Topic: Cofunction identities

Question: Find an angle θ that satisfies the equation.

$$\cos\left(\frac{4\pi}{9}\right) = \sin\theta$$

Answer choices:

A $-\dfrac{\pi}{18}$

B $\dfrac{4\pi}{9}$

C $\dfrac{5\pi}{18}$

D $\dfrac{\pi}{18}$

Solution: D

The equation we're given tells us that the sine of some angle is equivalent to cosine of the angle $4\pi/9$. Sine and cosine are cofunctions, which means we can plug into the cofunction identity for cosine that relates them.

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{\pi}{2} - \frac{4\pi}{9}\right)$$

Find a common denominator.

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{\pi}{2}\left(\frac{9}{9}\right) - \frac{4\pi}{9}\left(\frac{2}{2}\right)\right)$$

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{9\pi}{18} - \frac{8\pi}{18}\right)$$

$$\cos\left(\frac{4\pi}{9}\right) = \sin\left(\frac{\pi}{18}\right)$$

So the angle θ that satisfies the equation is $\theta = \pi/18$.

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Question: Find θ.

$$\sec\left(\frac{3\pi}{8}\right) = \csc\left(\frac{\pi}{2} - \theta\right)$$

Answer choices:

A $\dfrac{3\pi}{8}$

B $\dfrac{\pi}{8}$

C $\dfrac{\pi}{4}$

D $\dfrac{\pi}{2}$

Solution: A

Secant and cosecant are cofunctions, which means we can plug into the cofunction identity for secant that relates the two functions.

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\left(\frac{3\pi}{8}\right) = \csc\left(\frac{\pi}{2} - \frac{3\pi}{8}\right)$$

So the angle θ that satisfies the equation is $\theta = 3\pi/8$.

Topic: Cofunction identities

Question: Find an acute angle that satisfies the equation.

$$\tan(3\alpha + 27°) = \cot(\alpha + 3°)$$

Answer choices:

A   60°

B   18°

C   15°

D   72°

Solution: C

The equation matches the form of the cofunction identity for tangent, $\tan\theta = \cot(90° - \theta)$, where $\theta = 3\alpha + 27°$. Then

$$\alpha + 3° = 90° - (3\alpha + 27°)$$

$$\alpha + 3° = 90° - 3\alpha - 27°$$

$$\alpha + 3° = 63° - 3\alpha$$

$$4\alpha = 60°$$

$$\alpha = 15°$$