Trigonometry Workbook Solutions

Angles

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MATH

NAMING ANGLES

1. What do we call an angle of $\theta = 180^\circ$?

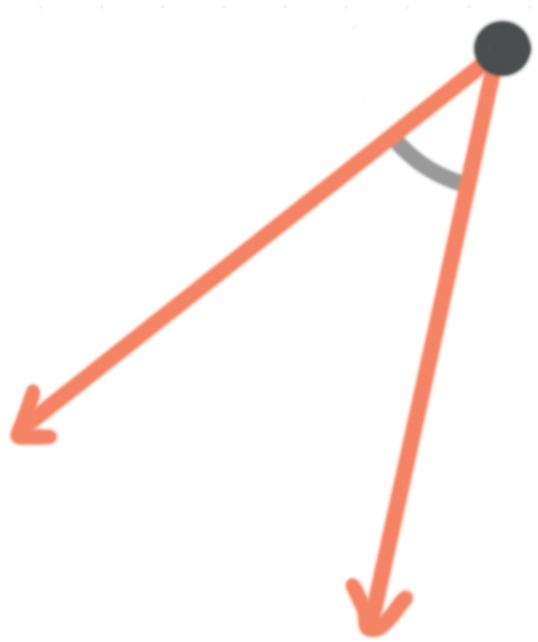
Solution:

Half-circle angles, or 180° angles, are called "straight" angles.

2. Sketch an acute angle.

Solution:

Any angle that's less than a quarter circle, or $0^\circ < \theta < 90^\circ$, is an acute angle.



3. What is the measure of a straight angle?

Solution:

A straight angle has a measure of 180°.

4. Name the angle $\theta = 6^\circ$.

Solution:

Any angle that's less than a quarter circle, or $0^\circ < \theta < 90^\circ$, is an acute angle. Therefore, $\theta = 6^\circ$ is acute angle.

5. Sketch a 360° angle.

Solution:

A 360° angle, or complete angle, is a full circle, which we sketch as

6. Give the full set of obtuse angles.

Solution:

Any angle that's greater than a quarter circle but less than a half circle, or $90^\circ < \theta < 180^\circ$, is an obtuse angle.

COMPLEMENTARY AND SUPPLEMENTARY ANGLES

■ 1. Find the supplement θ of $7\pi/8$.

Solution:

The angle θ and the angle $7\pi/8$ are supplementary, which means their sum is π.

$$\theta + \frac{7\pi}{8} = \pi$$

$$\theta = \pi - \frac{7\pi}{8}$$

$$\theta = \frac{8\pi}{8} - \frac{7\pi}{8}$$

$$\theta = \frac{\pi}{8}$$

■ 2. Angles $\angle 1$ and $\angle 2$ are complementary. Find the supplement of $\angle 2$.

$$m\angle 1 = (2x + 5)^\circ$$

$$m\angle 2 = (x + 4)^\circ$$

Solution:

Because $\angle 1$ and $\angle 2$ are complementary, that means they sum to 90°.

$$m\angle 1 + m\angle 2 = 90^\circ$$

$$2x + 5 + x + 4 = 90^\circ$$

$$3x + 9 = 90^\circ$$

$$3x = 81$$

$$x = 27$$

Plugging this back into $\angle 2$ gives

$$m\angle 2 = (27 + 4)^\circ = 31^\circ$$

The supplement of $\angle 2$ is therefore

$$180^\circ - 31^\circ$$

$$149^\circ$$

■ 3. The complement of θ is $\pi/6$. Find the supplement of θ.

Solution:

Complementary angles sum to $\pi/2$. So

$$\theta + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \frac{\pi}{6}$$

$$\theta = \frac{3\pi}{6} - \frac{\pi}{6}$$

$$\theta = \frac{\pi}{3}$$

Since supplementary angles sum to π, the supplement of θ is

$$\pi - \frac{\pi}{3}$$

$$\frac{3\pi}{3} - \frac{\pi}{3}$$

$$\frac{2\pi}{3}$$

4. Find the complement of 42°.

Solution:

Complementary angles sum to 90°, so if we call θ the complement of 42°, then

$$\theta + 42^\circ = 90^\circ$$

$$\theta = 90^\circ - 42^\circ$$

$$\theta = 48^\circ$$

■ 5. Find the angle that's supplementary to $2\pi/3$.

Solution:

Supplementary angles sum to π. So if we call the supplementary angle θ, then we can say

$$\theta + \frac{2\pi}{3} = \pi$$

$$\theta = \pi - \frac{2\pi}{3}$$

$$\theta = \frac{3\pi}{3} - \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

■ 6. True or False? If x and y are complementary angles, then $2(x + y) = 180^\circ$.

