Topic: Coterminal angles

Question: Which angle is coterminal with $-150°$?

Answer choices:

A $120°$

B $-420°$

C $570°$

D $230°$

Solution: C

We'll check each of the answer choices. We're looking for the angle that differs from $-150°$ by an integer multiple of $360°$.

There's only one answer choice that's a negative angle, so we'll check the negative angle first. If we subtract $360°$ from $-150°$, we get

$$-150° - 360°$$

$$-510°$$

So $-420°$ isn't coterminal with $-150°$.

Now let's check the three positive angle choices just by adding $360°$ a few times to $-150°$.

$$-150° + 1(360°) = 210°$$

$$-150° + 2(360°) = 570°$$

$$-150° + 3(360°) = 930°$$

The only angle that matches up is $570°$.

Topic: Coterminal angles

Question: Which angle is not coterminal with $-8\pi/3$?

Answer choices:

A $\frac{4\pi}{3}$

B $-\frac{4\pi}{3}$

C $-\frac{26\pi}{3}$

D $\frac{22\pi}{3}$

Solution: B

To find angles coterminal with $-8\pi/3$, we'll add or subtract multiples of 2π from the angle. Because the angle $-8\pi/3$ is given in thirds (the denominator is 3), it'll be easier to express 2π in thirds, so we'll rewrite 2π as

$$2\pi\left(\frac{3}{3}\right)$$

$$\frac{6\pi}{3}$$

So let's start by subtracting multiples of $6\pi/3$ from $-8\pi/3$ to find negative coterminal angles.

$$-\frac{8\pi}{3}-1\left(\frac{6\pi}{3}\right)=-\frac{14\pi}{3}$$

$$-\frac{8\pi}{3}-2\left(\frac{6\pi}{3}\right)=-\frac{20\pi}{3}$$

$$-\frac{8\pi}{3}-3\left(\frac{6\pi}{3}\right)=-\frac{26\pi}{3}$$

No we'll add multiples of $6\pi/3$ to $-8\pi/3$ to find more coterminal angles.

$$-\frac{8\pi}{3}+1\left(\frac{6\pi}{3}\right)=-\frac{2\pi}{3}$$

$$-\frac{8\pi}{3}+2\left(\frac{6\pi}{3}\right)=\frac{4\pi}{3}$$

$$-\frac{8\pi}{3}+3\left(\frac{6\pi}{3}\right)=\frac{10\pi}{3}$$

$$-\frac{8\pi}{3}+4\left(\frac{6\pi}{3}\right)=\frac{16\pi}{3}$$

$$-\frac{8\pi}{3}+5\left(\frac{6\pi}{3}\right)=\frac{22\pi}{3}$$

We've seen that each of the answer choices is coterminal with $-8\pi/3$, except $-4\pi/3$.

Topic: Coterminal angles

Question: Which angle α is coterminal with $71\pi/16$ if we find α by starting at $71\pi/16$ and making two full negative rotations?

Answer choices:

A $\alpha = \frac{7\pi}{16}$

B $\alpha = -\frac{7\pi}{16}$

C $\alpha = \frac{39\pi}{16}$

D $\alpha = -\frac{25\pi}{16}$

Solution: A

One full negative rotation is given by -2π, so two full negative rotations is given by -4π. Therefore, if we start with $71\pi/16$ and make two full negative rotations, we'll get an angle of

$$\alpha = \frac{71\pi}{16} - 4\pi$$

$$\alpha = \frac{71\pi}{16} - 4\pi\left(\frac{16}{16}\right)$$

$$\alpha = \frac{71\pi}{16} - \frac{64\pi}{16}$$

$$\alpha = \frac{71\pi - 64\pi}{16}$$

$$\alpha = \frac{7\pi}{16}$$