Topic: Distances and midpoints

Question: Use the distance formula to find the distance between the complex numbers $u = 3 + 4i$ and $z = 2 - 3i$.

Answer choices:

A $d = \sqrt{15}$

B $d = 5\sqrt{2}$

C $d = \sqrt{26}$

D $d = 4$

Solution: B

The distance formula is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The x-coordinates are the constants of the complex numbers and the y-coordinates are the coefficients of the imaginary numbers. Substitute the values into the distance formula and evaluate.

$$d = \sqrt{(3 - 2)^2 + (4 - (-3))^2}$$

$$d = \sqrt{(3 - 2)^2 + (4 + 3)^2}$$

$$d = \sqrt{1^2 + 7^2}$$

$$d = \sqrt{1 + 49}$$

$$d = \sqrt{50}$$

$$d = \sqrt{25 \cdot 2}$$

$$d = 5\sqrt{2}$$

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Question: Find the distance between the two complex numbers, $u = -3 - 3i$ and $z = -4 + 6i$, by graphing and using the Pythagorean theorem.

Answer choices:

A $c = \sqrt{10}$

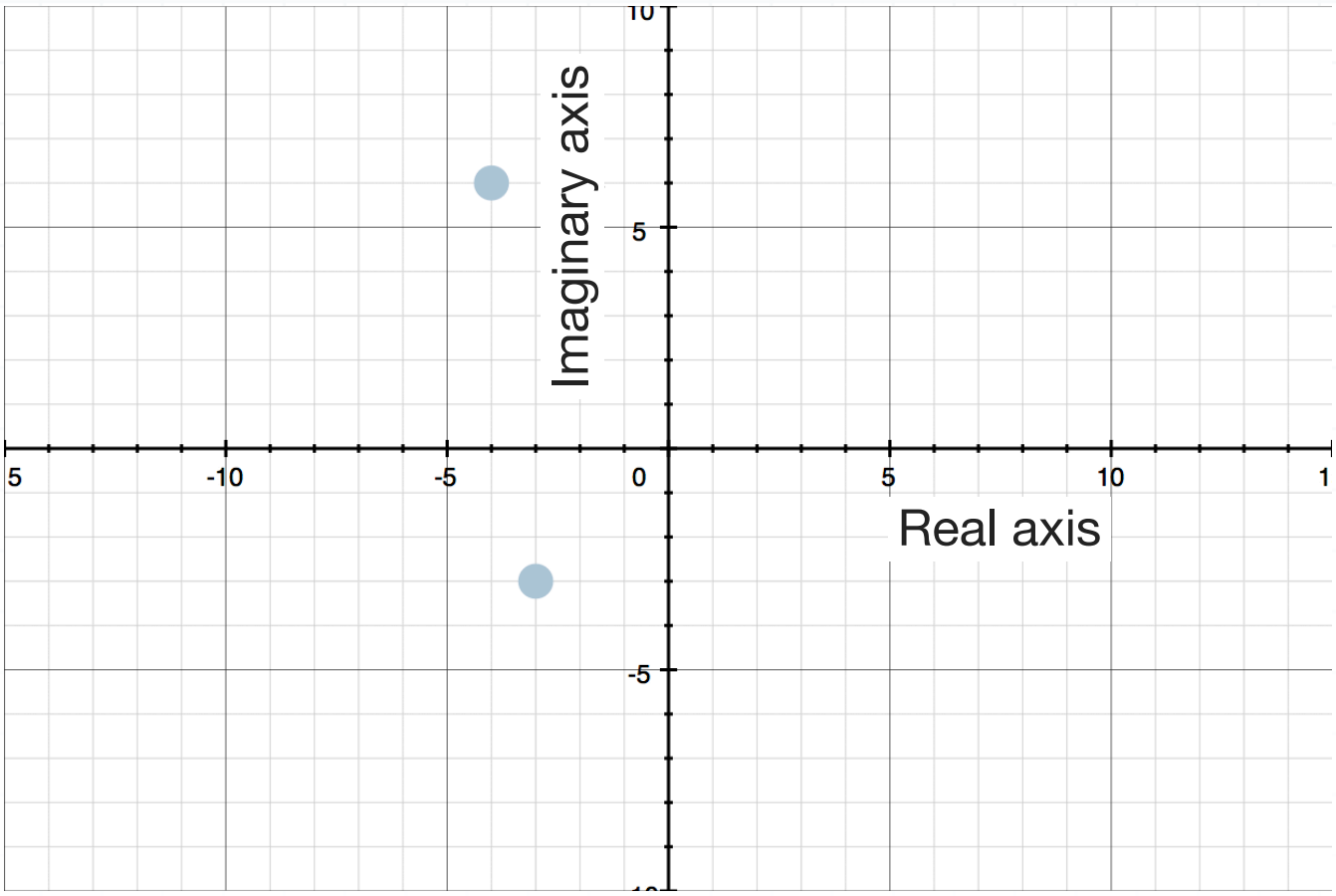
B $c = \sqrt{58}$

C $c = \sqrt{82}$

D $c = \sqrt{130}$

Solution: C

Graph $u = -3 - 3i$ and $z = -4 + 6i$ in the complex plane.



To find the distance between $u = -3 - 3i$ and $z = -4 + 6i$, start by finding the difference between the real parts and the imaginary parts.

The distance between the real parts is $-3 - (-4) = -3 + 4 = 1$, and the distance between the imaginary parts is $-3 - 6 = -9$. Then by the Pythagorean theorem, the distance between $u = -3 - 3i$ and $z = -4 + 6i$ is

$$1^2 + (-9)^2 = c^2$$

$$1 + 81 = c^2$$

$$82 = c^2$$

$$c = \sqrt{82}$$

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Question: Find the midpoint between $u = -3 - 3i$ and $z = -4 + 6i$.

Answer choices:

A $m = -3.5 + 1.5i$

B $m = -2 + i$

C $m = -1.5 + 1.5i$

D $m = -1 - 2i$

Solution: A

To find the midpoint between complex numbers, we find the midpoint of the real parts, and separately the midpoint of the imaginary parts.

The distance between the real parts of $u = -3 - 3i$ and $z = -4 + 6i$ is $-3 - (-4) = -3 + 4 = 1$. Half of that distance is $1/2 = 0.5$, so we look for the value that's 0.5 units from -3 and 0.5 units from -4, so the midpoint between those real parts must be -3.5.

The distance between the imaginary parts of $u = -3 - 3i$ and $z = -4 + 6i$ is $-3 - 6 = -9$. Half of that distance is $-9/2 = -4.5$, so we look for the value that's -4.5 units from -3 and -4.5 units from -6, so the midpoint between those imaginary parts must be 1.5.

So the midpoint between $u = -3 - 3i$ and $z = -4 + 6i$ is $m = -3.5 + 1.5i$. If we graph all three of these in the complex plane, we get