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Signals and Systems Lab Report 2

Part 1

Aim of this part is to plot real, imaginary, magnitude and phase versus t graphs of the given signal:

$$x_s(t) = \sum_{i=1}^{M} A_i e^{j\omega_i t}$$

From observing the sigal it can be interpreted that A and ω arrays are 1XM matrixes however in the following instructions it is specified that the dimensions of amplitude and real frequency arrays should be 1xn where n is mod(22101962,41)=10. The matlab code for calculating $x_s(t)$ is as following:

```
function [xs] = SUMCS(t, A, omega)


    M = length(A);
    xs = zeros(size(t));


    for i = 1:M
        xs = xs + A(i) * exp(1j * omega(i) * t);
    end
end
```

After establishing a function, amplitude and real frequency arrays with 1xn dimensions are generated by using rand functions. As the final step code for desired 4 plots (real-t, imaginary-t, magnitude-t, phase-t) is written. The matlab code and output plots can be observed in the following figures.

```
t=[0:0.001:1];
n=mod(22101962,41);
A_real=3*rand(1,n);
A_imaginary=3*rand(1,n);
A= A_real + j*A_imaginary
n=length(A);
omega=pi*rand(1,n)


xs = SUMCS(t, A, omega);
real_XS = real(xs);
imaginary_XS = imag(xs);
amp = abs(xs);
phase = angle(xs);


plot(t, real_XS);
xlabel('Time (s)');
ylabel('Real Part');
title('Real Part of xs(t)');
```

```
plot(t, imaginary_XS);
xlabel('Time (s)');
ylabel('Imaginary Part');
title('Imaginary Part of xs(t)');

plot(t, amp);
xlabel('Time (s)');
ylabel('Magnitude');
title('Magnitude of xs(t)');

plot(t, phase);
xlabel('Time (s)');
ylabel('Phase (radians)');
title('Phase of xs(t)');
```

```
A = 1×10 complex
    0.8978 + 1.9343i   1.6836 + 1.1288i   2.6456 + 0.5728i   2.0075 + 1.2848i   0.5713 + 1.4461i ...

omega = 1×10
    0.7911    0.9124    1.9386    0.8334    2.5809    3.0871    2.2041    1.0883    1.8340    0.3386
```

