N-Body Simulations with REBOUND

Lab course protocol

Group 3+10

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Abstract

This is optional, but never longer than half a page.

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Very short summary what the experiment is about and why the subject plays a role in astronomy/astrophysics.

2 Theory

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2.2.1 Leapfrog

2.2.2 IAS15

2.2.3 WHFast

2.2.4 Gragg-Bulirsch-Stoer

2.3 REBOUND

To simulate the N-body problem in various astrophysical contexts, we use the REBOUND software package developed by Professor Hanno Rein. REBOUND can simulate particles under the influence of their gravities. These particles can represent astrophysical bodies like stars, planets, moons, asteroids, dust particles etc[RL12]. The documentation for REBOUND can be found at: https://rebound.readthedocs.io/en/latest/. It provides convenient tools to study the properties and evolution of an N-body system like the energy, angular momentum and orbital elements.
REBOUND runs natively on windows, mac and linux. This can be run in either C or Python. For our purposes we will stick with the latter. REBOUND for python can be easily installed by `pip install rebound`.

2.3.1 REBOUNDX

3 Experiment

3.1 Two Body Problem

We use the simple two body problem to test various integrators in REBOUND (Leapfrog, IAS15, WHFast, Gragg-Bulirsch-Stoer) and compare the quality of the resulting outputs. We also test the quality of the results as we change the timestep from 1 to 10^{-6}
In this two body problem we simulate a moon orbiting a planet or a planet orbiting a star. Here, one body will be significantly heavier than the other. The energy and the angular momentum of the system should remain constant and are given as:

$$E = -\mu \frac{GM}{2a} \tag{1}$$

$$L = \mu \sqrt{GMa(1-e^2)} \tag{2}$$

Where $\mu = \frac{m_1 m_2}{M}$ is the reduced mass of the system and $M = m_1 + m_2$ is the total mass. From the above equations we can derive that the semi major axis and the eccentricity of the system should also remain constant as we integrate the system over time.
The two body problem is simulated with REBOUND as per the following procedure:

1. Initialize the simulation with a chosen integrator and timestep
2. Add the two bodies to the system with $m_1 = 1$ and $m_2 = 0.3, a = 1, e = 0.3$
3. The simulation is integrated for one orbit and 250 steps. At each step the positions, energies and orbital parameters of the system are stored.
4. The orbit can be plotted from the stored positions of the two bodies. Various properties of the system can be plotted as a function of time.

The code where the following procedure is implemented is given in the appendix 4.
Here is the orbit we obtain with the leapfrog integrator and a timestep of 10^{-3}:

