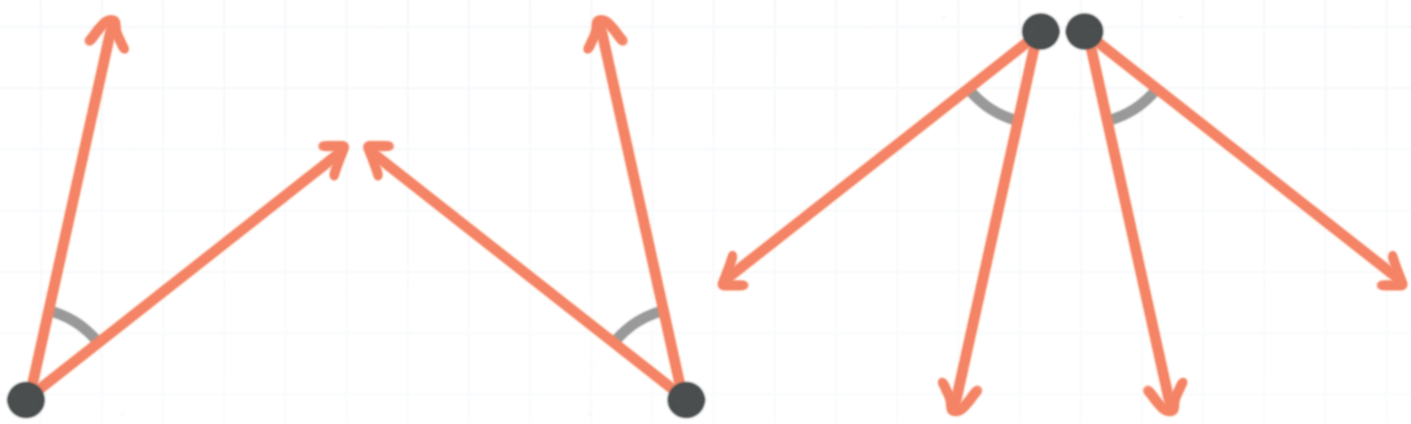
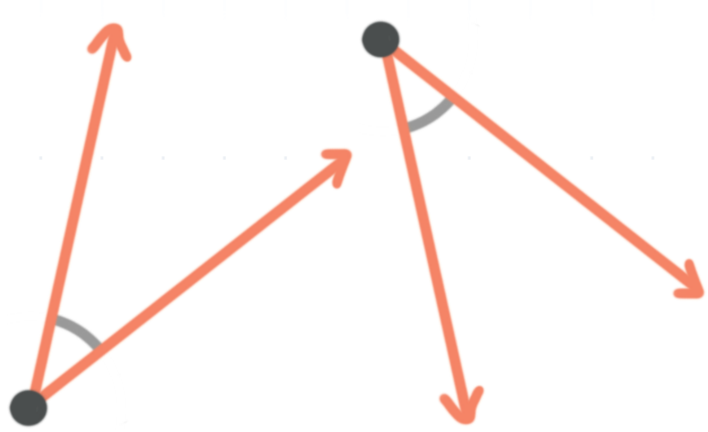
Positive and negative angles

Angles don't necessarily have to have the same orientation in order to have the same measure. For instance, even though they're positioned different ways, each of these angles have the same measure:



If we sketch angles without any kind of standard orientation, it can be difficult for us to visually compare the angles. After all, how are we supposed to know that these two angles



are really the same? One of them might be a few degrees narrower or wider than the other, but it's really hard for us to tell, since the angles are oriented differently.

