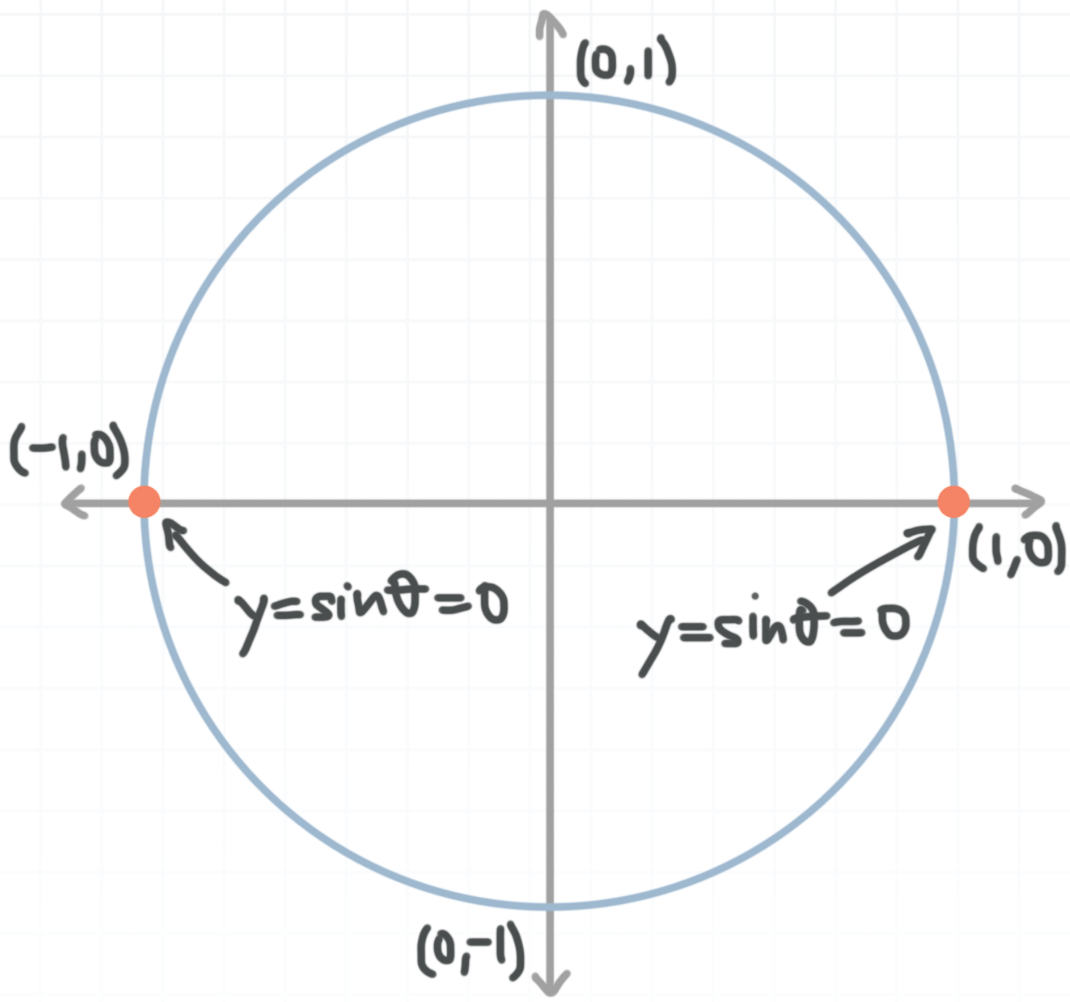
When the trig functions are undefined

In the last lesson, we saw that we could find the values of all six trig functions, with only the value of one of them and the quadrant of the angle.

