Precalculus Formulas

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Conversion between cartesian and polar coordinates

$$x = r\cos\theta \qquad y = r\sin\theta \qquad x^2 + y^2 = r^2$$

Eccentricity

Let F be a fixed point (called the focus) and l be a fixed line (called the directrix) in a plane. Let e be a fixed positive number (called the eccentricity). The set of all points P in the plane such that

$$e = \frac{|PF|}{|Pl|}$$

(that is, the ratio of the distance from F to the distance from l is the constant e) is a conic section. The conic is

an ellipse if $e < 1$

a parabola if $e = 1$

a hyperbola if $e > 1$

Polar equation of the conic section

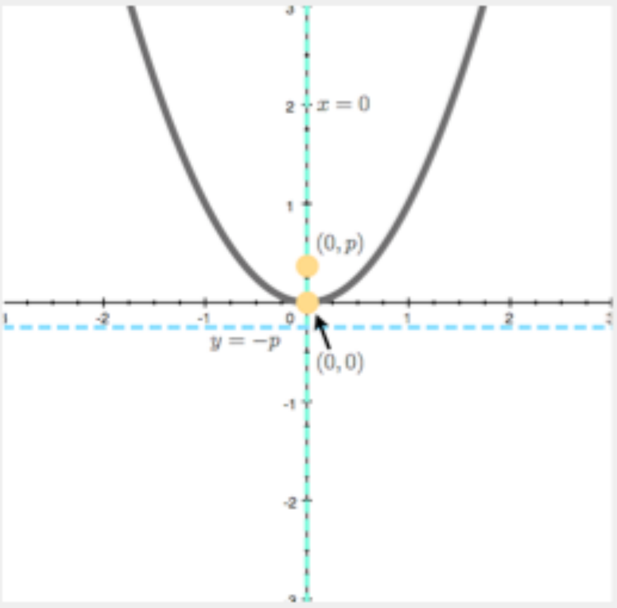
The polar equation of the conic section is

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad \text{or} \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

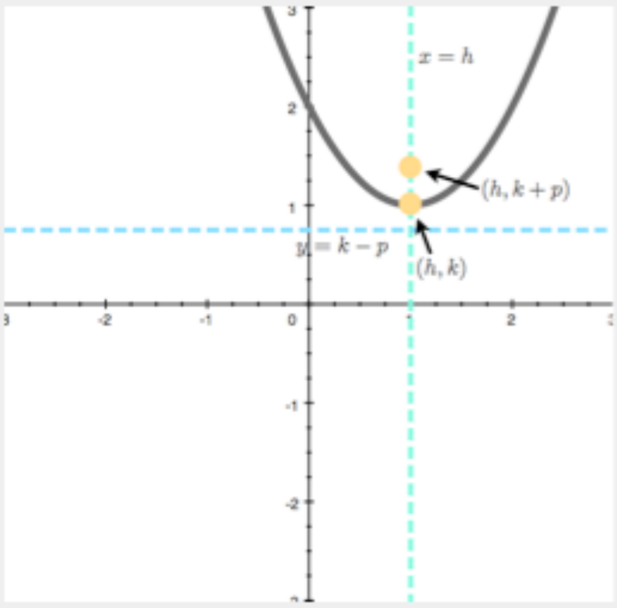
Analytic geometry of the parabola

	Equation	Vertex	Axis	Focus	Directrix
Parabolas with vertex at the origin					
Opens up	$x^2 = 4py$	$(0,0)$	$x = 0$	$(0,p)$	$y = -p$
Opens down	$x^2 = -4py$	$(0,0)$	$x = 0$	$(0,-p)$	$y = p$
Opens right	$y^2 = 4px$	$(0,0)$	$y = 0$	$(p,0)$	$x = -p$
Opens left	$y^2 = -4px$	$(0,0)$	$y = 0$	$(-p,0)$	$x = p$
Shifted parabolas with vertex off the origin					
Opens up	$(x-h)^2 = 4p(y-k)$	(h,k)	$x = h$	$(h,k+p)$	$y = k-p$
Opens down	$(x-h)^2 = -4p(y-k)$	(h,k)	$x = h$	$(h,k-p)$	$y = k+p$
Opens right	$(y-k)^2 = 4p(x-h)$	(h,k)	$y = k$	$(h+p,k)$	$x = h-p$
Opens left	$(y-k)^2 = -4p(x-h)$	(h,k)	$y = k$	$(h-p,k)$	$x = h+p$

