Precalculus Workbook

Matrices

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MATH

MATRIX DIMENSIONS AND ENTRIES

■ 1. Give the dimensions of the matrix.

$$D = \begin{bmatrix} 11 & 9 \\ -4 & 8 \end{bmatrix}$$

■ 2. Give the dimensions of the matrix.

$$A = \begin{bmatrix} 3 & 5 & -2 & 1 & 8 \end{bmatrix}$$

■ 3. Given matrix J, find $J_{4,1}$.

$$J = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 1 \end{bmatrix}$$

■ 4. Given matrix C, find $C_{1,2}$.

$$C = \begin{bmatrix} 3 & 12 \\ 1 & 4 \\ 9 & 5 \\ -3 & 2 \end{bmatrix}$$

5. Given matrix N, state the dimensions and find $N_{1,3}$.

$$N = \begin{bmatrix} 1 & 5 & 9 \\ 14 & -8 & 6 \end{bmatrix}$$

6. Given matrix S, state the dimensions and find $S_{3,4}$.

$$S = \begin{bmatrix} 3 & 6 & -7 & 1 & 0 \\ 0 & 9 & 15 & 3 & 4 \\ 4 & 0 & 2 & 11 & 8 \\ -5 & 8 & 7 & 9 & 2 \end{bmatrix}$$

REPRESENTING SYSTEMS WITH MATRICES

■ 1. Represent the system with an augmented matrix called A.

$-2x + 5y = 12$

$6x - 2y = 4$

■ 2. Represent the system with an augmented matrix called D.

$9y - 3x + 12 = 0$

$8 - 4x = 11y$

■ 3. Represent the system with an augmented matrix called H.

$4a + 7b - 5c + 13d = 6$

$3a - 8b = -2c + 1$

■ 4. Represent the system with an augmented matrix called M.

$-2x + 4y = 9 - 6z$

$7y + 2z - 3 = -3t - 9x$

■ 5. Represent the system with an augmented matrix called A.

$3x - 8y + z = 7$

$2z = 3y - 2x + 4$

$5y = 12 - 9x$

■ 6. Represent the system with an augmented matrix called K.

$-4b + 2c = 3 - 7a$

$9c = 4 - 2b$

$8a - 2c = 5b$

SIMPLE ROW OPERATIONS

■ 1. Write the new matrix after $R_1 \leftrightarrow R_2$.

$$\begin{bmatrix} 2 & 6 & -4 & 1 \\ 8 & 2 & 1 & -5 \end{bmatrix}$$

■ 2. Write the new matrix after $R_2 \leftrightarrow R_4$.

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 6 & 1 & 5 & -4 \\ -7 & 7 & 0 & 3 \\ 9 & 2 & 8 & 3 \end{bmatrix}$$

■ 3. Write the new matrix after $R_1 \leftrightarrow 3R_2$.

$$\begin{bmatrix} 9 & 2 & -7 \\ 1 & 6 & 4 \end{bmatrix}$$

■ 4. Write the new matrix after $3R_2 \leftrightarrow 3R_4$.

$$\begin{bmatrix} 0 & 11 & 6 \\ 7 & -3 & 9 \\ 8 & 8 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

5. Write the new matrix after $R_1 + 2R_2 \to R_1$.

$$\begin{bmatrix} 6 & 2 & 7 \\ 1 & -5 & 15 \end{bmatrix}$$

6. Write the new matrix after $4R_2 + R_3 \to R_3$.

$$\begin{bmatrix} 13 & 5 & -2 & 9 \\ 8 & 2 & 0 & 6 \\ 4 & 1 & 7 & -3 \end{bmatrix}$$

GAUSS-JORDAN ELIMINATION AND REDUCED ROW-ECHELON FORM

■ 1. Use Gauss-Jordan elimination to solve the system.

$x + 2y = -2$

$3x + 2y = 6$

■ 2. Use Gauss-Jordan elimination to solve the system.

$2x + 4y = 22$

$3x + 3y = 15$

■ 3. Use Gauss-Jordan elimination to solve the system.

$x - 3y - 6z = 4$

$y + 2z = -2$

$-4x + 12y + 21z = -4$

■ 4. Use Gauss-Jordan elimination to solve the system.

$2y + 4z = 4$

$x + 3y + 3z = 5$

$2x + 7y + 6z = 10$

■ 5. Use Gauss-Jordan elimination to solve the system.