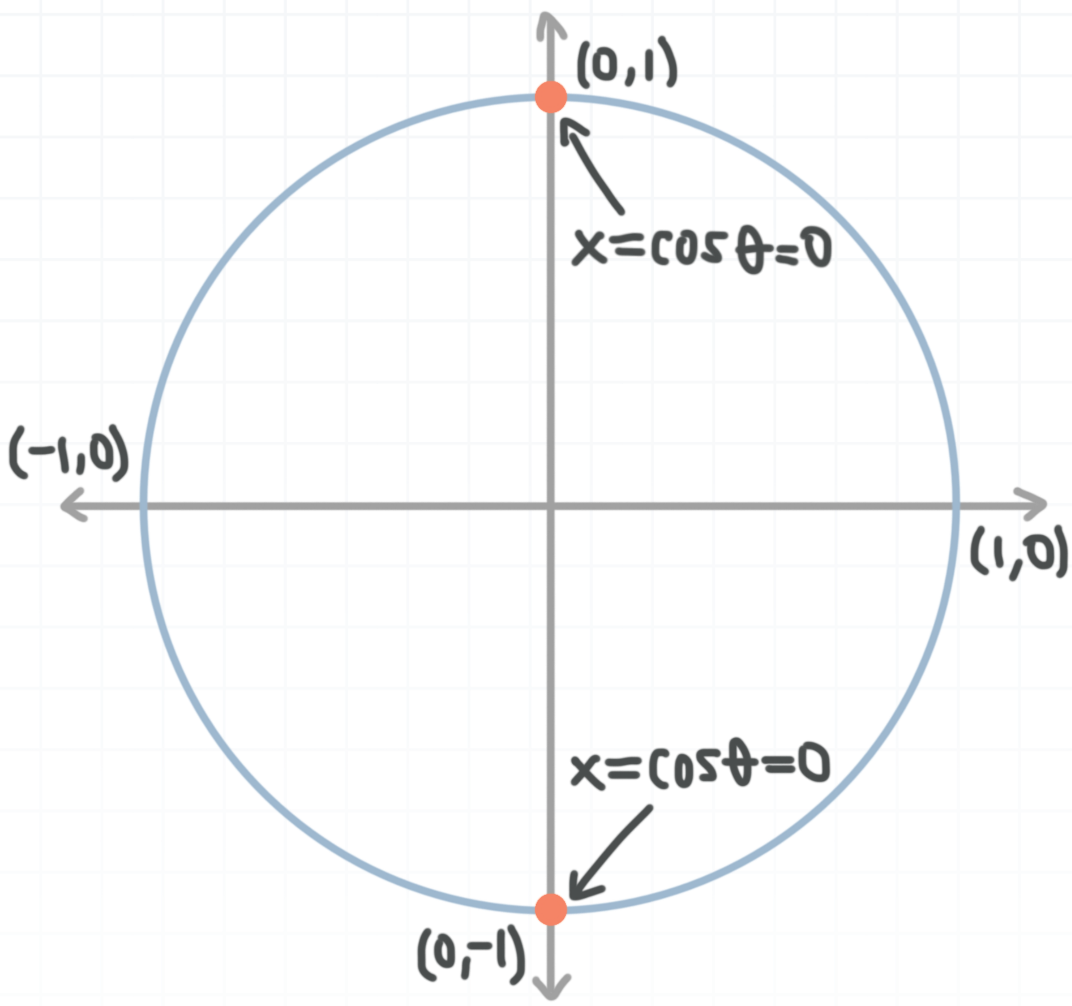
But not every one of the six trig functions is necessarily defined at every angle. The angles where we'll run into trouble will always be the quadrantal angles, where either sine or cosine of the angle is 0.

When sine and cosine are 0

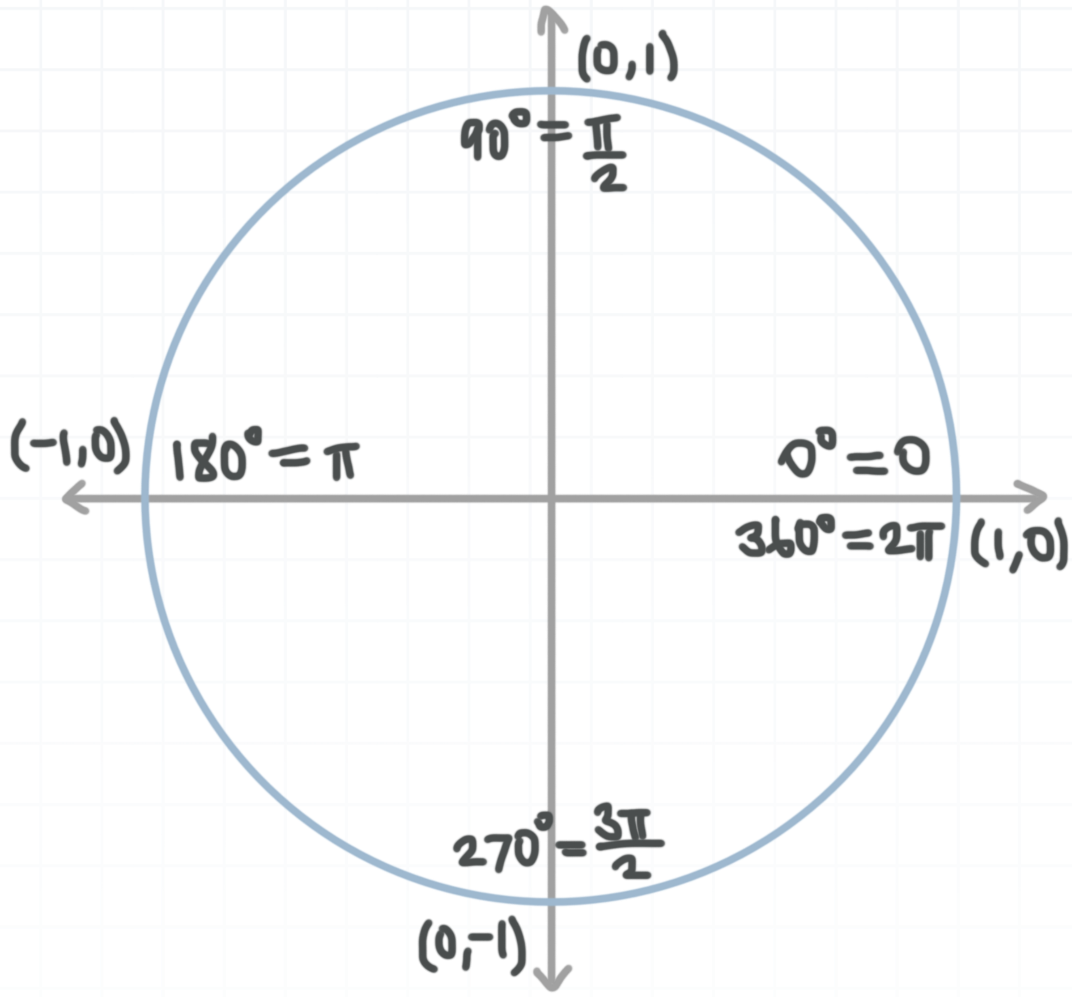
Remember that we learned previously that $\cos\theta$ represents the x-value in a coordinate point, and that $\sin\theta$ represents a y-value in a coordinate point. That means that, for a circle centered at the origin with radius 1, $\sin\theta = 0$ at $(1,0)$ and $(-1,0)$, which are the two points where the circle intersects the horizontal axis.



