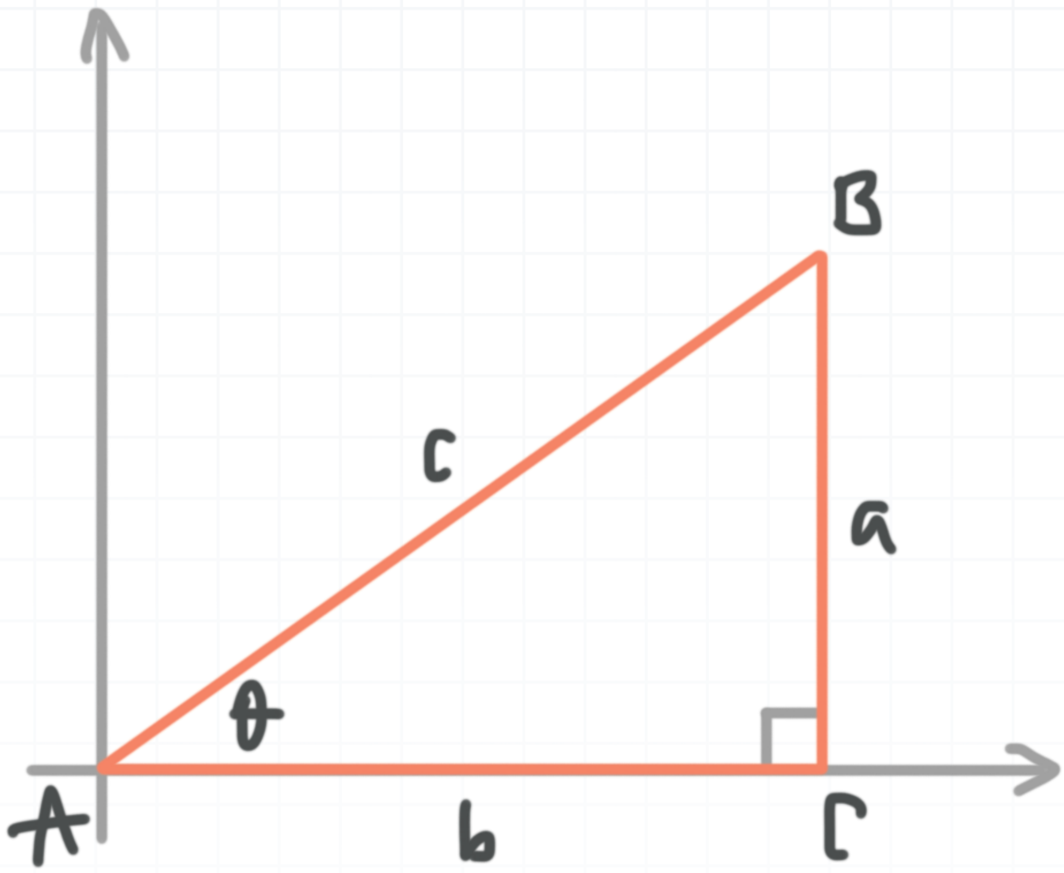
# Angles of elevation and depression

Right triangles can be used to solve all kinds of real-world problems. Now that we know how to solve right triangles, in this lesson we'll look at a really common application, angles of elevation and depression.

