y . You do not need to solve the system.

$$3x + 4y = 11$$

$$2x - 3y = -4$$

- 4. Use Cramer's rule to find the expression that would give the solution for y . You do not need to solve the system.



$$ax + by = e$$

$$cx + dy = f$$

■ 5. Use Cramer's rule to solve for x .

$$3x + 2y = 1$$

$$6x + 5y = 4$$

■ 6. Use Cramer's rule to solve for y .

$$3x + 2y = 1$$

$$6x + 5y = 4$$

■ 7. Use Cramer's rule to solve for x .

$$3x + 5y = 6$$

$$9x + 10y = 14$$

■ 8. Use Cramer's rule to solve for y .

$$3x + 5y = 6$$

$$9x + 10y = 14$$



