



Trigonometry Workbook

The unit circle

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MATH

THE UNIT CIRCLE

■ 1. What is the coordinate point associated with $\theta = 2\pi/3$ along the unit circle?

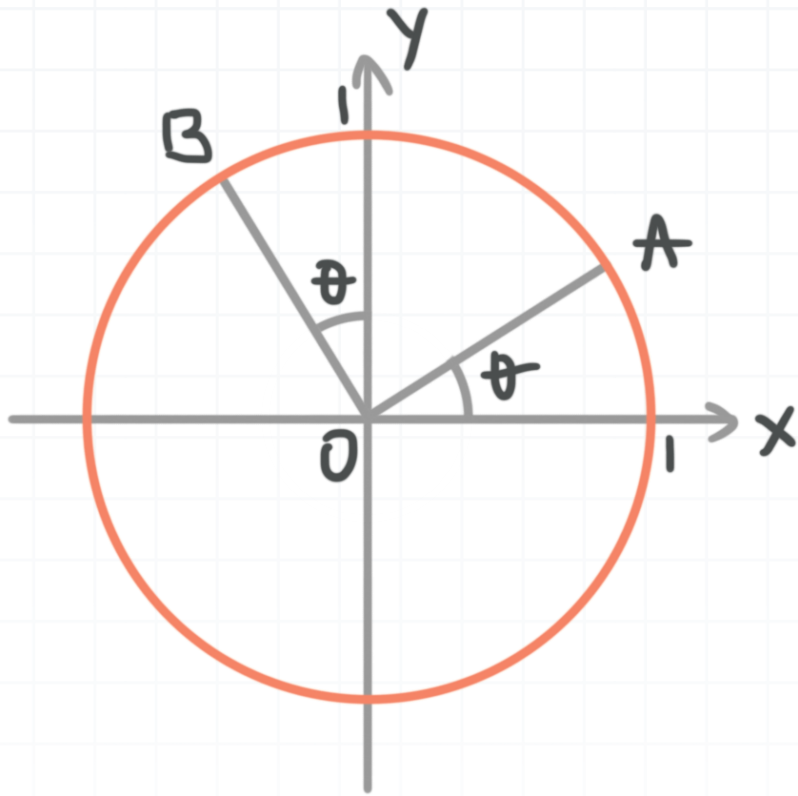
■ 2. The terminal side of the angle θ in $[0, 2\pi)$ intersects the unit circle at the given point. Find the measure of θ in degrees.

$$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

■ 3. Find $\sin \theta$ if $\theta \in [0, 2\pi)$ and $\cos \theta = \sin \theta$.

■ 4. The points A and B lie on the unit circle in quadrants I and II respectively. The angle between OA and the positive x -axis is θ . The angle between OB and the positive y -axis is θ . Find the sine of $\angle AOB$.





■ 5. Evaluate the expression.

$$2 \csc \left(\frac{49\pi}{6} \right) - 3 \cos \left(\frac{13\pi}{3} \right) + \tan \left(\frac{25\pi}{4} \right)$$

■ 6. Find the angle θ in the interval $[0, 2\pi)$.

$$\sin \theta = \frac{1}{2} \text{ and } \cos \theta = -\frac{\sqrt{3}}{2}$$



NEGATIVE ANGLES AND ANGLES MORE THAN ONE ROTATION

- 1. θ has a measure of 42° . If it's rotated 6 full rotations in the negative direction, find its measure after the rotations.
- 2. Find the values of the six trig functions at $\theta = -11\pi/3$.
- 3. Find the angle in the interval $[0^\circ, 360^\circ)$ that's coterminal with -427° .
- 4. Find the angle in the interval $[0, 2\pi)$ that's coterminal with $\theta = -65\pi/6$.
- 5. θ has a measure of $5\pi/3$. If it's rotated 8 full rotations in the negative direction, find its measure after the rotations.
- 6. Find the value of all six trig functions at $\theta = -23\pi/6$.