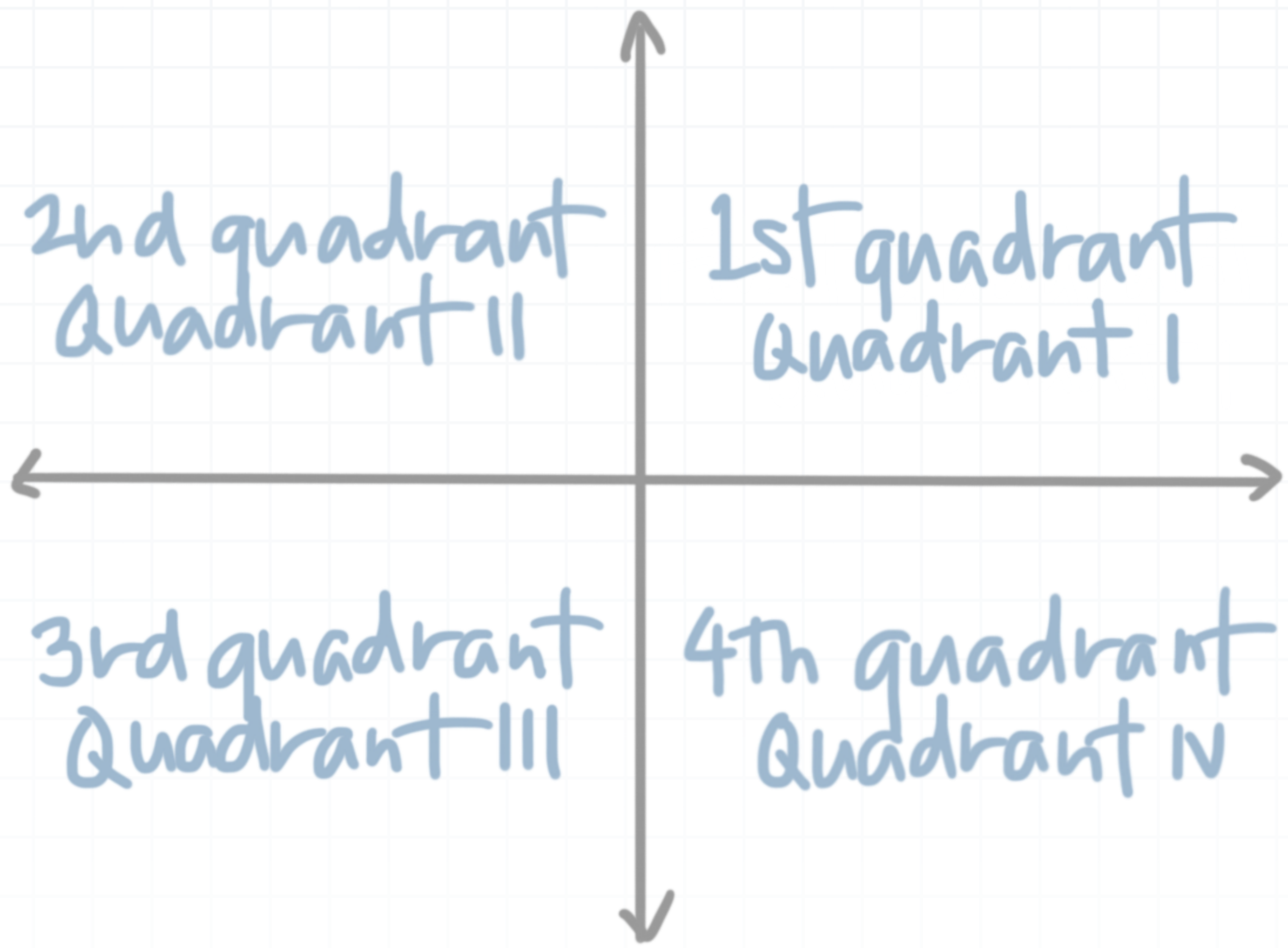
To solve this problem, we normally prefer to sketch angles in **standard position**, which means that we align the angle's initial side with the positive direction of the x-axis, placing its vertex at the origin.