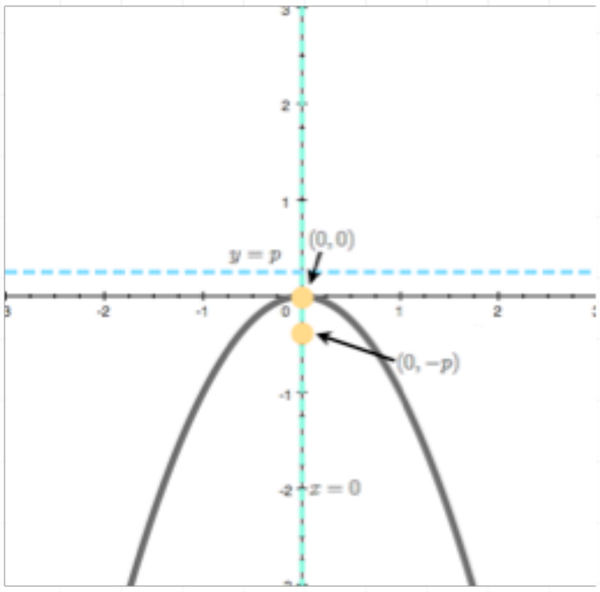
parabola (up)



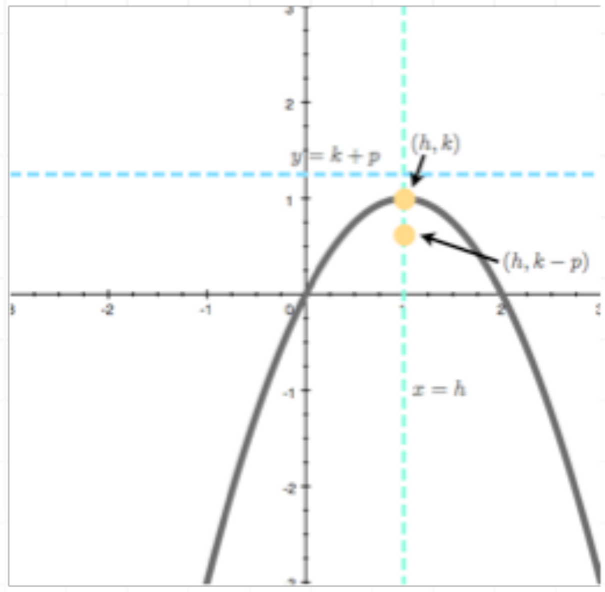
shifted parabola (up)



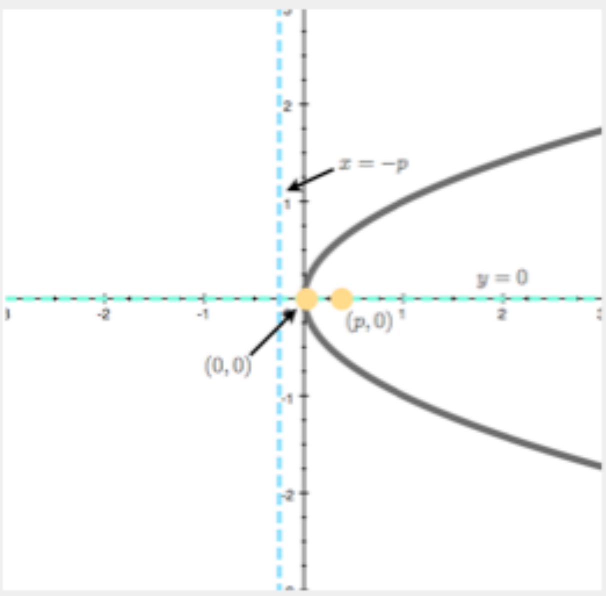
parabola (down)



