Degrees, radians, and DMS

We've already introduced both degrees and radians as two different ways to measure angles. In this lesson, we'll look at **DMS** (degrees, minutes, seconds) as a third angle-measurement system, and then we'll talk about how to convert between all three systems.

The DMS system

In the same way that we can express angles in degrees or in radians, we can also express them in DMS (degrees, minutes, seconds).

In degrees, we already know that one full rotation is $360°$. What that really means is that we're splitting up one full circle into three-hundred-sixty $1°$ angles.

In DMS, each of those $1°$ angles represents the "**degree**" part of degrees-minutes-seconds. If we then zoom in and divide that single $1°$ angle into 60 parts, each of those parts is the "**minute**" part of DMS, and then if we zoom in even more and divide each of those minutes into another 60 parts, each of those parts is the "**second**" part of DMS.

Despite the fact that we use the words "minutes" and "seconds," DMS angle measurement actually has nothing to do with time other than the fact that there are 60 seconds in a minute and 60 minutes in a degree, in the same way that there are 60 seconds in a minute and 60 minutes in an hour when we're talking about time.

As an example, a DMS angle of 36 degrees 40 minutes 7 seconds, will be written as $36°40′7″$. In other words, we use $″$ for seconds, $′$ for minutes, and, just like in degree measurement, $°$ for degrees.

Converting between degrees, radians, and DMS

Degrees and radians

Remember that one complete circle is measured with either $360°$ or 2π radians, so $360° = 2\pi$. Which means that if we want to convert an angle from degrees to radians, we multiply it by $\pi/180°$. For instance, we convert $45°$ into radians like this:

$$45° \left(\frac{\pi}{180°}\right)$$

$$\frac{45\pi}{180}$$

$$\frac{\pi}{4}$$

So $45° = \pi/4$. And to convert from radians to degrees, we flip the fraction and multiply the radian angle by $180°/\pi$. For instance, we convert $3\pi/2$ into degrees like this:

$$\frac{3\pi}{2}\left(\frac{180°}{\pi}\right)$$

$$\frac{3 \cdot 180°}{2}$$

$$270^\circ$$

Let's convert another angle from degrees to radians.

Example

Convert 68° to radians.

Since we're converting from degrees to radians, we'll multiply by $\pi/180^\circ$.

$$68^\circ \left(\frac{\pi}{180^\circ}\right)$$

$$\frac{68\pi}{180}$$

$$\frac{17\pi}{45}$$

So 68° is equivalent to $17\pi/45$ radians.

Let's show how to multiply a radian angle into a degree angle by multiplying by $180^\circ/\pi$.

Example

Convert $4\pi/5$ to degrees.

Since we're converting from radians to degrees, we'll multiply by $180°/\pi$.