Solution:

If x and y are complementary, then they sum to 90°.

$$x + y = 90^\circ$$

Then if we multiply both sides by 2, we get

$$2(x + y) = 180^\circ$$

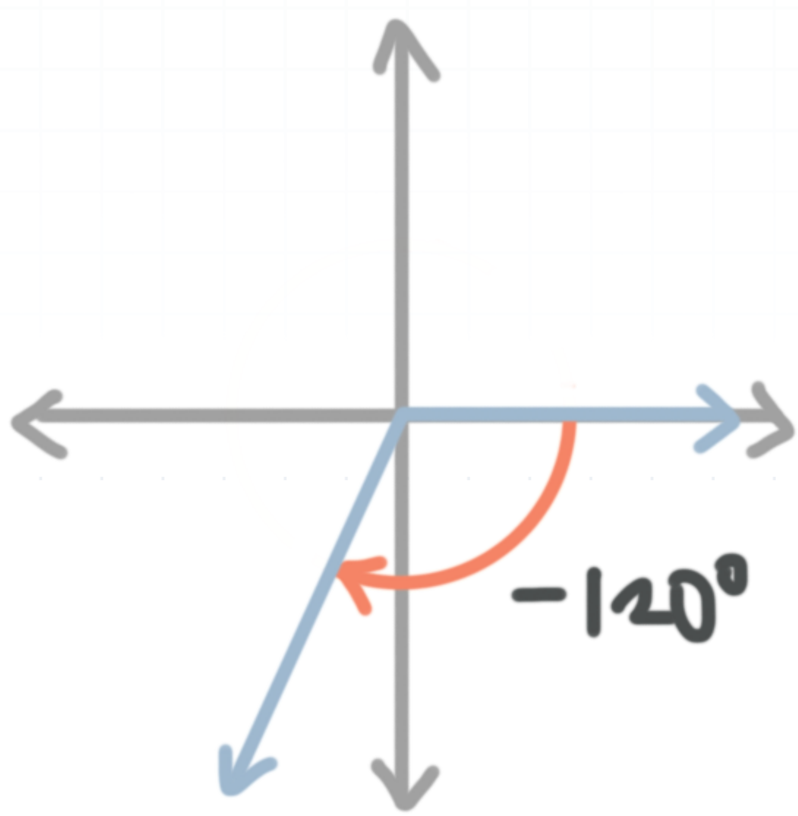
So that statement is true.

POSITIVE AND NEGATIVE ANGLES

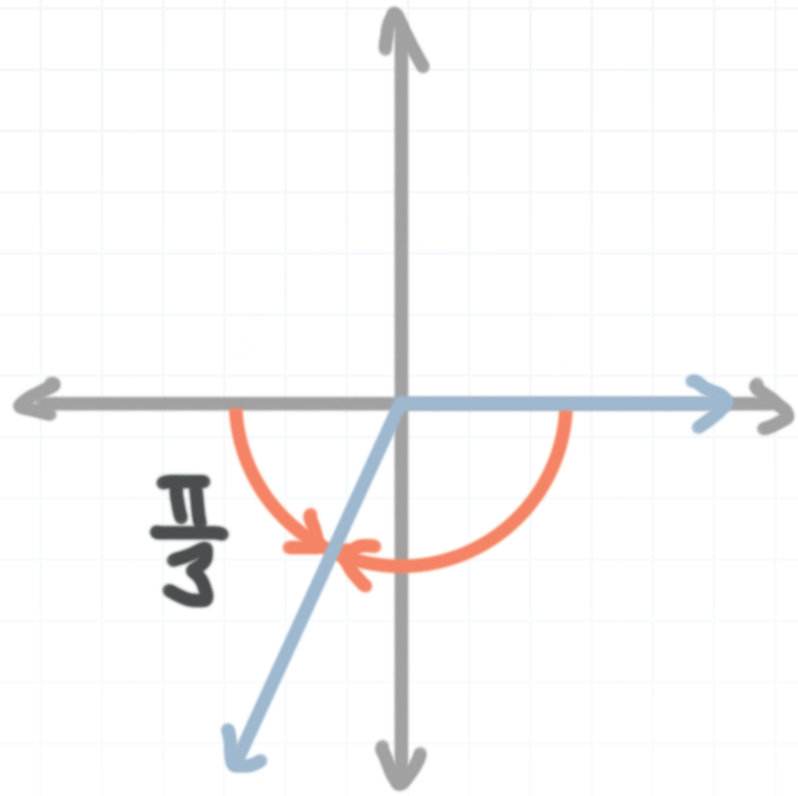
■ 1. Sketch -120° in standard position.

Solution:

The terminal side of the angle is found by rotating 120° in a negative (clockwise) direction. Since 120° is less than 180° but greater than 90°, the terminal side lies in quadrant III.



■ 2. Find the measure of the unknown negative angle in radians.



