Topic: The Pythagorean identities

Question: If $\cos(236^\circ) = -0.559$, find the value of $\sin(236^\circ)$.

Answer choices:

A $\sin(236^\circ) \approx 0.688$

B $\sin(236^\circ) \approx -0.688$

C $\sin(236^\circ) \approx 0.829$

D $\sin(236^\circ) \approx -0.829$

Solution: D

We'll use a rewritten form of the Pythagorean identity with sine and cosine.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2(236^\circ) = 1 - \cos^2(236^\circ)$$

$$\sin^2(236^\circ) = 1 - (-0.559)^2$$

$$\sin^2(236^\circ) \approx 1 - 0.312$$

$$\sin^2(236^\circ) \approx 0.688$$

$$\sin(236^\circ) \approx \pm\sqrt{0.688}$$

The angle 236° lies in the third quadrant, and all points in the third quadrant have a negative y-value, which means that the sine of any angle in the third quadrant will be negative. So we ignore the positive value and say

$$\sin(236^\circ) \approx -\sqrt{0.688}$$

$$\sin(236^\circ) \approx -0.829$$

Topic: The Pythagorean identities

Question: Which equation is not a Pythagorean identity?

Answer choices:

A $\sec^2\theta - \tan^2\theta = 1$

B $a^2 + b^2 = c^2$

C $\cot^2\theta + 1 = \csc^2\theta$

D $\sin^2\theta + \cos^2\theta = 1$

Solution: B

We know that the three Pythagorean identities are

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

We can rewrite the third identity as

$$\sec^2\theta - \tan^2\theta = 1$$

The equation $a^2 + b^2 = c^2$ is the Pythagorean Theorem, which tells us that, for any right triangle, the sum of the squares of the side lengths is equal to the square of the length of the hypotenuse. But the Pythagorean Theorem is not specifically one of the three Pythagorean identities.

Topic: The Pythagorean identities

Question: Find the value of the expression.

$$\frac{1}{\cos^2(25^\circ)} - \tan^2(25^\circ)$$

Answer choices:

A -1

B 0

C 1

D $\frac{\sqrt{2}}{2}$

Solution: C

Rewrite the expression using the reciprocal identity for secant.

$$\frac{1}{\cos^2(25^\circ)} - \tan^2(25^\circ)$$

$$\sec^2(25^\circ) - \tan^2(25^\circ)$$

Then the Pythagorean identity $1 + \tan^2\theta = \sec^2\theta$, which can be rewritten as $\sec^2\theta - \tan^2\theta = 1$, tells us that the value of the expression is 1.