The simple moving average model

TIME SERIES ANALYSIS IN R

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The simple moving average model

The simple moving average (MA) model:

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

More formally:

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

where ϵ_t is mean zero white noise (WN).

Three parameters:

- The mean μ
- The slope θ
- The WN variance σ^2

MA processes - I

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- If slope θ is zero then:

$$Y_t = \mu + \epsilon_t$$

And Y_t is White Noise (μ, σ_ϵ^2)

MA processes - II

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

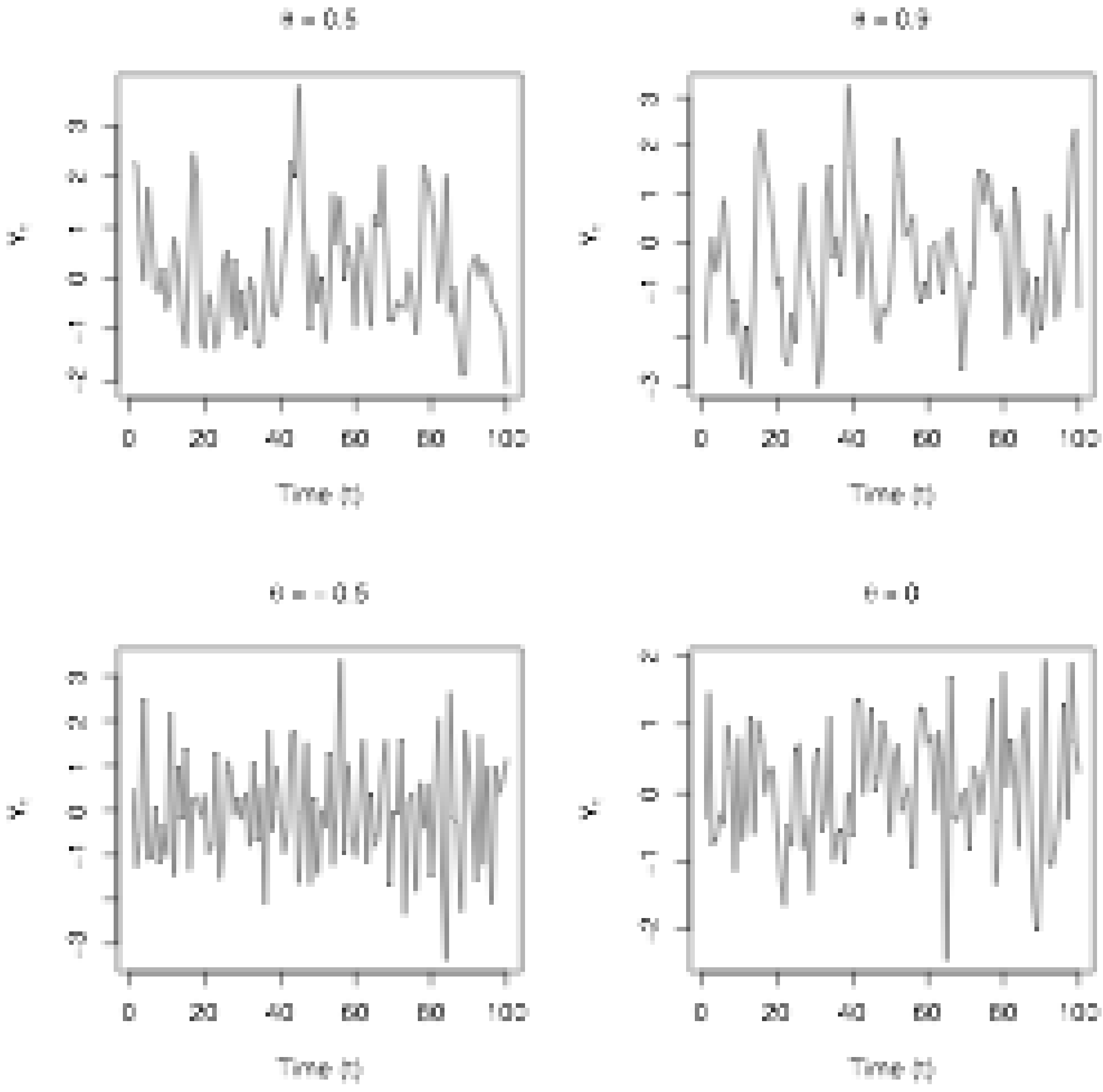
$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- If slope θ is **not** zero then Y_t depends on both ϵ_t and ϵ_{t-1}

