Eray Gündoğdu 22101962

Signals and Systems Lab 6

Part 1

Throughout the lab the following LCCDE is primarily investigated and applications are done accordingly.

$$y[n] = \sum_{l=1}^{N} a[l]y[n-l] + \sum_{k=0}^{M} b[k]x[n-k] \tag{1}$$

① $y[n] = a[1].y[n-1] + a[2].y[n-2] + a[3].y[n-3] ... + b[0].x[n] + b[1].x[n-1] ...$

$x[n] = y[n]$ if $n<0$, if $n=0$, $y[0] = a[1]\underbrace{y[-1]}_{0} + a[2].\underbrace{y[-2]}_{0} ... + \underbrace{b[0]x[0] + ...}_{0}$

$y[0] = b[0]\,x[0]$

if $n=1$, $y[1] = a[1].y[0] + \underbrace{....}_{0} + b[0].x[1] + b[1].x[0] + \underbrace{....}_{0}$

$y[1] = a[1].y[0] + b[0].x[1] + b[1].x[0]$

It can be concluded that

$y[n]\Big|_0 = \sum_{k=0}^{0} b[k].x[n]\Big|$

$y[n]\Big|_1 = \sum_{l=1}^{1} a[l].\underbrace{\sum_{k=0}^{0} b[k].x[n]\Big|_0}_{y[0]} + \sum_{k=0}^{1} b[k].x[n-k]\Big|_1$

② $Y(z).\sum_{q=0}^{Q} c_d[q]z^{-q} = X(z)\sum_{p=0}^{P} c_n[p].z^{-p}$, for some c

$x[n-c] \xrightarrow{Z} X(z).z^{-c}$. By taking this property as refference from Eq. 1 it can be said that,

$\sum_{p=0}^{P} c_n[p].X(z).z^{-p} \xrightarrow{\text{Inverse Z}} \sum_{p=0}^{P} c_n[p].x[n-p]$ hence,

$P = M$, $c_n[p] = b[k]$, $Y(z).z^{-0}.c_d[0] = -\sum_{q=1}^{Q} c_d[q].z^{-q}.Y(z) + \underbrace{...}_{x \text{ terms}}$

(X terms are not written to the previous eq)

Apply Inverse Z transforms, $y[n].c_d[0] = -\sum_{q=1}^{Q} c_d[q].y[n-q]$

From Eq. 1, $q = l$, $c_d[q] = -a[l]$, $Q = N$, where $q \geq 1$ and $l \geq 1$

$c_d[0] = 1$.

The desired DTLTI function that is responsible for implementing Eq.1 to MATLAB is as following.

```
function [y]=DTLTI(a,b,x,Ny)
N = length(a);
M = length(b) - 1;
y = zeros(1, Ny);


    for n = 0:Ny-1
        for l = 1:N
            if (n+1-l) < 1
                y(n+1) = y(n+1);
            elseif (n+1-l) >= 1
                y(n+1) = a(l)*y(n+1-l) + y(n+1);
            end
        end


        for k = 0:M
            if (n+1-k) < 1
                 y(n+1) = y(n+1);
            elseif (n+1-k) >= 1
                y(n+1) = b(k+1)*x(n+1-k) + y(n+1);
            end
        end
    end
end
```

Part 2

a)

