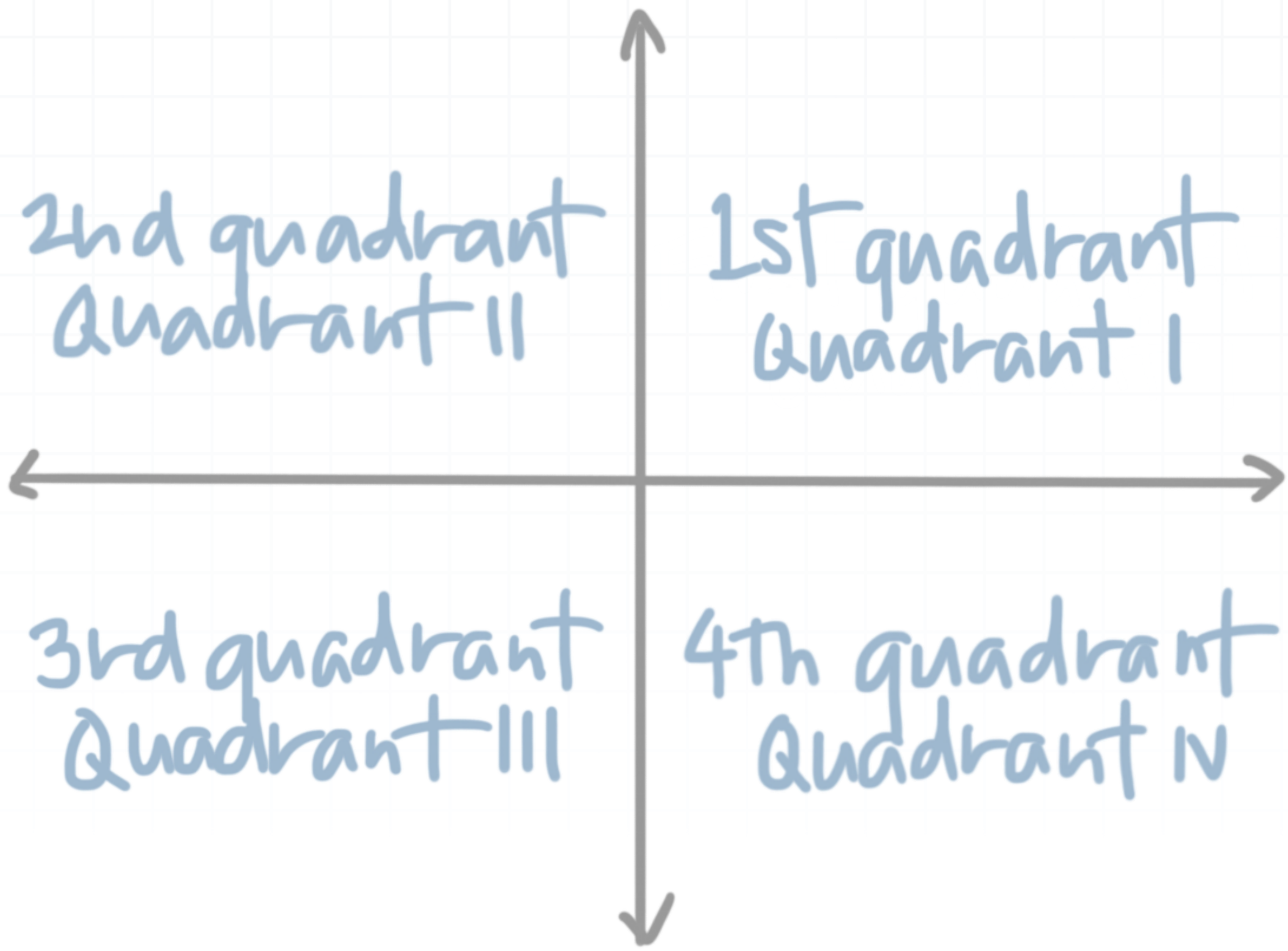
Quadrant of the angle

In the last lesson, we briefly touched on the four quadrants of the Cartesian coordinate system.



In this lesson, we'll go in depth a little more with each of the four quadrants and the axes that divide them.

The quadrant in which the angle lies

When you hear that an angle is "in the third quadrant" that means that when you sketch it in standard position, the terminal side is somewhere in the third quadrant. Or when an angle "lies in the fourth quadrant," that means its terminal side falls somewhere in the fourth quadrant when it's sketched in standard position.

