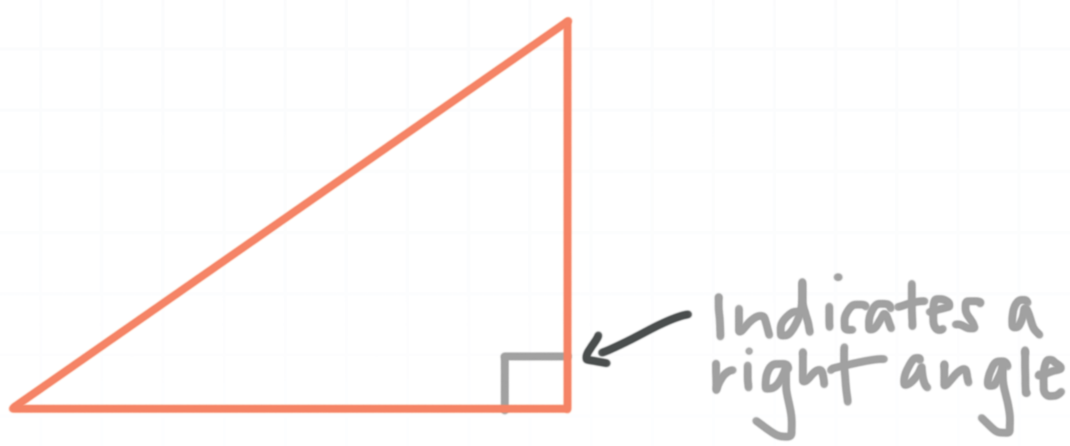
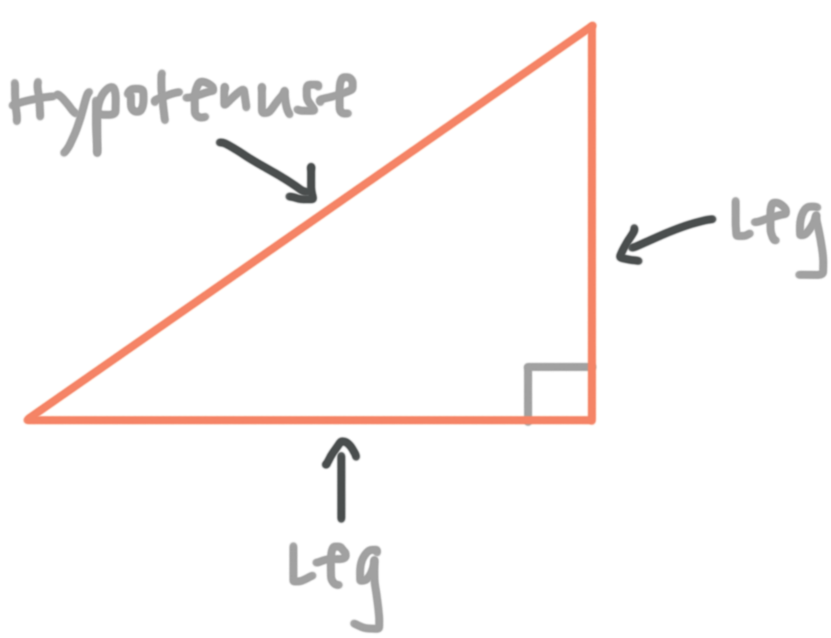
Sine, cosine, and tangent

Remember previously that we talked about different kinds of angles, including right angles, which were angles that measured exactly 90° or $\pi/2$ radians.

Based on this definition of a right angle, if we say that a triangle is a **right triangle**, that means the triangle includes exactly one 90° interior angle. We indicate a right angle with a little square.



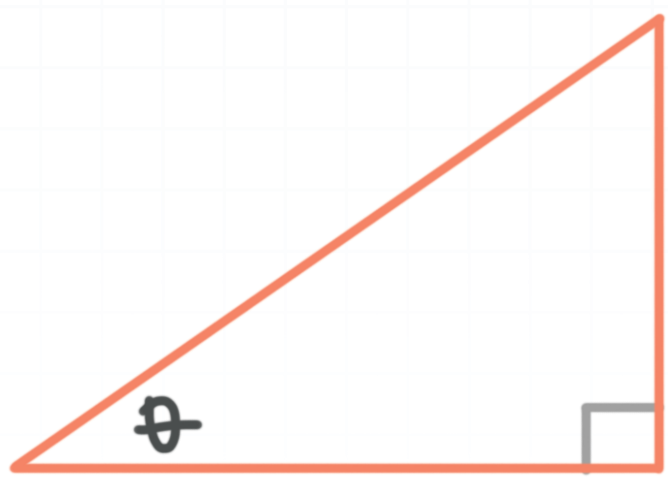
The side opposite the right angle is the **hypotenuse**, and it will always be the longest side. The other two sides are the legs.



