**Topic**: Multiplying and dividing polar forms

**Question**: What is the product $z_1 z_2$ of the complex numbers in polar form?

$$z_1 = \sqrt{2}\left(\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}\right)$$

$$z_2 = \frac{7}{3\sqrt{2}}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

**Answer choices**:

A $\dfrac{7}{\sqrt{6}}\left(\cos\dfrac{21\pi}{20} + i\sin\dfrac{21\pi}{20}\right)$

B $\dfrac{7}{3}\left(\cos\dfrac{13\pi}{10} + i\sin\dfrac{13\pi}{10}\right)$

C $\dfrac{3\sqrt{2}}{7}\left(\cos\dfrac{23\pi}{20} + i\sin\dfrac{23\pi}{20}\right)$

D $\dfrac{7}{3}\left(\cos\dfrac{3\pi}{20} + i\sin\dfrac{3\pi}{20}\right)$

**Solution**: D

Plug the complex numbers into the formula for the product of complex numbers.

$$z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)\right]$$

$$z_1 z_2 = \left(\sqrt{2} \cdot \frac{7}{3\sqrt{2}}\right)\left[\cos\left(\frac{7\pi}{5} + \frac{3\pi}{4}\right) + i \sin\left(\frac{7\pi}{5} + \frac{3\pi}{4}\right)\right]$$

Simplify.

$$z_1 z_2 = \frac{7}{3}\left[\cos\left(\frac{28\pi}{20} + \frac{15\pi}{20}\right) + i \sin\left(\frac{28\pi}{20} + \frac{15\pi}{20}\right)\right]$$

$$z_1 z_2 = \frac{7}{3}\left(\cos\frac{43\pi}{20} + i \sin\frac{43\pi}{20}\right)$$

You could leave the answer this way, but ideally we'd like to reduce the angle to one that's coterminal, but in the interval $[0,2\pi)$. If we subtract $2\pi$ from the angle, we get

$$z_1 z_2 = \frac{7}{3}\left[\cos\left(\frac{43\pi}{20} - \frac{40\pi}{20}\right) + i \sin\left(\frac{43\pi}{20} - \frac{40\pi}{20}\right)\right]$$

$$z_1 z_2 = \frac{7}{3}\left(\cos\frac{3\pi}{20} + i \sin\frac{3\pi}{20}\right)$$



**Topic**: Multiplying and dividing polar forms

**Question**: What is the quotient $z_1/z_2$ of the complex numbers in polar form?

$$z_1 = 16\left(\cos\frac{9\pi}{13} + i\sin\frac{9\pi}{13}\right)$$

$$z_2 = \frac{5}{\sqrt{3}}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

**Answer choices**:

A $\dfrac{16\sqrt{3}}{5}\left(\cos\dfrac{41\pi}{78} + i\sin\dfrac{41\pi}{78}\right)$

B $\dfrac{80}{\sqrt{3}}\left(\cos\dfrac{17\pi}{39} + i\sin\dfrac{17\pi}{39}\right)$

C $\dfrac{16}{5\sqrt{3}}\left(\cos\dfrac{9\pi}{78} + i\sin\dfrac{9\pi}{78}\right)$

D $\dfrac{16\sqrt{3}}{5}\left(\cos\dfrac{17\pi}{39} + i\sin\dfrac{17\pi}{39}\right)$

**Solution**: A

Plug the complex numbers into the formula for the quotient of complex numbers.

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{16}{\frac{5}{\sqrt{3}}}\left[\cos\left(\frac{9\pi}{13} - \frac{\pi}{6}\right) + i\sin\left(\frac{9\pi}{13} - \frac{\pi}{6}\right)\right]$$

Simplify.

$$\frac{z_1}{z_2} = 16 \cdot \frac{\sqrt{3}}{5}\left[\cos\left(\frac{54\pi}{78} - \frac{13\pi}{78}\right) + i\sin\left(\frac{54\pi}{78} - \frac{13\pi}{78}\right)\right]$$

$$\frac{z_1}{z_2} = \frac{16\sqrt{3}}{5}\left(\cos\frac{41\pi}{78} + i\sin\frac{41\pi}{78}\right)$$

**Topic**: Multiplying and dividing polar forms

**Question**: Suppose that a complex number $z$ is the quotient $z_1/z_2$ of the given complex numbers. If $z$ is expressed in polar form, $r(\cos\theta + i\sin\theta)$, where is $\theta$ located?

$$z_1 = 4\left(\cos\frac{13\pi}{9} + i\sin\frac{13\pi}{9}\right)$$

$$z_2 = \frac{17}{3}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

**Answer choices**:

A In the first quadrant

B On the negative horizontal axis

C In the second quadrant

D On the positive vertical axis

**Solution**: C

Plug the complex numbers into the formula for the quotient of complex numbers.

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{4}{\frac{17}{3}}\left[\cos\left(\frac{13\pi}{9} - \frac{5\pi}{6}\right) + i\sin\left(\frac{13\pi}{9} - \frac{5\pi}{6}\right)\right]$$

Simplify.

$$\frac{z_1}{z_2} = 4 \cdot \frac{3}{17}\left[\cos\left(\frac{78\pi}{54} - \frac{45\pi}{54}\right) + i\sin\left(\frac{78\pi}{54} - \frac{45\pi}{54}\right)\right]$$

$$\frac{z_1}{z_2} = \frac{12}{17}\left(\cos\frac{33\pi}{54} + i\sin\frac{33\pi}{54}\right)$$

The fraction $33/54$ is approximately equal to $0.6$, so the angle is about $0.6\pi$, which is in the second quadrant.