

Topic: Sketching tangent and cotangent**Question:** What are the vertical asymptotes for of the function?

$$y = 3 \cot \left(\frac{x}{2} \right)$$

Answer choices:

- A $\dots, -\pi, \pi, 3\pi, \dots$
- B $\dots, 0, 2\pi, 4\pi, \dots$
- C $\dots, 0, \pi, 2\pi, \dots$
- D There are no vertical asymptotes



Solution: B

To find adjacent vertical asymptotes for $y = a \cot(bx)$, solve $bx = 0$ and $bx = \pi$. The function $y = 3 \cot(x/2)$ is in the form $y = a \cot(bx)$ where $a = 3$ and $b = 1/2$, so we find

$$\frac{x}{2} = 0$$

$$x = 0$$

and

$$\frac{x}{2} = \pi$$

$$x = 2\pi$$

Since b is positive and the period of $\cot \theta$ is π , the period is

$$\frac{\pi}{|b|} = \frac{\pi}{\left|\frac{1}{2}\right|} = \pi \left(\frac{2}{1}\right) = 2\pi$$

So based on everything we've found so far, we know the other asymptotes will be 2π more or less from the previous one, so we'll have asymptotes at

$$0 - 2\pi = -2\pi$$

$$2\pi + 2\pi = 4\pi$$

$$4\pi + 2\pi = 6\pi$$



Therefore, the asymptotes for the function are

$$\dots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \dots$$

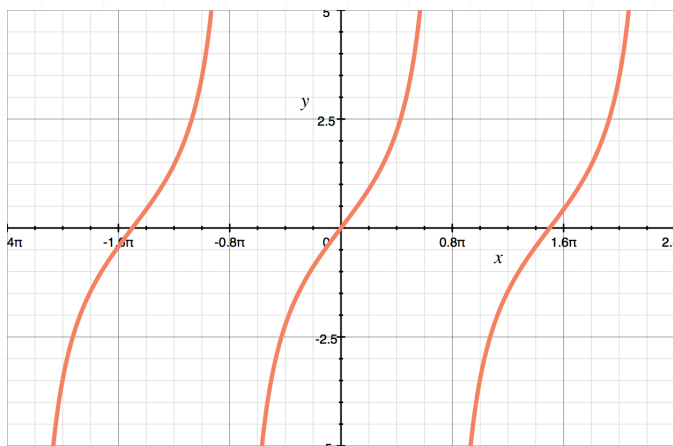


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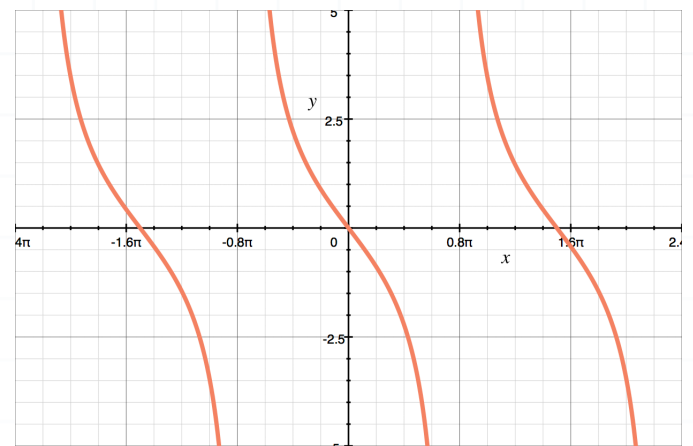
Question: Identify the graph of $y = 2 \tan(2\theta/3)$.