$3x + 12y + 42z = -27$

$x + 2y + 8z = -5$

$2x + 5y + 16z = -6$

■ 6. Use Gauss-Jordan elimination to solve the system.

$4x + 8y + 4z = 20$

$4x + 6y = 4$

$3x + 3y - z = 1$

MATRIX ADDITION AND SUBTRACTION

■ 1. Add the matrices.

$$\begin{vmatrix} 7 & 6 \\ 17 & 9 \end{vmatrix} + \begin{vmatrix} 0 & 8 \\ -2 & 5 \end{vmatrix}$$

■ 2. Add the matrices.

$$\begin{vmatrix} 8 & 3 \\ -4 & 7 \\ 6 & 0 \\ 1 & 13 \end{vmatrix} + \begin{vmatrix} 6 & 7 \\ 2 & -3 \\ 9 & 11 \\ 7 & -2 \end{vmatrix}$$

■ 3. Subtract the matrices.

$$\begin{vmatrix} 7 & 9 \\ 4 & -1 \end{vmatrix} - \begin{vmatrix} 3 & 8 \\ 12 & -3 \end{vmatrix}$$

■ 4. Subtract the matrices.

$$\begin{vmatrix} 8 & 11 & 2 & 9 \\ 6 & 3 & 16 & 8 \end{vmatrix} - \begin{vmatrix} 6 & 11 & 7 & -4 \\ 5 & 8 & 1 & 15 \end{vmatrix}$$

5. Solve for m.

$$\begin{vmatrix} 6 & 5 \\ 9 & -9 \end{vmatrix} + \begin{vmatrix} 3 & 7 \\ 1 & 6 \end{vmatrix} = m + \begin{vmatrix} 7 & 12 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 8 \\ 4 & -7 \end{vmatrix}$$

6. Solve for n.

$$\begin{vmatrix} 4 & 12 \\ 9 & 8 \end{vmatrix} - \begin{vmatrix} 0 & 3 \\ 9 & 9 \end{vmatrix} = n - \begin{vmatrix} 6 & 3 \\ 5 & 11 \end{vmatrix} + \begin{vmatrix} 7 & -4 \\ -18 & 1 \end{vmatrix}$$

SCALAR MULTIPLICATION AND ZERO MATRICES

■ 1. Use scalar multiplication to simplify the expression.

$$\frac{1}{4}\begin{vmatrix} 12 & 8 & 3 \\ 2 & -16 & 0 \\ 1 & 5 & 7 \end{vmatrix}$$

■ 2. Solve for y.

$$4\begin{vmatrix} 2 & 9 \\ -5 & 0 \end{vmatrix} + y = 5\begin{vmatrix} 1 & -3 \\ 6 & 8 \end{vmatrix}$$

■ 3. Solve for n.

$$-2\begin{vmatrix} 6 & 5 \\ 0 & 11 \end{vmatrix} = n - 4\begin{vmatrix} 2 & 4 \\ -1 & 9 \end{vmatrix}$$

■ 4. Add the zero matrix to the given matrix.

$$\begin{vmatrix} 8 & 17 \\ -6 & 0 \end{vmatrix}$$

■ 5. Find the opposite matrix.

$$\begin{vmatrix} 6 & 8 & 0 \\ 2 & -3 & 11 \\ 4 & 12 & 9 \end{vmatrix}$$

■ 6. Multiply the matrix by a scalar of 0.

$$\begin{vmatrix} 14 & -1 & 7 & 5 \\ 3 & 7 & 18 & -4 \end{vmatrix}$$

MATRIX MULTIPLICATION

■ 1. If matrix A is 3×3 and matrix B is 3×4, say whether AB or BA is defined, and give the dimensions of any product that's defined.

■ 2. Find the product of matrices A and B.

$$A = \begin{bmatrix} 2 & 6 \\ -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 0 \\ 5 & -4 \end{bmatrix}$$

■ 3. Find the product of matrices A and B.

$$A = \begin{bmatrix} 5 & -1 \\ 0 & 11 \\ 7 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 1 & 8 \\ -3 & 0 & 4 \end{bmatrix}$$

■ 4. Find the product of matrices A and B.