Solution:

The two angles shown are supplementary, which means they sum to π radians. So the measure of the unknown angle is

$$\pi - \frac{\pi}{3}$$

$$\frac{3\pi}{3} - \frac{\pi}{3}$$

$$\frac{2\pi}{3}$$

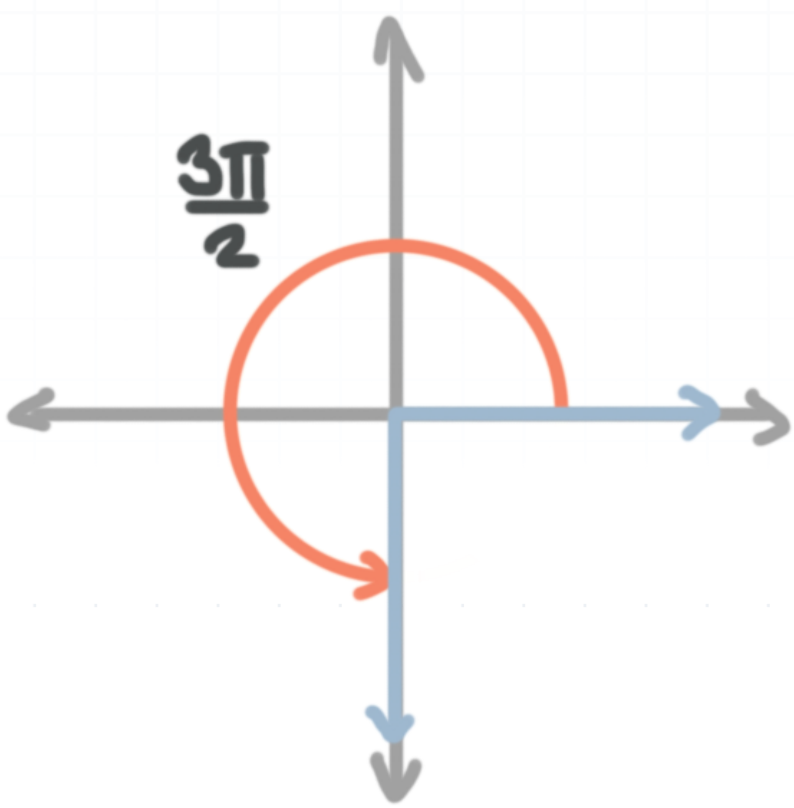
But because the angle shows clockwise rotation, that means it's the negative angle

$$-\frac{2\pi}{3}$$

■ 3. Sketch $3\pi/2$ in standard position.

Solution:

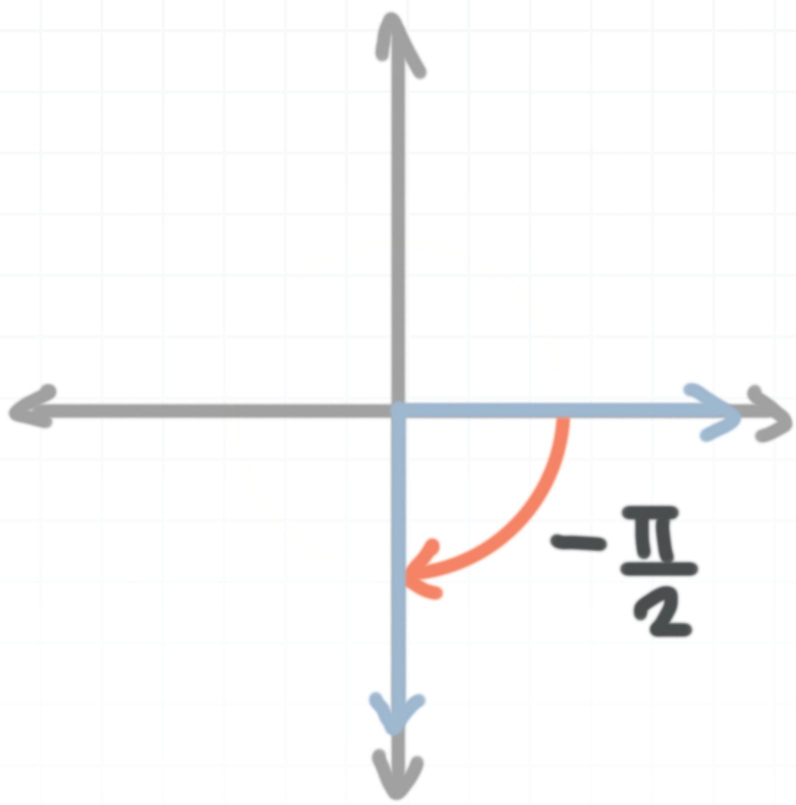
The angle $3\pi/2$ is positive, which means we rotate counterclockwise from the positive horizontal axis to the negative y-axis. Therefore, a sketch of the angle is



■ 4. Sketch $-\pi/2$ in standard position.