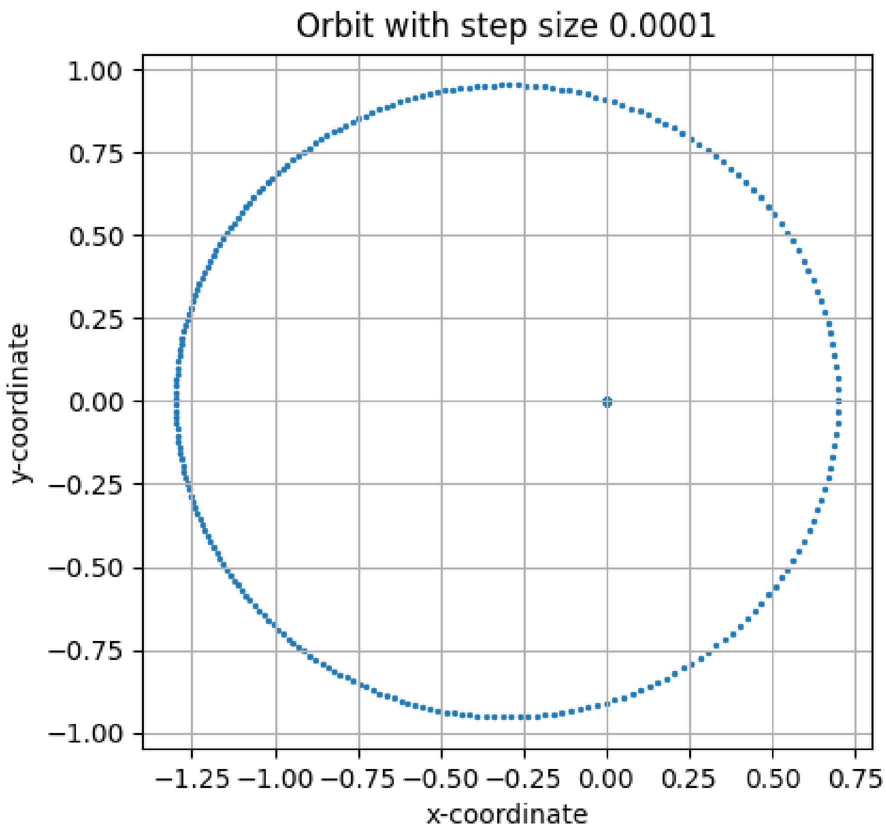
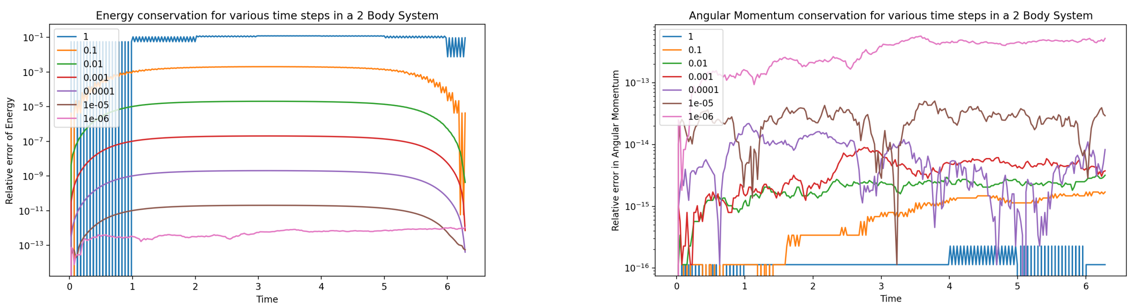


Figure 1: Orbit of a two body system with leapfrog integrator and timestep of 10^{-3}. The number of data points are less dense closer to the pericenter, which means that m_2 is moving faster. This follows Kepler's second law.

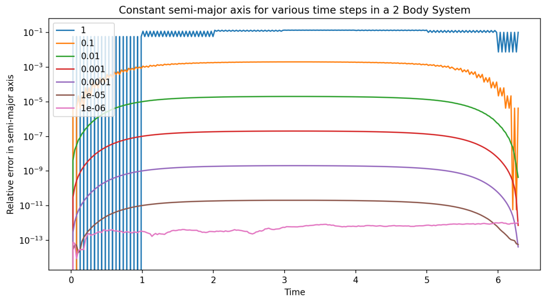
Effect of smaller timesteps is tested with the leapfrog integrator with timesteps of 1, 10^{-1},10^{-2}, 10^{-3},10^{-4}, 10^{-5} and 10^{-6}.



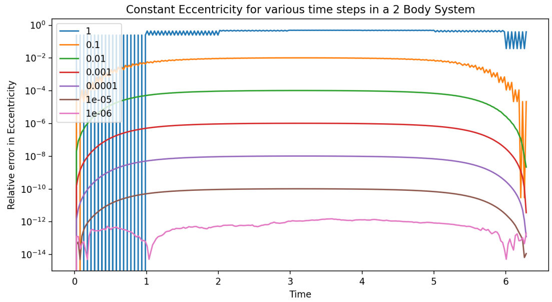
(a) Conservation of Energy at various timesteps. As we increase the timestep, the energy deviation from the initial value increases.

(b) Conservation of Angular Momentum at various timesteps. Conservation of angular momentum gets worse as we decrease the timestep.

