Experiences with topic-based workshops in first-year teaching

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Why do I need this?

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Textbooks often contain examples of applications

With large, mixed cohorts someone will be the odd one out

Many mathematicians are <u>awful</u> at coming up with applications

Why do I need this?

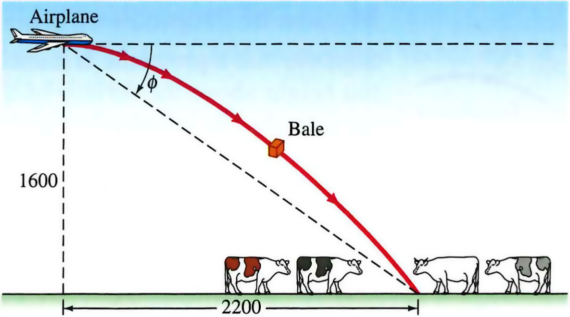


FIGURE 11.5.13 Trajectory of the hay bale of Example 9.

Edwards & Penney, *Calculus: Early Transcendentals (7ed)*; p. 860

Part I

Course structure

Idea of First.Math 2.0

The maths needs exemplification through *field specific* applications

Chosen in collaboration with the individual departments

Must...

- ...represent a relevant problem to the field
- ...cover an appropriate part of the curriculum

Students do *not* need the ability to solve the full problem

Course structure

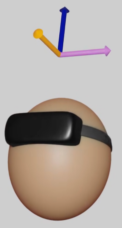
Each course is split into 4 ‘blocks’

Every block contains

- 2 or 4 lectures
- A programme-specific workshop

Each study board selects blocks for their students

What is a workshop?



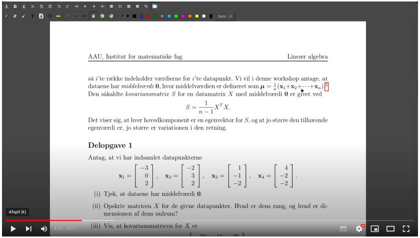
4-hour session

Exercises based on the contents of a block

Inspired by the programme-specific problem

Introductory video

Examiner and TA’s available



AAU, Institut for matematiske fag — Lineær algebra

så i'te række indeholder værdierne for i'te datapunkt. Vi vil i denne workshop antage, at dataene har *middelværdi* $\mathbf{0}$, hvor middelværdien er defineret som $\boldsymbol{\mu} = \frac{1}{n}(\mathbf{x}_1+\mathbf{x}_2+\cdots+\mathbf{x}_n)$.[2] Den såkaldte *kovariansmatrix* S for en datamatrix X med middelværdi $\mathbf{0}$ er givet ved

$$S = \frac{1}{n-1}X^T X.$$

Det viser sig, at hver hovedkomponent er en egenvektor for S, og at jo større den tilhørende egenværdi er, jo større er variationen i den retning.

Delopgave 1

Antag, at vi har indsamlet datapunkterne

$$\mathbf{x}_1 = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \quad \mathbf{x}_4 = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}.$$

(i) Tjek, at dataene har middelværdi $\mathbf{0}$.

(ii) Opskriv matricen X for de givne datapunkter. Hvad er dens rang, og hvad er dimensionen af dens nulrum?

(iii) Vis, at kovariansmatricen for X er

Assessment

Priority: Workshops are integral to the exam

Our solution:
Oral exam with workshops as starting point (15 min.)

Part II

Problem examples

Typical structure

Introduction highlighting relevance

1. Toy-example
2. Theoretical exercise
3. MATLAB/Python exercise

Linear algebra

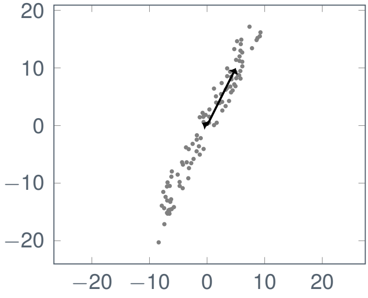
Block 2

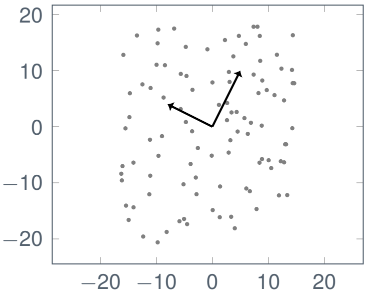
Subspaces, bases, eigenvalues, and diagonalization

Energy, Wind turbines

Eigenvectors appear in principal component analysis (PCA)

Suggestion: use this to detect anomalies in wind turbines





Energy, Exercise 1

Assume ‘toy observations’

$$\mathbf{x}_1 = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \quad \mathbf{x}_4 = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}.$$

