CS5100 Foundations of Artificial Intelligence

Module 02 Lesson 03

Uninformed Search
and Informed Search

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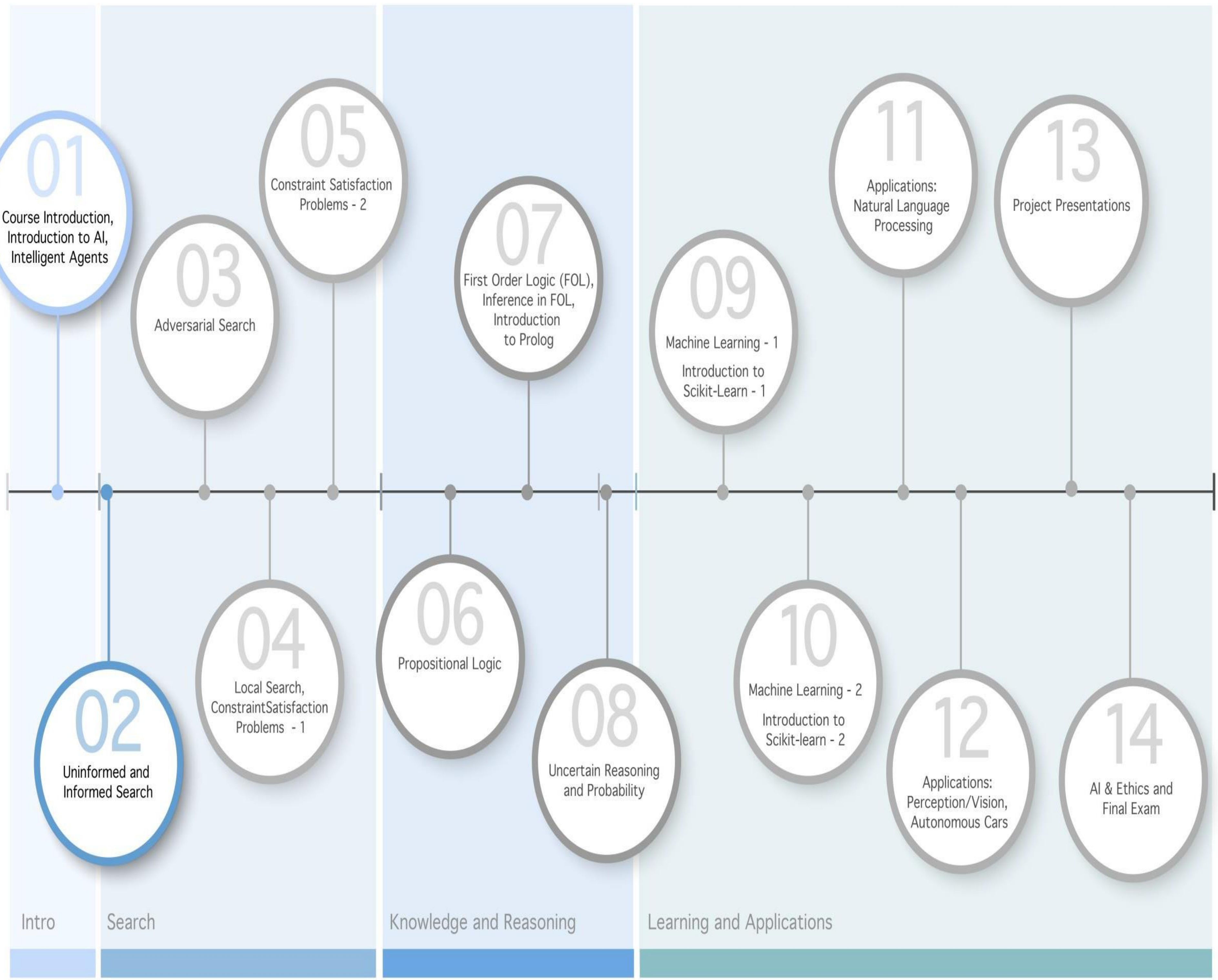
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Overview

- Search Problems
- Uninformed Search Methods
- Depth-First Search
- Breadth-First Search
- Uniform-Cost Search

In this module…

- We consider
  - Problem-solving agents
  - Using atomic representations

  - Simplest environments: episodic, single agent, fully observable, deterministic, static, discrete, and known
  - In later weeks we will look at unknown environments and multi-agent ones