Fig1.1 randomly generated A and ω arrays

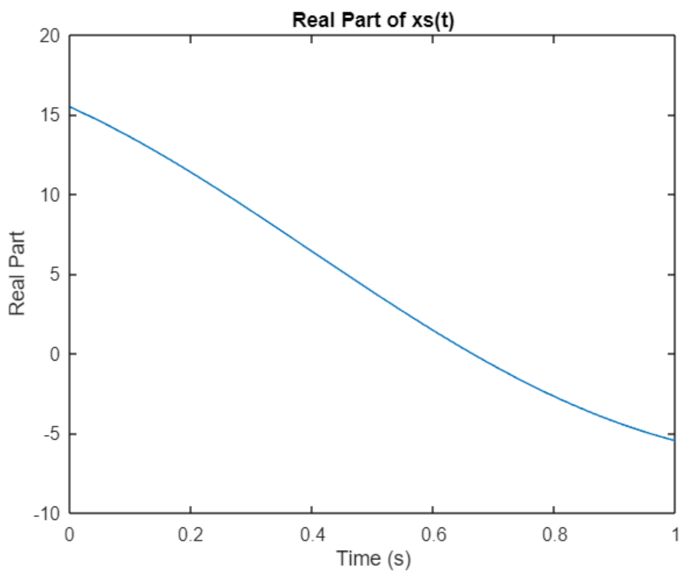

Fig1.2 real part of $x_s(t)$ vs t

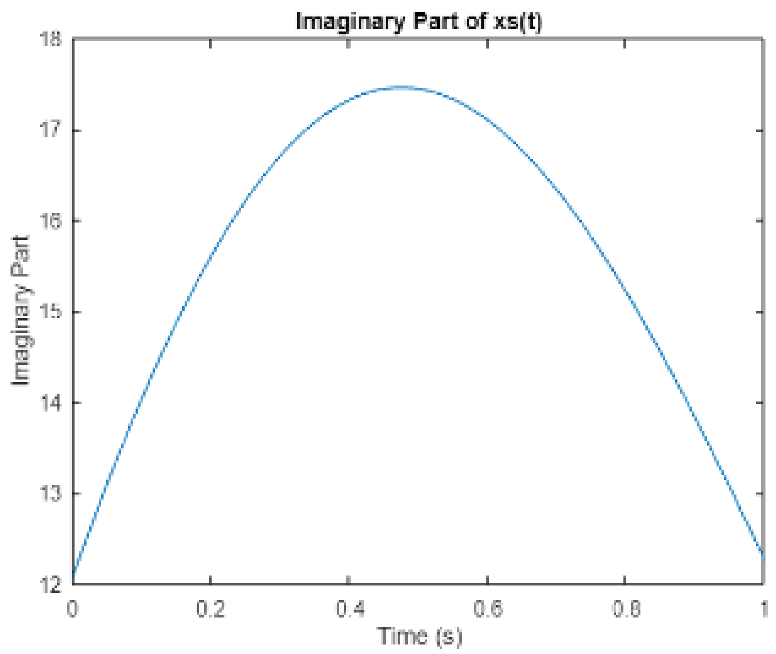

Fig1.3 imaginary part of $x_s(t)$ vs t

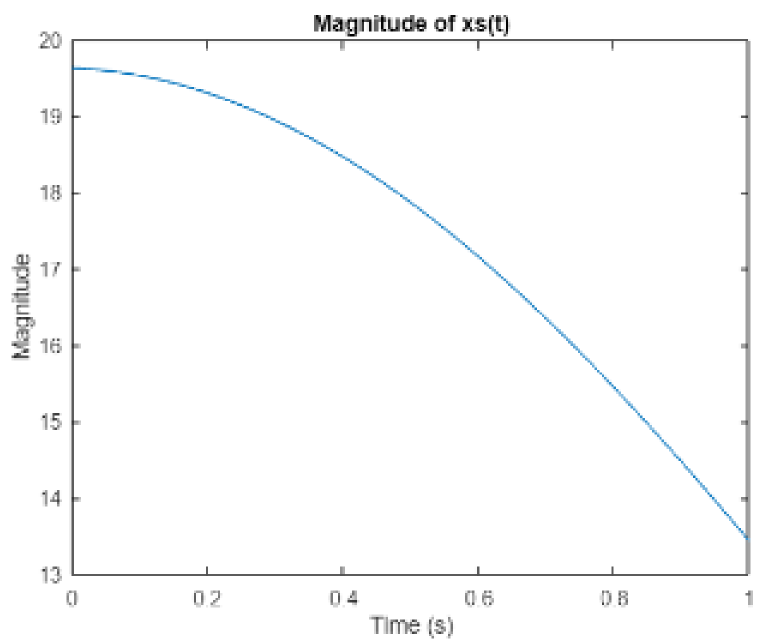

Fig 1.4 magnitude of $x_s(t)$ vs t

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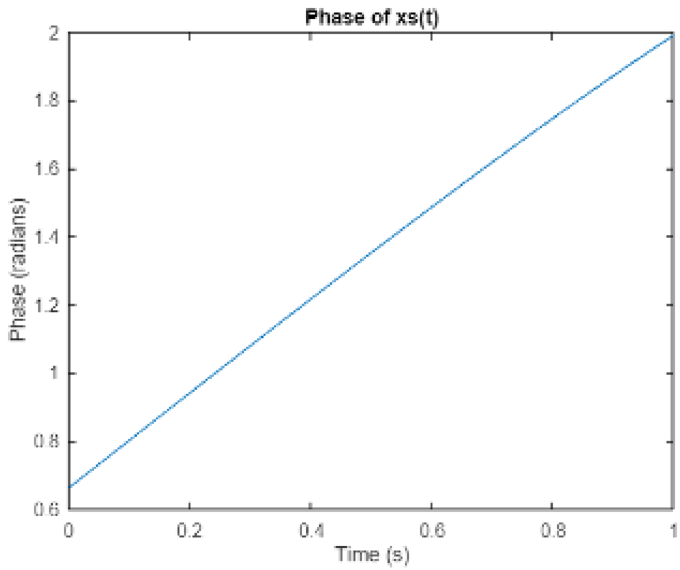

Fig1.5 phase of $x_s(t)$ vs t

Part 2

It is desired to sketch the graph of signal $x(t) = \begin{cases} 1 - 2t^2, \frac{-W}{2} < t < \frac{W}{2} \\ 0 \quad , otherwise \end{cases}$ where $t \in [-T/2, T/2)$

While sketching the graph W = 1.0 and time interval is -1.5T < T < 1.5T where T=2.

