Precalculus Workbook

Complex numbers

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COMPLEX NUMBERS

■ 1. Simplify the imaginary number.

i^{437}

■ 2. Simplify the imaginary number.

$i^{2,314}$

■ 3. Name the real and imaginary parts of the complex number.

$z = -5 + 17i$

■ 4. Name the real and imaginary parts of the complex number.

$z = \sqrt{7} - 4\pi i$

■ 5. How can the numbers be classified?

$z = -3 + 9i$

$z = 0 - 15i$

$z = 6 + 0i$

6. How can the numbers be classified?

$z = 0 - \pi i$

$z = -\sqrt{5} + 0i$

$z = -11 + \frac{2}{3}i$

COMPLEX NUMBER OPERATIONS

■ 1. Find the sum and difference of the complex numbers.

$\frac{7}{5} - \frac{2}{3}i$

$\frac{7}{2} - \frac{8}{3}i$

■ 2. Find the product of the complex numbers.

$-7i$

$-5 + 9i$

■ 3. Find the product of the complex numbers.

$5 - 2i$

$6 - 11i$

■ 4. Divide the complex number $-4 + 15i$ by the imaginary number $5i$.

■ 5. Find the complex conjugate of each complex number.

$9 - 9i$

$-3 + 13i$

$11 - 22i$

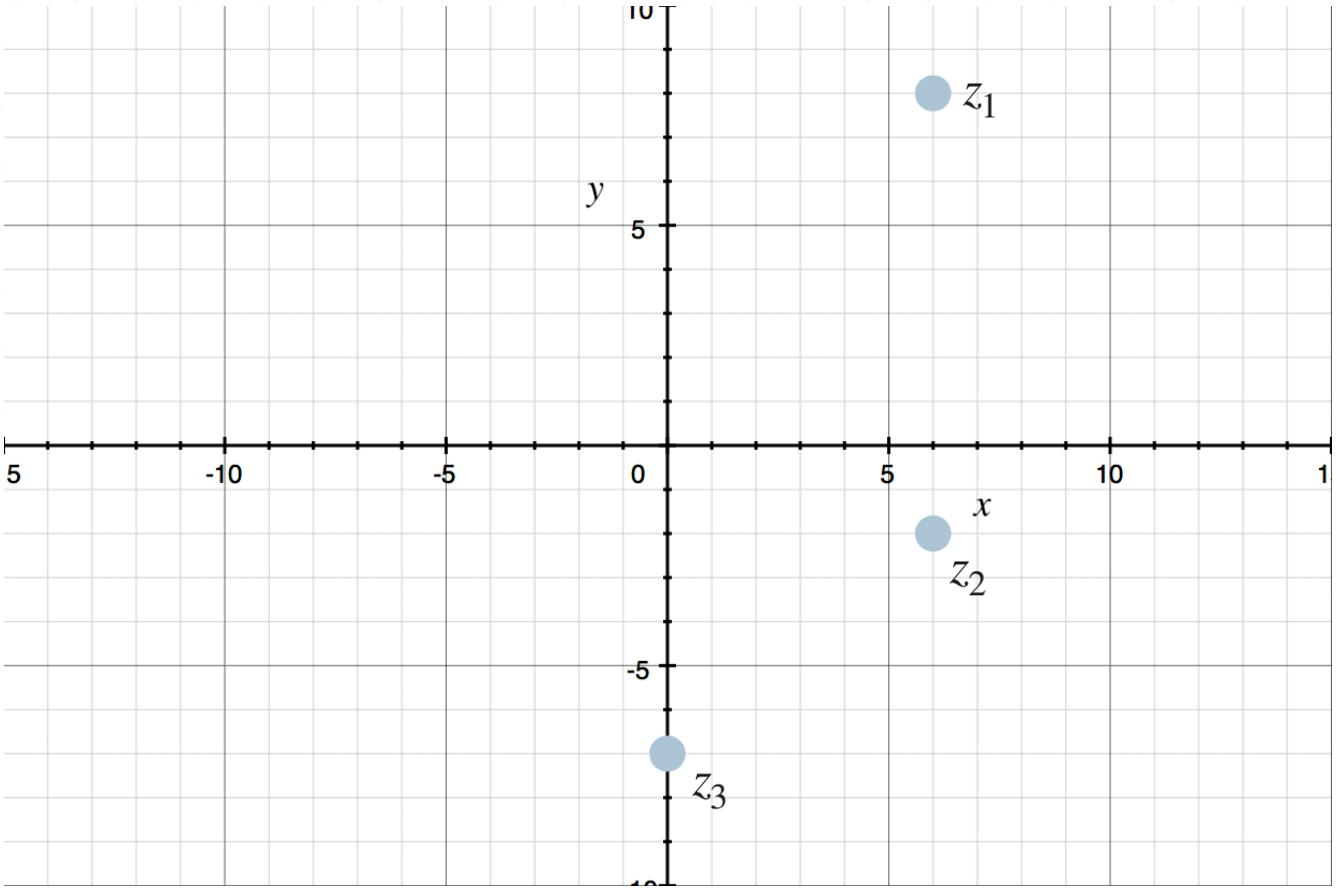
■ 6. Express the fraction in the form $a + bi$ where a and b are real numbers.

