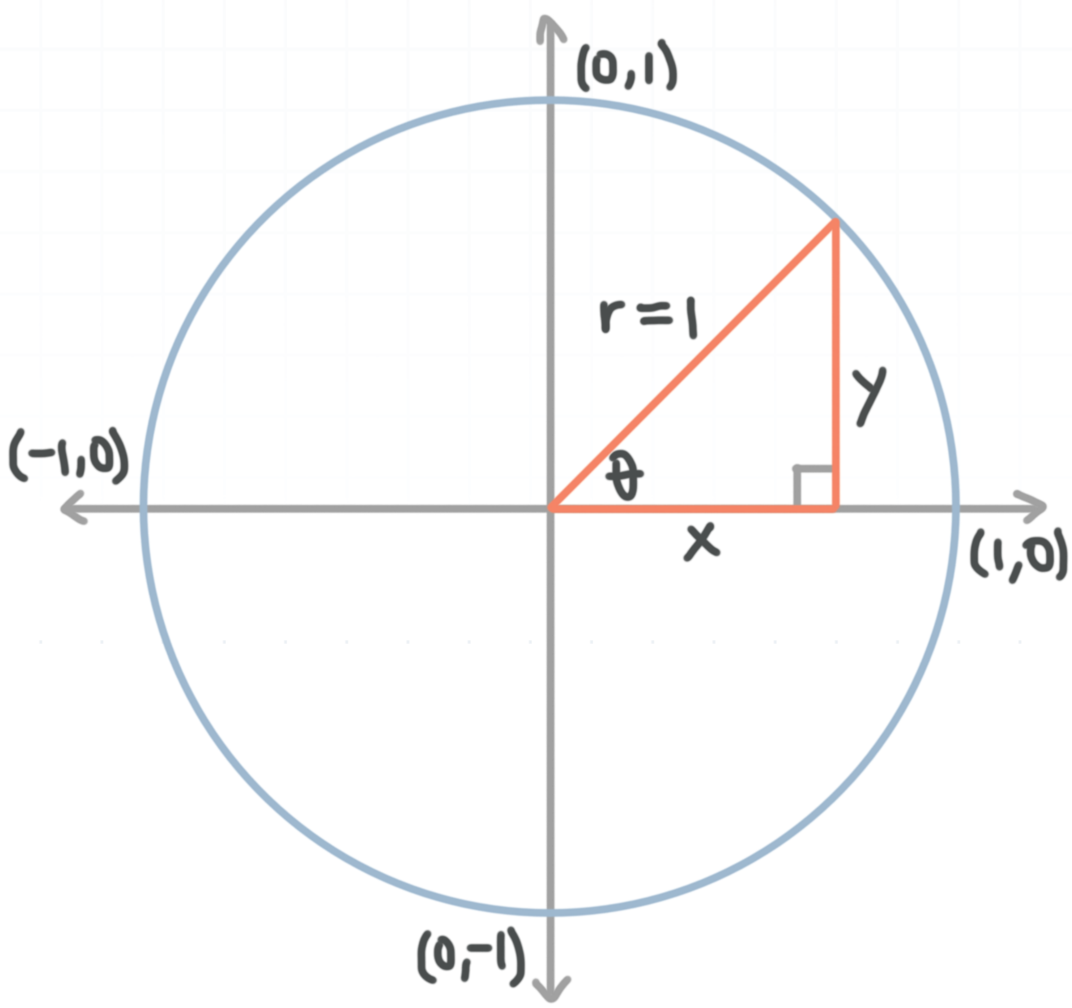
Signs by quadrant

When we know the value of one of the six trig functions for a particular angle, and we know the quadrant where the angle is located, we'll always be able to find the values of the other five trig functions for the same angle.

Remember what it looked like when we placed the right triangle in the coordinate plane:



Putting the right triangle in the coordinate plane let us define the six trig functions in terms of the horizontal leg x, the vertical leg y, and the hypotenuse as the radius of the circle r.

