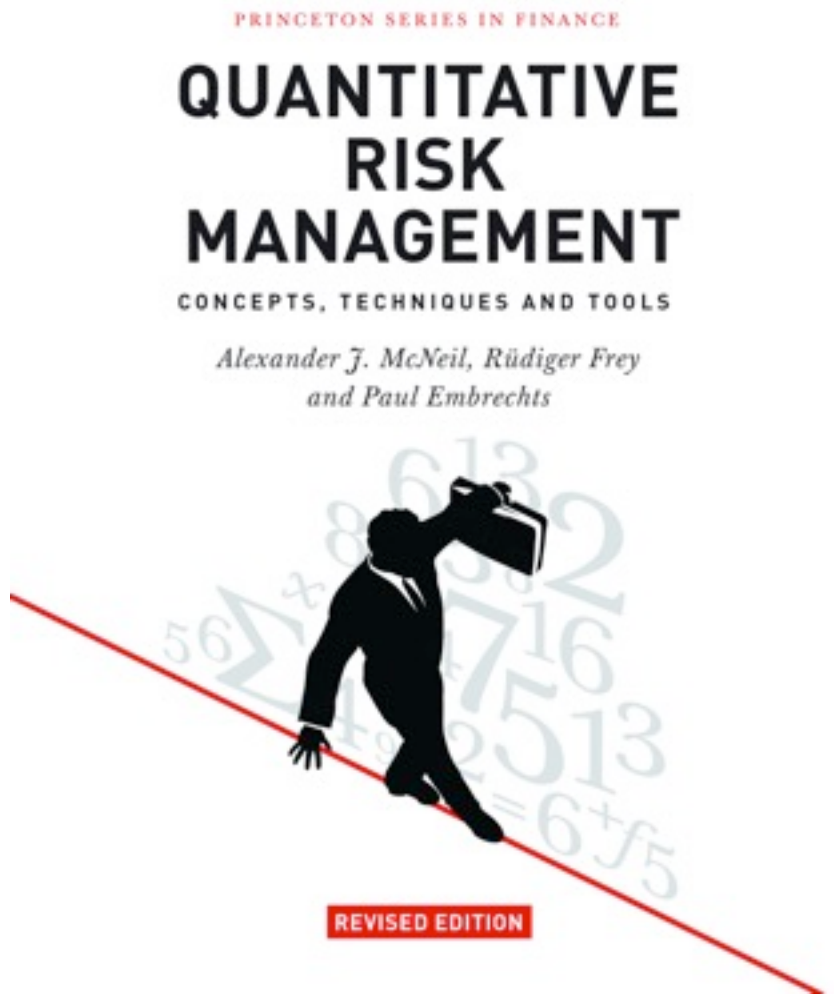
QUANTITATIVE RISK MANAGEMENT IN R

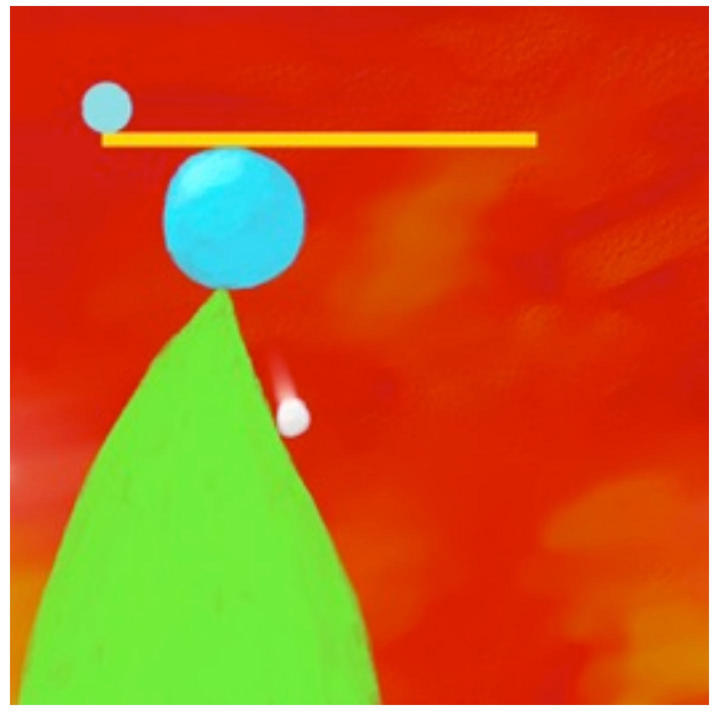
Welcome to the course!