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 8 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 \\ 4 & 8 \end{bmatrix}$$

■ 5. Use the distributive property to find $A(B+C)$.

$$A = \begin{bmatrix} 2 & 0 \\ 4 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 \\ 5 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & 1 \\ 3 & -1 \end{bmatrix}$$

■ 6. Find the product of matrices A and B.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -2 & 8 & 1 \\ 7 & 3 & 5 & 2 \end{bmatrix}$$

IDENTITY MATRICES

■ 1. Write the identity matrix I_4.

■ 2. If we want to find the product IA, where I is the identity matrix and A is a 4×2, then what are the dimensions of I?

■ 3. If we want to find the product IA, where I is the identity matrix and A is a 3×4, then what are the dimensions of I?

■ 4. If we want to find the product IA, where I is the identity matrix and A is given, then what are the dimensions of I? What is the product IA?

$$A = \begin{bmatrix} 2 & 8 \\ -2 & 7 \\ 3 & 5 \end{bmatrix}$$

■ 5. If we want to find the product IA, where I is the identity matrix and A is given, then what are the dimensions of I? What is the product IA?

$$A = \begin{bmatrix} 7 & 1 & 3 & -2 \\ 5 & 5 & 2 & 9 \end{bmatrix}$$

6. If A is a 2×4 matrix what are the dimensions of the identity matrix that make the equation true?

$$A \cdot I = A$$

TRANSFORMATIONS

■ 1. Find the resulting vector $\overrightarrow{b}$ after $\overrightarrow{a}=(1,6)$ undergoes a transformation by matrix M.

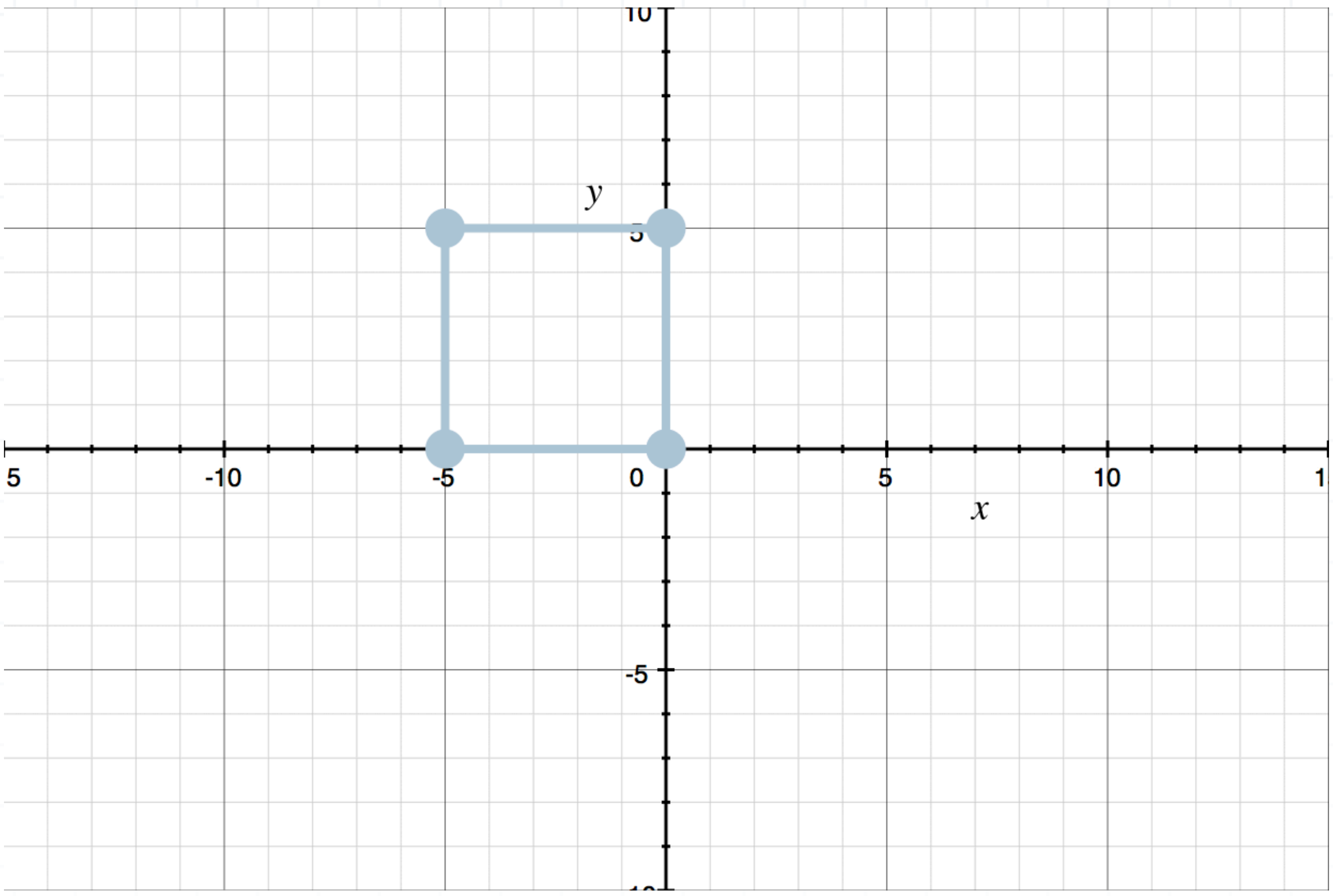
$$M=\begin{bmatrix}-7 & 1\\ 0 & -2\end{bmatrix}$$