$$\frac{-3 + 7i}{4 - 5i}$$

GRAPHING COMPLEX NUMBERS

■ 1. Graph $-3+5i$, $2-4i$, and 5 in the complex plane.

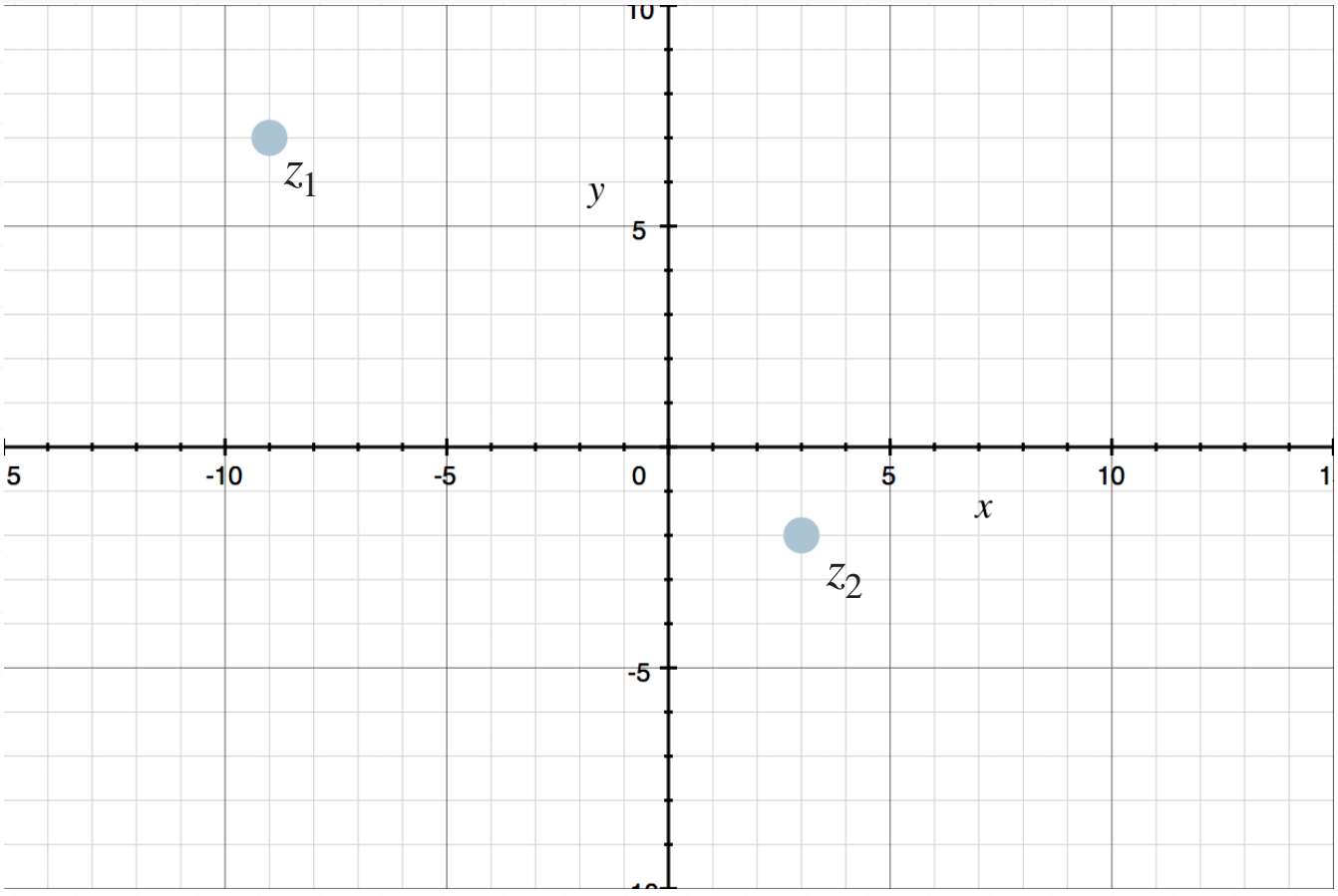
■ 2. Which three complex numbers are represented in the graph?