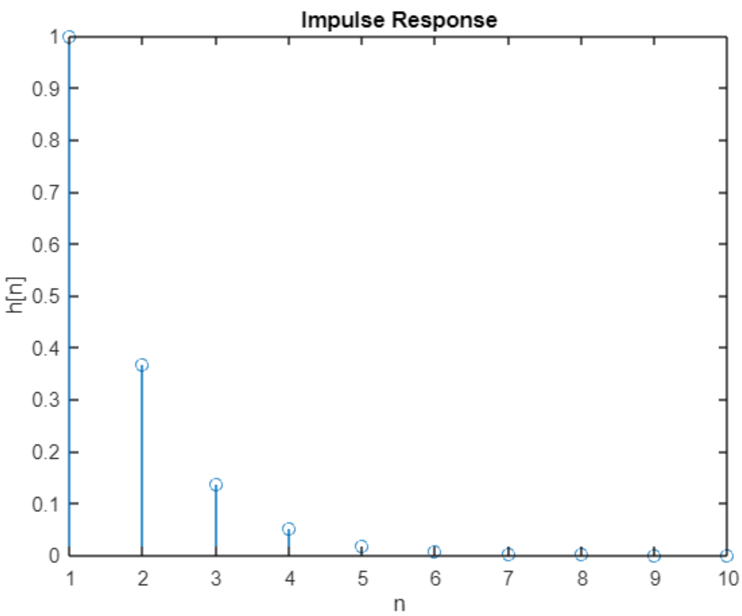


Figure 1 impulse response vs n

The necessary code for displaying impulse response against n:

```
D=22101962;
D4=mod(D,4);
M=5+D4;
k = [0:M];
b=exp(-k);
x = zeros(1,10);
x(1) = 1;
N=10;
a=zeros(1,10);
Ny=length(x);
h=DTLTI(a,b,x,Ny);
stem(h);
title('Impulse Response');ylabel('h[n]');xlabel('n');
```

b) In order to calculate impulse response x[n] = δ[n] and ,a[l]=0 for this part, hence the output becomes shifted impulses multiplied with b[k]. It can be interpreted that the pattern of the impulse response is the same with b[k]= e^{-k}.

c)This Impulse Response have finite lenght which is dependent on value of M meaning that size of impulse response is M+1 since the summation starts from zero.In addition this situation can be observed in Figure 1, h[n]=0 when n=9 and n=10 where M=7.

d)

$$Y(z) = \sum_{k=0}^{M} b[k] . X(z) . z^{-k}, \quad \frac{Y(z)}{X(z)} = \sum_{k=0}^{M} b[k] . z^{-k} = H(z)$$

take $z = r . e^{j\Omega}$ where $r=1$,

$$\sum_{k=0}^{M-1} (e . z)^{-k} = \frac{1-(ez)^{-M}}{1-(e . z)^{-1}}, \text{ substitute } z = e^{j\Omega}, \quad H(e^{j\Omega}) = \frac{1-e^{-M(j\Omega+1)}}{1-e^{-1-j\Omega}}$$

e)