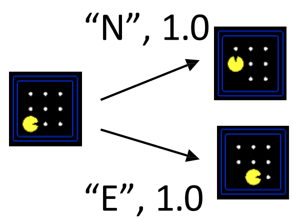
Outline

- Problem-solving agents
- Example Problems
- Problem formulation
- Search Algorithms
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies
- Heuristic Functions

Search Problems

- A search problem consists of:
  - A state space

  - A successor function (with actions, costs)



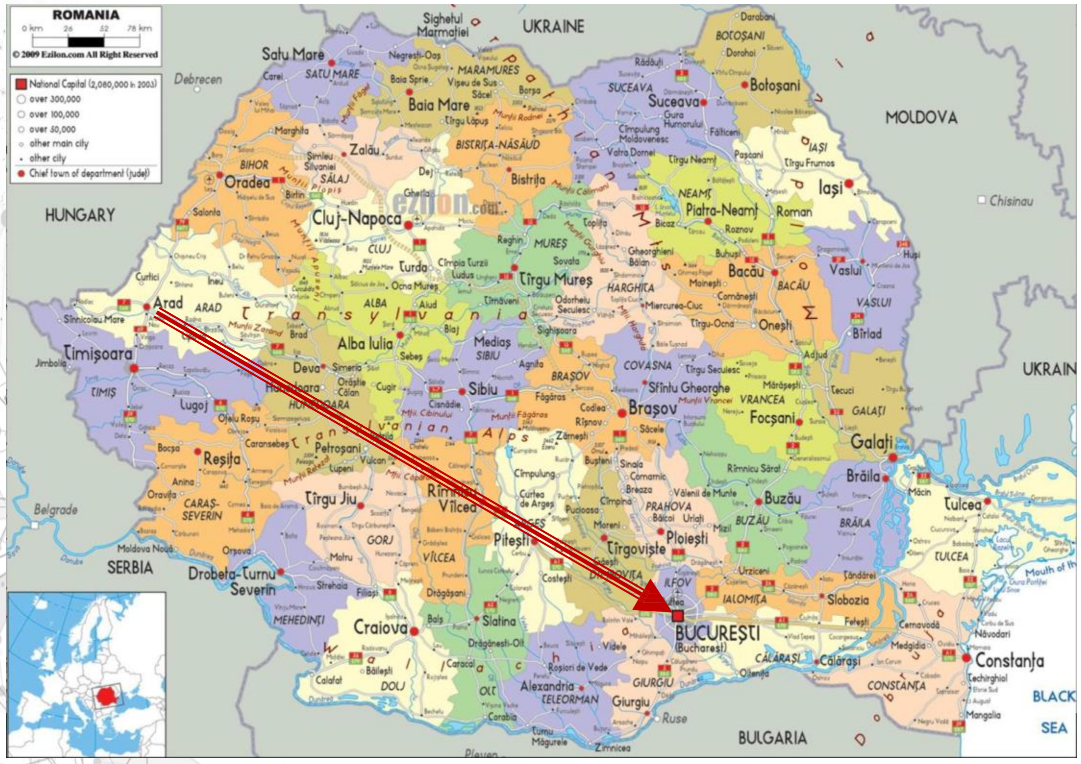
  - A start state and a goal test

- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Other equivalent ways to formulate search problems in other texts/editions

