# Feedback Control Systems Lab 2 Report

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## 1. Introduction

This lab work mainly focuses on magnitude and phase bode plots, input output relations of a DC motor system. The content consists of three questions in which it is desired to plot magnitude and phase bode plots of the estimated transfer function in lab 1, obtain the transfer function experimentally by applying sinusoidal inputs and, regenerating the estimated transfer function by taking delay of the hardware system into consideration. The primary objectives of this lab work are to obtaining the transfer function by applying sinusoidal inputs and comparing the three previously mentioned magnitude and phase bode plots.

## 2. Laboratory Content

In the first question it is desired to plot the magnitude and phase bode plots of the estimated transfer function in the first lab work. The estimated transfer function is as following. The bode plots are obtained by plugging the necessary coefficients to the provided code for question one.

$$G(s) = \frac{13.6}{0.0914s+1}$$

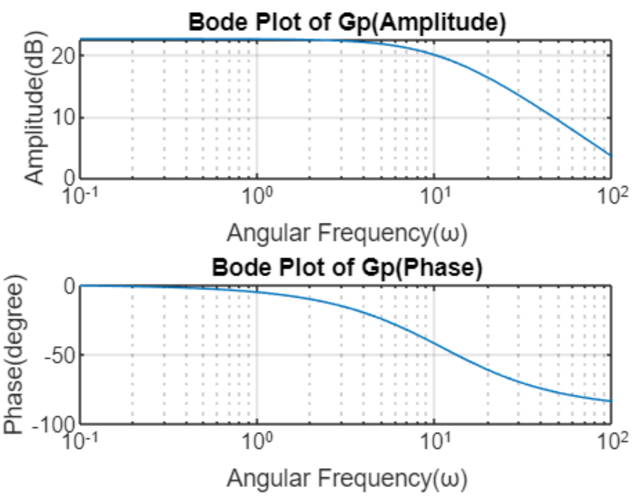


Figure 1 Bode plots of magnitude and phase of the estimated transfer function

In the second question of this lab work the aim is to obtain the transfer function by applying sinusoidal inputs with different frequencies and different durations. The durations of the inputs with different frequency values can be investigated from Table 1. By taking the FFT of the input and output for every sinusoidal input using Matlab fft command, phase and magnitude of the input and the output is obtained. After obtaining the phase and magnitude values a the magnitude and phase values for the transfer function is obtained by writing a code that utilizes the input output relations in Figure 2. Finally a magnitude and phase array is generated with seven sample points which are later plotted and compared with the plots in the first question.

| Angular Frequency | Simulation Duration (s) |
|---|---|
| **0.1** | 70 |
| **0.2** | 70 |
| **0.3** | 70 |
| **0.6** | 70 |
| **1** | 25 |
| **2** | 25 |
| **3** | 25 |
| **6** | 25 |
| **10** | 10 |
| **20** | 10 |
| **30** | 10 |
| **60** | 10 |
| **100** | 10 |

Table 1 Input angular frequency and duration

$$y(t) = A \times |G(jw)| \times \cos(wt + \angle G(jw)) \quad \text{when} \quad x(t) = A \times \cos(wt)$$