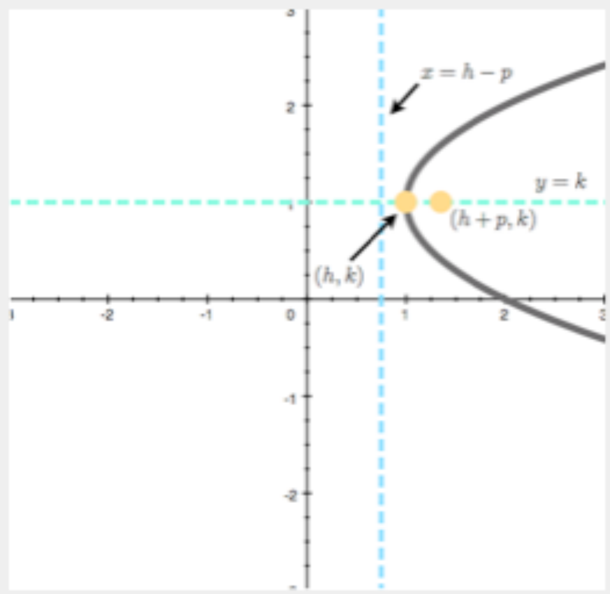
shifted parabola (down)



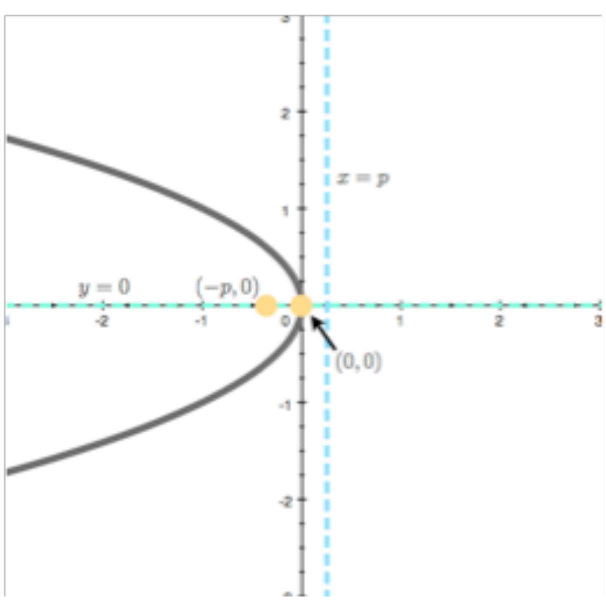
parabola (right)



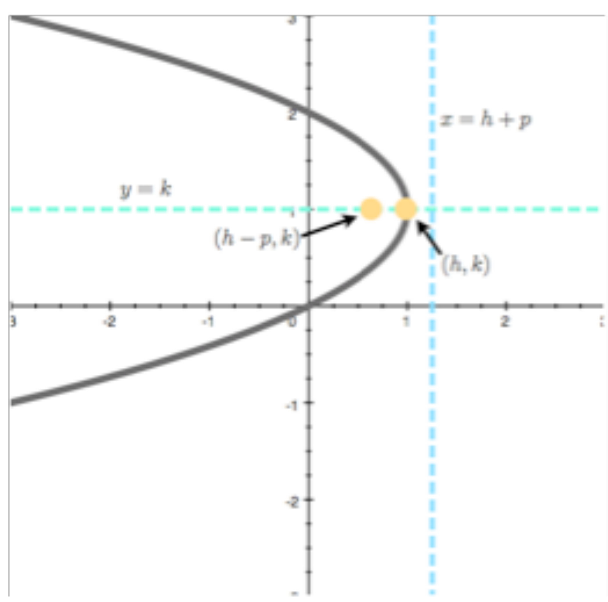
shifted parabola (right)



parabola (left)



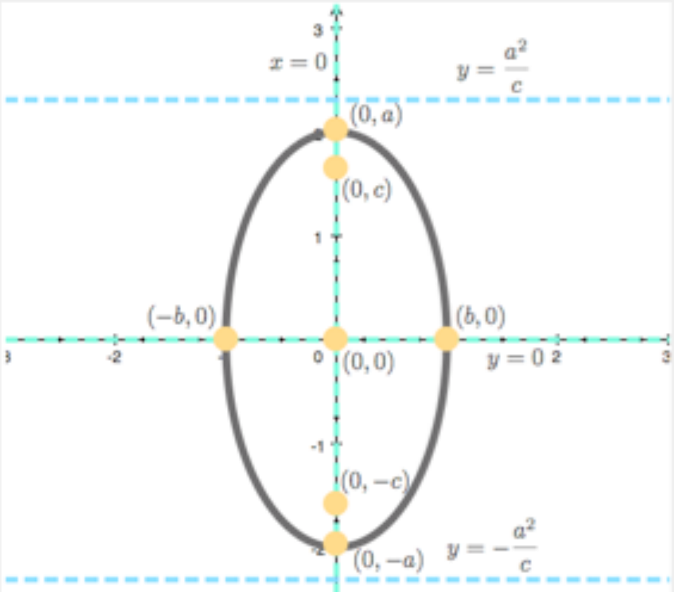
shifted parabola (left)