COTERMINAL ANGLES IN A PARTICULAR INTERVAL

- 1. Find the angle in the interval $[-90^\circ, 90^\circ]$ that's coterminal with -748° .
- 2. Find the angle in $[-4\pi, -2\pi]$ that's coterminal with $-21\pi/4$.
- 3. Find the values of the six trig functions at the angle in the interval $[-\pi/6, 11\pi/6]$ that's coterminal with $43\pi/2$. $\cot\left(\frac{3\pi}{2}\right) = \frac{0}{1} = 0$
- 4. Find the angle in the interval $[-550^\circ, -190^\circ]$ that's coterminal with 367° .
- 5. Find the angle in $[7\pi, 9\pi]$ that's coterminal with $7\pi/6$.
- 6. Find the values of the six trig functions at the angle in the interval $[-\pi, \pi]$ that's coterminal with $-22\pi/3$.



REFERENCE ANGLES

- 1. Find the reference angle for $\theta = -16\pi/3$.
- 2. Which of the angles 19° , 119° , $1,019^\circ$, and $2,019^\circ$ has the smallest reference angle?
- 3. Sketch the angle $4\pi/3$ in standard position and find its reference angle.
- 4. Find the reference angle for $\theta = 438^\circ$.
- 5. Which of the angles 68° , 168° , $1,068^\circ$, or $2,068^\circ$ has the largest reference angle?
- 6. Find the values of the six trig functions at the reference angle for $\theta = -27\pi/4$.



SYMMETRY ACROSS AXES

- 1. Use the unit circle to find the angle that has the same cosine as the angle $\theta = 11\pi/6$.
- 2. If θ is an angle such that $\cos \theta = -0.567$, what is a possible value of $\cos(\theta + 90^\circ)$?
- 3. If θ is an angle in the fourth quadrant such that $\cos \theta = 2/5$, find $\cos(\theta - 5\pi)$ and $\sin(\theta - 5\pi)$.
- 4. Use the unit circle to find an angle that has the same sine as the angle $\theta = 225^\circ$.
- 5. If θ is an angle such that $\cos \theta = 3/5$, what are two possible values of $\sin(\theta + 9\pi)$?
- 6. If θ is an angle in the third quadrant such that $\sin \theta = -0.255$, what is the value of $\cos(\theta - 11\pi)$?



EVEN-ODD IDENTITIES

- 1. Which of the six trig functions are even?

- 2. Evaluate the expression.

$$\sec(-45^\circ) \cdot \csc(-45^\circ) \cdot \tan(-45^\circ) \cdot \cot(-45^\circ)$$

- 3. List the values in order from smallest to largest.

$$\sin\left(-\frac{\pi}{6}\right), \cos\left(-\frac{\pi}{6}\right), \tan\left(-\frac{\pi}{6}\right)$$

- 4. Evaluate the expression.

$$\cos(-30^\circ) + \cos(-45^\circ) + \sin(-45^\circ) + \sin(-60^\circ)$$

- 5. Find the values of cotangent and cosecant at $\theta = -29\pi/2$.

- 6. Which of the six trig functions has the largest value at $\theta = -\pi/3$?



THE SET OF ALL POSSIBLE ANGLES

- 1. A trigonometric equation has two solutions in $[0, 2\pi)$, which are $\pi/4$ and $5\pi/4$. Give the complete solution set.

- 2. Solve the trigonometric equation.

$$\cos(3x + 5\pi) = 0$$

- 3. Find the set of coordinate points in the xy -plane that satisfy the equation.

$$\sin x \cos y = 0$$

- 4. Solve the equation $\cos \theta + \sqrt{2} = -\cos \theta$ for all possible values of θ .

- 5. Solve the trigonometric equation.

$$\sin\left(\frac{x}{2}\right) = -\frac{1}{2}$$

- 6. Solve the trigonometric equation.