Positive and negative angles

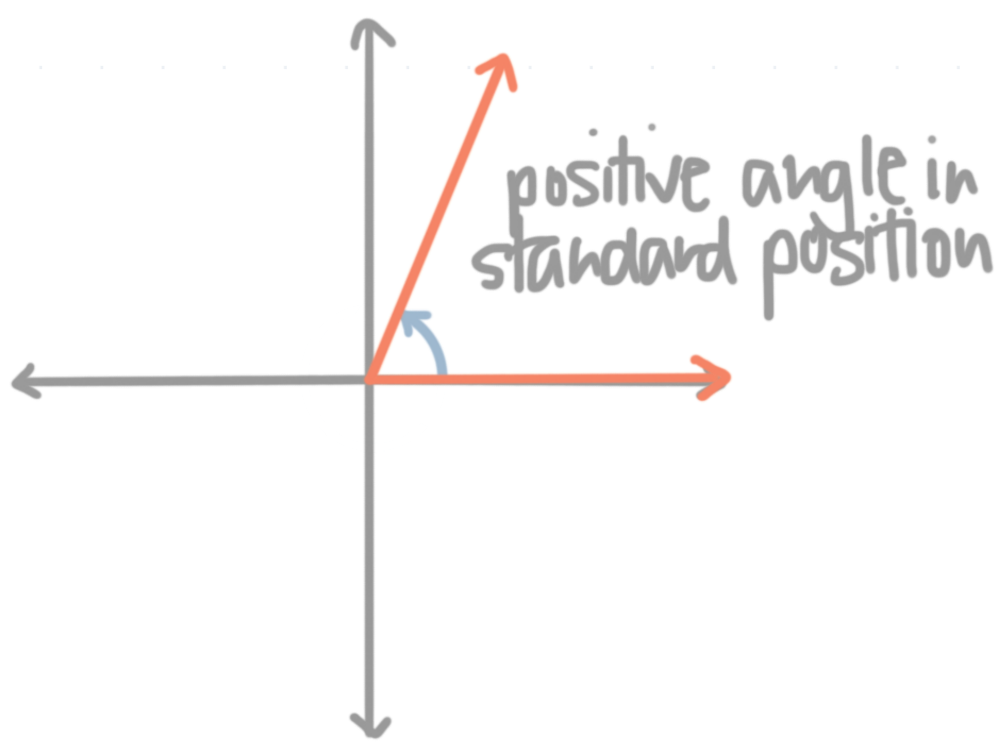
Previously, we hadn't distinguished between the two sides of the angle. We only looked at the degree or radian measure between the two rays. But if we define one side as the **initial side**, the side where the angle begins (the ray on the positive direction of the x-axis), and the other as the **terminal side**, the side where the angle ends, then we can distinguish between positive and negative angles.

The angle is measured by the amount of rotation from the initial side to the terminal side. Starting from the initial side, we have a **positive angle** when we have to rotate counterclockwise to get to the terminal side. We have a **negative angle** when we have to rotate clockwise to get to the terminal side.

If we remember the four quadrants of the Cartesian coordinate system,

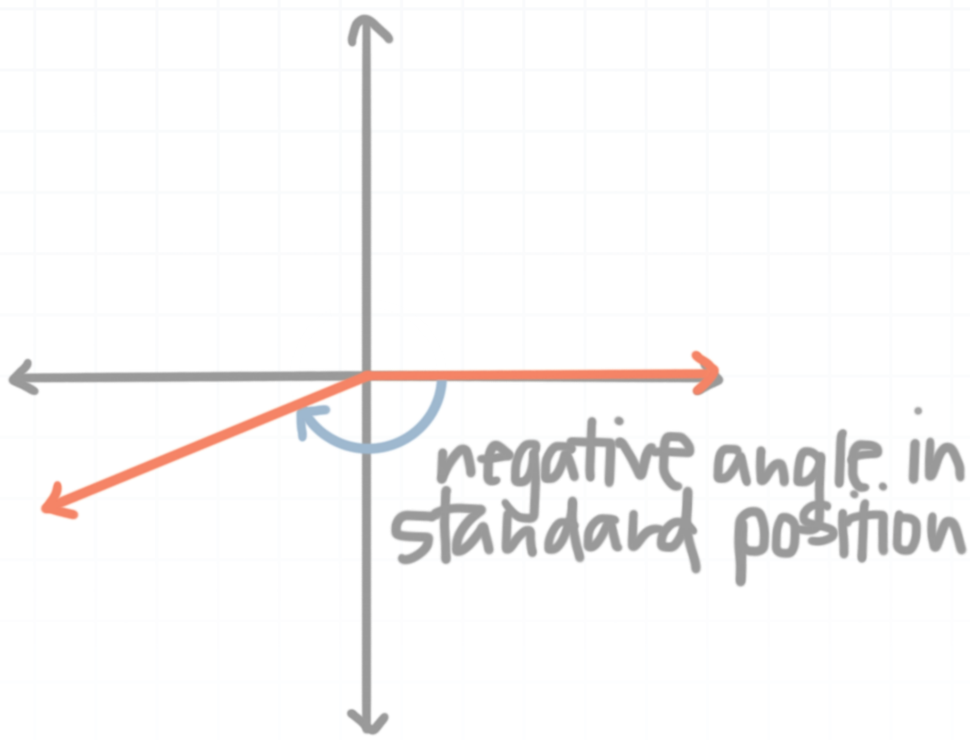


then we can say that a positive angle in standard position has its initial side on the positive direction of the x-axis, and opens up toward the first quadrant,



because this kind of angle is a counterclockwise rotation from the positive direction of the x-axis.