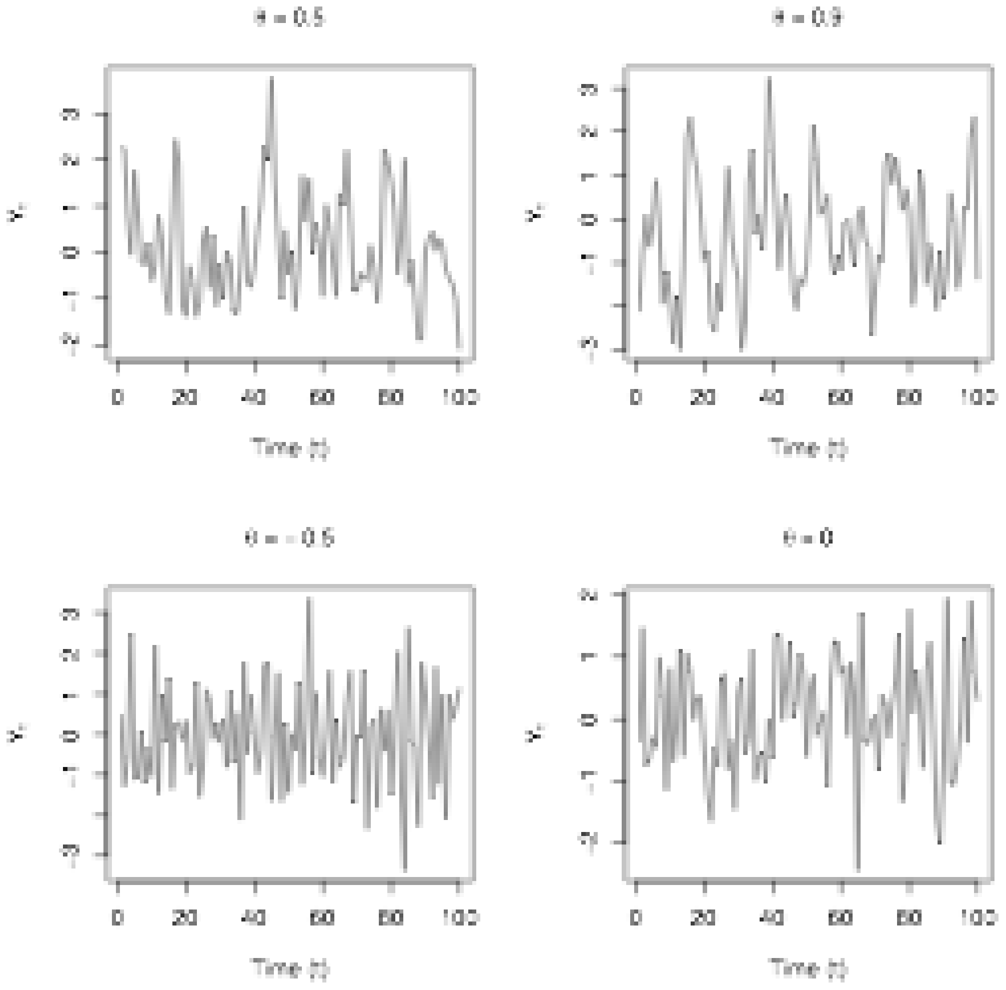
And the process Y_t is autocorrelated

- Large values of θ lead to greater autocorrelation
- Negative values of θ result in oscillatory time series

MA examples



Autocorrelations



Only lag 1 autocorrelation non-zero for the MA model.

Let's practice!

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MA model estimation and forecasting

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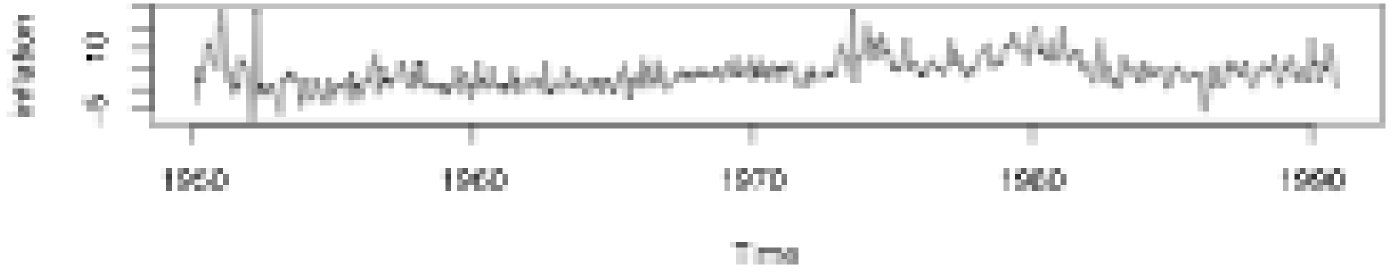
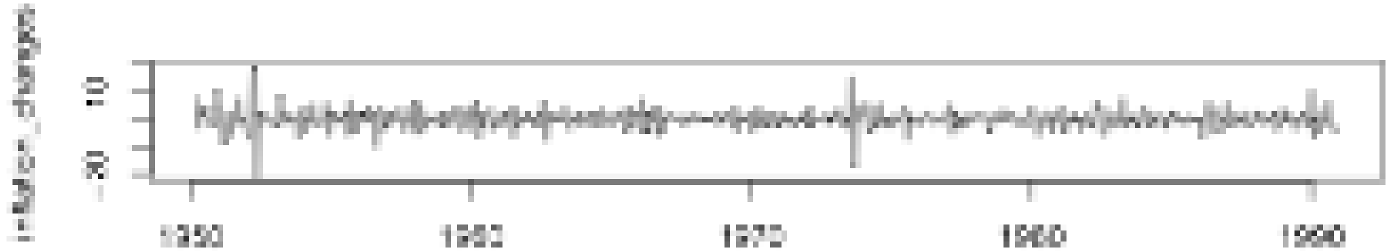
- One-month US inflation rate (in percent, annual rate)
- Monthly observations from 1950 through 1990

```
data(Mishkin, package = "Ecdat")
inflation <- as.ts(Mishkin[, 1])
inflation_changes <- diff(inflation)
ts.plot(inflation) ; ts.plot(inflation_changes)
```

