Linear and angular velocity

Linear and angular velocity are great real-world applications of what we've already learned about angles in circles. But before we jump into talking about these velocities, we need to understand the difference between speed and velocity, and their relationship to one another.

Speed and velocity

Velocity is defined by two factors, magnitude and direction. So a large positive velocity tells us that we're moving forward at high speed, whereas a small negative velocity tells us that we're moving backward at low speed.

In contrast, **speed** is only the magnitude portion of velocity. Speed tells us how fast we're moving, but doesn't tell us the direction of movement. This makes sense, too, because speed is the absolute value of velocity, which means speed is always a positive number. A small positive value for speed tells us we're moving slowly, whereas a large positive value for speed tells us we're moving quickly.

Linear and angular velocity

With this in mind, we'll define linear and angular velocity, which are both related to the arcs and circles that we've been learning about.

Linear velocity tells us how fast the length of an arc is changing. Imagine the arc of a circle. If the angle that creates the arc is growing, such that the length of the arc is increasing, then linear velocity will be positive, because the length of the arc is increasing over time. We use the formula

$$v = \frac{s}{t}$$

where v is linear velocity, s is arc length, and t is time.

On the other hand, while linear velocity gives the rate of change of the arc length, **angular velocity** tells us the rate of change of the interior angle (the rate at which the central angle is swept out as we move around the circle). The formula we use for angular velocity is

$$\omega = \frac{\theta}{t}$$

where ω (omega) is angular velocity, and θ is the radian measure of the interior angle at time t. Since θ is an angle measure and t is time, ω will be an angle measure per unit time, like radians per second, degrees per minute, etc.

Angular speed and angular velocity use the same formula; the difference between the two is that angular speed is a scalar quantity, while angular velocity is a vector quantity.

Let's do some examples, starting with one where we solve for angular velocity.

Example

What is the angular velocity, in radians per second, of a disc that rotates at a constant rate and sweeps out an angle of 36.4π radians in 8.39 seconds?

To find the angular velocity ω, we'll divide the total angle swept out θ by the total time t.

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{36.4\pi \text{ radians}}{8.39 \text{ seconds}}$$

$$\omega = \frac{36.4\pi}{8.39} \text{ radians per second}$$

$$\omega \approx 4.34\pi \text{ radians per second}$$

We always want to be careful about units, and make sure that we have matching units of time. If we don't, we'll need to do some conversions.

Example

What is the angular velocity, in radians per second, of a wheel that rotates at a constant rate and sweeps out an angle of 72.7π radians in 3.2 minutes?

Since we're asked to find angular velocity in radians per second, but we're given the rotation in minutes, we'll need to first convert the minutes into seconds.

$$t = (3.2 \text{ minutes})\left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right)$$

$$t = 3.2(60) \text{ seconds}$$

$$t = 192 \text{ seconds}$$

Now we can find angular velocity.

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{72.7\pi \text{ radians}}{192 \text{ second}}$$

$$\omega = \frac{72.7\pi}{192} \text{ radians per second}$$

$$\omega \approx 0.379\pi \text{ radians per second}$$

Let's do an example where we need to convert between radians and degrees.

Example

Find the angular velocity, in radians per second, of an object that rotates at a constant rate and sweeps out an angle of $1{,}043^\circ$ in 5.9 seconds.

We'll first convert the angle $1{,}043^\circ$ to radians.

$$\theta = 1{,}043^\circ \left(\frac{\pi \text{ radians}}{180^\circ}\right)$$

$$\theta = \frac{1{,}043\pi}{180} \text{radians}$$

Now we'll find angular velocity.

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{\frac{1{,}043\pi}{180} \text{ radians}}{5.9 \text{ seconds}}$$

$$\omega = \frac{1{,}043\pi}{180(5.9)} \text{ radians per second}$$

$$\omega \approx 0.982\pi \text{ radians per second}$$

We also often express angular velocity in *revolutions* per unit of time. When we want to convert from one set of units to another, we'll need to remember that there's 1 revolution per 2π radians.

Example

Express an angular velocity of 31 radians per second in units of revolutions per minute.

We know the angular velocity ω, and we just need to convert it to different units.

$$\omega = 31\ \frac{\text{radians}}{\text{second}}$$

$$\omega = \left(31\ \frac{\text{rad}}{\text{sec}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)\left(\frac{60\ \text{sec}}{1\ \text{min}}\right)$$

$$\omega = \frac{31(60)}{2\pi}\ \text{revolutions per minute}$$

If we say $\pi \approx 3.14$, we get

$$\omega \approx 296\ \text{revolutions per minute}$$

Here's an example of a conversion in the opposite direction.

Example

Express angular velocity of 86.3 revolutions per minute in units of radians per second.

We already know angular velocity, we just need to convert the units.

$$\omega = 86.3\ \frac{\text{revolutions}}{\text{minute}}$$

$$\omega = \left(86.3\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)$$

$$\omega = \frac{86.3(2)\pi}{60}\ \text{radians per second}$$

$$\omega \approx 2.88\pi\ \text{radians per second}$$