Solution:

The angle $-\pi/2$ is negative, which means we rotate clockwise from the positive horizontal axis to the negative y-axis. Therefore, a sketch of the angle is



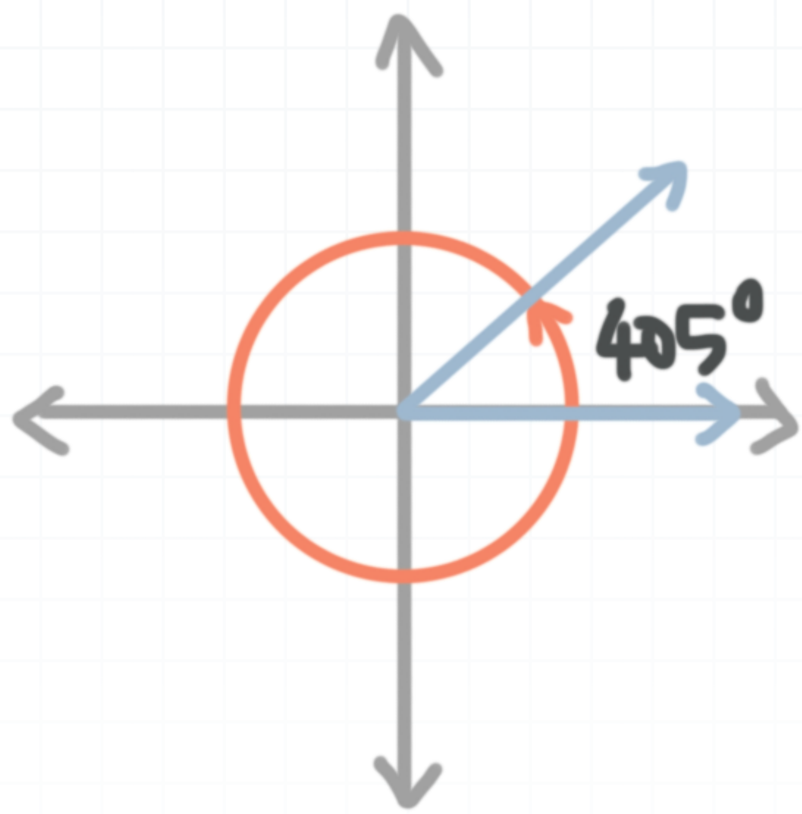
■ 5. Sketch 405° in standard position.

Solution:

Since $360^\circ < 405^\circ$, the angle 405° is more than one full rotation. We'll find out how much more by finding the difference between the angles.

$$405^\circ - 360^\circ = 45^\circ$$

So to sketch the angle, we'll put the initial side along the positive direction of the x-axis. Then we'll rotate counterclockwise, toward the first quadrant, and rotate one full rotation all the way around the circle, but then an additional 45°. Because 45° would normally land us in the first quadrant, we'll land in the first quadrant for the 405° angle as well.



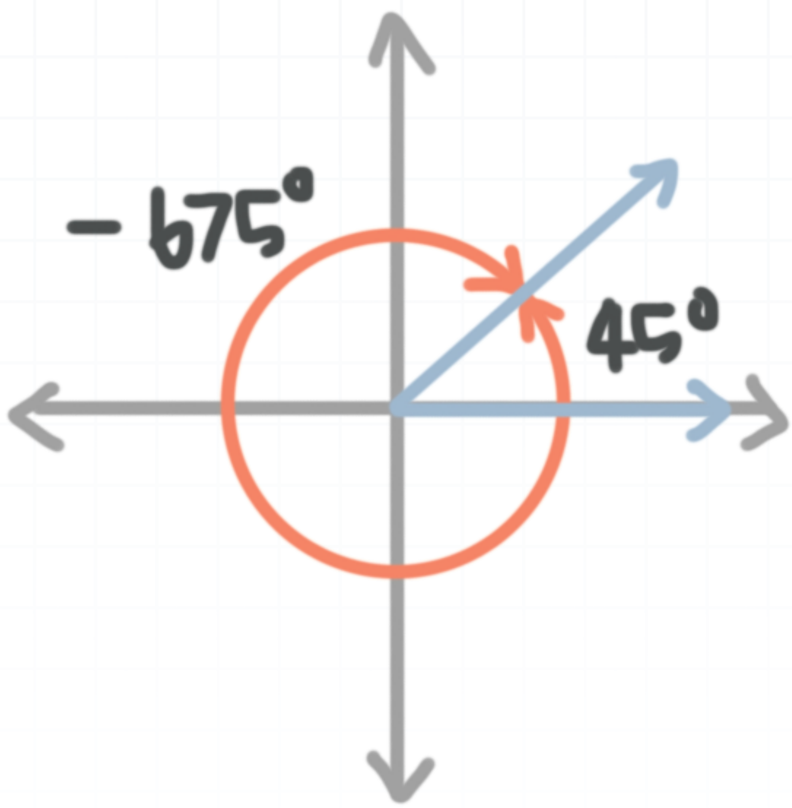
6. Find an angle between 0° and 360° that has the same terminal side as a -675° angle.

