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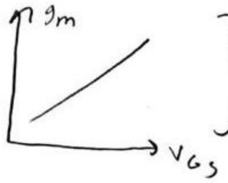
EEE415 Spring 2024/25 Homework 1

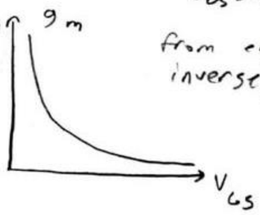
Q1) $g_m = \overset{①}{\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{th})} = \overset{②}{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}} = \overset{③}{\frac{2I_D}{V_{GS}-V_{th}}}$

(a)



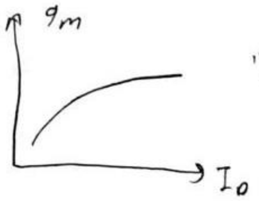
from eq. ①
linear relation

(b)



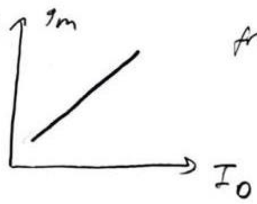
from eq. ③
inversely parabolic

(c)



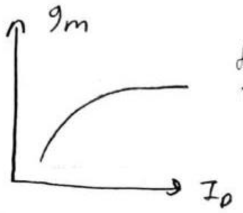
from eq ②
"sqrt" relation

(d)



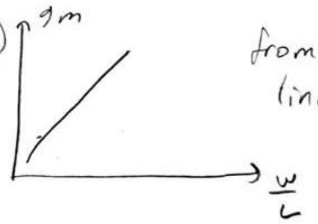
from eq ③
linear.

(e)



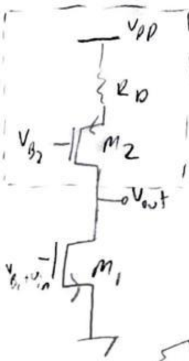
from eq ②
"Sqrt" relation

(f)



from eq ①
linear.

Q2) a

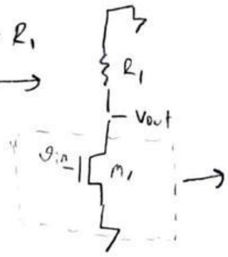


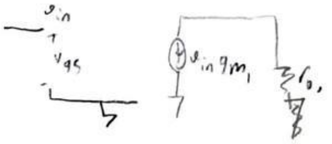
→ Looking from drain of pmos,

$R_1 = (1+(g_{m_2}+g_{mb_2})R_D)r_{o_2} + R_D \approx (g_{m_2}+g_{mb_2})R_D r_o$

$\uparrow R_1$

substitute R_1 →

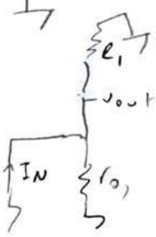




open ckt $V_{th} = -v_{in}g_{m_1}r_{o_1}$

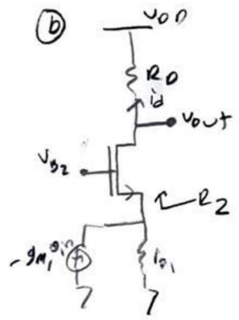
$R_{eq} = r_{o_1}$ (kill v_{in})

$I_N = -v_{in}g_{m_1}$

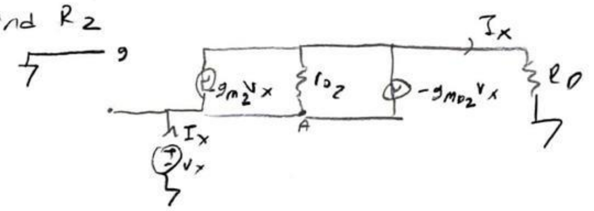


$V_{out} = -v_{in}g_{m_1} \cdot \dfrac{r_{o_1}}{r_{o_1}+R_1} \cdot R_1$, $A_v = -g_{m_1} \cdot (r_{o_1} \parallel R_1)$

b



Find R_2



KCL @ A: $-I_x + g_{m_2}V_x + \dfrac{V_x - I_x R_D}{r_{o_2}} + g_{mb_2}V_x = 0$

$V_x\left(g_{m_2}+g_{mb_2}+\dfrac{1}{r_{o_2}}\right) = I_x\left(1+\dfrac{R_D}{r_{o_2}}\right)$, $R_2 = \dfrac{r_{o_2}+R_D}{r_{o_2}(g_{m_2}+g_{mb_2})+1}$

transform the circuit →

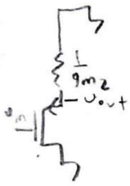
$i_d = \dfrac{r_{o_1}}{r_{o_1}+R_2}(-g_{m_1}v_{in})$, $-i_d R_D = v_{out}$