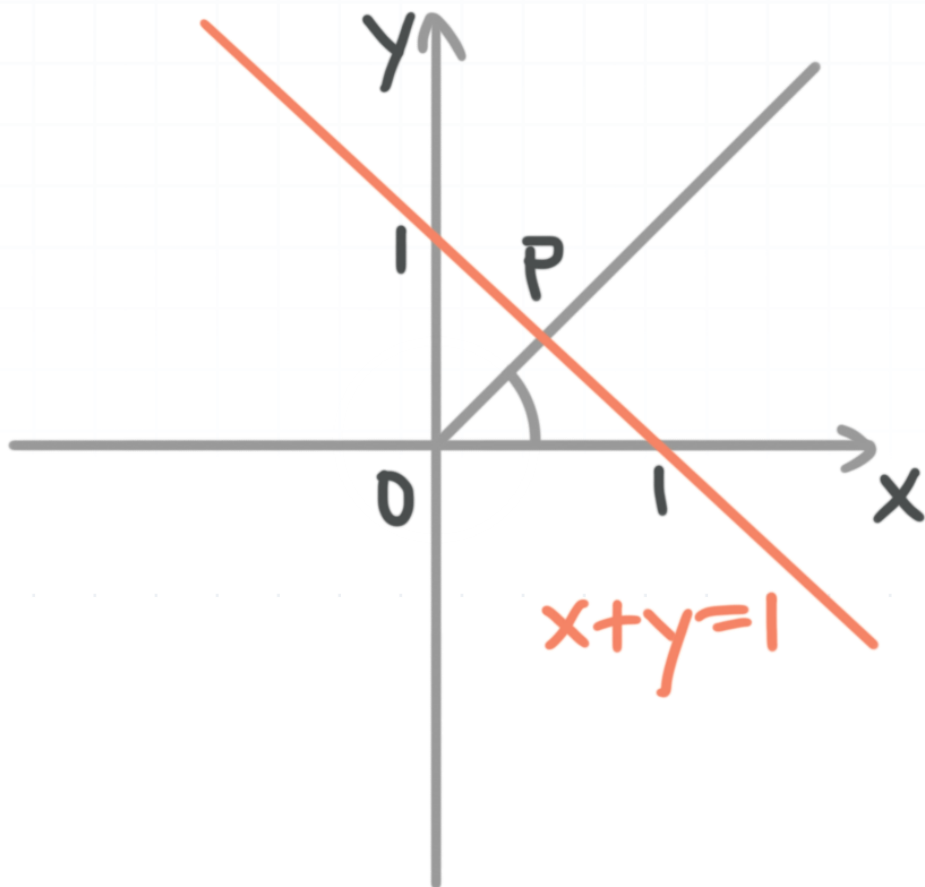
$$4 \cos^2 \theta = 1$$

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POINTS NOT ON THE UNIT CIRCLE

- 1. On the circle with center at the origin and radius 5, find the point(s) where $\cos \theta = -0.6$.
- 2. Find the point P on the line $x + y = 1$ where sine of the angle between P and the positive direction of the x -axis is $\sqrt{2}/2$.



