Topic: Double-angle identities

Question: If θ is an angle in the fourth quadrant whose sine is $-4/5$, what are the values of $\sin 2\theta$ and $\cos 2\theta$?

Answer choices:

A $\sin 2\theta = \frac{3}{5}$ $\cos 2\theta = -\frac{4}{5}$

B $\sin 2\theta = -\frac{24}{25}$ $\cos 2\theta = -\frac{7}{25}$

C $\sin 2\theta = \frac{2\sqrt{6}}{5}$ $\cos 2\theta = -\frac{1}{5}$

D $\sin 2\theta = \frac{7}{25}$ $\cos 2\theta = -\frac{24}{25}$

Solution: B

Substitute $\sin\theta = -4/5$ into the rewritten form of the Pythagorean identity with sine and cosine.

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - \left(-\frac{4}{5}\right)^2$$

$$\cos^2\theta = 1 - \frac{16}{25}$$

$$\cos^2\theta = \frac{9}{25}$$

$$\cos\theta = \pm\sqrt{\frac{9}{25}}$$

Since θ is in the fourth quadrant, $\cos\theta$ is positive, so

$$\cos\theta = \sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

Substituting $\cos\theta = 3/5$ and $\sin\theta = -4/5$ into the double-angle identity for sine, $\sin 2\theta = 2\sin\theta\cos\theta$, we get

$$\sin 2\theta = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$\sin 2\theta = -\frac{24}{25}$$

And using the double-angle identity for cosine, we get

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$$

$$\cos 2\theta = \frac{9}{25} - \frac{16}{25}$$

$$\cos 2\theta = -\frac{7}{25}$$

Topic: Double-angle identities

Question: Let θ be an angle in the third quadrant whose cosine is $-\sqrt{10}/11$. Which set of equations is true?

Answer choices:

A $\quad \sin 2\theta = \frac{2\sqrt{1{,}110}}{121} \qquad \cos 2\theta = -\frac{101}{121}$

B $\quad \cos 2\theta = -\frac{101}{121} \qquad \tan 2\theta = -\frac{\sqrt{1{,}110}}{101}$

C $\quad \sin 2\theta = \frac{101}{121} \qquad \tan 2\theta = -\frac{2\sqrt{1{,}110}}{101}$

D $\quad \sin 2\theta = \frac{1}{121} \qquad \cos 2\theta = -\frac{101}{121}$

Solution: A

Substitute $\cos\theta = -\sqrt{10}/11$ into the rewritten form of the Pythagorean identity with sine and cosine.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(-\frac{\sqrt{10}}{11}\right)^2$$

$$\sin^2\theta = 1 - \frac{10}{121}$$

$$\sin^2\theta = \frac{111}{121}$$

Substituting $\cos\theta = -\sqrt{10}/11$ and $\sin^2\theta = 111/121$ into the double-angle identity for cosine, $\cos 2\theta = \cos^2\theta - \sin^2\theta$, we get

$$\cos 2\theta = \left(-\frac{\sqrt{10}}{11}\right)^2 - \frac{111}{121}$$

$$\cos 2\theta = \frac{10}{121} - \frac{111}{121}$$

$$\cos 2\theta = -\frac{101}{121}$$

From the value of $\sin^2\theta$ we found earlier, we can say

$$\sin\theta = \pm\sqrt{\frac{111}{121}}$$

Since θ is in the third quadrant, $\sin\theta$ is negative. Therefore,

$$\sin\theta = -\sqrt{\frac{111}{121}} = -\frac{\sqrt{111}}{\sqrt{121}} = -\frac{\sqrt{111}}{11}$$

Apply the double-angle identity for sine, $\sin 2\theta = 2\sin\theta\cos\theta$.

$$\sin 2\theta = 2\left(-\frac{\sqrt{111}}{11}\right)\left(-\frac{\sqrt{10}}{11}\right)$$

$$\sin 2\theta = \frac{2\sqrt{111}\sqrt{10}}{121}$$

$$\sin 2\theta = \frac{2\sqrt{1,110}}{121}$$

Then tangent of the double angle will be

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{2\sqrt{1,110}}{121}}{-\frac{101}{121}} = -\frac{2\sqrt{1,110}}{101}$$

Topic: Double-angle identities

Question: Use double-angle identities to find the exact value of $\sin 240^\circ$.

Answer choices:

A $\frac{\sqrt{3}}{2}$

B $\sqrt{3}$

C $-\frac{1}{2}$

D $-\frac{\sqrt{3}}{2}$

Solution: D

We can rewrite $\sin 240^\circ$ as $\sin(2 \cdot 120^\circ)$ and use the double-angle identity for sine, $\sin 2\theta = 2 \sin \theta \cos \theta$. We'll substitute and get

$$\sin 240^\circ = \sin(2 \cdot 120^\circ) = 2 \sin 120^\circ \cos 120^\circ$$

Now we can use the double-angle identity for sine and cosine one more time, or if we remember $\sin 120^\circ = \sqrt{3}/2$ and $\cos(-1/2)$, we get

$$\sin 240^\circ = 2 \left(\frac{\sqrt{3}}{2} \right) \left(-\frac{1}{2} \right)$$

$$\sin 240^\circ = -\frac{\sqrt{3}}{2}$$