## Angle of elevation

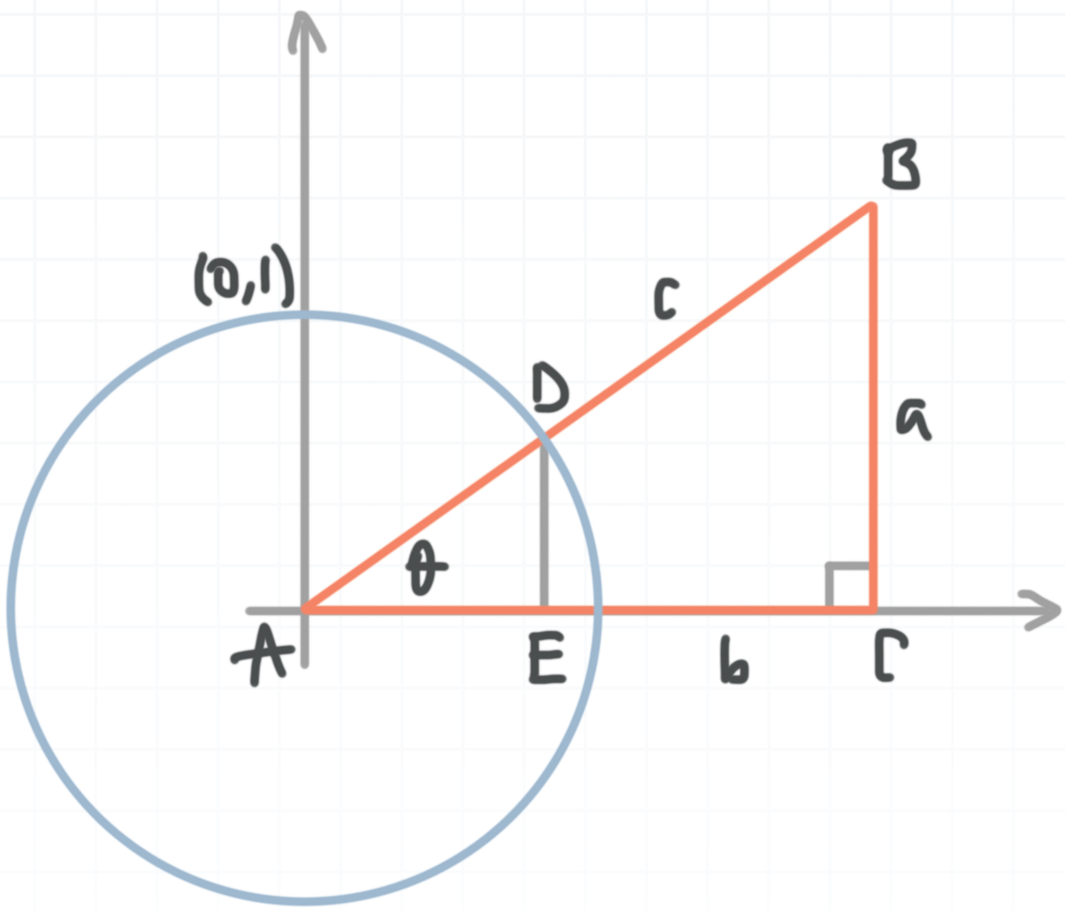
Think about an **angle of elevation** as a positive angle in standard position. The angle of elevation is always measured from the horizontal side of the angle along the positive side of the $x$-axis, *up to* the terminal side of the angle.

To translate this to the real world, let's imagine that we're standing on level ground and looking up at a bird in the air. If we place ourselves at $A$ and the bird at $B$, then we can call $C$ the point on the ground directly below the bird, then we have a right triangle in which $\theta$ is the angle of elevation.




With the figure set up this way, we can now use trigonometry to find the distance from us to the bird, or from the bird to the ground, both of which might be interesting values that we'd want to know.

To help us solve for some of these values, we can always draw the unit circle over the top of the triangle, in order to form a new triangle $ADE$, and then use the fact that triangles $ADE$ and $ABC$ are similar.




Because they're similar triangles, of course we can set up a proportion of the side lengths,

$$\frac{a}{\overline{DE}} = \frac{b}{\overline{AE}} = \frac{c}{1}$$

$$\frac{a}{\overline{DE}} = \frac{b}{\overline{AE}} = c$$

Since $D$ is on the unit circle, its coordinates are $(\cos\theta, \sin\theta)$. The coordinates of $E$ are $(\cos\theta, 0)$ and the coordinates of $A$ are $(0,0)$. Therefore, the lengths of $\overline{DE}$ and $\overline{AE}$ will always be

