Eray Gündoğdu
22101962

EEE - 321: Signals and Systems Lab Assignment 1

Part 1

a) When a=[3.2 34/7 -6 24] is written a 1x4 matrix is obtained as the output, if a is assigned as

[3.2; 34/7; -6; 24] a 4x1 matrix is obtained as the output.

b) In the first part, at first a is assigned as 1x4 matrix and 4x1 matrix consequetively therefore as the final output a remains as a 4x1 matrix. Different from the first part if a=[3.2 34/7 -6 24]; and b=[3.2; 34/7; -6; 24]; is written two different matrixes, a as 1x4 and b as 4x1, are obtained however outputs are not displayed and put to the workspace(memory) for further usage due to semicolons.

c)The time difference between variables a=[3.2 34/7 -6 24] and a=[3.2 34/7 -6 24]; is calculated as -0.011973 seconds by using tic and toc commands. Semicolons are useful if an output is not desired in the display section.

d) Error occurs due to the incorrect dimensions for matrix multiplication. a is 1x4 matrix, b is also 1x4 matrix therefore matrix multiplication can not be performed.

e) The "." syntax before multiplication is necessary for elementwise multiplication.This operation gives a matrix with smae dimensions with the input matrixes. Result of c=a.*b is displayed as c = 10^3 × [0.0186 0.0233 -0.0300 -2.4480]. The output remains the same when c is assigned as c=b.*a since the the operation is elementwise multiplication.

f) MATLAB performs a 1x4 and 4x1 matrix multiplication. 1x4 matrix cannbe classifed as a row vector similarly, 4x1 matrix can be considered as a column vector therefore result of the multiplication is scalar. The result of the multiplication is obtained as c = -2.4361e+03.

g) MATLAB performs a matrix multiplication between a 4x1 and 1x4 matrixes resulting in a 4x4 matrix. The output for c is displayed as c = 4×4
10^3 ×

0.0186	0.0154	0.0160	-0.3264
0.0282	0.0233	0.0243	-0.4954
-0.0348	-0.0288	-0.0300	0.6120
0.1392	0.1152	0.1200	-2.4480

h) The operation a=[1:0.01:2] counts from 1 to 2 with a unit step of 0.01 and displayes all counted numbers as output.

i) Elapsed time is calculated as 0.0045195 seconds by using tic and toc commands.

j) *The elapsed time is larger compared to i due to for loop usage. The elapsed time is displayed as 0.007871.

k) *The elapsed time in this process is 0.30385 seconds. It can be interpereted that creating memory location by using zeros and ones commands made the process longer compared to j. The elapsed times are i, j, k in an ascending order. Command in i can be considered as the most efficient one.

l) If the question is approached mathematically, writing a vector in sin function does not mean anything. A vector(array) a is generated that consists of all values between 0 and 2π incremented with a step size of π/8 also includes 0 and 2π. Since there is no mathematical meaning, MATLAB puts all values inside the array consequetively to sin function and outputs a 1x17 matrix that consists of sin function values.

m) Plot(x) plots the cos function of cos($\frac{3\pi t}{4} + \frac{\pi}{6}$) where t is assigned as t=[1:0.02:4]. Plot(x,t) command plots a graph where x values are on the x axis and t values are on y axis lastly, plot(t,x) command plots a graph where x values are labeled on y axis and t values are labeled on x axis.

n) Plot(t,x,'-+') outputs a graph similar to the plotted graph from the command plot(t,x) the only difference is every data point is marked with + sign. Plot(t,x,'+') plots a garph by only marking each data point with + sign without connecting the markes hence outputs a rough plot with a similar waveform compared to the previously displayed plot.

o) There are 26 time values in the array of t=[0:0.04:1].

p) Since there are 151 values in the array by using t = linspace(0, 1, 151) command same array, which is generated by t=[1:0.02:4], is generated.

q) x is displayed as a 1x26 matrix which consists of sin function values obtained by substituting the t values respectively.

s)Now x is displayed as a 1x101 matrix meaning that 101 time points are avaible.

v) After observing three plots the most accurate sine plot is the first plot which has 101 time points. Due to having 101 time points, the graph is plotted with 101 different sine function values which makes the plot smoother and continous likewise compared to the plots with 11 and 6 time points.

w) Plot command fills the space between data points by connecting them with a straight line hence the more data values are entered the plot becomes smoother.

x) The difference between stem and plot commands is, plot command plots a graph by connecting the y values of the function whereas stem command plots a graph by connecting y values with the x axis.

