Chapter 5: Video 1 - Supplemental slides

Simple moving average (MA) processes

Let $\epsilon_1, \epsilon_2, \ldots$ be White Noise(0,σ_ϵ^2) innovations, with variance σ_ϵ^2

Then, $Y_1, Y_2, \ldots$ is an MA process if for some constants μ and θ,

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- We focus on this 1st order case, the simplest MA process

Simple moving average (MA) processes

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- μ is the mean of the $\{Y_t\}$ process

- If $\theta = 0$, then $Y_t = \mu + \epsilon_t$, such that Y_t is White Noise(μ, σ_ϵ^2)

- If $\theta \neq 0$, then observations Y_t depend on both ϵ_t and ϵ_{t-1}

- And the process $\{Y_t\}$ is autocorrelated

- If $\theta \neq 0$, then ϵ_{t-1} is fed forward into Y_t

- θ determines its impact

- Larger values of $|\theta|$ result in greater impact

$$\begin{aligned}
\text{Corr}(Y_t, Y_{t-1}) &= \rho(1) = \frac{\theta}{1+\theta^2} \\
\text{Corr}(Y_t, Y_{t-h}) &= \rho(h) = 0 \qquad for\ all\ h > 1
\end{aligned}$$
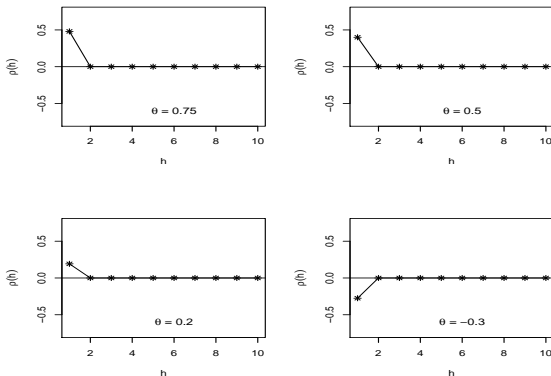

Figure: *Autocorrelation functions of MA processes with θ equal to 0.75, 0.5, 0.2, and -0.3.*