Topic: Law of cosines

Question: Solve the triangle that has side lengths $a = 28$ and $b = 37$, with an included angle $C = 110°$.

Answer choices:

A $c \approx 12.3$, and the other two angles are $11.9°$ and $58.1°$

B $c \approx 39.8$ and the other two angles are $23.1°$ and $46.9°$

C $c \approx 53.5$ and the other two angles are $29.4°$ and $40.6°$

D $c \approx 66.2$ and the other two angles are $51.3°$ and $18.7°$

Solution: C

Plugging what we know into the law of cosines with $\cos C$ gives

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 28^2 + 37^2 - 2(28)(37)\cos 110^\circ$$

$$c^2 = 784 + 1{,}369 - 56(37)\cos 110^\circ$$

$$c^2 = 2{,}153 - 2{,}072(-0.342)$$

$$c^2 \approx 2{,}862$$

$$c \approx 53.5$$

Rewrite the law of cosines with $\cos A$ by solving it for $\cos A$. (We could also use the law of sines.)

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Plug in what we know to find A.

$$\cos A \approx \frac{28^2 - (37^2 + 53.5^2)}{-2(37)(53.5)}$$

$$\cos A \approx \frac{784 - (1{,}369 + 2{,}862)}{-74(53.5)}$$

$$\cos A \approx \frac{784 - 4{,}231}{-3{,}959}$$

$$\cos A \approx \frac{-3{,}447}{-3{,}959}$$

$$\cos A \approx 0.871$$

$$A \approx \arccos 0.871$$

$$A \approx 29.4^\circ$$

The third angle is therefore

$$B \approx 180^\circ - 110^\circ - 29.4^\circ$$

$$B \approx 40.6^\circ$$

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Question: If the side lengths of a triangle are $a = 17$, $b = 24$, and $c = 31$, what are the measures of its three interior angles?

Answer choices:

A $A \approx 33.0^\circ$, $B \approx 50.3^\circ$, and $C \approx 96.7^\circ$

B $A \approx 46.3^\circ$, $B \approx 55.6^\circ$, and $C \approx 78.1^\circ$

C $A \approx 37.4^\circ$, $B \approx 63.5^\circ$, and $C \approx 79.1^\circ$

D $A \approx 33.9^\circ$, $B \approx 61.9^\circ$, and $C \approx 84.2^\circ$

Solution: A

Rewrite the law of cosines with $\cos A$ by solving it for $\cos A$.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Plugging in all three side lengths gives

$$\cos A \approx \frac{17^2 - (24^2 + 31^2)}{-2(24)(31)}$$

$$\cos A \approx \frac{289 - (576 + 961)}{-48(31)}$$

$$\cos A \approx \frac{289 - 1{,}537}{-1{,}488}$$

$$\cos A \approx \frac{-1{,}248}{-1{,}488}$$

$$\cos A \approx 0.839$$

$$A \approx \arccos 0.839$$

$$A \approx 33.0^\circ$$

Rewrite the law of cosines with $\cos B$ by solving it for $\cos B$. (We could also use the law of sines.)

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{b^2 - (a^2 + c^2)}{-2ac}$$

Plugging in all three side lengths gives

$$\cos B \approx \frac{24^2 - (17^2 + 31^2)}{-2(17)(31)}$$

$$\cos B \approx \frac{576 - (289 + 961)}{-34(31)}$$

$$\cos B \approx \frac{576 - 1{,}250}{-1{,}054}$$

$$\cos B \approx \frac{-674}{-1{,}054}$$

$$\cos B \approx 0.639$$

$$B \approx \arccos 0.639$$

$$B \approx 50.3^\circ$$

Then the third angle is

$$C = 180^\circ - 33.0^\circ - 50.3^\circ$$

$$C = 96.7^\circ$$

Topic: Law of cosines

Question: Find the length of a in the triangle that has side lengths $b = 14$ and $c = 31$, with an included angle $A = 135°$.

Answer choices:

A $1{,}771$

B 23.3

C 14.3

D 42.1

Solution: D

Plugging what we know into the law of cosines with $\cos C$ gives

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 14^2 + 31^2 - 2(14)(31)\cos 135^\circ$$

$$a^2 = 196 + 961 - 28(31)\cos 135^\circ$$

$$a^2 = 1{,}157 - 868(-0.707)$$

$$a^2 \approx 1{,}771$$

$$a \approx 42.1$$