■ 2. Sketch triangle $\triangle ABC$ with vertices $(2,3)$, $(-3,-1)$, and $(1,-4)$, and the transformation of $\triangle ABC$ after it's transformed by matrix L.

$$L=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$$

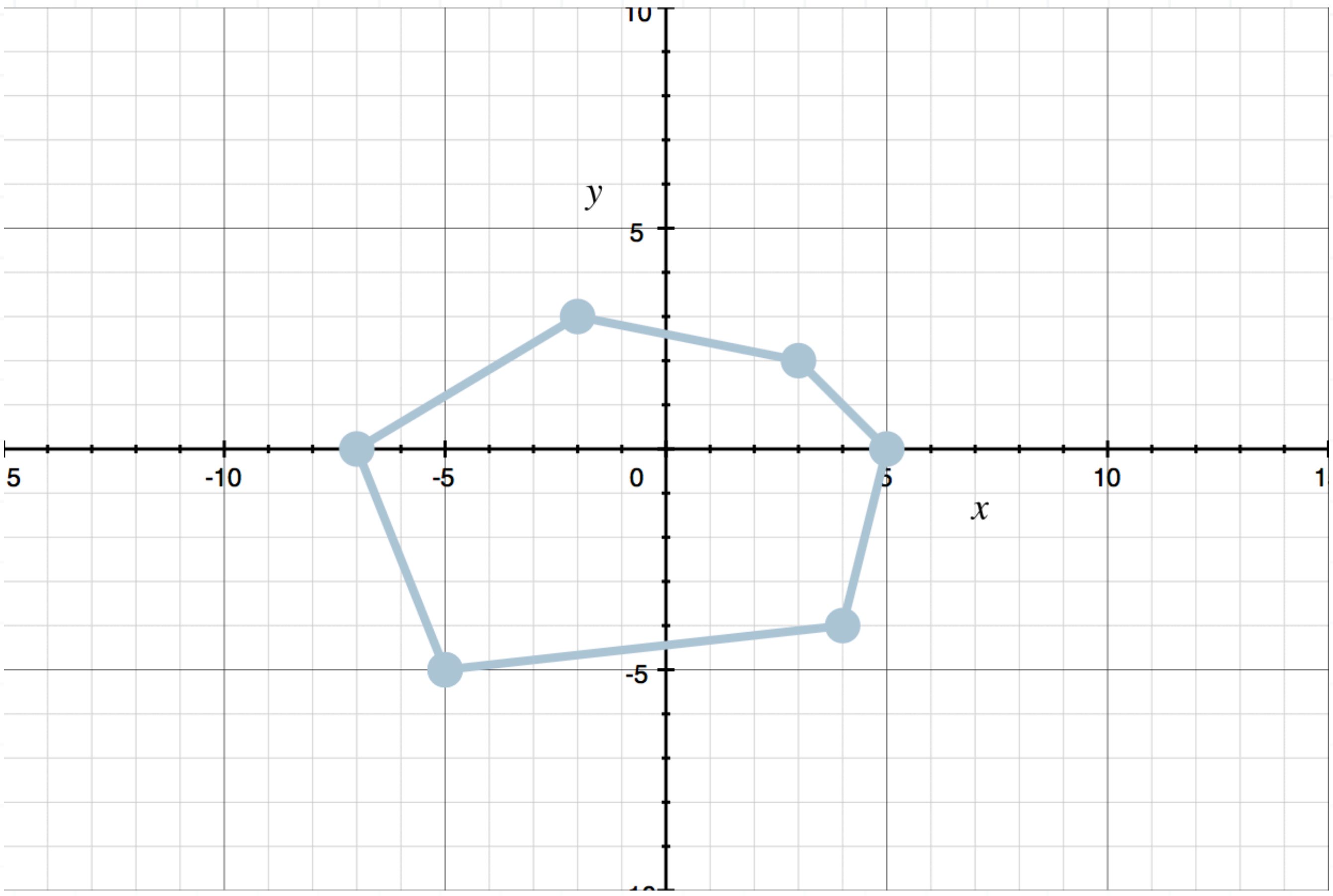
■ 3. Sketch the transformation of the square in the graph after it's transformed by matrix Z.