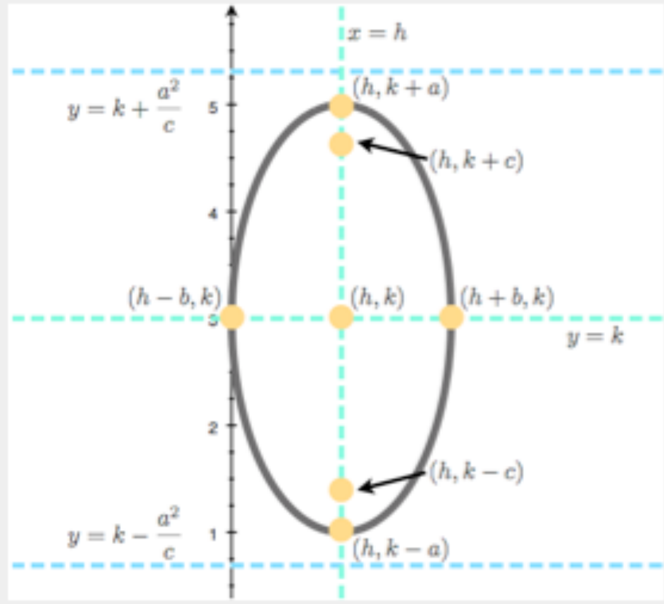
Analytic geometry of the ellipse

	Center at the origin $(0,0)$		Center off the origin at (h,k)	
	Tall	Wide	Tall	Wide
	$a \geq b > 0$	$a \geq b > 0$	$a \geq b > 0$	$a \geq b > 0$
Equation	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
Vertices	$(0, \pm a)$	$(\pm a, 0)$	$(h, k \pm a)$	$(h \pm a, k)$
	$(\pm b, 0)$	$(0, \pm b)$	$(h \pm b, k)$	$(h, k \pm b)$
Axes	Major: $x = 0$	Major: $y = 0$	Major: $x = h$	Major: $y = k$
	Minor: $y = 0$	Minor: $x = 0$	Minor: $y = k$	Minor: $x = h$
Foci	$(0, \pm c)$ with	$(\pm c, 0)$ with	$(h, k \pm c)$ with	$(h \pm c, k)$ with
	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$
Directrices	$y = \pm \frac{a^2}{c}$	$x = \pm \frac{a^2}{c}$	$y = k \pm \frac{a^2}{c}$	$x = h \pm \frac{a^2}{c}$

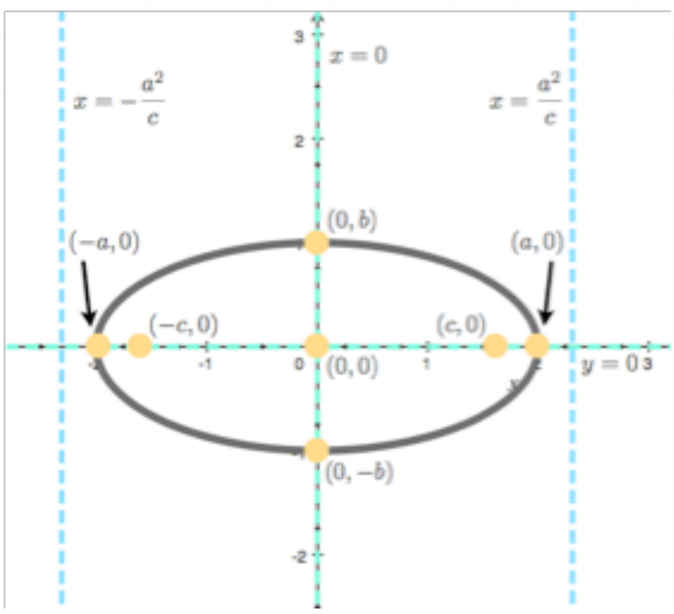
ellipse (tall)



