Trigonometry Final Exam Solutions

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Trigonometry Final Exam Answer Key

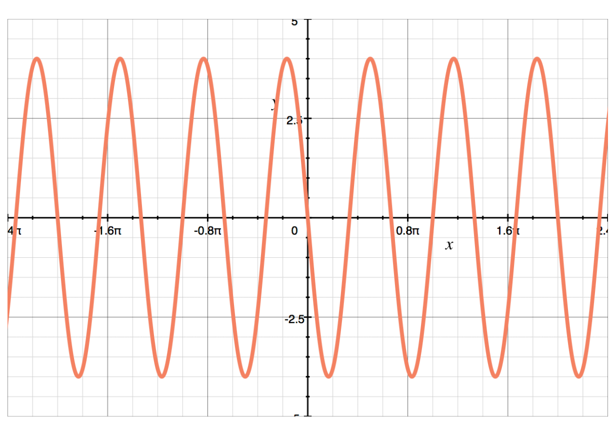
1. (5 pts)	A	B	C	■	E
2. (5 pts)	■	B	C	D	E
3. (5 pts)	A	■	C	D	E
4. (5 pts)	A	B	C	D	■
5. (5 pts)	A	B	■	D	E
6. (5 pts)	A	B	C	■	E
7. (5 pts)	■	B	C	D	E
8. (5 pts)	A	■	C	D	E

9. (15 pts) 817 ft

10. (15 pts) $c \approx 38.21$, $A \approx 23.98^\circ$, $B = 42.02^\circ$

11. (15 pts) $\sin(-22^\circ) = (-\sin 60^\circ)(\cos 38^\circ) + (\cos 60^\circ)(\sin 38^\circ)$

12. (15 pts)



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1. D. Use Heron's formula to find the area of the triangle. If a, b, and c are the lengths of the sides of the triangle, then half the perimeter is

$$s = \frac{1}{2}(a + b + c)$$

$$s = \frac{1}{2}(9 + 10 + 11)$$

$$s = \frac{1}{2}(30)$$

$$s = 15$$

Plugging this value into Heron's formula, we get

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

$$A = \sqrt{15(15 - 9)(15 - 10)(15 - 11)}$$

$$A = \sqrt{15(6)(5)(4)}$$

$$A = \sqrt{1{,}800}$$

$$A \approx 42$$

2. A. The angular velocity is

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$$\omega = \frac{12 \text{ mi}}{12 \text{ hr-in}}$$

Convert to revolutions per second.

$$\omega = \frac{12 \text{ mi}}{12 \text{ hr-in}} \cdot \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ rev}}{2\pi}$$

$$\omega = \frac{12 \cdot 5{,}280 \cdot 12 \text{ rev}}{12 \cdot 60 \cdot 60 \cdot 2\pi \text{ sec}}$$

$$\omega = \frac{44 \text{ rev}}{5\pi \text{ sec}}$$

$\omega \approx 2.80$ revolutions per second

3. B. The law of sines is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We know that $B = 70^\circ$, $C = 45^\circ$, $b = 15$, and $c = x$. Since the sum of the interior angles of a triangle must be 180°,

$$A = 180^\circ - 70^\circ - 45^\circ$$

$$A = 65^\circ$$

Substitute into the law of sines.

$$\frac{15}{\sin 70^\circ} = \frac{x}{\sin 45^\circ}$$

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$$x \sin 70^\circ = 15 \sin 45^\circ$$

$$x = \frac{15 \sin 45^\circ}{\sin 70^\circ}$$

$$x \approx 11.3$$

4. E. Since the central angle θ is in degrees, the area of the circular sector is

$$A = \pi r^2 \left(\frac{\theta}{360}\right)$$

Since the diameter is 6 ft, the radius is 3 ft.

$$A = \pi(3)^2\left(\frac{70}{360}\right)$$

$$A = 9\pi\left(\frac{7}{36}\right)$$

$$A = \frac{7}{4}\pi$$

5. C. The period of a cosecant function is $2\pi/b$ where b is the coefficient on θ.

$$\frac{2\pi}{\frac{1}{12}}$$

$$2\pi\left(\frac{12}{1}\right)$$

$$24\pi$$

6. D. Use the product identity for $\cos\theta\sin\alpha$.

$$\cos\theta\sin\alpha = \frac{1}{2}[\sin(\theta+\alpha) - \sin(\theta-\alpha)]$$

$$\cos 135^\circ \sin 15^\circ = \frac{1}{2}[\sin(135^\circ + 15^\circ) - \sin(135^\circ - 15^\circ)]$$

