Control of a Stationary Self-Balancing Two-wheeled Vehicle

Important note: The due date is 15/11/2022. Late submission is absolutely not allowed as the grades have to be submitted to the department very soon after the final exam. You may work together with your classmates. But do write your report independently. And the results are supposed to be different from each other as the parameters are based upon your matriculation numbers.

1 Background

We all had a tough time when learning to ride a bicycle when we are a teenager. It usually takes months to master that skill after crashing into walls for hundreds of times. Needless to say even after that, it is still difficult for us to ride on uneven surface or turn when riding in a high speed. Would that be excited if such two-wheeled vehicle comes to the market that it can self-balance itself to improve its stability and driving safety?

Self-sustaining two-wheeled vehicle not only is a proof of how control theory has been developed during the past decades, but also has a huge market potential. Therefore, a lot of researchers from universities and companies are working on related topics. Although most of the study are still in experimental stage, there are research groups and startups that have already published demonstration video online, such as the C-1 motorcycle from Lit Motors [1].

Figure 1 is a screenshot from a demonstration video on YouTube. As we can see, the vehicle looks like a motorcycle from outside, but inside the vehicle the driver drives as if it is a car. The vehicle self-balances itself when running on the road or even when it is still. This two-wheeled self-balancing vehicle is said to combine the virtues of both the car and the motor: safety and low cost.

Figure 1 Two-wheeled self-balancing electric car/motor [1]

Since there are many more dynamics involved when the vehicle is running, in this mini-project we only consider the self-balance of the two-wheeled vehicle when it is stationary. We will try to balance this vehicle using the control methods we have learned in *Linear Systems*.

2 Modelling

For model-based control, the first step is to build an effective dynamic model for our target plant, i.e., the two-wheeled vehicle in this project. The detailed procedures to model this vehicle can be found in [2] and [3]. Here we only give a short introduction and the resulted state space model.

An experimental system for the two-wheeled vehicle prototype is shown in Figure 2. The two-wheeled vehicle consists of three parts. There is a cart system that corresponds to the rider's center-of-gravity movement, a steering system (a front part) for steering, and a body (a rear part). The front part and the rear part are structures that are movable through a steering axis. A cart system and a steering system are driven by DC servo motor, and DC motors are controlled by servo amplifier which contains the velocity control system. Handle angle and cart position are measured by encoders. Attitude angles of the two-wheeled vehicle (roll angle and yaw angle) are measured by gyroscopes.

