Distances and midpoints

We've learned in the past how to find the distance between two points in rectangular coordinate space. We do it algebraically or graphically.

Algebraically, given the points $(3,2)$ and $(-1,-3)$, the distance between them can be found using the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(3 - (-1))^2 + (2 - (-3))^2}$$

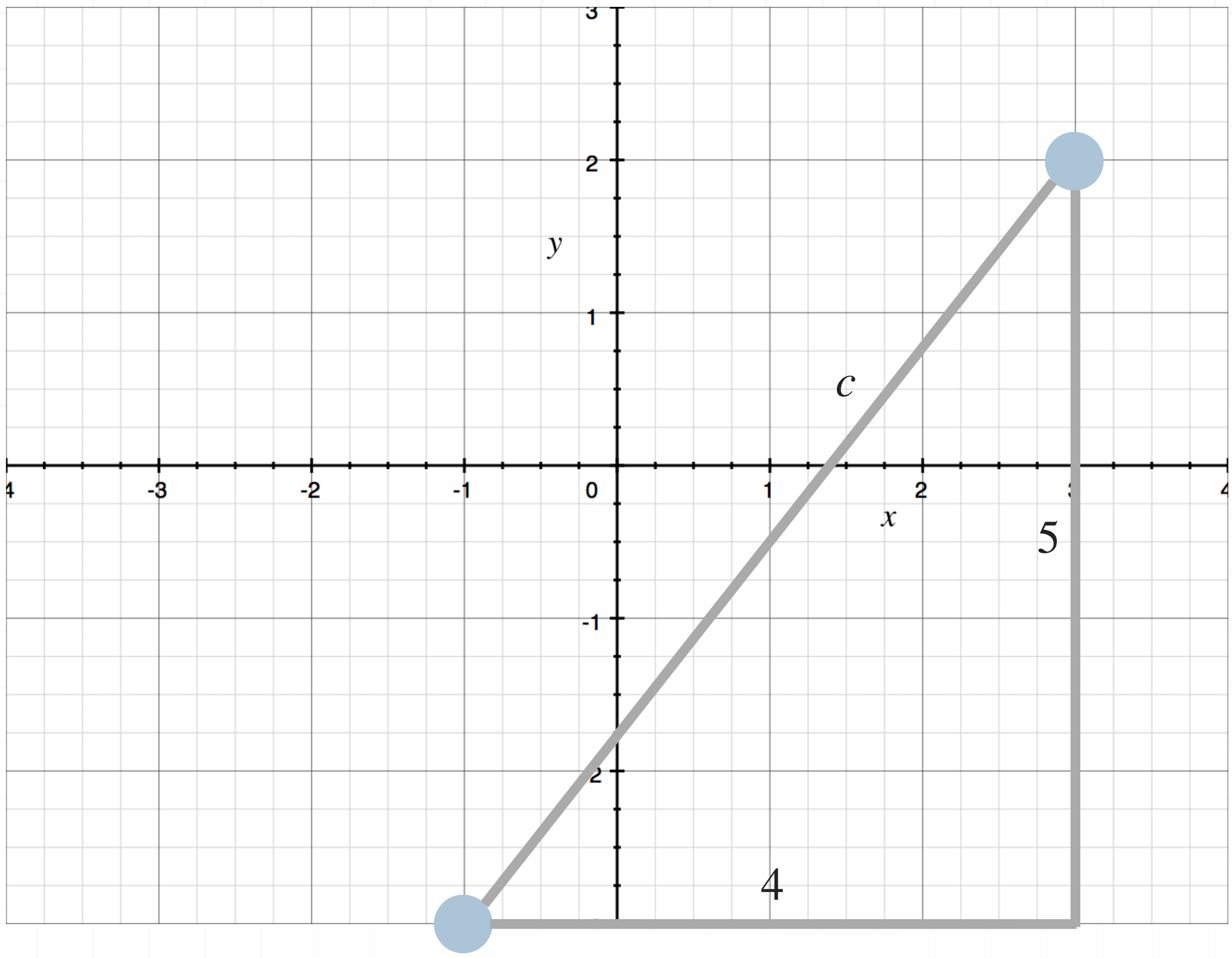
$$d = \sqrt{(3 + 1)^2 + (2 + 3)^2}$$

$$d = \sqrt{4^2 + 5^2}$$

$$d = \sqrt{16 + 25}$$

$$d = \sqrt{41}$$

Or we can do this graphically using the Pythagorean theorem. If we graph $(3,2)$ and $(-1,-3)$, and measure the horizontal and vertical distances between them, we get