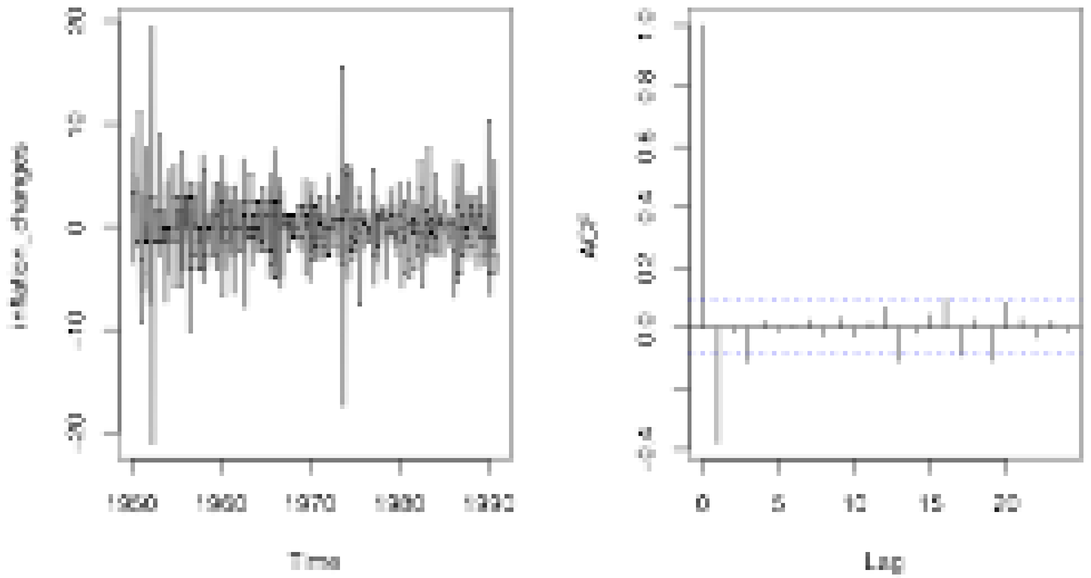




MA processes: changes in inflation tate - II

- `Inflation_changes` : changes in one-month US inflation rate
- Plot the series and its sample ACF:

```
ts.plot(inflation_changes)
acf(inflation_changes, lag.max = 24)
```



$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

$$\epsilon_t \; WhiteNoise(0, \sigma_\epsilon^2)$$

```
MA_inflation_changes <- arima(inflation_changes,
                              order = c(0, 0, 1))
print(MA_inflation_changes)
```

```
Coefficients:
          ma1  intercept
      -0.7932     0.0010
s.e.   0.0355     0.0281
sigma^2 estimated as 8.882
```

ma1 = $\hat{\theta}$, intercept = $\hat{\mu}$, sigma^2 = $\hat{\sigma}_\epsilon^2$

