Topic: Hyperbolic identities

Question: Which of the following hyperbolic trigonometric identities is false?

Answer choices:

A $\text{csch}(x) = \dfrac{1}{\sinh(x)}$

B $\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$

C $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$

D $\cosh^2(x) + \sinh^2(x) = 1$

Solution: D

The following list includes the basic hyperbolic identities.

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \text{csch}(x) = \frac{1}{\sinh(x)}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \text{sech}(x) = \frac{1}{\cosh(x)}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \coth(x) = \frac{1}{\tanh(x)}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\tanh^2(x) + \text{sech}^2(\text{x}) = 1$$

$$\coth^2(\text{x}) - \text{csch}^2(\text{x}) = 1$$

Answer choices A, B and C are all known identities, but answer choice D is not. It's similar to the identity $\cosh^2(x) - \sinh^2(x) = 1$, but the sign is wrong, so answer choice D is the correct answer.

Topic: Hyperbolic identities

Question: Which of the following hyperbolic trigonometric identities is true?

Answer choices:

A $\cosh(x) = \dfrac{e^x - e^{-x}}{2}$

B $\text{sech}(x) = \dfrac{1}{\cosh(x)}$

C $\text{coth}(x) = \dfrac{1}{\text{csch}(x)}$

D $\text{coth}^2(x) + \text{csch}^2(x) = 1$

Solution: B

The following list includes the basic hyperbolic identities

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh(x)}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \coth(x) = \frac{1}{\tanh(x)}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\tanh^2(x) + \operatorname{sech}^2(\mathrm{x}) = 1$$

$$\coth^2(\mathrm{x}) - \operatorname{csch}^2(\mathrm{x}) = 1$$

Let's look at our answer choices.

Answer choice A, $\cosh(x) = \dfrac{e^x - e^{-x}}{2}$ should be $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$

Answer choice B, sech$(x) = \dfrac{1}{\cosh(x)}$ is correct

Answer choice C, coth$(x) = \dfrac{1}{\text{csch}(x)}$ should be coth$(x) = \dfrac{1}{\tanh(x)}$

Answer choice D, coth$^2(x)$ + csch$^2(x) = 1$ should be coth$^2(x)$ – csch$^2(x) = 1$

So answer choice B is the correct choice.

Topic: Hyperbolic identities

Question: Is the identity true or false?

$$\cosh^2(x) - \sinh^2(x) = \tanh^2(x) + \text{sech}^2(x)$$

Answer choices:

A True

B False

Solution: A

The following list includes the basic hyperbolic identities.

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \text{csch}(x) = \frac{1}{\sinh(x)}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \text{sech}(x) = \frac{1}{\cosh(x)}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \coth(x) = \frac{1}{\tanh(x)}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\tanh^2(x) + \text{sech}^2(x) = 1$$

$$\coth^2(x) - \text{csch}^2(x) = 1$$

Given

$$\cosh^2(x) - \sinh^2(x) = \tanh^2(x) + \text{sech}^2(x)$$

we can use the identity $\cosh^2(x) - \sinh^2(x) = 1$ to substitute into the given equation, and we get

$$1 = \tanh^2(x) + \text{sech}^2(x)$$

Then, we'll use the identity $\tanh^2(x) + \text{sech}^2(x) = 1$ to substitute into the right side of the equation and we get

$$1 = 1$$

Therefore, we've proven that the given equation is true.