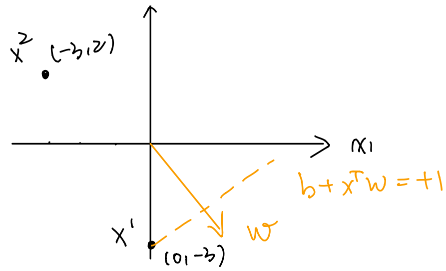
$x^1 = (0,-3) \qquad y_1 = 1$

$x^2 = (-3,2) \qquad y_2 = -1$



Q 2.1

$$\mathcal{L}(w,b,\alpha) = \frac{1}{2}(w_1^2+w_2^2)^2 - \alpha_1 \times (0,-3)\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \alpha_2 \cdot (-3,2)\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + b(\alpha_1-\alpha_2) + \alpha_1+\alpha_2$$

- w^*

$$\frac{\partial L}{\partial w} = 0 \iff w^* = \sum_{i=1}^{2} \alpha_i y_i x^i = \alpha_1 \begin{bmatrix} 0 \\ -3 \end{bmatrix} - \alpha_2 \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3\alpha_2 \\ -3\alpha_1 - 2\alpha_2 \end{bmatrix}$$

- Constrain:

$$\frac{\partial L}{\partial b} = 0 \iff \sum_{i=1}^{2} y_i \alpha_i = \alpha_1 - \alpha_2 = 0 \iff \alpha_1 = \alpha_2$$

- $$\alpha_1^*, \alpha_2^* = \max_{\alpha_1,\alpha_2} \left\{ -\frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \alpha_i \alpha_j y_i y_j x^{i^T} x^j + \sum_{i=1}^{2} \alpha_i \right\}$$

$x^{1^T} \cdot x^1 = 9 \qquad x^{2^T} \cdot x^2 = 13. \qquad x^{1^T} x^2 = -6$

$$= \max_{\alpha_1 \alpha_2} \left\{ -\frac{1}{2}\left(9\alpha_1^2 + 2\cdot(-6)(-1)\alpha_1\alpha_2 + 13\alpha_2^2\right) + \alpha_1 + \alpha_2 \right\}.$$

solve this by the constrain $\alpha_1 = \alpha_2 \geq 0$

$$= \max_{\alpha_1} \left\{ -17\alpha_1^2 + 2\alpha_1 \right\} \quad \Rightarrow \quad \alpha_1 = \frac{1}{17} = \alpha_2$$

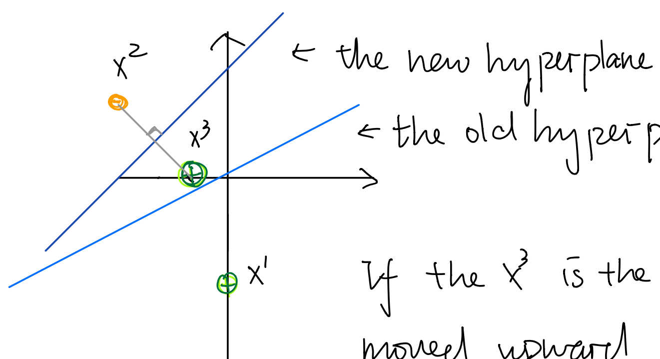
Hence $w^* = \begin{bmatrix} \frac{3}{17} \\ -\frac{5}{17} \end{bmatrix}$

Then the bias term b^* can be computed from x^1

$$y_1(x^{1T}w+b)=1$$

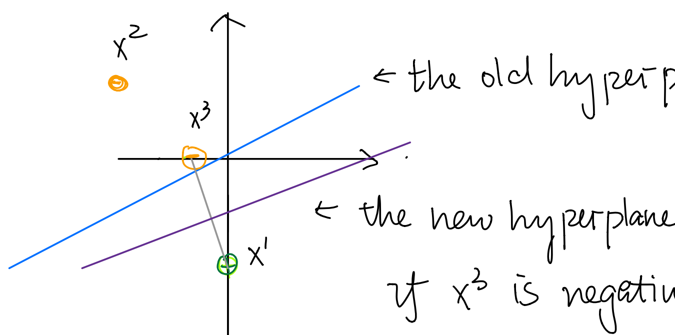
$$\frac{15}{17}+b=1 \Rightarrow b=\frac{2}{17}$$

Q3.2. ① if $x^3=\binom{-1}{0}$ is positive. then it will looks like



If the x^3 is the positive, the hyperplane will be moved upward and get a larger slope.

② if x^3 negative:



If x^3 is negative, the hyperplane will be moved downward. and have a less slope

Q3.3. If we have $x^{test} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, we need to transpose it by the weigh comes from the estimated matrix, that is

$$b + [x^{test\,T}] \cdot w^{est} = \frac{2}{17} + [4\ -1]\begin{bmatrix} \frac{3}{17} \\ -\frac{5}{17} \end{bmatrix} = \frac{19}{17} > 0$$

Hence, the predict label of x should be 1.