Part 2

Both of the sound and soundsc commands are appropriate for listening the discrete version of the signals. After observing different frequencies 440, 687, 883 respectively, it is observed that pitch of the signal increases due to applying a higher frequency.

For the continuing part, a new signal $x_2(t)$ is generated. This signal is basically multiplication of $x_1(t)$ with a decaying exponential due to this reason the graph of $x_1(t)$ is now squeezed between two decaying exponentials and can be observed in the following figures.

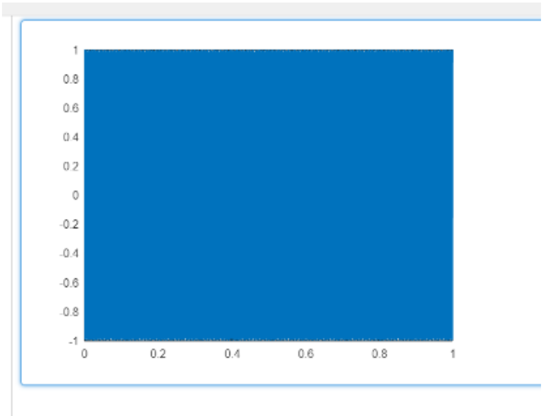

Fig 1.1 plot of $x_1(t)$

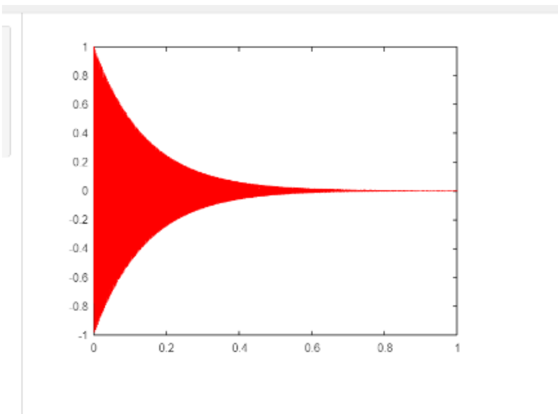

Fig 1.2 plot of $x_2(t)$

The code for the plot and listening of $x_2(t)$ is as following:

```
t=[0:1/8192:1];
x1=cos(2*pi*330*t);
x2=exp(-11*t).*cos(2*pi*330*t);
sound(x2)
plot(t,x2,'r')
```

$x_1(t)$ is referred as first sigfnal and $x_2(t)$ is referred as second signal.Both of the signals last for 1 seconds however there are differences between the sounds of the signals. The sound of first signal lasts for 1 second with a constant amplitude whereas second lasts for 1 seconds with a decreasing amplitude. This can be also observed by looking at the graphs. Moreover, since the frequencies are 440 and 330 repectively, the pitch of the first signal is higher than the second signal. It can be interpreted that second is more likely to sound like a piona and the first signal is more likely to sound like a flute. If α increases the decay rate of the function increases meaning that the amplitude of the signal decreases swiftly for larger α values. The duration of signals are limited between the range of [0.1] therefore all signals last for 1 seconds however as α increases the end of volume range that our ears can hear becomes closer to 0 seconds.

The code for computing $x_3(t)$ is as following:

```
t=[0:1/8192:1];
x3= cos(2*pi*4*t).*cos(2*pi*510*t);
sound(x3)
plot(t,x3,'r')
```

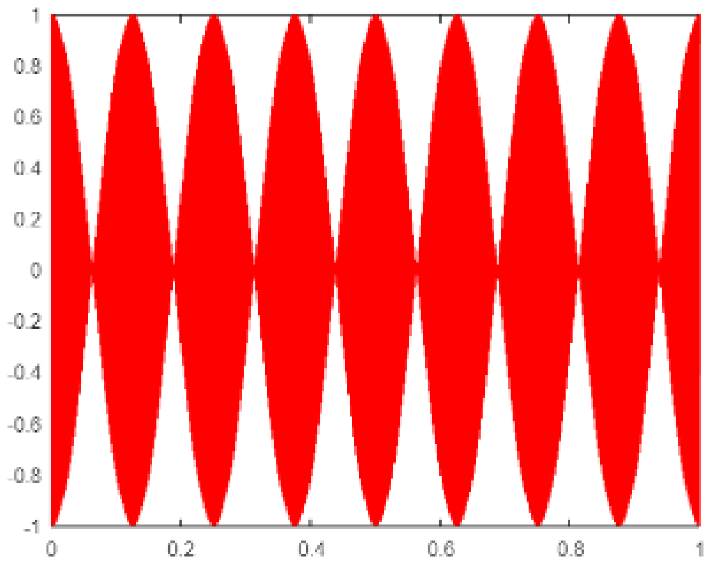

Fig1.3 plot of $x_3(t)$

Multiplication with a low frequency cosine term squeezes the the signal $x_1(t)$ between a low frequency cosine wave and x axis therefore the signal sounds wavey (high and low amplitude). The increase in f_1 increases the overall frequency therefore the sound fluctuates faster if $f_1$is decreased the sound fluctuates slower. $x_3(t)$ can be written as following by using cosine sum:

$$x_3(t) = 0.5(\cos(2\pi f_1 t + 2\pi f_0 t) + \cos(2\pi f_1 t - 2\pi f_0 t)) = \cos(2\pi f_1 t)\cos(2\pi f_0 t)$$

It can be interpreted that change in f_1 directly affects the frequency of the output therefore, as explained previously, lower $f_1$values decreases the fluctuation rate (frequency of the signal) and higher f_1 values increases the rate of fluctuation.

Part 3

The following equations show that signal $x_1(t)$ has an instantenous frequency of f_0.

$$x_1(t) = \cos(2\pi f_0 t) = \cos(2\pi \text{Ø}(\text{t}))$$

$$f_{ins}(t) = \frac{\text{dØ(t)}}{\text{dt}}$$

$$\text{Ø(t)} = f_0 t$$

$$\frac{\text{dØ(t)}}{\text{dt}} = f_0$$

Now the instantenous frequency of $\cos(\pi \alpha t^2)$ is investigated.

$$x_4(t) = \cos(\pi \alpha t^2) = \cos(\frac{\pi}{2} * 2\alpha t^2)$$

$$\text{Ø(t)} = 2\alpha t^2$$

$$f_{ins}(t) = \frac{\text{dØ(t)}}{\text{dt}}$$

$$f_{ins}(t) = \alpha t$$

The instantenous frequency of $x_4(t)$ is obtained as αt. At $t = 0$ instantenous frequency is 0 since $\alpha * 0 = 0$. At time t_0 instantenous frequency is αt_0. From the referred website α is selected as 1789. Now $f_{ins}(t) = 1789t$ meaning that the frequency range is [0,1789]. In order to listen $x_4(t)$ the following code is written:

```
t=[0:1/8192:1];
x4= cos(pi*t.*t*1789);
sound(x4)
plot(t,x4)
```

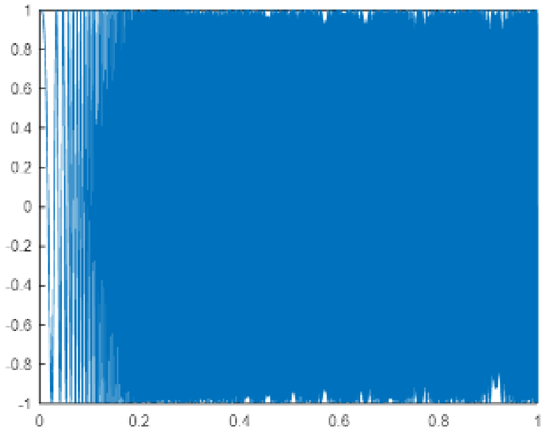

Fig1.4 plot of $x_4(t)$

From just hearing the sound it can be stated that frequency is increasing linearly. Now it is desired to make observations when α is increase and decreased with the new values of $\alpha_1 = \alpha/2$ and $\alpha_2 = 2\alpha$. Changing α alternates the slope of the instantenoues frequency meaning that when α_1 is applied the rate of change of the frequency is divided into 2 and reaches a maximum frequency of 1789/2. On the other hand when α_2 is applied the rate of change of the frequency is doubled and the frequency reaches a maximum of 3578. The lowest frequency for both cases reamin as 0. Now $x_5(t) = \cos(2\pi(-500t^2 + 1600t))$. In order to listen to this signal following code is written:

```
t=[0:1/8192:2];
x5= cos(2*pi*(-500*(t.*t)+1600*t));
sound(x5)
plot(t,x5)
```

