



Precalculus Formulas

Conversion between cartesian and polar coordinates

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x^2 + y^2 = r^2$$

Eccentricity

Let F be a fixed point (called the focus) and l be a fixed line (called the directrix) in a plane. Let e be a fixed positive number (called the eccentricity). The set of all points P in the plane such that

$$e = \frac{|PF|}{|Pl|}$$

(that is, the ratio of the distance from F to the distance from l is the constant e) is a conic section. The conic is

an ellipse if $e < 1$

a parabola if $e = 1$

a hyperbola if $e > 1$

Polar equation of the conic section

The polar equation of the conic section is

$$r = \frac{ed}{1 \pm e \cos \theta} \quad \text{or} \quad r = \frac{ed}{1 \pm e \sin \theta}$$



Analytic geometry of the parabola

Equation

Vertex

Axis

Focus

Directrix

Parabolas with vertex at the origin

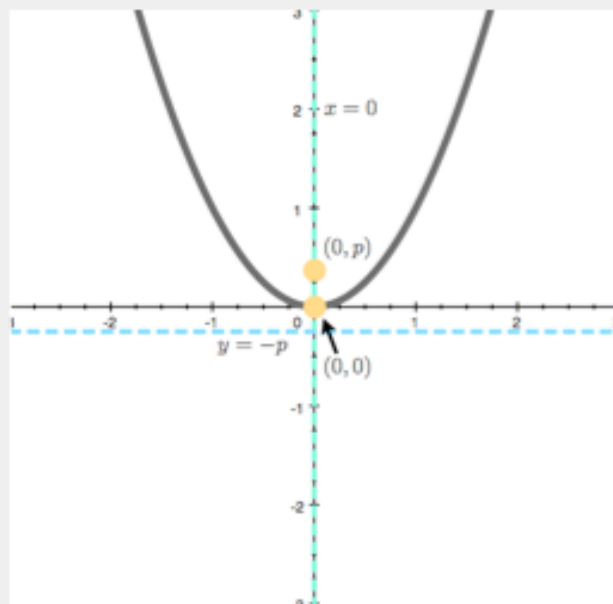
Opens up	$x^2 = 4py$	$(0,0)$	$x = 0$	$(0,p)$	$y = -p$
Opens down	$x^2 = -4py$	$(0,0)$	$x = 0$	$(0, -p)$	$y = p$
Opens right	$y^2 = 4px$	$(0,0)$	$y = 0$	$(p,0)$	$x = -p$
Opens left	$y^2 = -4px$	$(0,0)$	$y = 0$	$(-p,0)$	$x = p$

Shifted parabolas with vertex off the origin

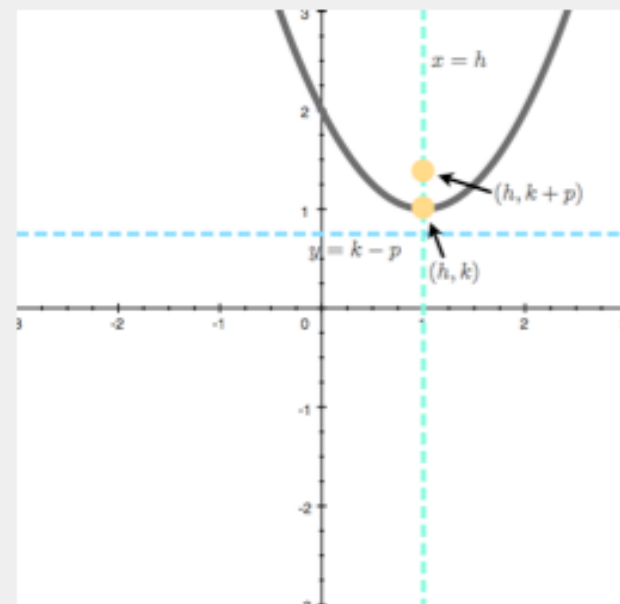
Opens up	$(x - h)^2 = 4p(y - k)$	(h, k)	$x = h$	$(h, k + p)$	$y = k - p$
Opens down	$(x - h)^2 = -4p(y - k)$	(h, k)	$x = h$	$(h, k - p)$	$y = k + p$
Opens right	$(y - k)^2 = 4p(x - h)$	(h, k)	$y = k$	$(h + p, k)$	$x = h - p$
Opens left	$(y - k)^2 = -4p(x - h)$	(h, k)	$y = k$	$(h - p, k)$	$x = h + p$



