N-Body Simulations with REBOUND

Lab course protocol

Group 3+10

Pratyush Singh,
Proshmit Dasputpa,
Erasyl Telman

07/03/2025

Advanced lab course in astronomy
Eberhard Karls Universität Tübingen

WiSe 2024/25

Abstract

This is optional, but never longer than half a page.

Contents

1 Introduction

Very short summary what the experiment is about and why the subject plays a role in astronomy/astrophysics.

2 Theory

2.1 Classical N-Body Problem

2.2 Time Integrators

2.2.1 Leapfrog

2.2.2 IAS15

2.2.3 WHFast

2.2.4 Gragg-Bulirsch-Stoer

2.3 REBOUND

3 Experiment

3.1 Two Body Problem

We use the simple two body problem to test various integrators in REBOUND (Leapfrog, IAS15, WHFast, Gragg-Bulirsch-Stoer) and compare the quality of the resulting outputs. We also test the quality of the results as we change the timestep from 1 to 10^{-6}
In a two body problem we simulate a moon orbiting a planet, or a planet orbiting a star. The energy and the angular momentum of the system will

remain constant. They are given as:

$$E = -\mu \frac{GM}{2a} \tag{1}$$

$$L = \mu \sqrt{GMa(1 - e^2)} \tag{2}$$

Where $\mu = \frac{m_1 m_2}{M}$ is the reduced mass of the system and $M = m_1 + m_2$ is the total mass.
From the above equations we can derive that the semi major axis and the eccentricity of the system should also remain constant when we simulate the system.
The two body problem is simulated with REBOUND as per the following procedure:

1. We initialize the simulation with a chosen integrator and timestep
2. Next, add the two bodies to the system with $m_1 = 1$ and $m_2 = 0.3, a = 1, e = 0.3$
3. Then the simulation is integrated for one orbit and 250 steps. At each step the positions, energies and orbital parameters of the system are stored.
4. The orbit can be plotted from the stored postions. Various properties of the system can be plotted as a function of time.

The code where the following procedure is implemented is given in the appendix 4.
Here is the orbit we obtain with the leapfrog integrator and a timestep of 10^{-3}:

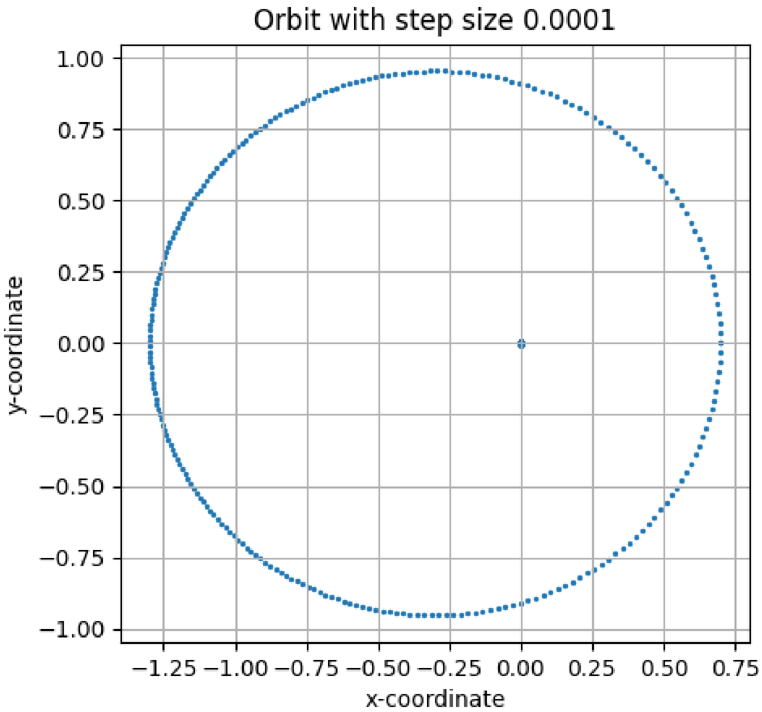
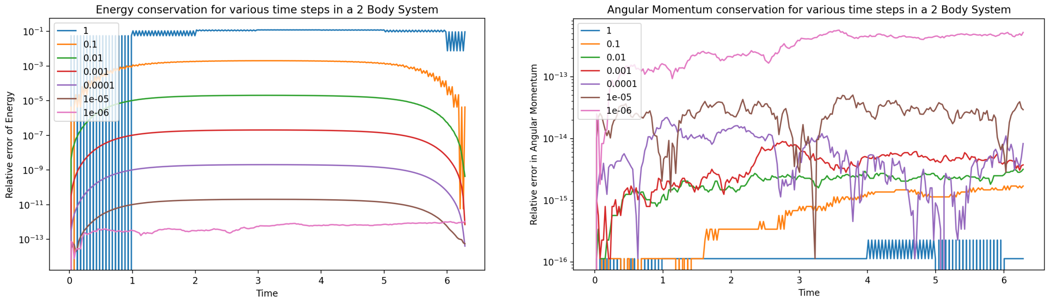


Figure 1: Orbit of a two body system with leapfrog integrator and timestep of 10^{-3}. The number of data points are less dense closer to the pericenter, which means that m_2 is moving faster. This follows Kepler's second law.