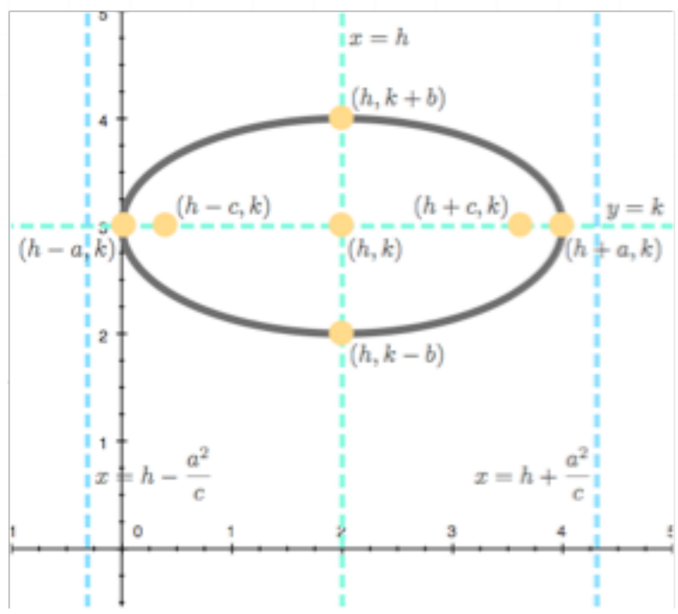
shifted ellipse (tall)



ellipse (wide)



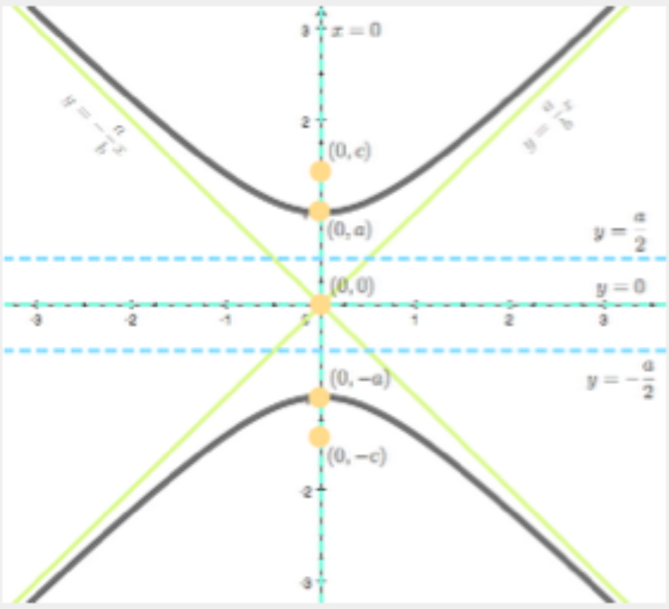
shifted ellipse (wide)