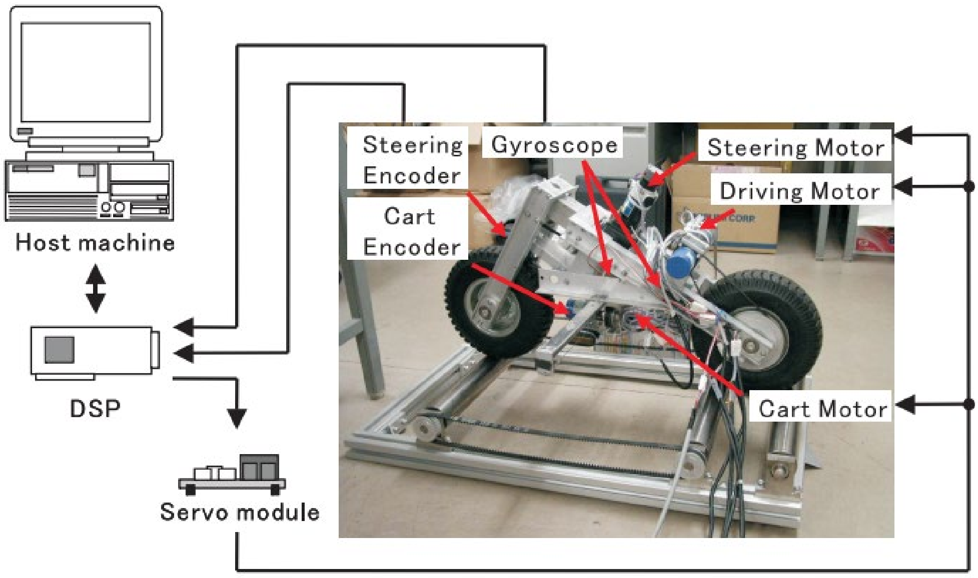

Figure 2 Composition of experimental system

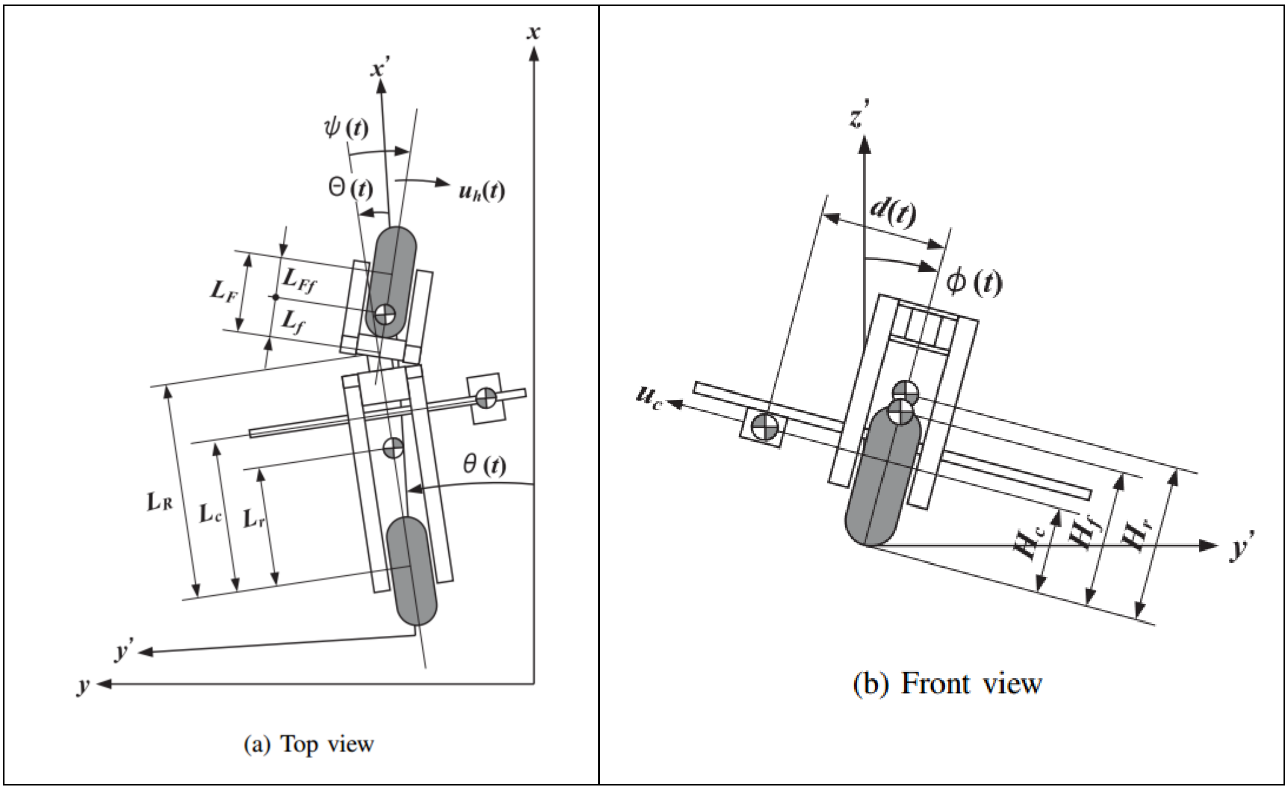

Figure 3 Two-wheeled vehicle structure model

The mechanical structure for the two-wheeled vehicle is given in Figure 3. The two-wheeled vehicle is stabilized by moving the cart position $d(t)$ and adjusting the handle angle $\psi(t)$. The control inputs are the voltages $u_c(t)$ and $u_h(t)$ to two DC servo motors, which drives the cart system and the steering system correspondingly.

For the dynamic model, the relevant symbols are defined in Table 1.

Table 1 Definition of Symbols

M_f, M_r, M_c	Mass of each part
H_f, H_r, H_c	Vertical length from a floor to a center-of-gravity of each part
L_{Ff}, L_F	Horizontal length from a front wheel rotation axis to a center-of-gravity of part of front wheel and steering axis.
L_r, L_R	Horizontal length from a rear wheel rotation axis to a center-of-gravity of part of rear wheel and steering axis.
L_c	Horizontal length from a rear wheel rotation axis to a center-of-gravity of the cart system
J_x	Moment of inertia around center-of-gravity x axially
J_{fz}	Moment of inertia for part of front wheel z axially.
J_z	Moment of inertia for part of rear wheel that contains cart system z axially.
μ_x	Viscous coefficient around x axis.
μ_{fz}	Viscous coefficient for part of front wheel around z axis.
μ_z	Viscous coefficient for part of rear wheel that contains cart system around z axis.
μ_c	A viscosity coefficient of a movement direction of the cart system
Subscript f, r, c	Part of front wheel, rear wheel, and cart system respectively
$d(t)$, $\phi(t)$, $\psi(t)$	Cart position, handle angle and bike angle

In [2], the state space linear model for the two-wheeled vehicle is derived to be

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where the state variable is

$$x = \begin{bmatrix} d(t) & \phi(t) & \psi(t) & \dot{d}(t) & \dot{\phi}(t) & \dot{\psi}(t) \end{bmatrix}^T \tag{2}$$

and the matrices and the input vector are[1]