Solution:

First we need to sketch -675° in standard position. Since $360^\circ < 675^\circ$, the angle 675° is more than one full rotation. We'll find out how much more by finding the difference between the angles.

$$675^\circ - 360^\circ = 315^\circ$$

So to sketch the angle, we'll put the initial side along the positive direction of the x-axis. Then we'll rotate clockwise, toward the fourth quadrant, and rotate one full rotation all the way around the circle, but then an additional 315°. $315^\circ = 270^\circ + 45^\circ$, so we would land in the first quadrant for a -315° angle, and we'll land in the first quadrant for the -675° angle as well.



An angle measuring 45° has the same terminal side as an angle of -675°.

QUADRANT OF THE ANGLE

1. In which quadrant is the angle $\pi/5$ located?

Solution:

The angle $\pi/5$ is less than the angle $\pi/2$, which means $\pi/5$ has to lie in the first quadrant.

2. In which quadrant is the angle -820° located?

Solution:

One full rotation is -360°, so we know -820° is more than one full rotation. To figure out how many rotations are made by -820°, divide -820° by -360°.

$$\frac{-820^\circ}{-360^\circ}$$

$$2.28$$

So -820° is almost 2 and 0.3 rotations in the negative direction. If we start along the positive x-axis in standard position and make 2 full rotations in the negative direction, we'll end up right back in the same place, on the positive x-axis.

Two full rotations is

$$2(-360^\circ)$$

$$-720^\circ$$

and once we've made a -720° rotation, to get to -820°, we'll need an additional -100° of rotation. From standard position, we know -90° puts us along the negative vertical axis, and then -180° puts us along the negative horizontal axis. So a rotation of -100° will put us in the third quadrant, just past the negative vertical axis.

3. In which quadrant is the angle $-13\pi/4$ located?

Solution:

Remember that, in radians, one full rotation is 2π. So to determine how many full rotations are included in $-13\pi/4$, divide $-13\pi/4$ by 2π.

$$\frac{-\frac{13\pi}{4}}{2\pi} = -\frac{13\pi}{4} \cdot \frac{1}{2\pi} = -\frac{13\pi}{8\pi} = -1.625$$

This tells us that $-13\pi/4$ includes 1 full rotation in the negative direction, plus an additional 0.625 rotation in the negative direction. We just need to figure out how much is 0.625 of 2π.