$$\cos 135^\circ \sin 15^\circ = \frac{1}{2}(\sin 150^\circ - \sin 120^\circ)$$

$$\cos 135^\circ \sin 15^\circ = \frac{1}{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$$

$$\cos 135^\circ \sin 15^\circ = \frac{1}{2}\left(\frac{1-\sqrt{3}}{2}\right)$$

$$\cos 135^\circ \sin 15^\circ = \frac{1-\sqrt{3}}{4}$$

7. A. Use the sum of cosines to find the exact value, since $120^\circ + 45^\circ = 165^\circ$.

$$\cos(\theta+\alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$$

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$$\cos 165^\circ = \cos(120^\circ + 45^\circ)$$

$$\cos 165^\circ = \cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ$$

$$\cos 165^\circ = -\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$\cos 165^\circ = -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\cos 165^\circ = -\frac{\sqrt{2}+\sqrt{6}}{4}$$

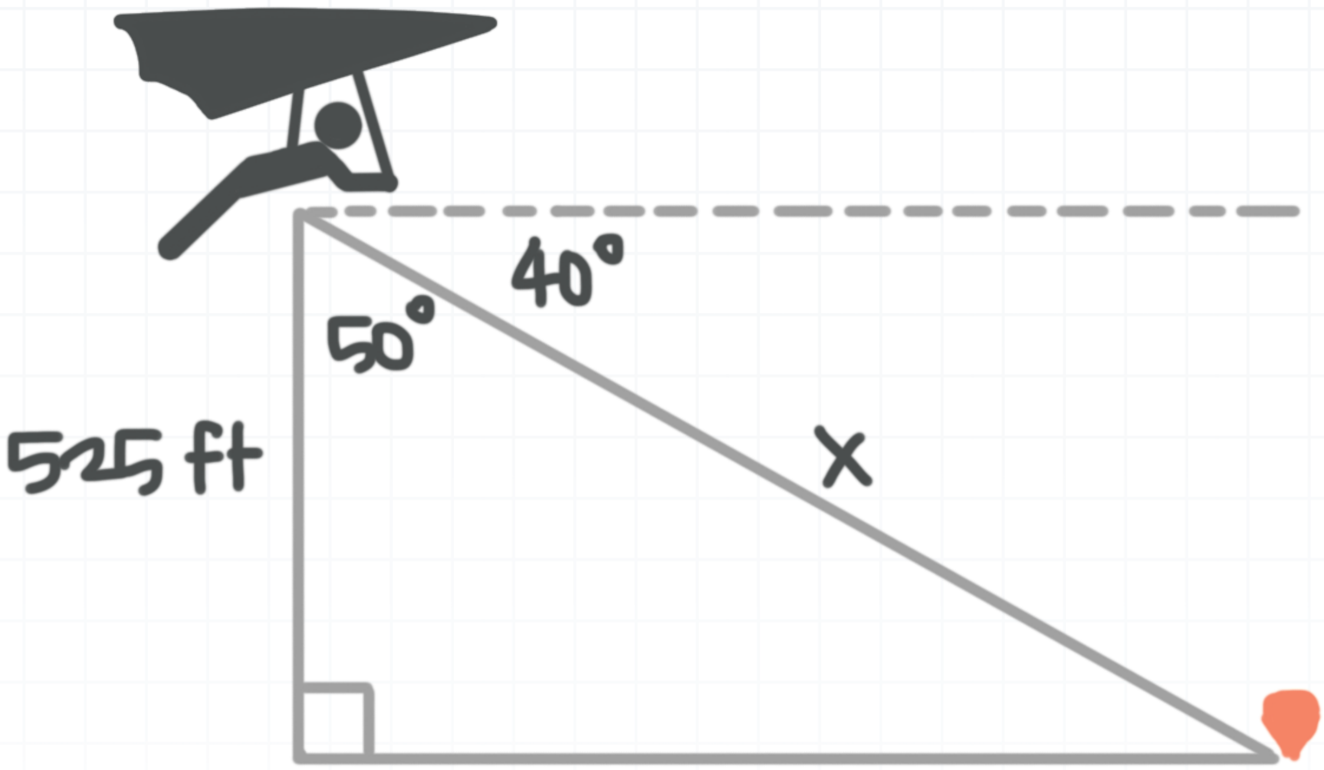
8. B. Remember that a sine function $y = a\cos(b\theta + c) + d$ has a vertical stretch/compression of a, horizontal stretch/compression of b, a horizontal shift of c, and a vertical shift of d.