Then the Pythagorean theorem, the hypotenuse of the triangle, or the distance between the points, is

$$4^2 + 5^2 = c^2$$

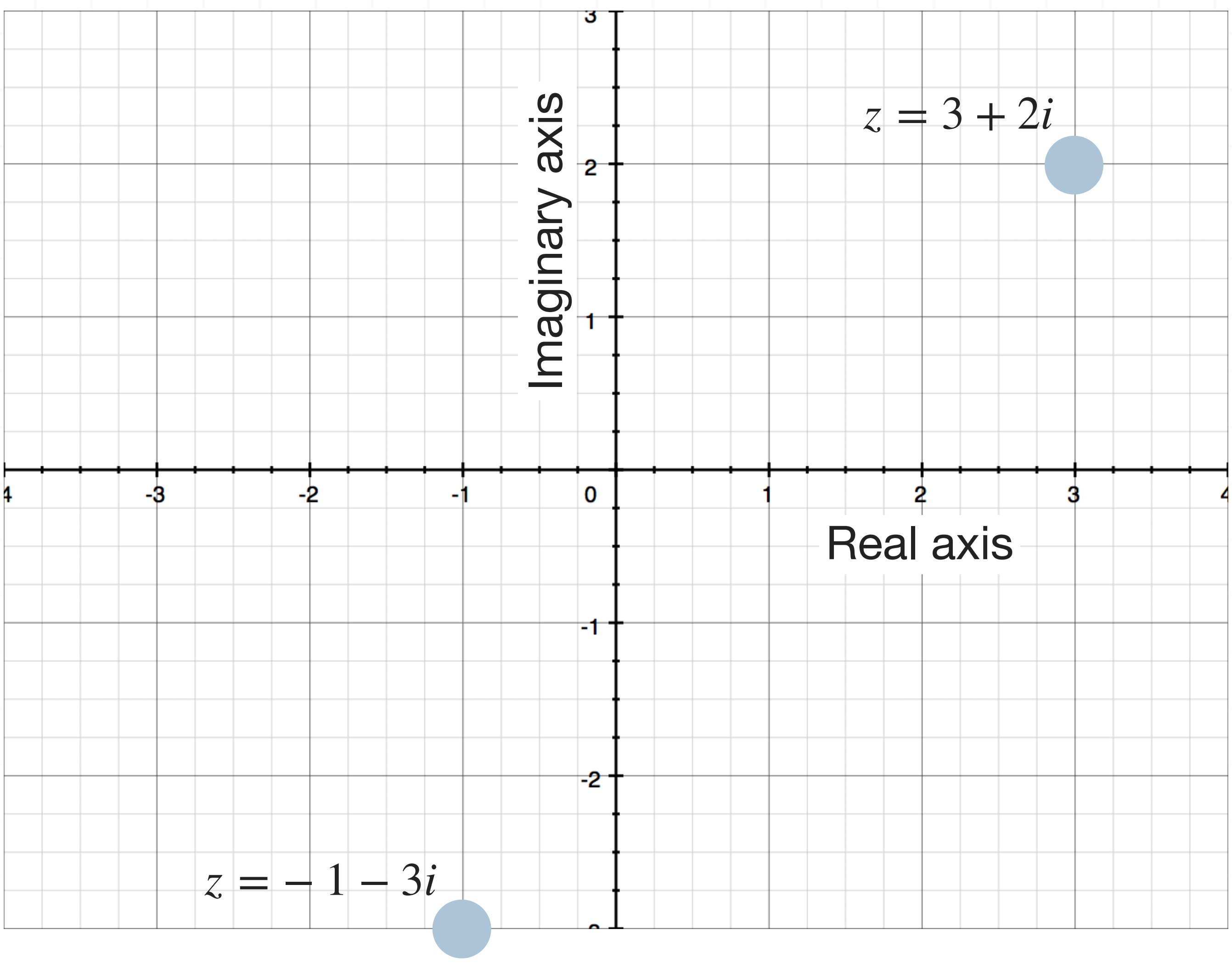
$$16 + 25 = c^2$$

$$41 = c^2$$

$$c = \sqrt{41}$$

The distance between complex numbers

Finding the distance between complex numbers follows exactly the same process. For instance, given the complex numbers $z = -1 - 3i$ and $z = 3 + 2i$,



we can find the distance between them by finding the difference between the real parts and the imaginary parts. The distance between the real parts is $3 - (-1) = 3 + 1 = 4$, and the distance between the imaginary parts is $2 - (-3) = 2 + 3 = 5$. Then by the Pythagorean them, the distance between $z = -1 - 3i$ and $z = 3 + 2i$ is