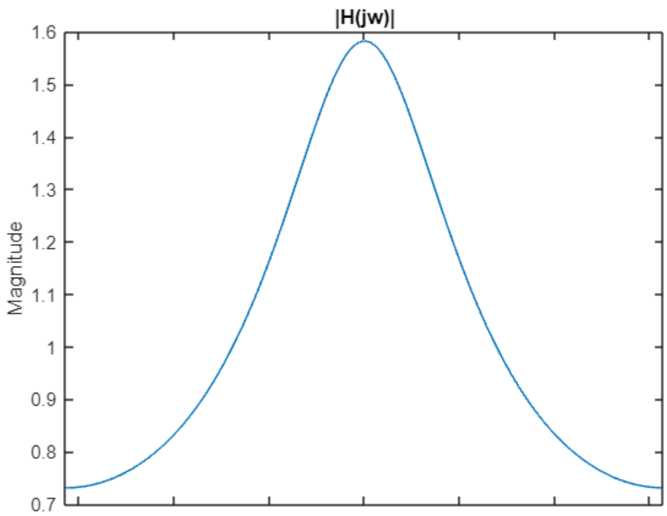


Figure 2 magnitude vs frequency of the impulse

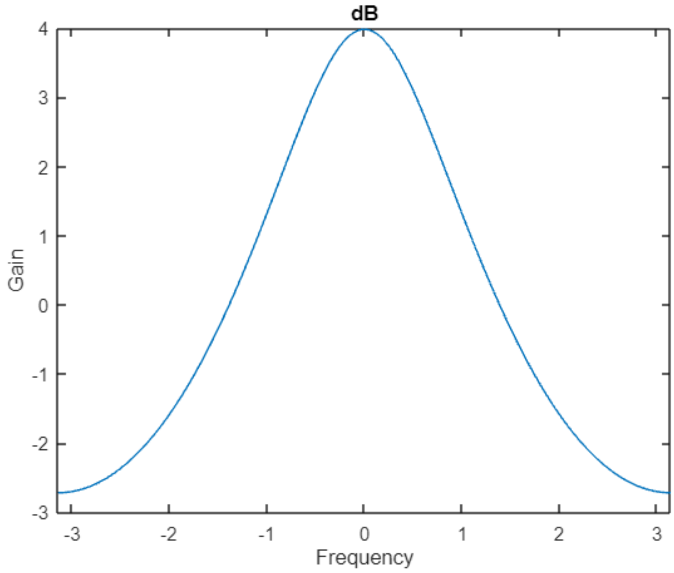


Figure 3 gain vs frequency(bode plot)

This is a LPF. 3db bandwidth indicates the frequency range which consists of frequencies that correspond to values between 3 db less than peak gain. The code for the gain and magnitude is accordingly.

```
w=linspace(-pi,pi,length(hFT));
hFT=(1-exp(M*(-j*w-1)))./(1-exp(-j*w-1));

plot (w,abs(hFT));

title('|H(jw)|');ylabel('Magnitude');xlabel('Frequency');
xlim([-pi,pi]);
gain=20*log10(abs(hFT));
max(gain)
w(457)/3.1416
w(945)/3.1416
plot (w,gain);
title('dB');ylabel('Gain');xlabel('Frequency');
xlim([-pi,pi]);
```

As it can be observed from the code, initially the max gain is computed later on by indexing the frequency matrix the low cutoff and high cutoff frequency is computed.
ans = 3.9761 is the peak point in dB.
ans = -0.3486π is the low cutoff frequency.
ans = 0.3486π is the high cut off frequency.
Hence 3db bandwidth is 0.3486π*2=0.6972 π. It should be highlighted that in Figure 2.3 x axis is the numerical expression of π, In the code the indexed frequency values are divided by numerical expression of π hence the values can be calculated in term of π.

f) Now chirp signal applications will be done using DTLTI in order to determine the frequency response of the system. The MATLAB code is as following.

```
b=exp(-k);
N=10;
a=zeros(1,10);
Ts=1/1400;
t=0:1/1400:1;
f=350*t.^2;
x=cos(2*pi*f);
y=DTLTI(a,b,x,length(x));
w=linspace(0,pi,length(y));
figure;
subplot(3,1,1);
plot(w,abs(y));title('0<t<1');xlabel('Frequency');ylabel('Magnitude');
xlim([0,pi]);
t1=0:1/1400:10;
f1=35*t1.^2;
x1=cos(2*pi*f1);
y1=DTLTI(a,b,x1,length(x1));
w1=linspace(0,pi,length(y1));
subplot(3,1,2);
plot(w1,abs(y1));title('0<t<10');xlabel('Frequency');ylabel('Magnitude');
xlim([0,pi]);
t2=0:1/1400:1000;
f2=0.35*t2.^2;
x2=cos(2*pi*f2);
y2=DTLTI(a,b,x2,length(x2));
w2=linspace(0,pi,length(y2));
subplot(3,1,3);
plot(w2,abs(y2));title('0<t<1000');xlabel('Frequency');ylabel('Magnitude');
xlim([0,pi]);
```

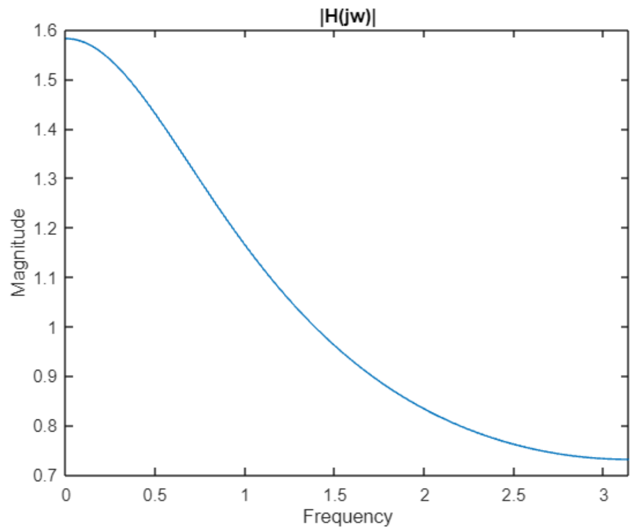


Figure 4 limited x-axis magnitude vs frequency plot from [0, π].

