Topic: Express the polar point multiple ways

Question: Convert the polar point to rectangular coordinates.

$$(r,\theta) = \left(-14, \frac{5\pi}{6}\right)$$

Answer choices:

A $(x,y) = \left(7, -7\sqrt{3}\right)$

B $(x,y) = \left(-\frac{7\sqrt{3}}{2}, 7\right)$

C $(x,y) = \left(7\sqrt{3}, -7\right)$

D $(x,y) = \left(\frac{7\sqrt{3}}{2}, -7\right)$

Solution: C

To convert a rectangular point to a polar point, we use the conversion formulas

$$x = r\cos\theta$$

$$y = r\cos\theta$$

Plugging the point into the conversion formulas give

$$x = -14\left(\cos\frac{5\pi}{6}\right)$$

$$y = -14\left(\sin\frac{5\pi}{6}\right)$$

We know that

$$\pi - \frac{5\pi}{6} = \frac{\pi}{6}$$

and by the reference angle theorem and the fact that an angle of measure $5\pi/6$ is in the second quadrant, we can say

$$\cos\left(\frac{5\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

Substituting these results, we get

$$x = -14\left(-\frac{\sqrt{3}}{2}\right) = 7\sqrt{3}$$

$$y = -14\left(\frac{1}{2}\right) = -7$$

Therefore, the point in rectangular coordinates is

$$\left(7\sqrt{3}, -7\right)$$

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Question: If one pair of polar coordinates of a point is $(-4.2, 41\pi/12)$, what is the pair of polar coordinates (r, θ) of that point if θ is in the interval $[-6\pi, -5\pi)$? Express θ in radians.

Answer choices:

A $(r, \theta) = (27.2, 4.08)$

B $(r, \theta) = (20.7, 54.1)$

C $(r, \theta) = \left(4.2, -\dfrac{67\pi}{12}\right)$

D $(r, \theta) = (13.6, 5.51)$

Solution: C

If (r, θ) is the pair of coordinates for this point in which θ is in the interval $[-6\pi, -5\pi)$, then θ must differ from $41\pi/12$ by $n\pi$ for some integer n. Note that

$$3\pi = \frac{36\pi}{12} < \frac{41\pi}{12} < \frac{48\pi}{12} = 4\pi$$

Now

$$-6\pi = 3\pi - 9\pi \text{ and } -5\pi = 4\pi - 9\pi$$

Therefore,

$$\theta = \frac{41\pi}{12} - 9\pi = \frac{1(41\pi) - 12(9\pi)}{12} = \frac{41\pi - 108\pi}{12} = -\frac{67\pi}{12}$$

Since 9 is an odd integer and the first coordinate in the given pair of polar coordinates of this point is -4.2 (hence negative), r must be positive (hence $r = 4.2$). Thus the indicated pair of polar coordinates of this point is $(4.2, -67\pi/12)$.

Topic: Express the polar point multiple ways

Question: Which of the following most closely approximates the polar coordinates (r,θ) of the point that has rectangular coordinates $(x,y)=(-9.6,-4.5)$ if θ is in the interval $[19\pi,20\pi)$? Express θ in radians.

Answer choices:

A $(r,\theta)=(13.9,57.0)$

B $(r,\theta)=(10.6,60.1)$

C $(r,\theta)=(-17.2,63.7)$

D $(r,\theta)=(-10.6,60.1)$

Solution: B

For the point P, both x and y are negative, so P is in the third quadrant. First, we'll find the basic polar coordinates of P.

Since the first of the two basic polar coordinates of any point other than the pole must be positive, and the second of its two basic polar coordinates must be an angle whose terminal side is in the same quadrant as the point itself, the basic polar coordinates of P are (s, α) for some positive number s and some angle α in the interval $[\pi, 3\pi/2)$.

In general, we know that

$$s = \sqrt{x^2 + y^2}$$

$$s = \sqrt{(-9.6)^2 + (-4.5)^2}$$

$$s = \sqrt{92.16 + 20.25}$$

$$s = \sqrt{112.41}$$

$$s \approx 10.6$$

Recall that the fact that α is in the third quadrant tells us that

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right) + \pi$$

With the help of a calculator, we find that

$$\frac{y}{x} = \frac{-4.5}{-9.6} = \frac{4.5}{9.6} \approx 0.469$$

and so

$$\tan^{-1}\left(\frac{y}{x}\right) \approx \tan^{-1}(0.469) \approx 0.439 \text{ radians}$$

So

$$\alpha \approx (0.439 + \pi) \text{ radians} \approx 3.58 \text{ radians}$$

Since the interval $[\pi, 3\pi/2)$ is a subset of the interval $[\pi, 2\pi)$, α is in the interval $[\pi, 2\pi)$. Note that

$$19\pi = \pi + 18\pi \text{ and } 20\pi = 2\pi + 18\pi$$

Therefore,

$$\theta = \alpha + 18\pi \approx (3.58 + 18\pi) \approx 60.1 \text{ radians}$$

Since 18 is even, we know that r must be of the same sign as s (hence positive), and thus that $r = s$ (hence $r \approx 10.6$).