MA processes: fitted values - I

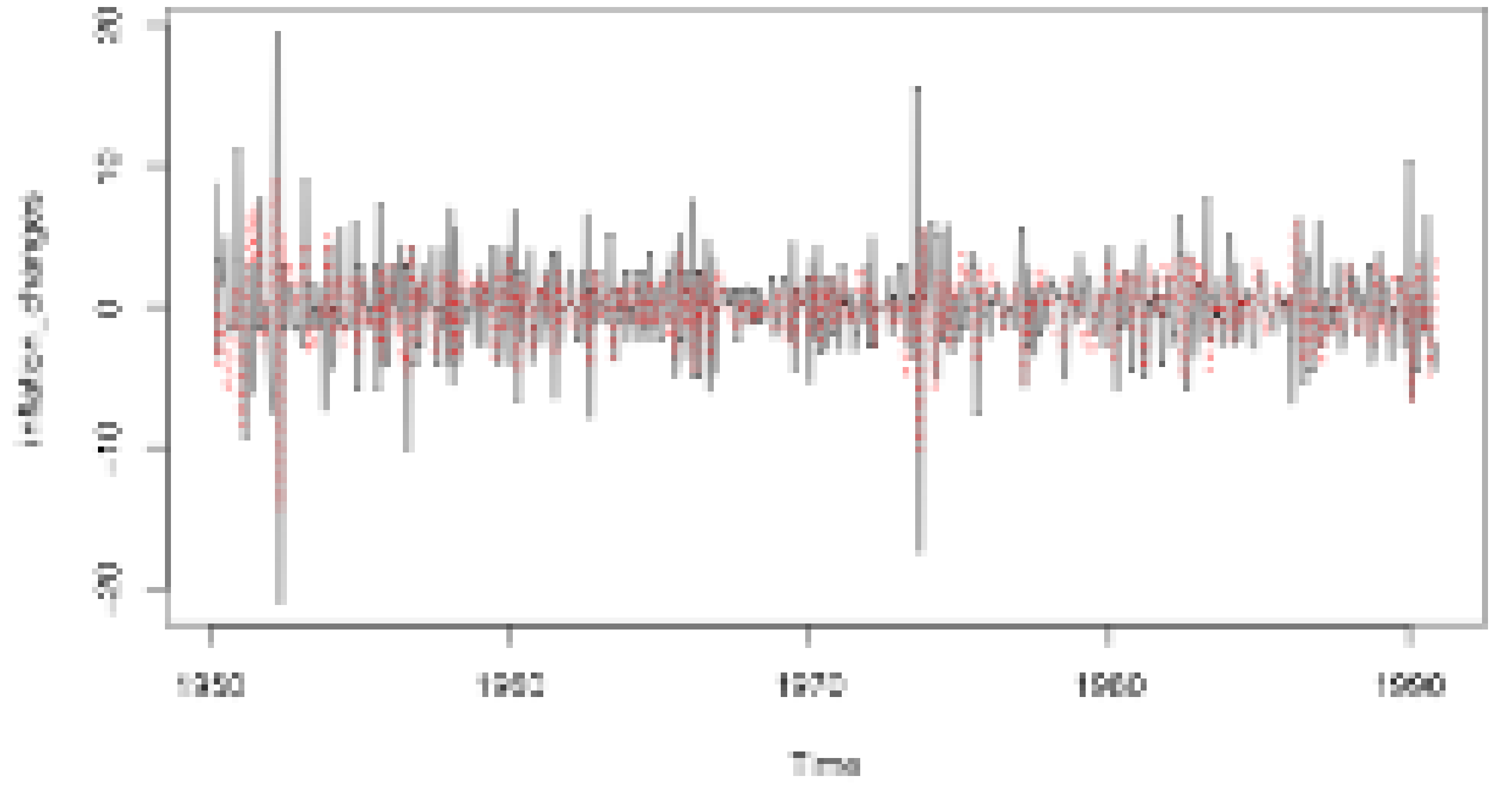
- MA fitted values:

$$\widehat{Today} = \widehat{Mean} + \widehat{Slope} * Yesterday\widehat{'sNoise}$$
$$\hat{Y}_t = \hat{\mu} + \hat{\theta}\hat{\epsilon_{t-1}}$$

- Residuals =

$$Today - \widehat{Today}$$
$$\hat{\epsilon}_t = Y_t - \hat{Y}_t$$

```r
ts.plot(inflation_changes)
MA_inflation_changes_fitted <-
    inflation_changes - residuals(MA_inflation_changes)
points(MA_inflation_changes_fitted, type = "l",
       col = "red", lty = 2)
```



Forecasting

- 1-step ahead forecasts:

```
predict(MA_inflation_changes)$pred
```

```
Jan
1991 4.831632
```

```
predict(MA_inflation_changes)$se
```

```
Jan
1991 2.980203
```

Forecasting (cont.)

- h-step ahead forecasts:

```
predict(MA_inflation_changes, n.ahead = 6)$pred
```

```
          Jan      Feb      Mar      Apr      May      Jun
1991 4.831632 0.001049 0.001049 0.001049 0.001049 0.001049
```

```
predict(MA_inflation_changes, n.ahead = 6)$se
```

```
          Jan      Feb      Mar      Apr      May      Jun
1991 2.980203 3.803826 3.803826 3.803826 3.803826 3.803826
```

Let's practice!