Answer choices:

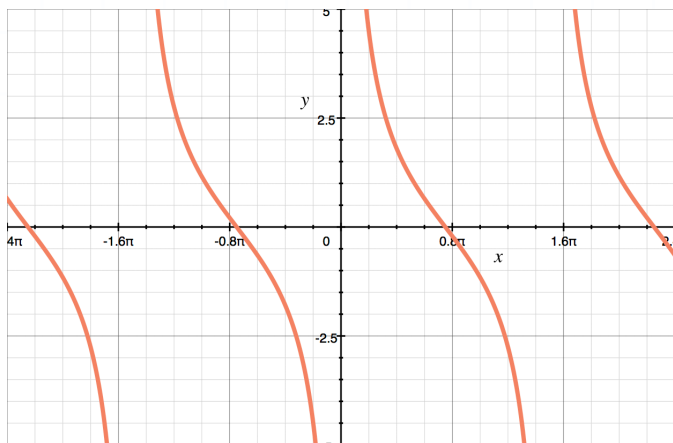
A



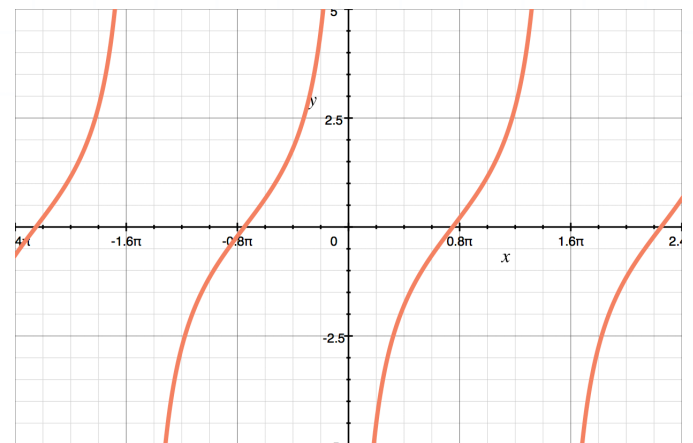
B



C



D



Solution: A

We'll find two adjacent vertical asymptotes by solving $bx = -\pi/2$ and $bx = \pi/2$ for x . With $b = 2/3$, we get