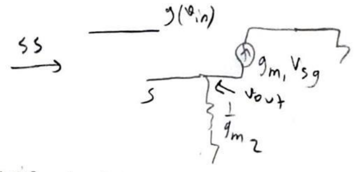
$-\dfrac{r_{o_1}}{r_{o_1}+R_2} \cdot g_{m_1}v_{in} \cdot R_D = v_{out}$, $A_v = \dfrac{-r_{o_1} \cdot g_{m_1} \cdot R_D}{r_{o_1}+R_2}$

Q3) (a) check $V_{DS} > V_{GS} - V_{th}$ and $V_{SD} > V_{SG} - |V_{thp}|$

$0 > -V_{th} \rightarrow m_2$ is SAT $\quad V_{in} > -V_{th} \rightarrow m_1$ is SAT

For on-off conditions, $m_1 \rightarrow V_{out} - V_{in} > |V_{thp}| = V_{th}$, $m_2 \rightarrow V_{DD} - V_{out} > V_{th}$





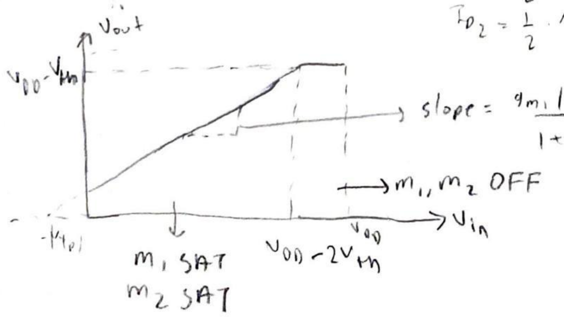
$g_{m_1}(v_{in} - v_{out}) \cdot \frac{1}{g_{m_2}} = v_{out}$

$A_V = \frac{v_{out}}{v_{in}} = \frac{g_{m_1}/g_{m_2}}{1 + g_{m_1}/g_{m_2}} = \frac{g_{m_1}}{g_{m_2} + g_{m_1}}$, $A_V \to 1 \Rightarrow g_{m_1} \gg g_{m_2}$

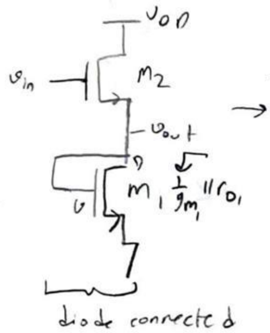
for unity gain $g_{m_1} \gg g_{m_2}$
$(\gamma = 0, \lambda = 0)$ choose $\left(\frac{W}{L}\right)_1 \gg \left(\frac{W}{L}\right)_2$

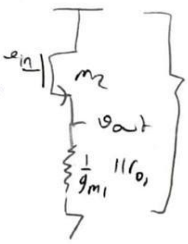
$I_{D_1} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_1 (V_{out} - V_{in} - V_{th})^2$

$I_{D_2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{th})^2$



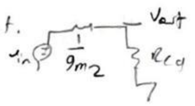
(b) Design a source follower with 2nmos.





source follower with diode connected nmos

from Thevenin ckt.



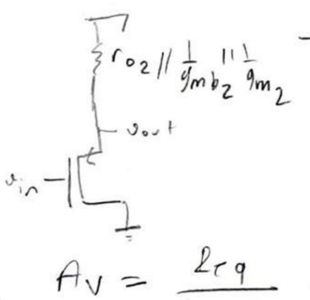
$A_V = \frac{R_{eq}}{\frac{1}{g_{m_2}} + R_{eq}}$

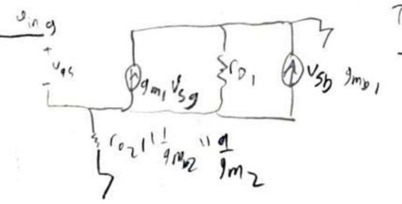
$R_{eq} = r_{o_1} \| r_{o_2} \| \frac{1}{g_{mb_2}} \| \frac{1}{g_{m_1}}$

If $\gamma, \lambda = 0$, $A_V = \frac{g_{m_2}}{g_{m_2} + g_{m_1}} \rightarrow$ same with part (a) only transistor numbers are different.