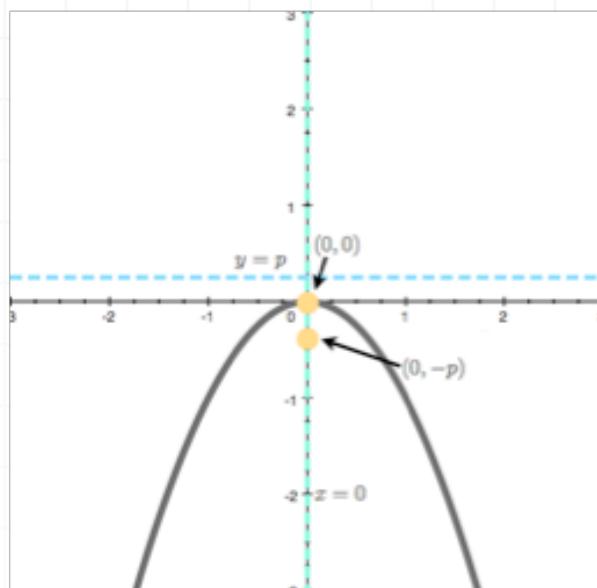
parabola (up)



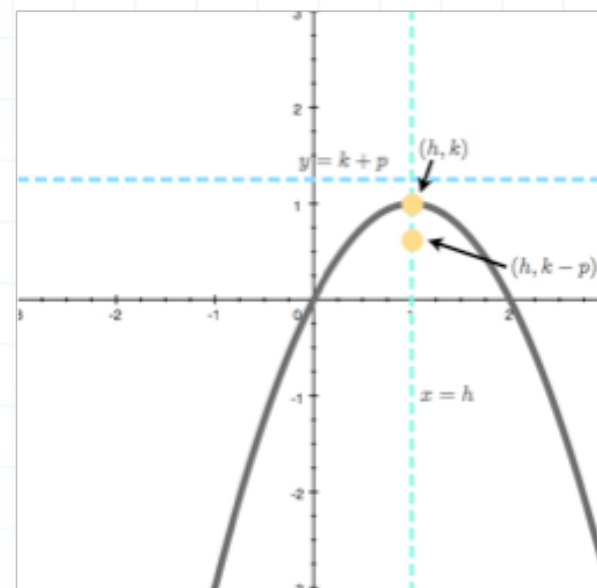
shifted parabola (up)



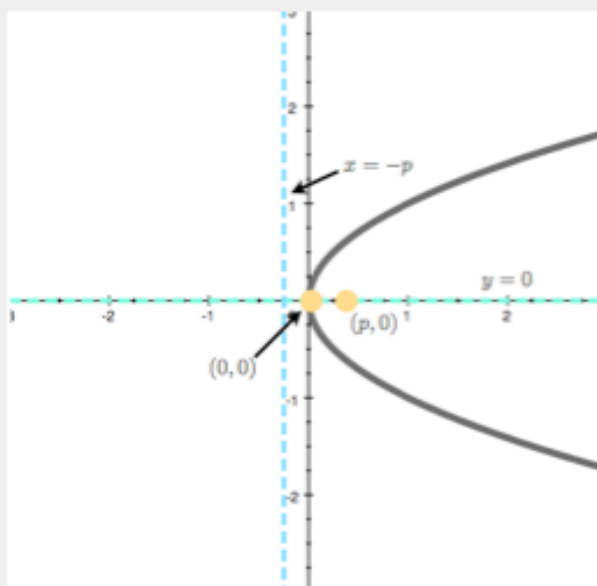
parabola (down)