[1] Some additional coupling terms are fabricated to facilitate our design.

$$A=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 6.5 & -10 & -\alpha & 0 & 0\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56}\\ 5 & -3.6 & 0 & 0 & 0 & -\gamma\end{bmatrix},\quad B=\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \beta & 11.2\\ b_{51} & b_{52}\\ 40 & \delta\end{bmatrix} \tag{3}$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix},\quad u=\begin{bmatrix}u_c(t), & u_h(t)\end{bmatrix}^T \tag{4}$$

The parameters in (3) can be calculated as

$$a_{51}=-\frac{M_c g}{den},\quad a_{52}=\frac{(M_f H_f+M_r H_r+M_c H_c)g}{den},\quad a_{53}=\frac{(M_r L_r L_F+M_c L_c L_F+M_f L_{Ff} L_R)g}{(L_R+L_F)den}$$
$$a_{54}=-\frac{M_c H_c \alpha}{den},\quad a_{55}=-\frac{\mu_x}{den},\quad a_{56}=\frac{M_f H_f L_{Ff}\gamma}{den} \tag{5}$$
$$b_{51}=\frac{M_c H_c \beta}{den},\quad b_{52}=-\frac{M_f H_f L_{Ff}\delta}{den},\quad den=M_f H_f^2+M_r H_r^2+M_c H_c^2+J_x$$

where g is the gravitational acceleration, $g\approx 9.8m/s^2$.

The physical parameters in (5) can be measured directly or identified by experiments. The value of all these physical parameters is summarized in Table 2.

Table 2 Physical parameters of the two-wheeled vehicle

Parameter	Value	Parameter	Value
M_f $[kg]$	$2.14+c/20$	H_f $[m]$	0.18
M_r $[kg]$	$5.91-b/10$	H_r $[m]$	0.161
M_c $[kg]$	1.74	H_c $[m]$	0.098
L_{Ff} $[m]$	0.05	L_F $[m]$	0.133
L_r $[m]$	0.128	L_R $[m]$	$0.308+(a-d)/100$
L_c $[m]$	0.259		
J_x $[kgm^2]$	$0.5+(c-d)/100$	μ_x $[kgm^2/s]$	$3.33-b/20+ac/60$
α	$15.5-a/3+b/2$	β	$27.5-d/2$
γ	$11.5+(a-c)/(b+d+3)$	δ	$60+(a-b)c/10$

where in Table 2 a, b, c, d represent the last four digits in your matriculation number. For example, if your matriculation number is A0162903M, then $a=2,\ b=9,\ c=0,\ d=3$ and one of the parameters can be computed as $\mu_x=3.33-9/20+2*0/60=2.88$.

3 Control System Design

After all, we get a linear state space model (1) for the stationary two-wheeled vehicle. In the following, different control strategies will be explored to stabilize this vehicle to achieve its self-balance. We will target both the regulation and set point tracking problems. The initial condition for the two-wheeled vehicle system (1) is assumed to be $x_0 = \left[0.2, \quad -0.1, \quad 0.15, \quad -1, \quad 0.8, \quad 0\right]^T$.

3.1 Design specifications

The transient response performance specifications for all the outputs y in state space model (1) are as follows:

1) The overshoot is less than 10%.
2) The 2% settling time is less than 5 seconds.

Note: (a) This transient response is checked by giving a step reference signal for each input channel, i.e., [1, 0] and [0, 1], with zero initial conditions; (b) For all the following task 1) to 5), your control system should satisfy this performance specification and you are supposed to finish the required investigation for each task as well.

3.2 Tasks

Your study should include, but not limited to

1) Assume that you can measure all the six state variables, design a state feedback controller using the pole place method, simulate the designed system and show all the six state responses to non-zero initial state with zero external inputs. Discuss effects of the positions of the poles on system performance, and also monitor control signal size. In this step, both the disturbance and set point can be assumed to be zero. (10 points)

2) Assume that you can measure all the six state variables, design a state feedback controller using the LQR method, simulate the designed system and show all the state responses to non-zero initial state with zero external inputs. Discuss effects of weightings Q and R on system performance, and also monitor control signal size. In this step, both the disturbance and set point can be assumed to be zero. (10 points)

3) Assume you can only measure the three outputs. Design a state observer, simulate the resultant observer-based LQR control system, monitor the state estimation error, investigate effects of observer poles on state estimation error and closed-loop control performance. In this step, both the disturbance and set point can be assumed to be zero. (10 points)

4) Suppose we are only interested in the two outputs $d(t)$ and $\psi(t)$, i.e., a new output matrix is

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

then we get a 2-input-2-output system. Design a decoupling controller with closed-loop stability and simulate the step set point response of the resultant control system to verify decoupling performance with stability. In this step, the disturbance can be assumed to be zero. Is the decoupled system internally stable? Please provide both the step (transient) response with zero initial states and the initial response with respect to x_0 of the decoupled system to support your conclusion. (10 points)

5) Assume that the operating set point for the three outputs is

$$y_{sp} = -\frac{1}{10} C A^{-1} B \begin{bmatrix} -0.5 + (a-b)/20 \\ 0.1 + (b-c)/(a+d+10) \end{bmatrix}$$

where a, b, c, d are still the last four digits in your matriculation number, as defined above. Therefore, the objective of the controller is to maintain the output vector around this operating set point as close as possible.