$$4^2 + 5^2 = c^2$$

$$16 + 25 = c^2$$

$$41 = c^2$$

$$c = \sqrt{41}$$

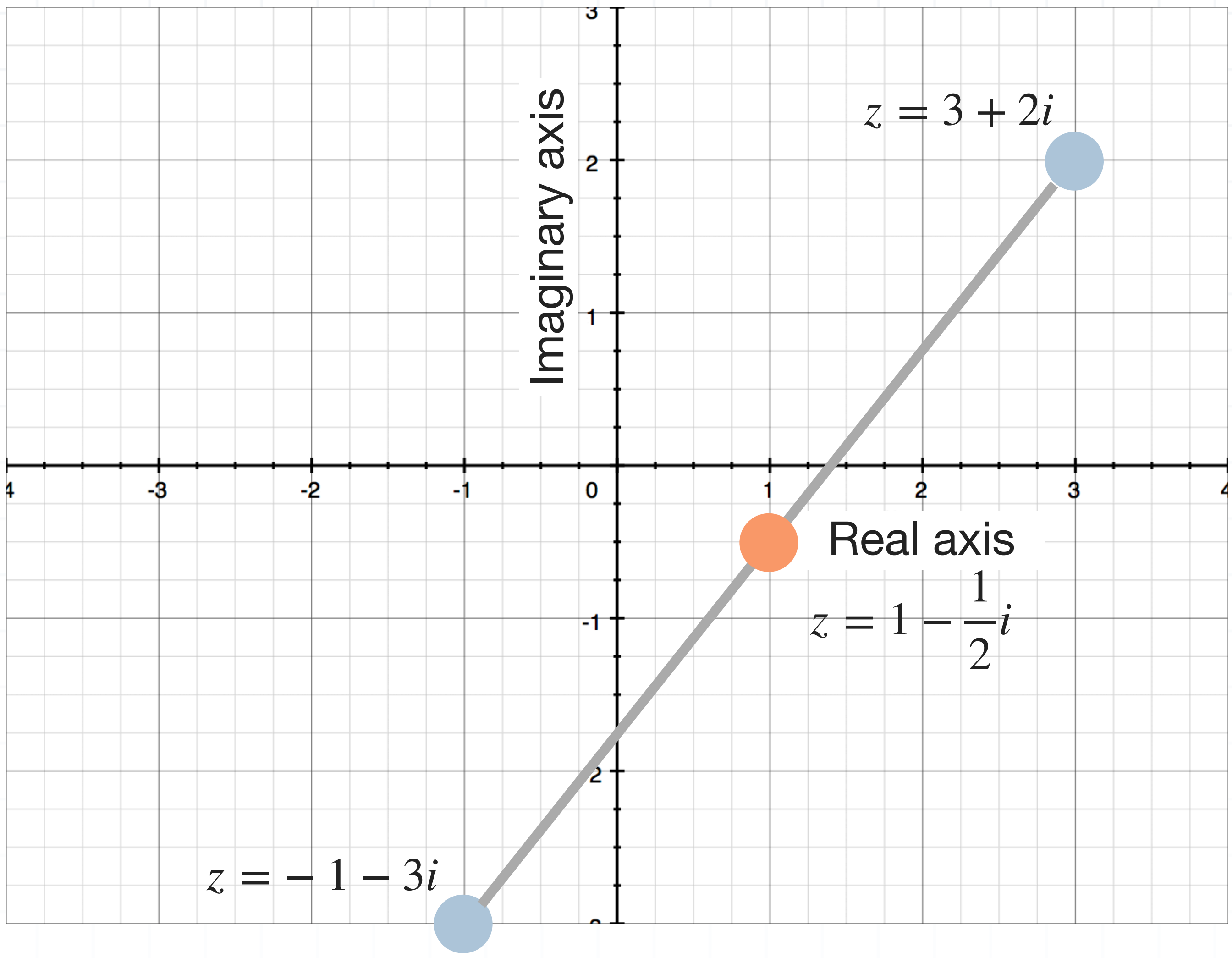
The midpoint between complex numbers

To find the midpoint between complex numbers, we just find the midpoint of the real parts, and separately the midpoint of the imaginary parts.

The distance between the real parts of $z = -1 - 3i$ and $z = 3 + 2i$ is $3 - (-1) = 3 + 1 = 4$. Half of that distance is $4/2 = 2$, so we look for the value that's 2 units from -1 and 2 units from 3, so the midpoint between those real parts must be 1.

The distance between the imaginary parts of $z = -1 - 3i$ and $z = 3 + 2i$ is $2 - (-3) = 2 + 3 = 5$. Half of that distance is $5/2 = 2.5$, so we look for the value that's 2.5 units from -3 and 2.5 units from 2, so the midpoint between those imaginary parts must be -0.5.

So the midpoint between $z = -1 - 3i$ and $z = 3 + 2i$ is $z = 1 - (1/2)i$. If we graph all three of these in the complex plane, we get


$z = 3 + 2i$
Imaginary axis
Real axis
$z = 1 - \frac{1}{2}i$
$z = -1 - 3i$