Topic: Sum-difference identities for sine and cosine

Question: Simplify the sum.

$$(-\sin 70^\circ)(\cos 42^\circ) + (\cos 70^\circ)\sin(42^\circ)$$

Answer choices:

A $\cos 112^\circ$

B $\sin 28^\circ$

C $\sin(-28^\circ)$

D $\sin(-112^\circ)$

Solution: C

The expression is in the form

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

Substitute the angles from the expression.

$$\sin(-70^\circ + 42^\circ) = (\sin(-70^\circ))(\cos 42^\circ) + (\cos(-70^\circ))(\sin 42^\circ)$$

By the odd identity $\sin\theta = -\sin(-\theta)$ and the even identity $\cos\theta = \cos(-\theta)$, this equation becomes

$$\sin(-70^\circ + 42^\circ) = (-\sin 70^\circ)(\cos 42^\circ) + (\cos 70^\circ)(\sin 42^\circ)$$

$$\sin(-28^\circ) = (-\sin 70^\circ)(\cos 42^\circ) + (\cos 70^\circ)(\sin 42^\circ)$$

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Question: Let θ be an angle in the second quadrant whose sine is $1/3$, and let α be an angle in the fourth quadrant whose cosine is $2/\sqrt{5}$. What are the exact values of $\sin(\theta+\alpha)$ and $\cos(\theta-\alpha)$?

Answer choices:

A $\quad \sin(\theta+\alpha)=\dfrac{2+2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=-\dfrac{4\sqrt{2}+1}{3\sqrt{5}}$

B $\quad \sin(\theta+\alpha)=\dfrac{2-2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=-\dfrac{4\sqrt{2}+1}{3\sqrt{5}}$

C $\quad \sin(\theta+\alpha)=\dfrac{2+2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=\dfrac{4\sqrt{2}-1}{3\sqrt{5}}$

D $\quad \sin(\theta+\alpha)=-\dfrac{2+2\sqrt{2}}{3\sqrt{5}} \qquad \cos(\theta-\alpha)=\dfrac{4\sqrt{2}-1}{3\sqrt{5}}$

Solution: A

Rewrite the Pythagorean identity with sine and cosine $\sin^2\theta + \cos^2\theta = 1$ as

$$\cos^2\theta = 1 - \sin^2\theta$$

Substitute $\sin\theta = 1/3$.

$$\cos^2\theta = 1 - \left(\frac{1}{3}\right)^2$$

$$\cos^2\theta = 1 - \frac{1}{9}$$

$$\cos^2\theta = \frac{8}{9}$$

$$\cos\theta = \pm\sqrt{\frac{8}{9}}$$

Since θ is in the second quadrant, $\cos\theta$ is negative.

$$\cos\theta = -\sqrt{\frac{8}{9}} = -\frac{\sqrt{8}}{\sqrt{9}} = -\frac{\sqrt{4(2)}}{3} = -\frac{2\sqrt{2}}{3}$$

Rewrite the Pythagorean identity with sine and cosine $\sin^2\alpha + \cos^2\alpha = 1$ as

$$\sin^2\alpha = 1 - \cos^2\alpha$$

Substitute $\cos\alpha = 2/\sqrt{5}$.

$$\sin^2\alpha = 1 - \left(\frac{2}{\sqrt{5}}\right)^2$$

$$\sin^2\alpha = 1 - \frac{4}{5}$$

$$\sin^2\alpha = \frac{1}{5}$$

$$\sin\alpha = \pm\sqrt{\frac{1}{5}}$$

Since α is in the fourth quadrant, $\sin\alpha$ is negative.

$$\sin\alpha = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}}$$

By the sum identity for the sine function,

$$\sin(\theta+\alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

$$\sin(\theta+\alpha) = \left(\frac{1}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(-\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{\sqrt{5}}\right)$$

$$\sin(\theta+\alpha) = \frac{2}{3\sqrt{5}} + \frac{2\sqrt{2}}{3\sqrt{5}}$$

$$\sin(\theta+\alpha) = \frac{2+2\sqrt{2}}{3\sqrt{5}}$$

By the difference identity for the cosine function,

$$\cos(\theta-\alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha$$

$$\cos(\theta - \alpha) = \left(-\frac{2\sqrt{2}}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{1}{3}\right)\left(-\frac{1}{\sqrt{5}}\right)$$

$$\cos(\theta - \alpha) = -\frac{4\sqrt{2}}{3\sqrt{5}} - \frac{1}{3\sqrt{5}}$$

$$\cos(\theta - \alpha) = -\frac{1 + 4\sqrt{2}}{3\sqrt{5}}$$

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Question: Simplify the expression.

$$\cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{11\pi}{6}\right)+\sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{11\pi}{6}\right)$$

Answer choices:

A $-\frac{\sqrt{3}}{2}$

B $-\frac{1}{2}$

C 1

D $\frac{1}{2}$

Solution: A

The expression is in the form of the difference identity for cosine,

$$\cos(\theta - \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha$$

so we'll substitute the angles from the expression.

$$\cos\left(\frac{11\pi}{6} - \frac{2\pi}{3}\right) = \cos\left(\frac{11\pi}{6}\right)\cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{11\pi}{6}\right)\sin\left(\frac{2\pi}{3}\right)$$

$$\cos\left(\frac{11\pi}{6} - \frac{4\pi}{6}\right) = \cos\left(\frac{11\pi}{6}\right)\cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{11\pi}{6}\right)\sin\left(\frac{2\pi}{3}\right)$$

$$\cos\left(\frac{7\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$