$$\frac{4\pi}{5}\left(\frac{180°}{\pi}\right)$$

$$\frac{4(180°)}{5}$$

$$\frac{720°}{5}$$

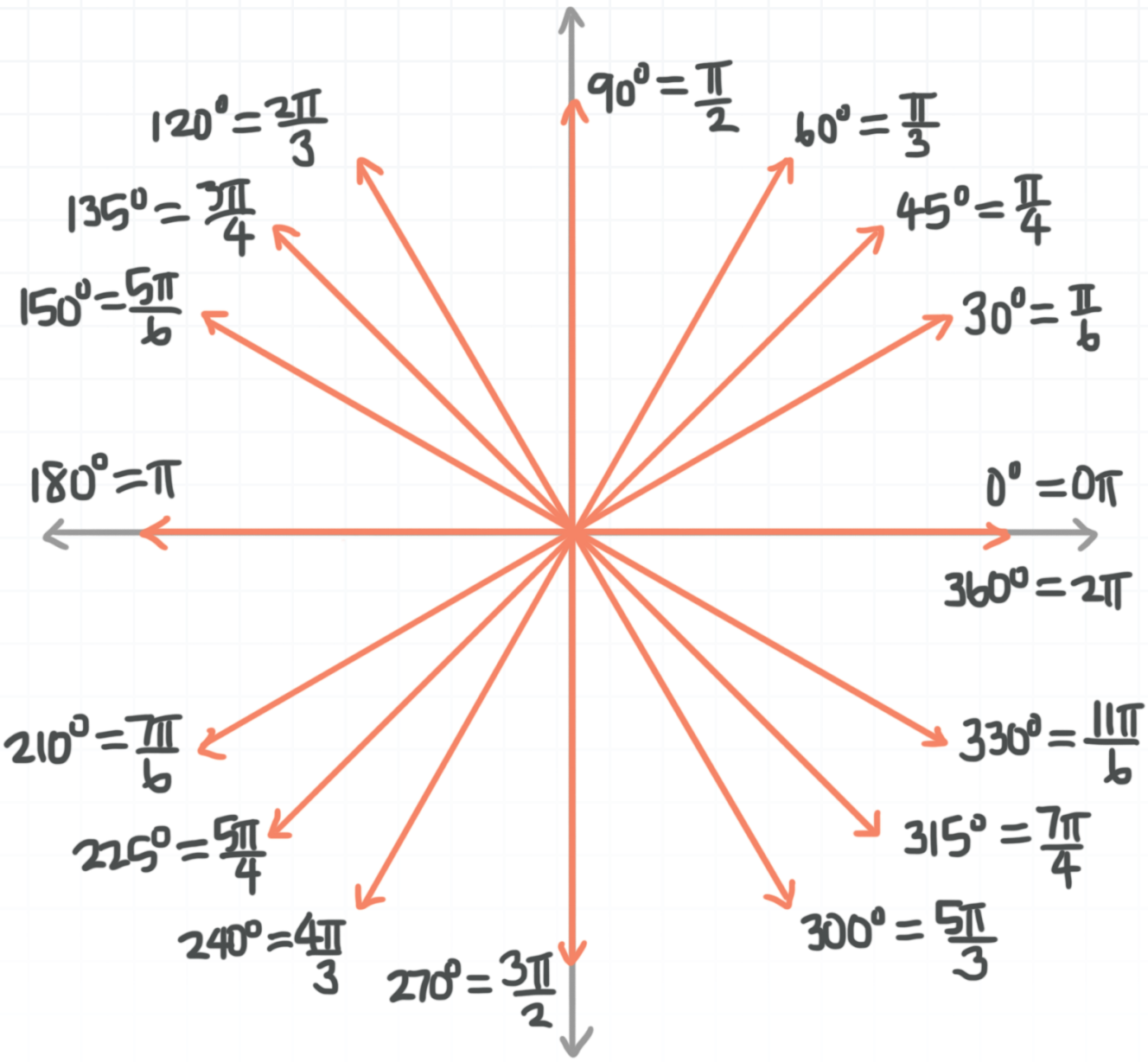
$$144°$$

There are certain angles that we commonly use in Trigonometry, and it's helpful to memorize their values in both degrees and radians.

Degrees	Radians	Location
$0°$	0	Positive horizontal axis
$30°$	$\pi/6$	1st quadrant
$45°$	$\pi/4$	1st quadrant
$60°$	$\pi/3$	1st quadrant
$90°$	$\pi/2$	Positive vertical axis
$120°$	$2\pi/3$	2nd quadrant
$135°$	$3\pi/4$	2nd quadrant
$150°$	$5\pi/6$	2nd quadrant

180°	π	Negative horizontal axis
210°	$7\pi/6$	3rd quadrant
225°	$5\pi/4$	3rd quadrant
240°	$4\pi/3$	3rd quadrant
270°	$3\pi/2$	Negative vertical axis
300°	$5\pi/3$	4th quadrant
315°	$7\pi/4$	4th quadrant
330°	$11\pi/6$	4th quadrant
360°	2π	Positive horizontal axis

We can sketch out these angles in a circle centered at the origin.



This image is the beginning of the “unit circle” which is a really important topic that we’ll cover later on in the course. For now, we can just use this diagram to visualize the locations of these common angles.

Degrees and DMS

If we’re given an angle in degrees that’s an integer, then the angle will look exactly the same in DMS, because the degrees part of DMS is equivalent to the the way we measure angles with degrees. So an angle of $72°$ in degrees is also $72°$ in DMS.

But if we’re given a non-integer angle in degrees, like $72.75°$, then converting to DMS requires us to consider minutes and seconds. The angle

$72.75°$ says that we have $72°$ and 0.75 of one more degree. Remember that each individual degree can be divided into 60 minutes, so $72.75°$ tells us that we have 0.75 of 60 minutes. Then to find out how many minutes we have, we multiply the decimal number by $60'$.

$$0.75(60')$$

$$45'$$

So $72.75°$ in degrees converts to $72°45'0''$, or just $72°45'$, in DMS. If we'd had $72.7525°$ in degrees instead, we'd have ended up with a non-zero seconds part. We'd first identify that the 72 converts to $72°$. Then we'd take the decimal 0.7525 and multiply by $60'$ to find minutes.

$$0.7525(60')$$

$$45.15'$$

Then we'd identify that the 45 converts to $45'$, and take the 0.15 and multiply by $60''$ to find seconds.

$$0.15(60'')$$

$$9''$$

Then the angle $72.7525°$ in degrees is equivalent to the angle $72°45'9''$.