$$\frac{2x}{3} = -\frac{\pi}{2}$$

$$x = -\frac{3\pi}{4}$$

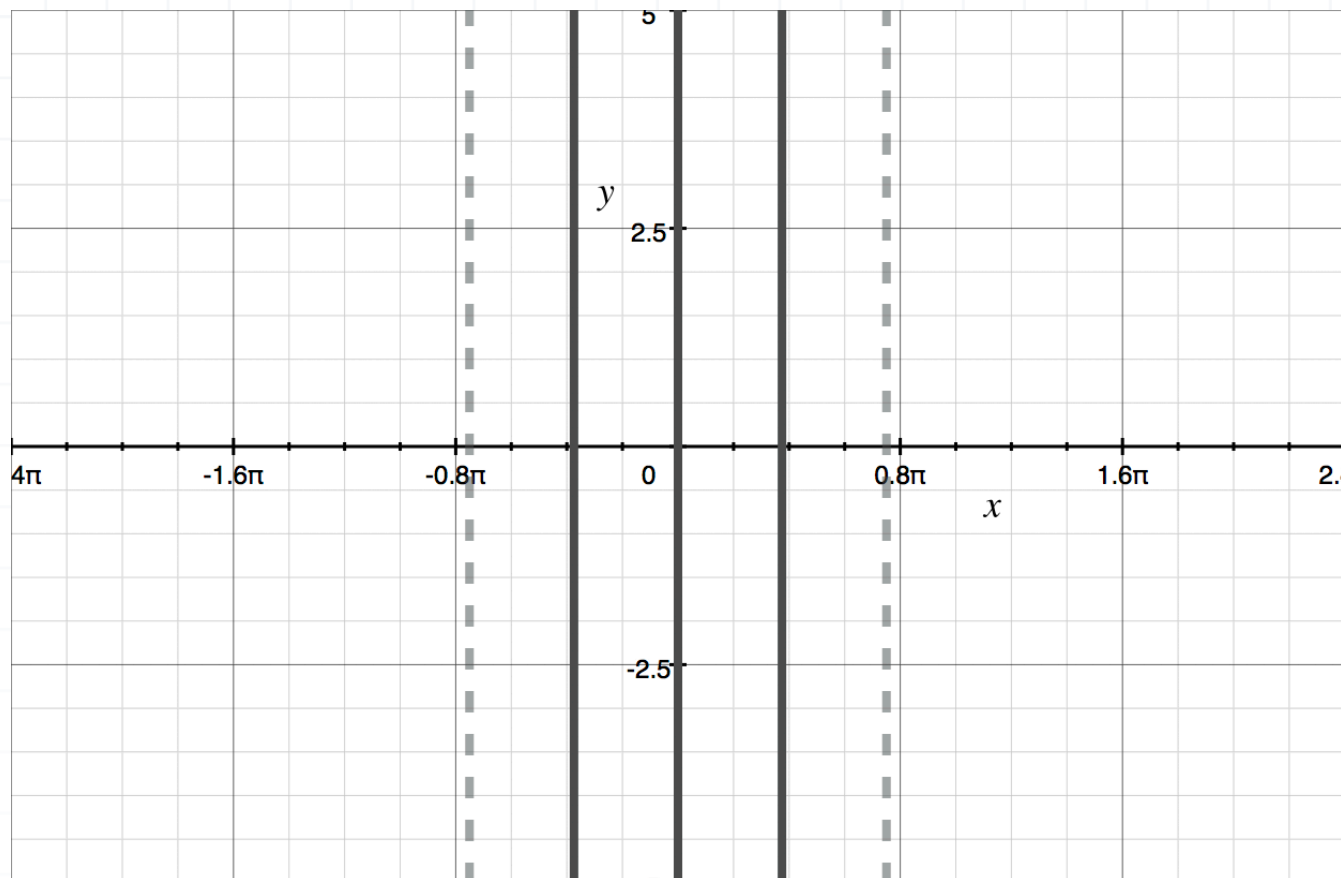
and

$$\frac{2x}{3} = \frac{\pi}{2}$$

$$x = \frac{3\pi}{4}$$

We'll sketch in the vertical asymptotes $x = -3\pi/4$ and $x = 3\pi/4$, and then divide the interval between $x = -3\pi/4$ and $x = 3\pi/4$ into four equal parts.