Search Problems are Models

Example: Traveling in Romania, from Arad to Bucharest



Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:
  be in Bucharest

Formulate problem:
  states: various cities
  actions: drive between cities

Find solution:
  sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

State space:
  Cities

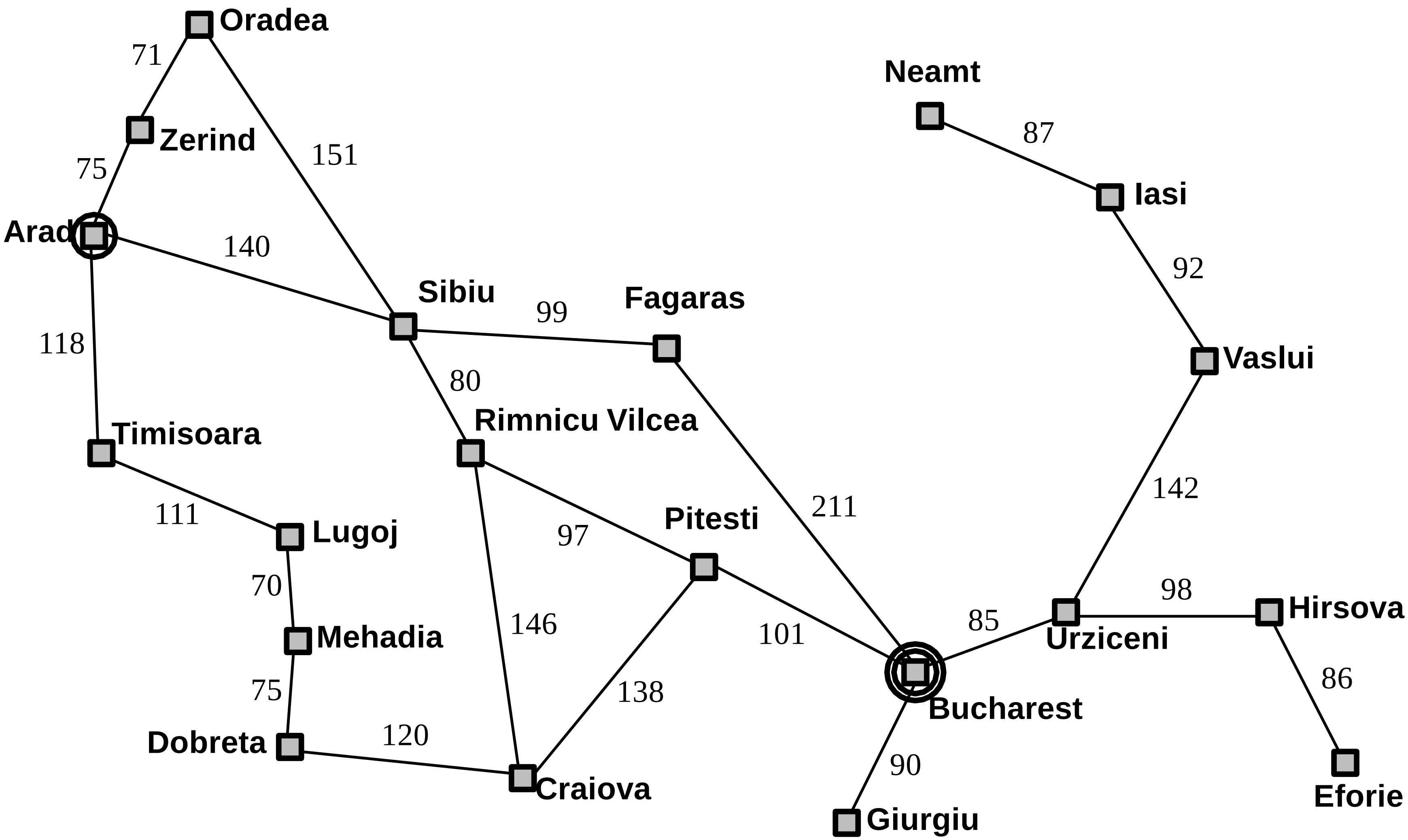
Successor function:
  Roads: Go to adjacent city with cost = distance

Start state:
  Arad

Goal test:
  Is current state = Bucharest?

Solution?

Example: Romania



State space:

Cities

Successor fu

Roads:
with co

Start state:

Arad

Goal test:

Is curre
Buchar

Solution?

Single-state problem formulation

A problem is defined by four items:

initial state   e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs

e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind\rangle, \ldots\}$

goal test, can be

explicit, e.g., x = "at Bucharest"

implicit, e.g., $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of actions executed, etc.