Figure 2 Input output relations in a control system

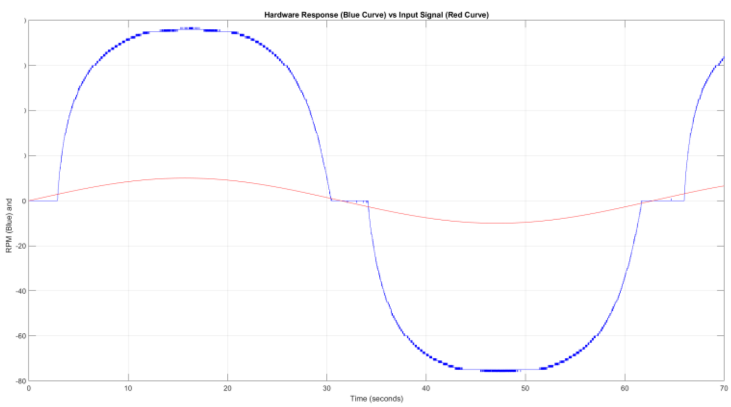


Figure 3 Input and output demonstration when angular frequency is 0.1

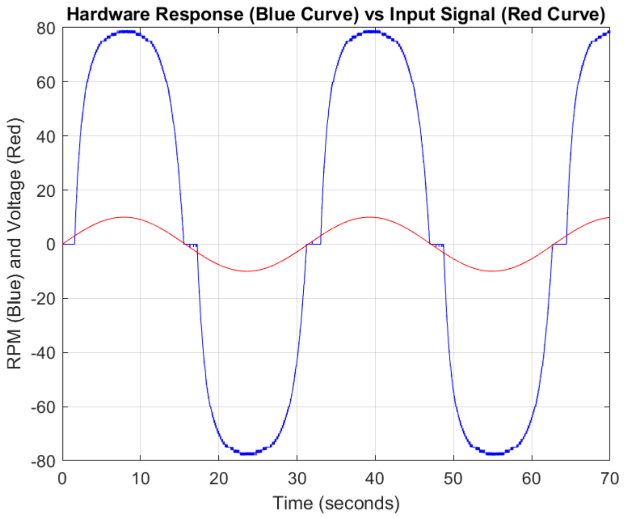


Figure 4 Input and output demonstration when angular frequency is 0.2 rad/s

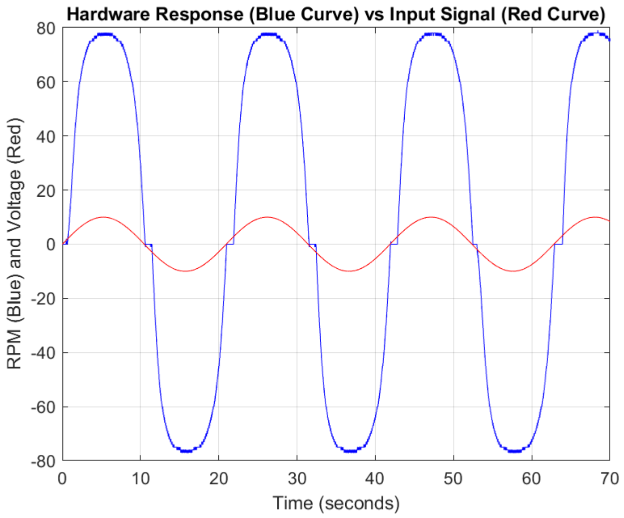


Figure 5 Input and output demonstration when angular frequency is 0.3 rad/s

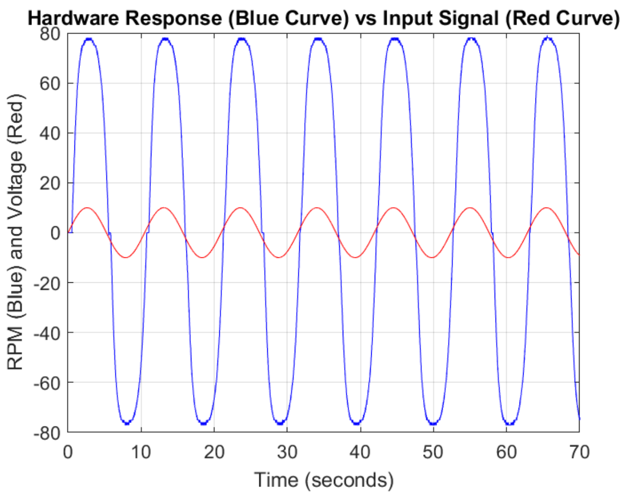


Figure 6 Input and output demonstration when angular frequency is 0.6

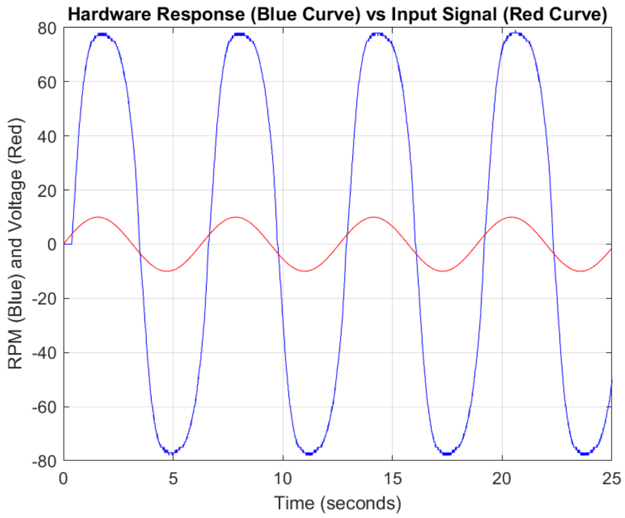


Figure 7 Input and output demonstration when angular frequency is 1 rad/s

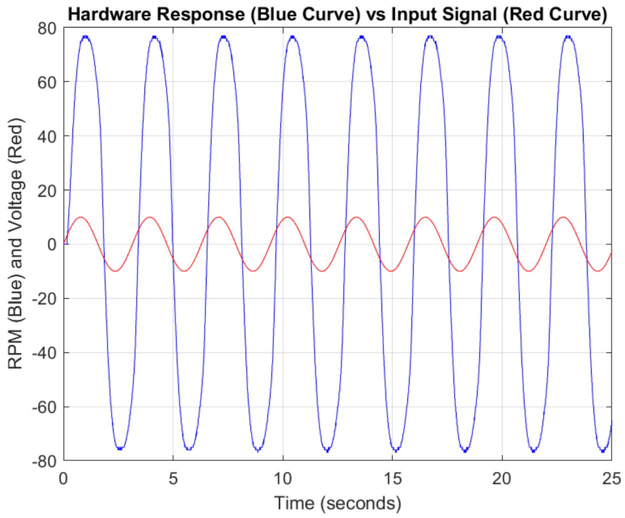


Figure 8 Input and output demonstration when angular frequency is 2 rad/s

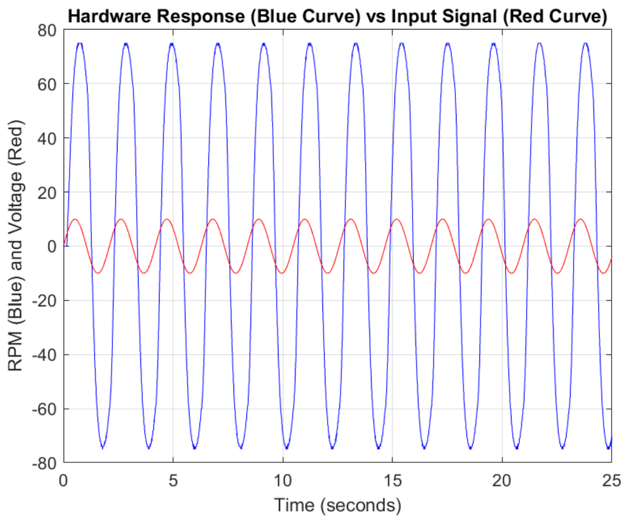


Figure 9 Input and output demonstration when angular frequency is 3 rad/s

