Topic: Vertex, axis, focus, directrix of a hyperbola

Question: Which set of parametric equations defines the hyperbola with the given vertices?

$$\left(\sqrt{10},0\right)$$

$$\left(-\sqrt{10},0\right)$$

Answer choices:

A $x=2t+\frac{1}{5t}$ and $y=5t-\frac{1}{2t}$

B $x=5t+\frac{1}{2t}$ and $y=6t-\frac{1}{2t}$

C $x=5t+\frac{1}{2t}$ and $y=2t-\frac{1}{2t}$

D $x=5t+\frac{1}{2t}$ and $y=5t-\frac{1}{2t}$

Solution: D

Choose the equations from answer choice D,

$$x = 5t + \frac{1}{2t}$$

$$y = 5t - \frac{1}{2t}$$

Square both sides of each equation, and simplify.

$$x = 5t + \frac{1}{2t}$$

$$x^2 = \left(5t + \frac{1}{2t}\right)^2$$

$$x^2 = 25t^2 + \frac{1}{4t^2} + 5$$

and

$$y = 5t - \frac{1}{2t}$$

$$y^2 = \left(5t - \frac{1}{2t}\right)^2$$

$$y^2 = 25t^2 + \frac{1}{4t^2} - 5$$

Subtract the equation for y^2 from the equation for x^2.

$$x^2 - y^2 = \left(25t^2 + \frac{1}{4t^2} + 5\right) - \left(25t^2 + \frac{1}{4t^2} - 5\right)$$

$$x^2 - y^2 = 10$$

Therefore, the vertices of the hyperbola are at $\left(\sqrt{10},0\right)$ and $\left(-\sqrt{10},0\right)$.

Topic: Vertex, axis, focus, directrix of a hyperbola

Question: A hyperbola is defined by the given functions. Where are the foci of the hyperbola?

$$x = \frac{m}{\cos\theta} - 2 \text{ and } y = n\tan\theta - 3$$

Answer choices:

A $\left(-2+\sqrt{m^2+n^2}, -3\right)$ and $\left(-2-\sqrt{m^2+n^2}, -3\right)$

B $\left(3+\sqrt{m^2+n^2}, 2\right)$ and $\left(3-\sqrt{m^2+n^2}, 2\right)$

C $\left(-2+\sqrt{m^2-n^2}, -3\right)$ and $\left(-2-\sqrt{m^2-n^2}, -3\right)$

D $\left(4+\sqrt{m^2+n^2}, 3\right)$ and $\left(4-\sqrt{m^2+n^2}, 3\right)$

Solution: A

Rewrite each of the given equations.

$$x = \frac{m}{\cos\theta} - 2$$

$$x + 2 = \frac{m}{\cos\theta}$$

$$\frac{(x+2)^2}{m^2} = \frac{1}{\cos^2\theta}$$

$$\frac{(x+2)^2}{m^2} = 1 + \tan^2\theta$$

and

$$y = n\tan\theta - 3$$

$$y + 3 = n\tan\theta$$

$$\frac{y+3}{n} = \tan\theta$$

$$\frac{(y+3)^2}{n^2} = \tan^2\theta$$

Subtract this second equation from the first.

$$\frac{(x+2)^2}{m^2} - \frac{(y+3)^2}{n^2} = 1 + \tan^2\theta - \tan^2\theta$$

$$\frac{(x+2)^2}{m^2} - \frac{(y+3)^2}{n^2} = 1$$

Therefore, the foci are at

$$\left(-2+\sqrt{m^2+n^2}, -3\right)$$

and

$$\left(-2-\sqrt{m^2+n^2}, -3\right)$$

Topic: Vertex, axis, focus, directrix of a hyperbola

Question: Which hyperbola opens left and right and has its axes at $x = -4$ and $y = -3$s?

Answer choices:

A $\quad x = \frac{7}{\cos t} - 4 \qquad y = 5\tan t - 3$

B $\quad x = \frac{5}{\cos t} - 4 \qquad y = 7\tan t - 3$

C $\quad x = \frac{3}{\cos t} + 4 \qquad y = 4\tan t - 3$

D $\quad x = \frac{6}{\cos t} - 4 \qquad y = 6\tan t + 3$

Solution: A

Check answer choice A by rewriting both equations.

$$x = \frac{7}{\cos t} - 4$$

$$x + 4 = \frac{7}{\cos t}$$

$$\frac{x+4}{7} = \frac{1}{\cos t}$$

$$\frac{(x+4)^2}{7^2} = \frac{1}{\cos^2 t}$$

$$\frac{(x+4)^2}{7^2} = \sec^2 t$$

$$\frac{(x+4)^2}{7^2} = 1 + \tan^2 t$$

and

$$y = 5\tan t - 3$$

$$y + 3 = 5\tan t$$

$$\frac{y+3}{5} = \tan t$$

$$\frac{(y+3)^2}{5^2} = \tan^2 t$$

Subtract this second equation from the first equation we found.

$$\frac{(x+4)^2}{7^2} - \frac{(y+3)^2}{5^2} = 1 + \tan^2 t - \tan^2 t$$

$$\frac{(x+4)^2}{7^2} - \frac{(y+3)^2}{5^2} = 1$$

The axes of this hyperbola are $x = -4$ and $y = -3$, which are the axes we're looking for, so answer choice A must be the correct choice.

Answer choice B also has its axes at $x = -4$ and $y = -3$, but it opens up and down.

$$\frac{(x+4)^2}{5^2} - \frac{(y+3)^2}{7^2} = 1$$

Answer choice C has its axes at $x = 4$ and $y = -3$, and it opens left and right.

$$\frac{(x-4)^2}{3^2} - \frac{(y+3)^2}{4^2} = 1$$

Answer choice D has its axes at $x = -4$ and $y = 3$, and it opens left and right.

$$\frac{(x+4)^2}{6^2} - \frac{(y-3)^2}{6^2} = 1$$