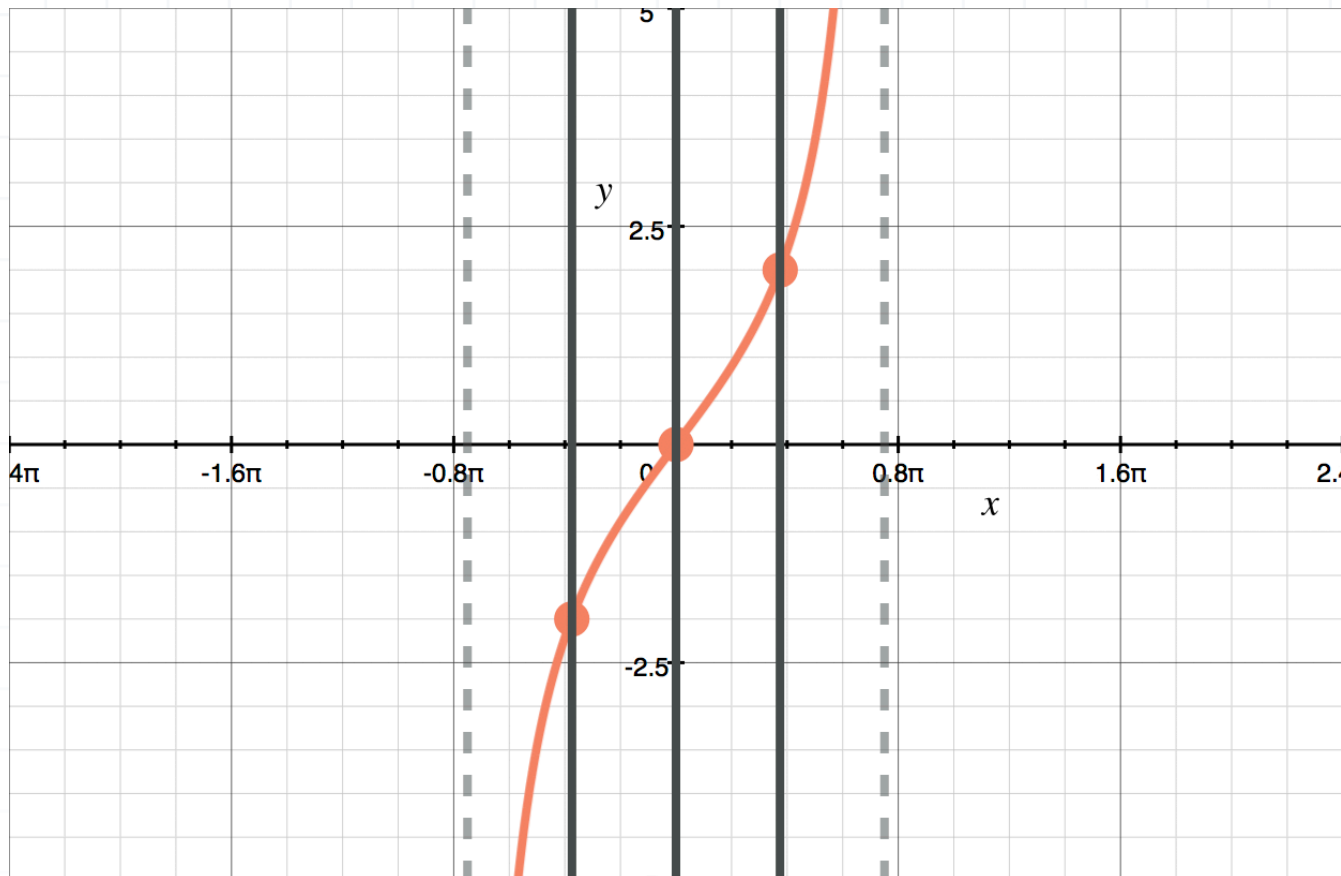


The dividing lines of each of these four sub-intervals are $x = -3\pi/8$, $x = 0$, and $x = 3\pi/8$, so we'll evaluate $y = 2 \tan(2\theta/3)$ at those three values, and we'll get