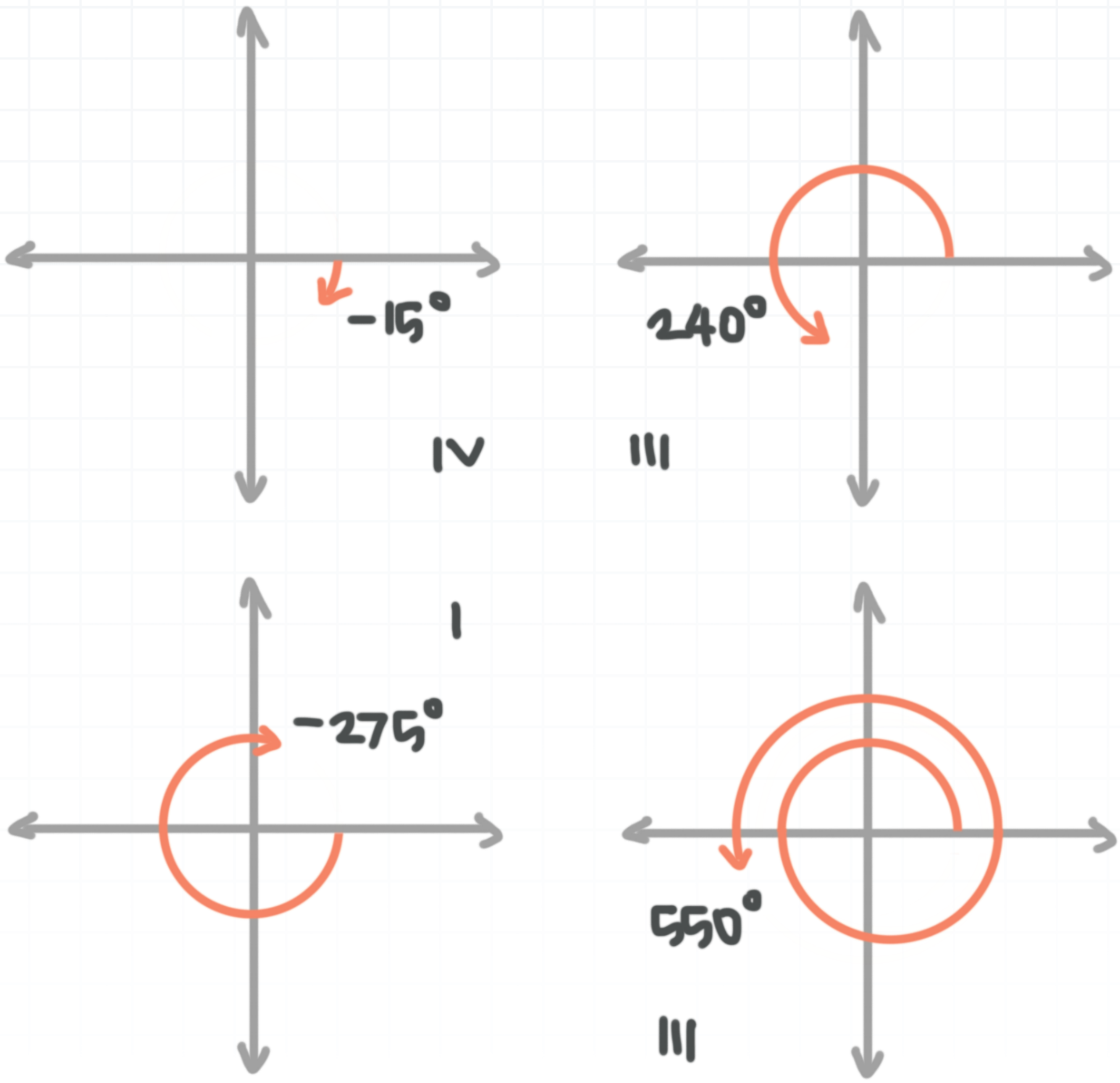
$$0.625(2\pi) = 1.25\pi$$

So, from the starting point of the positive direction of the x-axis, we complete 1 full rotation in the negative direction, which gets us back to the same starting point, and then we rotate an additional 1.25π in the negative direction, which is further than $-1.0\pi = -\pi$, but not as far as $-1.5\pi = -3\pi/2$. Which means the terminal side of the angle will land in the second quadrant.

■ 4. Of the angles -15°, 240°, -275°, and 550°, assuming all four are sketched in standard position, which one has its terminal side in the fourth quadrant?

Solution:

If we sketch each angle, we see that -15° lies in the fourth quadrant, $240^\circ = 180^\circ + 60^\circ$ lies in the third quadrant, $-275^\circ = -180^\circ - 90^\circ - 5^\circ$ lies in the first quadrant, and $550^\circ = 360^\circ + 180^\circ + 10^\circ$ lies in the third quadrant.



■ 5. In which quadrant is the angle $1{,}200^\circ$ located?

Solution:

Subtract multiples of 360° from $1{,}200^\circ$ until we find an angle in the interval $[0^\circ, 360^\circ)$.

$$1{,}200^\circ - 360^\circ = 840^\circ$$

$$840^\circ - 360^\circ = 480^\circ$$

$$480^\circ - 360^\circ = 120^\circ$$

This tells us that $1,200^\circ$ includes 3 full rotations in the positive direction, plus an additional 120° rotation in the positive direction.

The angle 120° falls between angles of 90° and 180°, which means 120° lies in the second quadrant. Therefore, $1,200^\circ$ also lies in the second quadrant.

■ 6. On which axis does the angle -7π lie?

Solution:

An angle of -7π is more than one full negative rotation, so we'll add multiples of 2π to -7π until we find an angle in the interval $[0,2\pi)$.

$$-7\pi + 2\pi = -5\pi$$

$$-5\pi + 2\pi = -3\pi$$

$$-3\pi + 2\pi = -\pi$$

This tells us that -7π includes 3 full rotations in the negative direction, plus an additional π rotation in the negative direction.

The angle $-\pi$ falls on the negative horizontal axis, which means -7π also lies on the negative horizontal axis.

DEGREES, RADIANS, AND DMS

1. Convert $65^\circ 13' 12''$ to a decimal angle.

Solution:

To convert from DMS to degrees, we only need to convert the minutes and seconds parts, since the degree part is already given in degrees.