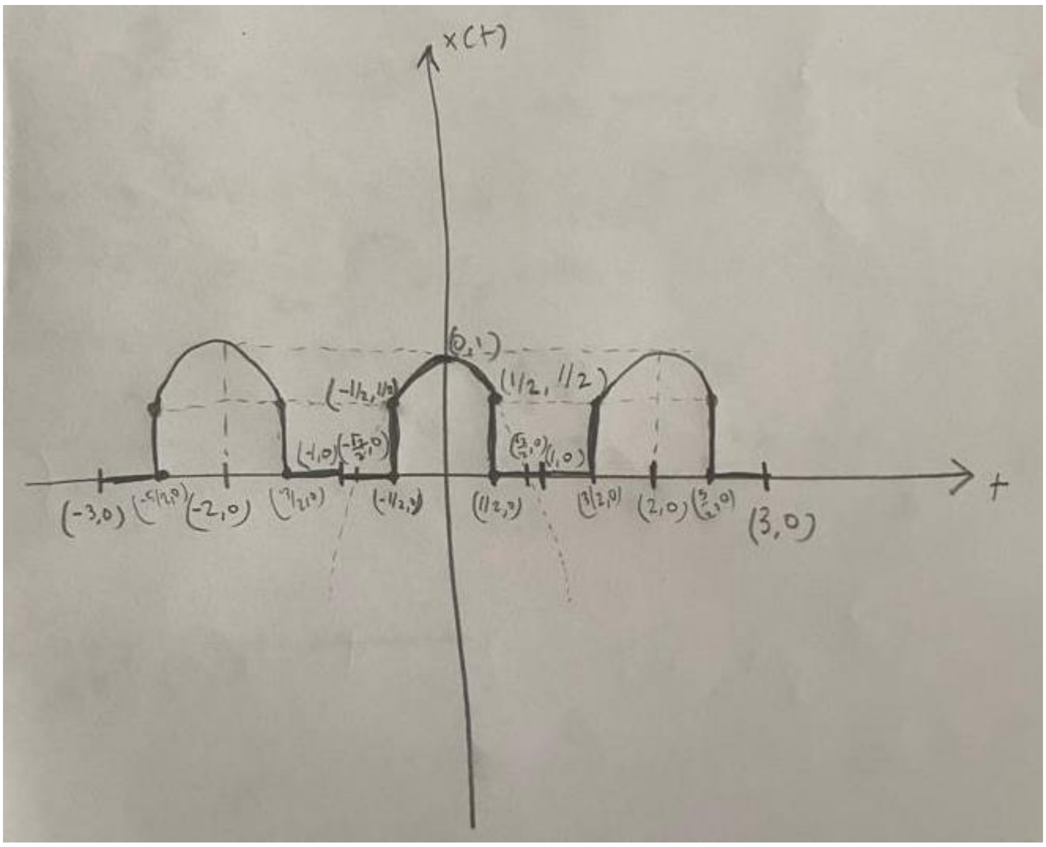

Fig2.1 desired sketch of $x(t)$

Now it is desired to determine X_k by hand. The calculations can be observed in the following calculations.

$$X_k = \frac{1}{T} \cdot \int_{-T/2}^{T/2} x(t) \cdot e^{-j\frac{2\pi k t}{T}} \cdot dt .$$

$$X_k = \frac{1}{T} \int_{-T/2}^{-w/2} x(t) \cdot e^{\frac{-j2\pi k t}{T}} \cdot dt + \frac{1}{T} \int_{-w/2}^{w/2} x(t) \cdot e^{\frac{-j2\pi k t}{T}} \cdot dt + \int_{w/2}^{T/2} x(t) \cdot e^{-\frac{j2\pi k t}{T}} dt$$

if $-T/2 \leq t \leq -w/2$ and $w/2 \leq t < T/2$, $x(t) = 0$

therefore $X_k = \frac{1}{T} \int_{-w/2}^{w/2} x(t) \cdot e^{-\frac{j2\pi k t}{T}} \cdot dt . = \frac{1}{T} \int_{-w/2} (1-2t^2) \cdot e^{-\frac{j2\pi k t}{T}} dt$