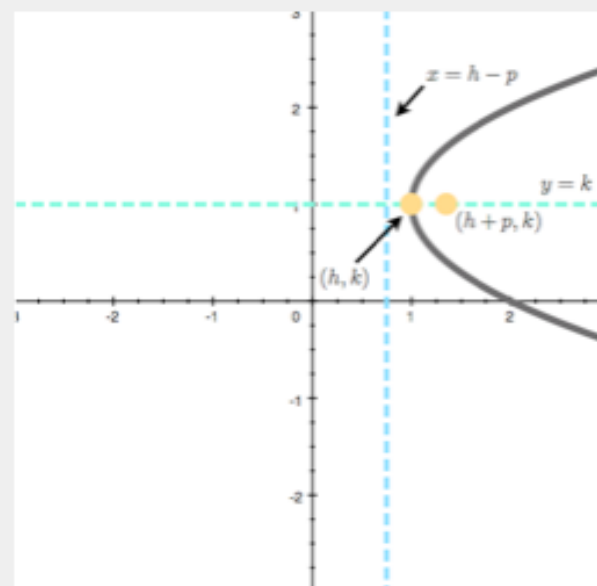
shifted parabola (down)



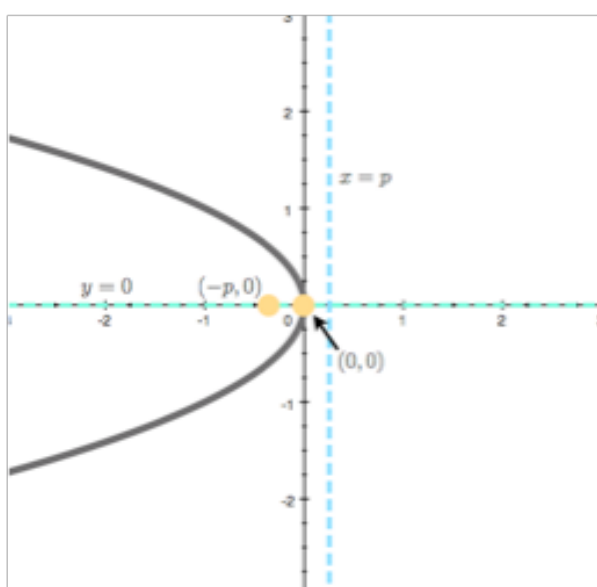
parabola (right)



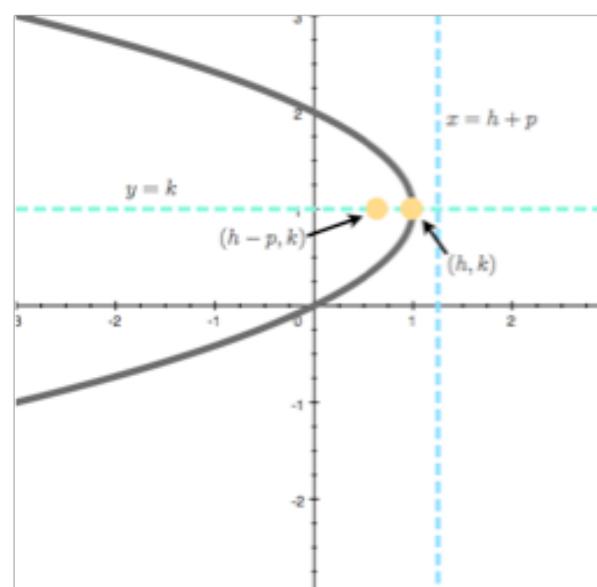
shifted parabola (right)



parabola (left)



shifted parabola (left)