$$\overline{DE} = \sin\theta - 0 = \sin\theta$$

$$\overline{AE} = \cos\theta - 0 = \cos\theta$$

Then the proportion becomes

$$\frac{a}{\overline{DE}} = \frac{b}{\overline{AE}} = c$$

$$\frac{a}{\sin\theta} = \frac{b}{\cos\theta} = c$$

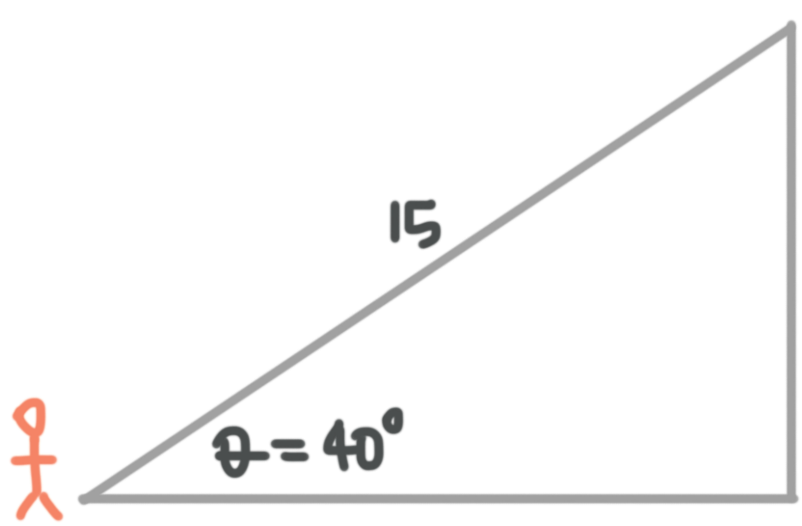
and we should be able to use this equation to calculate the lengths of the sides of the larger triangle.

Let's do a full example where we're standing on the ground and looking at a bird in the air.

### Example

If the angle of elevation of a bird with respect to the point where we're standing is $40^\circ$, and the distance between us and the bird is $15$ feet, find the height of the bird above the ground and our distance from the point on the ground that's directly below the bird.

Let's sketch out the situation.



Since the bird is $15$ feet away, $c = 15$ feet. The height of the bird above the ground is $a$, so using

$$\frac{a}{\sin\theta} = \frac{b}{\cos\theta} = c$$

we get

$$a = 15(\sin 40^\circ)$$

$$a \approx 15(0.643)$$

$$a \approx 9.65 \text{ feet}$$

Our distance from the point on the ground below the bird is $b$, so

$$b = 15(\cos 40^\circ)$$

$$b \approx 15(0.766)$$

$$b \approx 11.5 \text{ feet}$$

---

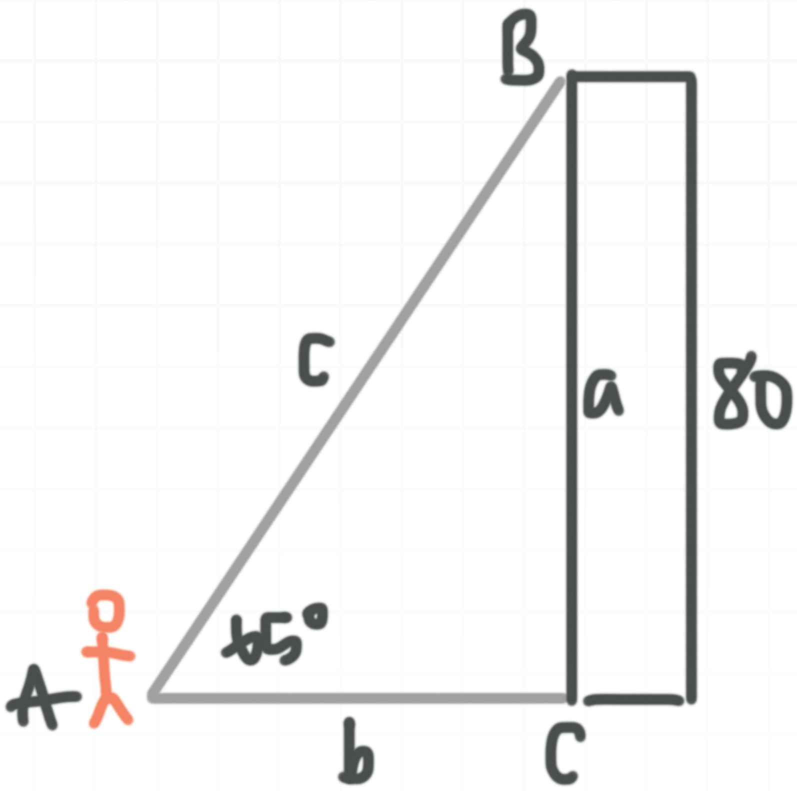
Next let's consider an example where we know the angle of elevation to the top of a building and the height of the building.

---

**Example**

If the angle of elevation from where we're standing on the ground to the top of an $80$-foot building is $65^\circ$, what's the distance between us and the top of the building, and how far do we have to walk to reach the building?

Let's sketch out the situation.