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

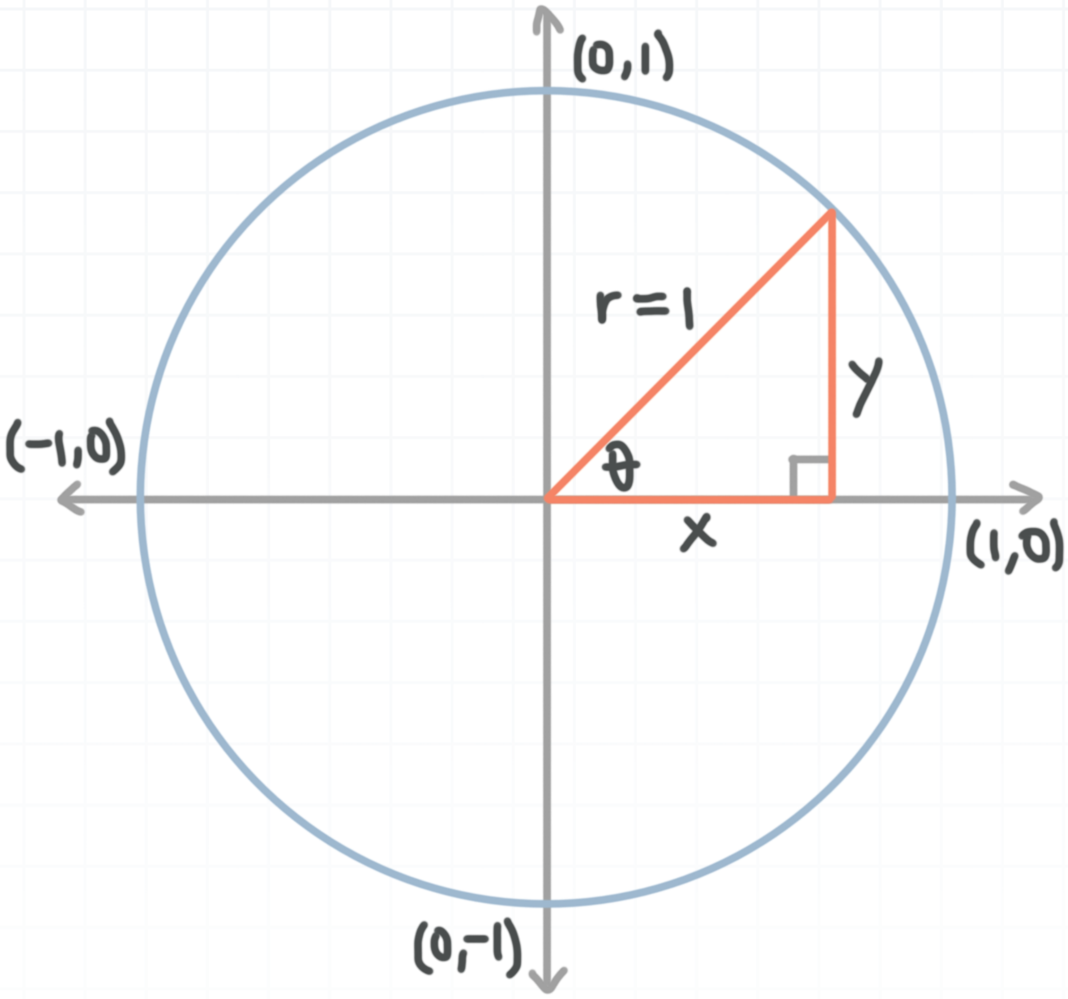
When we define the trig functions this way in terms of x, y, and r, we need to realize that r represents the radius, which means it's a distance, which means its value is always positive. But x and y are signed based on the quadrant of the angle.

When the triangle, and therefore the angle θ, lies in the first quadrant like in the image above, then x and y are positive. But the signs on x and y are different for the other three quadrants.