The equation

$$y = \frac{1}{5}\cos\left(3\theta + \frac{\pi}{3}\right) - 5$$

has a vertical stretch of $1/5$, a horizontal compression of 3, horizontal shift to the left of $\pi/3$, and vertical shift down of 5.

9. Draw a diagram.



Since the angle of depression is 40°, the angle inside the triangle is $90^\circ - 40^\circ = 50^\circ$. To find x, the slant distance from the hang glider to the barn, use

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

The adjacent is 525 ft and the hypotenuse is x.

$$\cos 50^\circ = \frac{525}{x}$$

$$x\cos 50^\circ = 525$$

$$x = \frac{525}{\cos 50^\circ}$$

$$x \approx 817 \text{ ft}$$

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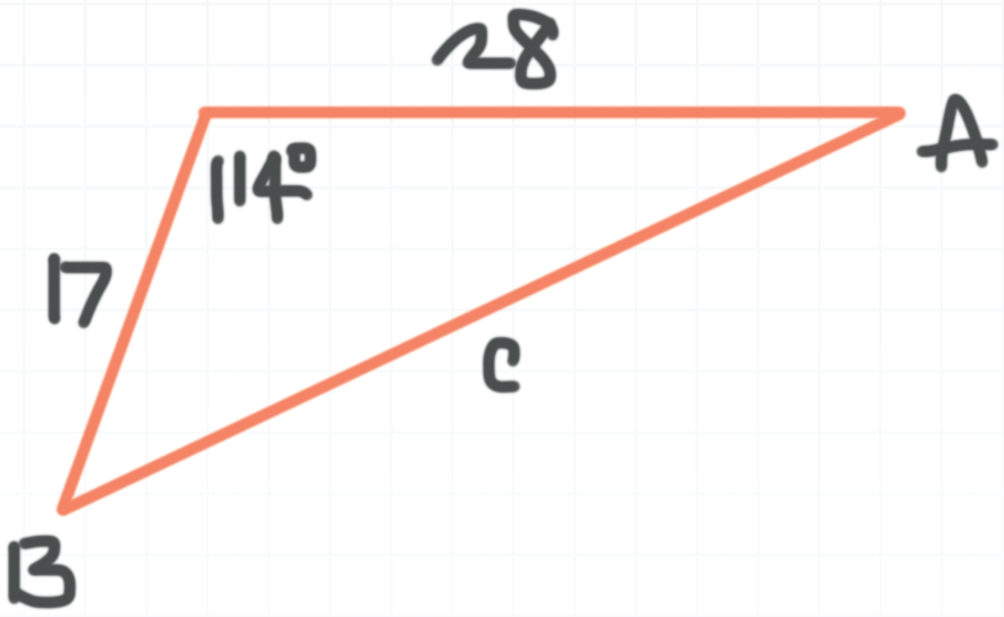
10. Use the law of cosines to find side c.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 17^2 + 28^2 - 2(17)(28)\cos 114^\circ$$

$$c^2 = 289 + 784 - 952\cos 114^\circ$$

$$c = \sqrt{289 + 784 - 952\cos 114^\circ}$$

$$c \approx 38.21$$

Use the law of sines to find angle A.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{17}{\sin A} = \frac{38.21}{\sin 114^\circ}$$

$$38.21\sin A = 17\sin 114^\circ$$

$$\sin A = \frac{17\sin 114^\circ}{38.21}$$

$$A = \sin^{-1}\left(\frac{17\sin 114^\circ}{38.21}\right)$$

$$A \approx 23.98^\circ$$

Since the sum of all the angles in a triangle is 180°, subtract the measures of angles A and C from 180° to find angle B.

$$B = 180^\circ - A - C$$

$$B = 180^\circ - 23.98^\circ - 114^\circ$$

$$B = 42.02^\circ$$

11. The expression $(-\sin 60^\circ)(\cos 38^\circ) + (\cos 60^\circ)\sin(38^\circ)$ is in the form

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

Substitute the angles from the expression.

$$\sin(-60^\circ + 38^\circ) = (\sin(-60^\circ))(\cos 38^\circ) + (\cos(-60^\circ))(\sin 38^\circ)$$

By the odd identity $\sin\theta = -\sin(-\theta)$ and the even identity $\cos\theta = \cos(-\theta)$, the equation becomes

$$\sin(-60^\circ + 38^\circ) = (-\sin 60^\circ)(\cos 38^\circ) + (\cos 60^\circ)(\sin 38^\circ)$$

$$\sin(-22^\circ) = (-\sin 60^\circ)(\cos 38^\circ) + (\cos 60^\circ)(\sin 38^\circ)$$