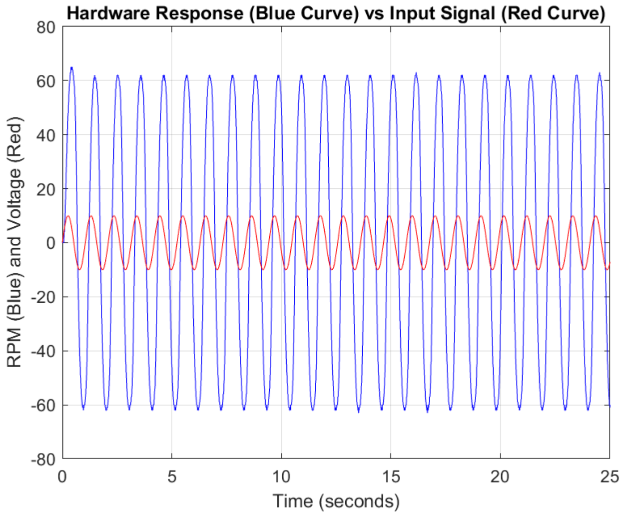

Figure 10 Input and output demonstration when angular frequency is 6 rad/s

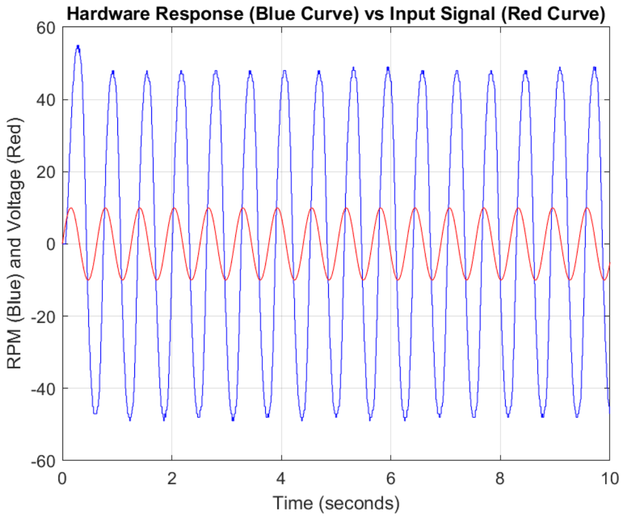

Figure 11 Input and output demonstration when angular frequency is 10 rad/s

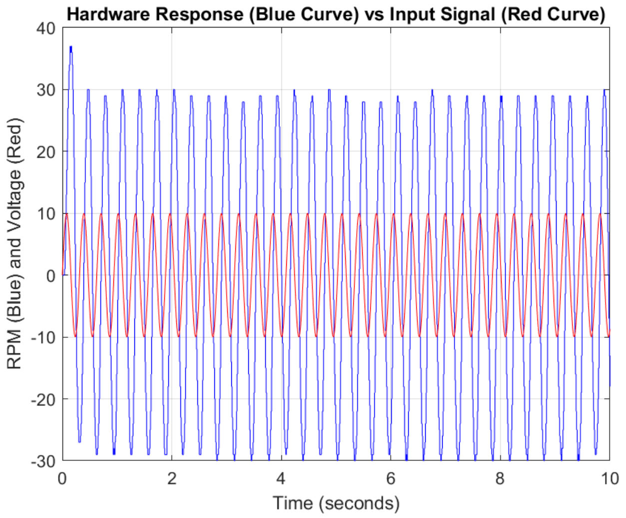

Figure 12 Input and output demonstration when angular frequency is 20 rad/s

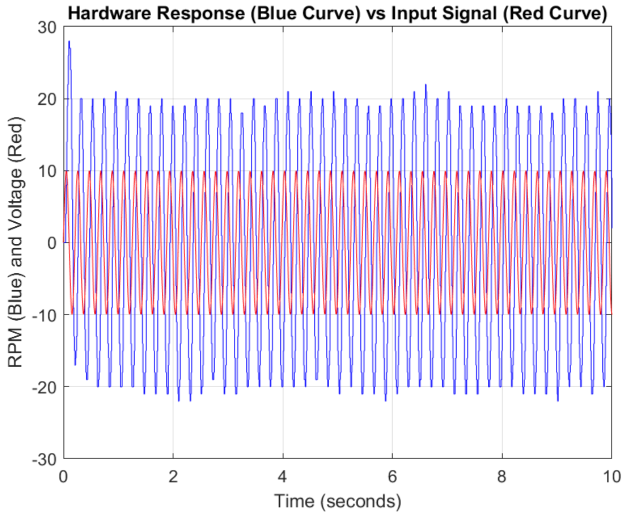

Figure 13 Input and output demonstration when angular frequency is 30 rad/s

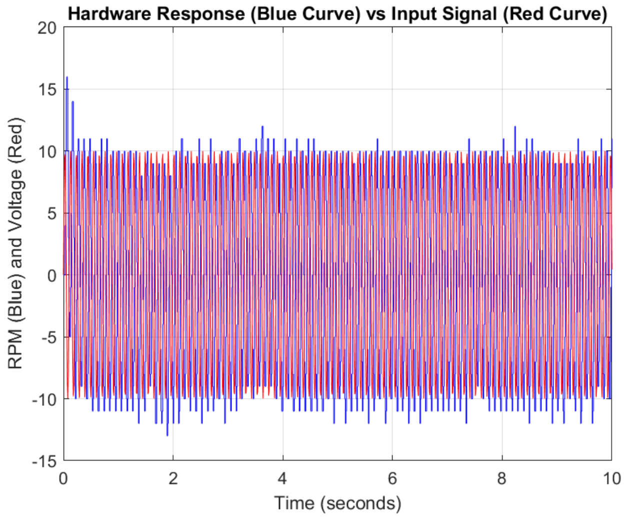

Figure 14 Input and output demonstration when angular frequency is 60 rad/s

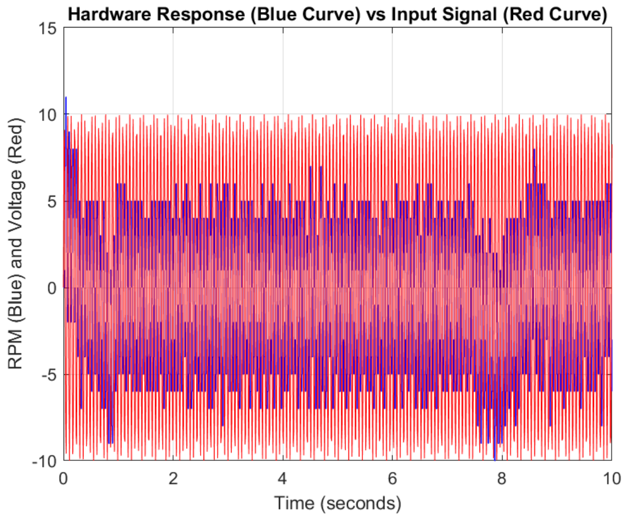

Figure 15 Input and output demonstration when angular frequency is 100 rad/s

