Topic: The set of all possible angles

Question: Which set of angles satisfies the equation $\sin\theta = \sin 60^\circ$?

Answer choices:

A $\theta = \left\{-\frac{2\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\}$

B $\theta = \left\{\frac{\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\} \cup \left\{\frac{2\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\}$

C $\theta = \left\{-\frac{5\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\} \cup \left\{-\frac{4\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\}$

D $\theta = \left\{\frac{4\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\} \cup \left\{\frac{5\pi}{3} + 2\pi k: \ k \in \mathbb{Z}\right\}$

Solution: B

The value of sine is positive, which means the y-coordinate is positive, and therefore that angles which satisfy the equation are found in the first and second quadrants.

We'll convert the degree angle to radians.

$$60^\circ = 60^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{60\pi}{180} = \frac{\pi}{3}$$

From the unit circle, we know that

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

So $\pi/3$ is one solution, but we need to include all other angles that are coterminal with $\pi/3$.

$$\left\{\frac{\pi}{3} + 2\pi k: \ k \in Z\right\}$$

But angles in the first and second quadrant have equivalent sine values, which means $2\pi/3$ is also a solution, as well as all angles that are coterminal with $2\pi/3$:

$$\left\{\frac{2\pi}{3} + 2\pi k: \ k \in Z\right\}$$

Topic: The set of all possible angles

Question: Which set of angles satisfies the equation?

$$\cos\theta = \cos\left(\frac{11\pi}{8}\right)$$

Answer choices:

A $\theta = \left\{-\frac{57\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

B $\theta = \left\{\frac{45\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

C $\theta = \left\{-\frac{75\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

D $\theta = \left\{\frac{29\pi}{8} + 2\pi k: \ k \in \mathbb{Z}\right\}$

Solution: C

The set that satisfies the equation could be the set of angles that are all coterminal with $11\pi/8$. So we'll check to see which of the angles given in the answer choices, if any, is coterminal with $11\pi/8$.

$$-\frac{57\pi}{8}-\frac{11\pi}{8}=\frac{(-57-11)\pi}{8}=-\frac{68\pi}{8}=-\frac{17\pi}{2}$$

$$\frac{45\pi}{8}-\frac{11\pi}{8}=\frac{(45-11)\pi}{8}=\frac{34\pi}{8}=\frac{17\pi}{4}$$

$$-\frac{75\pi}{8}-\frac{11\pi}{8}=\frac{(-75-11)\pi}{8}=-\frac{86\pi}{8}=-\frac{43\pi}{4}$$

$$\frac{29\pi}{8}-\frac{11\pi}{8}=\frac{(29-11)\pi}{8}=\frac{18\pi}{8}=\frac{9\pi}{4}$$

None of these results give an angle that's a multiple of 2π, which means none of the angles in the answer choices are coterminal with $11\pi/8$.

The given equation tells us that the cosines of the angles are equal, and we know that cosine always gives the x-value in our coordinate point in the unit circle.

Because the angle $11\pi/8$ is in the third quadrant, we can also find an equal cosine value in the second quadrant, where we're at the same point along the x-axis, but at the opposite point along the y-axis.

We know that $11\pi/8$ is $3\pi/8$ past π, which means by symmetry that the angle in the second quadrant with the same cosine is $3\pi/8$ short of π, or at

$$\pi-\frac{3\pi}{8}=\frac{8\pi}{8}-\frac{3\pi}{8}=\frac{5\pi}{8}$$

Which means that the correct angle set must be coterminal with $5\pi/8$. We'll check the answers again to see which angle is coterminal with $5\pi/8$.

$$-\frac{57\pi}{8}-\frac{5\pi}{8}=\frac{(-57-5)\pi}{8}=-\frac{62\pi}{8}=-\frac{31\pi}{4}$$

$$\frac{45\pi}{8}-\frac{5\pi}{8}=\frac{(45-5)\pi}{8}=\frac{40\pi}{8}=5\pi$$

$$-\frac{75\pi}{8}-\frac{5\pi}{8}=\frac{(-75-5)\pi}{8}=-\frac{80\pi}{8}=-10\pi$$

$$\frac{29\pi}{8}-\frac{5\pi}{8}=\frac{(29-5)\pi}{8}=\frac{24\pi}{8}=3\pi$$

Only $-75\pi/8$ gives an integer multiple of 2π (the angle is five full rotations in the negative direction, since $2\pi(-5)=-10\pi$).

Topic: The set of all possible angles

Question: Solve $\cos\theta = 1/2$ for all possible values of θ.

Answer choices:

A $\theta = \{60^\circ + (360^\circ)k: \ k \in Z\} \cup \{300^\circ + (360^\circ)k: \ k \in Z\}$

B $\theta = \{30^\circ + (360^\circ)k: \ k \in Z\} \cup \{330^\circ + (360^\circ)k: \ k \in Z\}$

C $\theta = \{60^\circ + (360^\circ)k: \ k \in Z\} \cup \{120^\circ + (360^\circ)k: \ k \in Z\}$

D $\theta = \{30^\circ + (360^\circ)k: \ k \in Z\} \cup \{150^\circ + (360^\circ)k: \ k \in Z\}$

Solution: A

The equation is telling us that the value of cosine is positive, which means the value of the x-coordinate is positive. Therefore, the solution will be limited to values of θ in the first and fourth quadrants.

From the unit circle, we know that

$$\cos 60^\circ = \frac{1}{2}$$

So 60° is one solution, but we need to include all other angles that are coterminal with 60°.

$$\{60^\circ \pm (360^\circ)k: \ k = 0,1,2,\ldots\} \text{ or } \ \theta = \{60^\circ + (360^\circ)k: \ k \in Z\}$$

We can find the other set of angles by symmetry. If 60° gives us one set of angles, and 60° is 60° "above" the x-axis, then the other set of angles is at 60° "below" the x-axis, so the other angle is $(360 - 60)^\circ = 300^\circ$. Which means the set

$$\{300^\circ \pm (360^\circ)k: \ k = 0,1,2,\ldots\} \text{ or } \theta = \{300^\circ + (360^\circ)k: \ k \in Z\}$$

also satisfies the equation. Combining these results, the complete solution set is

$$\theta = \{60^\circ + (360^\circ)k: \ k \in Z\} \cup \{300^\circ + (360^\circ)k: \ k \in Z\}$$