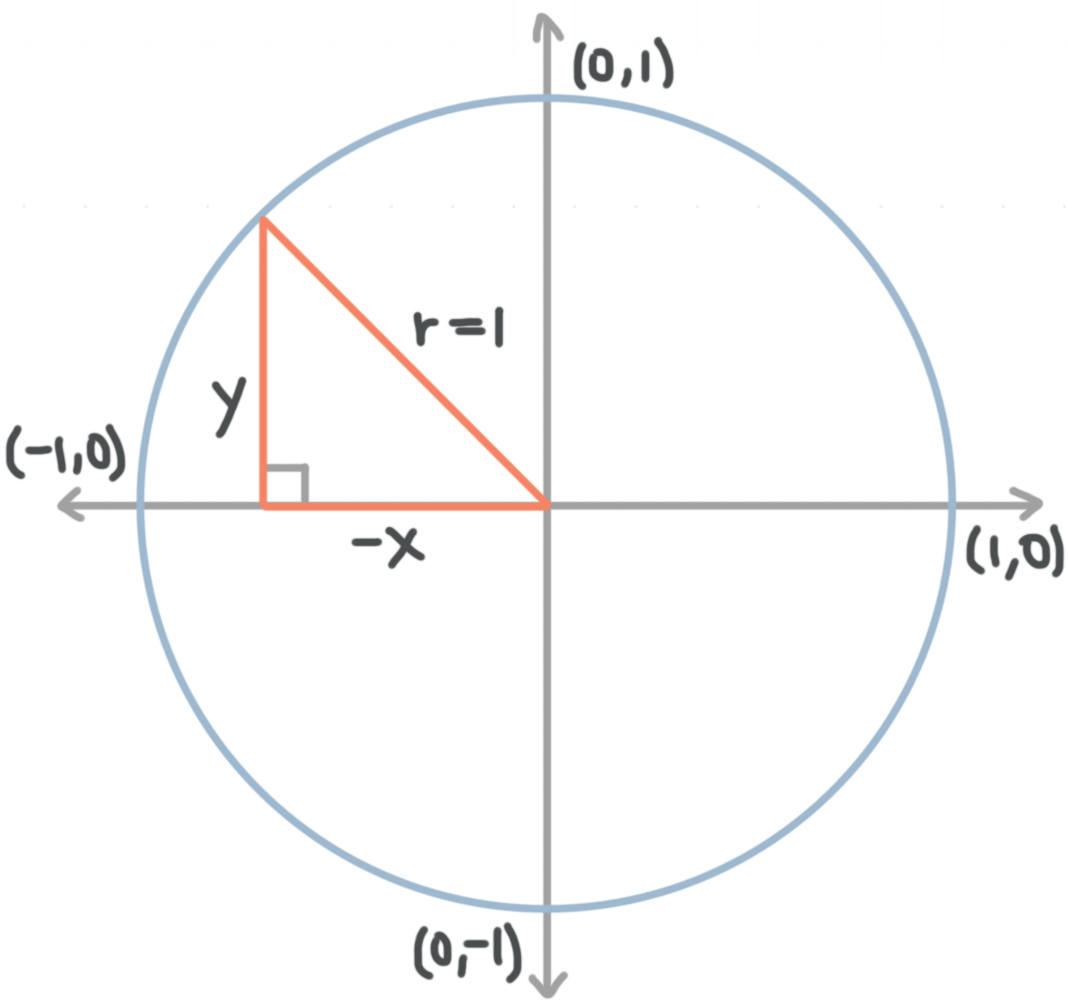
Quadrant	Sign on x	Sign on y	Sign on r
I	+	+	+
II	-	+	+
III	-	-	+
IV	+	-	+

We can also see this visually when we sketch a triangle in each quadrant.