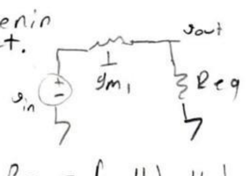
- In pMOS source follower m_1 and m_2 suffer from body effect whereas in 2 nMOS source follower only m_2 suffers from body effect.

(c) M_2 is in deep triode replace it with a resistor.





Thevenin ckt.

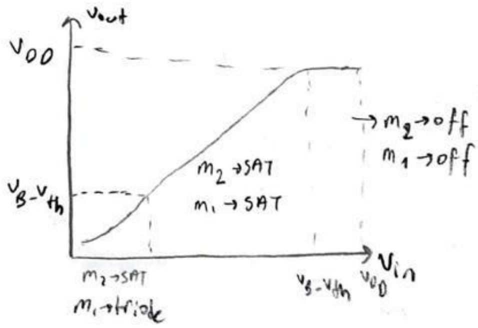


$R_{eq} = r_{o_2} \| \frac{1}{g_{mb_2}} \| \frac{1}{g_{mb_1}} \| r_{o_1} \| \frac{1}{g_{m_2}}$

$A_V = \frac{R_{eq}}{\frac{1}{g_{m_1}} + R_{eq}} \rightarrow$ if $\gamma, \lambda = 0$ $A_V = \frac{g_{m_1}}{g_{m_2} + g_{m_1}}$ holds with previous parts.

Q4)

(a) m_2 always in SAT. → $0 > -v_{th}$, $v_{in}\uparrow\ V_{gs_1}\downarrow\ I_{D_1}\downarrow \Rightarrow I_{D_2}\downarrow \Rightarrow V_{out}\uparrow$, positive gain.



$V_o - V_{in} > V_B - V_{in} - V_{th}$, $V_{DD} - V_o > V_{th}$

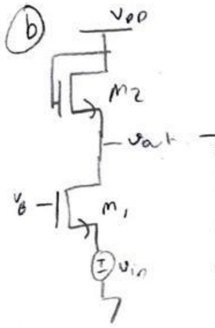
Around small v_{in}, m_1 produces large current hence V_{GS_2} should be large hence, V_{out} is small

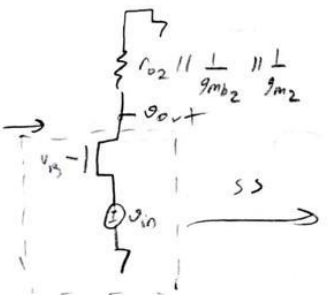
$V_o > V_B - V_{th} \Rightarrow m_1$ goes from triode to SAT

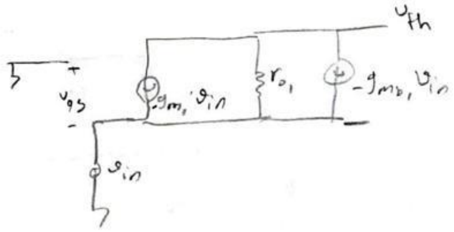
check $V_{GS1} > V_{in}$ → $V_B - V_{in} > V_{th}$, $V_B - V_{th} > V_{in}$ for m_1

for $v_{in} > v_B - V_{th}$ m_1 turns off. Also since $V_{GS_2} = 0 < V_{th}$ ($V_{out} = V_{DD}$), m_2 turns off

(b)



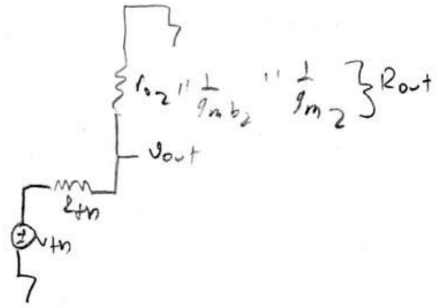




$-g_{m_1} v_{in} - g_{mb_1} v_{in} + \frac{v_{th} - v_{in}}{r_{o_1}} = 0$

$\frac{v_{th}}{r_{o_1}} = v_{in}\left(g_{m_1} + g_{mb_1} + \frac{1}{r_{o1}}\right)$, $v_{th} = v_{in}\left(r_{o_1}(g_{m_1}+g_{mb_1}) + 1\right)$

