Management Science

Group Assignment

Due date: December 5, 23:59

Instructions

1. Each group should submit a pdf file and a notebook. The pdf file should have all names of the group.

2. Your model code (but not pre-processing / post-processing code) **should also be included in the pdf using screenshots**. You should not assume we will read the notebook— but you should still submit one.

3. Handwritten equations are ok, as long as they are clear. Consider using Latexit (MACOS) or KLatexFormula (Windows) to typeset equations.

4. Points awarded are not necessarily proportional to the effort needed to answer a question.

5. Any questions on the assignment should be posted on the corresponding discussion forum in Canvas and not be sent via email.

6. Collaboration across groups is not allowed.

7. There is a limit on the size of problems you can solve with the 3-month license provided with Gurobi's installation. At least one member of each group will have to install a license (after

connecting to the university via VPN) following the instructions in the introductory slides of week 1.

Introduction

Great Wall Airlines (GWA) has been operating exclusively in Asia for the last 10 years with great success. GWA now plans to expand in Europe and has to soon decide how many planes (and of which type) per week to devote to each potential route. GWA has hired a consulting company to research the market and come up with an estimate on daily demands for each origin-destination pair. The estimated demands d_{ij}, along with the distance r_{ij} of each flight in miles is given in the file **flights.csv**.

GWA has access to 3 types of planes, small, medium and large that can accomodate 50, 100 and 300 passengers respectively. The corresponding flying costs per mile are \$4.5, \$8 and \$20.

GWA is planning to operate a complex revenue management system to set prices, but for the purpose of setting up the flight schedule it assumes it can charge a flat price price of \$0.1 per mile, per seat for direct flights, and the aforementioned demands have been estimated under this assumption.

1 Direct flights only (80 points)

Allowing only for direct flights determine how many planes of each type should GWA devote for each origin-destination pair to maximize profit. Note that GWA does not have to satisfy all demand and for some routes might even decide not to allocate a plane.

1. **Modeling** Using mathematical notation, write down an integer program expressing the problem. Clearly explain the meaning of every group of constraints.

Hint: Use integer variable x_{ij} to indicate the number of passengers that will travel daily from i to j, and integer variables s_{ij}, m_{ij}, l_{ij}, to indicate the number of small/medium/large plane flights per day from i to j. Use $\mathcal{D}$ as the set of all direct connections (i, j).

2. **Data Preparation** Read in the csv file using python. Prepare data in a convenient way to ease the implementation of your model in GUROBI ' **Hint**: One, but not the only way, is to create the following dictionaries:

Dictionary name	Example key	Example value
aeroplaneSizes	'small'	50
aeroplaneCost	'medium'	100
DirectDistance	('Berlin','Milan')	523
Demand	('Moscow','Berlin')	169

3. **Implement and solve in GUROBI** Implement your model in GUROBI and solve it.

4. **Analyze Solution** For the optimal solution

   (a) What is the daily profit?

   (b) What is the total number of planes of each type used?

   (c) What is the daily revenue and what is the daily cost?

   (d) What is the profit margin?

   (e) Calculate the utilization of the allocated capacity $\dfrac{\sum_{(i,j)} x_{ij}}{\sum_{(i,j)} (50 s_{ij} + 100 m_{ij} + 300 l_{ij})}$

   (f) Calculate the percentage of lost demand $\dfrac{\sum_{(i,j)} d_{ij} - \sum_{(i,j)} x_{ij}}{\sum_{(i,j)} d_{ij}}$

2 Introducing a hub and spoke system (20 points)

GWA is considering using one airport as a hub, allowing for indirect flights with one stop. GWA only considers indirect flights that increase the traveling distance by less than 30%. Indirect tickets

from i to j via k are sold at a discount for \$0.08 per mile, where the direct distance from i to j is used for the price calculation. GWA assumes that with this discount it can retain the same level of demand even if it redirects part of it to indirect flights. Note that a plane can accommodate passengers of different itineraries. A plane flying from Berlin to Milan, for example, can at the same time accommodate passengers that are flying directly from Berlin to Milan, passengers that are flying from Manchester or Helsinki to Milan via Berlin and passengers that are flying from Berlin to Moscow via Milan.

GWA wants to decide which airport to use as a hub and to recalculate the allocation of planes on each flight leg. The hope is that the increase in efficiency by grouping passengers together will more than compensate for the discounted indirect tickets.

5. **Modeling** Adjust the model of the previous section to reflect the new situation.

   Hints: In addition to the previous variables, use integer variable f_{ikj} to indicate the number of passengers that will travel daily indirectly from i to j via k, and binary variables h_k indicating whether a hub is located at airport k. Use $\mathcal{I}$ as the set of all pottential indirect routes (i, k, j). Assume that $\mathcal{I}$ has been filtered and for every $(i, k, j) \in \mathcal{I}$ there holds

   - $(i, k) \in \mathcal{D}$ and $(k, j) \in \mathcal{D}$
   - $r_{ik} + r_{kj} \leq 1.3 r_{ij}$

   If convenient, you can use $\mathcal{C}$ as the set of all cities. Remember

   - to indicate that the variable f_{ikj} can be strictly positive only if a hub is located at k
   - to ensure that only one hub is permitted.
   - to ensure the total number of customers flying indirectly or directly from i to j does not exceed the total demand.
   - to ensure the total number of people that are passing from the connection (i, j) can be accommodated by the planes allocated on this arc.

6. **Data Preparation** Prepare the data in a convenient way to ease the implementation of your new model in GUROBI.

Hint: You might find it helpful to create an extra dictionary such as:

Dictionary name	Example key	Example value
IndirectRevenuePerCustomer	('Frankfurt','Milan','Madrid')	71.92

It might be convenient if the dictionary only has entries of triplets (i, k, j) with $d_{ik} + d_{kj} < 1.3 \cdot d_{ij}$. That is, ('Frankfurt','Madrid','Milan') should not be in the dictionary. The revenue that a customer pays to fly from Frankfurt to Madrid via Milan was calculated above via $899 \cdot 0.08 = 71.92$ where 899 is the direct distance between Frankfurt and Madrid.

Hint2: You might find useful the following code as a first step

```
df2=df.set_index('departureCity').join(df.set_index('departureCity'),
            lsuffix="first",rsuffix="second").reset_index()
df2=df2[df2.arrivalCityfirst!=df2.arrivalCitysecond]
df2["distance"]=df2.Distancefirst+df2.Distancesecond
df2=df2[["departureCity","arrivalCityfirst","arrivalCitysecond","distance"]]
df2.columns=["hub","arrival","departure","distance"]
```

7. **Solve in GUROBI** Implement your model in GUROBI. If it is taking too long you may limit the solver to 15 minutes (900 seconds) using the model parameter *Params.timeLimit*. In this case, report the percentile gap between the objective value of the solution you computed and the tightest upper bound on the optimal value.

8. **Analyze the solution** Where would you place the hub? Compute the KPIs of Q4.

9. **Analyze the solution** Repeat the exercise for 2 hubs–where indirect flights can still have at most one stop. Only report what you have to change in the model, and the lines of code that you changed in the implementation. Run it for 900 seconds. Where would you place the hubs? Compute the KPIs of Q4.

10. For each KPI create a bar chart visualizing the changes from "no hub" to "1 hub" to "2 hubs". Compare and discuss.