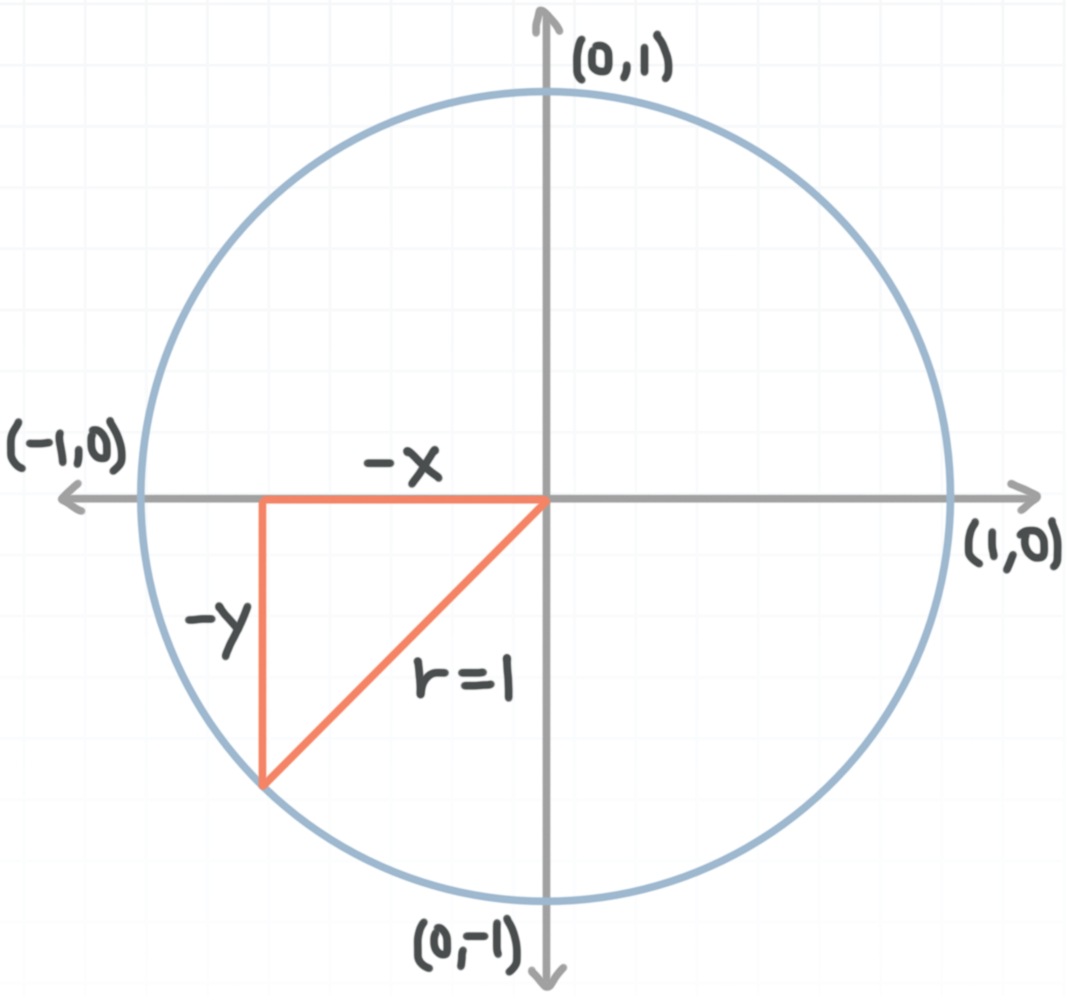
In quadrant I, x, y, and r are all positive.



