Topic: Proving the trig equation

Question: Choose the equivalent expression.

$$\frac{\sin\theta + \sin(3\theta)}{\cos\theta + \cos(3\theta)}$$

Answer choices:

A $\sin(2\theta)$

B $\tan(2\theta)$

C $\cos(2\theta)$

D $\cot(2\theta)$

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Solution: B

Work on the numerator of the expression. By a sum-to-product identity,

$$\sin\theta + \sin(3\theta) = 2\sin\left(\frac{\theta + 3\theta}{2}\right)\cos\left(\frac{\theta - 3\theta}{2}\right)$$

$$\sin\theta + \sin(3\theta) = 2\sin\left(\frac{4\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)$$

$$\sin\theta + \sin(3\theta) = 2\sin(2\theta)\cos(-\theta)$$

Work on the denominator of the expression. By a sum-to-product identity,

$$\cos\theta + \cos(3\theta) = 2\cos\left(\frac{\theta + 3\theta}{2}\right)\cos\left(\frac{\theta - 3\theta}{2}\right)$$

$$\cos\theta + \cos(3\theta) = 2\cos\left(\frac{4\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)$$

$$\cos\theta + \cos(3\theta) = 2\cos(2\theta)\cos(-\theta)$$

Replacing both the numerator and denominator with these results gives

$$\frac{\sin\theta + \sin(3\theta)}{\cos\theta + \cos(3\theta)} = \frac{2\sin(2\theta)\cos(-\theta)}{2\cos(2\theta)\cos(-\theta)}$$

$$\frac{\sin\theta + \sin(3\theta)}{\cos\theta + \cos(3\theta)} = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$\frac{\sin\theta + \sin(3\theta)}{\cos\theta + \cos(3\theta)} = \tan(2\theta)$$

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Topic: Proving the trig equation

Question: Choose the equivalent expression.

$$\sin\left(\frac{\theta}{4}\right)\cos\left(\frac{3\theta}{4}\right)$$

Answer choices:

A $\quad \frac{1}{2}\left[\sin\theta+\sin\left(\frac{\theta}{2}\right)\right]$

B $\quad \frac{1}{2}\left[\sin\theta-\sin\left(\frac{\theta}{2}\right)\right]$

C $\quad \frac{1}{2}\left[\cos\theta-\cos\left(\frac{\theta}{2}\right)\right]$

D $\quad \frac{1}{2}\left[\cos\theta+\cos\left(\frac{\theta}{2}\right)\right]$

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Solution: B

Using a product-to-sum identity, we can rewrite the expression as

$$\frac{1}{2}\left[\sin\left(\frac{\theta}{4}+\frac{3\theta}{4}\right)+\sin\left(\frac{\theta}{4}-\frac{3\theta}{4}\right)\right]$$

$$\frac{1}{2}\left[\sin\left(\frac{\theta+3\theta}{4}\right)+\sin\left(\frac{\theta-3\theta}{4}\right)\right]$$

$$\frac{1}{2}\left[\sin\left(\frac{4\theta}{4}\right)+\sin\left(\frac{-2\theta}{4}\right)\right]$$

$$\frac{1}{2}\left[\sin\theta+\sin\left(-\frac{\theta}{2}\right)\right]$$

Using the odd identity for sine, we get

$$\frac{1}{2}\left[\sin\theta-\sin\left(\frac{\theta}{2}\right)\right]$$

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Topic: Proving the trig equation

Question: Choose the equivalent expression.

$$\tan^2 x - \sin^2 x$$

Answer choices:

A $\sin^2 x \tan^2 x$

B $\cos^2 x \cot^2 x$

C $(1 - \sin^2 x)(\tan^2 x)$

D $\sin x \tan x$

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Solution: A

Use the quotient identity to rewrite the tangent function.

$$\tan^2 x - \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

Factor out the sine function, then find a common denominator and combine the fractions.

$$\sin^2 x \left(\frac{1}{\cos^2 x} - 1 \right)$$

$$\sin^2 x \left(\frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} \right)$$

$$\sin^2 x \left(\frac{1 - \cos^2 x}{\cos^2 x} \right)$$

Use the Pythagorean identity with sine and cosine for the numerator of the fraction.

$$\sin^2 x \left(\frac{\sin^2 x}{\cos^2 x} \right)$$

$$\sin^2 x \tan^2 x$$

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