Topic: Coterminal angles in a particular interval

Question: Which angle in the interval $(-3\pi/5, 7\pi/5]$ is coterminal with $67\pi/5$?

Answer choices:

A $-\frac{2\pi}{5}$

B $-\frac{3\pi}{5}$

C $\frac{7\pi}{5}$

D $-\frac{\pi}{5}$

Solution: C

We'll let $\theta = 67\pi/5$ and α be the angle within $(-3\pi/5, 7\pi/5]$ that's coterminal with θ. We'll use $\alpha = \theta + n(2\pi)$ and solve for the value of n that makes α lie in that interval.

$$-3\pi/5 < \alpha \leq 7\pi/5$$

$$-3\pi/5 < \theta + n(2\pi) \leq 7\pi/5$$

$$-\frac{3\pi}{5} < \frac{67\pi}{5} + n(2\pi) \leq \frac{7\pi}{5}$$

$$-\frac{70\pi}{5} < n(2\pi) \leq -\frac{60\pi}{5}$$

$$-14\pi < n(2\pi) \leq -12\pi$$

$$-7 < n \leq -6$$

Because n has to be an integer, we know $n = -6$. To find α, we'll substitute $n = -6$ into $\theta + n(2\pi)$.

$$\alpha = \frac{67\pi}{5} + (-6)(2\pi)$$

$$\alpha = \frac{67\pi}{5} - \frac{60\pi}{5}$$

$$\alpha = \frac{7\pi}{5}$$

Topic: Coterminal angles in a particular interval

Question: Which angle in the interval $(380^\circ, 740^\circ]$ is coterminal with 145°?

Answer choices:

A 380°

B -215°

C 740°

D 505°

Solution: D

The interval $(380^\circ, 740^\circ]$ is a full 360° rotation. Notice how, because we have a parenthesis around the 380° and a bracket around the 740°, it means that the angle 380° exactly isn't included in the interval, but the angle 740° exactly *is* included.

We'll let $\theta = 145^\circ$, and then we'll say that α is the coterminal angle that lies within $(380^\circ, 740^\circ]$. Then we can say

$$380^\circ < \alpha \leq 740^\circ$$

But since α is coterminal with θ, we substitute $\alpha = \theta + n(360^\circ)$ into the inequality.

$$380^\circ < \theta + n(360^\circ) \leq 740^\circ$$

$$380^\circ < 145^\circ + n(360^\circ) \leq 740^\circ$$

$$235^\circ < n(360^\circ) \leq 595^\circ$$

$$0.65 < n \leq 1.65$$

Remember, n has to be an integer, which means $n = 1$. And therefore, to find α, we'll substitute $n = 1$ into $\alpha = \theta + n(360^\circ)$.

$$\alpha = 145^\circ + 1(360^\circ)$$

$$\alpha = 145^\circ + 360^\circ$$

$$\alpha = 505^\circ$$

Topic: Coterminal angles in a particular interval

Question: Which angle in the interval $[25\pi/4, 33\pi/4)$ is coterminal with $-33\pi/4$?

Answer choices:

A $\frac{31\pi}{4}$

B $\frac{25\pi}{4}$

C $\frac{23\pi}{4}$

D $\frac{33\pi}{4}$

Solution: A

We'll let $\theta = -33\pi/4$ and α be the angle within $[25\pi/4, 33\pi/4)$ that's coterminal with θ. We'll use $\alpha = \theta + n(2\pi)$ and solve for the value of n that makes α lie in that interval.

$$\frac{25\pi}{4} \le \alpha < \frac{33\pi}{4}$$

$$\frac{25\pi}{4} \le \theta + n(2\pi) < \frac{33\pi}{4}$$

$$\frac{25\pi}{4} \le -\frac{33\pi}{4} + n(2\pi) < \frac{33\pi}{4}$$

$$\frac{29\pi}{2} \le n(2\pi) < \frac{33\pi}{2}$$

$$7.25 \le n < 8.25$$

Because n has to be an integer, we know $n = 8$. To find α, we'll substitute $n = 8$ into $\theta + n(2\pi)$.

$$\alpha = \frac{-33\pi}{4} + 8(2\pi)$$

$$\alpha = -\frac{33\pi}{4} + \frac{64\pi}{4}$$

$$\alpha = \frac{31\pi}{4}$$