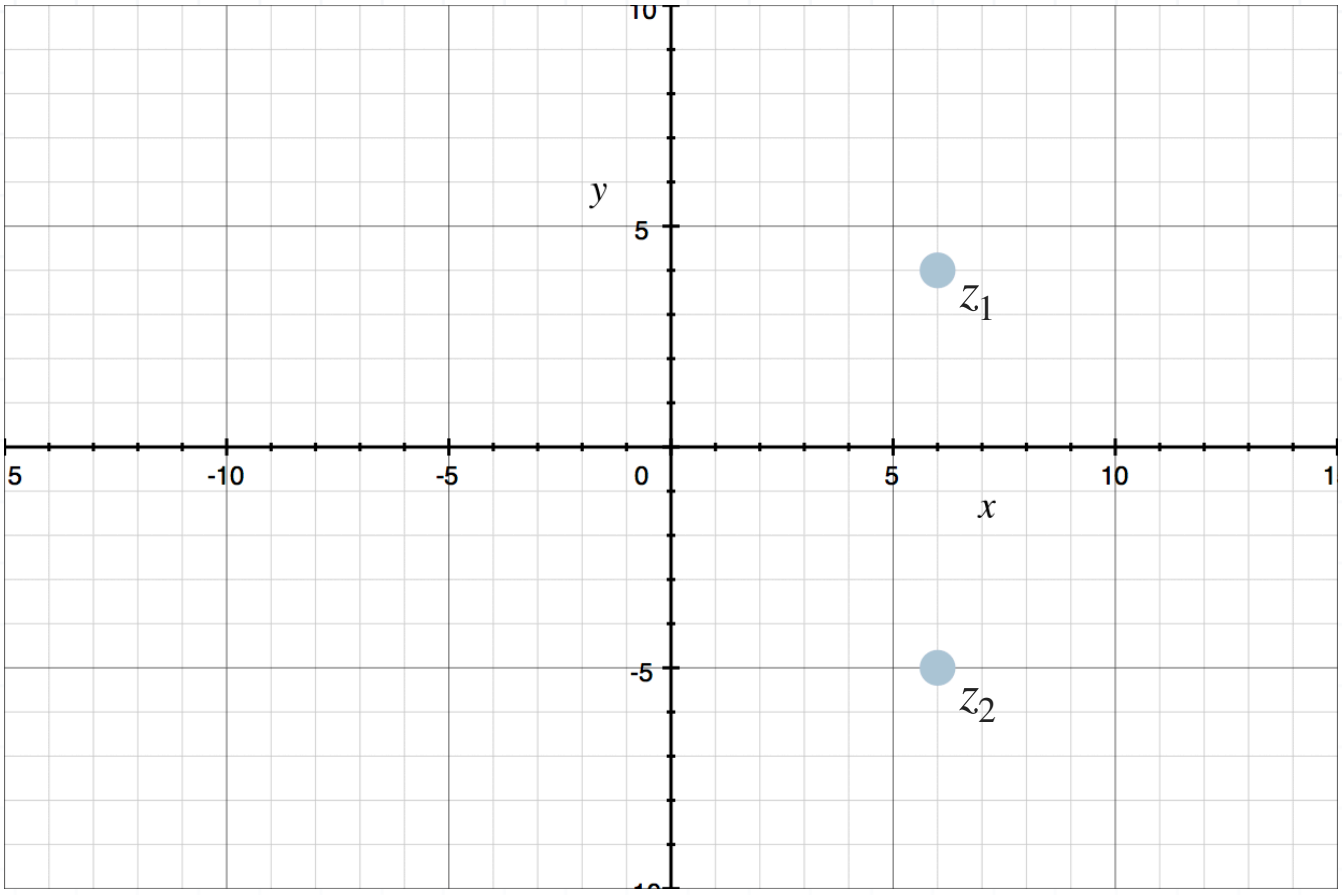
■ 3. Graph the sum of the complex numbers $5-4i$ and $-1+10i$.