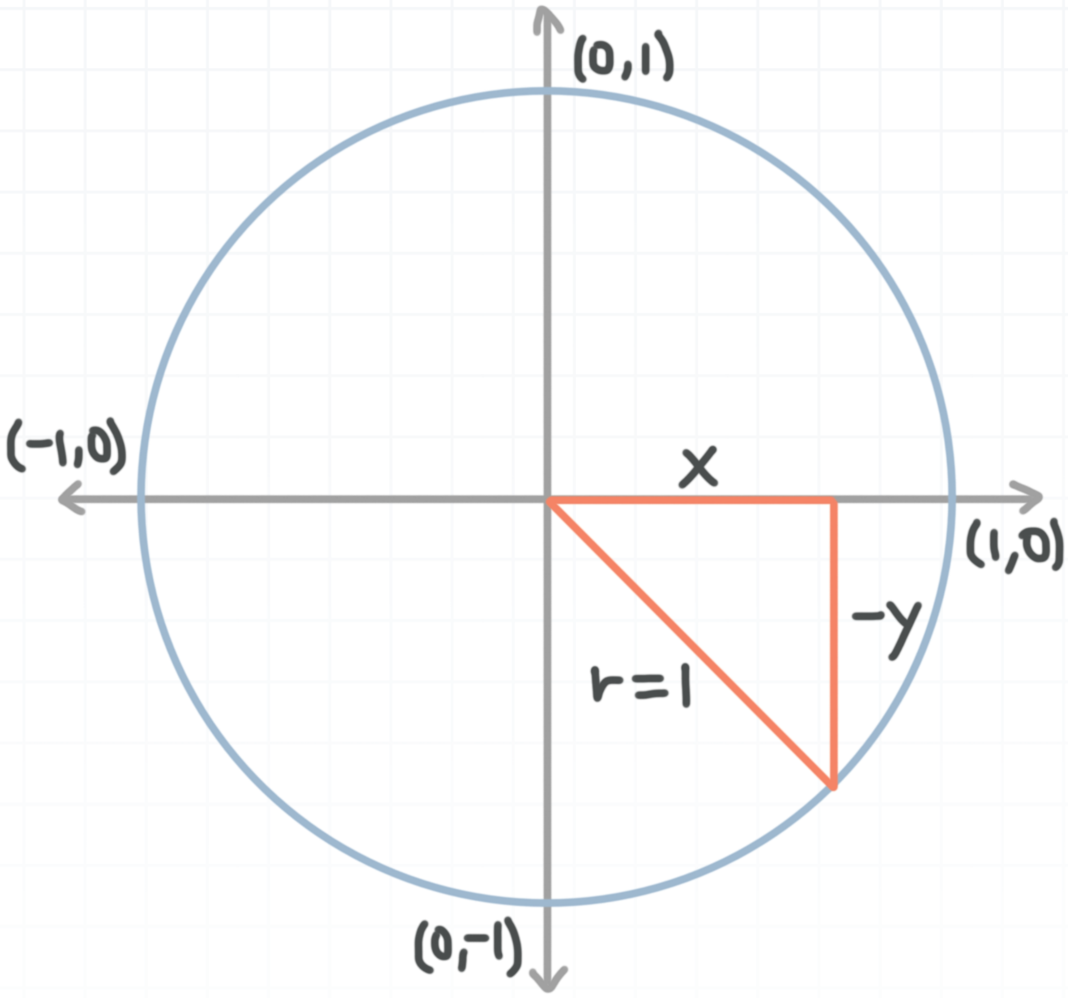
In quadrant II, x is negative, and y and r are positive.



In quadrant III, x and y are negative and r is positive.



In quadrant IV, x is positive, y is negative, and r is positive.



Signs of the trig functions

Now that we understand the signs of x, y, and r in each quadrant, we can plug these signs into formulas in terms of these variables for the six trig functions. For instance, in quadrant I, the signs of the six trig functions are

$$\sin\theta = \frac{y}{r} = \frac{+}{+} = + \qquad \csc\theta = \frac{r}{y} = \frac{+}{+} = +$$

$$\cos\theta = \frac{x}{r} = \frac{+}{+} = + \qquad \sec\theta = \frac{r}{x} = \frac{+}{+} = +$$

