Topic: Graph the polar curve, cardioid

Question: Which of the following is a pair of polar coordinates (r, θ) of some point of the cardioid?

$$r = 6(1 - \sin\theta)$$

Answer choices:

A $(r, \theta) = \left(6 + 3\sqrt{3}, \frac{\pi}{3}\right)$

B $(r, \theta) = \left(6 - 2\sqrt{2}, \frac{\pi}{6}\right)$

C $(r, \theta) = \left(6 + 3\sqrt{3}, \frac{5\pi}{3}\right)$

D $(r, \theta) = \left(6 + 2\sqrt{2}, -\frac{\pi}{4}\right)$

Solution: C

If we substitute the angle $\theta = 5\pi/3$ from answer choice C into the polar equation $r = 6(1 - \sin\theta)$ we get

$$r = 6\left[1 - \sin\left(\frac{5\pi}{3}\right)\right]$$

Now

$$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

so

$$r = 6\left[1 - \left(-\frac{\sqrt{3}}{2}\right)\right] = 6\left[1 + \frac{\sqrt{3}}{2}\right] = 6 + 3\sqrt{3}$$

Thus the point with polar coordinates

$$\left(6 + 3\sqrt{3}, \frac{5\pi}{3}\right)$$

is a point of the cardioid $r = 6(1 - \sin\theta)$.

Now we'll show that none of the other answer choices is correct.

If we substitute the angle $\theta = \pi/3$ from answer choice A into the polar equation $r = 6(1 - \sin\theta)$, we get

$$r = 6\left[1 - \sin\left(\frac{\pi}{3}\right)\right]$$

Now

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

so

$$r = 6\left(1 - \frac{\sqrt{3}}{2}\right) = 6 - 3\sqrt{3} \neq 6 + 3\sqrt{3}$$

Thus the point with polar coordinates

$$\left(6 + 3\sqrt{3}, \frac{\pi}{3}\right)$$

is not a point of the cardioid $r = 6(1 - \sin\theta)$.

For answer choice B, we'll substitute $\pi/6$ for θ, which gives

$$r = 6\left[1 - \sin\left(\frac{\pi}{6}\right)\right]$$

Now

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

so

$$r = 6\left(1 - \frac{1}{2}\right) = 6 - 3 = 3 \neq 6 - 2\sqrt{2}$$

Thus the point with polar coordinates

$$\left(6 - 2\sqrt{2}, \frac{\pi}{6}\right)$$

is not a point of the cardioid $r = 6(1 - \sin\theta)$.

For answer choice D, we'll substitute $-\pi/4$ for θ, which gives

$$r = 6\left[1 - \sin\left(-\frac{\pi}{4}\right)\right]$$

Now

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

so

$$r = 6\left[1 - \left(-\frac{\sqrt{2}}{2}\right)\right] = 6\left(1 + \frac{\sqrt{2}}{2}\right) = 6 + 3\sqrt{2} \neq 6 + 2\sqrt{2}$$

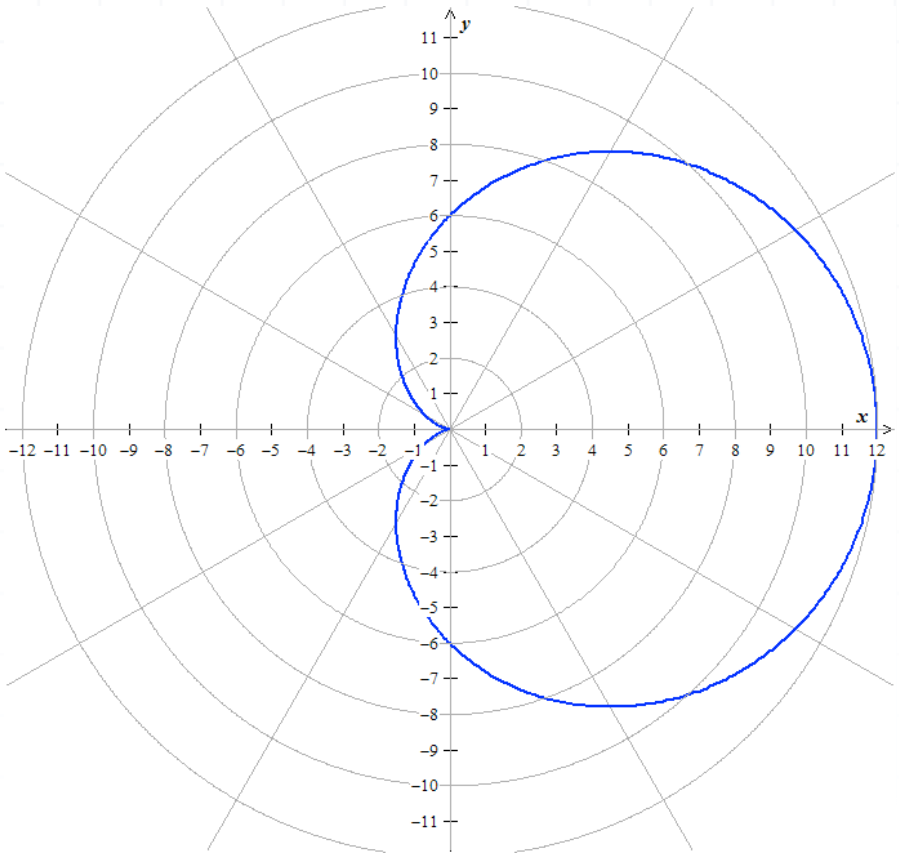
This shows that the point with polar coordinates

$$\left(6 + 2\sqrt{2}, -\frac{\pi}{4}\right)$$

is not a point of the cardioid $r = 6(1 - \sin\theta)$.

