Topic: When the trig functions are undefined

Question: For what angle is $\sec\theta$ undefined in the interval?

$$\left(-\frac{53\pi}{6}, -\frac{25\pi}{3}\right)$$

Answer choices:

A $\theta = -9\pi$

B $\theta = -\frac{17\pi}{2}$

C $\theta = -\frac{15\pi}{2}$

D $\theta = -8\pi$

Solution: B

We need to start by first identifying an interval that's coterminal with the given interval.

We know that one full rotation is given by 2π radians. Temporarily, let's express 2π as $12\pi/6$, since the lower bound of the interval was given to us in sixths as $-53\pi/6$. If one full rotation is $12\pi/6$, then four full rotations is $48\pi/6$. Because $53 - 48 = 5$, we know that an angle of $-53\pi/6$ is four full negative rotations, followed by another $-5\pi/6$ rotation. So $-53\pi/6$ is coterminal with $-5\pi/6$.

Similarly, let's temporarily express 2π as $6\pi/3$, since the upper bound of the interval was given to use in thirds as $-25\pi/3$. If one full rotation is $6\pi/3$, then four full rotations is $24\pi/3$. Because $25 - 24 = 1$, we know that an angle of $-25\pi/3$ is four full negative rotations, followed by another $-\pi/3$ rotation. So $-25\pi/3$ is coterminal with $-\pi/3$.

Therefore, we can rewrite the given interval as

$$\left(-\frac{5\pi}{6}, -\frac{\pi}{3}\right)$$

That's an interval that spans from the fourth quadrant, past the negative vertical axis, and into the third quadrant.

We know by the reciprocal identity

$$\sec\theta = \frac{1}{\cos\theta}$$

that secant is undefined when cosine is 0, and we know that cosine is 0 when $x = 0$. We know we'll have $x = 0$ along the vertical axis, which means that, in the interval

$$\left(-\frac{5\pi}{6}, -\frac{\pi}{3}\right)$$

secant will be undefined at $\theta = -\pi/2$. But remember we found that the original interval included four more full negative rotations. So the angle within the original interval that makes secant undefined is

$$\theta = -\frac{\pi}{2} - 4(2\pi)$$

$$\theta = -\frac{\pi}{2} - 8\pi$$

$$\theta = -\frac{\pi}{2} - \frac{16\pi}{2}$$

$$\theta = -\frac{17\pi}{2}$$

Therefore, $\theta = -17\pi/2$ is the only angle in the interval $(-53\pi/6, -25\pi/3)$ at which $\sec\theta$ is undefined.

Topic: When the trig functions are undefined

Question: Say whether $\tan(29\pi/6)$ is defined.

Answer choices:

A $\quad \tan\left(\frac{29\pi}{6}\right)$ is defined

B $\quad \tan\left(\frac{29\pi}{6}\right)$ is undefined

C $\quad$ We can't determine whether or not $\tan\left(\frac{29\pi}{6}\right)$ is defined

D $\quad \tan\left(\frac{5\pi}{6}\right)$ is undefined

Solution: A

From the quotient identity for tangent,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

we know that $\tan\theta$ is undefined when $\cos\theta = 0$. We know $\cos\theta = 0$ anywhere along the vertical axis. So we need to check to see whether or not $29\pi/6$ lies along the vertical axis.

If we know one full rotation is $2\pi = 12\pi/6$, then we can say $29\pi/6$ is two full rotations ($24\pi/6$) and then an additional $5\pi/6$ rotations. The angle $5\pi/6$ doesn't fall on the vertical axis, which means $\tan(29\pi/6)$ will be defined.

Topic: When the trig functions are undefined

Question: At which angle is the cotangent function undefined?

Answer choices:

A $\theta = 3\pi$

B $\theta = \frac{5\pi}{3}$

C $\theta = \frac{7\pi}{2}$

D $\theta = \frac{9\pi}{4}$

Solution: A

We know from the quotient identity for cotangent,

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

cotangent of an angle θ is undefined when $\sin \theta = 0$. We know that the sine function, which represents the y-coordinate, is 0 at angles along the horizontal axis. Angles along the horizontal axis include the full set of angles coterminal with 0,

$$\theta = 0, \ \pm 2\pi, \ \pm 4\pi, \ \pm 6\pi, \ \dots$$

and the full set of angles coterminal with π.

$$\theta = \pm \pi, \ \pm 3\pi, \ \pm 5\pi, \ \pm 7\pi, \ \dots$$

Of the answer choices, the only angle contained in either of these sets is the angle $\theta = 3\pi$.