$\frac{1}{T} \left(\frac{-T}{j2\pi k} \cdot e^{\frac{-j2\pi k t}{T}} \Big|_{-w/2}^{w/2} \right) \longrightarrow$ first integral

Substituting $T=2$ and $w=1$, and dividing integral into 2 parts ($1 \cdot exp$ and $-2t^2 \cdot exp$). After substituting values to the first integral mentioned above

$\swarrow$ result of 1st integral

$$\frac{e^{j \cdot 0.5\pi k} - e^{-j0.5\pi k}}{2j\pi k} = \boxed{\frac{\sin\left(\frac{\pi k}{2}\right)}{\pi k}}$$

Substituting w, T values to second integral $\rightarrow -\frac{1}{2} \int_{-0.5}^{0.5} t^2 \cdot e^{-j\pi k t} dt$

Applying integration by parts

$u = t^2$, $du = 2t \cdot dt$, $dv = e^{-\pi k t} \cdot dt$, $v = \frac{e^{-j\pi k t}}{-j\pi k}$

$\downarrow$

$$-\int_{-0.5}^{0.5} t^2 \cdot e^{-j\pi t} \cdot dt = -\left[\underbrace{\frac{t^2 e^{-j\pi k t}}{j\pi k} \Bigg|_{+0.5}^{-0.5}}_{\frac{\sin(0.5\pi k)}{2\pi k}} + \underbrace{\frac{2}{j\pi k} \int_{-0.5}^{0.5} t \cdot e^{-j\pi k t} \cdot dt}_{\downarrow} \right]$$

Another integration by parts

$u = t$

$du = dt$

$dv = e^{-j\pi k t} \cdot dt$

$v = \frac{e^{-j\pi k t}}{-j\pi k}$

Applying by parts

$$\frac{2}{j\pi k}\int_{-0.5}^{0.5} t.e^{-j\pi k t}.dt = \frac{2}{j\pi k}\left[\frac{t.e^{-j\pi k t}}{j\pi k}\Bigg|_{0.5}^{-0.5}\right] + \frac{1}{j\pi k}\int_{-0.5}^{0.5} e^{-j\pi k t}.dt$$

$$= \frac{2}{j\pi k}\left[\frac{-0.5.e^{j0.5\pi k} - 0.5\,e^{-j0.5\pi k}}{j\pi k}\right] + \frac{2}{j^2\pi^2 k^2}.e^{-\frac{j\pi k t}{j\pi k}}\Bigg|_{0.5}^{-0.5}$$

$$= \frac{2\cos(\frac{\pi k}{2})}{\pi^2.k^2} - \frac{4\sin(\frac{\pi k}{2})}{\pi^3 k^3}$$

Now returning to the first by part and substituting result of second by part

$$-\int_{-0.5}^{0.5} t^2.e^{-j\pi t} = -\left[\frac{\sin(\pi k/2)}{2\pi k} + \frac{2\cos(\pi k/2)}{\pi^2 k^2} - \frac{4\sin(\pi k/2)}{\pi^3 k^3}\right]$$

Adding first integral and second integral

$$\frac{\sin(\pi k/2)}{\pi k} - \frac{\sin(\pi k/2)}{2\pi k} - \frac{2\cos(\pi k/2)}{\pi^2 k^2} + \frac{4\sin(\pi k/2)}{\pi^3 k^3}$$

$$X_k = \frac{\sin(\pi k/2)}{2\pi k} - \frac{2\cos(\pi k/2)}{\pi^2 k^2} + \frac{4\sin(\pi k/2)}{\pi^3 k^3}$$