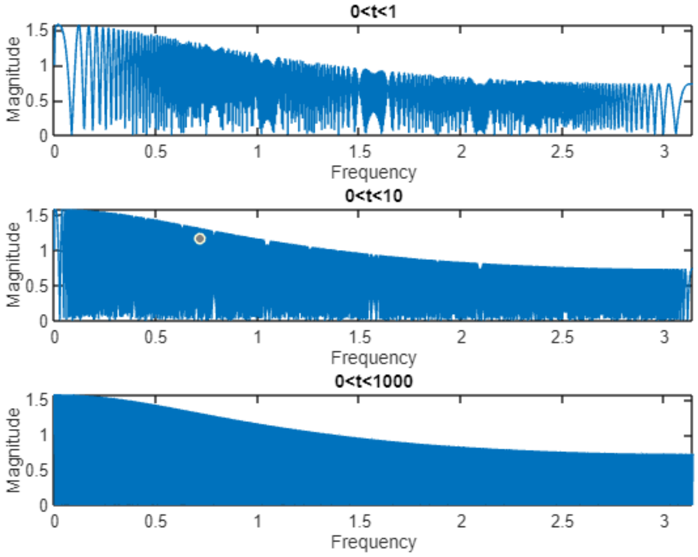


Figure 5 reconstructed freqency responses by using chirp signal with different t

The reconstructed frequency responses have the almost exact behaviour with Figure 4 which indicates that frequency response of the system is obtained by using chirp signal. In other Word general trend is significantly similar to Figure 4. As t increases te amount of sample increases hence the general trend can be observed more accurately with increasing t due to increase in samples.

Part 3

z_1, p_1, p_2 values will be calculated using MATLAB

(a) $$H(z) = \frac{z - z_1}{z^2 - z(p_1+p_2) + p_1 p_2}$$

(b) $$H(z) = \frac{z^{-1} - z^{-2} \cdot z_1}{1 - z^{-1}(p_1+p_2) + p_1 p_2 \cdot z^{-2}}, \quad Y(z) - z^{-1} Y(z)(p_1+p_2) + Y(z) p_1 p_2 z^{-2} = X(z) z^{-1} - z_1 X(z) \cdot z^{-2}$$

Apply Inverse z transform, $y[n] = (p_1+p_2) y[n-1] - p_1 p_2 y[n-2] + x[n-1] - z_1 x[n-2]$

$$y[n] = \sum_{l=1}^{N=2} a[l] \cdot y[n-l] + \sum_{k=0}^{M=2} b[k] \cdot x[n-k]$$

where $N=2$, $M=2$, $a[1] = p_1+p_2$, $a[2] = -p_1 p_2$, $b[0] = 0$, $b[1] = 1$

$b[2] = -z_1$

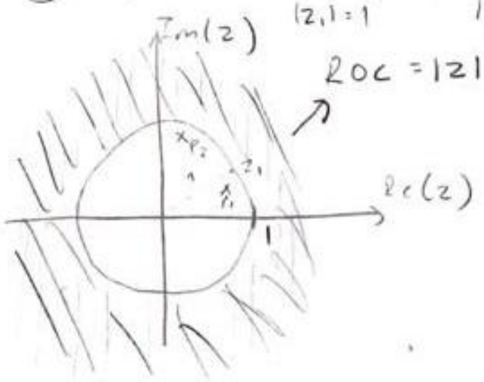
(c) $x[n] = \delta[n]$, $h[n] = (p_1+p_2) h[n-1] - p_1 p_2 h[n-2] + \delta[n-1] - z_1 \delta[n-2]$

