Feedback Control Systems Lab 3 Report

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1. Introduction

The aim of this lab work is to familiarize students with determining phase, gain and delay margins of aa system. This lab work consists of two parts. In the first part it is desired to estimate previously mentioned margins by investigating phase and magnitude bode plots whereas in the second part it is desired to verify the determined estimations of margins by utilizing hardware.

2. Laboratory Content

The laboratory content consists of two parts which are margin estimation and verification respectively.

Part 1: Margin Estimation

The first order transfer function of the DC motor is pade approximated, which will be plant, and cascaded with the specified controller. Margin estimations will be done by investigating the bode plots of the cascaded system.

$$G_p(s) = \frac{13.6}{0.091s + 1}$$

$$G_{pade}(s) = \frac{13.6}{0.091s + 1} * \frac{1 - 0.005s}{1 + 0.005s}$$

$$G_c(s) = \frac{0.1471s + 11.76}{s^2 + 32.97s}$$

$$G(s) = G_c(s) * G_{pade}(s)$$

$$G(s) = \frac{-0.01s^2 + 1.2s + 160}{0.000455s^4 + 0.111s^3 + 4.165s^2 + 32.97s}$$

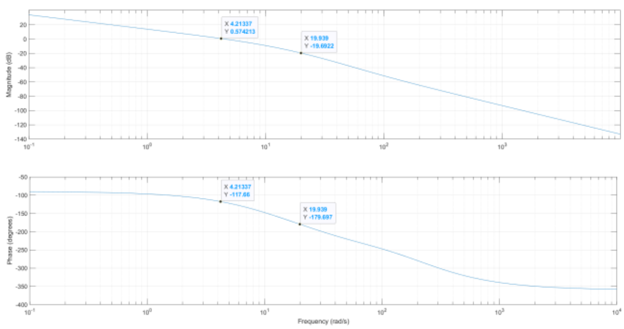

Figure 1 Magnitude and Phase Bode plots of G(s)

From Figure 1 it can be seen that when the phase is -180 degrees, the angular frequency is 19.939rad/s. So GM (gain margin) can be calculated as following,

$$\omega_1 = 19.939\frac{rad}{s}$$

$$20\log(|G\omega_1|) = -19.692$$

$$20\log(GM) = 19.692$$

$$GM = 9.65$$

For PM (phase margin), the angular velocity where the gain is 0dB is necessary which is observable in Figure 1.

$$PM = 180° + \angle G(j\omega_2)$$

$$\omega_2 = 4.21\, rad/s$$

$$PM = 180° - 117.66° = 62.34°$$

Finally, the DM (delay margin) can be expressed as following,

$$DM = \frac{PM}{\omega_2} = \frac{62.34}{4.21} * \frac{\pi}{180} = 0.2584s$$

According to Matlab, margins are calculated as,

```
ans = struct with fields:
      GainMargin: 9.7746
     GMFrequency: 20.0667
     PhaseMargin: 60.8235
     PMFrequency: 4.4629
     DelayMargin: 0.2379
     DMFrequency: 4.4629
          Stable: 1
```

	GM	PM (degrees)	DM (seconds)
Estimated Results (from Bode)	9.65	62.34	0.2584
Software Result (Matlab)	9.77	60.82	0.2379

Table 1 Comparison of calculated results and Matlab results

As it can be seen from Table 1 calculated results are consistent with the Matlab's software results hence the margin s will be verified by taking estimated values as references.

Part 2: Margin Verification

Now, by looking at the K values in which the system is stable, marginally stable and unstable, a maximum K value that makes the system stable will be determined.