Part 3

The code for fourier synthesis of $x(t)$ is as following:

```
t=[-5:0.001:5];
D11= mod(22101962,11);
T=2;
W=1;
K=2+D11;
j=1i;
xt=FSWave(t,K,T,W);
real_xt = real(xt);
imaginary_xt = imag(xt);
a=max(real_xt);
c=min(real_xt);
b=max(imaginary_xt);
d=min(imaginary_xt);
plot(t, real_xt);
xlabel('Time (s)');
ylabel('Real Part');
title('Real Part of x(t)');
plot(t, imaginary_xt);
xlabel('Time (s)');
```

```
ylabel('Imaginary Part');
title('Imaginary Part of x(t)')
 function [xs] = FSWave(t,K,T,W)
xs = zeros(1,length(t));
fun = @(t,k) (1-2*(t.*t))./exp(pi*j*k*t);


for k = -K:K
   xk = integral(@(t) fun(t,k),-0.5,0.5);
  xs = xs + SUMCS(t,xk,pi*k);
end


 end
```

```
function [xs] = SUMCS(t, A, omega)


    M = length(A);
    xs = zeros(size(t));


    for i = 1:M
        xs = xs + A(i) * exp(1j * omega(i) * t);
    end
end
```

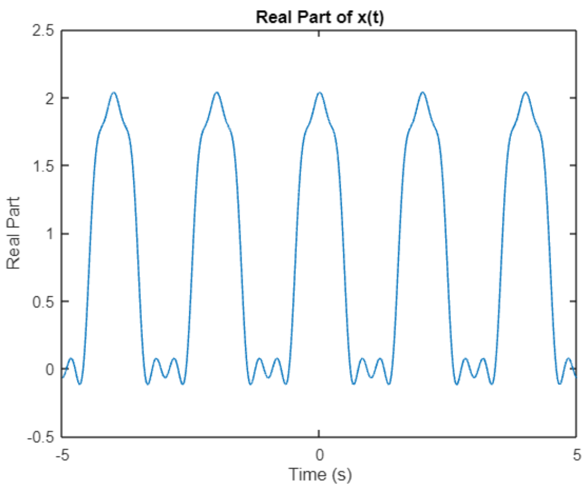

Fig3.1 real part of foruier synthesis

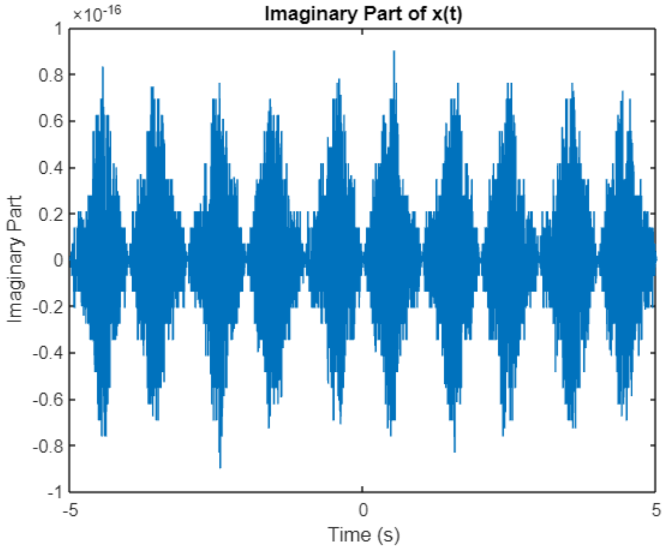

Fig3.2 imaginary part of foruier synthesis

The max,min value for real part is 2.0381 and c = -0.1171 respectively. On the other hand max, min value for imaginary part is b = 9.0206e-17 and d = -9.0206e-17 respectiveley It is expected that the signal should not have any imaginary part however according to MATLAB imaginary part is present due to being able to consider at most 16 digits after the fraction . In order to provide a better perception, MATLAB outputs -5.5511e-17 as the result of the operation sin(pi/6)-0.5 which is 0 in theory. Now influence of K value will be observed by substituting K as 2+D5, 7+D5, 15+D5, 50+D5, 100+D5.