About me

- Professor in mathematical statistics, actuarial science, and quantitative finance
- Author of *Quantitative Risk Management: Concepts, Techniques & Tools* with R. Frey and P. Embrechts



- Creator of qrmtutorial.org with M. Hofert
- Contributor to R packages including `qrmdata` and `qrmtools`



The objective of QRM

- In **quantitative risk management** (QRM), we quantify the risk of a portfolio
- Measuring risk is first step towards managing risk
- Managing risk:
  - Selling assets, diversifying portfolios, implementing hedging with derivatives
  - Maintaining sufficient capital to withstand losses
- **Value-at-risk** (VaR) is a well-known measure of risk

Risk factors

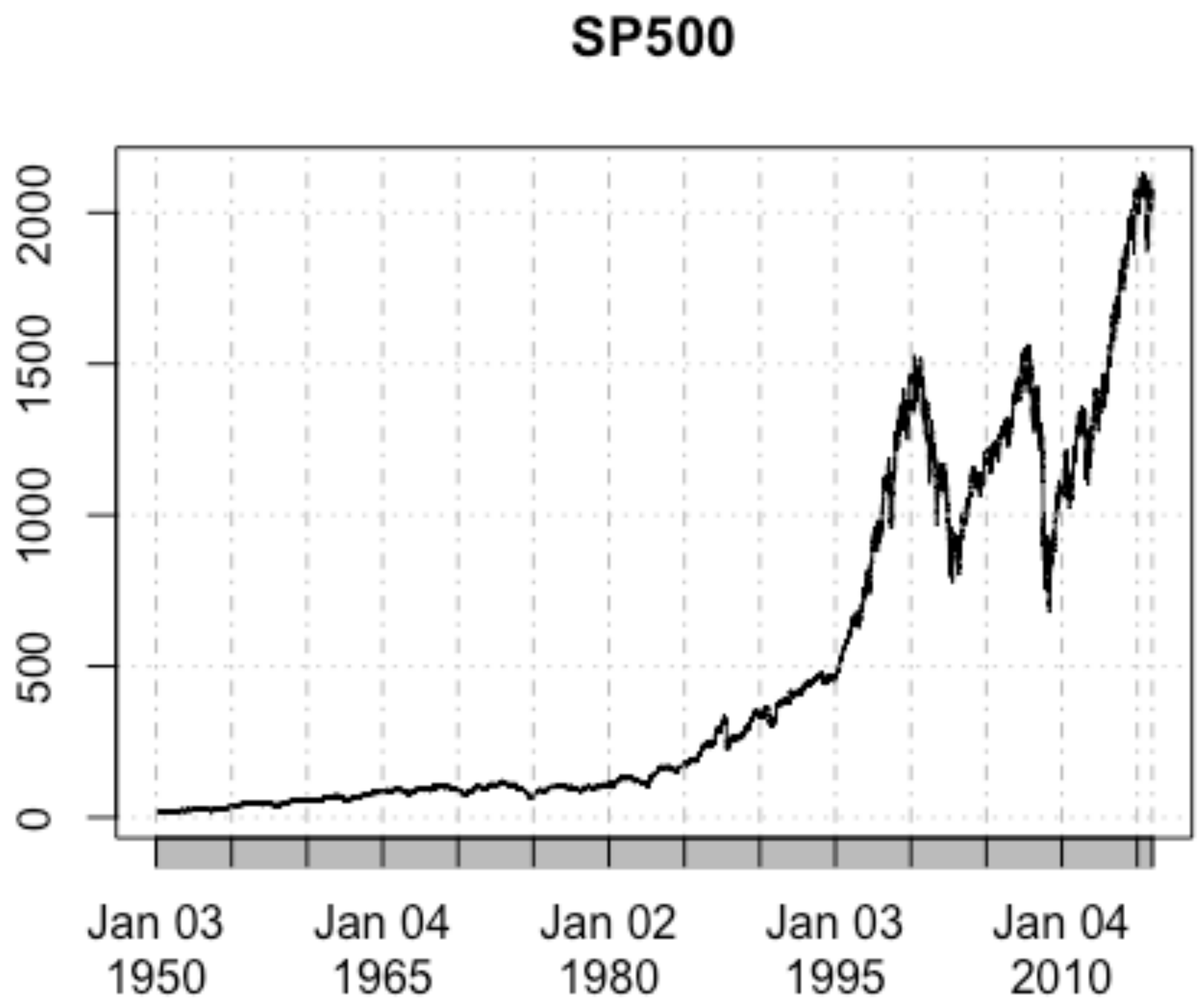
- Value of a portfolio depends on many **risk factors**
- Examples: equity indexes/prices, FX rates, interest rates
- Let's look at the S&P 500 index