Assume that you only have three cheap sensors to measure the output. Design a controller such that the plant (vehicle) can operate around the set point as close as possible at steady state even when step disturbances are present at the plant input. Plot out both the control and output signals. In your simulation, you may assume the step disturbance for the two inputs, $w = [-1,\ 1]^T$ takes effect from time $t_d = 10s$ afterwards. (10 points)

6) We have learned about the multivariable integral control using state space model in Chapter 9. It is a classical way to solve the set point tracking problem even when a constant disturbance is involved. Now for the two-wheeled vehicle, can we maintain the three outputs at an arbitrary constant set point with zero steady-state error? You can try the integral control method or any other method you figure out. You can use simulations to test various set points and see the results. Please give a formal mathematical analysis/proof for your conclusion. (10 points)

Note that there are no unique answers to all the above design questions. For the tasks in our project, you can assume that the control input is unlimited. However, in practice all the physical actuators can only provide a limited drive capacity. You need to make your own judgement assuming you are the engineer responsible for the control system design in the real world. There are three major factors you should consider when you design and justify your controller:

- Speed --- Transient response
- Accuracy --- Steady state error
- Cost ---- Size of the control signals

4 Reference

[1] 2014 Lit Motors C-1 - YouTube More videos can be found on YouTube such as https://www.youtube.com/watch?v=zb51CvptTt4.

[2] Satoh, H. and Namerikawa, T., 2006. Modeling and robust attitude control of stationary self-sustaining two-wheeled vehicle. Nippon Kikai Gakkai Ronbunshu C Hen (Transactions o f the Japan Society of Mechanical Engineers Part C)(Japan), 18(7), pp.2130-2136.

[3] Satoh, H. and Namerikawa, T., 2007, October. Robust stabilization of running self-sustai ning two-wheeled vehicle. In 2007 IEEE International Conference on Control Applications (pp. 539-544). IEEE.

5 Format of Reports

Your report should mainly contain the plant description, control and observer design method description, your design details, simulation results, possible comparison, comments and discussion, modification and refinements.

The report should include the following and be organized in the following sequence:

- A cover paper to indicate "Assignment for EE5101/ME5401 (or your specialization code if else) Linear Systems", a title of your report at your choice, your full name, your Matriculation number, email address and date;
- An abstract of 50-100 words on a separate page;
- A contents table on a separate page;
- Section 1 Introduction
- The major materials of your report organized nicely in a few sections each with specific focus. Label your equations, tables, and figures with number and caption for reference in the text. Your figure size and figure quality should be high enough to facilitate the verification of your results.
- The last section on conclusions.
- A list of reference books/papers if any;
- Appendices if any each on a separate page. Your MATLAB code should be in this appendix. If you use Simulink, a screenshot of your Simulink model should be inserted at proper position in the above major materials part as figures.

Pay attention to your presentation (English writing, organization, and layout et al). Make the report formal, complete and readable. It is also advisable to write your report with a word-processing software such as Word or LaTeX.

The final point to note about your report: it is the content that matters not the length. Keep in mind that there are only TWENTY SEVEN pages in John Nash's PhD thesis, which leaded to his Nobel Prize. Therefore, you will be penalized if you put too much "copy and paste" material in your report.

6 A Note on Access and Use of MATLAB

To complete the project, you are supposed to use SIMULINK and MATLAB. The easy way is to learn how to build various block diagrams in SIMULINK first, and then try to solve the control systems design for the mini-project. An excellent *Control Tutorial for MATLAB and Simulink* can be found at http://ctms.engin.umich.edu/CTMS/index.php?aux=Home. Besides, a Matlab manual is provided in IVLE for the first timers.

If you don't have MATLAB on your PC currently, you can access MATLAB in either of the following two ways:

1) Go to PC clusters located at the third floor of E2: http://www.eng.nus.edu.sg/eitu/pc.html.
2) Download MATLAB from NUS information technology center: every NUS student can have a license. https://nusit.nus.edu.sg/services/software_and_os/software/software-student/#install-matlab.

Hint on MATLAB/SIMULINK:

A. You can use functions such as *step*, *initial* and *lsim* to simulate the system's corresponding response. Also, all these simulations can be done with SIMULINK.

B. In some cases, it may be easier to use SIMULINK for the simulation, for example, question 5).