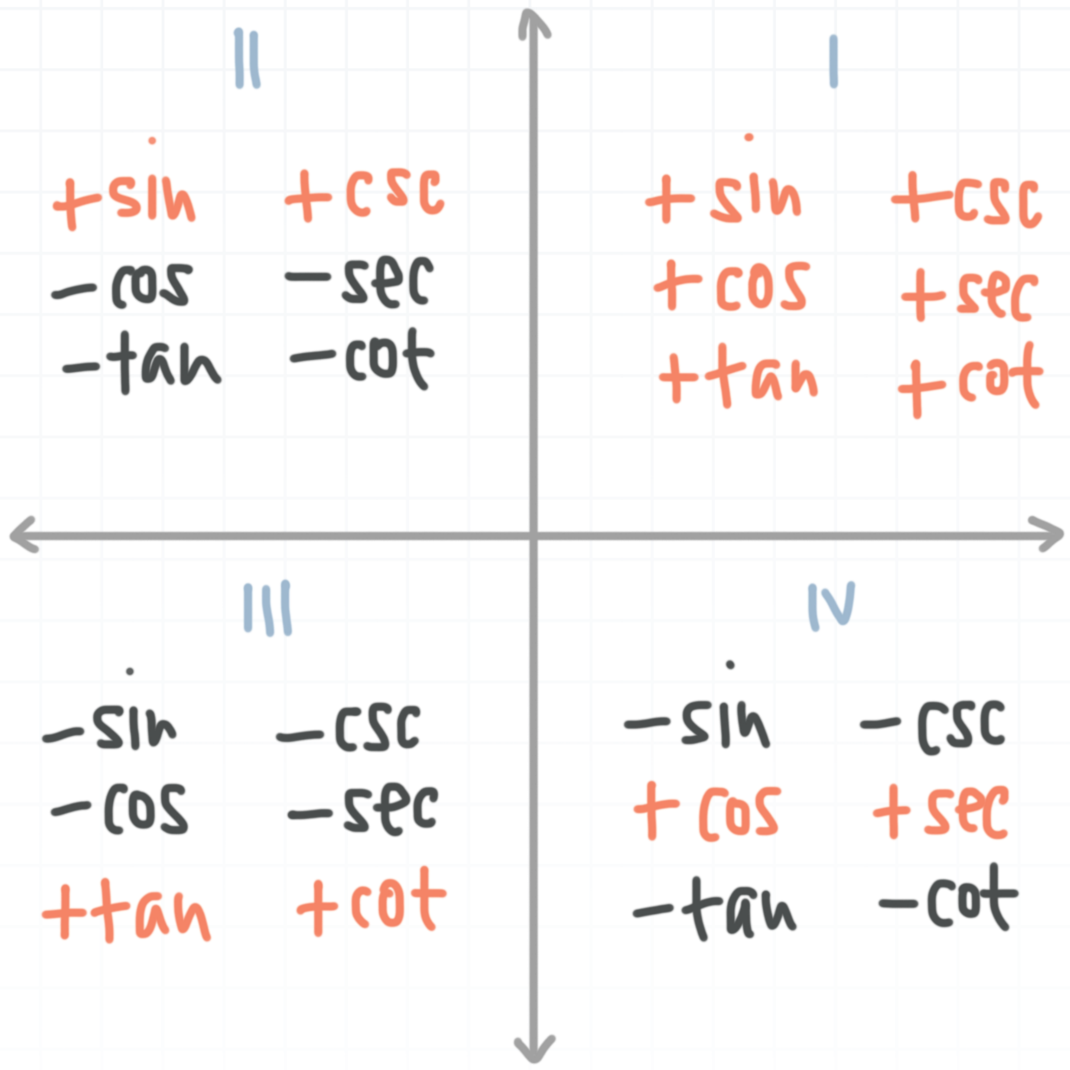
$$\tan\theta = \frac{y}{x} = \frac{+}{+} = + \qquad \cot\theta = \frac{x}{y} = \frac{+}{+} = +$$

In other words, all six of the trig functions are positive in the first quadrant. We can summarize the signs of all six trig functions in each quadrant in another table.

	I	II	III	IV
sin	+	+	-	-
csc	+	+	-	-
cos	+	-	-	+
sec	+	-	-	+
tan	+	-	+	-
cot	+	-	+	-

Notice how the signs match for sine and cosecant, for cosine and secant, and for tangent and cotangent. That makes sense, since each of those pairs are reciprocals of one another (which we saw previously when we talked about the reciprocal identities).

If we sketched out these signs in the coordinate plane, we'd see that all six trig functions are positive in quadrant I, that sine and cosecant are positive in quadrant II, that tangent and cotangent are positive in quadrant III, and that cosine and secant are positive in quadrant IV.



Let's do an example where we use one trig function and the angle's quadrant in order to figure out the values of the other five trig functions.

Example

For an angle θ in the third quadrant with $\sec\theta = -2.53$, find the values of the other five trig functions at θ.