Of course, we can work backwards through this same process to convert from DMS to degrees. Given the DMS-angle $72°45'9''$, we divide $9''$ by $60'$ to get

$$\frac{9''}{60'}$$

$$0.15'$$

Then the angle $72°45'9''$ becomes $72°45.15'$ Then we divide $45.15'$ by $60°$ to get

$$\frac{45.15'}{60°}$$

$$0.7525°$$

Then the angle $72°45.15'$ becomes $72.7525°$, and we've finished converting from DMS back to degrees.

Let's do a few more conversions between degrees and DMS before we talk about converting between radians and DMS.

Example

Convert from degrees to DMS.

$$149.3°$$

The angle in degrees is $149.3°$, so the degrees part in DMS is $149°$. All we have to do is convert $0.3°$ to minutes and seconds. First, we'll convert $0.3°$ to minutes, and then if we get a decimal for the minutes, convert the remaining part to seconds.

There are $60'$ in $1°$, so we can multiply $0.3°$ by $60'/1°$ to convert the degrees to minutes.

$$0.3° \left(\frac{60'}{1°}\right)$$

$$(0.3(60))'$$

$$18'$$

We've found that $0.3°$ converts to $18'$. Since 18 is an integer, there's nothing left to convert to seconds, so the angle in DMS is $149°18'$.

Let's try converting from DMS to degrees.

Example

Express $85°31'22''$ in degrees.

We'll convert the seconds part first. We need to convert $22''$ from seconds to minutes. We know that $1' = 60''$, so we'll multiply $22''$ by $1'/60''$ in order to cancel the seconds and be left with just minutes.

$$22''\left(\frac{1'}{60''}\right)$$

$$\left(\frac{22}{60}\right)'$$

$$\left(\frac{11}{30}\right)'$$

Then the total minutes in $85°31'22''$ is

$$\left(31+\frac{11}{30}\right)'$$

$$\left(31\left(\frac{30}{30}\right)+\frac{11}{30}\right)'$$

$$\left(\frac{930}{30}+\frac{11}{30}\right)'$$

$$\left(\frac{941}{30}\right)'$$

To convert this value for minutes into degrees, we'll multiply by $1°/60'$ in order to cancel the minutes and be left with just degrees.

$$\left(\frac{941}{30}\right)'\left(\frac{1°}{60'}\right)$$

$$\left(\frac{941}{30(60)}\right)°$$

$$\left(\frac{941}{1{,}800}\right)°$$

Putting this together with the $85°$ from the original angle, we get approximately

$$\left(85+\frac{941}{1{,}800}\right)°$$

$$(85+0.5228)°$$

$$85.5228°$$

Let's look at how to convert an angle from DMS to degrees when the seconds part is a decimal number.

Example

Convert $74°10'3.6''$ to degrees.

We'll convert the seconds part first. We need to convert $3.6''$ from seconds to minutes. We know that $1' = 60''$, so we'll multiply $3.6''$ by $1'/60''$ in order to cancel the seconds and be left with just minutes.

$$3.6''\left(\frac{1'}{60''}\right)$$

$$\left(\frac{3.6}{60}\right)'$$

$$0.06'$$

Then the total minutes in $74°10'3.6''$ is

$$(10 + 0.06)'$$

$$10.06'$$

To convert this value for minutes into degrees, we'll multiply by $1^\circ/60'$ in order to cancel the minutes and be left with an approximate value for degrees.

$$10.06'\left(\frac{1^\circ}{60'}\right)$$

$$\left(\frac{10.06}{60}\right)^\circ$$

$$0.1677^\circ$$

Putting this together with the 74° from the original angle, we get approximately

$$(74+0.1677)^\circ$$

$$74.1677^\circ$$

Radians and DMS

To convert from radians to DMS, we'll first convert from radians to degrees, and then to DMS. Similarly, to convert the other way from DMS to radians, we'll convert the DMS angle to a degree angle, and then convert the degree angle to a radian angle.

radians → degrees → DMS

DMS → degrees → radians

We can summarize all the conversions we've learned into a table.

	To radians	To degrees	To DMS
From radians	-	Multiply by $180^\circ/\pi$	Convert through degrees first
From degrees	Multiply by $\pi/180^\circ$	-	Convert fraction of a degree to minutes, then fraction of a minute to a second
From DMS	Convert through degrees first	Convert seconds to minutes, then minutes to degrees	-