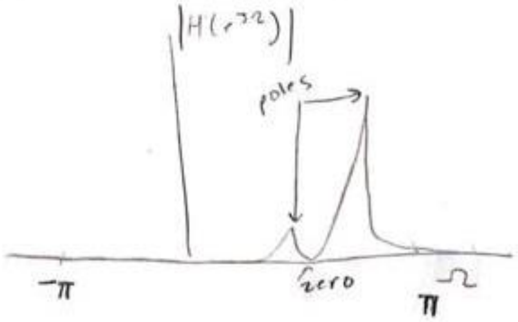
$y[n] = h[n] = h[n] * \delta[n]$

(d) $z_1 = 0.8 + 0.6j$, $p_1 = 0.7845 + 0.5883j$, $p_2 = 0.3405 + 0.9364j$

$|z_1| = 1$ $\quad |p_1| < 1$ $\quad |p_2| < 1$

ROC $= |z| > 1$





(e) $|H(z)| = \left|\sum_{n=-\infty}^{\infty} h[n] \cdot z^{-n}\right| = \sum_{n=-\infty}^{\infty} |h[n] z^{-n}|$ converges for $|z| > 1$ hence the system is not stable.

(f) The system is FIR since M and N are $\in \mathbb{Z}$ and does not approach to infinity.

(g) $z = r \cdot e^{j\Omega}$ where $r = 1$ hence $H(e^{j\Omega}) = \dfrac{e^{j\Omega} - z_1}{e^{2j\Omega} - e^{j\Omega}(p_1 + p_2) + p_1 p_2}$

h)

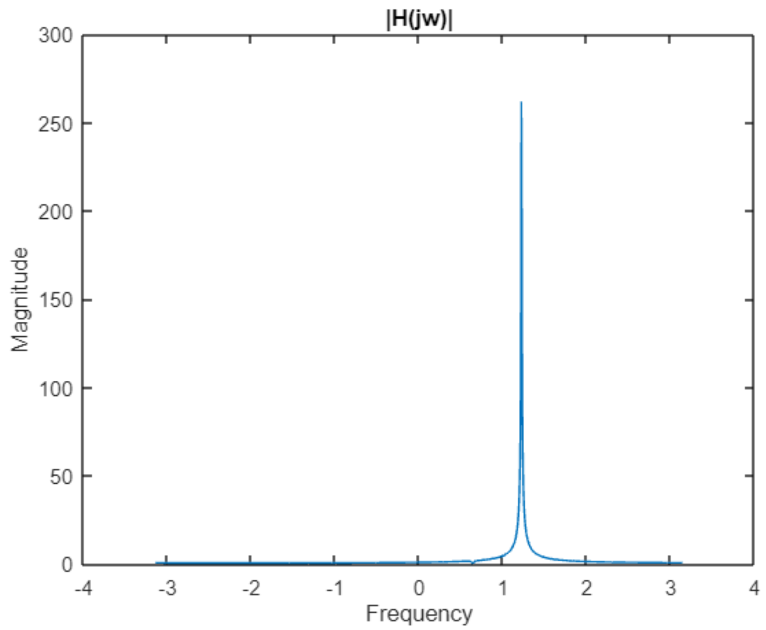


Figure 6 the magnitude vs frequency plot of the frequency response(Band Pass Filter)

It should be highlighted that according to the given instructions the z transform has 2 poles and one zero. Since these poles and zeros are determined according to my ID number, first polse and the zero are remarkably close to each other. Also this situation can be observed from the zero-pole plot, hence the magnitude graph of the frequency respone is likely to have one pole.

