Topic: Complex numbers in polar form

Question: If the complex number $-3 - 7i$ is expressed in polar form, which quadrant contains the angle θ?

Answer choices:

A In the first quadrant

B On the negative vertical axis

C In the third quadrant

D On the positive horizontal axis

Solution: C

If we set the complex number equal to its polar form, we get

$$-3 - 7i = r(\cos\theta + i\sin\theta)$$

$$-3 - 7i = r\cos\theta + ri\sin\theta$$

From this equation, we know that

$$-3 = r\cos\theta$$

$$\cos\theta = -\frac{3}{r}$$

The value of r is always positive, since r represents a distance, so $-3/r$ has to be less than 0, which means $\cos\theta$ has to be negative.

We also know from $-3 - 7i = r\cos\theta + ri\sin\theta$ that

$$-7 = r\sin\theta$$

$$\sin\theta = -\frac{7}{r}$$

Because the value of r is always positive, $-7/r$ has to be less than 0, which means $\sin\theta$ has to be negative.

The values of $\cos\theta$ and $\sin\theta$ are negative in the third quadrant.

Topic: Complex numbers in polar form

Question: What is the polar form of the complex number?

$$-4+6i$$

Answer choices:

A $2\sqrt{5}\left[\cos(0.98)+i\sin(0.98)\right]$

B $2\sqrt{13}\left[\cos(2.16)+i\sin(2.16)\right]$

C $2\sqrt{13}\left[\cos(5.30)+i\sin(5.30)\right]$

D $2\sqrt{5}\left[\cos(3.14)+i\sin(3.14)\right]$

Solution: B

If we write the complex number $-4+6i$ as $a+bi$, we get $a=-4$ and $b=6$, so

$$r=\sqrt{a^2+b^2}=\sqrt{(-4)^2+6^2}=\sqrt{16+36}=\sqrt{52}=\sqrt{4(13)}=2\sqrt{13}$$

and

$$\tan\theta=\frac{b}{a}=\frac{6}{-4}=-\frac{3}{2}$$

Using the trigonometric identity $\sec^2\theta=1+\tan^2\theta$, we get

$$\sec^2\theta=1+\left(-\frac{3}{2}\right)^2=1+\frac{9}{4}=\frac{4(1)+1(9)}{4}=\frac{13}{4}$$

$$\cos^2\theta=(\cos\theta)^2=\left(\frac{1}{\sec\theta}\right)^2=\frac{1}{\sec^2\theta}=\frac{1}{\left(\frac{13}{4}\right)}=\frac{4}{13}$$

So

$$\cos\theta=\pm\sqrt{\frac{4}{13}}=\pm\frac{2}{\sqrt{13}}$$

The real part of z is $a=-4$, and the imaginary part is $b=6$, which puts the complex number in the second quadrant. Since the cosine of every angle in the second quadrant is negative, we get

$$\cos\theta=-\frac{2}{\sqrt{13}}$$

$$\arccos(\cos\theta) = \arccos\left(-\frac{2}{\sqrt{13}}\right)$$

$$\theta = \arccos\left(-\frac{2}{\sqrt{13}}\right)$$

$\theta \approx 2.16$ radians

Substituting the values of r and θ into the polar form for a complex number, we get

$$r(\cos\theta + i\sin\theta)$$

$$2\sqrt{13}\left[\cos(2.16) + i\sin(2.16)\right]$$

Topic: Complex numbers in polar form

Question: Write the complex number in polar form.

$$-14i$$

Answer choices:

A $\quad -14\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$

B $\quad -14\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

C $\quad 14\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

D $\quad 14\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$

Solution: D

The complex number $-14i$ can be written as $0 - 14i$, so its real part is 0, which means the number is located on the imaginary axis. Because $a = 0$ and $b = -14$, the distance of $0 - 14i$ from the origin is

$$r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (-14)^2} = \sqrt{0 + 196} = \sqrt{196} = 14$$

Since the imaginary part of $0 - 14i$ is -14, which is negative, $0 - 14i$ is located on the negative imaginary axis, so $\theta = 3\pi/2$. In polar form, we get

$$r(\cos\theta + i\sin\theta)$$

$$14\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$