In this case, $\theta = 65^\circ$ and $a = 80$ feet. The distance between $A$ and $B$ is $c$, and the distance from $A$ to the bottom of the building is $b$. If we start with

$$\frac{a}{\sin\theta} = \frac{b}{\cos\theta} = c$$

then to find $b$, we can use

$$\frac{a}{\sin\theta} = \frac{b}{\cos\theta}$$

Multiplying both sides by $\cos\theta$, we get

$$b = \frac{a}{\sin\theta}(\cos\theta)$$

$$b \approx \frac{80}{0.906}(0.423)$$

$$b \approx 37.4 \text{ feet}$$

Now find $c$.

$$c = \frac{a}{\sin\theta}$$

$$c \approx \frac{80}{0.906}$$

$$c \approx 88.3 \text{ feet}$$

Therefore, the distance between us and the top of the building is about $88.3$ feet, and the distance between us and the bottom of the building (the distance we'd have to walk to reach it) is about $37.4$ feet.

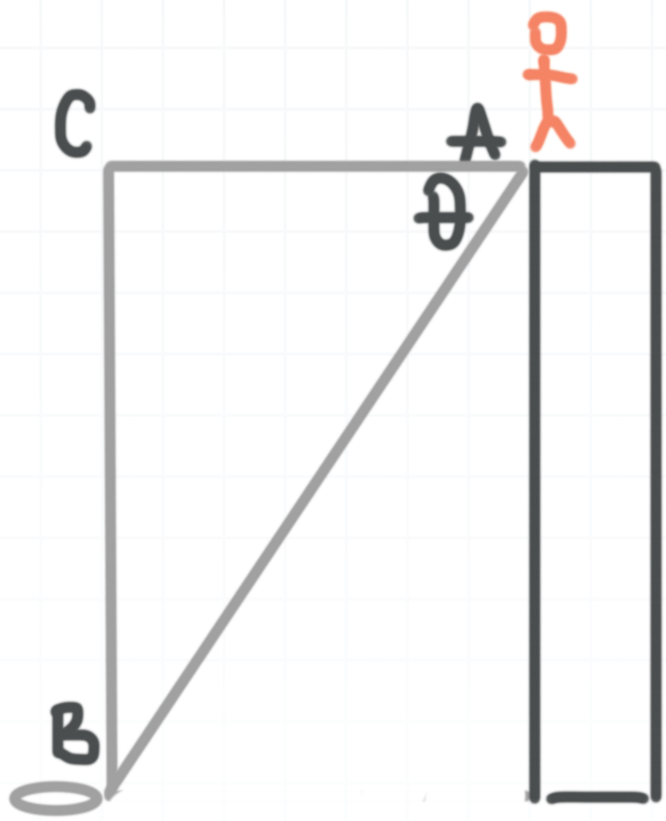
---

## Angle of depression

An **angle of depression**, in contrast to an angle of elevation, is like a negative angle in standard position.

