NTNU

Kunnskap for en bedre verden

Teaching contextual and computational mathematics to large groups of engineering students

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Mathematics in Engineering Education

- Mathematics has “always” been a central subject in engineering education
  - Its role has been discussed since many years (Bajpai, 1985; Scanlan, 1985).
- And still …
  - Mathematics is not seen as relevant by the students (Faulkner et al., 2020; Loch & Lamborn, 2016)
  - Mathematics is not visible in the engineering courses (González-Martín et al., 2021)
  - There is a rupture between mathematics as taught in mathematics courses and courses in the other disciplines (González-Martín et al., 2021)
  - Challenging to make connections between mathematics and engineering (Faulkner et al., 2019)
  - Students struggle to use mathematics when they need it in their engineering courses (Harris et al., 2015)
- Similar issues also in other user programmes, e.g., Economics (Langärds-Tarvoll, 2024)

Mathematics and Engineering Education at NTNU

- 16 five-year Master of Engineering (ME) programmes
  - $\approx$ 1600 new students each year
  - Campus Trondheim
- 16 three-year Bachelor of Engineering (BE) programmes
  - $\approx$ 900 new students each year
  - Campus Trondheim, Gjøvik and Ålesund
- Basic courses in mathematics for BE and ME are separate
- **I will only talk about the ME programmes**

Mathematics and Engineering Education at NTNU for the ME programmes

- Up to now: Four (7.5 ECTS) compulsory mathematics courses distributed over the first 3 or 4 semesters
  - Minor differences between the programmes
- Mathematical content is rather general
  - Some applications are included (modelling examples and modelling tasks)
  - Connection to applications often superficial and/or artificial
- Some numerical methods included from the beginning – more advanced methods assembled in a separate (the fourth) course

Mathematics and Engineering Education at NTNU for the ME programmes

- From 2025 a revision of the courses:
  - Greater variation between the programmes
  - More emphasis on computational (numerical) methods – throughout all courses
  - Stronger links between mathematics and the engineering programmes
- Revision inspired by CDIO ideas

CDIO (Concieve – Develop – Implement – Operate) (http://www.cdio.org/)

- The CDIO-initiative: An innovative educational framework for producing the next generation of engineers
  - Examples include believable situations that students recognize as being important to their current or possible future lives
  - Concepts … are presented in the context of their use (Crawley et al., 2014)
- Simulation-based mathematics (Optional standard 2)
  - Mathematics curriculum with programming, numerical modelling and simulation
  - Mathematics courses include more authentic and complex problems.
  - Reinforce the connection to science and engineering courses
- A better understanding of what advanced mathematics can be used for and how that it carried out strengthens student motivation. (CDIO, 2022).