$c(x, a, y)$ is the step cost, assumed to be ≥ 0

A solution is a sequence of actions
leading from the initial state to a goal state

Selecting a state space

Real world is absurdly complex
⇒ state space must be abstracted for problem solving

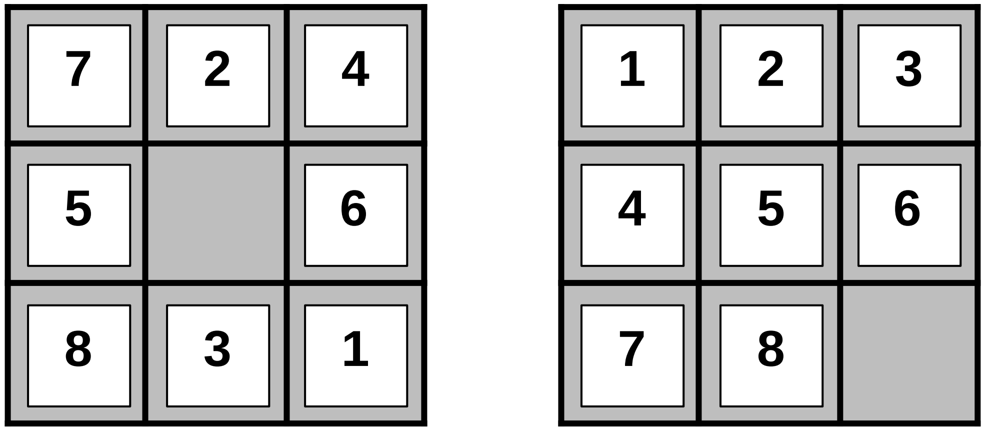
(Abstract) state = set of real states

(Abstract) action = complex combination of real actions
e.g., "Arad → Zerind" represents a complex set
of possible routes, detours, rest stops, etc.
For guaranteed realizability, any real state "in Arad"
must get to some real state "in Zerind"

(Abstract) solution =
set of real paths that are solutions in the real world

Each abstract action should be "easier" than the original problem!

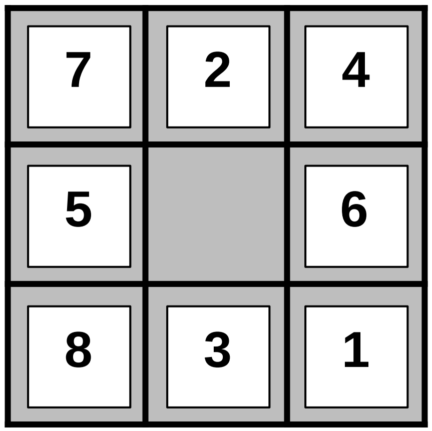
Example: The 8-puzzle



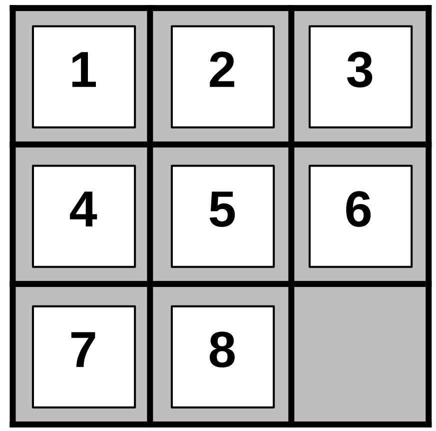
Start State **Goal State**

states??
actions??
goal test??
path cost??

Example: The 8-puzzle



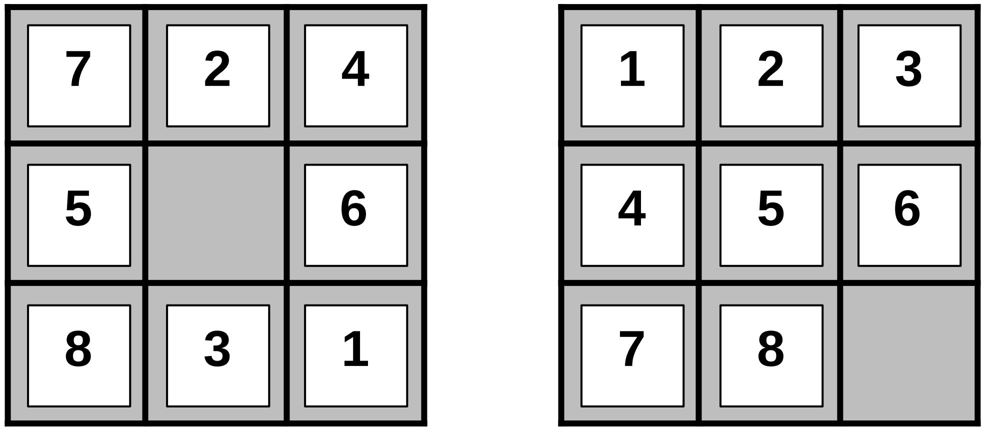
Start State



Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??
goal test??
path cost??

