Topic: Converting polar coordinates to rectangular

Question: What are the rectangular coordinates (x, y) of the point that has polar coordinates $(r, \theta) = (4,\pi)$?

Answer choices:

A $(x, y) = (4,0)$

B $(x, y) = (0, -4)$

C $(x, y) = (0,4)$

D $(x, y) = (-4,0)$

Solution: D

Here, $r = 4$ and $\theta = \pi$. Since $r > 0$ and an angle of measure π is in the interval $[0,2\pi)$, the polar coordinates $(4,\pi)$ are the "basic" polar coordinates of the point in question. Thus the distance of this point from the pole is $r = 4$. Since (the terminal side of) an angle of measure π is on the negative horizontal axis, this point is on the negative horizontal axis (hence its y coordinate is 0) and it lies 4 units to the left of the pole (hence its x coordinate is -4), so its rectangular coordinates are $(-4,0)$.

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Question: Which of the following most closely approximates the rectangular coordinates (x, y) of the point whose polar coordinates are $(r, \theta) = (5, 11\pi/7)$?

Answer choices:

A $(x, y) = (-1.24, -2.38)$

B $(x, y) = (1.12, -4.88)$

C $(x, y) = (-4.87, 1.11)$

D $(x, y) = (1.24, -2.38)$

Solution: B

Here, $r = 5$ and $\theta = 11\pi/7$. Note that

$$\frac{3\pi}{2} = \frac{21\pi}{14} < \frac{22\pi}{14} = \frac{11\pi}{7} < \frac{14\pi}{7} = 2\pi$$

Thus an angle of measure $11\pi/7$ is not only in the interval $[0,2\pi)$ but in the fourth quadrant. Since r is positive, $(5,11\pi/7)$ are the "basic" polar coordinates of the point in question, so the point is in the fourth quadrant. Therefore, its x coordinate is positive and its y coordinate is negative.

Using the general equations

$$x = r\cos\theta$$

$$y = r\sin\theta$$

we get

$$x = 5\cos\frac{11\pi}{7}$$

and

$$y = 5\sin\frac{11\pi}{7}$$

With the help of a calculator, we find that $\cos(11\pi/7) \approx 0.223$ and $\sin(11\pi/7) \approx -0.975$, so $x \approx 5(0.223) \approx 1.12$ and $y \approx 5(-0.975) \approx -4.88$.

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Question: What are the rectangular coordinates (x, y) of the point that has polar coordinates $(r, \theta) = (21{,}9\pi/8)$?

Answer choices:

A $\quad (x, y) = \left(-21\left(\frac{\sqrt{2}+1}{2\sqrt{2}}\right), -21\left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right)\right)$

B $\quad (x, y) = \left(-21\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}, -21\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}\right)$

C $\quad (x, y) = \left(-21\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}, -21\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}\right)$

D $\quad (x, y) = \left(-21\left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right), -21\left(\frac{\sqrt{2}+1}{2\sqrt{2}}\right)\right)$

Solution: C

Here, $r = 21$ and $\theta = 9\pi/8$. Note that

$$\pi = \frac{8\pi}{8} < \frac{9\pi}{8} = \frac{27\pi}{24} < \frac{36\pi}{24} = \frac{3\pi}{2}$$

Thus an angle of measure $9\pi/8$ is not only in the interval $[0,2\pi)$ but in the third quadrant. Since r is positive, $(21,9\pi/8)$ are the “basic” polar coordinates of the point in question, so the point is in the third quadrant. Therefore, its x and y coordinates are both negative.

To determine x and y, we'll use the equations

$$x = r\cos\theta = 21\cos\frac{9\pi}{8}$$

$$y = r\sin\theta = 21\sin\frac{9\pi}{8}$$

and the half-angle identities for cosine and sine. By the half-angle identity for cosine,

$$\cos^2\frac{9\pi}{8} = \frac{1}{2}\left\{1 + \cos\left[2\left(\frac{9\pi}{8}\right)\right]\right\} = \frac{1}{2}\left[1 + \cos\left(\frac{9\pi}{4}\right)\right]$$

By the half-angle identity for sine,

$$\sin^2\frac{9\pi}{8} = \frac{1}{2}\left\{1 - \cos\left[2\left(\frac{9\pi}{8}\right)\right]\right\} = \frac{1}{2}\left[1 - \cos\left(\frac{9\pi}{4}\right)\right]$$

Now

$$\frac{9\pi}{4} = \frac{8\pi + \pi}{4} = 2\pi + \frac{\pi}{4}$$

Thus an angle of measure $9\pi/4$ is coterminal with an angle of measure $\pi/4$.

Recall that

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Thus

$$\cos\frac{9\pi}{4} = \frac{1}{\sqrt{2}}$$

Substituting this result, we obtain

$$\cos^2\frac{9\pi}{8} = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

and

$$\sin^2\frac{9\pi}{8} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$$

Since x is negative, $\cos(9\pi/8)$ is negative. Therefore,

$$x = 21\cos\frac{9\pi}{8} = 21\left(-\sqrt{\cos^2\frac{9\pi}{8}}\right)$$

$$x = -21\sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)}$$

$$x = -21\sqrt{\frac{1}{2}\left(\frac{\sqrt{2}(1)+1(1)}{\sqrt{2}}\right)}$$

$$x = -21\sqrt{\frac{1}{2}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)}$$

$$x = -21\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

And since y is negative, $\sin(9\pi/8)$ is negative, so

$$y = 21\sin\frac{9\pi}{8} = 21\left(-\sqrt{\sin^2\frac{9\pi}{8}}\right)$$

$$y = -21\sqrt{\frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)}$$

$$y = -21\sqrt{\frac{1}{2}\left(\frac{\sqrt{2}(1)-1(1)}{\sqrt{2}}\right)}$$

$$y = -21\sqrt{\frac{1}{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)}$$

$$y = -21\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$