The first three trigonometric functions

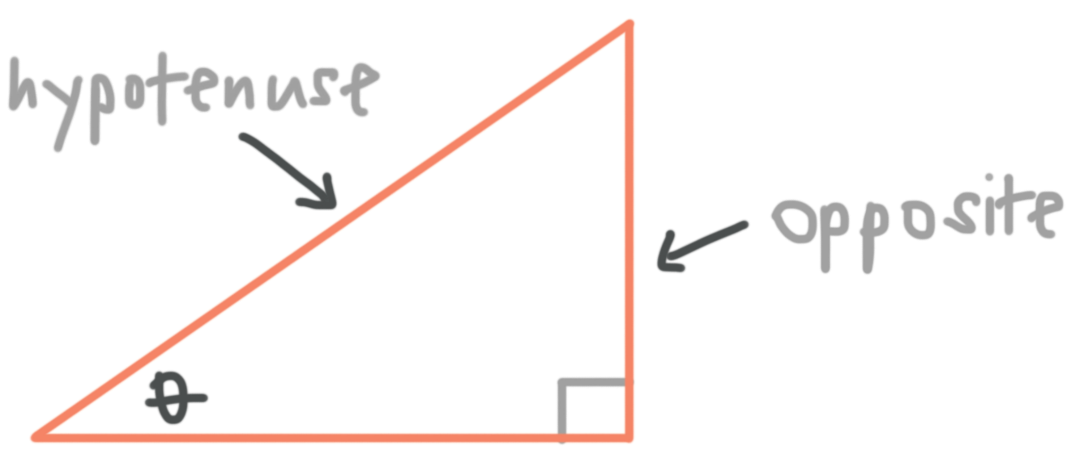
The word "**trigonometry**" literally means "the study of triangles." And remember that a function like $y = x^2$ is an equation that gives the relationship between the variables x and y. So if we bring those ideas together, we can say that a **trigonometric function** is a function that gives the relationship between different parts of a triangle.

In fact, there are six trigonometric functions, and we can easily define the first three using the parts of a right triangle. In a right triangle like this,

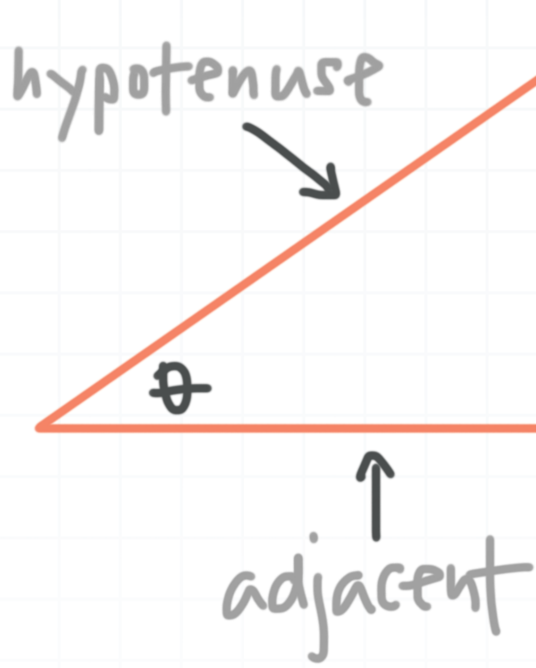


where we show the right angle, and define the angle θ as one of the other angles, then the sine of that angle θ is equivalent to the length of the side opposite the angle θ, divided by the length of the hypotenuse.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