When the terminal side of an angle falls exactly on one of the axes, it's called a **quadrantal angle**, and it's technically not in any quadrant since the axes aren't in a quadrant.

Remember that an angle in standard position is always positioned with its initial side along the positive direction of the x-axis, so think about angles as "starting" along the positive direction of the x-axis. That means any zero-angle will also have its terminal side on the positive direction of the x-axis.

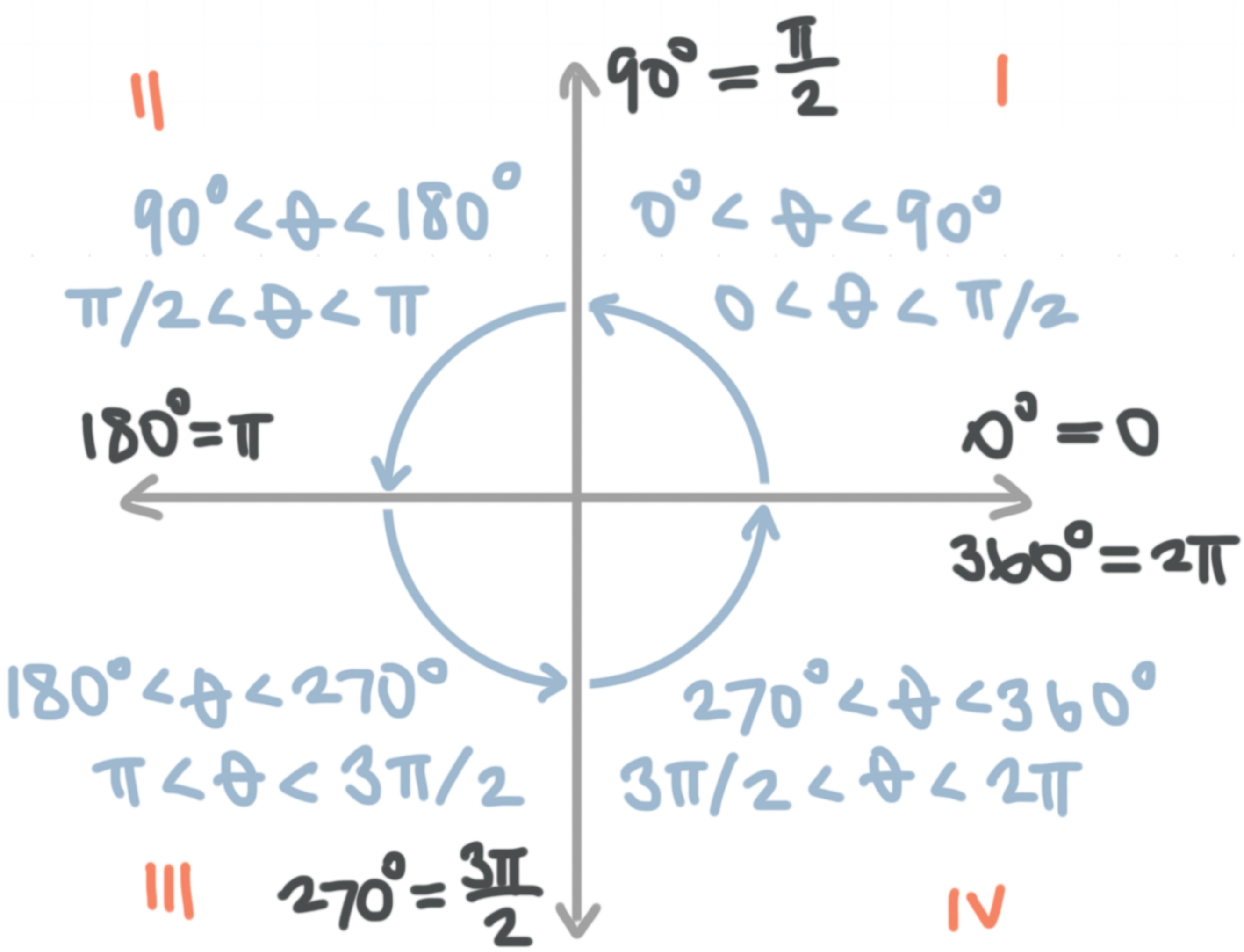
From there, we rotate counterclockwise, with positive rotation, toward the positive y-axis, then the negative x-axis, then the negative y-axis, and finally back around to the positive x-axis. Every axis is separated from the previous by 90° or $\pi/2$. So the angles associated with each axis are