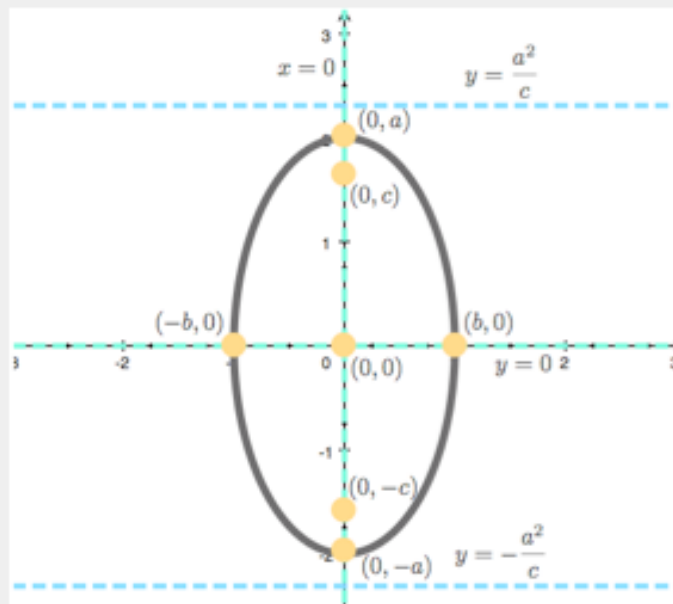


Analytic geometry of the ellipse

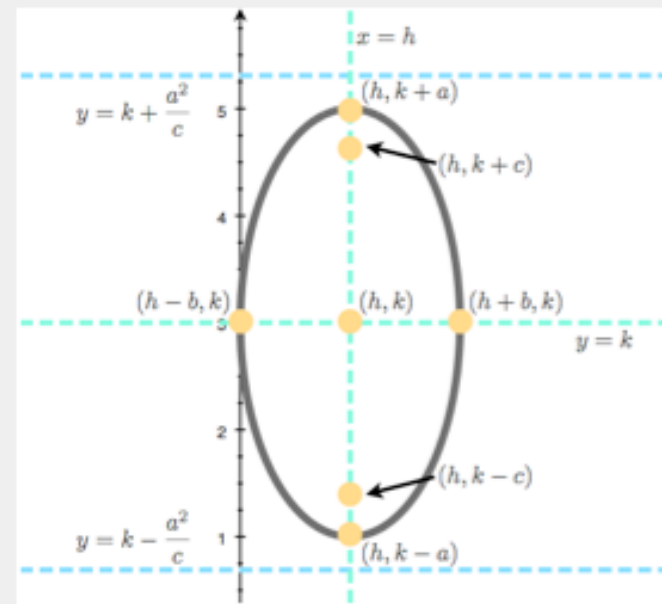
	Center at the origin (0,0)		Center off the origin at (h,k)	
	Tall	Wide	Tall	Wide
	$a \geq b > 0$	$a \geq b > 0$	$a \geq b > 0$	$a \geq b > 0$
Equation	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
Vertices	$(0, \pm a)$	$(\pm a, 0)$	$(h, k \pm a)$	$(h \pm a, k)$
	$(\pm b, 0)$	$(0, \pm b)$	$(h \pm b, k)$	$(h, k \pm b)$
Axes	Major: $x = 0$	Major: $y = 0$	Major: $x = h$	Major: $y = k$
	Minor: $y = 0$	Minor: $x = 0$	Minor: $y = k$	Minor: $x = h$
Foci	$(0, \pm c)$ with	$(\pm c, 0)$ with	$(h, k \pm c)$ with	$(h \pm c, k)$ with
	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$
Directrices	$y = \pm \frac{a^2}{c}$	$x = \pm \frac{a^2}{c}$	$y = k \pm \frac{a^2}{c}$	$x = h \pm \frac{a^2}{c}$