TIME SERIES ANALYSIS IN R

Compare AR and MA models

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MA and AR processes

- MA model:

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- AR model:

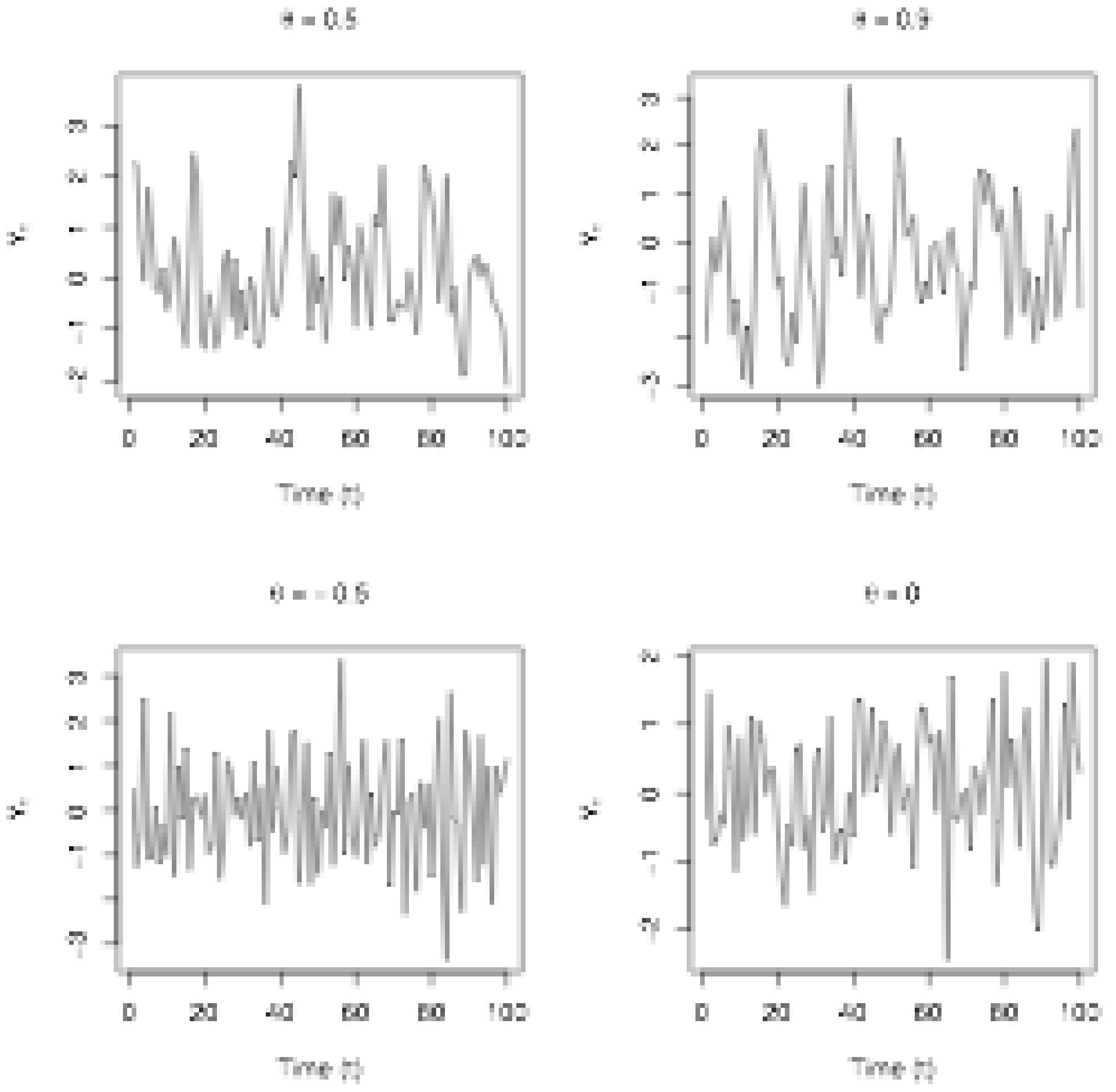
$$(Today - Mean) = Slope * (Yesterday - Mean) \\ +Noise$$