On the other hand, a negative angle in standard position has its initial side on the positive direction of the x-axis, and opens up toward the fourth quadrant,



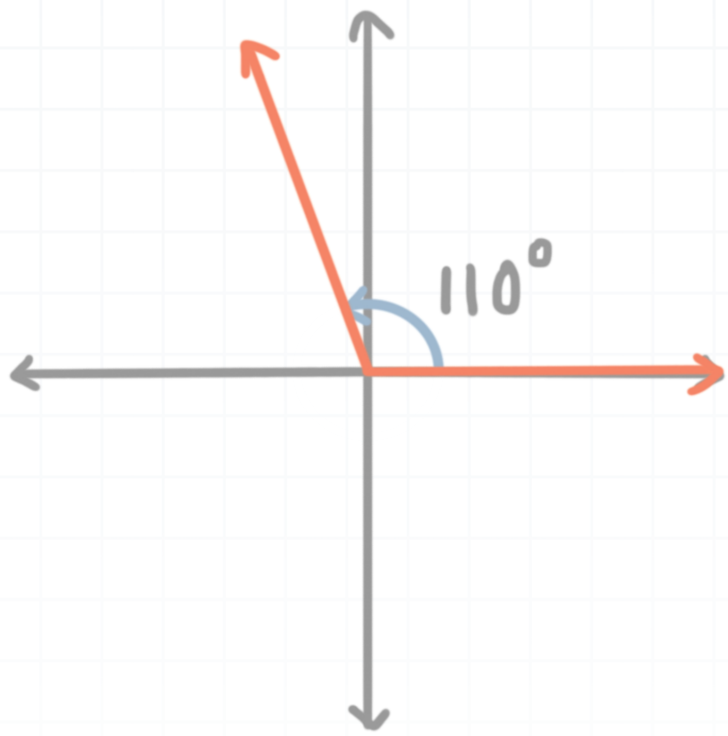
because this kind of angle is a clockwise rotation from the positive direction of the x-axis.

Let's look at how to sketch a positive angle in standard position.

Example

Sketch a 110° angle in standard position.

Since the angle is larger than 90° but smaller than 180°, it's obtuse. We put the initial side along the positive direction of the x-axis, and since the angle is positive, open up counterclockwise, into the first quadrant, then past the positive direction of the y-axis (which is at 90°) until we get to 110°.

