

# Solving systems with Cramer's rule

Cramer's rule is a simple little rule that lets us use determinants to solve a system of equations. The rule says that you can solve for any variable in the system by calculating

$$\frac{D_v}{D}$$

where  $D_v$  is the determinant of the coefficient matrix with the answer column values in the variable column you're trying to solve, and where  $D$  is the determinant of the coefficient matrix.

Which means that, if we want to find the value of  $x$ , we need to find  $D_x/D$ , and if we want to find the value of  $y$ , we need to find  $D_y/D$ . All that sounds tricky, but let's look at an example to break it down.

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## Example

Solve for  $x$  in the system.

$$2x - 3y = 5$$

$$3x + 12y = -8$$

Because we're looking for the value of  $x$ , we want to find  $D_x/D$ . We need to start with the coefficient matrix for the system.



$$\begin{bmatrix} 2 & -3 \\ 3 & 12 \end{bmatrix}$$

The answer column matrix is the constants from the right side of the system,

$$\begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

$D_x$  is the determinant of the coefficient matrix with the answer column matrix substituted into the  $x$ -column, so

$$D_x = \begin{vmatrix} 5 & -3 \\ -8 & 12 \end{vmatrix}$$

$D$  is the determinant of the coefficient matrix, so

$$D = \begin{vmatrix} 2 & -3 \\ 3 & 12 \end{vmatrix}$$

Then, putting these values together, Cramer's rule tells us that the value of  $x$  in the system is

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 5 & -3 \\ -8 & 12 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 3 & 12 \end{vmatrix}}$$

$$x = \frac{(5)(12) - (-3)(-8)}{(2)(12) - (3)(-3)}$$

$$x = \frac{36}{33} = \frac{12}{11}$$



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Let's do another example where we use Cramer's rule to solve for  $y$ .

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### Example

Use Cramer's rule to solve for the value of  $y$  that satisfies the system.

$$3x - 2y = 7$$

$$5x - 8y = 21$$

The coefficient matrix is

$$\begin{bmatrix} 3 & -2 \\ 5 & -8 \end{bmatrix}$$

The answer column matrix is

$$\begin{bmatrix} 7 \\ 21 \end{bmatrix}$$

Then  $D_y$  is what we get when we plug the answer column matrix into the second column of the coefficient matrix, and then take the determinant of the result.

$$D_y = \begin{vmatrix} 3 & 7 \\ 5 & 21 \end{vmatrix}$$

The determinant of the coefficient matrix is



$$D = \begin{vmatrix} 3 & -2 \\ 5 & -8 \end{vmatrix}$$

Then, putting these values together, Cramer's rule tells us that the value of  $y$  in the system is

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 3 & 7 \\ 5 & 21 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 5 & -8 \end{vmatrix}}$$

$$y = \frac{(3)(21) - (7)(5)}{(3)(-8) - (-2)(5)}$$

$$y = \frac{28}{-14} = -2$$

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