- Collect observations in matrix form X. Determine rank and dimension of null space
- Show that the covariance matrix is

$$\frac{1}{3}\begin{bmatrix} 30 & -15 & -20 \\ -15 & 14 & 12 \\ -20 & 12 & 16 \end{bmatrix}.$$

Energy, Exercise 2

Students now consider a covariance matrix

$$S = \begin{bmatrix} 5 & 1 & 4 \\ 1 & 5 & 4 \\ 4 & 4 & 10 \end{bmatrix}$$

with ‘nice’ eigenvalues and -vectors

They are asked to

- Check that a vector is an eigenvector
- Determine remaining eigenvalues and eigenspaces
- Perform diagonalization of S

Energy, Exercise 3

Asked to assume that $\mathbb{R}^m$ has a basis $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_m\}$ of eigenvectors of S such that

$$\begin{cases} \mathbf{v}_i^T \mathbf{v}_i = 1 & \text{for all } i \\ \mathbf{v}_i^T \mathbf{v}_j = \mathbf{v}_j^T \mathbf{v}_i = 0 & \text{for all pairs } (i, j) \text{, where } i \neq j \end{cases}$$

(They don't know orthogonality or the spectral theorem yet)

We guide them to show that for unit vector $\mathbf{w} = \sum c_i \mathbf{v}_i$

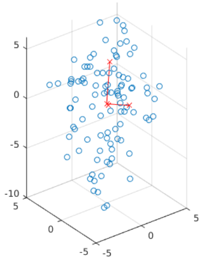
$$\mathrm{Cov}(X\mathbf{w}) = c_1^2 \lambda_1 + c_2^2 \lambda_2 + \cdots + c_m^2 \lambda_m$$

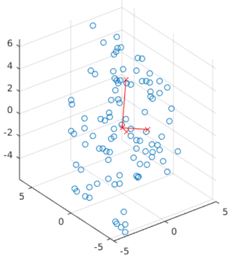
which is maximized for $\mathbf{w} = \mathbf{v}_1$.

Energy, Exercise 4

Students are provided with

- Two datasets
- A script performing PCA and plotting the result





Asked to determine which turbine needs servicing first

Linear algebra

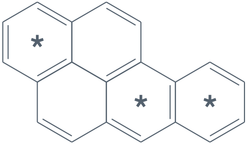
Block 5

Linear programming

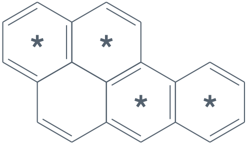
Chemistry: The Fries number

Polycyclic aromatic hydrocarbons (PAH's)

Fries number: Max. benzene ring count (potentially overlapping)



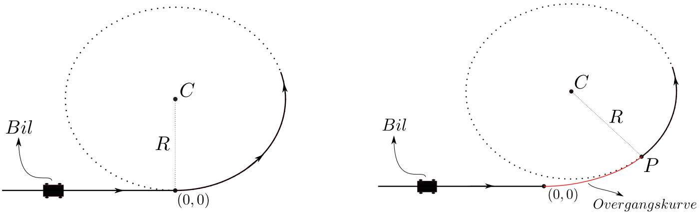
Can be formulated as integer optimization, but can be solved using linear programming



Calculus

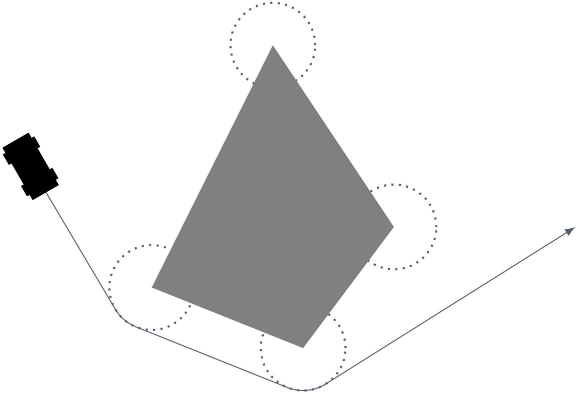
Block 2
Space curves

Land surveing: Road geometry



The transition curve forms part of a *clothoid*

Robotics: ‘Road geometry’



Part III

Challenges and CTM connection

Initial workload

Creating workshops takes **a lot** of time

Several iterations may be necessary

Later attempts require less effort
(E.g. I have made 30 video introductions)

Exam workload

Oral vs. written exam

But certain knowledge is easier to assess orally

Prerequisites

Computational aspects are fairly basic

Most students have no prior programming experience – and the course is *not* a programming course

Tutorials from CTM might bring us closer to computational thinking

Questions?

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