Simulation of US Interest Rate (and Risk Premia) Paths

Objective: Create realistic simulation scenarios of US market interest rates (the entire yield curve AND risk premia) at monthly frequency (market trading days only) out to 30 years from now. Approx. 10,000 simulations needed.

Approach 1a: Cox-Ingersoll-Ross Model (or Chen Model) for 3-factor PCA

1. Get Historical Data of (zero-coupon) Interest Rate Paths
    a. Go to Bloomberg (BBG)
    b. Download Daily Data (with dates matched) for the following tenors:
        i. Overnight Fed Funds Effective Rate
        ii. 1-month LIBOR/SOFR (If BBG doesn't have a single index to capture this, please paste together LIBOR's past history with SOFR's more recent history after 2021).
        iii. 3-month LIBOR/SOFR
        iv. 6-month Treasury
        v. 1-Year Treasury (USGGY01 Index is the ticker?)
        vi. 2-Year Treasury
        vii. 5-Year Treasury
        viii. 10-Year Treasury
        ix. 30-Year Treasury
    c. As a check, please compare the BBG Treasury data with the Fed's estimated data here:
        i. https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html
2. Do Principal Components Analysis (PCA) on the entire Yield Curve (vector of 10 interest rates)
    a. Get historical data on estimates for first 3 components
        i. Level, Slope, and Curvature
    b. Check which components can turn negative and which remain positive
        i. I think Level should always be positive
        ii. Slope and Curvature can be negative
3. Simulate Level, Slope, and Curvature according to the CIR Model (or more advanced "Chen Model")
    a. Wiki reference: https://en.wikipedia.org/wiki/Cox%E2%80%93Ingersoll%E2%80%93Ross_model
    b. Wiki reference: https://en.wikipedia.org/wiki/Chen_model
    c. Use historical data to estimate hyperparameters
    d. Then simulate CIR or Chen Model-style stochastic Brownian process for the LEVEL principal component
    e. For the components like slope and curvature that can become NEGATIVE, drop the square-root component from the stochastic component, i.e. just use Ornstein-Uhlenbeck mean-reverting process
    f. https://en.wikipedia.org/wiki/Ornstein%E2%80%93Uhlenbeck_process

g. Some code: https://towardsdatascience.com/stochastic-processes-simulation-the-cox-ingersoll-ross-process-c45b5d206b2b

4. Reconvert the simulated 3 principal components and expand them back into the simulated yield curves (a vector of ten interest rates) using historical loadings
5. Create monthly averages of the yield curve
6. Create monthly estimates of the risk-neutral yields and risk premia inside each simulation using Adrian-Crump-Moench (ACM) estimation technique
    a. Reference link: https://www.newyorkfed.org/research/data_indicators/term-premia-tabs#/overview
    b. Some code: https://github.com/miabrahams/PricingTermStructure
7. Generate estimates of risk-neutral yields and risk premia for each tenor
8. Check RMSE of forecasting power of risk-neutral yields

Approach 1b: Cox-Ingersoll-Ross Model (or Chen Model) for 5-factor PCA

9. Same as step 1 above.
10. Do Principal Components Analysis (PCA) on the entire Yield Curve (vector of 10 interest rates)
    a. Get historical data on estimates for first FIVE components
    b. Check which components can turn negative and which remain positive
        i. First component SHOULD always be positive
        ii. Others can be negative
11. Simulate First Component ONLY according to the CIR Model (or more advanced “Chen Model”)
    a. Wiki reference: https://en.wikipedia.org/wiki/Cox%E2%80%93Ingersoll%E2%80%93Ross_model
    b. Wiki reference: https://en.wikipedia.org/wiki/Chen_model
    c. Use historical data to estimate hyperparameters
    d. Then simulate CIR or Chen Model-style stochastic Brownian process for the FIRST principal component
    e. For remaining four components that can become NEGATIVE, drop the square-root component from the stochastic component, i.e. just use Ornstein-Uhlenbeck mean-reverting process
    f. https://en.wikipedia.org/wiki/Ornstein%E2%80%93Uhlenbeck_process
    g. Some code: https://towardsdatascience.com/stochastic-processes-simulation-the-cox-ingersol
12. Create monthly averages of the 5 principal components
13. Create monthly estimates of the risk-neutral yields and risk premia inside each simulation using Adrian-Crump-Moench (ACM) estimation technique
    a. Given that we already have the 5 principal components, we can skip the PCA step in ACM

    b. Reference link: https://www.newyorkfed.org/research/data_indicators/term-premia-tabs#/overview
    c. Some code: https://github.com/miabrahams/PricingTermStructure
14. Generate estimates of risk-neutral yields and risk premia for each tenor
15. Check RMSE of forecasting power of risk-neutral yields

Approach 2: Adrian-Crump-Moench VAR model shock simulation

1. Simulate the 5 (or 3) PCA factors by iterating the VAR inside the ACM Model with new Gaussian shocks