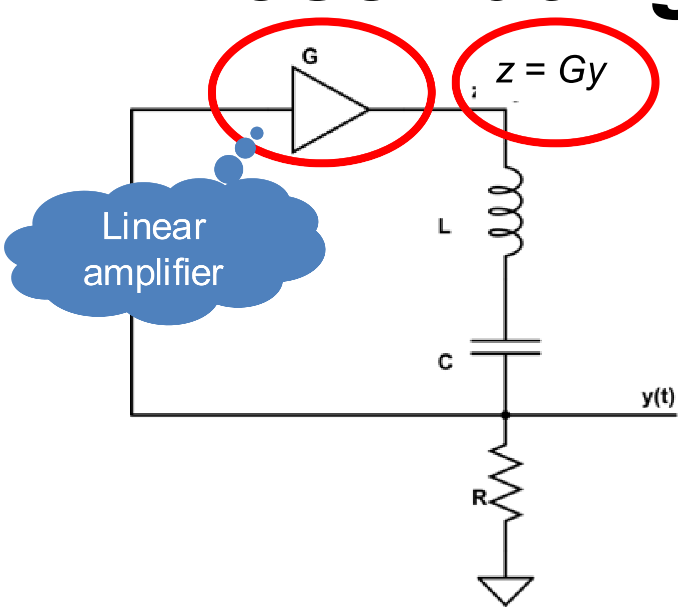
Revising the ME programmes at NTNU

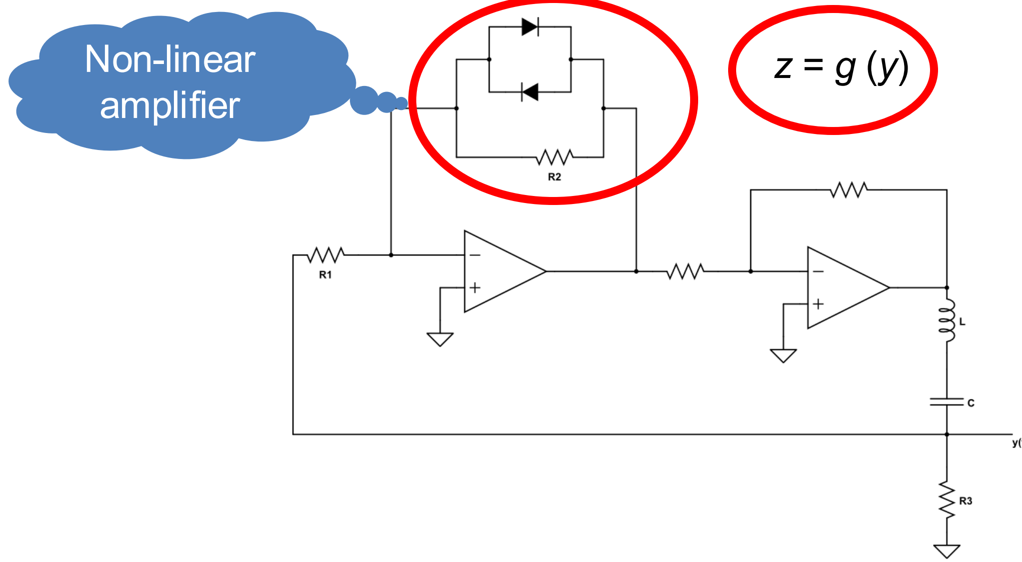
- The project *Technology Studies for the Future* (FTS, 2022)
- Inspired by CDIO and the CDIO standards
- Ten principles acting as guidelines for developing the technology programmes at NTNU
  - Principle 3: *Contextual learning* as a pedagogical principle should permeate the technological study programmes at NTNU
- The project *Mathematics as a Thinking Tool* (MARTA)
  - Strengthen the connections between mathematics and engineering by using mathematics in realistic engineering situations

From MARTA to full-scale implementation

- Phase 1 (2020): Electronic Systems and Innovation
- Phase 2 (2022): Electronic Systems and Innovation, Cybernetics and Robotics, and Chemistry and Biotechnology
- Aim: Achieve stronger integration between mathematics and engineering subjects (contextual learning)
- Hope: Enhance students' perceived relevance of mathematics
- Phase 3 (2025): Computational and context-based mathematics for all ME programmes
- What possibilities and challenges can be observed in the various phases?

An oscillating circuit (from Phase 1)



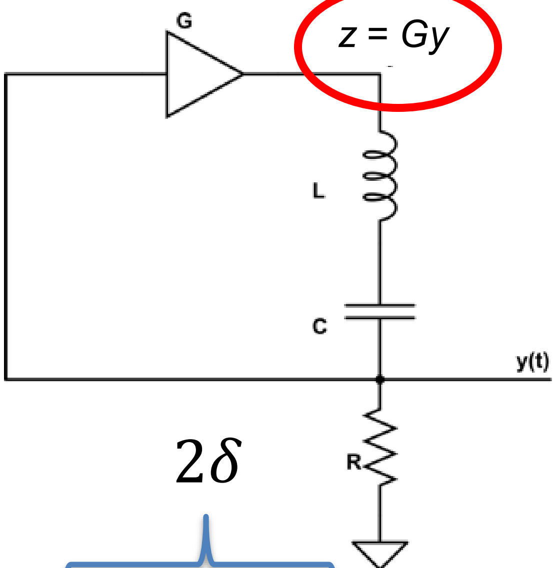


- Simple circuit
- Can be built, but will not give the desired output
- Mathematically simple to describe and analyse (linear diff.eq.)