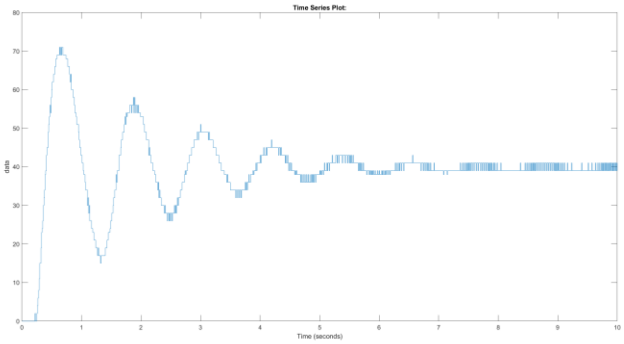

Figure 2 Stable output when K=1

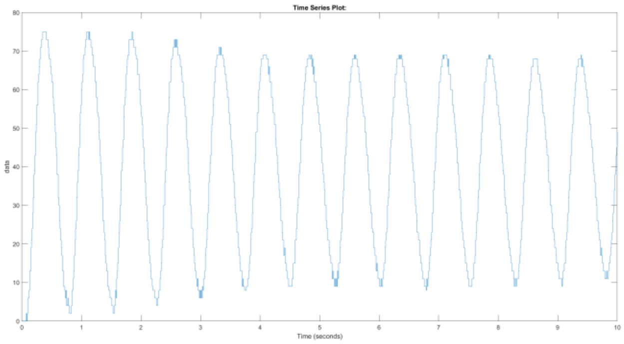

Figure 3 Marginally Stable output when K=2.7

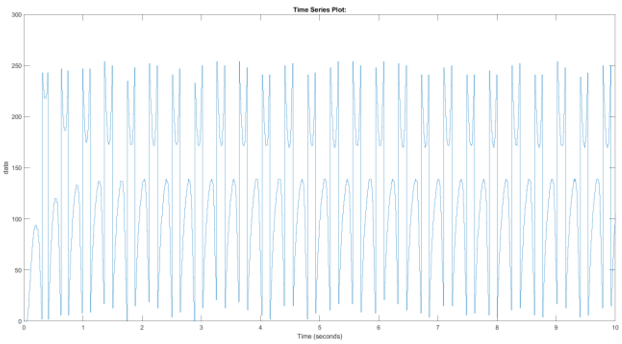

Figure 4 Unstable output when K=10

The waveforms in the previous figures, directly indicate the stability of the system. When K=1 the output approaches to a finite value in steady state which makes the system stable. In Figure 3, the output is indecisive about approaching to a finite value or increasing hence has a uniform peak value which makes the system marginally stable. In Figure 4 the output is corrupted and tends to increase as time passes which makes the system unstable. According to this experiment, the maximum K value which makes the system stable is K=2.7

Now the DM will be verified with the same logic used in verifying GM.

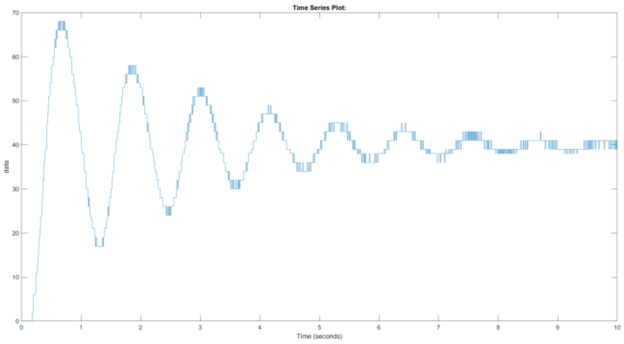

Figure 5 Stable output when delay=0.01s

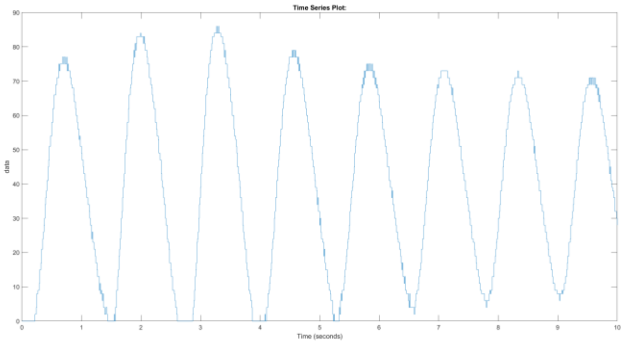

Figure 6 Marginally stable output when delay=0.04s

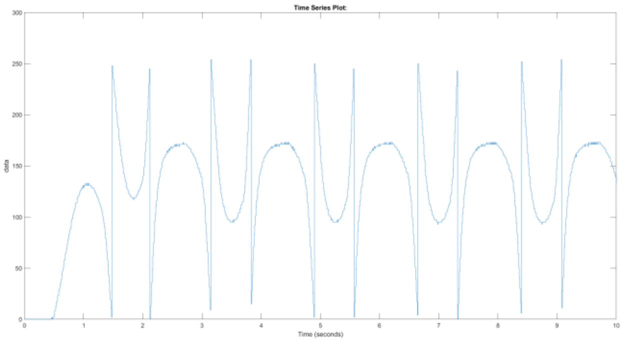

Figure 7 Unstable output when delay=0.23s

The DM is can be determined as 0.04 seconds.

	GM	DM (seconds)
Estimated Results (from Bode)	9.65	0.2584
Software Result (Matlab)	9.77	0.2379
Calculated Resulsts (Experiment)	2.7	0.04

Table 2 Comparison of estimated, calculated and software results of GM and DM

From Table 2, it can be clearly seen that there are significant differences between estimated and verified margins. Since this is a real life application it is expectable to have differences between estimated and verified margins. This may be a result of approximate point selection from bode plots, excessive energy loss in hardware configuration due to internal resistance of cables and encoder capacity of Arduino board.

3. Conclusion

This lab work provides a better perception on determining delay, gain and phase margins of the system. In the first part of this lab work, the PM, GM and DM are estimated by investigating the magnitude and phase bode plots of the specified transfer function. After completing the estimations the verification part is done by applying different K and delays to the system. After observing the outputs with different K and delay values they are classified as stable, marginally stable and, unstable. After the classification the maximum value of delay and K is determined by checking the marginally stability. The maximum values of these parameters can be referred as hardware results of GM and DM. After comparing the estimated, calculated values it is concluded that the estimated and calculated values are different from each other and the reasons of this situation are mentioned in the very last paragraph which can be summed up as hardware insufficiency.

REFERENCES

1. 1. Richard C. Dorf, and Robert H. Bishop. Modern Control Systems. Pearson Prentice Hall, 2017

Appendix

Matlab Code:

```
Gc=tf((2/13.6)*[1,80],[1,3/0.091,0])
Gp=tf(13.6,[0.091,1])
Gpade=tf([-0.005,1],[0.005,1])
G_p=Gp*Gpade
G=Gc*G_p
bode(G);allmargin(G)
```