Example: The 8-puzzle



Start State **Goal State**

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??
path cost??

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??: 1 per move

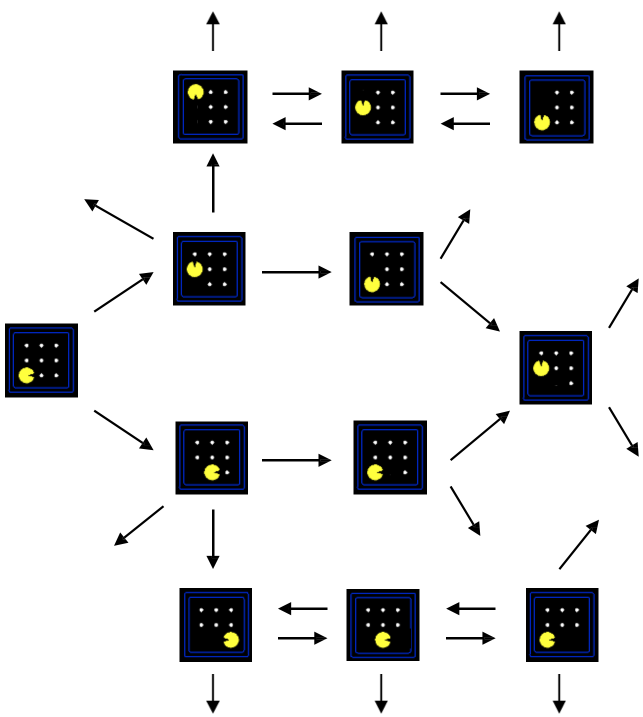
State Space Graphs

State space graph: A mathematical representation of a search problem

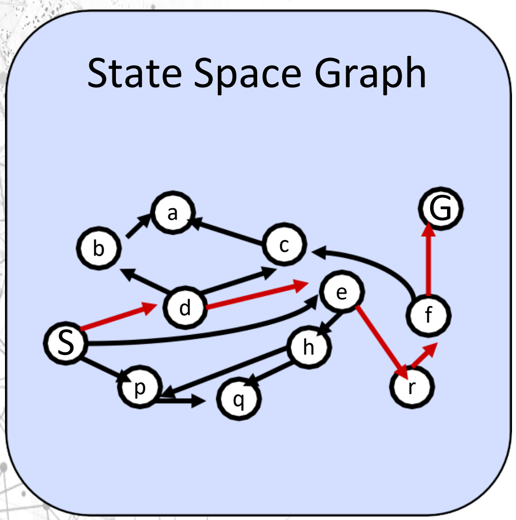
- Nodes are (abstracted) world configurations
- Arcs represent successors (action results)
- The goal test is a set of goal nodes (one or more)

In a state space graph, each state occurs only once!

Usually too big to build in memory, but an useful idea

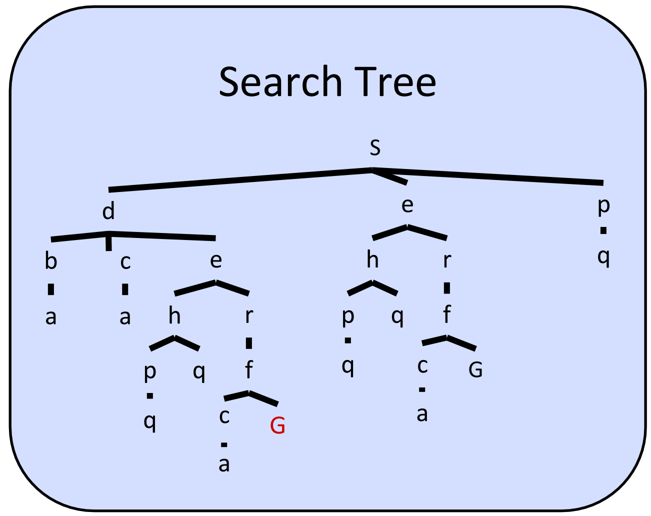


State Space Graphs versus Search Trees



Each NODE in in the search tree is an entire PATH in the state space graph.