Axis	**Degrees**	**Radians**
Positive x-axis	0°	0
Positive y-axis	90°	$\pi/2$
Negative x-axis	180°	π
Negative y-axis	270°	$3\pi/2$
Positive x-axis	360°	2π

Given these values, we know the angle measures that divide each quadrant, which means we can give an inequality that defines all the angles contained within each quadrant:

Quadrant	Degrees	Radians
First	$0^\circ < \theta < 90^\circ$	$0 < \theta < \pi/2$
Second	$90^\circ < \theta < 180^\circ$	$\pi/2 < \theta < \pi$
Third	$180^\circ < \theta < 270^\circ$	$\pi < \theta < 3\pi/2$
Fourth	$270^\circ < \theta < 360^\circ$	$3\pi/2 < \theta < 2\pi$

We can illustrate these quadrantal angles and the interval of angles which are defined in each quadrant.



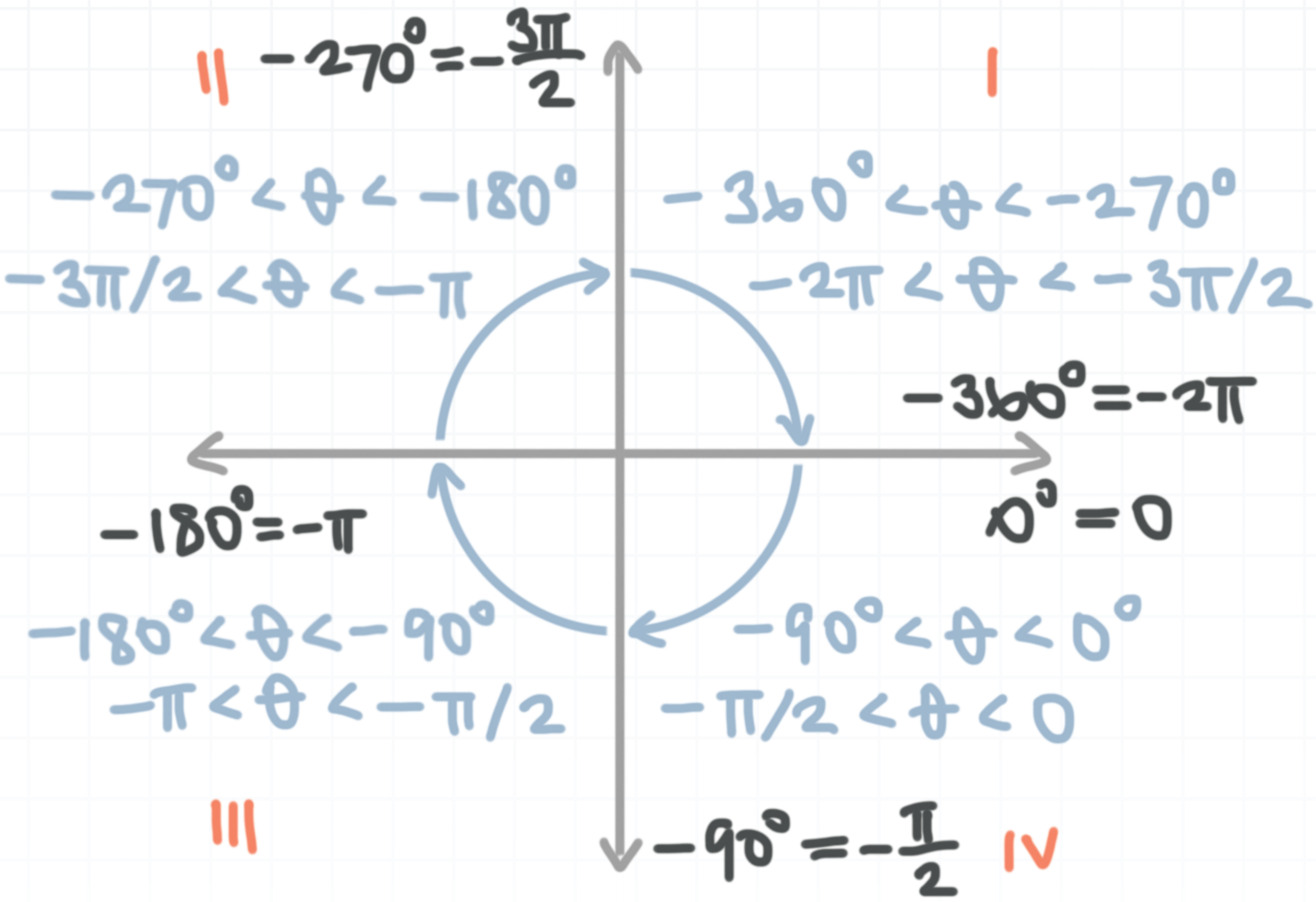
We can also define a similar set of negative angles and inequalities. We start from the positive x-axis and rotate clockwise, with negative rotation, toward the negative y-axis, then the negative x-axis, then the positive y-axis, and finally back around to the positive x-axis. So the angles associated with each axis are