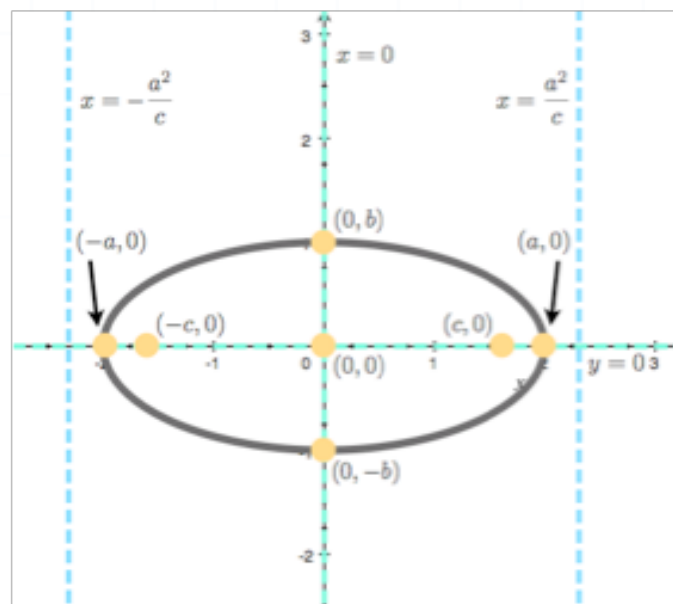
ellipse (tall)



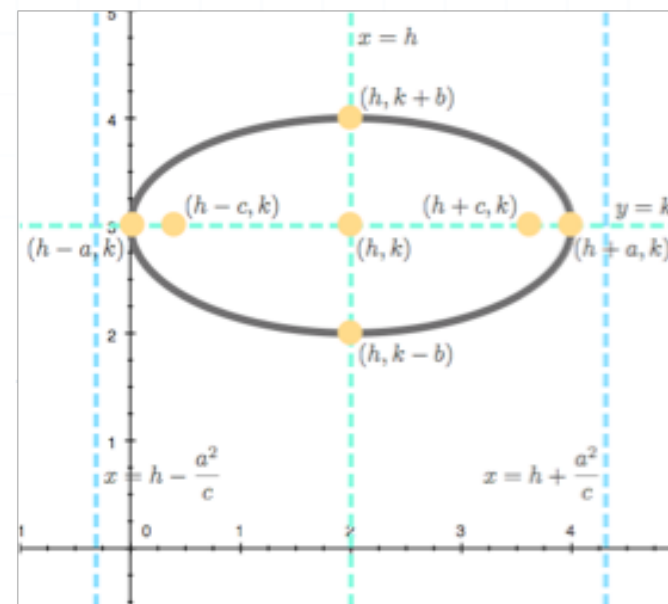
shifted ellipse (tall)



ellipse (wide)



shifted ellipse (wide)