Likewise, $\cos\theta = 0$ at $(0,1)$ and $(0, -1)$, which are the two points where the circle intersects the vertical axis.



Remember also that, for angles sketched in standard position, these points along the major axes represent angles in degrees of $\theta = 0^\circ,\ 90^\circ,\ 180^\circ,\ 270^\circ,\ 360^\circ$ or angles in radians of $\theta = 0,\ \pi/2,\ \pi,\ 3\pi/2,\ 2\pi$, and all other angles that are coterminal with those sets.



In other words, we'll only have undefined trig functions at the quadrantal angles.

When the trig functions are undefined

The only question now is, which trig functions are undefined at which quadrantal angles? Well, if we remember the reciprocal identities for cosecant and secant, as well as the quotient identities for tangent and cotangent,

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

and we remember the fact that the value of a denominator can never be 0, then we can say that

- $\csc\theta$ and $\cot\theta$ are undefined when $\sin\theta = 0$

- $\sec\theta$ and $\tan\theta$ are undefined when $\cos\theta = 0$

Since $\sin\theta = 0$ only along the x-axis, and $\cos\theta = 0$ only along the y-axis, we could also write this as

- $\csc\theta$ and $\cot\theta$ are undefined for angles on the x-axis (anything coterminal with $\theta = 0$ or $\theta = \pi$)

- $\sec\theta$ and $\tan\theta$ are undefined for angles on the y-axis (anything coterminal with $\theta = \pi/2$ or $\theta = 3\pi/2$)

Let's summarize these in a table.

	sin	csc	cos	sec	tan	cot
0°=0	0	Undefined	1	1	0	Undefined
90°=π/2	1	1	0	Undefined	Undefined	0
180°=π	0	Undefined	-1	-1	0	Undefined
270°=3π/2	-1	-1	0	Undefined	Undefined	0
360°=2π	0	Undefined	1	1	0	Undefined

Let's look at an example where we're asked to figure out which trig functions are undefined for a particular angle.

Example

Find the values of all six trig functions at $\theta = 3\pi/2$, and say whether or not any of them are undefined at that angle.

The angle $\theta = 3\pi/2$ falls on the negative y-axis. So in a circle centered at the origin with radius 1,

$$y = \sin\frac{3\pi}{2} = -1$$

$$x = \cos\frac{3\pi}{2} = 0$$

Use the reciprocal identities to find cosecant and secant of the angle.

$$\csc\frac{3\pi}{2} = \frac{1}{\sin\frac{3\pi}{2}} = \frac{1}{-1} = -1$$

$$\sec\frac{3\pi}{2} = \frac{1}{\cos\frac{3\pi}{2}} = \frac{1}{0}$$

Use the quotient identities to find tangent and cotangent of the angle.

$$\tan\frac{3\pi}{2} = \frac{\sin\frac{3\pi}{2}}{\cos\frac{3\pi}{2}} = \frac{-1}{0}$$

$$\cot\frac{3\pi}{2} = \frac{\cos\frac{3\pi}{2}}{\sin\frac{3\pi}{2}} = \frac{0}{-1} = 0$$

So to summarize, because we got a 0 value in the denominator of the secant and tangent values, we know these two trig functions are undefined at $\theta = 3\pi/2$. The other four trig functions are defined.

$$\sin\theta = -1 \qquad \csc\theta = -1$$

$$\cos\theta = 0 \qquad \sec\theta = \text{undefined}$$

$$\tan\theta = \text{undefined} \qquad \cot\theta = 0$$