We can immediately use the reciprocal identity to find cosine of the angle.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-2.53} \approx -0.395$$

If we rewrite the Pythagorean identity for sine and cosine,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

then we can plug the value we just found for $\cos\theta$ into this equation in order to find sine of the angle.

$$\sin^2\theta \approx 1 - (-0.395)^2$$

$$\sin^2\theta \approx 1 - 0.156$$

$$\sin^2\theta \approx 0.844$$

$$\sin\theta \approx \pm\sqrt{0.844}$$

$$\sin\theta \approx \pm 0.919$$

Since the sine of any angle in the third quadrant is negative, we know $\sin\theta \approx -0.919$. Then we can use the reciprocal identity to find cosecant of the angle.

$$\csc\theta = \frac{1}{\sin\theta} \approx \frac{1}{-0.919} \approx -1.09$$

Now we'll plug the values of $\sin\theta$ and $\cos\theta$ into the quotient identity to find tangent of the angle.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \approx \frac{-0.919}{-0.395} \approx 2.33$$

Use the reciprocal identity to find cotangent of the angle.

$$\cot\theta = \frac{1}{\tan\theta} \approx \frac{1}{2.33} \approx 0.430$$

Let’s summarize the values we found for all six trig function of the angle θ in the third quadrant whose secant was given as $\sec\theta = -2.53$.

$$\sin\theta \approx -0.919 \qquad \csc\theta \approx -1.09$$

$$\cos\theta \approx -0.395 \qquad \sec\theta = -2.53$$

$$\tan\theta \approx 2.33 \qquad \cot\theta \approx 0.430$$

To double-check ourselves, we can confirm that the sign of each of these trig functions matches the sign table we made earlier.

	I	II	III	IV
sin			-	
csc			-	
cos			-	
sec			-	
tan			+	
cot			+	

Let’s do an example where we’re starting with the tangent function instead of the secant function.

Example

For an angle θ in the fourth quadrant whose tangent is -6.79, find the values of the other five trig functions.

Use the value of $\tan\theta$ in the Pythagorean identity with tangent and secant to find the secant of the angle.

$$1+\tan^2\theta=\sec^2\theta$$

$$1+(-6.79)^2=\sec^2\theta$$

$$1+46.10\approx\sec^2\theta$$

$$\sec^2\theta\approx 47.10$$

$$\sec\theta=\pm\sqrt{47.10}$$

$$\sec\theta=\pm 6.863$$

The secant of any angle in the fourth quadrant is positive, so $\sec\theta=6.86$. Since we have tangent, we can use the reciprocal identity to find the cotangent.

$$\cot\theta=\frac{1}{\tan\theta}$$

$$\cot\theta=\frac{1}{-6.79}\approx-0.15$$

We'll use the value of $\cot\theta$ in the Pythagorean identity with cotangent and cosecant to find the cosecant of the angle.

$$1+\cot^2\theta=\csc^2\theta$$

$$1+(-0.147)^2=\csc^2\theta$$

$$1+0.022\approx\csc^2\theta$$

$$\csc\theta = \pm\sqrt{1.022}$$

$$\csc\theta = \pm\, 1.01$$

The cosecant of any angle in the fourth quadrant is negative, so $\csc\theta = -\,1.01$.

And now that we have secant and cosecant, we can use the reciprocal identities to find cosine and sine.

$$\sin\theta = \frac{1}{\csc\theta} \approx \frac{1}{-1.01} \approx -\,0.99$$

$$\cos\theta = \frac{1}{\sec\theta} \approx \frac{1}{6.86} \approx 0.15$$

Let's summarize the values we found for all six trig function of the angle θ in the fourth quadrant whose tangent was given as $\tan\theta = -\,6.79$.

$$\sin\theta \approx -\,0.99 \qquad \csc\theta \approx -\,1.01$$

$$\cos\theta \approx 0.15 \qquad \sec\theta \approx 6.86$$

$$\tan\theta = -\,6.79 \qquad \cot\theta \approx -\,0.15$$

To double-check ourselves, we can confirm that the sign of each of these trig functions matches the sign table we made earlier.

	I	II	III	IV
sin				-
csc				-
cos				+
sec				+
tan				-
cot				-