■ 4. Graph the difference of the complex numbers $8-7i$ and $13-4i$.

5. Graph the sum of the complex numbers z_1 and z_2.



6. Graph the difference of the complex numbers z_1 and z_2.



DISTANCES AND MIDPOINTS

- 1. Find the distance between $s = 5 + 3i$ and $t = 1 - i$.

- 2. Find the distance between $u = -5 - 3i$ and $v = 4 + 2i$.

- 3. Find the distance between $w = 2 + 6i$ and $z = -2 - 6i$.

- 4. Find the midpoint between $s = 5 + 3i$ and $t = 1 - i$.

- 5. Find the midpoint between $u = -7 - 5i$ and $z = 2 + 2i$.

- 6. Graph the midpoint between $w = 6 + 8i$ and $z = 2 + 4i$.

COMPLEX NUMBERS IN POLAR FORM

■ 1. If the complex number $6 - 2i$ is expressed in polar form, which quadrant contains the angle θ?

■ 2. Find r for the complex number.

$-9 - 3i$

■ 3. What is the polar form of the complex number?

$5 + 12i$

■ 4. Write the complex number in polar form.

$11i$

■ 5. What is the polar form of the complex number?

$$z = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

6. Write the complex number in polar form.

-5

MULTIPLYING AND DIVIDING POLAR FORMS

■ 1. What is the product $z_1 z_2$ of the complex numbers in polar form?

$$z_1 = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$z_2 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

■ 2. What is the product $z_1 z_2$ of the complex numbers in polar form?

$$z_1 = \sqrt{3}\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right)$$

$$z_2 = \frac{\sqrt{5}}{3}\left(\cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}\right)$$

■ 3. What is the quotient z_1/z_2 of the complex numbers in polar form?

$$z_1 = 12\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$z_2 = 15\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

■ 4. What is the quotient z_1/z_2 of the complex numbers in polar form?

$$z_1 = \sqrt{7}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

$$z_2 = \frac{1}{\sqrt{2}}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

■ 5. What is the product $z_1 z_2$ of the complex numbers in polar form?

$$z_1 = \frac{\sqrt{15}}{4}\left(\cos\frac{7\pi}{2} + i\sin\frac{7\pi}{2}\right)$$

$$z_2 = \frac{1}{\sqrt{5}}\left(\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}\right)$$

■ 6. Suppose that a complex number z is the product $z_1 \cdot z_2$ of the given complex numbers. If z is expressed in polar form, $r(\cos\theta + i\sin\theta)$, where is θ located?

$$z_1 = 3\sqrt{5}\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$$

$$z_2 = 6\left(\cos\frac{7\pi}{10} + i\sin\frac{7\pi}{10}\right)$$

POWERS OF COMPLEX NUMBERS AND DE MOIVRE'S THEOREM

■ 1. Find z^5 in polar form.

$$z = 2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

■ 2. Find z^7 in polar form.

$$z = \sqrt{5}\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$$

■ 3. Find z^6 in rectangular form $a + bi$.

$$z = \frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$$

■ 4. Find z^3 in rectangular form $a + bi$.

$$z = 2\sqrt{6}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$

■ 5. Find z^5 in polar form.

$z = -4 - 4i$

■ 6. Find z^4 in rectangular form $a + bi$.

$z = \sqrt{6} - \sqrt{2}i$

COMPLEX NUMBER EQUATIONS

■ 1. Find the solutions of the complex equation.

$z^2 = 49$

■ 2. Find the solution of the complex equation that lies in the third quadrant.

$z^3 = 216$

■ 3. Find the solutions of the complex equation.

$z^4 = 256$

■ 4. Find the solutions of the complex equation.

$z^6 = 729$

■ 5. Find the solutions of the complex equation.

$z^5 = 32$

■ 6. How many solutions of the complex equation lie in the second quadrant?

$z^8 = 256$

ROOTS OF COMPLEX NUMBERS

■ 1. Find the cube roots of the complex number.

$$z = 27\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

■ 2. Find the 4th root of the complex number.

$$z = 256\left(\cos 60^\circ + i\sin 60^\circ\right)$$

■ 3. Find the 5th roots of the complex number that lies in the first quadrant of the complex plane.

$$z = 25\left(\cos 80^\circ + i\sin 80^\circ\right)$$

■ 4. Find the 4th roots of the complex number.