- Modified circuit
- Can be built, and will give the desired output (stable oscillations)
- Mathematically more complicated to describe and analyse (non-linear diff.eq.)

The simple circuit

$$e^{-\delta t}$$

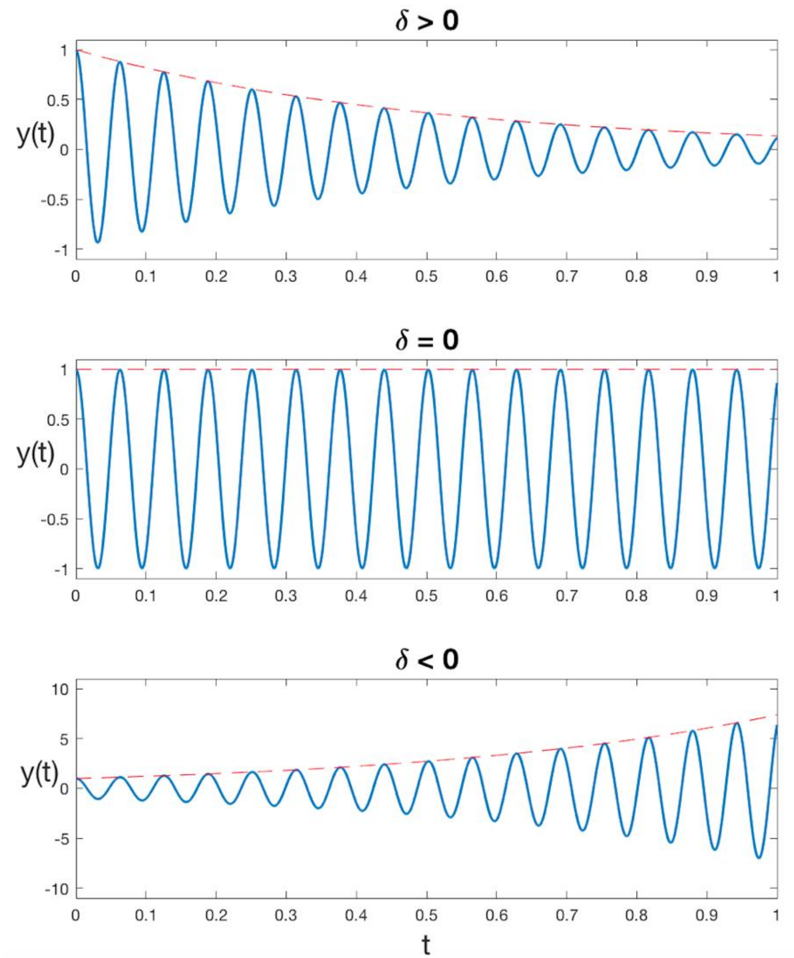


$$y'' + \underbrace{(1-G)\frac{R}{L}}_{2\delta} y' + \frac{1}{LC} y = 0$$

$G < 1\ (\delta > 0)$