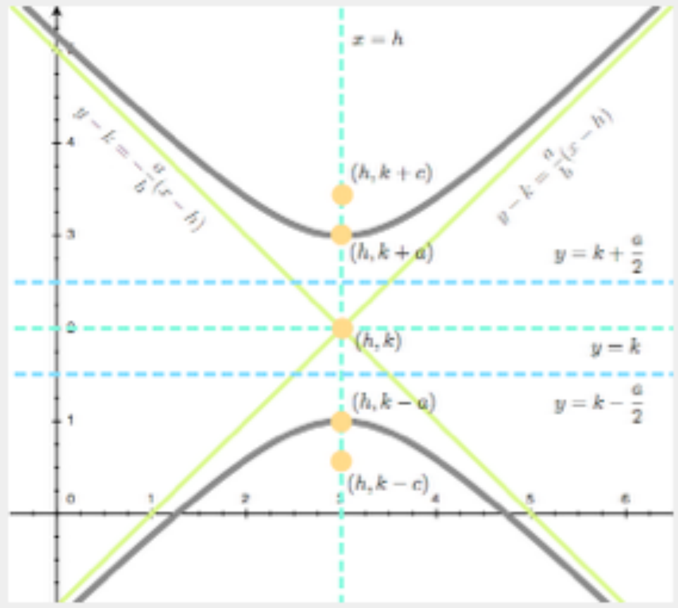
Analytic geometry of the hyperbola

	Center at the origin $(0,0)$		*Center off the origin at* (h,k)	
	Up/down	Left/right	Up/down	Left/right
Equation	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$
Vertices	$(0,\pm a)$	$(\pm a,0)$	$(h,k\pm a)$	$(h\pm a,k)$
Axes	Major: $x=0$	Major: $y=0$	Major: $x=h$	Major: $y=k$
	Minor: $y=0$	Minor: $x=0$	Minor: $y=k$	Minor: $x=h$
Foci	$(0,\pm c)$ with	$(\pm c,0)$ with	$(h,k\pm c)$ with	$(h\pm c,k)$ with
	$c^2=a^2+b^2$	$c^2=a^2+b^2$	$c^2=a^2+b^2$	$c^2=a^2+b^2$
Directrices	$y=\pm\frac{a}{2}$	$x=\pm\frac{a}{2}$	$y=k\pm\frac{a}{2}$	$x=h\pm\frac{a}{2}$
Asymptotes	$y=\pm\frac{a}{b}x$	$y=\pm\frac{b}{a}x$	$y-k=\pm\frac{a}{b}(x-h)$	$y-k=\pm\frac{b}{a}(x-h)$

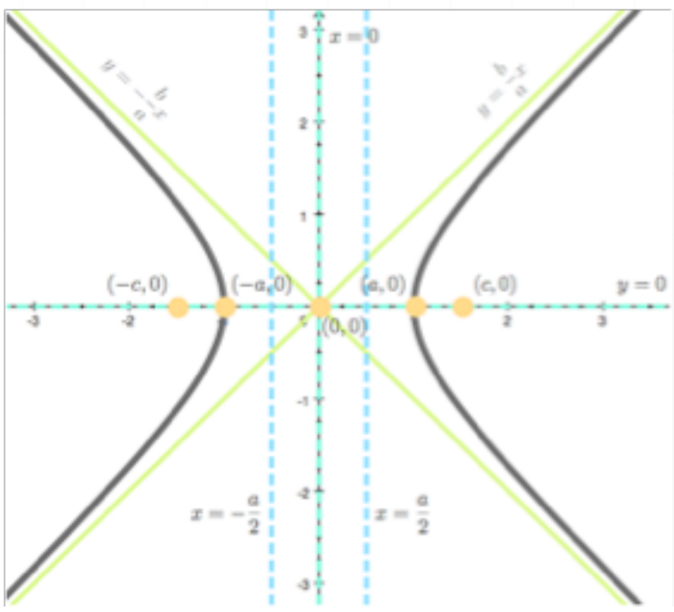
hyperbola (up-down)



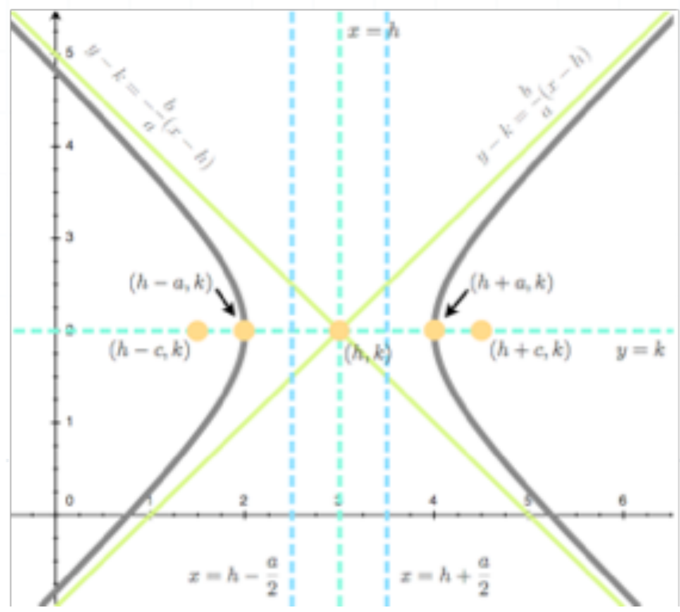
shifted hyperbola (up-down)



hyperbola (right-left)



shifted hyperbola (right-left)



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