Analyzing risk factors with R

```
> library(qrmdata)

> data(SP500)

> head(SP500,  n = 3)
                     ^GSPC
1950-01-03 16.66
1950-01-04 16.85
1950-01-05 16.93

> tail(SP500, n = 3)
                        ^GSPC
2015-12-29 2078.36
2015-12-30 2063.36
2015-12-31 2043.94
```

Plotting risk factors

```
> plot(SP500)
```



Let’s practice!

Risk-factor returns

Risk-factor returns

- Changes in risk factors are **risk-factor returns** or **returns**
- Let (Z_t) denote a time series of risk factor values
- Common definitions of returns (X_t) :

$$X_t = Z_t - Z_{t-1} \qquad \text{(simple returns)}$$

$$X_t = \frac{Z_t - Z_{t-1}}{Z_{t-1}} \qquad \text{(relative returns)}$$

  - 0.02 = 2% gain, -0.03 = 3% loss

$$X_t = \ln(Z_t) - \ln(Z_{t-1}) \qquad \text{(log-returns)}$$

Properties of log-returns

- Resulting risk factors cannot become negative
- Very close to relative returns for small changes:

$$\ln(Z_t) - \ln(Z_{t-1}) \approx \frac{Z_t - Z_{t-1}}{Z_{t-1}}$$

- Easy to aggregate by summation to obtain longer-interval log-returns
- Independent normal if risk factors follow **geometric Brownian motion** (GBM)

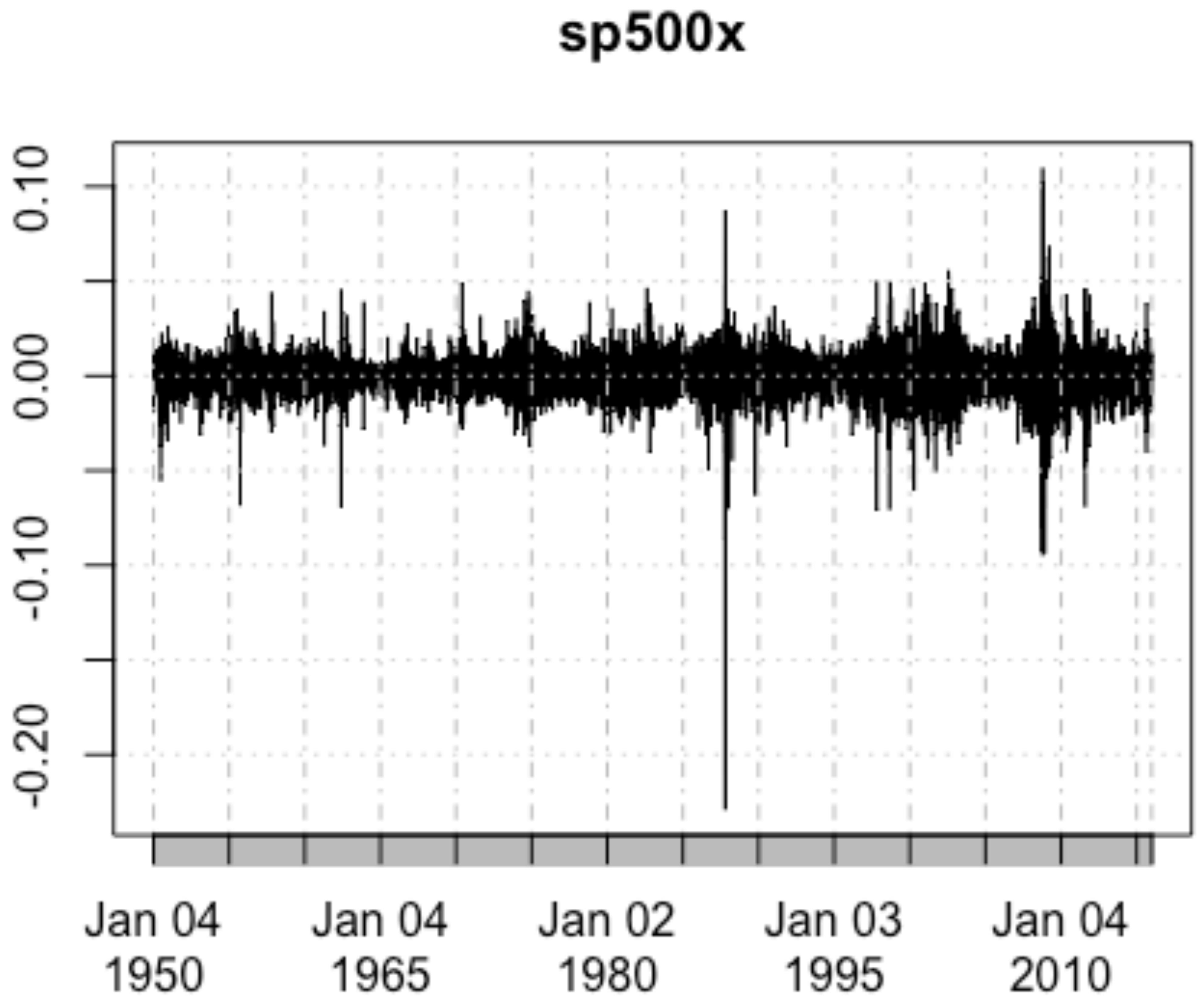
Log-returns in R

```
> sp500x <- diff(log(SP500))
> head(sp500x, n = 3)  # note the NA in first position
                          ^GSPC
1950-01-03          NA
1950-01-04 0.011340020
1950-01-05 0.004736539
```

```
> sp500x <- diff(log(SP500))[-1]
> head(sp500x)
                           ^GSPC
1950-01-04  0.011340020
1950-01-05  0.004736539
1950-01-06  0.002948985
1950-01-09  0.005872007
1950-01-10 -0.002931635
1950-01-11  0.003516944
```

