Topic: Signs by quadrant

Question: If θ is an angle in the second quadrant and $\csc\theta = \sqrt{7}$, what is the value of $\sec\theta$?

Answer choices:

A $\sec\theta = -\frac{\sqrt{42}}{7}$

B $\sec\theta = -\frac{6}{7}$

C $\sec\theta = -\frac{7}{6}$

D $\sec\theta = -\frac{\sqrt{42}}{6}$

Solution: D

Use the reciprocal identity to find the value of $\sin\theta$.

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\sin\theta = \frac{1}{\sqrt{7}}$$

Then we can use the value of sine in the Pythagorean identity to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - \left(\frac{1}{\sqrt{7}}\right)^2$$

$$\cos^2\theta = 1 - \frac{1}{7}$$

$$\cos^2\theta = \frac{6}{7}$$

$$\cos\theta = \pm\sqrt{\frac{6}{7}}$$

The cosine of any angle in the second quadrant is negative, so

$$\cos\theta = -\sqrt{\frac{6}{7}}$$

Then we'll use the reciprocal identity to find $\sec\theta$.

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec\theta = \frac{1}{-\sqrt{\frac{6}{7}}}$$

$$\sec\theta = -\sqrt{\frac{7}{6}}$$

$$\sec\theta = -\frac{\sqrt{7}}{\sqrt{6}}$$

Rationalize the denominator by multiplying both the numerator and denominator by $\sqrt{6}$.

$$\sec\theta = -\frac{\sqrt{7}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$\sec\theta = -\frac{\sqrt{42}}{6}$$

Topic: Signs by quadrant

Question: If $\cos(11°) \approx 0.981627$, what's the approximate value of $\cot(11°)$?

Answer choices:

A $\cot\theta \approx 4.64°$

B $\cot\theta \approx 4.98°$

C $\cot\theta \approx 5.14°$

D $\cot\theta \approx 5.26°$

Solution: C

Start with a rewritten form of the Pythagorean identity with sine and cosine.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sqrt{\sin^2\theta} = \pm\sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$$

Plug $\cos(11°) = 0.98$ into the equation we've created.

$$\sin\theta \approx \pm\sqrt{1 - 0.981627^2}$$

$$\sin\theta \approx \pm\sqrt{1 - 0.963592}$$

$$\sin\theta \approx \pm\sqrt{0.036408}$$

$$\sin\theta \approx \pm 0.190809$$

An angle of $11°$ is in the first quadrant. Since the sine of any angle in the first quadrant is positive, we get

$$\sin\theta \approx 0.190809$$

Plug this value for $\sin\theta$ into the quotient identity for $\cot\theta$.

$$\cot\theta \approx \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta \approx \frac{0.981627}{0.190809} \approx 5.14$$

Topic: Signs by quadrant

Question: What is the value of $\tan\theta$ if $\cos\theta = -0.218$ and θ is an angle in the third quadrant?

Answer choices:

A $\tan\theta \approx -5.46$

B $\tan\theta \approx \pm 4.48$

C $\tan\theta \approx 4.48$

D $\tan\theta \approx \pm 5.46$

Solution: C

We need to find any possible values of $\sin\theta$ using the Pythagorean identity with sine and cosine.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - (-0.218)^2$$

$$\sin^2\theta \approx 1 - 0.0475$$

$$\sin^2\theta = 0.953$$

$$\sin\theta = \pm\sqrt{0.953}$$

$$\sin\theta = \pm 0.976$$

Since θ is an angle in the third quadrant, $\sin\theta$ is negative and $\sin\theta \approx -0.976$.

If we plug this value of sine into the quotient identity for tangent, along with $\cos\theta = -0.218$, we get:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \approx \frac{-0.976}{-0.218} \approx 4.48$$