Again chirp signal application will be used, the necessary code for this part is as following.

```
j=1i;
ni = [ 2 2 1 0 1 9 6 2 ];
ni = ni +2;
z1 = (ni(2)+1i*ni(3))/((ni(2)^2+ni(3)^2)^(1/2));
p1 = (ni(1)+1i*ni(5))/((1 + ni(1)^2+ni(5)^2)^(1/2));
p2 = (ni(8)+1i*ni(6))/((1 + ni(8)^2+ni(6)^2)^(1/2));
hFT=(exp(j*w)-z1)./((exp(j*w)-p1).*(exp(j*w)-p2));
w0=linspace(-pi,pi,length(hFT));
figure;
%subplot(4,1,1);
plot(w0,abs(hFT));title('|H(jw)|');ylabel('Magnitude');xlabel('Frequency');
a=[(p1+p2) , (-p1*p2)];
b=[0, 1 ,-z1];
Ts=1/1400;
t=0:Ts:1;
f=700*t.^2-700*t-700;
x=exp(j*2*pi*f);
y=DTLTI(a,b,x,length(x));
w=linspace(-pi,pi,length(y));
subplot(4,1,2);
plot(w,abs(y));title('0<t<1');xlabel('Frequency');ylabel('Magnitude');
xlim([-pi,pi]);
t1=0:Ts:10;
f1=70*t1.^2-700*t1-700;
x1=exp(j*2*pi*f1);
y1=DTLTI(a,b,x1,length(x1));
w1=linspace(-pi,pi,length(y1));
subplot(4,1,3);
plot(w1,abs(y1));title('0<t<10');xlabel('Frequency');ylabel('Magnitude');
xlim([-pi,pi]);
t2=0:Ts:1000;
f2=0.7*t2.^2-700*t2-700;
x2=exp(j*2*pi*f2);
y2=DTLTI(a,b,x2,length(x2));
w2=linspace(-pi,pi,length(y2));
subplot(4,1,4);
plot(w2,abs(y2));title('0<t<1000');xlabel('Frequency');ylabel('Magnitude');
xlim([-pi,pi]);

plot(w0,angle(y));
title('0<t<1');xlabel('Frequency');ylabel('Phase');
xlim([-pi,pi]);
```