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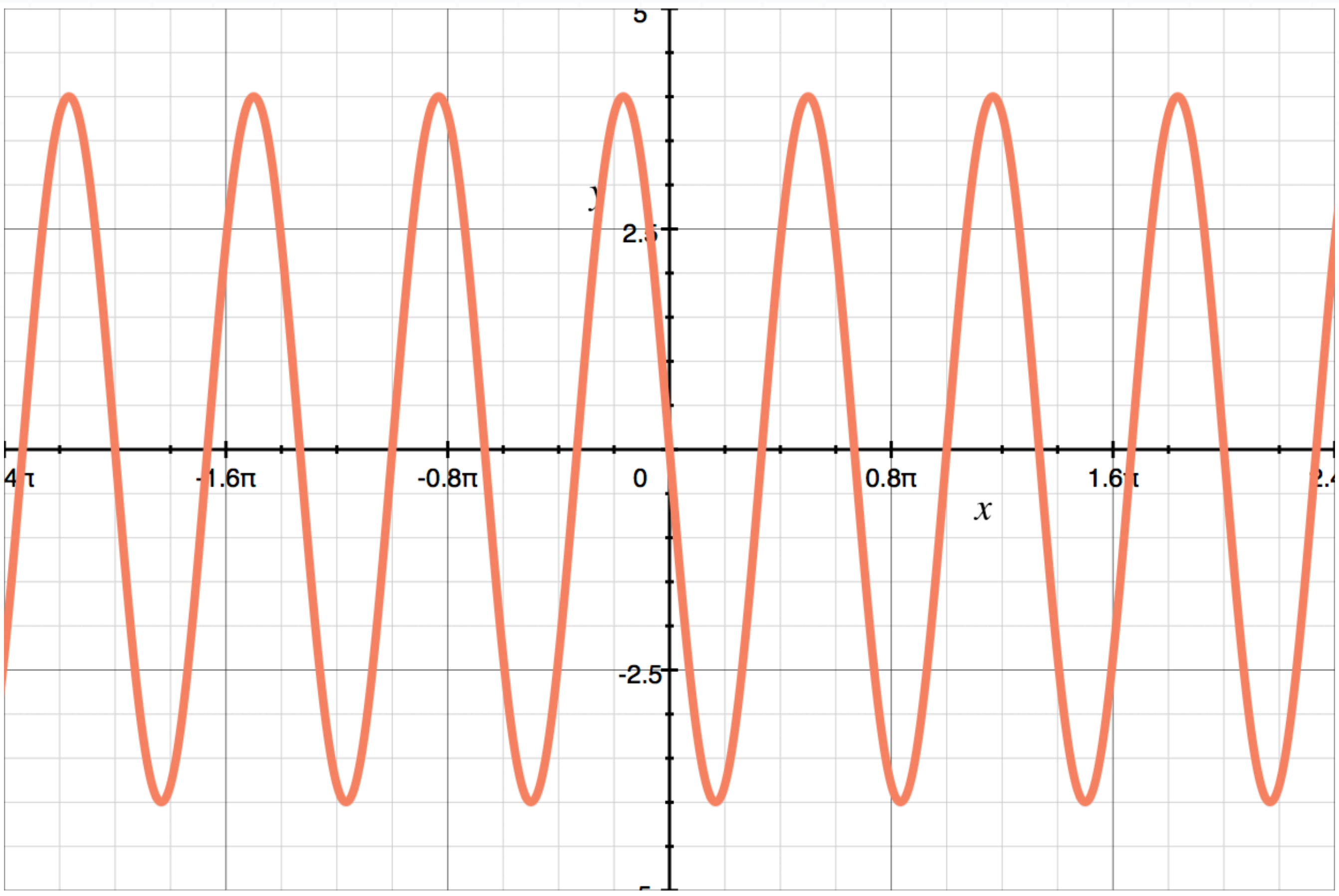
12. Since the graph is the sine function, it will go through the origin. From the origin it will go down to -4 since the amplitude is 4 and the function is negative. The period is

$$\frac{2\pi}{|b|} = \frac{2\pi}{3}$$

There are minimums at $-\pi/2$ and $\pi/6$ and maximums at $-\pi/6$ and $\pi/2$. The zeros are at $-2\pi/3$, $-\pi/3$, 0, $\pi/3$, and $2\pi/3$.



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