Figure 2: Conservation of Energy and Angular Momentum at various timesteps



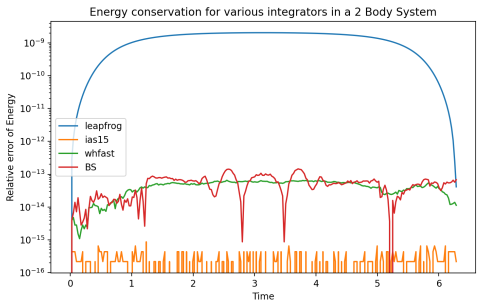
(a) Semi Major Axis at various timesteps



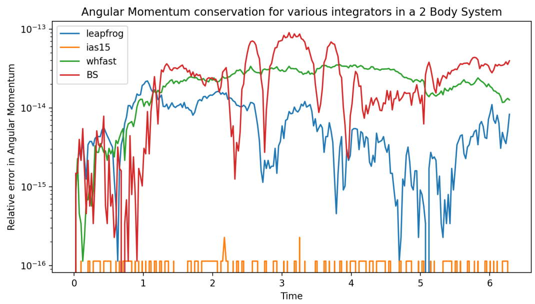
(b) Eccentricity at various timesteps

Figure 3: Constant semi major axis and eccentricity at various timesteps

Effect of different integrators is tested with a timestep of 10^{-3} using the leapfrog, IAS15, WHFast and Gragg-Bulirsch-Stoer integrators

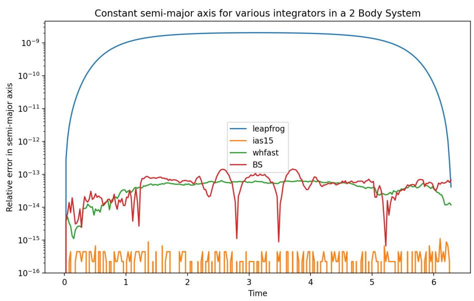


(a) Conservation of Energy using different integrators

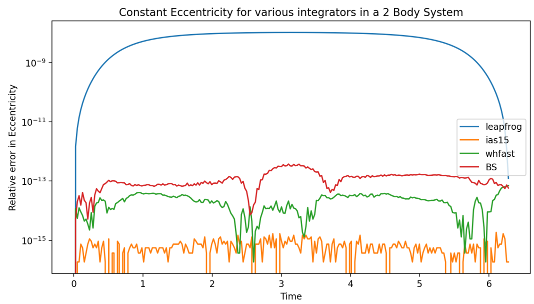


(b) Conservation of Angular Momentum using different integrators

Figure 4: Conservation of Energy and Angular Momentum using different integrators



(a) Semi Major Axis using different integrators



(b) Eccentricity using different integrators

Figure 5: Constant semi major axis and eccentricity using different integrators

3.2 Three Body Problem and Stability of the Planet System

Here we will simulate a coplanar 3 body system with a large central mass (m_1) and two smaller masses (m_2, m_3) in circular orbits. The two small objects will start opposite to each other (at a phase difference of π). We vary the distance between m_2 and m_3 to study the stability of the system.
The eccentricity and the semi major axis of the bodies will vary due to gravitational perturbations from the other bodies. For bodies that are closer, the gravitational perturbations will be larger which will result in stronger changes for the eccentricities and semi major axis. For initially circular orbits, Gladman[Gla93] defined a criterion for the stability of the system:

$$\Delta_c \simeq 2.40(\mu_2 + \mu_3)^{1/3} \tag{3}$$

At distances greater than Δ_c, the system is considered to be stable.

3.3 Jupiter and Kirkwood Gaps

In this part of the lab work we must observe Kirkwood gaps - gaps in the asteroid belt - by simulationg and observing the evolution of Sun-Jupiter-Mars system with 10000 test particles. Using REBOUND we can simulate these objects and get semi-major axis and eccentricity of all bodies up to 1 million years of evolution. Additionally, we check the change of distibution at every 100000 years.

Kirkwood gaps are caused by the resonant interaction between the asteroids and Jupiter. The resonant interaction leads to the instability of the orbits of the asteroids. Moreover, the instabilities cause the increase of eccentricity of the asteroids, and the asteroids with higher eccentricity are more likely to collide with inner planets. The main Kirkwood gaps are at 2.50, 2.82, 2.95, 3.27 AU corresponding to the resonant periods of 3:1, 5:2, 7:3, 2:1 respectively.

