Topic: Symmetry across axes

Question: Use the unit circle to find the angle that has the same sine as the angle $\theta = 123^\circ$.

Answer choices:

A -123°

B -57°

C 57°

D 33°

Solution: C

For any angle θ, the value of $\sin \theta$ is equal to the y-coordinate of the point at which the terminal side of θ intersects the unit circle.

Since $\theta = 123^\circ$ lies in the second quadrant, the sine of the angle will be positive. The only other quadrant in which the sine is positive is the first quadrant. Because $\theta = 123^\circ$ is $123^\circ - 90^\circ = 33^\circ$ past the positive y-axis, we need an angle that's 33° short of the positive y-axis, which is the angle

$$90^\circ - 33^\circ = 57^\circ$$

Topic: Symmetry across axes

Question: If θ is an angle such that $\sin\theta = 0.439$, what are two possible values of $\cos(\theta + 540^\circ)$?

Answer choices:

A $\cos(\theta + 540^\circ) = \pm 0.193$

B $\cos(\theta + 540^\circ) = \pm 0.807$

C $\cos(\theta + 540^\circ) = \pm 0.327$

D $\cos(\theta + 540^\circ) = \pm 0.898$

Solution: D

Use the Pythagorean identity with sine and cosine, and the given value $\sin\theta = 0.439$, to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$(0.439)^2 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - (0.439)^2$$

$$\cos^2\theta \approx 1 - 0.193$$

$$\cos^2\theta \approx 0.807$$

$$\cos\theta \approx \pm\sqrt{0.807}$$

$$\cos\theta \approx \pm 0.898$$

Since $\sin\theta$ is positive, θ is in either the first or second quadrant. In the first quadrant we'll have $\cos\theta \approx 0.898$, and in the second quadrant we'll have $\cos\theta \approx -0.898$.

We've been asked about the angle $\theta + 540^\circ$. If we realize that $540^\circ = 360^\circ + 180^\circ$, we realize that a 360° rotation puts us right back at the same angle θ, but that the additional 180° flips us across the y-axis and then across the x-axis, which means the signs on both x and y will flip.

Which means that the angle θ in the second quadrant associated with $(-0.898, 0.439)$ will become the angle $\theta + 540^\circ$ associated with $(0.898, -0.439)$. And the angle θ in the first quadrant associated with $(0.898, 0.439)$ will become the angle $\theta + 540^\circ$ associated with $(-0.898, -0.439)$.

Therefore, for an angle $\theta + 540^\circ$, the possible values of cosine of the angle are the x-values from $(0.898, -0.439)$ and $(-0.898, -0.439)$.

$$\cos(\theta + 540^\circ) = \pm 0.898$$

Topic: Symmetry across axes

Question: If θ is an angle in the second quadrant such that $\cos\theta = -0.713$, what is the value of $\sin(\theta + 5\pi)$?

Answer choices:

A $\sin(\theta + 5\pi) = 0.508$

B $\sin(\theta + 5\pi) = -0.713$

C $\sin(\theta + 5\pi) = -0.701$

D $\sin(\theta + 5\pi) = 0.693$

Solution: C

Use the Pythagorean identity with sine and cosine, and the given value $\cos\theta = -0.713$, to find $\sin\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + (-0.713)^2 = 1$$

$$\sin^2\theta = 1 - (-0.713)^2$$

$$\sin^2\theta \approx 1 - 0.508$$

$$\sin^2\theta \approx 0.492$$

$$\sin\theta \approx \pm\sqrt{0.492}$$

Since θ is in the second quadrant, sine of the angle must be positive.

$$\sin\theta \approx \sqrt{0.492}$$

$$\sin\theta \approx 0.701$$

We've been asked about the angle $\theta + 5\pi$. If we realize that $5\pi = 4\pi + \pi$, we realize that a 4π rotation puts us right back at the same angle θ, but that the additional π flips us across the y-axis and then across the x-axis, which means the signs on both x and y will flip.

Which means that the angle θ in the second quadrant associated with $(-0.714, 0.701)$ will become the angle $\theta + 5\pi$ associated with $(0.714, -0.701)$.

Therefore, for an angle $\theta + 5\pi$, the value of sine of the angle is the y-value from $(0.714, -0.701)$.

$$\sin(\theta + 5\pi) = -0.701$$