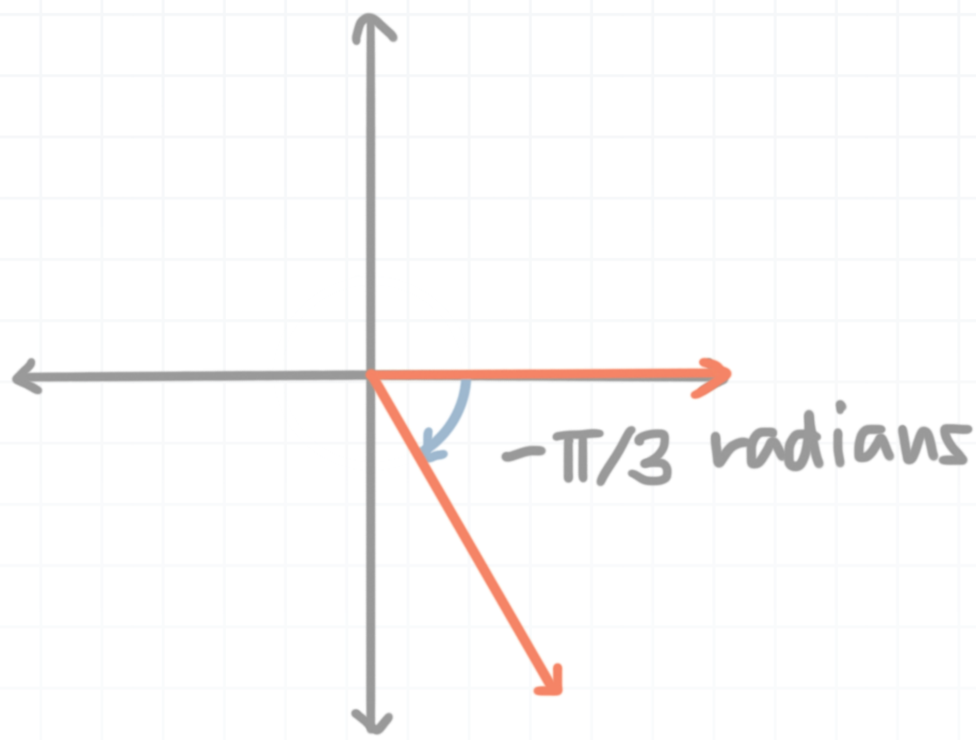


Let's do an example with a negative angle in radians.

Example

Sketch $-\pi/3$ in standard position.

Since $\pi/3$ is less than $\pi/2$, it's an acute angle. We'll put the initial side along the positive direction of the x-axis, and since the angle is negative, open up clockwise, into the fourth quadrant, until we get to $-\pi/3$.



Up to now we've been talking about angles that are less than one full rotation around a circle. A full circle is 360° or 2π radians, and we very often handle angles that are smaller than one full rotation around a circle.

But we can also have angles that are greater than one full rotation around a circle. For example, the angle 600° is more than a full rotation, because $600^\circ > 360^\circ$. Because $600^\circ - 360^\circ = 240^\circ$, the angle 600° just means that we're rotating a full 360°, but then continuing on another 240°.

Let's do an example so that we can see what it looks like to sketch one of these larger angles.

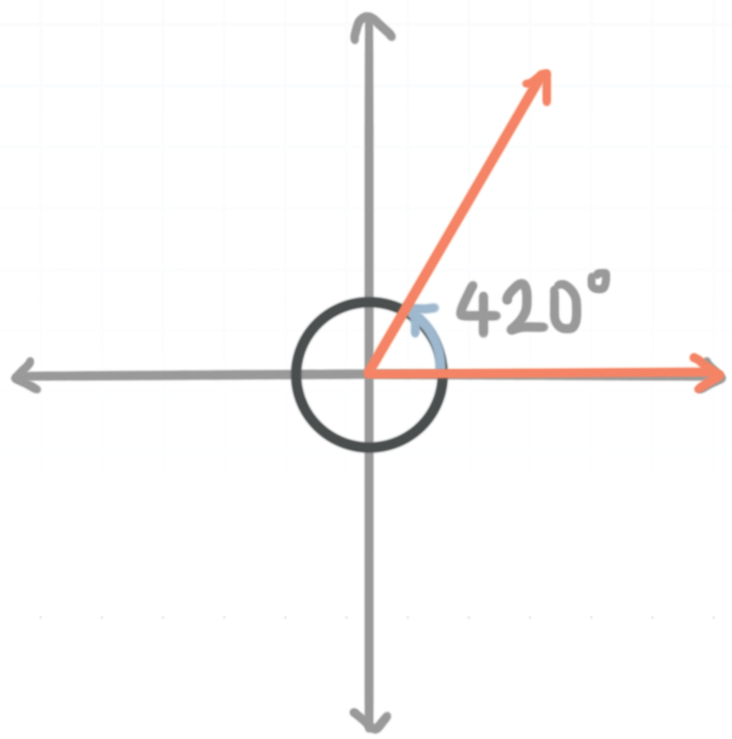
Example

Sketch 420° in standard position.

Since $360^\circ < 420^\circ$, the angle 420° is more than one full rotation. We'll find out how much more by finding the difference between the angles.

$$420^\circ - 360^\circ = 60^\circ$$

So to sketch the angle, we'll put the initial side along the positive direction of the x-axis. Then we'll rotate counterclockwise, toward the first quadrant, and rotate one full rotation, all the way around the circle, but then an additional 60°. Because 60° would normally land us in the first quadrant, we'll land in the first quadrant for the 420° angle as well.



The angle looks like a normal 60°, but in the figure, the dark gray arc shows the first 360°, and the blue arc shows the extra 60° of the rotation.