$$Z=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$$



■ 4. Sketch the transformation of the hexagon after it’s transformed by matrix Y.

$$Y = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$$



■ 5. What happens to the unit vector $\overrightarrow{a} = (1,0)$ after the transformation given by matrix K.

$$K = \begin{bmatrix} 3 & -5 \\ -1 & 0 \end{bmatrix}$$

■ 6. What happens to the unit vector $\overrightarrow{b} = (0,1)$ after the transformation given by matrix K.

$$K = \begin{bmatrix} 3 & -5 \\ -1 & 0 \end{bmatrix}$$

MATRIX INVERSES, AND INVERTIBLE AND SINGULAR MATRICES

■ 1. Find the determinant of the matrix.

$$B = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

■ 2. Find the determinant of the matrix.

$$B = \begin{bmatrix} 1 & -6 \\ 5 & 5 \end{bmatrix}$$

■ 3. Find the inverse of matrix G.

$$G = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

■ 4. Find the inverse of matrix N.

$$N = \begin{bmatrix} 11 & -4 \\ 5 & -3 \end{bmatrix}$$

■ 5. Is the matrix invertible or singular?

$$Z = \begin{bmatrix} 4 & 2 \\ -2 & -1 \end{bmatrix}$$

■ 6. Is the matrix invertible or singular?

$$Y = \begin{bmatrix} 0 & 6 \\ 2 & -1 \end{bmatrix}$$

SOLVING SYSTEMS WITH INVERSE MATRICES

■ 1. Use an inverse matrix to solve the system.

$-4x + 3y = -14$

$7x - 4y = 32$

■ 2. Use an inverse matrix to solve the system.

$6x - 11y = 2$

$-10x + 7y = -26$

■ 3. Use an inverse matrix to solve the system.

$13y - 6x = -81$

$7x + 17 = -22y$

■ 4. Sketch a graph of vectors to visually find the solution to the system.

$3x = 3$

$x - y = -2$

5. Sketch a graph of vectors to visually find the solution to the system.

$-y = -4$

$2x - y = -2$

6. Sketch a graph of vectors to visually find the solution to the system.

$x - y = 0$

$x + y = 2$

SOLVING SYSTEMS WITH CRAMER'S RULE

■ 1. Use Cramer's rule to find the expression that would give the solution for x. You do not need to solve the system.

$2x - y = 5$

$x + 3y = 15$

■ 2. Use Cramer's rule to find the expression that would give the solution for x. You do not need to solve the system.

$ax + by = e$

$cx + dy = f$

■ 3. Use Cramer's rule to find the expression that would give the solution for y. You do not need to solve the system.

$3x + 4y = 11$

$2x - 3y = -4$

■ 4. Use Cramer's rule to find the expression that would give the solution for y. You do not need to solve the system.

$ax + by = e$

$cx + dy = f$

■ 5. Use Cramer's rule to solve for x.

$3x + 2y = 1$

$6x + 5y = 4$

■ 6. Use Cramer's rule to solve for y.

$3x + 2y = 1$

$6x + 5y = 4$

■ 7. Use Cramer's rule to solve for x.

$3x + 5y = 6$

$9x + 10y = 14$

■ 8. Use Cramer's rule to solve for y.

$3x + 5y = 6$

$9x + 10y = 14$

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