Topic: Negative angles and angles more than one rotation

Question: Which angle in $[0°,360°)$ is coterminal with $-116°$?

Answer choices:

A $116°$

B $326°$

C $360°$

D $244°$

Solution: D

The angle $\theta = -116^\circ$ is close enough to the interval $[0^\circ, 360^\circ)$ that we'll just add 360° to $\theta = -116^\circ$ to find the coterminal angle.

$$-116^\circ + 360^\circ$$

$$244^\circ$$

Topic: Negative angles and angles more than one rotation

Question: Which angle in the interval $[0,2\pi)$ is coterminal with $-9\pi/4$?

Answer choices:

A $\frac{9\pi}{4}$

B $-\frac{7\pi}{2}$

C $\frac{7\pi}{4}$

D $\frac{5\pi}{4}$

Solution: C

To find the number of full rotations included in $\theta = -9\pi/4$, we'll divide the angle by 2π.

$$\frac{-\frac{9\pi}{4}}{2\pi}$$

$$-\frac{9\pi}{4} \cdot \frac{1}{2\pi}$$

$$-\frac{9\pi}{8\pi}$$

$$-1.125$$

So $\theta = -9\pi/4$ is 1 full rotation in the negative direction, and then an additional 0.125 of one more rotation in the negative direction. So to find a coterminal angle, we'll get rid of the 1 full rotation by adding $1(2\pi)$ to the angle.

$$-\frac{9\pi}{4} + 1(2\pi)$$

$$-\frac{9\pi}{4} + 2\pi$$

$$-\frac{9\pi}{4} + \frac{8\pi}{4}$$

$$-\frac{\pi}{4}$$

Now we have an angle that's less than one full rotation, but we'd still like to find a positive coterminal angle that's less than one full rotation. So we'll add 2π one more time.

$$-\frac{\pi}{4}+2\pi$$

$$-\frac{\pi}{4}+\frac{8\pi}{4}$$

$$\frac{7\pi}{4}$$

Therefore, we can say that $7\pi/4$ is coterminal with $\theta = -9\pi/4$ in the interval $[0,2\pi)$.

Topic: Negative angles and angles more than one rotation

Question: Find the value of sine of the angle $\theta = -71\pi/4$.

Answer choices:

A 1

B $\frac{\sqrt{2}}{2}$

C $-\frac{1}{2}$

D $-\frac{\sqrt{2}}{2}$

Solution: B

First we need to find the angle in the interval $[0,2\pi)$ that's coterminal with $\theta = -71\pi/4$.

To find the number of full rotations included in $\theta = -71\pi/4$, we'll divide the angle by 2π.

$$\frac{-\frac{71\pi}{4}}{2\pi}$$

$$-\frac{71\pi}{4} \cdot \frac{1}{2\pi}$$

$$-\frac{71\pi}{8\pi}$$

$$-8.875$$

So $\theta = -71\pi/4$ is 8 full rotations in the negative direction, and then an additional 0.875 of one more rotation in the negative direction. So to find a coterminal angle, we'll get rid of the 8 full rotations by adding $8(2\pi)$ to the angle.

$$-\frac{71\pi}{4} + 8(2\pi)$$

$$-\frac{71\pi}{4} + 16\pi$$

$$-\frac{71\pi}{4} + \frac{64\pi}{4}$$

$$-\frac{7\pi}{4}$$

Now we have an angle that's less than one full rotation, but we'd still like to find a positive coterminal angle that's less than one full rotation. So we'll add 2π one more time.

$$-\frac{7\pi}{4}+2\pi$$

$$-\frac{7\pi}{4}+\frac{8\pi}{4}$$

$$\frac{\pi}{4}$$

Therefore, we can say that $\pi/4$ is coterminal with $\theta = -71\pi/4$, and therefore that

$$\sin\left(-\frac{71\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$