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

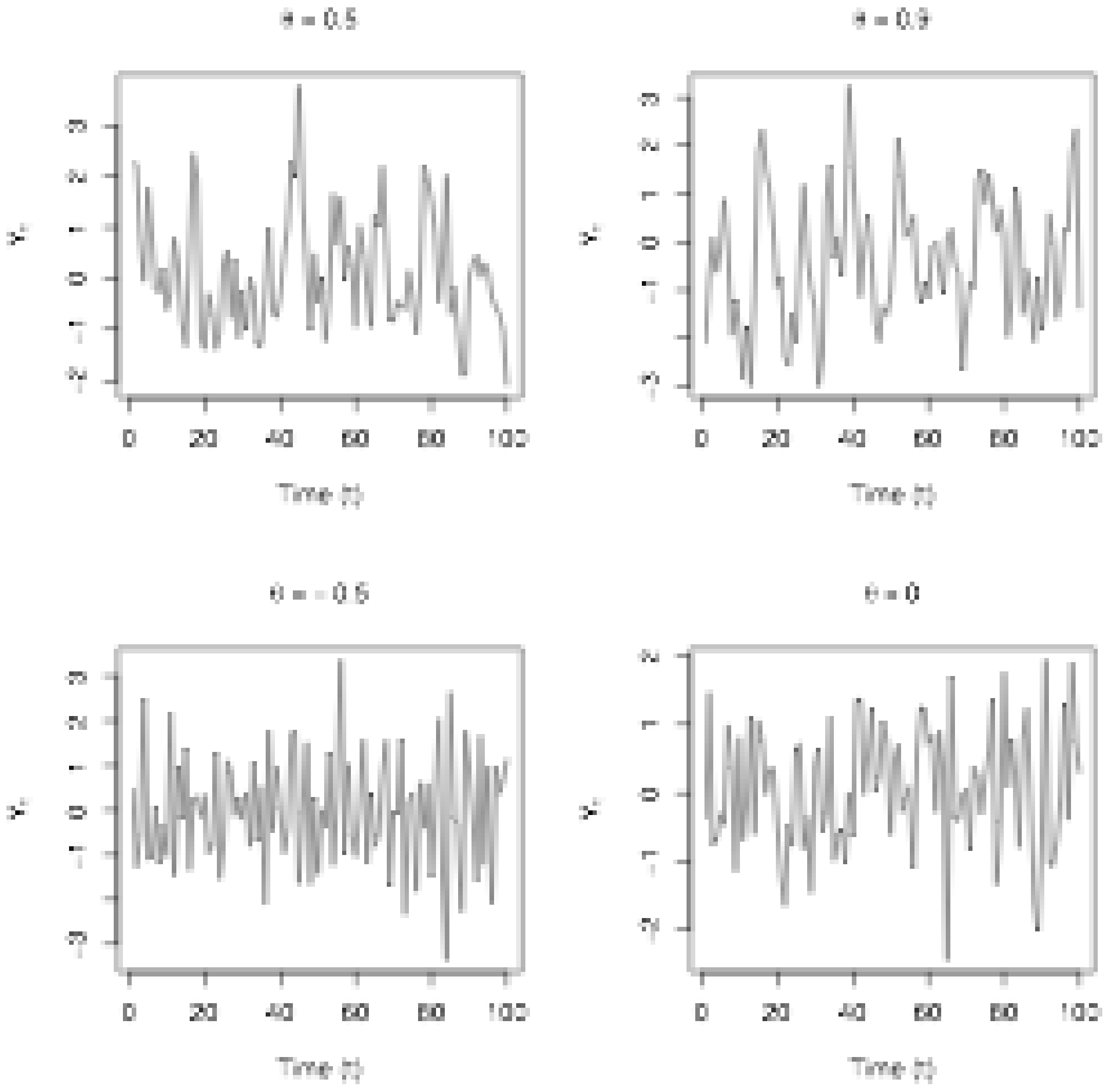
- Where:

$$\epsilon_t \sim WhiteNoise(0, \sigma_t^2)$$

MA and AR processes: autocorrelations

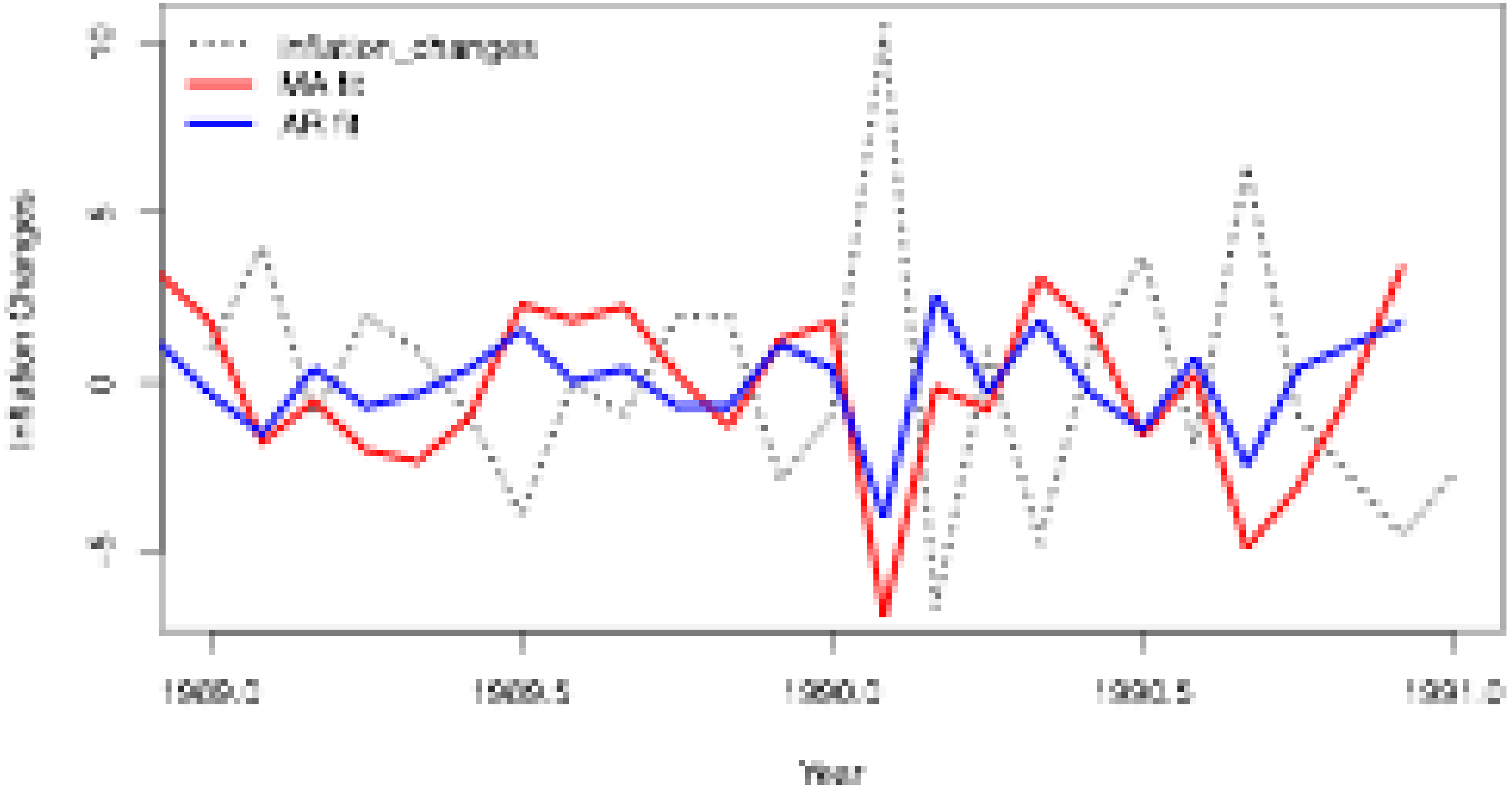


MA and AR processes: simulations



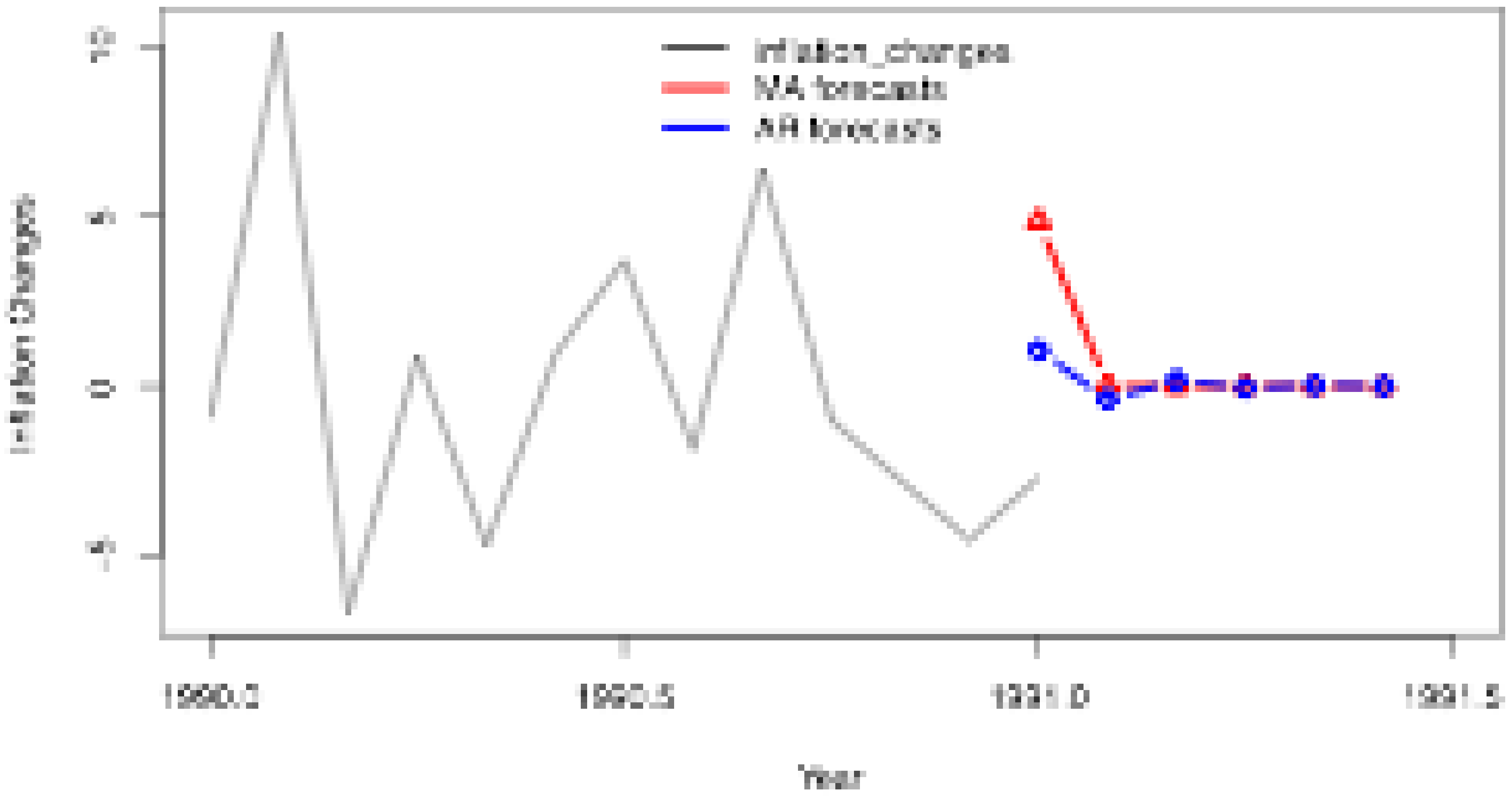
MA and AR processes: fitted values

- Changes in one-month US inflation rate



MA and AR processes: forecasts

- Changes in one-month US inflation rate



```
MA_inflation_changes <-
arima(inflation_changes,
order = c(0,0,1))
```

```
         ma1 intercept
     -0.7932    0.0010
s.e.  0.0355    0.0281
sigma^2 estimated as 8.882:
log likelihood = -1230.85,
aic = 2467.7
```

```
AIC(MA_inflation_changes)
BIC(MA_inflation_changes)
```

```
2467.703
2480.286
```

```
AR_inflation_changes <-
arima(inflation_changes,
order = c(1,0,0))
```