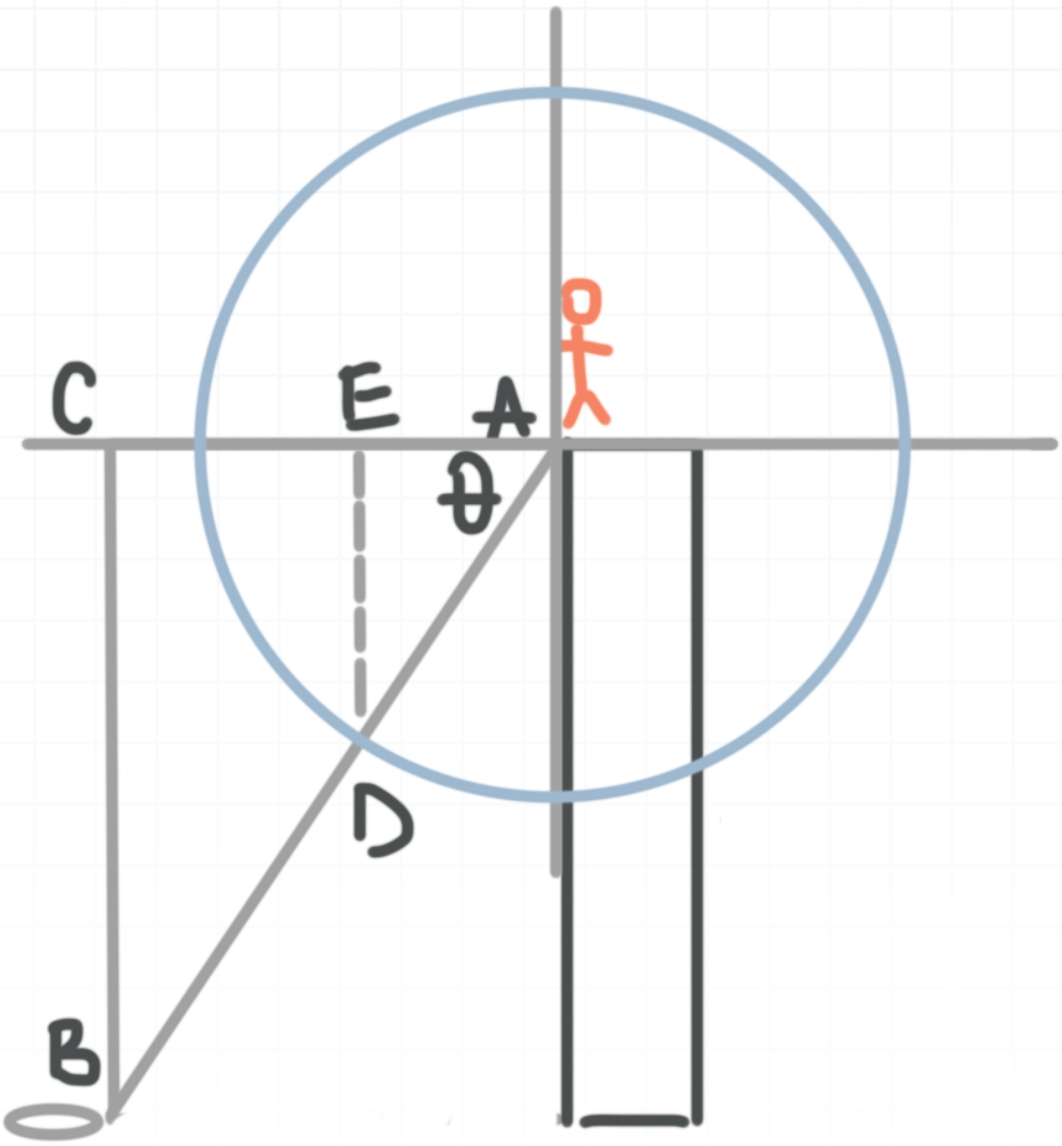
The angle of depression is always measured from the horizontal side of the angle along the positive side of the $x$-axis, *down to* the terminal side of the angle.

To translate this to the real world, let's imagine that we're standing on the top of a building and looking down at a coin on the ground. If we place ourselves at $A$ and the coin at $B$, then we can call $C$ the point in the air directly above the coin, then we have a right triangle in which $\theta$ is the angle of depression.




With the figure set up this way, we can now use trigonometry to find the distance from us to the coin, or the perfectly vertical distance between us and the coin, both of which might be interesting values that we'd want to know.

To help us solve for some of these values, we can always draw the unit circle over the top of the triangle, in order to form a new triangle $ADE$, and then use the fact that triangles $ADE$ and $ABC$ are similar.




Because they're similar triangles, of course we can set up the same proportion of the side lengths that we did for the angle of elevation triangle.

$$\frac{a}{\overline{DE}} = \frac{b}{\overline{AE}} = \frac{c}{1}$$

$$\frac{a}{\overline{DE}} = \frac{b}{\overline{AE}} = c$$

Since $D$ is on the unit circle, its coordinates are $(\cos\theta, \sin\theta)$. The coordinates of $E$ are $(\cos\theta, 0)$ and the coordinates of $A$ are $(0,0)$. Therefore, the lengths of $\overline{DE}$ and $\overline{AE}$ will still always be,

$$\overline{DE} = \sin\theta - 0 = \sin\theta$$

$$\overline{AE} = \cos\theta - 0 = \cos\theta$$

so the proportion becomes

$$\frac{a}{\overline{DE}} = \frac{b}{\overline{AE}} = c$$

$$\frac{a}{\sin\theta} = \frac{b}{\cos\theta} = c$$

and we should be able to use this equation to calculate the lengths of the sides of the larger triangle.