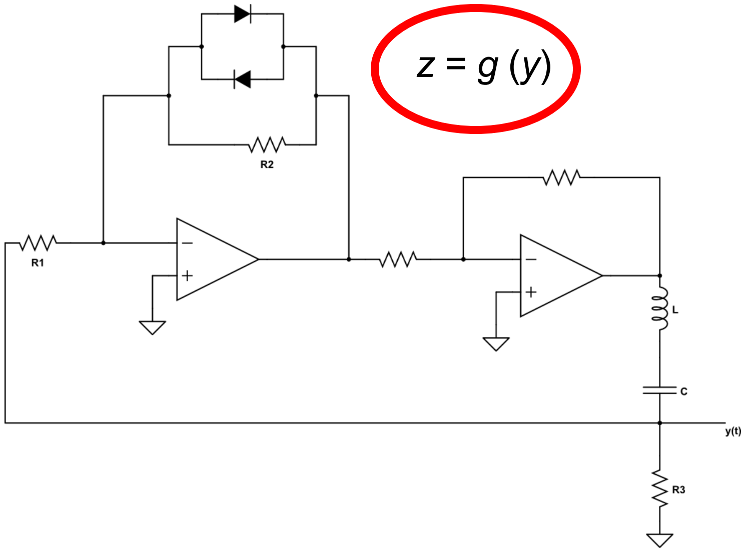
$G = 1\ (\delta = 0)$

$G > 1\ (\delta < 0)$



$$\delta = (1-G)\frac{R}{2L}$$

The modified circuit

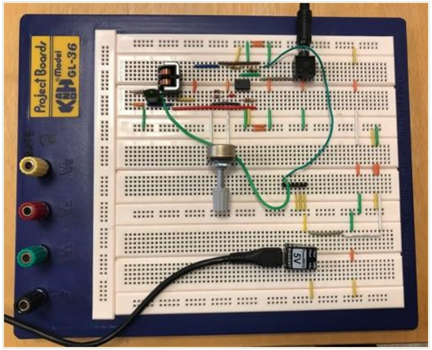


$$\frac{d^2y}{dt^2} + \frac{R_3}{L} f(y) \frac{dy}{dt} + \frac{1}{LC} y = 0$$

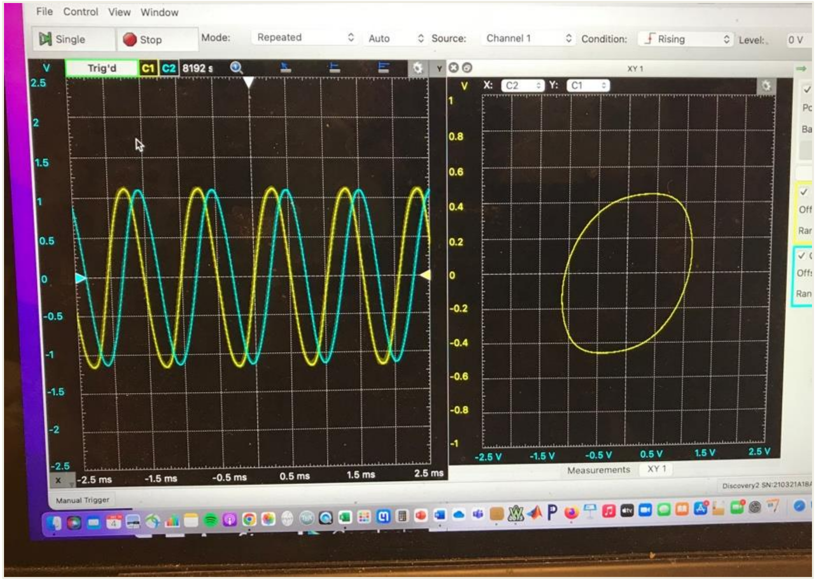
where $f(y) = 1 - \frac{dg(y)}{dy}$, and