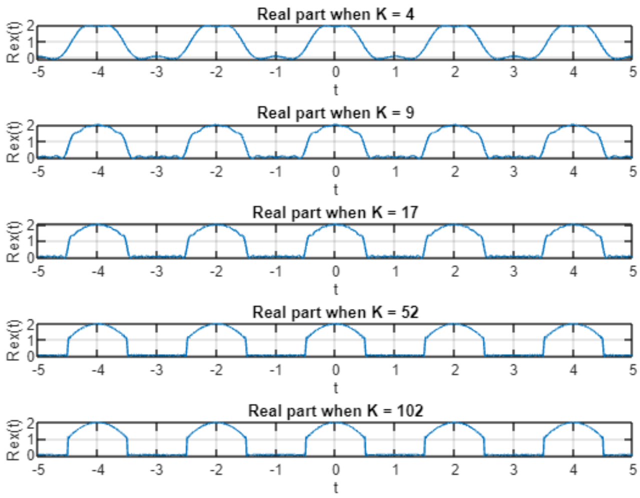

Fig 3.3 fourier synthesis graphs with ascending K values.

As it can be observed as K increases due to addition of greater value of sinusoids the plot gets tighter. It can be interpreted that as K increases a better approximation of $x(t)$ is performed. The waveform in the last plot is closer to the sketched graph compared to other 4 plots with a smaller K value.

Part 4

a)

Modificaiton: Done to fun function

```
fun = @(t,k) (1-2*(t.*t))./exp(pi*j*-k*t);
```

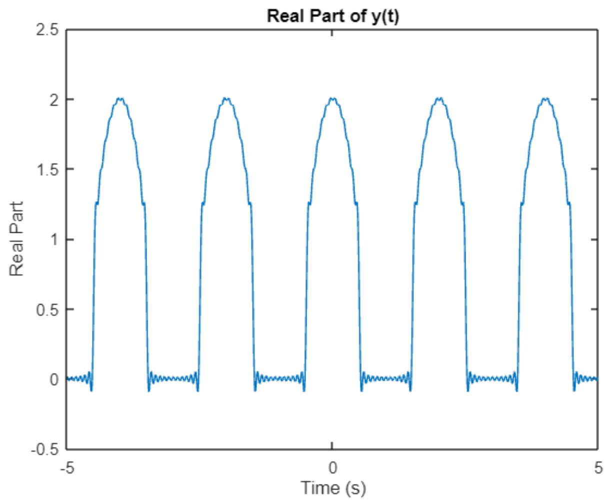

Fig4.1 real part of $y(t)$ vs t

Explanation:

It can be interpreted that the output should be same with the condition where expansion coefficient is taken as X_k because now the first coefficient will be the last cofficient of fourier expansion with X_k hence the result of summation does not change only the order of the coefficients change. In other words the signal remains the same due to being an even function.

b)

Modification: Done to for loop

```
for k = -K:K
    xk= integral(@(t) fun(t,k),-0.5,0.5);
    yk=xk*exp(-j*2*pi*k*0.3);
  xs = xs + SUMCS(t,yk,pi*k);
end
```

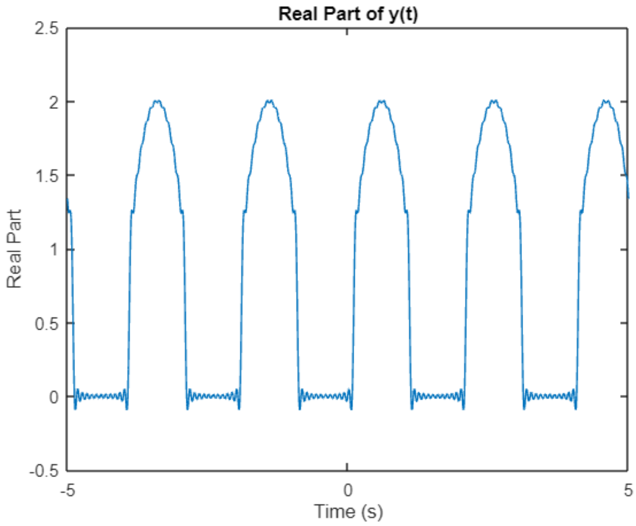

Fig4.2 real part of $y(t)$ vs t

Explanation: This operation performs a phase shift determined by t_0 value therefore the output waveform remains the same with a phase shift.

c)

Modification: done to for loop

```
for k = -K:K
    xk= integral(@(t) fun(t,k),-0.5,0.5);
    yk=xk*(j*k*2*pi/2);
  xs = xs + SUMCS(t,yk,pi*k);
end
```