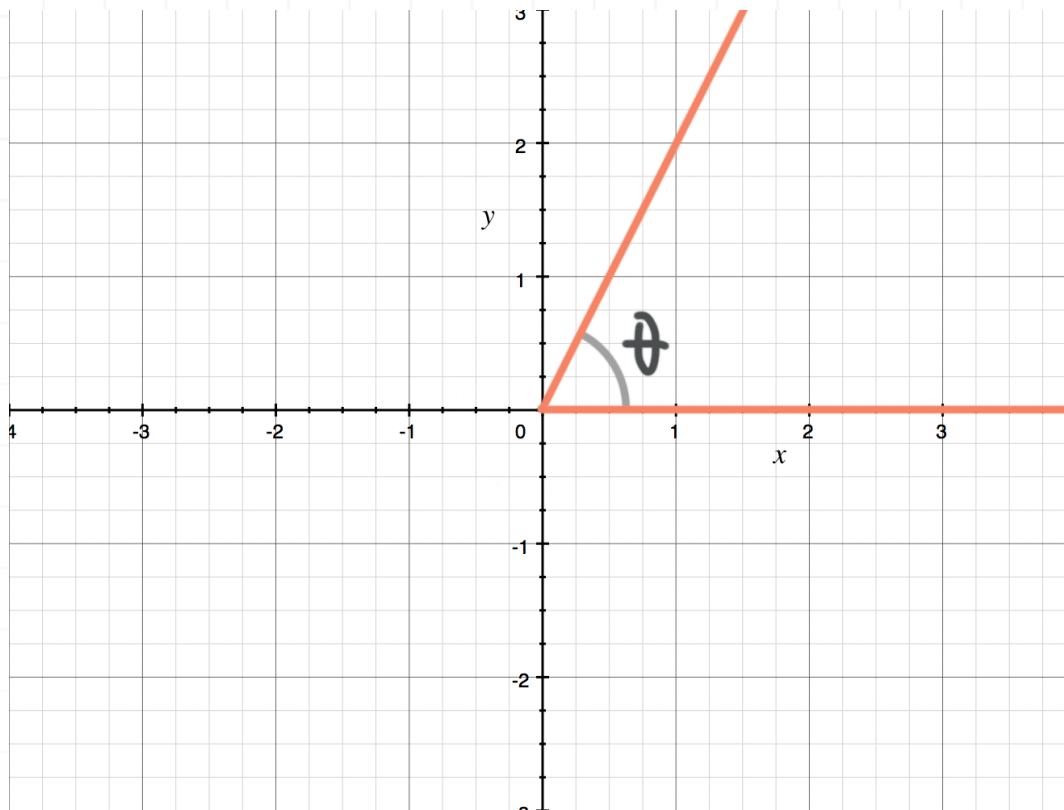
- 3. An airplane flew 3 hours North-North-East (NNE) with a constant velocity of 120 km/h. How far north is the plane from the airport and how far east is the plane from the airport?





- 4. An airplane flew due East with a constant velocity and reached its destination in 12 minutes. Then the plane turned North and flew with the same velocity an additional 5 minutes. Find the cosine of the angle between the line passing through the initial and current positions of the plane, and the direction of due East.
- 5. Find the sine of the acute angle between the line $y = 2x$ and the positive direction of the x -axis.





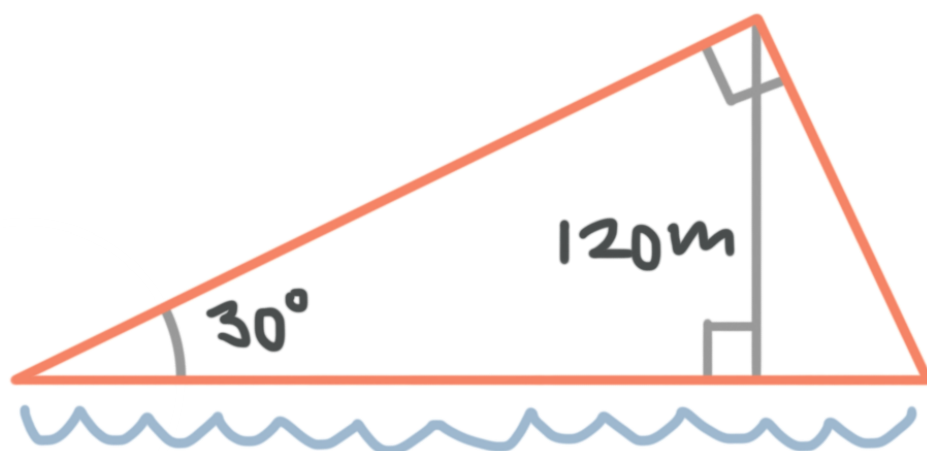
- 6. Find the set of all possible angles θ that pass through the point $P(-3, -3\sqrt{3})$.