```
         ar1 intercept
     -0.3849    0.0038
s.e.  0.0420    0.1051
sigma^2 estimated as 10.37:
log likelihood = -1268.34,
aic = 2542.68
```

```
AIC(AR_inflation_changes)
BIC(AR_inflation_changes)
```

```
2542.679
2555.262
```

Let's practice!

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Congratulations!

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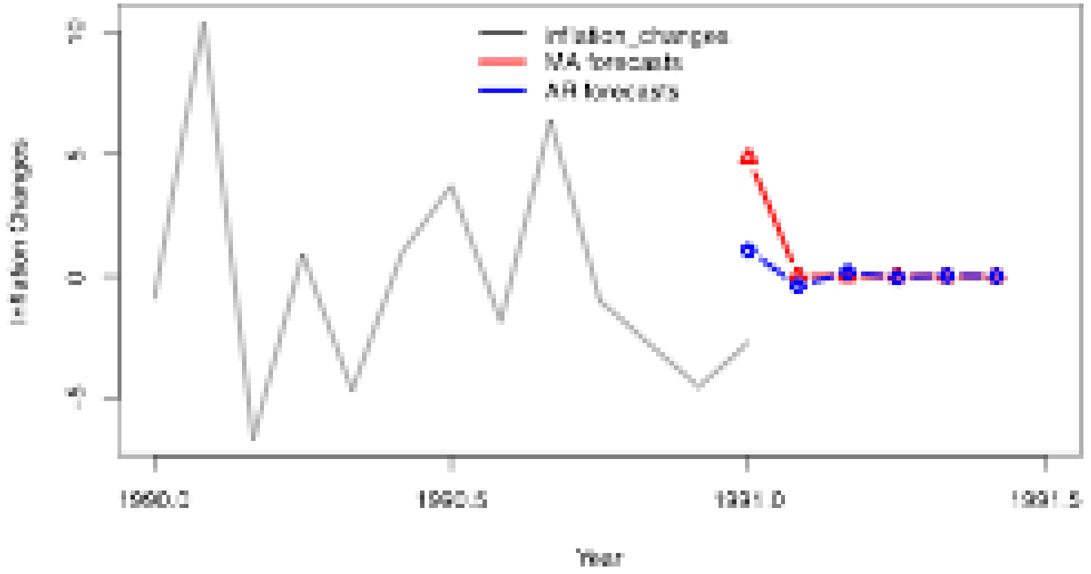
What you've learned

- Manipulating `ts` objects, including `log()` and `diff()`
- Time series models: white noise, random walk, autoregression, simple moving average
- Time series simulation (`arima.sim`), fitting (`arima`), and forecasting (`predict`).



Let's practice!

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