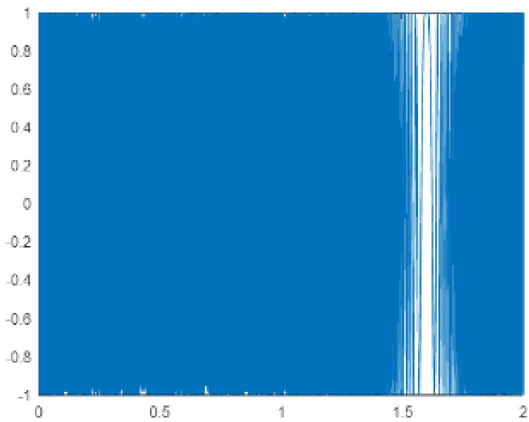

Fig 1.5 plot of $x_5(t)$

Just by the experimental comments the frequency follows an increasing-decreasing-increasing path. Now this experimental comment will be proofed mathematically.

$$x_5(t) = \cos(2\pi(-500t^2 + 1600t))$$

$$\emptyset(\mathrm{t}) = -500t^2 + 1600t$$

$$f_{ins}(t) = \frac{\mathrm{d}\emptyset(\mathrm{t})}{\mathrm{dt}} = -1000t + 1600$$

When t=0, t=1, t=2 frequency is 1600Hz, 600Hz, -400Hz respectively. When $f_{ins} = 0$ frequency gets the lowest value which is at t= 1.6 seconds with a value of 0Hz.

Part 4

As $x(t) = \cos(2\pi * 1789t + \varphi) = \cos\left(2\pi(1789t + \frac{\varphi}{2\pi}\right)$ is investigated with different φ values the pitch and volüme is observed the same in all 5 different cases. The reason behind this result is, volume depends on the coefficient of cos which is 1 in all cases and $\frac{\mathrm{d}\emptyset(\mathrm{t})}{\mathrm{dt}} = 1789Hz$ for all t therefore the sound remains the same in all cases φ values are only responsible for phase shift.

Part 5

In this part it is unknown that A_1 and A_2 is equal or not therefore in order to obtain a valid result for all values phasor approach on $x(t)$ functions is used.

$$x_1(t) = A_1 co\, s(2\pi f_0 t + \varphi_1) = A_1 e^{j(2\pi f_0 t + \varphi_1)}$$

$$x_2(t) = A_2 co\, s(2\pi f_0 t + \varphi_2) = A_2 e^{j(2\pi f_0 t + \varphi_2)}$$

$$x_1(t) + x_2(t) = x_3(t)$$

$$A_1 e^{j(2\pi f_0 t + \varphi_1)}\} + A_2 e^{j(2\pi f_0 t + \varphi_2)} = e^{j(2\pi f_0 t)}(A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2})$$

$$x_3(t) = e^{j(2\pi f_0 t)}(A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2})$$

$$A_3 e^{j(2\pi f_3 t + \varphi_3)} = e^{j(2\pi f_0 t)}(A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2})$$

$$f_3 = f_0$$

Now we are left with:

$$A_3 e^{j\varphi_3} = A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2}$$

Taking absolute values(magnitude of phasors) of both sides in order to obtain A_3 value:

$$A_3 = |A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2}|$$

The magnitude of A_3 can be calculated with cosine theorem with 2 vector with a magnitude of A_1 and A_2.

$$A_3{}^2 = A_1{}^2 + A_2{}^2 - 2A_1A_2\cos(|\varphi_2 - \varphi_1|)$$

If A_3 will be maximized $|\varphi_2 - \varphi_1| = (2n + 1)$ π and $A_3 = |A_1 + A_2|$ where n is an integer.

If A_3 will be minimized $|\varphi_2 - \varphi_1| = 2\pi n$ and $A_3 = |A_1 - A_2| = 0$ as minimum value where n is an integer.

After obtaining A_3 and f_3 values, φ_3 will be derived by equating real and imaginary parts of the equation $A_3 e^{j\varphi_3} = A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2}$.

Equating real parts:

$$A_3 \cos(\varphi_3) = A_1 \cos(\varphi_1) + A_2 \cos(\varphi_2)$$

Equating imaginary parts:

$$A_3 \sin(\varphi_3) = A_1 \sin(\varphi_1) + A_2 \sin(\varphi_2)$$

Dividing second equation with the first equaiton:

$$\tan(\varphi_3) = \frac{A_1 \sin(\varphi_1) + A_2 \sin(\varphi_2)}{A_1 \cos(\varphi_1) + A_2 \cos(\varphi_2)}$$

$$\varphi_3 = \arctan(\frac{A_1 \sin(\varphi_1) + A_2 \sin(\varphi_2)}{A_1 \cos(\varphi_1) + A_2 \cos(\varphi_2)})$$

Min and max value for A_3,value of f_3 and φ_3 are obtained in terms of A_1, A_2, φ_1, φ_2, f_0.