Topic: Product-to-sum identities

Question: Rewrite $\cos(14\theta)\sin(-5\theta)$ as a sum.

Answer choices:

A $\frac{1}{2}\left[\sin(19\theta)+\sin(-5\theta)\right]$

B $\frac{1}{2}\left[\sin(9\theta)+\sin(-19\theta)\right]$

C $\frac{1}{2}\left[\sin(19\theta)+\sin(-14\theta)\right]$

D $\frac{1}{2}\left[\sin(14\theta)+\sin(-9\theta)\right]$

Solution: B

Using the product-to-sum identity,

$$\cos\theta\sin\alpha = \frac{1}{2}\left[\sin(\theta+\alpha) - \sin(\theta-\alpha)\right]$$

we can set $\theta = 14\theta$ and $\alpha = -5\theta$ and rewrite the product as

$$\frac{1}{2}\left[\sin(14\theta + (-5\theta)) - \sin(14\theta - (-5\theta))\right]$$

$$\frac{1}{2}\left[\sin(14\theta - 5\theta) - \sin(14\theta + 5\theta)\right]$$

$$\frac{1}{2}\left[\sin(9\theta) - \sin(19\theta)\right]$$

Use the even-odd identity $\sin(-\theta) = -\sin\theta$ to rewrite the difference as a sum.

$$\frac{1}{2}\left[\sin(9\theta) + \sin(-19\theta)\right]$$

Topic: Product-to-sum identities

Question: Find the value of the expression.

$$\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$$

Answer choices:

A $\frac{\sqrt{3}+2}{2}$

B $\frac{\sqrt{3}-2}{2}$

C $\frac{1}{2}$

D $\frac{1}{4}$

Solution: D

Using the product-to-sum identity,

$$\sin\theta\cos\alpha = \frac{1}{2}\left[\sin(\theta+\alpha)+\sin(\theta-\alpha)\right]$$

we can set $\theta = \pi/12$ and $\alpha = \pi/12$ and rewrite the product as

$$\frac{1}{2}\left[\sin\left(\frac{\pi}{12}+\frac{\pi}{12}\right)+\sin\left(\frac{\pi}{12}-\frac{\pi}{12}\right)\right]$$

$$\frac{1}{2}\left(\sin\frac{\pi}{6}+\sin 0\right)$$

$$\frac{1}{2}\left(\frac{1}{2}+0\right)$$

$$\frac{1}{4}$$

Topic: Product-to-sum identities

Question: Which angle pair is a solution to the equation?

$$\cos\theta\cos\alpha = -\frac{2+\sqrt{2}}{4}$$

Answer choices:

A $(\theta, \alpha) = \left(\frac{17\pi}{8}, \frac{25\pi}{8}\right)$

B $(\theta, \alpha) = \left(\frac{11\pi}{8}, \frac{7\pi}{8}\right)$

C $(\theta, \alpha) = \left(\frac{5\pi}{8}, \frac{7\pi}{8}\right)$

D $(\theta, \alpha) = \left(\frac{9\pi}{8}, \frac{11\pi}{8}\right)$

Solution: A

We need to test each answer choice using the product-to-sum identity for the product of two cosine functions.

$$\cos\theta\cos\alpha=\frac{1}{2}\left[\cos(\theta+\alpha)+\cos(\theta-\alpha)\right]$$

Test answer choice A with $\theta=17\pi/8$ and $\alpha=25\pi/8$.

$$\frac{1}{2}\left[\cos\left(\frac{17\pi}{8}+\frac{25\pi}{8}\right)+\cos\left(\frac{17\pi}{8}-\frac{25\pi}{8}\right)\right]$$

$$\frac{1}{2}\left[\cos\left(\frac{42\pi}{8}\right)+\cos\left(-\frac{8\pi}{8}\right)\right]$$

$$\frac{1}{2}\left[\cos\left(\frac{21\pi}{4}\right)+\cos(-\pi)\right]$$

The angle $21\pi/4$ is coterminal with $5\pi/4$, so both angles will have the same cosine, and we can rewrite the expression as

$$\frac{1}{2}\left[\cos\left(\frac{5\pi}{4}\right)+\cos(-\pi)\right]$$

Using the unit circle, we can find the value of each of the cosine functions.

$$\frac{1}{2}\left[-\frac{\sqrt{2}}{2}+(-1)\right]$$

$$-\frac{\sqrt{2}}{4}-\frac{1}{2}$$

Find a common denominator.

$$-\frac{\sqrt{2}}{4}-\frac{2}{4}$$

$$\frac{-2-\sqrt{2}}{4}$$

$$-\frac{2+\sqrt{2}}{4}$$