$$g^{-1}(y) = 2R_1 I_0 \sinh\left(\frac{y}{V_0}\right) + \frac{1}{R_2} y$$

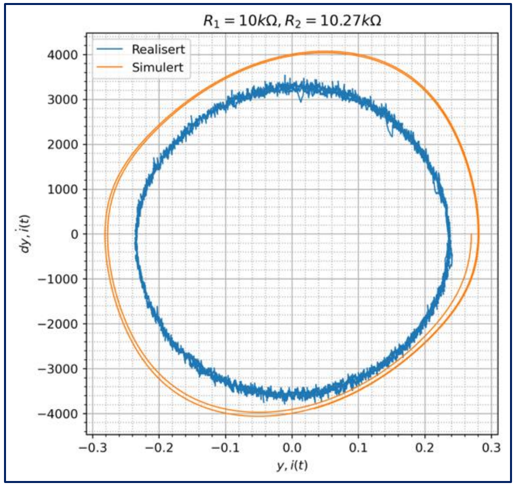
The modified circuit

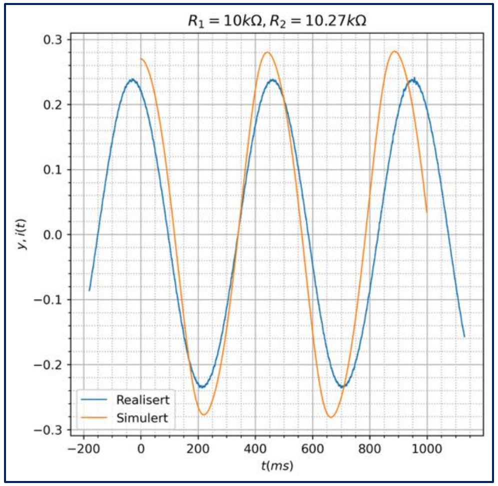


Building the circuit



Measuring the output





Computing the output

Opportunities and challenges

- The oscillating circuit was treated both in the mathematics class and in the electronics class (Phase 1)
- The close collaboration between mathematics and *one* programme made this possible (see Alpers, 2008; Enelund et al., 2011, and mechanical engineering)
- With more programmes involved (Phase 2 and onwards):
  - Providing separate lessons for a large number of programmes is expensive
  - The knowledge of the mathematics teachers will be a limitation
  - Necessary with close collaboration between mathematics and engineering programmes

Phase 3: 2025 –

- Revising the mathematics courses for ME
  - More emphasis on numerical methods and linear algebra
  - Greater variation between the programmes
  - Stronger connection between mathematics and the engineering programmes
- Dividing the 16 ME programmes in five clusters:
  - Cluster A: Electronic Systems and Cybernetics
  - Cluster B1 and B2: Civil engineering, mechanical engineering, marine engineering, chemistry, geology, ...
  - Cluster C: Information and communication technology
  - Cluster D: Physics and mathematics
- Each cluster will be divided in smaller groups for parts of the teaching

Course 1: Calculus and Linear Algebra

- Common for all programmes in clusters A, B and C
- Contextualisation through examples
- Emphasis on numerical methods/analysis

Time	Monday	Tuesday	Wednesday		Thursday		Friday
08.15 | 10.00	Cluster A	Cluster B2	Cluster A_1	Cluster $B1_1$	Cluster $B2_1$	Cluster C_1	
10.15 | 12.00	Cluster B1	Cluster C	Cluster A_2	Cluster $B1_2$	Cluster $B2_2$	Cluster C_2	
12.15 | 14.00				Cluster $B1_3$	Cluster $B2_3$		

Example:
A = Electronic systems (A_1) and Cybernetics (A_2)
B1 = Civil engineering ($B1_1$), Marine Engineering ($B1_2$), Mechanical engineering ($B1_3$)

Organisation

- Four main teachers in mathematics
  - Each with a special connection to one of the clusters A, B1, B2 and C

Cluster A, semester 1	
Electronics Systems Design and Innovation	**Cybernetics and Robotics**
Calculus and Linear Algebra	Calculus and Linear Algebra
Information Technology, Introduction	Information Technology, Introduction
Introduction to Analog and Digital Electronics	Introduction to Analog and Digital Electronics
Electronic System Design, Basic Course	Computerized Control, Introduction

- The main mathematics teacher and the other teachers in the cluster form a working group

The next courses

- Increasingly greater differences between the clusters
  - in content and in sequence of topics
- Examples:
  - Cluster A: Integral transforms and Fourier series in Course 2
  - Cluster B: Multivariate calculus in Course 2
  - Plus more Linear Algebra for both clusters

The issue of relevance

- Survey results indicate that perceived relevance of mathematics and visibility of mathematics differ between programmes
- Some programmes report that they use little mathematics in the first years – how to deal with this?

- How specialised must examples be to be motivating?
- What kind of engagement in mathematics can be expected from the engineering teachers?
- What kind of engagement in engineering can be expected from the mathematics teachers?

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