Topic: Equation of an elliptical conic section

Question: Which two parametric equations represent an ellipse centered at $(2, -1)$.

$$x = 2 - 3\sin t$$

$$x = t$$

$$x = 2 + 5\sin t$$

$$y = \cos t - 3$$

$$y = 4\cos t - 1$$

$$y = t^2 + 2$$

Answer choices:

A $x = 2 - 3\sin t$ and $y = 4\cos t - 1$

B $x = 2 + 5\sin t$ and $y = t^2 + 2$

C $x = t$ and $x = 2 + 5\sin t$

D $x = t$ and $y = t^2 + 2$

Solution: A

Choose the equations $x = 2 - 3\sin t$ and $y = 4\cos t - 1$, and solve them for $\sin^2 t$ and $\cos^2 t$ respectively.

$$x = 2 - 3\sin t$$

$$x - 2 = -3\sin t$$

$$\frac{(x-2)^2}{9} = \sin^2 t$$

and

$$y = 4\cos t - 1$$

$$\frac{y+1}{4} = \cos t$$

$$\frac{(y+1)^2}{16} = \cos^2 t$$

Because we know that $\sin^2 t + \cos^2 t = 1$, we can say

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1$$

Then this is the equation of an ellipse centered at $(2, -1)$.

Topic: Equation of an elliptical conic section

Question: Which type of conic section is represented by the parametric functions?

$$x = 11\sin t - 3$$

$$y = 3\cos t - 11$$

Answer choices:

A Circle

B Ellipse

C Hyperbola

D Parabola

Solution: B

Solve the equations $x = 11\sin t - 3$ and $y = 3\cos t - 11$ for $\sin^2 t$ and $\cos^2 t$ respectively.

$$x = 11\sin t - 3$$

$$x + 3 = 11\sin t$$

$$\frac{x+3}{11} = \sin t$$

$$\frac{(x+3)^2}{121} = \sin^2 t$$

and

$$y = 3\cos t - 11$$

$$y + 11 = 3\cos t$$

$$\frac{y+11}{3} = \cos t$$

$$\frac{(y+11)^2}{9} = \cos^2 t$$

Then we can say

$$\frac{(x+3)^2}{121} + \frac{(y+11)^2}{9} = \sin^2 t + \cos^2 t$$

$$\frac{(x+3)^2}{121} + \frac{(y+11)^2}{9} = 1$$

So, the given equations represent an ellipse.

Topic: Equation of an elliptical conic section

Question: Any point on an ellipse satisfies the parametric equations. Which single equation defines the given ellipse?

$$x = \frac{m}{(m-1)\sec t} - 1$$

$$y = \frac{n}{(n-1)\csc t} - 1$$

Answer choices:

A $\dfrac{(x+1)^2}{\frac{m+1}{m-1}} + \dfrac{(y+1)^2}{\frac{n+1}{n-1}} = 1$

B $\dfrac{(x+1)^2}{\frac{m^2}{(m-1)^2}} + \dfrac{(y+1)^2}{\frac{n^2}{(n-1)^2}} = 1$

C $\dfrac{(x+1)^2}{\frac{m-1}{m}} + \dfrac{(y+1)^2}{\frac{n-1}{n}} = 1$

D $\dfrac{(x+1)^2}{\frac{m+1}{m}} + \dfrac{(y+1)^2}{\frac{n+1}{n}} = 1$

Solution: B

Solve the given equations for $\sin t$ and $\cos t$. Then square both sides of each equation.

$$x = \frac{m}{(m-1)\sec t} - 1$$

$$x + 1 = \frac{m \cos t}{(m-1)}$$

$$\frac{(x+1)^2}{\frac{m^2}{(m-1)^2}} = \cos^2 t$$

and

$$y = \frac{n}{(n-1)\csc t} - 1$$

$$y + 1 = \frac{n \sin t}{n-1}$$

$$\frac{(y+1)^2}{\frac{n^2}{(n-1)^2}} = \sin^2 t$$

Then we can say

$$\frac{(x+1)^2}{\frac{m^2}{(m-1)^2}} + \frac{(y+1)^2}{\frac{n^2}{(n-1)^2}} = 1$$