SOLVING RIGHT TRIANGLES

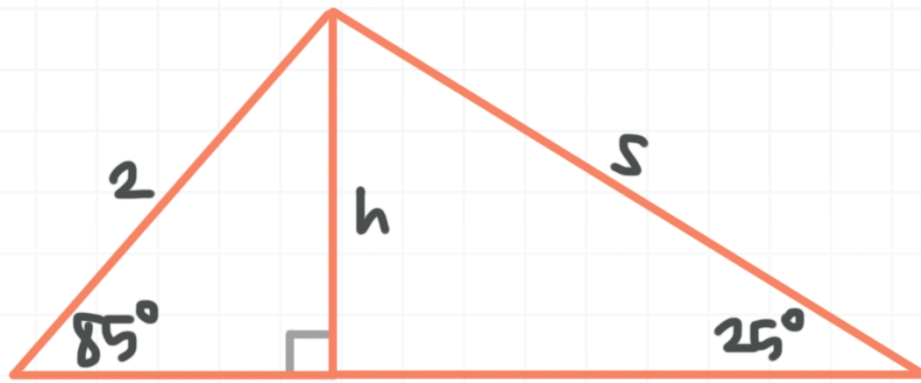
- 1. A right triangle has a hypotenuse with length 5 and one 35° interior angle. Solve the triangle.
- 2. A right triangle has a leg with length 7.5 which is opposite of an angle of 76° . Solve the triangle.
- 3. A plot of land shaped like a right triangle has its hypotenuse along a river. The angle between one of the legs and the river is 30° , and the distance from the river to the furthest point of the plot is 120 m.

The land owner decides to build a fence around the plot and needs to find the total length of the fence. Since the fence is sold by sections of 10 meters, round up to the nearest 10 meters.



- 4. Find the values of h and s , rounded to the nearest whole number.



