Topic: Polar equation of a parabolic conic section

Question: Convert the parabola into polar coordinates.

$$y = \frac{1}{2}x^2 - x$$

Answer choices:

A $r^2 = 2\left(1 + \tan^2\theta\right)(1 - \tan\theta)^2$

B $r^2 = 2\left(1 - \tan^2\theta\right)(1 + \tan\theta)^2$

C $r^2 = 4\left(1 + \tan^2\theta\right)(1 + \tan\theta)^2$

D $r^2 = 4\left(1 - \tan^2\theta\right)(1 + \tan\theta)^2$

Solution: C

The equation of the parabola is

$$y = \frac{1}{2}x^2 - x$$

$$2y = x^2 - 2x$$

$$x^2 = 2x + 2y$$

$$x = \frac{2x + 2y}{x}$$

$$x = \frac{2x}{x} + \frac{2y}{x}$$

$$x = 2 + 2\frac{y}{x}$$

Replace $x = r\cos\theta$ and $\tan\theta = y/x$, and simplify.

$$r\cos\theta = 2 + 2\tan\theta$$

$$r = 2\left(\frac{1}{\cos\theta}\right)(1 + \tan\theta)$$

$$r^2 = 4\left(\frac{1}{\cos^2\theta}\right)(1 + \tan\theta)^2$$

Knowing that

$$\frac{1}{\cos^2\theta} = 1 + \tan^2\theta$$

we can substitute and get

$$r^2 = 4\left(1 + \tan^2\theta\right)(1 + \tan\theta)^2$$

Topic: Polar equation of a parabolic conic section

Question: Convert the polar equation into a parabola in rectangular coordinates.

$$r\sin^2\theta - 2\cos\theta - 2 = 0$$

Answer choices:

A $x = \frac{1}{4}y^2 - 1$

B $x = \frac{1}{4}y^2 - 4$

C $x = \frac{1}{2}y^2 - 1$

D $x = \frac{1}{2}y^2 + 1$

Solution: A

Transform the given equation as follows:

$$r \sin^2 \theta - 2 \cos \theta - 2 = 0$$

$$r \sin^2 \theta = 2 \cos \theta + 2$$

$$r \sin^2 \theta = 2 (\cos \theta + 1)$$

$$r = \frac{2 (\cos \theta + 1)}{\sin^2 \theta}$$

We know that $\sin^2 \theta = 1 - \cos^2 \theta$.

$$r = \frac{2 (\cos \theta + 1)}{1 - \cos^2 \theta}$$

Factor the denominator, then cancel common factors.

$$r = \frac{2 (\cos \theta + 1)}{(1 - \cos \theta)(1 + \cos \theta)}$$

$$r = \frac{2}{(1 - \cos \theta)}$$

$$r (1 - \cos \theta) = 2$$

$$r - r \cos \theta = 2$$

Use the conversion equation $x = r \cos \theta$ to substitute.

$$r - x = 2$$

$$r = x + 2$$

$$r^2 = (x+2)^2$$

Use the conversion equation $x^2 + y^2 = r^2$ to substitute.

$$x^2 + y^2 = x^2 + 4x + 4$$

$$y^2 = 4x + 4$$

$$y^2 - 4 = 4x$$

$$x = \frac{1}{4}y^2 - 1$$

Topic: Polar equation of a parabolic conic section

Question: Which equation can be equivalent to the given function?

$$rf(\theta) + r - 8 = 0$$

Answer choices:

A $r = \dfrac{8}{1 + \sin\theta}$ with directrix $d = 8$.

B $r = \dfrac{8}{1 + \tan\theta}$ with directrix $d = 8$.

C $r = \dfrac{1}{8 + \sin\theta}$ with directrix $d = 1$.

D $r = \dfrac{8}{8 + \sin\theta}$ with directrix $d = 8$.

Solution: A

Transform the given function as follows:

$$rf(\theta) + r - 8 = 0$$

$$r\left(f(\theta) + 1\right) - 8 = 0$$

$$r\left(f(\theta) + 1\right) = 8$$

$$r = \frac{8}{1 + f(\theta)}$$

Replacing $f(\theta)$ by $\sin\theta$ results in

$$r = \frac{8}{1 + \sin\theta}$$

where its directrix is $d = 8$.