

Topic: Matrix inverses, and invertible and singular matrices

Question: Are the matrices inverses of one another?

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix}$$

Answer choices:

- A Yes
- B No
- C There's not enough information to know



Solution: A

To find the inverse of matrix A , plug it into the formula for the inverse matrix.

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix}} \begin{bmatrix} 1 & -5 \\ -3 & 2 \end{bmatrix}$$

Find the determinant in the denominator of the fraction.

$$A^{-1} = \frac{1}{(2)(1) - (5)(3)} \begin{bmatrix} 1 & -5 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2 - 15} \begin{bmatrix} 1 & -5 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{13} \begin{bmatrix} 1 & -5 \\ -3 & 2 \end{bmatrix}$$

Then distribute the scalar across the matrix.

$$A^{-1} = \begin{bmatrix} -\frac{1}{13}(1) & -\frac{1}{13}(-5) \\ -\frac{1}{13}(-3) & -\frac{1}{13}(2) \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix}$$



Because the value we found matches matrix B , it means that matrices A and B are inverses of one another.



Topic: Matrix inverses, and invertible and singular matrices

Question: Find the inverse of matrix M .

$$M = \begin{bmatrix} 0 & -2 \\ -4 & 5 \end{bmatrix}$$

Answer choices:

A $M^{-1} = \begin{bmatrix} 0 & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{5}{8} \end{bmatrix}$

B $M^{-1} = \begin{bmatrix} -\frac{5}{8} & -\frac{1}{4} \\ -\frac{1}{2} & 0 \end{bmatrix}$

C $M^{-1} = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{2} & -\frac{5}{8} \end{bmatrix}$

D $M^{-1} = \begin{bmatrix} -\frac{5}{8} & \frac{1}{4} \\ \frac{1}{2} & 0 \end{bmatrix}$



Solution: B

Plug the values from the matrix into the formula for the inverse matrix.

$$M^{-1} = \frac{1}{|M|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$M^{-1} = \frac{1}{\begin{vmatrix} 0 & -2 \\ -4 & 5 \end{vmatrix}} \begin{bmatrix} 5 & 2 \\ 4 & 0 \end{bmatrix}$$

Find the determinant in the denominator of the fraction.

$$M^{-1} = \frac{1}{(0)(5) - (-2)(-4)} \begin{bmatrix} 5 & 2 \\ 4 & 0 \end{bmatrix}$$

$$M^{-1} = \frac{1}{0 - 8} \begin{bmatrix} 5 & 2 \\ 4 & 0 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{8} \begin{bmatrix} 5 & 2 \\ 4 & 0 \end{bmatrix}$$

Then distribute the scalar across the matrix.

$$M^{-1} = \begin{bmatrix} -\frac{1}{8}(5) & -\frac{1}{8}(2) \\ -\frac{1}{8}(4) & -\frac{1}{8}(0) \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -\frac{5}{8} & -\frac{1}{4} \\ -\frac{1}{2} & 0 \end{bmatrix}$$



Topic: Matrix inverses, and invertible and singular matrices

Question: Classify the matrix.

$$L = \begin{bmatrix} 3 & 7 \\ 0 & -1 \end{bmatrix}$$

Answer choices:

- A The matrix is invertible
- B The matrix is singular
- C The matrix is invertible and singular
- D The matrix is neither invertible nor singular



Solution: A

A matrix is either invertible or singular, it can never be both. To determine whether a matrix in the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is invertible or singular, we need to look at the ratio of a to b , compared to the ratio of c to d .

For the given matrix L ,

$$\frac{a}{b} = \frac{3}{7}$$

$$\frac{c}{d} = \frac{0}{-1} = 0$$

Because these ratios aren't equivalent, that means matrix L is invertible. If the ratios had been equivalent, the matrix would have been singular.