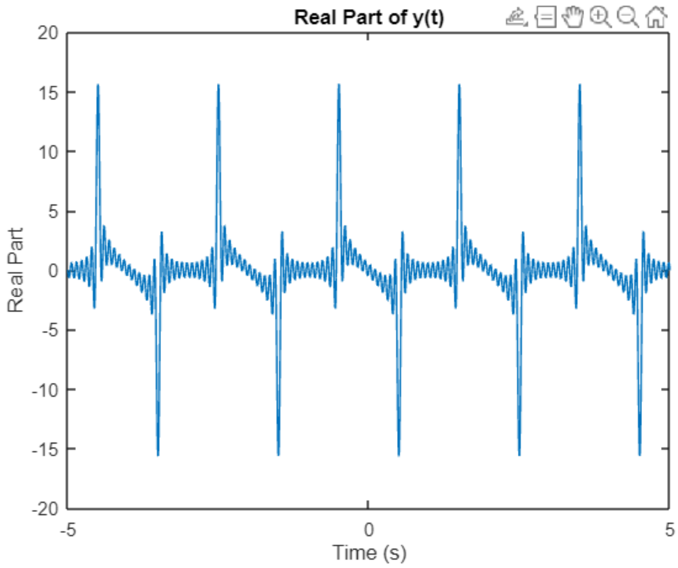

Fig4.3 real part of $y(t)$ vs t

Explanation: This is a derivative operation done to X_k. $Y_k = \frac{dX_k}{dt} = jk\frac{2\pi}{T}X_k$.

d) Modificaiton: Done to for loop

```
for k = -K:K
    if 0<k<K+1
        fun = @(t,k) (1-2*(t.*t))./exp(pi*j*(K+1-k)*t);
        xk = integral(@(t) fun(t,k),-0.5,0.5);
    end
    if k==0
        fun = @(t,k) (1-2*(t.*t))./exp(pi*j*k*t);
        xk= integral(@(t) fun(t,k),-0.5,0.5);
    end
    if-K-1<k<0
      fun = @(t,k) (1-2*(t.*t))./exp(pi*j*(K+1+k)*t);
      xk= integral(@(t) fun(t,k),-0.5,0.5);
    end
  xs = xs + SUMCS(t,xk,pi*k);
end
```

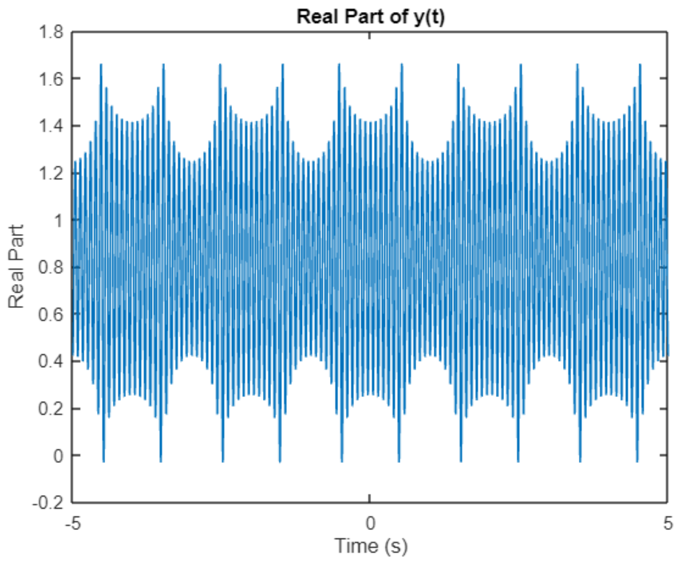

Fig4.4 real part of $y(t)$ vs t

Explanation: Synthesis of $y(t)$ is divided into 3 parts where in the first synthesis $Y_{1\ldots K} = X_{K\ldots 1}$, in the second synthesis $Y_0 = X_0$ and in the third synthesis $Y_{-K\ldots -1} = X_{-1\ldots -K}$. In this operation expansion coefficients are multiplied with euler with different k's. To be more clear, X_{-1} is multiplied with euler value with k=-1 however in this case $Y_{-K} = X_{-1}$ is multiplied by euler value with k=-K hence results in a different waveform and different amplitude.