$I_x = \frac{v_x}{r_{o_1}}$, $R_{th} = r_{o_1}$

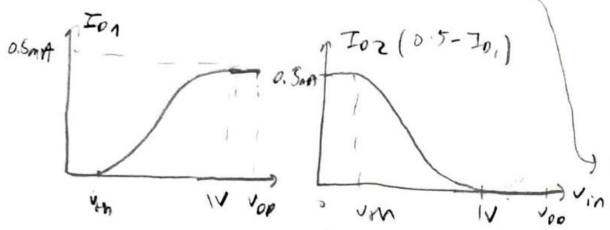


$v_{out} = v_{th} \cdot \frac{R_{out}}{R_{th} + R_{out}}$, $A_v = \left(r_{o_1}(g_{m_1} + g_{mb_1}) + 1\right) \cdot \frac{R_{out}}{R_{th} + R_{out}}$

Q5)

(a) $V_{in}=0 \Rightarrow M_1$ is off, $I_{D1}=0$, $I_{D2}=0.5mA$, $V_{out}=R_D \cdot I_{D2}=1.5V$

$\underset{0.5}{I_{D2}} = \frac{1}{2} \cdot \mu_p C_{ox}\left(\frac{W}{L}\right)_2 \cdot (V_x-1.5-0.5)^2$. Also $V_{in}\uparrow$ $I_{D1}\uparrow$ $I_{D2}\downarrow$ $V_x\downarrow$ $V_{out}\downarrow$ negative gain

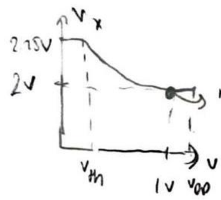


when M_2 turns off

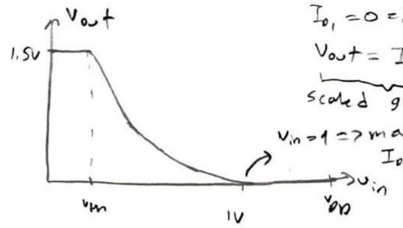
$V_x-1.5<0.5 \rightarrow V_x<2V$

$I_{D2}=\frac{1}{2}\mu_p C_{ox}\cdot\left(\frac{W}{L}\right)_2 (V_x-2)^2$, $V_x=2 \Rightarrow I_{D2}=0$ (min V_x)

$V_x=2.25V$ (max V_x value)



$\frac{1}{2}=\frac{1}{2}\cdot 0.4\cdot 10\cdot (V_{in}-0.5)^2 \rightarrow V_{in}=1V$



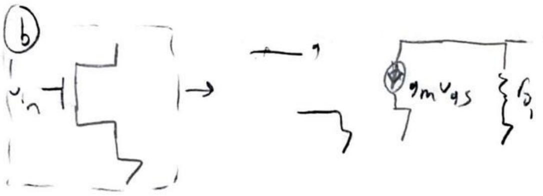
$I_{D1}=0 \Rightarrow V_{out}=0.5\cdot 3=1.5V$

$\underbrace{V_{out}=I_{D2}\cdot R_D}_{\text{scaled graph of } I_{D2}}$

$V_{in}=1 \Rightarrow$ max current (I_{D1})

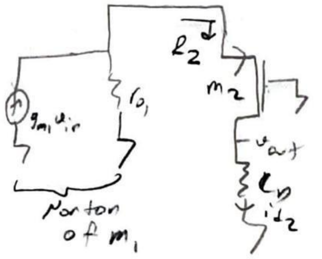
$I_{D2}=0$ hence $V_{out}=0$

(b)

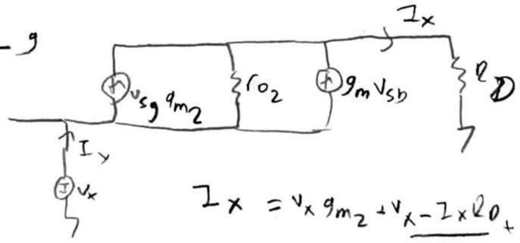


$I_N=g_{m1}V_{in}$

$R_{th}=r_{o1}$



Find R_2 →



$$I_x = V_x g_{m2} + \frac{V_x - I_x R_D}{r_{o2}} + g_{mb2}V_x = 0$$

$$R_2 = \frac{V_x}{I_x} = \left(1+\frac{R_D}{r_{o2}}\right)\cdot\left(g_{m2}+g_{mb2}+\frac{1}{r_{o2}}\right)^{-1} = \frac{r_{o2}+R_D}{1+r_{o2}(g_{m2}+g_{mb2})}$$

$i_{d2} = g_{m1}V_{in}\cdot\frac{r_{o1}}{R_2+r_{o1}}$, $V_{out}=i_{d2}\cdot R_D$, $A_V=\frac{g_{m1}r_{o1}R_D}{R_2+r_{o1}}$