Log-returns in R (2)

```
> plot(sp500x)
```



Let's practice!

Aggregating log-returns

Aggregating log-returns

- Just add them up!
- Assume (X_t) are daily log-returns calculated from risk-factor values (Z_t)
- Log-returns for a trading week is the sum of log-returns for each trading day:

$$\ln(Z_{t+5}) - \ln(Z_t) = \sum_{i=1}^{5} X_{t+i}$$

- Similar for other time horizons

Aggregating log-returns in R

- Use the `sum()` function within `apply.weekly()` and `apply.monthly()` in the `xts` package

```
> sp500x_w <- apply.weekly(sp500x, sum)
> head(sp500x_w, n = 3)
                          ^GSPC
1950-01-09  0.02489755
1950-01-16 -0.02130264
1950-01-23  0.01189081
```

```
> sp500x_m <- apply.monthly(sp500x, sum)
> head(sp500x_m, n = 3)
                          ^GSPC
1950-01-31 0.023139508
1950-02-28 0.009921296
1950-03-31 0.004056917
```

Let’s practice!

Exploring other kinds of risk factors

Exploring other kinds of risk factors

- So far we have looked at:
  - Calculating log-returns and aggregating log-returns over longer intervals
  - Equity data, indexes and single stocks, and **foreign-exchange** (FX) data
- Two other categories of risk factors:
  - Commodities prices
  - Yields of zero-coupon bonds

Commodities data and interest-rate data

- Commodities such as gold and oil prices
  - Do log-returns behave like stocks?
- Government bonds - value depends on interest rates
  - Consider **yields of zero-coupon bonds** as risk factors

Bond prices

- Let $p(t, T)$ denote the price at time small t of a zero-coupon bond paying one unit at maturity T
- $p(0, 10)$: price at $t = 0$ of bond maturing at $T = 10$
- $p(0, 5)$: price at $t = 0$ of bond maturing at $T = 5$
- $p(5, 10)$: price at $t = 5$ of bond maturing at $T = 10$

Yields as risk factors

- The **yield** y(t, T) is defined by the equation:

$$y(t,T) = \frac{-\ln p(t,T)}{T-t}$$

- y(t, 10): yield for a 10-year bond acquired at time t
- y(t, 5): yield for a 5-year bond acquired at time t
- Advantage of yields: comparable across maturities T
- The mapping T to y(t, T) is yield curve at time t
- Log-returns or simple returns of yields?

Let's practice!