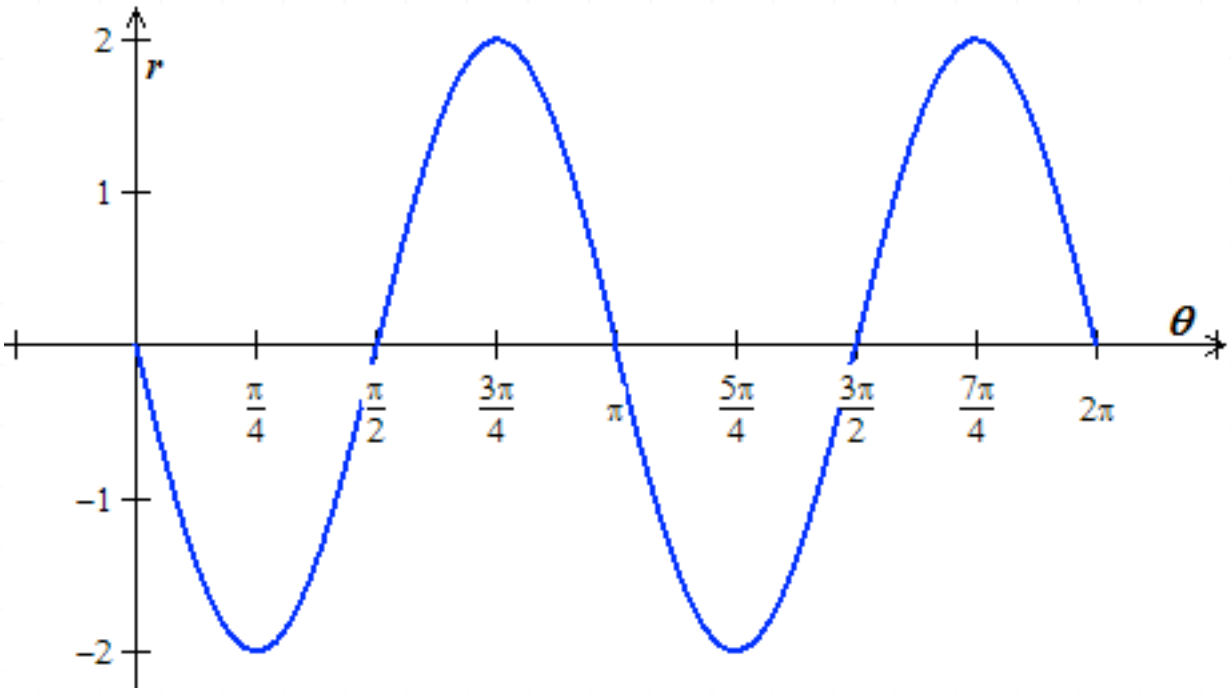
Topic: Graph the polar region in the xy-plane

Question: Which polar equation is it the graph of?



Answer choices:

A $r = 1 + 2\sin\theta$

B $r = \cos(2\theta)$

C $r = 2 + \cos\theta$

D $r = -2\sin(2\theta)$

Solution: D

Let's see if we can eliminate any of the answer choices.

First, notice that on the given rectangular graph the value of r at $\theta = 0$ is 0.

That tells us that answer choice A can't be correct, because if the given graph were the rectangular graph of the polar equation $r = 1 + 2\sin\theta$, then at $\theta = 0$ we would get

$$r = 1 + 2(\sin 0) = 1 + 2(0) = 1 + 0 = 1 \neq 0$$

The same thing applies to answer choice B, because there the value of r at $\theta = 0$ is

$$r = \cos(2(0)) = \cos(0) = 1 \neq 0$$

For the polar equation in answer choice C, we get the following at $\theta = 0$:

$$r = 2 + \cos(0) = 2 + 1 = 3 \neq 0$$

Thus the only answer choice that could be correct is D. Notice that if $r = -2\sin(2\theta)$, then at $\theta = 0$ we have

$$r = -2\sin(2(0)) = -2\sin(0) = -2(0) = 0$$

Another way to arrive at the correct answer would be to notice that the size and shape of the part of the given rectangular graph which corresponds to the interval $[\pi,2\pi]$ are identical to the size and shape of the part which corresponds to the interval $[0,\pi]$, and that the graph on each of those two intervals looks like a complete sine curve. Therefore, it appears that the given rectangular graph is that of a sine function that has period π.