The eight samples consist of 0.1rad/s, 0.3rad/s, 1rad/s, 3rad/s, 10rad/s, 30rad/s and 100rad/s. The seven data samples are compared with the bode plots for both phase and magnitude of the estimated transfer function G(s) in Figure 16. In Figure 16 hardware results are referred as Hardware Result.

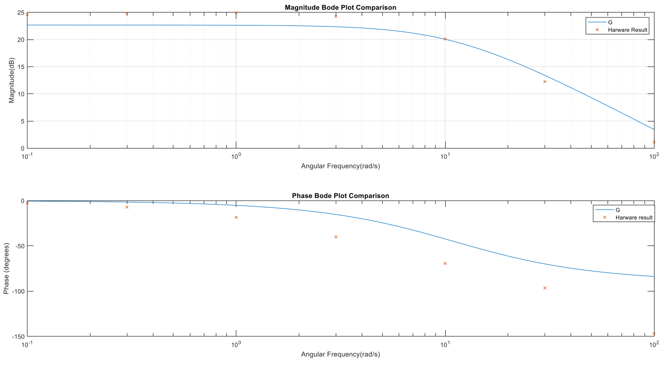


Figure 16 comparison of hardware result with the magnitude and phase bode plots

As it can be observed the hardware result is compatible with the magnitude and phase bode plots of G(s).

In question 3 it is desired to consider the delay of the harware system of 10ms by using Pade approximation. After tkaing the delay into consideration it is expected to observe increased similarity betweeen hardware result and newly approximated transfer function compared to the similarity between the initially estimated transfer function and harware result. The approximated, delayed transfer function is found as following.