Axis	**Degrees**	**Radians**
Positive x-axis	0°	0
Negative y-axis	-90°	$-\pi/2$
Negative x-axis	-180°	$-\pi$
Positive y-axis	-270°	$-3\pi/2$
Positive x-axis	-360°	-2π

And then we can give an inequality that defines all the negative angles contained within each quadrant:

Quadrant	**Degrees**	**Radians**
Fourth	$-90^\circ < \theta < 0^\circ$	$-\pi/2 < \theta < 0$
Third	$-180^\circ < \theta < -90^\circ$	$-\pi < \theta < -\pi/2$
Second	$-270^\circ < \theta < -180^\circ$	$-3\pi/2 < \theta < -\pi$
First	$-360^\circ < \theta < -270^\circ$	$-2\pi < \theta < -3\pi/2$

We'll also illustrate these negative angles in terms of the four quadrants.



Let's work through a few examples of how to use this information to determine the quadrant in which an angle is located.

Example

Determine the quadrant in which $\theta = 283^\circ$ is located.

We just need to compare the angle against the inequalities we set up to define each quadrant.

$$270^\circ < 283^\circ < 360^\circ$$

The angle $\theta = 283^\circ$ lies in the fourth quadrant, because it's a larger angle than 270°, but a smaller angle than 360°. We know that 270° is along the negative y-axis and that 360° is along the positive x-axis, so $\theta = 283^\circ$ falls between them in the fourth quadrant.

Let's look at an example with a negative radian angle that measures more than one full rotation.

Example

In which quadrant is $-(33/5)\pi$ located?

Remember that, in radians, one full rotation is 2π. So to determine how many full rotations are included in $-(33/5)\pi$, divide $-(33/5)\pi$ by 2π.

$$\frac{-\frac{33\pi}{5}}{2\pi} = -\frac{33\pi}{5} \cdot \frac{1}{2\pi} = -\frac{33\pi}{10\pi} = -3.3$$

This tells us that $-(33/5)\pi$ includes 3 full rotations in the negative direction, plus an additional 0.3 rotations in the negative direction. We just need to figure out how much is 0.3 of 2π.

$$0.3(2\pi) = 0.6\pi$$

So, from the starting point of the positive direction of the x-axis, we complete 3 full rotations in the negative direction, which gets us back to the same starting point, and then we rotate an additional 0.6π in the negative direction, which is further of a rotation than $-0.5\pi = -\pi/2$, but not as far as $-1.0\pi = -\pi$. Which means the terminal side of the angle will land in the third quadrant.

So we can say that $-(33/5)\pi$ is located in the third quadrant.

Now let's deal with an angle in radians whose measure isn't given in terms of π.

Example

In which quadrant is 21.9 radians located?

To put the angle in terms of π, we'll divide 21.9 by $\pi \approx 3.14$ to get

$$\frac{21.9}{3.14} = 6.97$$

so 21.9 radians $\approx 6.97\pi$. This angle is outside the interval $[0,2\pi)$ that represents one full rotation. We know that 6π represents 3 full rotations, so we know the angle is three full rotations in the positive direction, and then an additional 0.97π rotations in the positive direction.

Now all we need to do is figure out the quadrant of 0.97π. Since 0.97 is more than $1/2$, but less than 1, it means that 0.97π is more than $\pi/2$, but less than π, which leaves us in the second quadrant.

Therefore, 21.9 radians is in the second quadrant.