Topic: Law of sines

Question: If the measures of two interior angles of a triangle are 70° and 43°, and the length of the side opposite the 70° angle is 12, find the length b of the side opposite the 43° angle and the length c of the third side.

Answer choices:

A $b \approx 11.8$ and $c \approx 16.5$

B $b \approx 16.5$ and $c \approx 12.2$

C $b \approx 12.2$ and $c \approx 8.71$

D $b \approx 8.71$ and $c \approx 11.8$

Solution: D

We know the third interior angle has measure

$$180^\circ - 70^\circ - 43^\circ$$

$$67^\circ$$

Then the law of sines gives

$$\frac{12}{\sin 70^\circ} = \frac{b}{\sin 43^\circ} = \frac{c}{\sin 67^\circ}$$

Find b using the first two parts of this three-part equation.

$$\frac{12}{\sin 70^\circ} = \frac{b}{\sin 43^\circ}$$

$$b = \frac{12 \sin 43^\circ}{\sin 70^\circ} \approx \frac{12(0.682)}{0.940} \approx 8.71$$

Find c using the first and third parts of the three-part equation.

$$\frac{12}{\sin 70^\circ} = \frac{c}{\sin 67^\circ}$$

$$c = \frac{12 \sin 67^\circ}{\sin 70^\circ} \approx \frac{12(0.921)}{0.940} \approx 11.8$$

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Question: If the lengths of two sides of a triangle are 20 and 30, and the measure of the interior angle opposite the side of length 30 is $B = 95^\circ$, find the measures of angles A and C, where A is opposite the side of length 20.

Answer choices:

A $A \approx 48.9^\circ$, $C \approx 38.6^\circ$, and $c \approx 16.3$

B $A \approx 138.4^\circ$, $C \approx 22.9^\circ$, and $c \approx 45.2$

C $A \approx 37.6^\circ$, $C \approx 47.4^\circ$, and $c \approx 31.8$

D $A \approx 41.6^\circ$, $C \approx 43.4^\circ$, and $c \approx 20.7$

Solution: D

Plugging what we know into the law of sines gives

$$\frac{20}{\sin A} = \frac{30}{\sin 95^\circ} = \frac{c}{\sin C}$$

Find A using the first two parts of this three-part equation.

$$\frac{20}{\sin A} = \frac{30}{\sin 95^\circ}$$

$$\sin A = \frac{20 \sin 95^\circ}{30} \approx \frac{2(0.996)}{3} \approx 0.664$$

If A is acute, then $A = 41.6^\circ$, and if angle A is obtuse, then $A = 138.4^\circ$. But it's impossible to have $A = 138.4^\circ$ because the sum of interior angles A and B would be $138.4^\circ + 95^\circ = 233.4^\circ$, which exceeds 180°, so $A = 41.6^\circ$.

Then the third interior angle has measure

$$180^\circ - 95^\circ - 41.6^\circ$$

$$43.4^\circ$$

Find c using the second and third parts of the three-part equation.

$$\frac{30}{\sin 95^\circ} \approx \frac{c}{\sin 43.4^\circ}$$

$$c \approx \frac{30 \sin 43.4^\circ}{\sin 95^\circ} \approx \frac{30(0.687)}{0.996} \approx 20.7$$

Topic: Law of sines

Question: Solve the triangle with interior angles 68° and 79°, where the length of the side opposite the third angle is 18.

Answer choices:

A $C \approx 33^\circ$, $a \approx 30.6$, and $b \approx 32.4$

B $C \approx 112^\circ$, $a \approx 18$, and $b \approx 19.1$

C $C \approx 32^\circ$, $a \approx 31.5$, and $b \approx 9.7$

D $C \approx 101^\circ$, $a \approx 17$, and $b \approx 18$

Solution: A

We'll let angle $A = 68^\circ$ and angle $B = 79^\circ$, then we'll find the measure of the third angle.

$$A + B + C = 180^\circ$$

$$68^\circ + 79^\circ + C = 180^\circ$$

$$C = 180^\circ - 68^\circ - 79^\circ$$

$$C = 33^\circ$$

The known side is opposite this third angle C, so we'll say $c = 18$. Plugging this side length and all three angle measures into the law of sines gives

$$\frac{a}{\sin 68^\circ} = \frac{b}{\sin 79^\circ} = \frac{18}{\sin 33^\circ}$$

We'll use just the first and third parts of the three-part equation in order to solve for a.

$$\frac{a}{\sin 68^\circ} = \frac{18}{\sin 33^\circ}$$

$$a = \frac{18 \sin 68^\circ}{\sin 33^\circ}$$

$$a \approx 30.6$$

To solve for b, we'll use just the second and third parts of the three-part equation.

$$\frac{b}{\sin 79^\circ} = \frac{18}{\sin 33^\circ}$$

$$b = \frac{18 \sin 79^\circ}{\sin 33^\circ}$$

$$b \approx 32.4$$