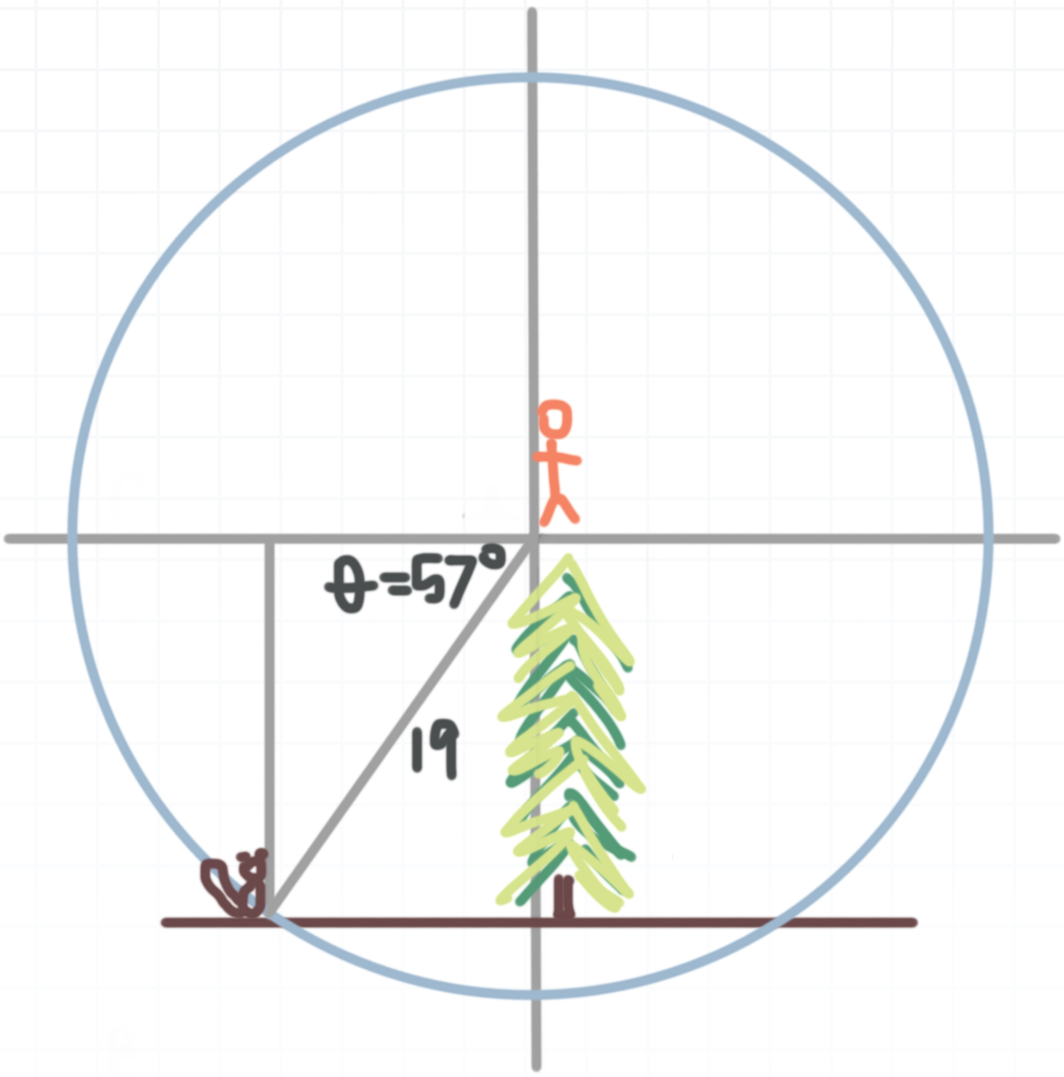
Let's look at an example where we use the angle of depression.

---

**Example**

We've climbed a tree and just spotted a squirrel on the ground. The angle of depression from us to the squirrel is $57^\circ$, and the distance between us is $19$ feet. How far are we from the squirrel, both vertically and horizontally?

The angle of depression is $\theta = 57^\circ$ and the distance from us to the squirrel is $c = 19$ feet.




The vertical distance between us in the tree and the squirrel on the ground is $a$, so

$$\frac{a}{\sin\theta} = c$$

$$\frac{a}{\sin 57^\circ} = 19$$

$$a = 19(\sin 57^\circ)$$

$$a \approx 19(0.839)$$

$a \approx 15.9$ feet

The horizontal distance from us to the squirrel is $b$, so

$$\frac{b}{\cos\theta} = c$$

$$\frac{b}{\cos 57^\circ} = 19$$

$$b = 19(\cos 57^\circ)$$

$$b \approx 19(0.545)$$

$b \approx 10.4$ feet

So your vertical distance from us in the three to the squirrel on the ground is $15.9$ feet, and the horizontal distance from us to the squirrel is $10.4$ feet.