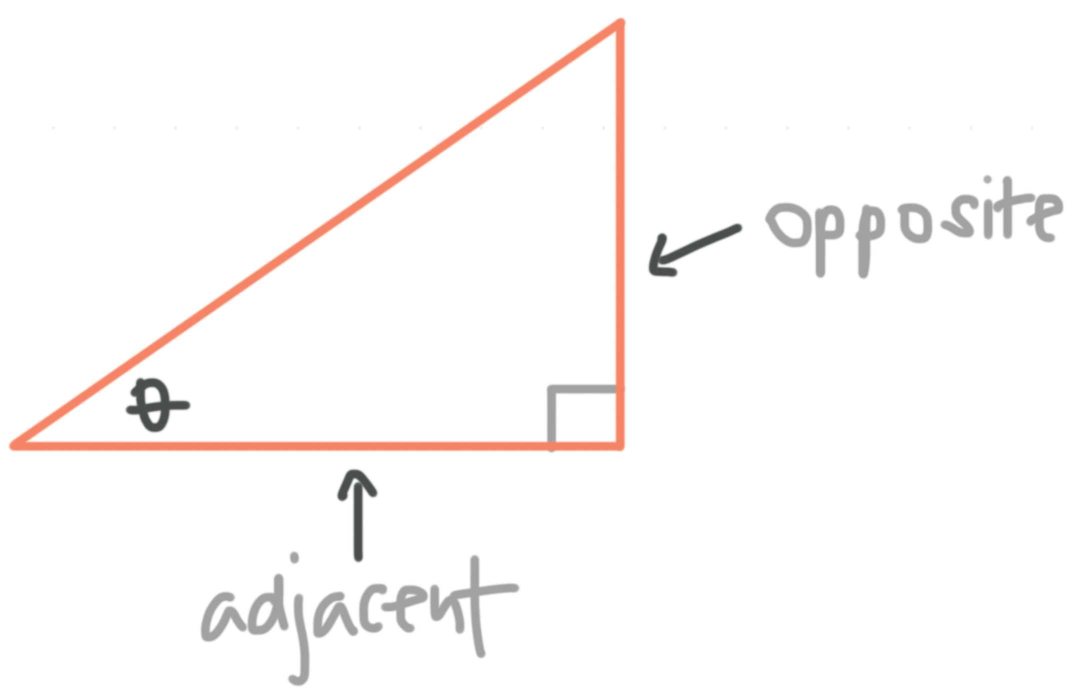


The cosine of that angle θ is equivalent to the length of the side adjacent to the angle θ, divided by the length of the hypotenuse.



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

And the tangent of that angle θ is equivalent to the length of the side opposite the angle θ, divided by the length of the side adjacent to the angle θ.



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Be careful here. These first three trig functions, sine, cosine, and tangent, which we abbreviate as $\sin$, $\cos$, and $\tan$, are functions, just like a function

$f(x)$. When we're given $f(x)$, it doesn't indicated that f is multiplied by x; instead it means that f is a function of x, or that we can plug x into f. In the same way, $\sin\theta$ doesn't mean that $\sin$ is multiplied by θ; instead it means that $\sin$ is a function of θ, or that we can plug θ into $\sin$.

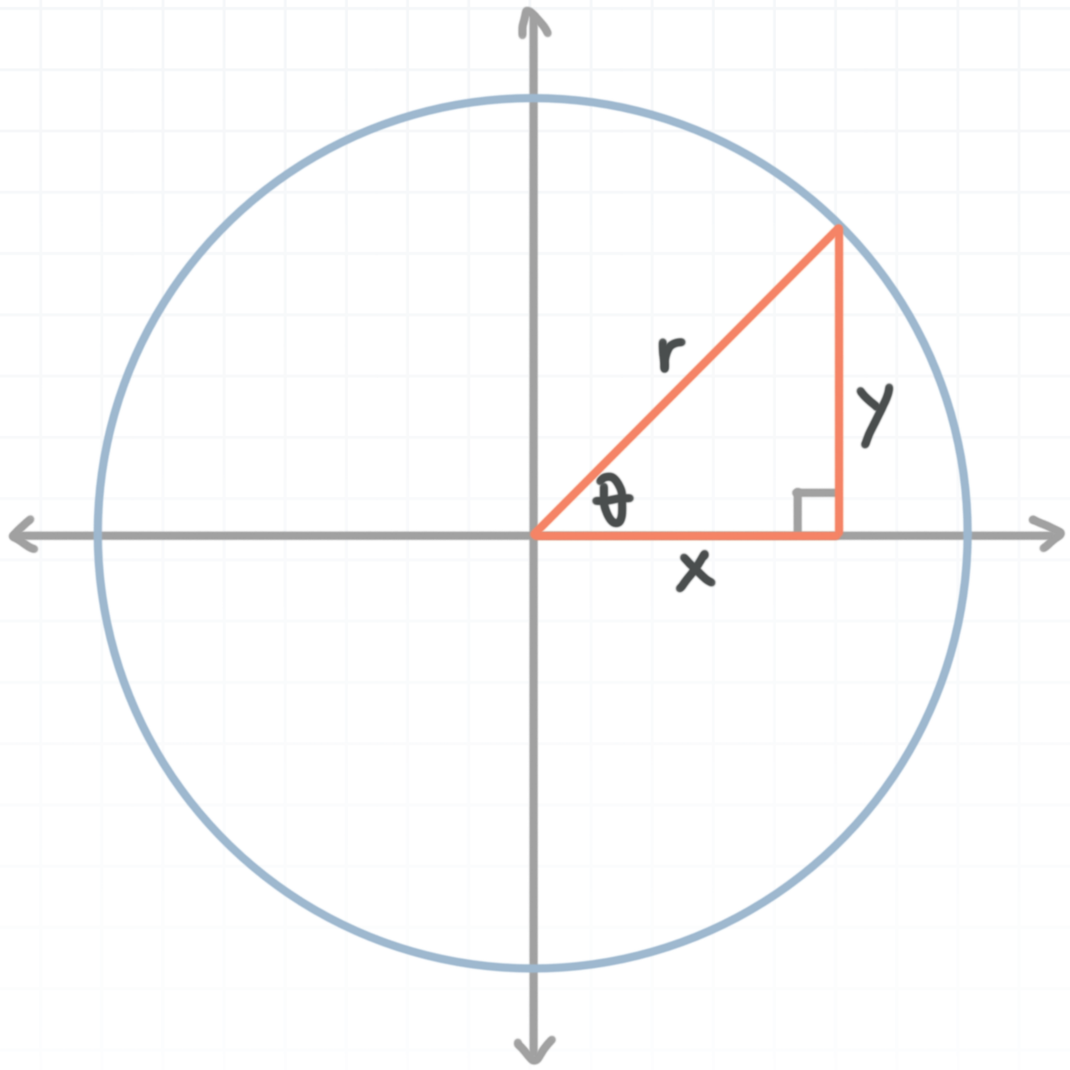
To remember the definition of these three trig functions, remember SOH-CAH-TOA.

