Topic: Equation of a parabolic conic section

Question: A parametric curve is defined by the equations $x = 4\sqrt{t}$ and $y = 24t$ within the interval $[0,\infty)$. Which statement describes the graph of this function?

Answer choices:

A The equations represent a half-parabola originating from the point $(6,0)$, and extending up into the first quadrant.

B The equations represent a half-parabola originating from the point $(0,6)$, and extending up into the second quadrant.

C The equations represent a half-parabola originating from the point $(0,0)$, and extending up into the second quadrant.

D The equations represent a half-parabola originating from the point $(0,0)$, and extending up into the first quadrant.

Solution: D

Square $x = 4\sqrt{t}$, and solve the result for t.

$$x^2 = \left(4\sqrt{t}\right)^2$$

$$x^2 = 16t$$

$$t = \frac{1}{16}x^2$$

Now, replace $t = (1/16)x^2$ in $y = 24t$.

$$y = 24\left(\frac{1}{16}x^2\right)$$

$$y = \frac{3}{2}x^2$$

Thus, the equations represent a half-parabola originating from the point $(0,0)$, and extending up above the x-axis and to the right of the y-axis.

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Question: For which values of m and n do the parametric equations $x = t - m$ and $y = 4t^2 - n$ represent the parabola $y = 4x^2 - 40x + 103$?

Answer choices:

A $m = 5$ and $n = 3$

B $m = -5$ and $n = -3$

C $m = -5$ and $n = 3$

D $m = 5$ and $n = -3$

Solution: B

Choosing $m = -5$ and $n = -3$ transforms the given parametric equations to the following forms:

$$x = t + 5$$

$$y = 4t^2 + 3$$

Eliminate 5 from the right side of $x = t + 5$, and then square the result.

$$x - 5 = t$$

$$(x - 5)^2 = t^2$$

Replace $(x - 5)^2 = t^2$ in $y = 4t^2 + 3$, and expand.

$$y = 4(x - 5)^2 + 3$$

$$y = 4\left(x^2 - 10x + 25\right) + 3$$

$$y = 4x^2 - 40x + 103$$

Topic: Equation of a parabolic conic section

Question: Which of the following parametric functions represents a full parabola?

Answer choices:

A $\sqrt[3]{x^2} = 3\sqrt[3]{t}$ and $y = 81t - 6$

B $x = 6\sqrt[4]{t}$ and $y = 4t - 5$

C $x = 12\sqrt{t}$ and $y = 2t - 3$

D $x = 4\sqrt{t} - 1$ and $y = 5t$

Solution: A

Assuming answer choice A is correct, solve $\sqrt[3]{x^2} = 3\sqrt[3]{t}$ for t.

$$\sqrt[3]{x^2} = 3\sqrt[3]{t}$$

$$\left(\sqrt[3]{x^2}\right)^3 = \left(3\sqrt[3]{t}\right)^3$$

$$x^2 = 27t$$

$$t = \frac{x^2}{27}$$

Replace $t = x^2/27$ in $y = 81t - 6$.

$$y = 81t - 6$$

$$y = 81\left(\frac{x^2}{27}\right) - 6$$

$$y = 3x^2 - 6$$

This function represents a graph within the domain $(-\infty, \infty)$.