Topic: The ambiguous case of the law of sines

Question: How many triangles are possible with side lengths 17 and 25, where the angle opposite the side with length 17 is 80°?

Answer choices:

A Two triangles are possible

B One triangle is possible

C No triangles are possible

D The number of triangles can't be determined

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Solution: C

Let $a=17$ and $b=25$, and let angle $A=80^\circ$. Then, plugging what we know into the law of sines gives

$$\frac{17}{\sin 80^\circ}=\frac{25}{\sin B}=\frac{c}{\sin C}$$

Find B using the first two parts of this three-part equation.

$$\frac{17}{\sin 80^\circ}=\frac{25}{\sin B}$$

$$\sin B=\frac{25\sin 80^\circ}{17}\approx 1.45$$

Since the sine of an angle can't be greater than 1, it's impossible to build a triangle with these properties.

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Question: A triangle has side lengths $a = 20$ and $c = 16$ and interior angle $C = 35^\circ$. How many triangles can be made with these properties?

Answer choices:

A Two triangles can be made

B One triangle can be made

C No triangles can be made

D The number of triangles can't be determined

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Solution: A

Plugging what we know into the law of sines gives

$$\frac{20}{\sin A} = \frac{b}{\sin B} = \frac{16}{\sin 35^\circ}$$

Find A using the first and third parts of this three-part equation.

$$\frac{20}{\sin A} = \frac{16}{\sin 35^\circ}$$

$$\sin A = \frac{20 \sin 35^\circ}{16} \approx \frac{5(0.574)}{4} \approx 0.718$$

If A is acute then $A = 45.9^\circ$, and if A is obtuse then $A = 134.1^\circ$. Both angle measures keep the sum of the first two interior angles at less than 180°, which means two triangles are possible.

We weren't asked in the question to solve the triangles, but if we do, we find that the two triangles are

1) a triangle with interior angles of 45.9°, 99.1°, and 35°, and corresponding side lengths 20, 27.5, and 16

2) a triangle with interior angles of 134.1°, 10.9°, and 35°, and corresponding side lengths 20, 5.27, and 16

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Topic: The ambiguous case of the law of sines

Question: A triangle has side lengths $b = 90$ and $c = 45$ and interior angle $C = 30^\circ$. How many triangles can be made with these properties?

Answer choices:

A One triangle can be made

B Two triangles can be made

C No triangles can be made

D The number of triangles can't be determined

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Solution: A

Plugging what we know into the law of sines gives

$$\frac{a}{\sin A} = \frac{90}{\sin B} = \frac{45}{\sin 30^\circ}$$

Find A using the first and third parts of this three-part equation.

$$\frac{90}{\sin B} = \frac{45}{\sin 30^\circ}$$

$$\sin A = \frac{90 \sin 30^\circ}{45} = \frac{90\left(\frac{1}{2}\right)}{45} = 1$$

$$\arcsin(1) = 90^\circ$$

Which means one triangle is possible.

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