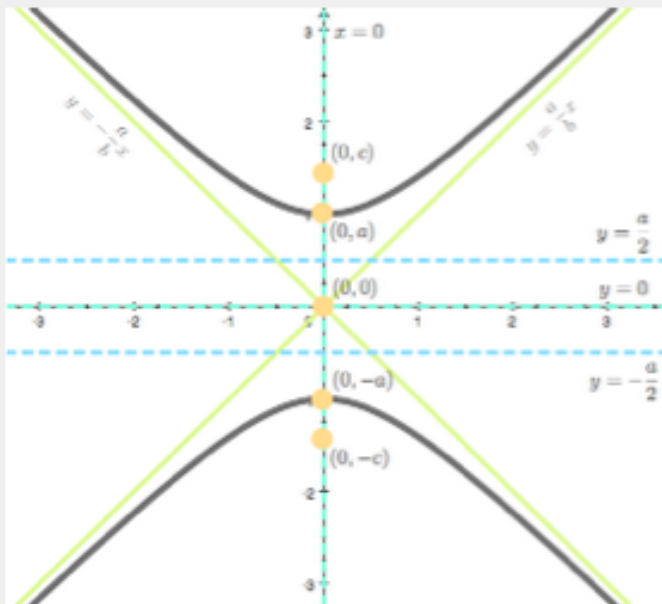


Analytic geometry of the hyperbola

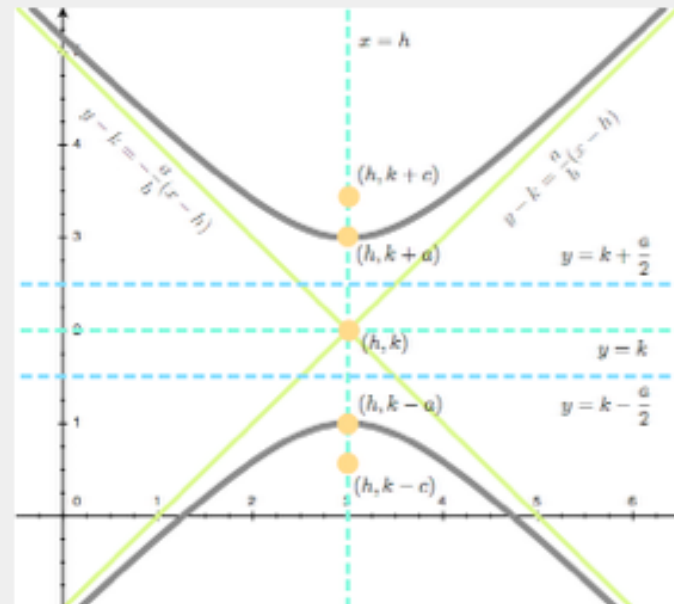
	Center at the origin (0,0)		Center off the origin at (h,k)	
	Up/down	Left/right	Up/down	Left/right
Equation	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Vertices	$(0, \pm a)$	$(\pm a, 0)$	$(h, k \pm a)$	$(h \pm a, k)$
Axes	Major: $x = 0$	Major: $y = 0$	Major: $x = h$	Major: $y = k$
	Minor: $y = 0$	Minor: $x = 0$	Minor: $y = k$	Minor: $x = h$
Foci	$(0, \pm c)$ with $c^2 = a^2 + b^2$	$(\pm c, 0)$ with $c^2 = a^2 + b^2$	$(h, k \pm c)$ with $c^2 = a^2 + b^2$	$(h \pm c, k)$ with $c^2 = a^2 + b^2$
Directrices	$y = \pm \frac{a}{2}$	$x = \pm \frac{a}{2}$	$y = k \pm \frac{a}{2}$	$x = h \pm \frac{a}{2}$
Asymptotes	$y = \pm \frac{a}{b}x$	$y = \pm \frac{b}{a}x$	$y - k = \pm \frac{a}{b}(x - h)$	$y - k = \pm \frac{b}{a}(x - h)$



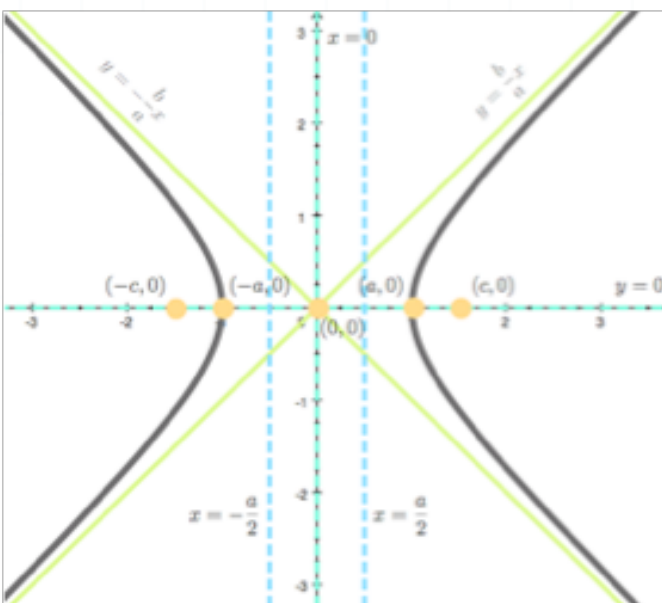
hyperbola (up-down)



shifted hyperbola (up-down)



hyperbola (right-left)



shifted hyperbola (right-left)

