Cofunction identities

Next, we'll look at the cofunction identities, which give us a relationship between sine and cosine, a relationship between secant and cosecant, and a relationship between tangent and cotangent.

If we take the graph of $y = \sin x$ and shift it to the left by $\pi/2$ units, it looks exactly like the graph of $y = \cos x$. The same is true for tangent and cotangent, as well as for secant and cosecant. That's the basic premise of co-function identities — the sine and cosine functions take on the same values, but those values are shifted slightly on the coordinate plane when we look at one function compared to the other.

In degrees, the cofunction identities are

$$\sin\theta = \cos(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

$$\cos\theta = \sin(90^\circ - \theta) \qquad \sec\theta = \csc(90^\circ - \theta)$$

$$\tan\theta = \cot(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta)$$

and of course, in radians these identities are

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) \qquad \sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

Notice how sine and co(sine) are cofunctions, tangent and co(tangent) are cofunctions, and secant and co(secant) are cofunctions. The fact that the functions are named the way they are makes it really easy to remember which pairs of trig functions are cofunctions.

The value of a trigonometric function of an angle is equal to the value of the cofunction of the angle's complement. Remember from geometry that complementary angles are angles that sum to 90°.

These identities are useful when we know sine of an angle and want to find cosine, or vice versa, when we know tangent of an angle and want to find cotangent, or vice versa, or when we know secant of an angle and want to find cosecant, or vice versa.

Let's do an example where we use a cofunction identity to find the value of a trig function from the value of another trig function at the same angle.

Example

Find an angle θ that satisfies the equation.

$$\sec\frac{3\pi}{4}=\csc\theta$$

The equation we're given tells us that the cosecant of some angle is equivalent to secant of $3\pi/4$. Secant and cosecant are cofunctions, which means we can plug into the cofunction identity for secant that relates them.

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\frac{3\pi}{4} = \csc\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)$$

Find a common denominator.

$$\sec\frac{3\pi}{4} = \csc\left(\frac{\pi}{2}\left(\frac{2}{2}\right) - \frac{3\pi}{4}\right)$$

$$\sec\frac{3\pi}{4} = \csc\left(\frac{2\pi}{4} - \frac{3\pi}{4}\right)$$

$$\sec\frac{3\pi}{4} = \csc\left(-\frac{\pi}{4}\right)$$

So the angle θ that satisfies the equation is $\theta = -\pi/4$. And this result tells us that secant of the angle $3\pi/4$ has the same value as cosecant of the angle $-\pi/4$.

Let's do one more example where we use a cofunction identity for sine and cosine.

Example

Find an angle θ that satisfies the equation.

$$\sin\left(-\frac{\pi}{6}\right) = \cos\theta$$

The equation we're given tells us that the cosine of some angle is equivalent to sine of $-\pi/6$. Sine and cosine are cofunctions, which means we can plug into the cofunction identity for sine that relates them.

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\sin\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2} - \left(-\frac{\pi}{6}\right)\right)$$

$$\sin\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$$

Find a common denominator.

$$\sin\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2}\left(\frac{3}{3}\right) + \frac{\pi}{6}\right)$$

$$\sin\left(-\frac{\pi}{6}\right) = \cos\left(\frac{3\pi}{6} + \frac{\pi}{6}\right)$$

$$\sin\left(-\frac{\pi}{6}\right) = \cos\frac{4\pi}{6}$$

$$\sin\left(-\frac{\pi}{6}\right) = \cos\frac{2\pi}{3}$$

So the angle θ that satisfies the equation is $\theta = 2\pi/3$. And this result tells us that sine of the angle $-\pi/6$ has the same value as cosine of the angle $2\pi/3$.