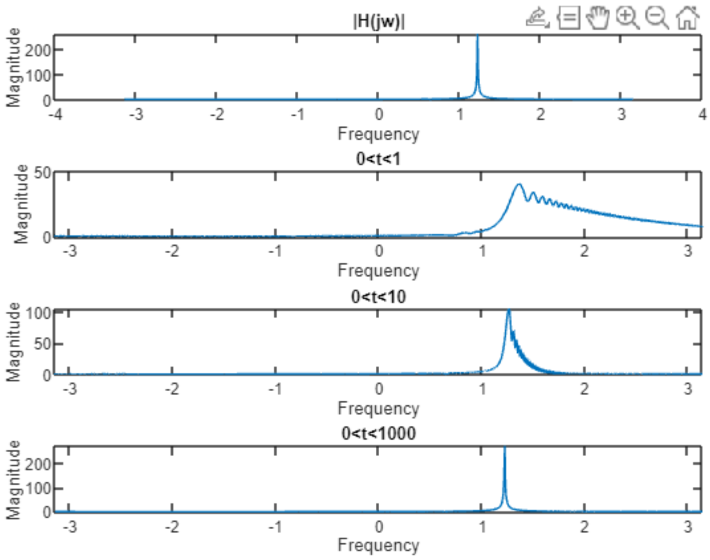


Figure 7 output of the chirp signal application with 3 different t intervals

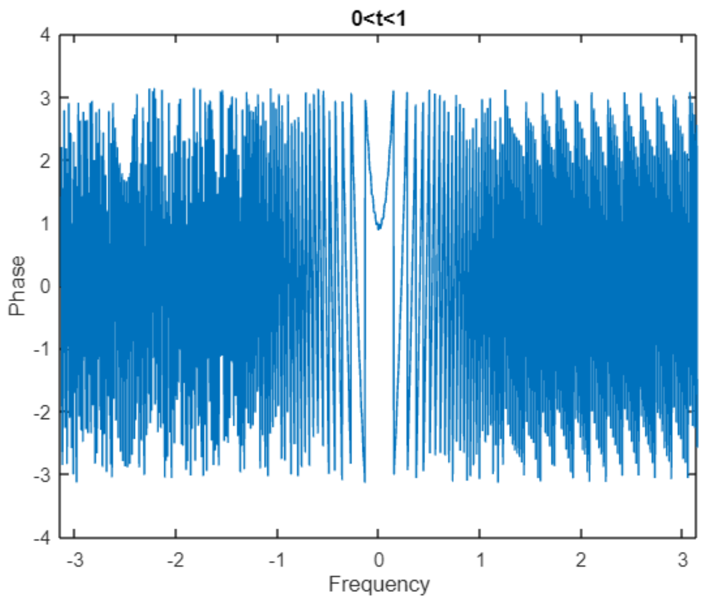


Figure 8 pahse of the output when 0<t<1

From figure 7 it can be observed that chirp signal application is successful since the frequency response can be observed by reconstructing it using chirp signal. As t interval increases amount of samples increase hence behaviour of the frequency response can be observed better with higher amount of samples. In addition it is desired to answer the question waht should be sampling frequency if chirp signal sweeps from -600Hz to 800 Hz. According Nyquist-Shannon criteria,

$$\omega_s > 2\omega_{max}$$

$$f_s > \Delta f_{max}$$

So sampling frequency should be larger than 1600Hz.

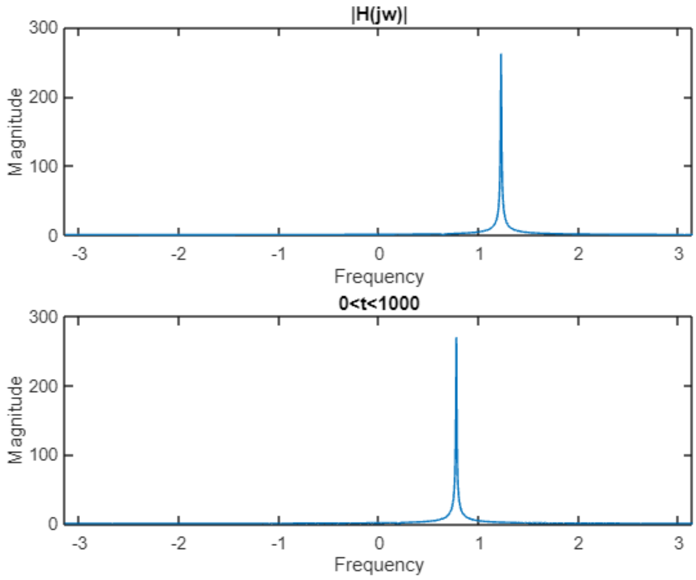


Figure 9 reconstruction using chirp signal with fs=1400Hz

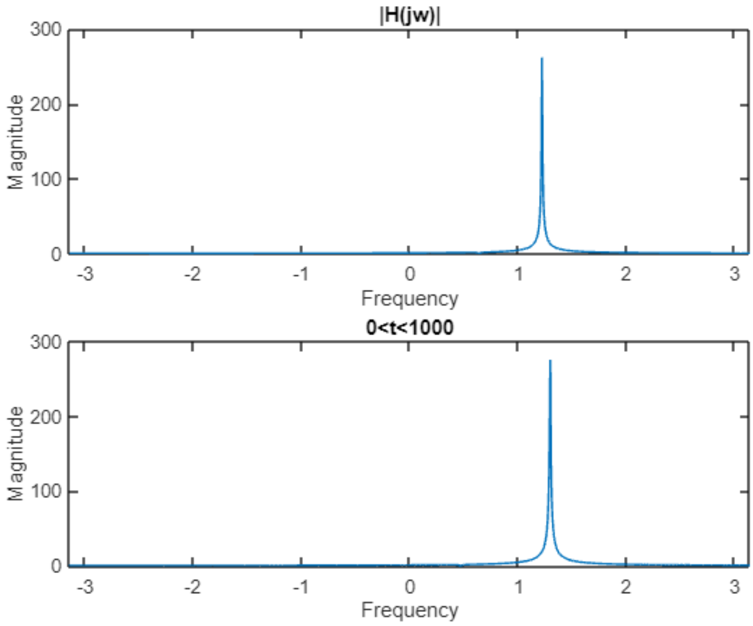


Figure 10 reconstruction using chirp signal with fs=2000Hz