$$G(s)_{delayed} = G(s)\,\frac{1-0.005s}{1+0.005s}$$

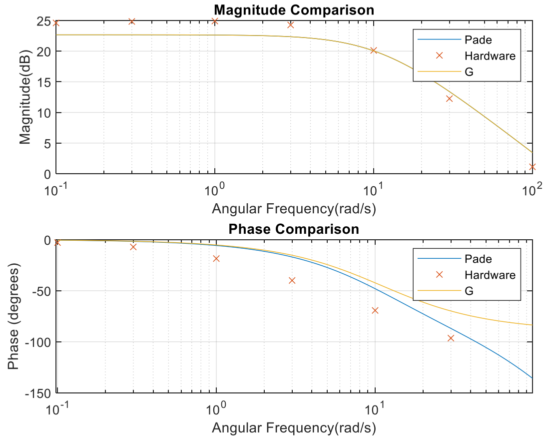


Figure 17 Comparison of magnitude and phase bode plots of hardware result, estimated transfer function and, pade approximated transfer function

In Figure 17, blue traces demonstrate the transfer function which is generated by using pade approximation whereas yellow traces demonstrate the original estimated transfer function and crosses demonstrate the hardware result. As it can be observed, similarity rate between newly generated transfer function and hardware result is higher than the similarity rate between the estimated transfer function and hardware result. Blue traces are closer to cross signs compared to yellow traces. Hence it can be concluded as pade approximation is performed successfully.