Topic: Graph the polar curve, cardioid

Question: The following curve is the graph of one of the polar equations given below. Which polar equation is it?



Answer choices:

A $r = 6(1 + \cos\theta)$

B $r = 12(1 + \cos\theta)$

C $r = 6(1 - \sin\theta)$

D $r = 12(1 + \sin\theta)$

Solution: A

This curve is a cardioid that's symmetric with respect to the horizontal axis, so it's a “cosine cardioid”. This means that we can eliminate answer choices C and D.

The point of the given curve which is furthest from the pole is at a distance of 12 units from the pole and has polar coordinates $(r, \theta) = (12,0)$. Now

$$\theta = 0 \Longrightarrow r = 6(1 + \cos\theta) = 6(1 + 1) = 6(2) = 12$$

and

$$\theta = 0 \Longrightarrow r = 12(1 + \cos\theta) = 12(1 + 1) = 12(2) = 24 \neq 12$$

Thus answer choice A is correct.

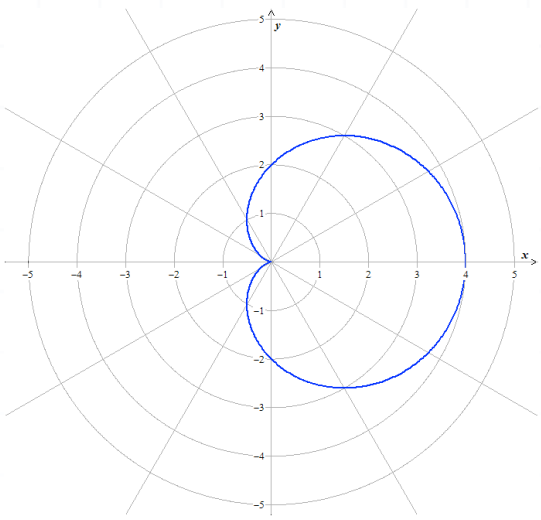
Topic: Graph the polar curve, cardioid

Question: Which of the following is the graph of the cardioid?

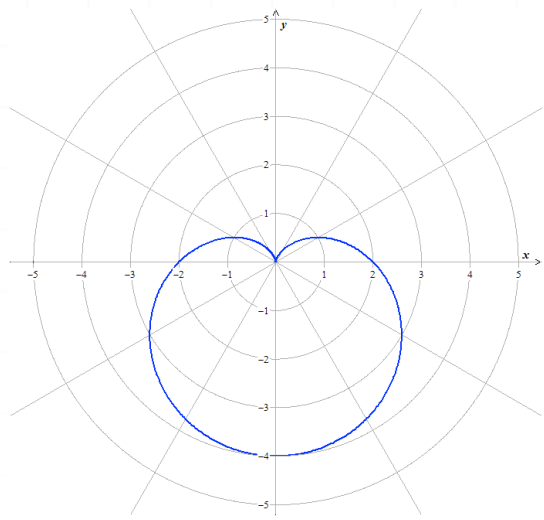
$$r = 4(1 + \sin\theta)$$

Answer choices:

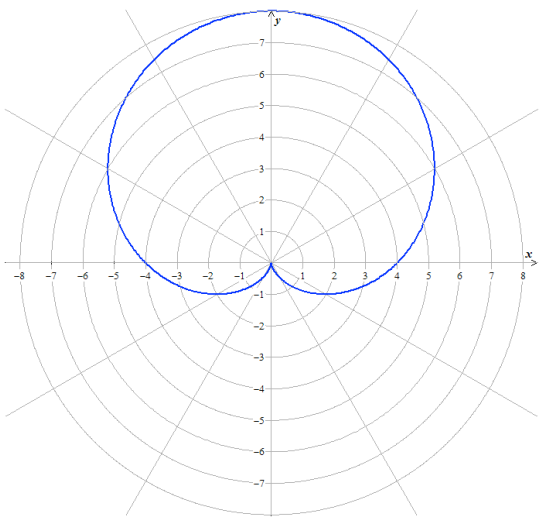
A



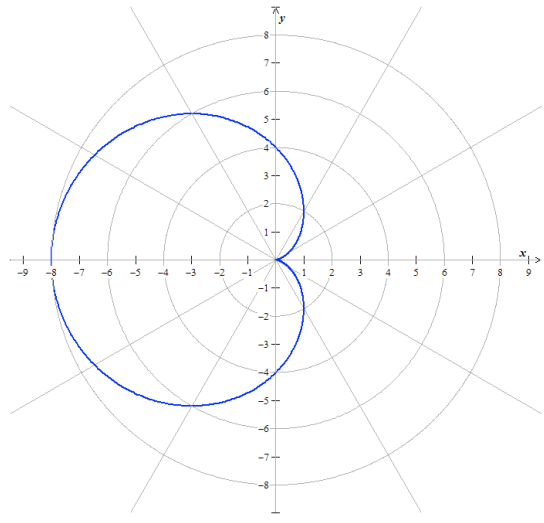
C



B



D



Solution: B

The equation $r = 4(1 + \sin\theta)$ is a "sine cardioid", so its graph is symmetric with respect to the vertical axis. Therefore, we can eliminate answer choices A and D, because those curves are symmetric with respect to the horizontal axis.

The point of the cardioid $r = 4(1 + \sin\theta)$ which is furthest from the pole is $8 = 2(4)$ units away from the pole. The curve in answer choice C doesn't include a point which is 8 units away from the pole, so B is the correct answer.