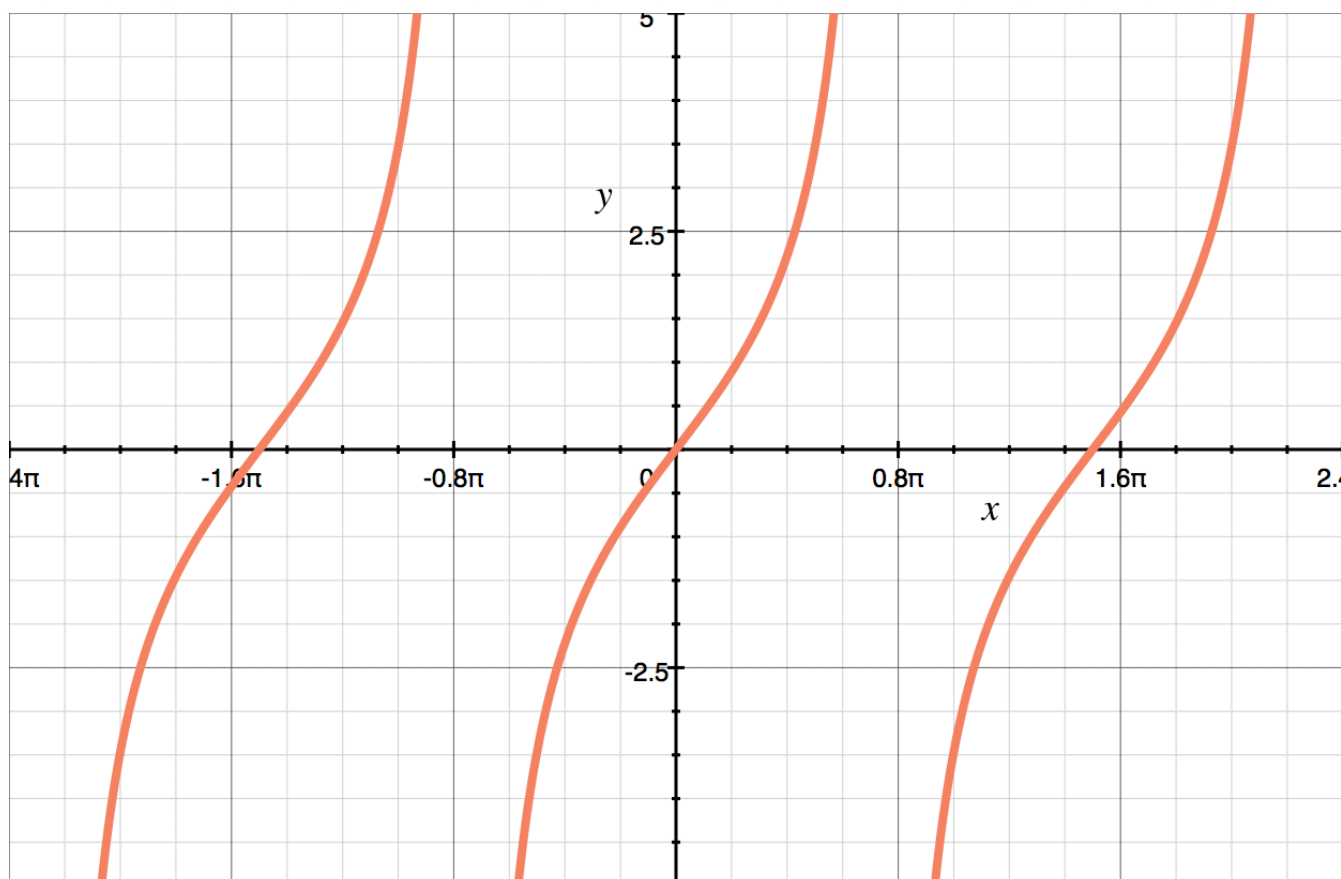
$$\left(-\frac{3\pi}{8}, -2\right), (0,0) \text{ and } \left(\frac{3\pi}{8}, 2\right)$$

We'll plot those points, and then connect them with a smooth curve, respecting the asymptotes.





Finally, we'll repeat that pattern both to the left and right, and take away the asymptotes and other guiding lines that we sketched, and we'll get the final graph of $y = 2 \tan(2\theta/3)$.



Topic: Sketching tangent and cotangent**Question:** Where are the first pair of asymptotes of the function?

$$y = 5 \tan \left(\frac{2\pi x}{15} \right)$$

Answer choices:

A $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

B $-\frac{15\pi}{4}$ and $\frac{15\pi}{4}$

C 0 and $\frac{15}{2}$

D $-\frac{15}{4}$ and $\frac{15}{4}$



Solution: D

To find adjacent vertical asymptotes for $y = a \tan(bx)$, solve the equations $bx = -\pi/2$ and $bx = \pi/2$. With $b = 2\pi/15$, we get

$$\frac{2\pi}{15}x = -\frac{\pi}{2}$$

$$x = -\frac{\pi}{2} \cdot \frac{15}{2\pi}$$

$$x = -\frac{15}{4}$$

and

$$\frac{2\pi}{15}x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} \cdot \frac{15}{2\pi}$$

$$x = \frac{15}{4}$$

Therefore, the first pair of asymptotes of the function are $-15/4$ and $15/4$.