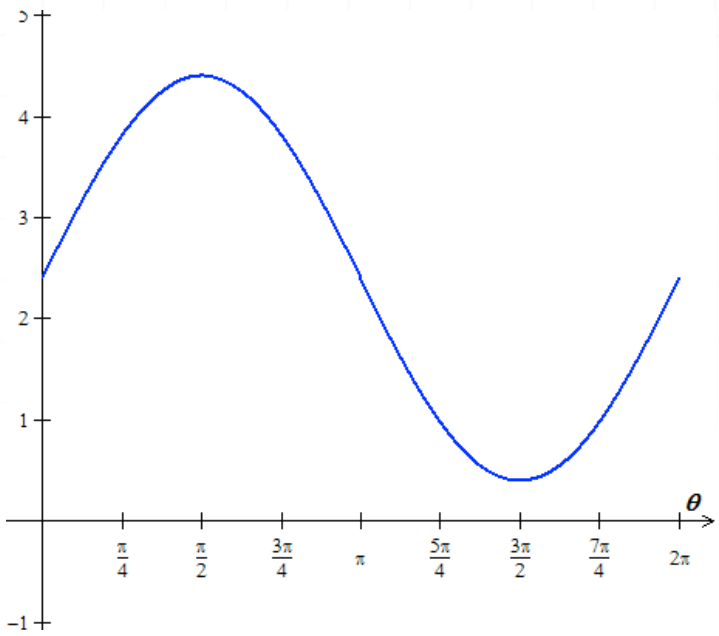
This is the case with the polar equation $r = -2\sin(2\theta)$, because the period of the function $-2\sin(2\theta)$ is $2\pi/b$ (where b is the coefficient of θ in the expression 2θ , which is 2), hence the period is $2\pi/2 = \pi$.

Topic: Graph the polar region in the xy-plane

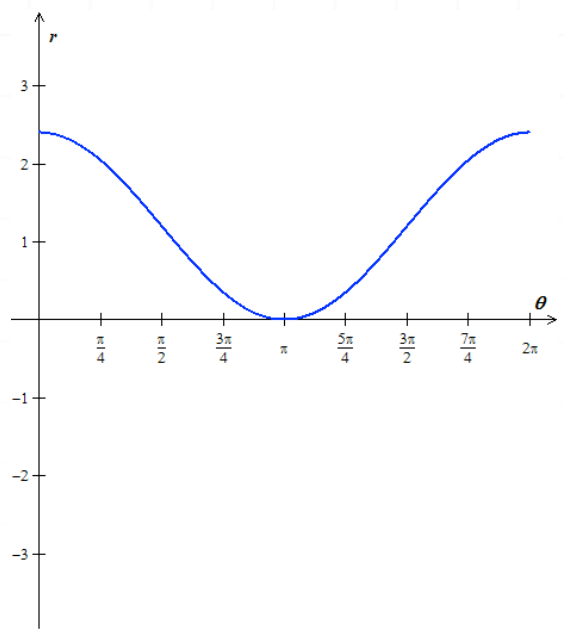
Question: Which of the following is the rectangular graph of the polar equation $r = 1.2(1 + \cos\theta)$ on the interval $[0,2\pi]$?

Answer choices:

