Relating linear and angular velocity

Now we're ready to discuss linear velocity in connection with angular velocity. To relate them to each other, we can use the arc length formula that we learned before, $s = r\theta$. Because $s = r\theta$, we'll substitute $r\theta$ into the linear velocity formula for s.

$$v = \frac{s}{t} = r\frac{\theta}{t}$$

Then, because we know that angular velocity is $\omega = \theta/t$, we can replace the θ/t in this linear velocity formula with ω.

$$v = \frac{s}{t} = r\frac{\theta}{t} = r\omega$$

So now we have a formula relating linear velocity directly to angular velocity, which tells us that linear velocity is equivalent to the product of the length of the radius and angular velocity,

$$v = r\omega$$

or that angular velocity is the quotient of linear velocity and the radius of the circle.

$$\omega = \frac{v}{r}$$

Let's do an example where we calculate linear velocities when we know the angular velocity.

Example

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What are the linear velocities, in inches per second, of points on a rotating disc that has an angular velocity of 9.4 radians per second if those points are located 1.3 and 2.6 inches, respectively, from the center of the disc?

For the point that's 1.3 inches from the center of the disc, we'll say $r = 1.3$ and $\omega = 9.4$.

$$v = r\omega$$

$$v = (1.3 \text{ inches})\left(\frac{9.4}{\text{second}}\right)$$

$$v = 1.3(9.4) \text{ inches per second}$$

$$v = 12.22 \text{ inches per second}$$

For the point that's 2.6 inches from the center of the disc, $r = 2.6$ but ω is still 9.4.

$$v = r\omega$$

$$v = (2.6 \text{ inches})\left(\frac{9.4}{\text{second}}\right)$$

$$v = 2.6(9.4) \text{ inches per second}$$

$$v = 24.44 \text{ inches per second}$$

Notice that the linear velocity of the second point is double the linear velocity of the first point. That's because the arc traced out by the second point is twice as long as the arc traced out by the first point.

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Sometimes we'll have to convert units before we can solve the problem.

Example

What is the linear velocity, in inches per second, of a point that's 5.6 centimeters from the center of the disc if the disc is rotating at 42.8 revolutions per minute?

We'll need to convert the lengths from centimeters to inches, and we'll need to convert the angular velocity from revolutions per minute to radians per second.

For the first conversion, we'll use the fact that 1 inch is about 2.54 centimeters.

$$r = 5.6 \text{ cm}$$

$$r \approx (5.6 \text{ cm})\left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)$$

$$r \approx \frac{5.6}{2.54} \text{ in}$$

$$r \approx 2.20 \text{ in}$$

To express the angular velocity in radians per second, we'll get

$$\omega = 42.8 \frac{\text{rev}}{\text{min}}$$

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$$\omega = \left(42.8\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)$$

$$\omega = \frac{42.8(2)\pi}{60} \text{ radians per second}$$

$$\omega \approx 1.43\pi \text{ radians per second}$$

Now we'll combine these two results to get the linear speed.

$$v = r\omega$$

$$v \approx (2.20\text{in})\left(\frac{1.43\pi}{\text{sec}}\right)$$

$$v \approx 2.20(1.43\pi) \text{ inches per second}$$

$$v \approx 9.88 \text{ inches per second}$$

Let's do an example where we asked to solve for an unknown central angle.

Example

An object moves at a constant linear speed of 15 m/sec around a circle of radius 5 m. How large of a central angle does it sweep out in 2.5 seconds?

The formula for linearly velocity is $v = r\omega$, and we know that $\omega = \theta/t$, so

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$$v = \frac{r\theta}{t}$$

Solve this equation for θ, then substitute the values from the question.

$$\theta = \frac{vt}{r} = \frac{15 \cdot 2.5}{5} = 7.5 \text{ radians}$$

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