First, Sun, Jupiter and Mars are added with their corresponding masses, semi-major axes and eccemtricities. Next, we add 10000 test particles in a range of 2 to 4 AU and random true anomaly and eccentricities. It is important to make only three major objects active with $\texttt{sim.N}_\texttt{a}\texttt{ctive} = \texttt{3}$ line so all test particles do not affect the system. Additionally, we simplify the solution by using leap-frog integrator due to the complicatied computation. The code records the values of semi-major axis a and eccentricity a of each body to .txt file for further visualization.

Fig. **??** shows the histograms of the number of asteroids at various distances every 10^5 years. End result does not show the exact Kirkwood gaps due to the short timespan of the simulation and simplifications in the model and integrator. However, there are smaller number of asteroids near the expected respnance locations, especially near 2.82 AU and 2.95 AU.

Further, we can plot the semi-major axis and eccentricity of the test particles to see the evolution of the system:

Fig. 7 shows the eccentricity vs semi-major axis of the asteroids over time. One can

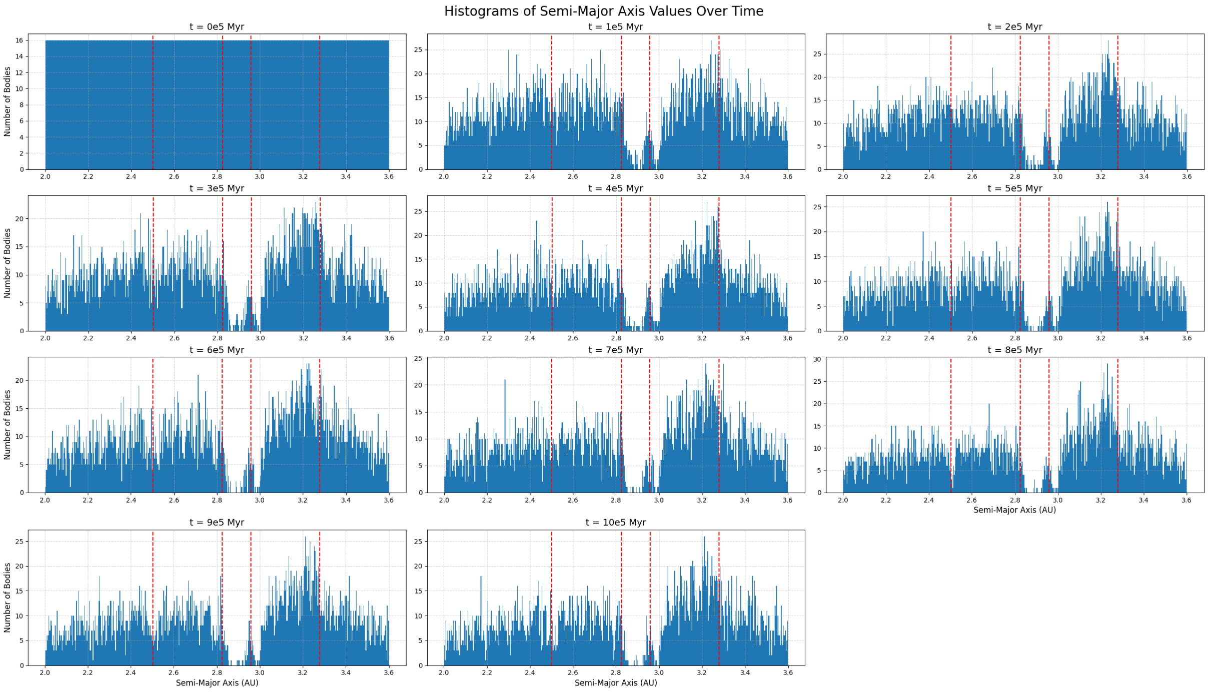


Figure 6: Histograms of the number of asteroids at various distances every 10^5 years. Red vertical lines correspond to actual Kirkwood gaps at 2.50, 2.82, 2.95, 3.27 AU.

see gaps near 2.82 and 2.95 AU as well as decrease of the eccentricity at smaller orbits. At smaller semi-major axis, objects with higher eccentricity more likely to interact with Mars, but at some even bigger eccentricities can keep being in the system at least for 10^6 million years.