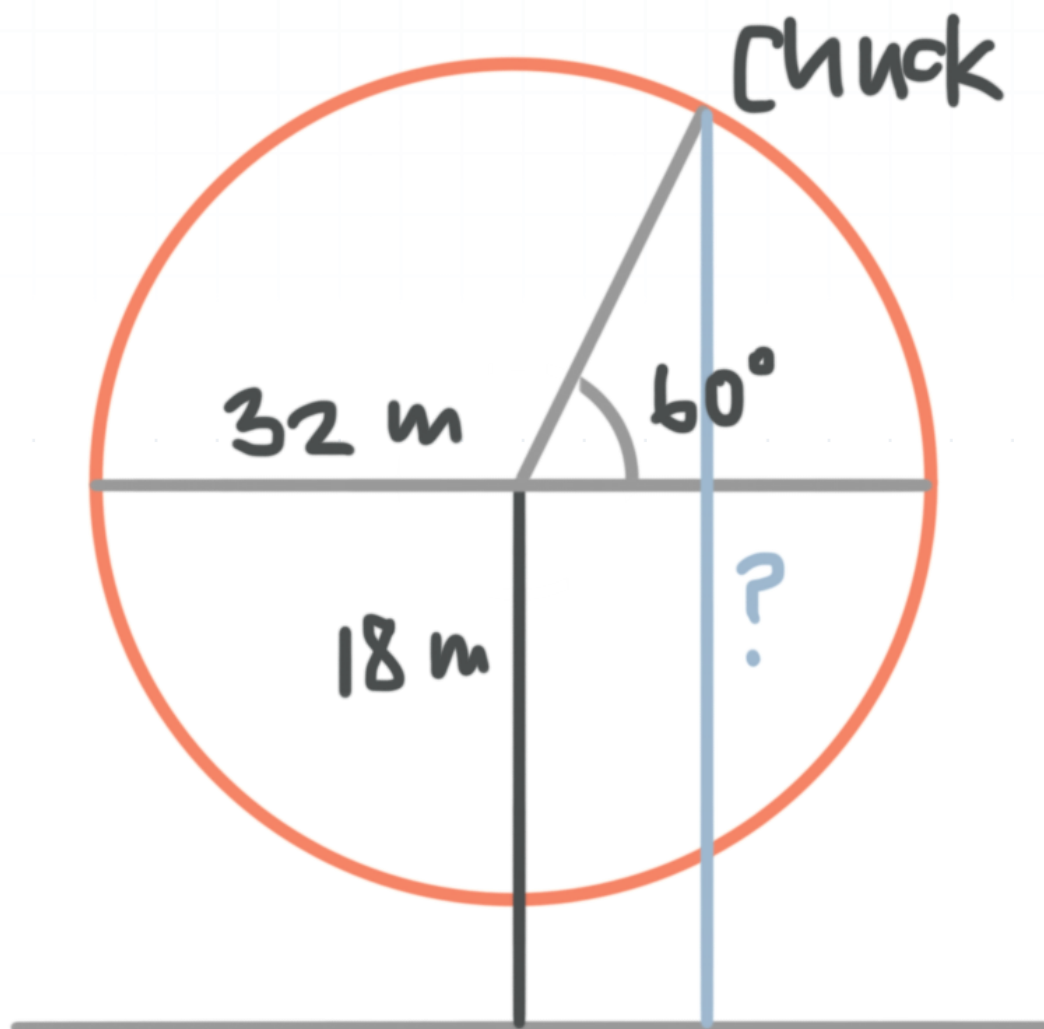


- 5. The length of one leg of a right triangle is 20, and the angle opposite the other leg has a measure of 40° . What is the length of the other leg?
- 6. A right triangle has a leg with length 6. The angle opposite that leg is 24° . Find the measures of all three interior angles and the lengths of all three sides.

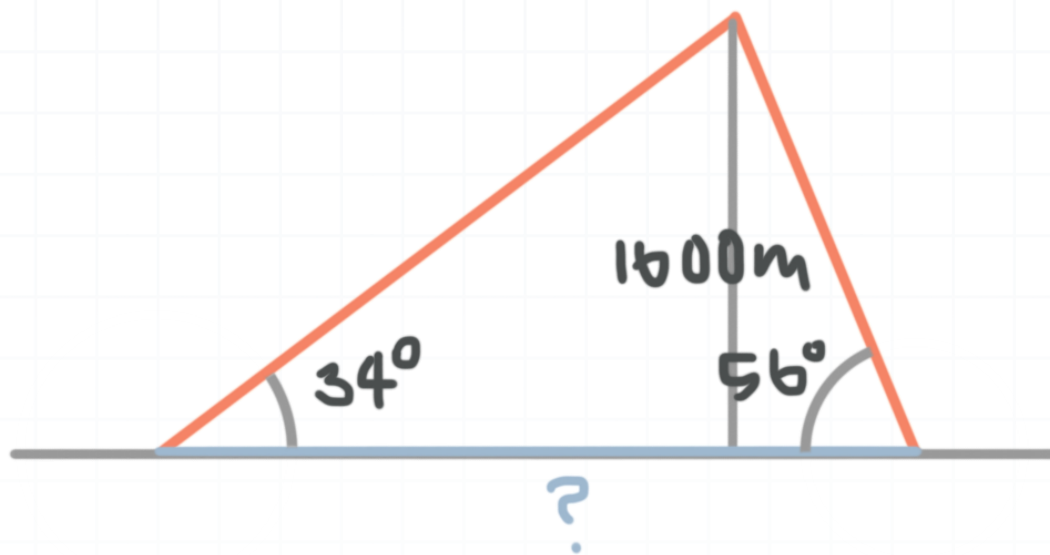


ANGLES OF ELEVATION AND DEPRESSION

- 1. Find $\cos \theta$, where θ is the angle of elevation of the point $(2,3)$ with respect to the point $(-1, -1)$.
- 2. Chuck rides a ferris wheel at a carnival. Find the distance from Chuck to the ground (rounded to the nearest meter), if the center of the wheel is 18 m from the ground, the diameter of the wheel is 32 m, and the angle of elevation of Chuck with respect to the center of the wheel is 60° .



- 3. A company considers building a tunnel through a mountain. The height of the mountain is 1,600 m above the ground, and the angles of elevation of the peak with respect to the left and right endpoints of the tunnel are 34° and 56° respectively. Find the length of the tunnel to the nearest meter.



- 4. Suppose you measure the horizontal distance of a cell-phone tower from a nearby point on the ground A to be 24 feet. If the angle of elevation of the top of the tower with respect to A is 78° , how far is the top of the tower from A , and what is the height of the tower?

- 5. Find $\sin \theta$, where θ is the angle of depression of the point $(3, -4)$ with respect to origin.

- 6. Tom stands on the roof of a 110 m high building, and he looks down at Jerry, who's standing on the roof of a neighboring 80 m high building. The



angle of depression of Jerry with respect to Tom is $\pi/6$. Find the distance between the buildings to the nearest meter.

