Topic: Graphing transformations

Question: Which transformations must be applied to $y = \sin\theta$ to get to the following function?

$$y = 6\left(\sin\left(\frac{2\theta}{5} - \frac{\pi}{10}\right) - 5\right)$$

Answer choices:

A A horizontal compression, a horizontal shift to the right, a vertical stretch, and a vertical shift down.

B A horizontal stretch, a horizontal shift to the left, a vertical compression, and a vertical shift up.

C A horizontal compression, a horizontal shift to the left, a vertical compression, and a vertical shift down.

D A horizontal stretch, a horizontal shift to the right, a vertical stretch, and a vertical shift down.

Solution: D

Rewrite the given function in the form $a\sin(b(\theta + c)) + d$.

$$y = 6\left(\sin\left(\frac{2\theta}{5} - \frac{\pi}{10}\right) - 5\right)$$

$$y = 6\sin\left(\frac{2\theta}{5} - \frac{\pi}{10}\right) - 30$$

$$y = 6\sin\left(\frac{2}{5}\left(\theta - \frac{\pi}{4}\right)\right) - 30$$

From this rewritten form, $a = 6$, $b = 2/5$, $c = -\pi/4$, and $d = -30$. Therefore, the series of transformations must be

1. Horizontally stretch $y = \sin\theta$ by a factor of $5/2$

2. Horizontally shift the result to the right by $\pi/4$

3. Vertically stretch the result by a factor of 6

4. Vertically shift the result down by 30

Topic: Graphing transformations

Question: Which function is the result of only a vertical shift and a horizontal compression applied to the basic cosine function?

Answer choices:

A $y = \cos\theta - 2.9$

B $y = 2\cos\left(\theta + \frac{\pi}{8}\right)$

C $y = \cos\frac{3\theta}{2} + 0.2$

D $y = \cos\left(\frac{\theta}{4} - \frac{\pi}{3}\right)$

Solution: C

Answer choices B and D don't have a vertical shift and answer choice A doesn't have a horizontal compression. We can rewrite the function in answer choice C as

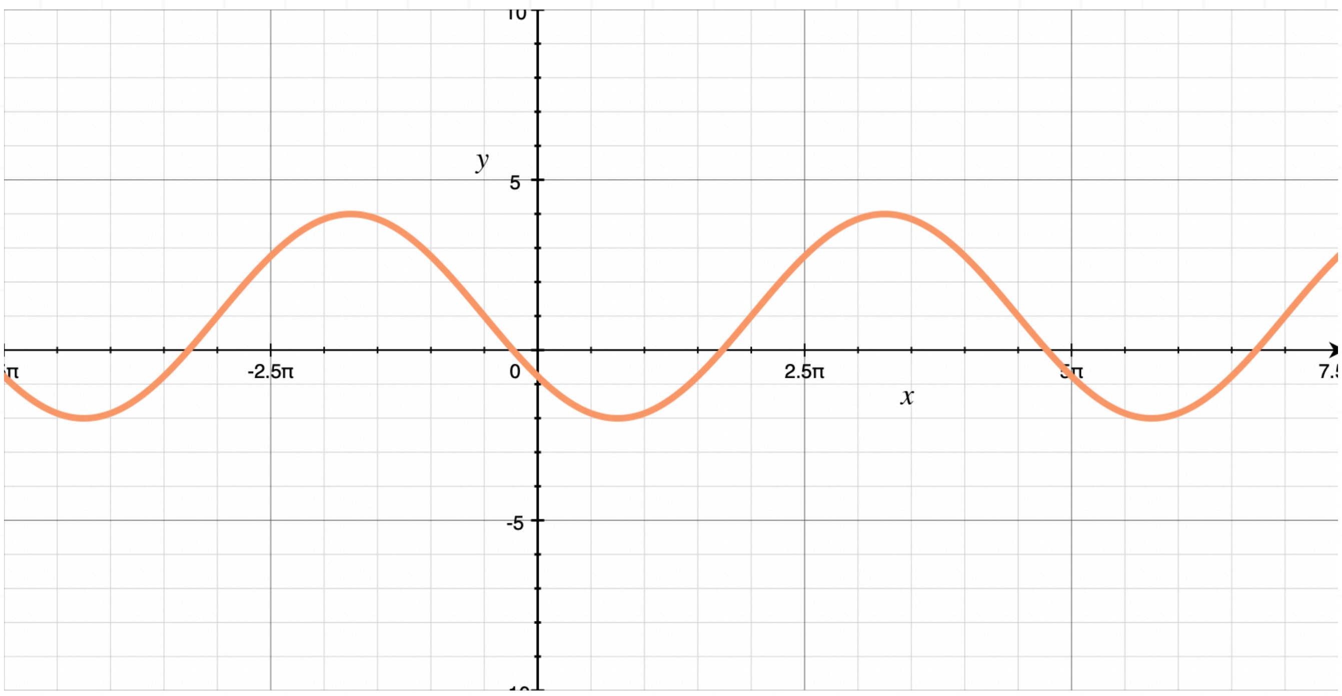
$$y = \cos\left(\frac{3}{2}(\theta + 0)\right) + 0.2$$

Now the function is in the form $y = a\cos(b(\theta + c)) + d$ with $a = 1$, $b = 3/2$, $c = 0$, and $d = 0.2$.

- Since $a = 1$, there's no vertical stretch or compression.
- Since $b = 3/2$, there's a horizontal compression.
- Since $c = 0$, there's no horizontal shift.
- Since $d = 0.2$, there is a vertical shift up.

Topic: Graphing transformations

Question: The graph shows a sine or cosine function with a period equal to 5π. What is the function?



Answer choices:

A $\quad 3\cos\left(\frac{\theta}{5}+\frac{\pi}{10}\right)-1$

B $\quad -3\sin\left(\frac{2\theta}{5}+\frac{\pi}{5}\right)+1$

C $\quad 2\sin\left(\frac{2\theta}{5}+\frac{\pi}{10}\right)+2$

D $\quad -2\cos\left(\frac{\theta}{10}+\frac{2\pi}{5}\right)-1$

Solution: B

The given curve ranges from -2 to 4, so the amplitude of the function must be

$$\frac{1}{2}(4-(-2))=\frac{1}{2}(6)=3$$

This rules out answer choices C and D, which both have an amplitude of 2.

If the amplitude is 3, and the curves ranges from -2 to 4, we can subtract the amplitude from the upper end of the range and add the amplitude to the lower end of the range.

$$4-3=1$$

$$-2+3=1$$

Because we get 1 in both cases, it tells us the function has a vertical shift of $+1$.

Since the period of sine and cosine functions is always given by $2\pi/|b|$, and the period of the given function is 5π, we set up an equation that lets us solve for the value of b.

$$\frac{2\pi}{|b|}=5\pi$$

$$|b|=\frac{2\pi}{5\pi}$$

$$|b|=\frac{2}{5}$$

$$b = \pm \frac{2}{5}$$

Answer choice B is the only function that meets both requirements.