Topic: Even-odd identities

Question: Use even-odd identities to find the values of $\sin(-855^\circ)$ and $\cos(-855^\circ)$.

Answer choices:

A $\sin(-855^\circ) = \frac{\sqrt{3}}{2}$ $\cos(-855^\circ) = -\frac{1}{2}$

B $\sin(-855^\circ) = -\frac{\sqrt{2}}{2}$ $\cos(-855^\circ) = \frac{\sqrt{2}}{2}$

C $\sin(-855^\circ) = \frac{1}{2}$ $\cos(-855^\circ) = -\frac{\sqrt{3}}{2}$

D $\sin(-855^\circ) = -\frac{\sqrt{2}}{2}$ $\cos(-855^\circ) = -\frac{\sqrt{2}}{2}$

Solution: D

Let's find an angle within $[0^\circ,360^\circ)$ that's coterminal with 855°. We're using 855° instead of -855° because using the even-odd identities tell us that

$$\sin(-855^\circ)=-\sin(855^\circ)$$

$$\cos(-855^\circ)=\cos(855^\circ)$$

and we'll deal with the negative sign using even-odd identities later.

$$855^\circ-360^\circ=495^\circ$$

$$495^\circ-360^\circ=135^\circ$$

From the unit circle we know the sine and cosine of 135°.

$$\sin(135^\circ)=\frac{\sqrt{2}}{2}$$

$$\cos(135^\circ)=-\frac{\sqrt{2}}{2}$$

Because 855° is coterminal with 135°, these two angles will have equal sine and cosine values, so

$$\sin(855^\circ)=\frac{\sqrt{2}}{2}$$

$$\cos(855^\circ)=-\frac{\sqrt{2}}{2}$$

Substituting the values we already found for $\sin(855^\circ)$ and $\cos(855^\circ)$ into the right sides of the even-odd equations, we get

$$\sin(-855^\circ) = -\frac{\sqrt{2}}{2}$$

$$\cos(-855^\circ) = -\frac{\sqrt{2}}{2}$$

Topic: Even-odd identities

Question: Use reference angles and even-odd identities to find the values of tangent and secant at $\theta = -5\pi/6$.

Answer choices:

A $\tan\left(-\frac{5\pi}{6}\right) = \sqrt{3}$ $\sec\left(-\frac{5\pi}{6}\right) = \frac{2\sqrt{3}}{3}$

B $\tan\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3}$ $\sec\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

C $\tan\left(-\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{3}$ $\sec\left(-\frac{5\pi}{6}\right) = -\frac{2\sqrt{3}}{3}$

D $\tan\left(-\frac{5\pi}{6}\right) = -\sqrt{3}$ $\sec\left(-\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Solution: C

Since the even-odd identities for sine and cosine are

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

we can find the reference angle for $5\pi/6$. The angle $5\pi/6$ is in the second quadrant and has a reference angle of $\pi/6$. We could also consider the angle $\pi/6$ in the first quadrant that has the same reference angle of $\pi/6$.

Remember that sine and the cosine are positive in the first quadrant, and sine is positive while cosine is negative in the second quadrant. Therefore, the reference angle theorem tells us that

$$\cos\left(\frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{5\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

Then by the even-odd identities for sine and cosine,

$$\cos\left(-\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{5\pi}{6}\right) = -\frac{1}{2}$$

Then tangent and secant of $\theta = -5\pi/6$ must be

$$\tan\left(-\frac{5\pi}{6}\right)=\frac{\sin\left(-\frac{5\pi}{6}\right)}{\cos\left(-\frac{5\pi}{6}\right)}=\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\sec\left(-\frac{5\pi}{6}\right)=\frac{1}{\cos\left(-\frac{5\pi}{6}\right)}=\frac{1}{-\frac{\sqrt{3}}{2}}=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

Topic: Even-odd identities

Question: Find the values of cosecant and tangent at $\theta = -49\pi/3$.

Answer choices:

A $\csc\left(-\frac{49\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$ $\tan\left(-\frac{49\pi}{3}\right) = -\sqrt{3}$

B $\csc\left(-\frac{49\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\tan\left(-\frac{49\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

C $\csc\left(-\frac{49\pi}{3}\right) = \frac{\sqrt{3}}{2}$ $\tan\left(-\frac{49\pi}{3}\right) = -\sqrt{3}$

D $\csc\left(-\frac{49\pi}{3}\right) = \frac{2\sqrt{3}}{3}$ $\tan\left(-\frac{49\pi}{3}\right) = \frac{\sqrt{3}}{3}$

Solution: A

Since the cosine function is even and the sine function is odd, we can say

$$\cos\left(-\frac{49\pi}{3}\right)=\cos\left(\frac{49\pi}{3}\right)$$

$$\sin\left(-\frac{49\pi}{3}\right)=-\sin\left(\frac{49\pi}{3}\right)$$

Then to find the cosine of this positive angle, we'll get the coterminal angle for $49\pi/3$ by dividing it by 2π.

$$\frac{\frac{49\pi}{3}}{2\pi}=\frac{49\pi}{3(2\pi)}=\frac{49}{6}\approx 8.167$$

So $\theta=49\pi/3$ is 8 full positive rotations, plus a little bit more. If we take away 8 full rotations, or 16π, from $\theta=49\pi/3$, we're left with

$$\frac{49\pi}{3}-16\pi=\frac{49\pi}{3}-\frac{48\pi}{3}=\frac{\pi}{3}$$

So $\theta=49\pi/3$ is coterminal with $\alpha=\pi/3$, which means the sine of these angles are equal, and the cosine of these angles are equal.

$$\sin\left(\frac{49\pi}{3}\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{49\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$$

Apply even-odd identities and reciprocal identities to find cosecant and tangent of $\theta=-49\pi/3$.

$$\csc\left(-\frac{49\pi}{3}\right) = -\csc\left(\frac{49\pi}{3}\right) = -\frac{1}{\sin\left(\frac{49\pi}{3}\right)} = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\tan\left(-\frac{49\pi}{3}\right) = -\tan\left(\frac{49\pi}{3}\right) = -\frac{\sin\left(\frac{49\pi}{3}\right)}{\cos\left(\frac{49\pi}{3}\right)} = -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$