Topic: Vertex, axis, focus, directrix of an ellipse

Question: Which statement describes the graph of the parametric functions?

$$x = \sin t$$

$$y = 5\cos t - 7$$

Answer choices:

A The equations represent an ellipse centered at $(0, -7)$, with a semimajor axis with a length of 7 along the y-axis, and with a semiminor axis with a length of 1 along with the line $y = -7$.

B The equations represent an ellipse centered at $(0, -7)$, with a semimajor axis with a length of 5 along the y-axis, and with a semiminor axis with a length of 1 along with the line $y = -5$.

C The equations represent an ellipse centered at $(0, -7)$, with a semimajor axis with a length of 5 along the y-axis, and with a semiminor axis with a length of 1 along with the line $y = -7$.

D The equations represent an ellipse centered at $(0,7)$, with a semimajor axis with a length of 5 along the y-axis, and with a semiminor axis with a length of 1 along with the line $y = -7$.

Solution: C

Solve $y = 5\cos t - 7$ for $\cos t$, and square the result.

$$y = 5\cos t - 7$$

$$y + 7 = 5\cos t$$

$$\frac{y+7}{5} = \cos t$$

$$\frac{(y+7)^2}{25} = \cos^2 t$$

Square $x = \sin t$ and add this to the result.

$$x^2 + \frac{(y+7)^2}{25} = \sin^2 t + \cos^2 t$$

$$x^2 + \frac{(y+7)^2}{25} = 1$$

Thus the equations represent an ellipse with the following properties:

- Centered at $(0, -7)$
- Semimajor axis with a length of 5 along the y-axis
- Semiminor axis with a length of 1 along with the line $y = -7$

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Question: The following ellipses are defined by parametric equations. The graph of which ellipse is closer to a circle than the other graphs?

Ellipse E:	$x = 2\sin t$	and	$y = 3\cos t - 1$
Ellipse F:	$x = 3\sin t$	and	$y = 5\cos t - 2$
Ellipse G:	$x = 2\sin t$	and	$y = \cos t - 1$
Ellipse H:	$x = \sin t$	and	$y = 4\cos t - 3$

Answer choices:

A Ellipse E is closer to a circle because its eccentricity, $e = \sqrt{5}/3 = 0.75$, is less than the eccentricities of the other ellipses.

B Ellipse F is closer to a circle because its eccentricity, $e = 6/7 = 0.88$, is less than the eccentricities of the other ellipses.

C Ellipse G is closer to a circle because its eccentricity, $e = \sqrt{5}/3 = 0.76$, is less than the eccentricities of the other ellipses.

D Ellipse G is closer to a circle because its eccentricity, $e = \sqrt{15}/4 = 0.97$, is greater than the eccentricities of the other ellipses.

Solution: A

Find a and b for each ellipse. Then calculate $e = c/a$.

For ellipse E, given by $x = 2\sin t$ and $y = 3\cos t - 1$, the eccentricity is

$$x^2 = 4\sin^2 t \text{ and } (y+1)^2 = 9\cos^2 t$$

$$\frac{x^2}{4} = \sin^2 t \text{ and } \frac{(y+1)^2}{9} = \cos^2 t$$

$$\frac{x^2}{4} + \frac{(y+1)^2}{9} = 1$$

$$e = \frac{\sqrt{5}}{3} = 0.75$$

For ellipse F, given by $x = 3\sin t$ and $y = 5\cos t - 2$, the eccentricity is

$$x^2 = 9\sin^2 t \text{ and } (y+2)^2 = 25\cos^2 t$$

$$\frac{x^2}{9} = \sin^2 t \text{ and } \frac{(y+2)^2}{25} = \cos^2 t$$

$$\frac{x^2}{9} + \frac{(y+2)^2}{25} = 1$$

$$e = \frac{4}{5} = 0.80$$

For ellipse G, given by $x = 2\sin t$ and $y = \cos t - 1$, the eccentricity is

$$\frac{x^2}{4} = \sin^2 t \text{ and } (y+1)^2 = \cos^2 t$$

$$\frac{x^2}{4} + (y+1)^2 = 1$$

$$e = \frac{\sqrt{3}}{2} = 0.87$$

For ellipse H, given by $x = \sin t$ and $y = 4\cos t - 3$, the eccentricity is

$x^2 = \sin^2 t$ and $\frac{(y+3)^2}{16} = \cos^2 t$

$$x^2 + \frac{(y+3)^2}{16} = 1$$

$$e = \frac{\sqrt{15}}{4} = 0.97$$

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Question: Which ellipses have foci with the same x-coordinates?

Answer choices:

A Ellipses $x = \sin t$, $y = 3\cos t - 4$ and $x = 4\sin t$, $y = 5\cos t - 3$

B Ellipses $x = 3\sin t$, $y = \cos t + 9$ and $x = -4\sin t$, $y = -5\cos t + 1$

C Ellipses $x = -3\sin t + 9$, $y = 4\cos t$ and $x = \sin t - 6$, $y = -5\cos t$

D Ellipses $x = 5\sin t$, $y = 3\cos t - 2$ and $x = 5\sin t$, $y = 3\cos t - 4$

Solution: D

Choose the ellipse given by $x = 5\sin t$, $y = 3\cos t - 2$:

$$\frac{x^2}{25} = \sin^2 t$$

$$\frac{(y+2)^2}{9} = \cos^2 t$$

Therefore

$$\frac{x^2}{25} + \frac{(y+2)^2}{9} = 1$$

The x-coordinates of its foci are $c = -4$ and $c = 4$.

Choose the ellipse given by $x = 5\sin t$, $y = 3\cos t - 4$:

$$\frac{x^2}{25} = \sin^2 t$$

$$\frac{(y+4)^2}{9} = \cos^2 t$$

Therefore

$$\frac{x^2}{25} + \frac{(y+4)^2}{9} = 1$$

The x-coordinates of its foci are $c = -4$ and $c = 4$.