3.4 Resonant Capture of a Planet

4 Conclusions

An important section in which you should critically review the experiment and its results. Mention also parts that did not work out as expected, but keep a neutral to positive view. This can span from a few sentences to half a page.

References

[Gla93] Brett Gladman. “Dynamics of Systems of Two Close Planets”. In: *Icarus* 106 (Nov. 1993), pp. 247–263. DOI: `10.1006/icar.1993.1169`.

[RL12] H. Rein and S. -F. Liu. “REBOUND: an open-source multi-purpose N-body code for collisional dynamics”. In: *aap* 537, A128 (Jan. 2012), A128. DOI: `10.1051/0004-6361/201118085`. arXiv: `1110.4876 [astro-ph.EP]`.

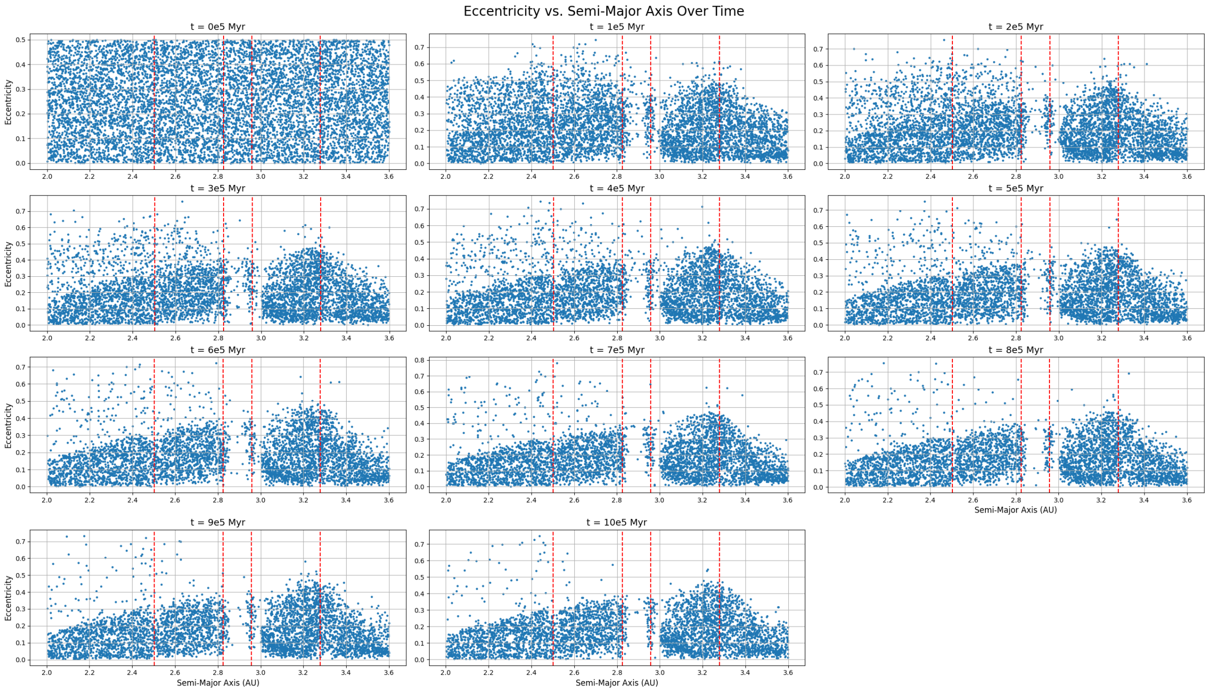


Figure 7: Eccentricity vs Semi-Major Axis of asteroids over time. The gaps and smaller eccentricity at smaller semi-major axis can be seen.

Appendix

Code

Please attach here your original handwritten notes and other documents created during the experiment.