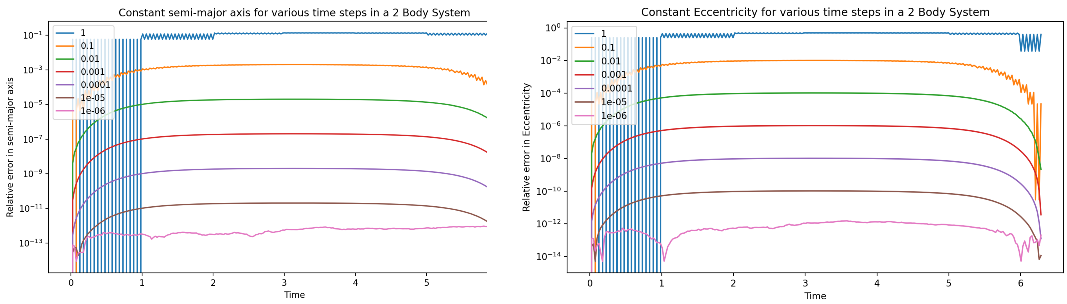
Effect of smaller timesteps is tested with the leapfrog integrator with timesteps of 1, 10^{-1},10^{-2}, 10^{-3},10^{-4}, 10^{-5} and 10^{-6}.



(a) Conservation of Energy at various timesteps

(b) Conservation of Angular Momentum at various timesteps

Figure 2: Conservation of Energy and Angular Momentum at various timesteps

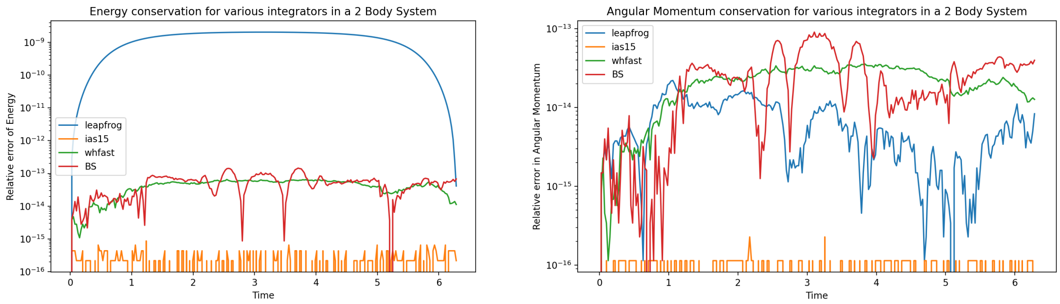


(a) Semi Major Axis at various timesteps

(b) Eccentricity at various timesteps

Figure 3: Constant semi major axis and eccentricity at various timesteps

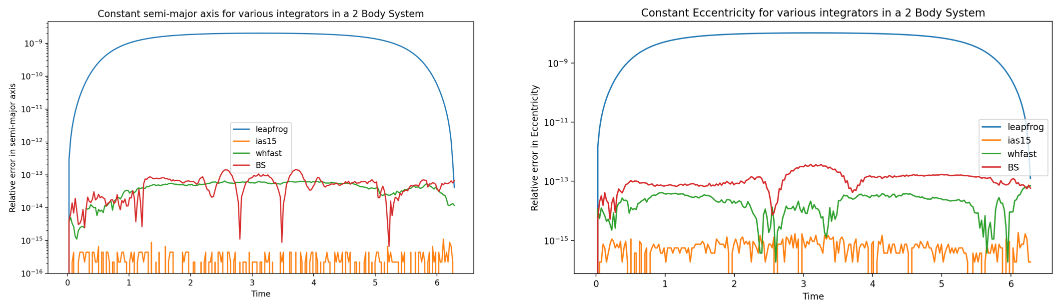
Effect of different integrators is tested with a timestep of 10^{-3} using the leapfrog, IAS15, WHFast and Gragg-Bulirsch-Stoer integrators



(a) Conservation of Energy using different integrators

(b) Conservation of Angular Momentum using different integrators

Figure 4: Conservation of Energy and Angular Momentum using different integrators



(a) Semi Major Axis using different integrators

(b) Eccentricity using different integrators

Figure 5: Constant semi major axis and eccentricity using different integrators

3.2 Three Body Problem and Stability of the Planet System

3.3 Jupiter and Kirkwood Gaps

3.4 Resonant Capture of a Planet

4 Conclusions

An important section in which you should critically review the experiment and its results. Mention also parts that did not work out as expected, but keep a neutral to positive view. This can span from a few sentences to half a page.

References

[1] Brown B, Aaron M (2001) The politics of nature. In: Smith J (ed) The rise of modern genomics, 3rd edn. Wiley, New York, p 234–295

[2] Dod J (1999) Effective Substances. In: The dictionary of substances and their effects. Royal Society of Chemistry. Available via DIALOG. http://www.rsc.org/dose/title of subordinate document. Cited 15 Jan 1999

[3] Slifka MK, Whitton JL (2000) Clinical implications of dysregulated cytokine production. J Mol Med, doi: 10.1007/s001090000086

[4] Smith J, Jones M Jr, Houghton L et al (1999) Future of health insurance. N Engl J Med 965:325–329

[5] South J, Blass B (2001) The future of modern genomics. Blackwell, London

Appendix

Code

Please attach here your original handwritten notes and other documents created during the experiment.