Topic: Complex number operations

Question: What are the sum and difference of the complex numbers?

$$2\frac{5}{6} - \frac{1}{3}i$$

$$-4\frac{1}{6} + \frac{1}{2}i$$

Answer choices:

A The sum is $-(11/6) + (1/6)i$

The difference is $(7/2) + (-1/3)i$

B The sum is $-(4/3) + (1/6)i$

The difference is $7 + (-5/6)i$

C The sum is $-(7/6) + (1/3)i$

The difference is $5 + (-2/3)i$

D The sum is $-1 + (1/3)i$

The difference is $(20/3) + (-1/6)i$

Solution: B

The sum of the complex numbers is

$$\left(\frac{17}{6}-\frac{1}{3}i\right)+\left(-\frac{25}{6}+\frac{1}{2}i\right)$$

$$\left(\frac{17}{6}-\frac{25}{6}\right)+\left(-\frac{1}{3}i+\frac{1}{2}i\right)$$

$$\left(\frac{17}{6}-\frac{25}{6}\right)+\left(-\frac{1}{3}+\frac{1}{2}\right)i$$

$$-\frac{8}{6}+\frac{1}{6}i$$

$$-\frac{4}{3}+\frac{1}{6}i$$

The difference of the complex numbers is

$$\left(\frac{17}{6}-\frac{1}{3}i\right)-\left(-\frac{25}{6}+\frac{1}{2}i\right)$$

$$\frac{17}{6}-\frac{1}{3}i+\frac{25}{6}-\frac{1}{2}i$$

$$\frac{17}{6}+\frac{25}{6}-\frac{1}{3}i-\frac{1}{2}i$$

$$\frac{42}{6}-\frac{5}{6}i$$

$$7-\frac{5}{6}i$$

Topic: Complex number operations

Question: What is the product of the complex numbers?

$$-9 - 5i$$

$$7 + 13i$$

Answer choices:

A $-63 - 65i$

B $48 + i$

C $-128 - 82i$

D $2 - 152i$

Solution: D

Use FOIL to find the product of the complex numbers.

$$(-9-5i)(7+13i)$$

$$(-9)(7)+(-9)(13i)+(-5i)(7)+(-5i)(13i)$$

$$-63+(-9)(13)i+(-5)(7)i+(-5)(13)(i^2)$$

$$-63-117i-35i+(-65)(i^2)$$

Using $i^2=-1$ in the last term, we get

$$-63-117i-35i+(-65)(-1)$$

$$-63-117i-35i+65$$

$$(-63+65)+(-117i-35i)$$

$$2+(-117-35)i$$

$$2-152i$$

Topic: Complex number operations

Question: Express the fraction in the form $a + bi$ where a and b are real numbers.

$$\frac{5 + 2i}{1 + 3i}$$

Answer choices:

A $-\frac{5}{4} + \frac{7}{4}i$

B $\frac{3}{5} - \frac{9}{10}i$

C $\frac{11}{10} - \frac{13}{10}i$

D $\frac{7}{8} + \frac{1}{8}i$

Solution: C

Multiply by the conjugate of the denominator.

$$\left(\frac{5+2i}{1+3i}\right)\left(\frac{1-3i}{1-3i}\right)$$

$$\frac{(5+2i)(1-3i)}{(1+3i)(1-3i)}$$

Use FOIL to expand the numerator and denominator.

$$\frac{5-15i+2i-6i^2}{1-3i+3i-9i^2}$$

$$\frac{5-13i-6i^2}{1-9i^2}$$

Using $i^2 = -1$ gives

$$\frac{5-13i-6(-1)}{1-9(-1)}$$

$$\frac{5-13i+6}{1+9}$$

$$\frac{11-13i}{10}$$

Split the fraction.

$$\frac{11}{10}-\frac{13}{10}i$$