Topic: Trig functions of inverse trig functions

Question: Find the value of the expression.

$$\cos^{-1}\left(\cos\left(\frac{3\pi}{8}\right)\right)$$

Answer choices:

A $\frac{3\pi}{8}$

B $\frac{8}{3\pi}$

C $-\frac{3\pi}{8}$

D $\frac{5\pi}{8}$

Solution: A

The inverse property $\cos^{-1}(\cos x) = x$ applies for every x in $[0,\pi]$. This value of $x = 3\pi/8$ lies in $[0,\pi]$, which is the domain of the cosine function. Therefore we can use the inverse property $\cos^{-1}(\cos x) = x$. Therefore,

$$\cos^{-1}\left(\cos\left(\frac{3\pi}{8}\right)\right) = \frac{3\pi}{8}$$

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Question: Find the value of the expression.

$$\cos\left(\sin^{-1}\left(\frac{8}{17}\right)\right)$$

Answer choices:

A $\frac{8}{15}$

B $\frac{8}{17}$

C $\frac{15}{17}$

D $\frac{17}{15}$

Solution: C

Let θ represent the angle in the interval $[-\pi/2,\pi/2]$ whose sine is $8/17$. Then we can say

$$\theta = \sin^{-1}\left(\frac{8}{17}\right)$$

$$\sin\theta = \frac{8}{17}$$

Because $\sin\theta$ is positive, θ must be an angle in $(0,\pi/2]$, so θ is a positive angle that lies in quadrant I and x and y are both positive.

$$\theta = \sin^{-1}\left(\frac{8 = \text{opposite}}{17 = \text{hypotenuse}}\right)$$

Given a triangle with opposite leg 8 and hypotenuse 17, the adjacent leg must be

$$a^2 + b^2 = c^2$$

$$a^2 + 8^2 = 17^2$$

$$a^2 = 289 - 64$$

$$a^2 = 225$$

$$a = 15$$

Because cosine is equivalent to adjacent/hypotenuse, we get

$$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$$

$$\cos\left(\sin^{-1}\left(\frac{8}{17}\right)\right) = \cos\theta = \frac{15}{17}$$

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Question: Find the value of $\csc(\sin^{-1} x)$.

Answer choices:

A x

B $\frac{1}{x}$

C $\frac{1}{\sqrt{1-x^2}}$

D $\sqrt{1-x^2}$

Solution: B

Set $\theta = \sin^{-1} x$. Then we can say

$$\theta = \sin^{-1}\left(\frac{x}{1}\right)$$

$$\theta = \sin^{-1}\left(\frac{x = \text{opposite}}{1 = \text{hypotenuse}}\right)$$

Because cosecant is equivalent to hypotenuse/opposite, we get

$$\frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{x}$$