Topic: Sum-difference identities for tangent

Question: Simplify the expression.

$$\frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

Answer choices:

A $\tan 55^\circ$

B $\tan 45^\circ$

C $\tan 119^\circ$

D $\tan 109^\circ$

Solution: B

The expression matches the form of the right side of the sum identity for tangent.

$$\tan(\theta + \alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$$

Substitute the angles from the expression.

$$\tan(82^\circ + (-37^\circ)) = \frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

$$\tan(82^\circ - 37^\circ) = \frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

$$\tan 45^\circ = \frac{\tan 82^\circ + \tan(-37^\circ)}{1 - \tan 82^\circ \tan(-37^\circ)}$$

Topic: Sum-difference identities for tangent

Question: Find the exact value of $\tan(13\pi/12)$.

Answer choices:

A $2+\sqrt{3}$

B $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

C $2-\sqrt{3}$

D $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

Solution: C

From just the unit circle, we wouldn't know the value of tangent at $13\pi/12$, but we can rewrite $13\pi/12$ as

$$\frac{13\pi}{12}=\frac{(10+3)\pi}{12}=\frac{10\pi}{12}+\frac{3\pi}{12}=\frac{5\pi}{6}+\frac{\pi}{4}$$

So the original expression can be rewritten as

$$\tan\left(\frac{13\pi}{12}\right)=\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)$$

and we can plug this right side into the sum identity for the tangent function.

$$\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{\tan\left(\frac{5\pi}{6}\right)+\tan\left(\frac{\pi}{4}\right)}{1-\tan\left(\frac{5\pi}{6}\right)\tan\left(\frac{\pi}{4}\right)}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{-\frac{\sqrt{3}}{3}+1}{1-\left(-\frac{\sqrt{3}}{3}\right)(1)}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{-\frac{\sqrt{3}}{3}+1}{1+\frac{\sqrt{3}}{3}}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{3-\sqrt{3}}{3+\sqrt{3}}$$

Multiply both the numerator and denominator by the conjugate of the denominator.

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{3-\sqrt{3}}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{9-6\sqrt{3}+3}{9-3}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\frac{12-6\sqrt{3}}{6}$$

$$\tan\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=2-\sqrt{3}$$

Topic: Sum-difference identities for tangent

Question: Find the exact values of $\tan(\theta + \alpha)$ if $\tan\theta = -1/2$ and $\tan\alpha = 3/4$.

Answer choices:

A $\frac{2}{5}$

B $-\frac{2}{5}$

C 2

D $\frac{2}{11}$

Solution: D

Use the sum identity for the tangent function, substituting the values we've been given.

$$\tan(\theta + \alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$$

$$\tan(\theta + \alpha) = \frac{-\frac{1}{2} + \frac{3}{4}}{1 - \left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)}$$

Simplify the right side.

$$\tan(\theta + \alpha) = \frac{\frac{-2+3}{4}}{1 + \frac{3}{8}}$$

$$\tan(\theta + \alpha) = \frac{\frac{1}{4}}{\frac{8+3}{8}}$$

$$\tan(\theta + \alpha) = \frac{\frac{1}{4}}{\frac{11}{8}}$$

$$\tan(\theta + \alpha) = \frac{1}{4} \cdot \frac{8}{11}$$

$$\tan(\theta + \alpha) = \frac{2}{11}$$