First we convert seconds to minutes, then minutes to degrees. We'll convert the seconds part first. We need to convert $12''$ from seconds to minutes. We know that $1' = 60''$, so we'll multiply $12''$ by $1'/60''$ in order to cancel the seconds and be left with just minutes.

$$12''\left(\frac{1'}{60''}\right)$$

$$\left(\frac{12}{60}\right)'$$

$$0.2'$$

Then the total minutes in $65^\circ 13' 12''$ is

$$(13 + 0.2)'$$

$$13.2'$$

To convert this value for minutes into degrees, we'll multiply by $1^\circ/60'$ in order to cancel the minutes and be left with an approximate value for degrees.

$$13.2'\left(\frac{1^\circ}{60'}\right)$$

$$\left(\frac{13.2}{60}\right)^\circ$$

$$0.22^\circ$$

Putting this together with the 65° from the original angle, we get approximately

$$(65 + 0.22)^\circ$$

$$65.22^\circ$$

■ 2. Find the sum $20.25^\circ + 20^\circ 2'5''$ in DMS. Note: The sum of two DMS angles is found by adding the two degree parts, adding the two minutes parts, adding the two seconds parts, and then combining the separate sums into one angle.

Solution:

First convert 20.25° to DMS.

$$20^\circ + 0.25^\circ$$

$$20^\circ + \left(\frac{25}{100}\right)^\circ \left(\frac{60'}{1^\circ}\right)$$

$$20^\circ + 15'$$

$$20^\circ 15'$$

Then find the sum.

$$20.25^\circ + 20^\circ 2' 5''$$

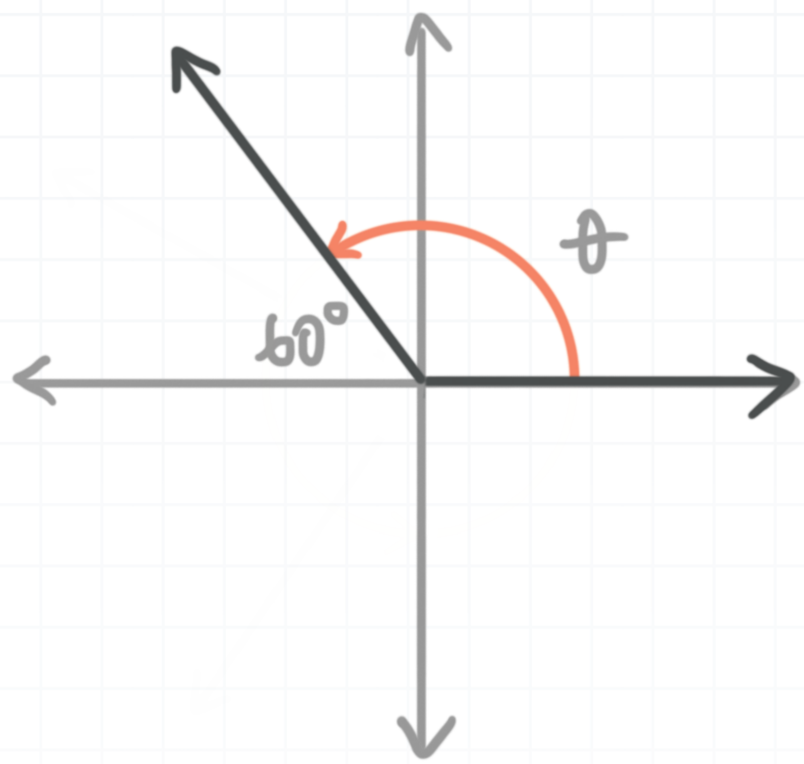
$$20^\circ 15' + 20^\circ 2' 5''$$

$$20^\circ + 20^\circ + 15' + 2' + 5''$$

$$40^\circ + 17' + 5''$$

$$40^\circ 17' 5''$$

■ 3. Find the measure of θ in radians.



Solution:

Because θ is supplementary with 60°, its measure in degrees is

$$180^\circ - 60^\circ$$

$$120^\circ$$

Convert this to radians.

$$120^\circ \left(\frac{\pi}{180^\circ}\right)$$

$$\frac{2\pi}{3}$$

■ 4. Convert the angle $-9\pi/2$ to degrees.

Solution:

Convert the radian angle measure to degrees.

$$-\frac{9\pi}{2}\left(\frac{180^\circ}{\pi}\right)$$

$$-\frac{9(180^\circ)}{2}$$

$$-9(90^\circ)$$

$$-810^\circ$$

■ 5. What is the measure of 152.34° in DMS?

Solution:

Break apart the degree angle.

$$152.34^\circ$$

$$152^\circ + 0.34^\circ$$

$$152^\circ + \left(\frac{34}{100}\right)^\circ$$

Since 152° is already in degrees, we only need to convert the $(34/100)^\circ$ into DMS.

$$\left(\frac{34}{100}\right)^\circ \left(\frac{60'}{1^\circ}\right)$$

$$\left(\frac{(34)(60)}{100}\right)'$$

$$\left(\frac{204}{10}\right)'$$

$$20.4'$$

The minutes part will be $20'$, and we'll convert $0.4'$ into seconds.

$$0.4'\left(\frac{60''}{1'}\right)$$

$$(0.4(60))''$$

$$24''$$

Therefore, the angle 152.34° in DMS is $152^\circ 20' 24''$.