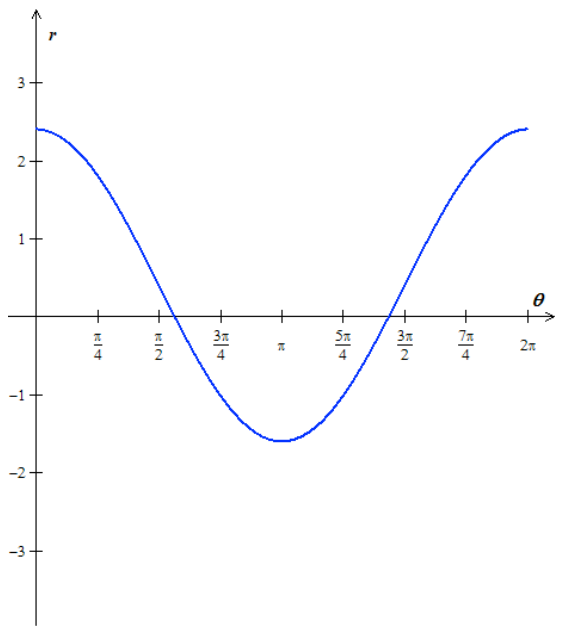
A



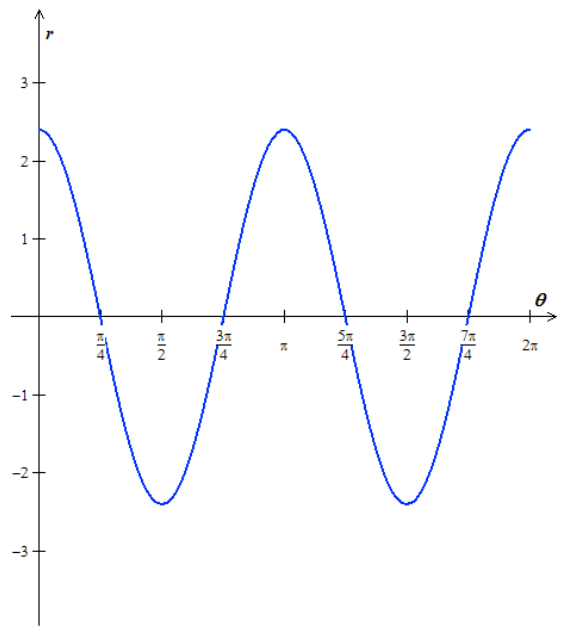
B



C



D



Solution: B

From the given polar equation, $r = 1.2(1 + \cos\theta)$, we find that the value of r at $\theta = 0$ is

$$r = 1.2(1 + \cos(0)) = 1.2(1 + 1) = 1.2(2) = 2.4$$

All four of these graphs look as though they would yield $r \approx 2.4$ at $\theta = 0$, so we'll have to consider some other value(s) of θ to eliminate the wrong answer choices.

In the rectangular graph given in answer choice A, the value of r at $\theta = \pi/2$ is greater than 4. From the given polar equation, $r = 1.2(1 + \cos\theta)$, what we find is that the value of r at $\theta = \pi/2$ is

$$r = 1.2\left[1 + \cos\left(\frac{\pi}{2}\right)\right] = 1.2(1 + 0) = 1.2(1) = 1.2$$

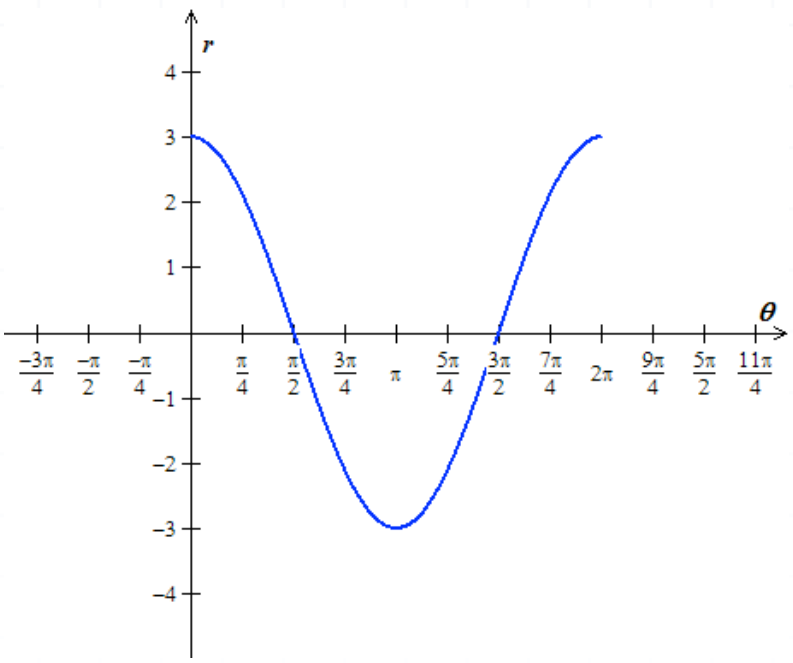
which is well below 4. Thus we can eliminate answer choice A.

In the rectangular graph given in answer choice C, the value of r at $\theta = \pi/2$ is positive but well below 1.2, so we can eliminate answer choice C.

In the rectangular graph given in answer choice D, the value of r at $\theta = \pi/2$ is negative, hence it isn't equal to 1.2, so we can eliminate answer choice D.

Topic: Graph the polar region in the xy-plane

Question: The following graph is the rectangular graph of a certain polar equation on the interval $[0,2\pi]$. Which type of curve would you get if you were to convert this rectangular graph to a polar graph?



Answer choices:

A Cardioid

B Circle

C Limacon

D Rose

Solution: B

Inspection of the given rectangular graph tells us that it's just the graph of the basic cosine function (over one complete period of length 2π) magnified by a factor of 3. Thus it's the rectangular graph of the polar equation $r = 3\cos\theta$ on the interval $[0,2\pi]$. The curve we get when we draw the polar graph of this equation is the circle whose center is on the horizontal axis and located $3/2 = 1.5$ units to the right of the pole.