SOH: sine, opposite, hypotenuse

CAH: cosine, adjacent, hypotenuse

TOA: tangent, opposite, adjacent

We can also place this same right triangle in the coordinate plane with the angle θ at the origin. If we sketch out a circle around the triangle, then the hypotenuse becomes the radius of the circle, and we can call the three sides of the triangle x, y, and r, where x is always the adjacent side, y is always the opposite side, and r is always the hypotenuse.



In this context, we can also define the first three trig functions as

$$\sin\theta = \frac{y}{r}$$

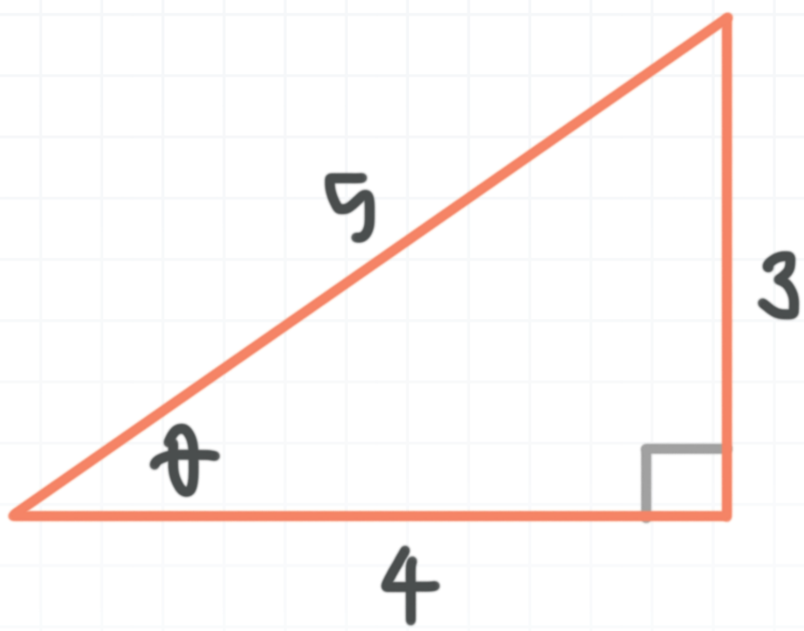
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

Let's calculate these first three trig functions for a particular triangle.

Example

Find the values of the sine, cosine, and tangent functions for θ.



Given the position of the angle θ in the right triangle, the length of the opposite side is 3, the length of the adjacent side is 4, and the length of the hypotenuse is 5.

Then the values of sine, cosine, and tangent for the angle are

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$