

# Minimum Spanning Tree

## Kruskal's Algorithm ( $\Theta(|E| \log |E|)$ )

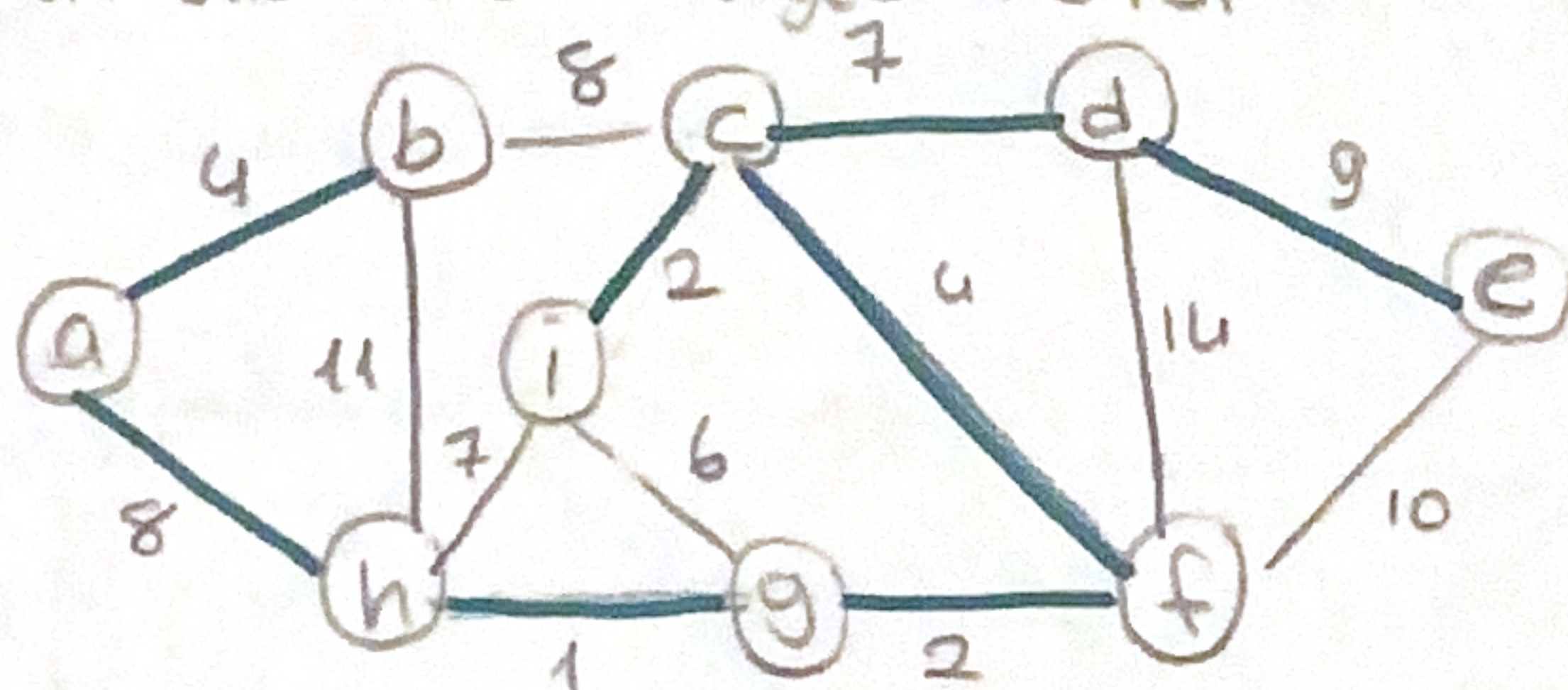
(loglinear)

sparse graph  $\rightarrow O(|V| \log |V|)$   
 dense graph  $\rightarrow O(|V|^2 \log |V|)$   
 better than Prim  
 worse than Prim

1. Sort all the edges from smallest to biggest in weights
2. Pick an edge
3. If the edge connects two vertices in disjoint subsets, merge the subsets, add the edge to F

If all subsets are merged  $\rightarrow$  STOP

exa



1. Assume every single vertex is the subset of the graph
2.  $S = \{\{a\}, \{b\}, \dots, \{i\}\} \rightarrow$  every node.

edges

hg  $\rightarrow$  Pick this 1st it  $F = \{hg\}$  2nd it  $F = \{hg, ci\}$

gf

3rd it

$F = \{hg, ci, gf\}$   
 $S = \{\{a\}, \{b\}, \{i, c\}, \{d\}, \{h, g, f\}, \{e\}\}$   
 subset

ci

cf

ab

gi

cd

hi

ah

bc

dc

cf

bn

4th it

$F = \{hg, ci, gf, ab\}$   
 $S = \{\{ab\}, \{i, c, f, g, h, d\}, \{e\}\}$

5th it

$F = \{hg, ci, gf, ab, cf\}$   
 $S = \{\{ab\}, \{i, c, f, g, h, d\}, \{e\}\}$

6th it

$F = \{hg, ci, gf, ab, cf, cd\}$   
 $S = \{\{ab\}, \{i, c, f, g, h, d\}, \{e\}\}$

7th it

$F = \{hg, ci, gf, ab, cf, cd, ah\}$   
 $S = \{\{ab, c, f, g, h, d\}, \{e\}\}$

8th it

$F = \{hg, ci, gf, ab, cf, cd, ah, de\}$   
 $S = \{a, b, c, d, e, f, g, h, i\}$