## 3. Conclusion

This lab work was important in terms of familiarizing observers with phase, magnitude bode plots and input output relations of a DC motor system. The ultimate aim of this lab work is to compare the magnitude and phase bode plots' of the initially estimated transfer function (lab1), the experimentally generated transfer function by applying sinusoidal inputs and, the last transfer function which is generated by using pade approximation. In Figure 16 it can be observed that experimentally generated transfer function has similar characteristics with the estimated transfer function which can be considered as successful. Moreover, as it can be observed form Figure 17, after delay of the hardware system is taken into consideration and pade approximation is applied to the estimated transfer function the similarity between the hardware result and the system transfer function increases as the blue traces get closer to the cross signs. Small errors may be a result of computational rounding of Matlab software and, unexpected friction that influences the hardware system.

# Appendix

**Matlab Code for Question 1:**

```
w = logspace(-1,2,100);
for k = 1:100
s = 1i * w(k);
G(k) = 13.6/ (0.091*s+1);
end
subplot(2,1,1)
semilogx(w,20*log10(abs(G)));
title('Bode Plot of Gp(Amplitude)')
ylabel('Amplitude(dB)')
xlabel('Angular Frequency(ω)')
hold on
grid on
subplot(2,1,2)
semilogx(w,angle(G)*180/pi)
title('Bode Plot of Gp(Phase)')
ylabel('Phase(degree)')
xlabel('Angular Frequency(ω)')
grid on
```

**Matlab Code for Question 2:**

```
duration = 10
angular_frequency = 100;
w = logspace(-1,2,100);
s = 1i * w;
t = 0:0.01:duration;
input = 10*sin(angular_frequency * t);
plot(out.velocity);
IN1 = fft(input);
OUT1 = fft(out.velocity.Data);
[amp1,freq1] = max(abs(OUT1));
[amp2,freq2] = max(abs(IN1));
%K1=amp1./ amp2;
%phase= angle(OUT1(freq1))-
angle(IN1(freq2));
K =
[8.5,8.74,8.81,8.1825,5.07,2.05,0.569];
Phs=[-0.05,-0.125,-0.323,-0.6978,-
1.21,4.6-2*pi,3.72-2*pi];
G = 13.6 ./ (0.091*s+1);
angular =[0.1,  0.3,  1,  3, 10,  30,
100];
subplot(2,1,1);
semilogx(w, 20 * log10(abs(G)));
hold on;
semilogx(angular, 20 *
log10(K(1:7)*2),'x');
title('Magnitude Bode Plot Comparison');
xlabel('Angular Frequency(rad/s)');
ylabel('Magnitude(dB)');
legend('G ','Hardware Reslt');
grid on;
hold off;
subplot(2,1,2);
semilogx(w, angle(G) * 180 / pi);
hold on;
semilogx(angular, Phs(1:7) * 180 /
pi,'x');
title('Phase Bode Plot Comparison');
xlabel(' Angular Frequency(rad/s)');
ylabel(['Phase(degrees)']);
legend('G ','Hardware Result');
grid on;
```

**Matlab Code for Question 3:**

```
w = logspace(-1,2,100); i = 1;
s = 1i * w;
G = 13.6 ./ (0.091*s + 1);
G_p = (1-0.005*s)./(1+0.005*s);
G_d = G_p.*G
K =
[8.5,8.74,8.81,8.1825,5.07,2.05,0.569];
Phs=[-0.05,-0.125,-0.323,-0.6978,-
1.21,4.6-2*pi,3.72-2*pi]
angular =[0.1,  0.3,  1,  3, 10,  30,
100];
subplot(2,1,1);
semilogx(w, 20 * log10(abs(G_d)));
hold on;
semilogx(angular, 20 *
log10(K(1:7)*2),'x');
title('Magnitude Bode Plot');
xlabel(' Angular Frequency(rad/s)');
ylabel('Magnitude(dB)');
grid on;
semilogx(w, 20 * log10(abs(G)));
hold off;
subplot(2,1,2);
semilogx(w, angle(G_d) * 180 / pi);
hold on;
semilogx(angular, Phs(1:7) * 180 /
pi,'x');
title('Phase Bode Plot');
xlabel('Angular Frequency(rad/s)');
ylabel(['Phase(Degrees)']);
semilogx(w, angle(G) * 180 / pi);
grid on;
```