Representing systems with matrices

You might remember from Algebra that we learned how to solve systems of linear equations using three methods:

1. Substitution

2. Elimination

3. Graphing

But we can also use matrices to solve systems of equations, too. When we use a matrix to represent a system of linear equations, we call it an **augmented matrix**. Each row in the augmented matrix represents one equation in the system, and each column represents a different variable, or the constants. For instance, given the linear system

$$3x + 2y = 7$$

$$x - 6y = 0$$

the augmented matrix for the system would be

$$\begin{bmatrix} 3 & 2 & 7 \\ 1 & -6 & 0 \end{bmatrix}$$

What you want to notice is that each row corresponds to one of the equations. In the first row, the values 3, 2, and 7 come from the first equation, $3x + 2y = 7$, and in the second row, the values 1, -6, and 0 come from the second equation, $x - 6y = 0$. When it comes to columns, the values from the first column are coefficients for the x's, the values from the second column are coefficients for the y's, and the values from the third column are the constants that come from the right side of each equation.

You can use augmented matrices to represent systems of any size. If we added two more equations to the system, we'd simply add two more rows to the matrix. Or if we added another variable to the system, like z, we'd simply add one more column to the matrix.

Let's do an example with a few more variables.

Example

Represent the system with an augmented matrix called M.

$$-2x + y - t = 7$$

$$x - y + z + 4t = 0$$

You always want to look at all the variables that are included in the system, not just the first equation, since the first equation may not include all the variables.

This particular system includes x, y, z, and t. Which means the augmented matrix will have four columns, one for each variable, plus a column for the constants, so five columns in total. Because there are two equations in the system, the matrix will have two rows. We could setup the matrix like this:

$$M = \begin{bmatrix} x_1 & y_1 & z_1 & t_1 & C_1 \\ x_2 & y_2 & z_2 & t_2 & C_2 \end{bmatrix}$$

Because there's no z-term in the first equation, the value of z_1 will be 0. If we fill in the matrix with that value and all the other coefficients and constants, we get

$$M = \begin{bmatrix} -2 & 1 & 0 & -1 & 7 \\ 1 & -1 & 1 & 4 & 0 \end{bmatrix}$$

As you're building out an augmented matrix, you want to be sure that you have all the variables in the same order, and all your constants grouped together on the same side of the equation. That way, with everything lined up, it'll be easy to make sure that each entry in a column represents the same variable or constant, and that each row in the matrix captures the entire equation.

Let's do an example where the terms aren't already in order.

Example

Express the system of linear equations as a matrix called B.

$$2x + 3y - z = 11$$

$$7y = 6 - x - 4z$$

$$-8z + 3 = y$$

Before we do anything, we want to put each equation in order, with x, then y, then z on the left side, and the constant on the right side.

$$2x + 3y - z = 11$$

$$x + 7y + 4z = 6$$

$$-y - 8z = -3$$

We could also recognize that there is no x-term in the third equation, but we could add in a 0 “filler” term.

$$2x + 3y - z = 11$$

$$x + 7y + 4z = 6$$

$$0x - y - 8z = -3$$

Plugging the coefficients and constants into an augmented matrix gives

$$B = \begin{bmatrix} x_1 & y_1 & z_1 & C_1 \\ x_2 & y_2 & z_2 & C_2 \\ x_3 & y_3 & z_3 & C_3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 & -1 & 11 \\ 1 & 7 & 4 & 6 \\ 0 & -1 & -8 & -3 \end{bmatrix}$$