■ 6. What is the measure of 0.2565π in DMS?

Solution:

To convert from radians to DMS, we'll first convert from radians to degrees, and then to DMS. We'll start by multiplying by $180^\circ/\pi$ to get an approximation of the radian angle in degrees.

$$0.2565\pi\left(\frac{180^\circ}{\pi}\right)$$

$$46.17^\circ$$

Break the degree angle apart.

$$46.17^\circ$$

$$46^\circ + 0.17^\circ$$

Since 46° is already in degrees, we only need to convert the 0.17° into DMS.

$$0.17^\circ\left(\frac{60'}{1^\circ}\right)$$

$$10.2'$$

The minutes part will be $10'$, and we'll convert $0.2'$ into seconds.

$$0.2'\left(\frac{60''}{1'}\right)$$

$$12''$$

Therefore, the angle 0.2565π in DMS is $46^\circ 10' 12''$.

COTERMINAL ANGLES

1. If we start at -270° and move three full rotations clockwise around the origin, at which angle will we arrive?

Solution:

A clockwise rotation is a negative rotation, so if we call the new angle α, then the measure of the new angle will be

$$\alpha = -270^\circ - 3(360^\circ)$$

$$\alpha = -270^\circ - 1{,}080^\circ$$

$$\alpha = -1{,}350^\circ$$

2. If we start at $5\pi/6$ and move two full rotations counterclockwise around the origin, at which angle will we arrive?

Solution:

A counterclockwise rotation is a positive rotation, so if we call the new angle α, then the measure of the new angle will be

$$\alpha = \frac{5\pi}{6} + 2(2\pi)$$

$$\alpha = \frac{5\pi}{6} + 4\pi$$

Find a common denominator.

$$\alpha = \frac{5\pi}{6} + 4\pi\left(\frac{6}{6}\right)$$

$$\alpha = \frac{5\pi}{6} + \frac{24\pi}{6}$$

$$\alpha = \frac{29\pi}{6}$$

3. Find the negative and positive coterminal angles that are one full rotation away from $\theta = 200^\circ$.

Solution:

To find angles that are coterminal with $\theta = 200^\circ$, we need to add and subtract 360°. So the coterminal angles that are one full rotation from $\theta = 200^\circ$ are

$$200^\circ - 1(360^\circ) = -160^\circ$$

$$200 + 1(360^\circ) = 560^\circ$$

4. Which angle in the interval $[0^\circ, 360^\circ)$ is coterminal with $-1{,}624^\circ$?

Solution:

We'll add 360° to the angle $-1{,}624^\circ$ until we get an angle in the interval $[0^\circ,360^\circ)$.

$$-1{,}624^\circ + 360^\circ = -1{,}264^\circ$$

$$-1{,}264^\circ + 360^\circ = -904^\circ$$

$$-904^\circ + 360^\circ = -544^\circ$$

$$-544^\circ + 360^\circ = -184^\circ$$

$$-184^\circ + 360^\circ = 176^\circ$$

The angle 176° is within the interval $[0^\circ,360^\circ)$.

■ 5. Find the angle in the interval $[0,2\pi)$ that's coterminal with $16\pi/3$.

Solution:

We'll subtract 2π from the angle $16\pi/3$ until we get an angle in the interval $[0,2\pi)$.

$$\frac{16\pi}{3} - 2\pi = \frac{16\pi}{3} - \frac{6\pi}{3} = \frac{10\pi}{3}$$

$$\frac{10\pi}{3} - 2\pi = \frac{10\pi}{3} - \frac{6\pi}{3} = \frac{4\pi}{3}$$

The angle $4\pi/3$ is within the interval $[0,2\pi)$.

■ 6. Which angle in $[-\pi,\pi)$ is coterminal with $-19\pi/2$?

Solution:

We'll add 2π to the angle $-19\pi/2$ until we get an angle in the interval $[-\pi,\pi)$.

$$-\frac{19\pi}{2}+2\pi=-\frac{19\pi}{2}+\frac{4\pi}{2}=-\frac{15\pi}{2}$$

$$-\frac{15\pi}{2}+2\pi=-\frac{15\pi}{2}+\frac{4\pi}{2}=-\frac{11\pi}{2}$$

$$-\frac{11\pi}{2}+2\pi=-\frac{11\pi}{2}+\frac{4\pi}{2}=-\frac{7\pi}{2}$$

$$-\frac{7\pi}{2}+2\pi=-\frac{7\pi}{2}+\frac{4\pi}{2}=-\frac{3\pi}{2}$$

$$-\frac{3\pi}{2}+2\pi=-\frac{3\pi}{2}+\frac{4\pi}{2}=\frac{\pi}{2}$$

The angle $\pi/2$ is within the interval $[-\pi,\pi)$.

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