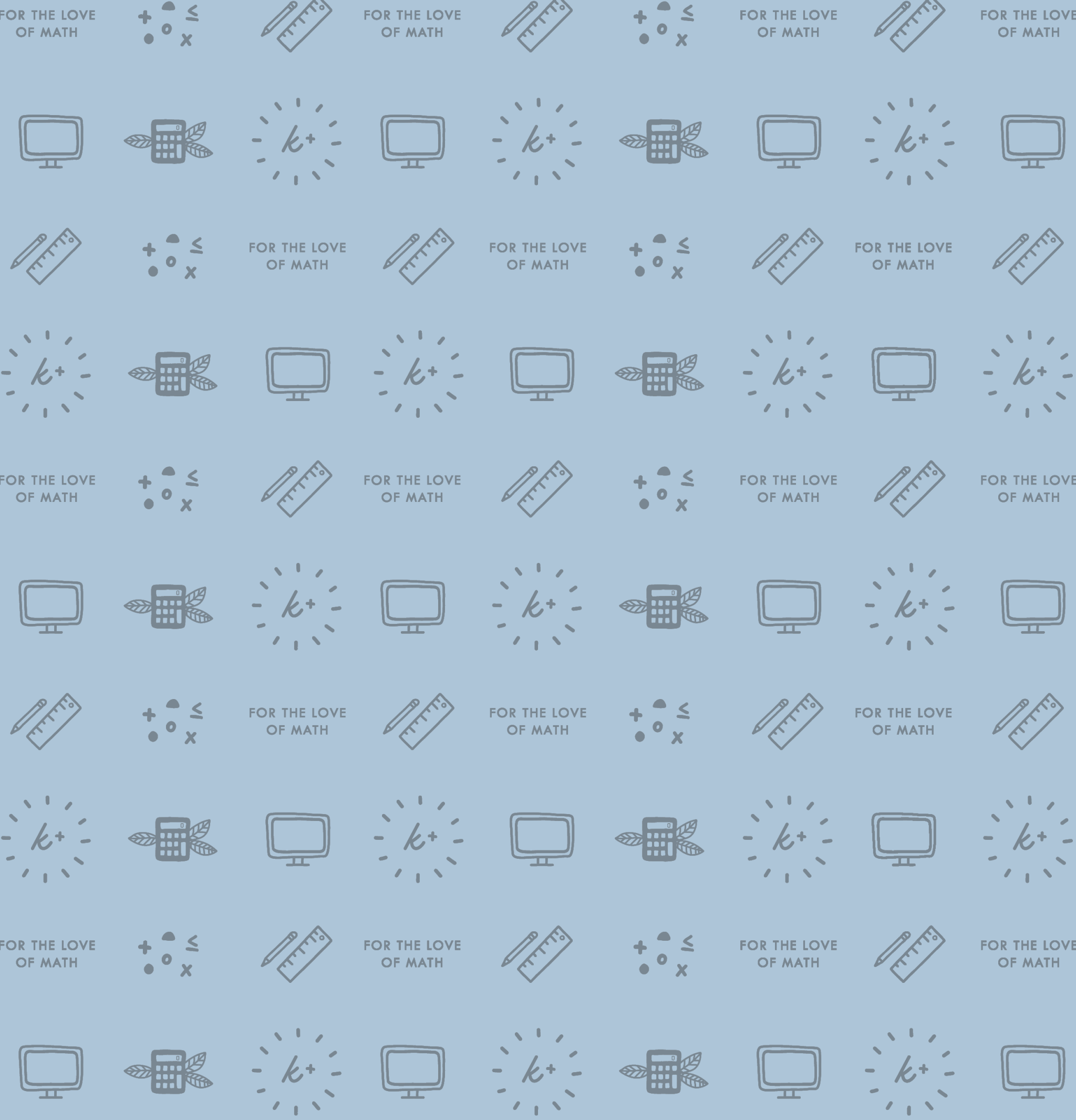
$$z = 34\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$$

■ 5. Find the 6th roots of the complex number that lie in the second quadrant of the complex plane.

$$z = 11\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

6. Find the 7th roots of the complex number.

$$z = 20\left(\cos 120^\circ + i\sin 120^\circ\right)$$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