We construct both on demand – and we construct as little as possible.



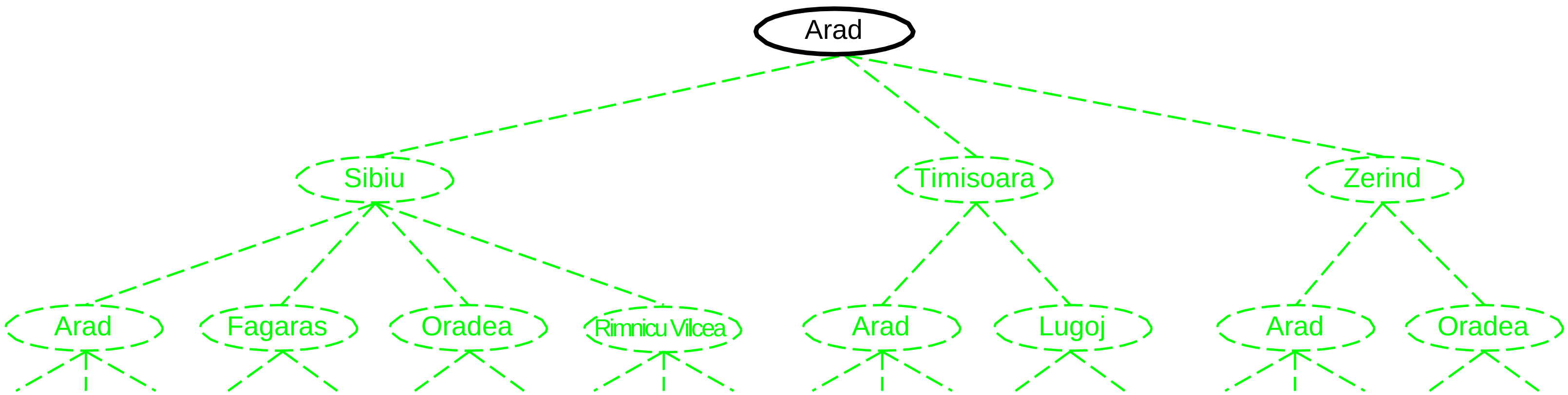
Tree search algorithms

Basic idea:

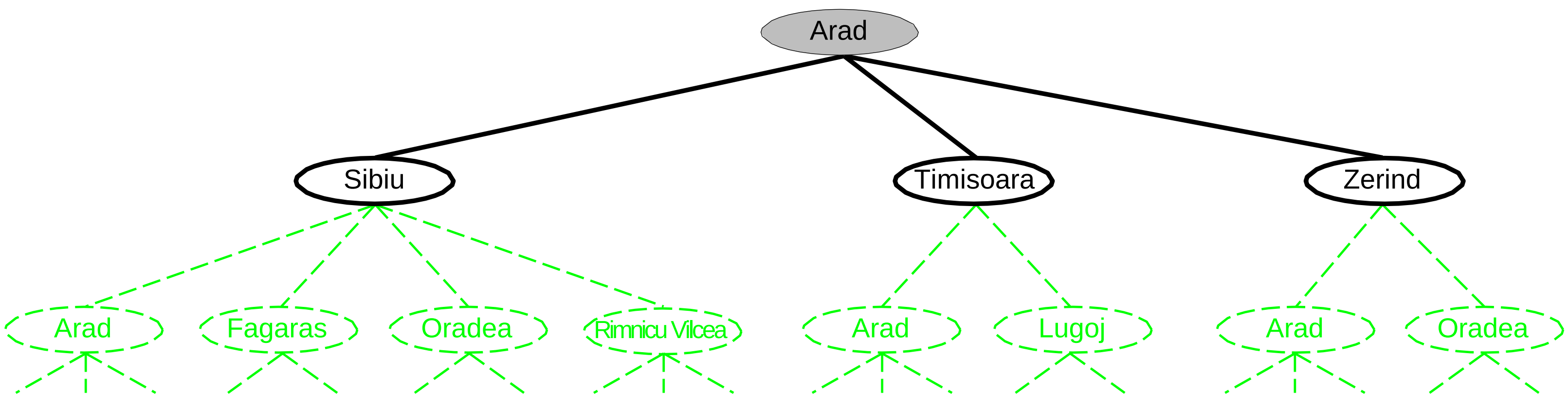
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

```
function Tree-Search( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

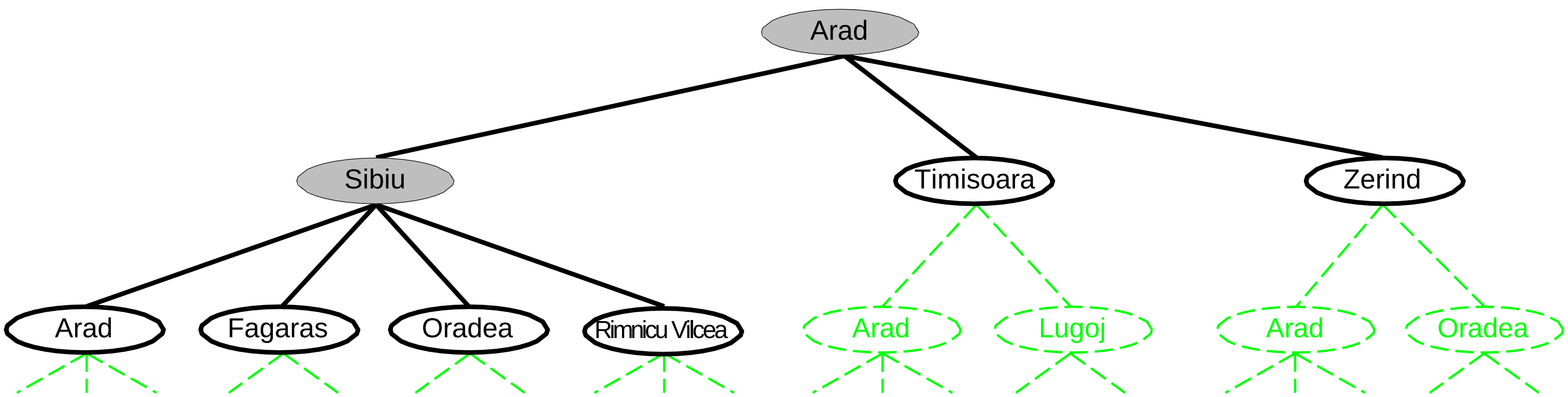
Tree search example



Tree search example

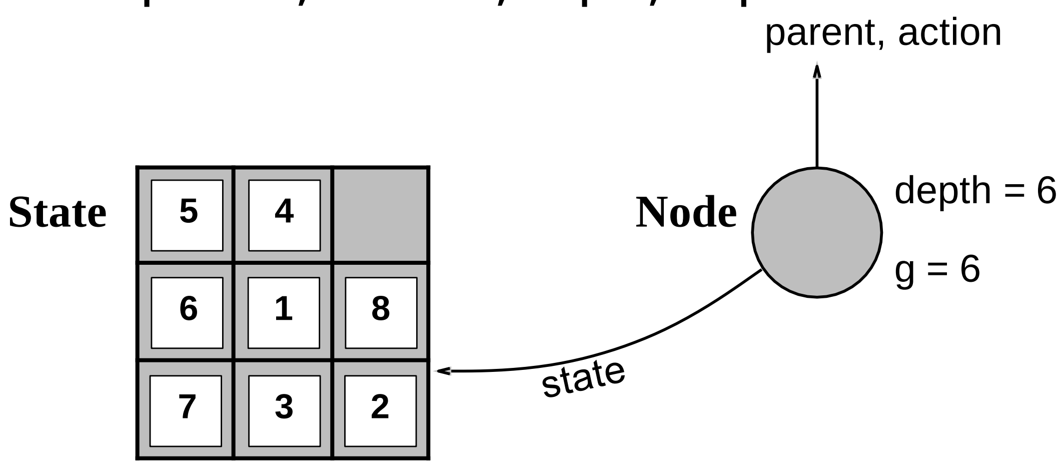


Tree search example



Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes parent, children, depth, path cost $g(x)$
States do not have parents, children, depth, or path cost!



The Expand function creates new nodes, filling in the various fields and using the SuccessorFn of the problem to create the corresponding states.

Implementation: general tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem, State(node)) then return node
        fringe ← InsertAll(Expand(node, problem), fringe)
```

```
function Expand(node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in Successor-Fn(problem, State[node]) do
        s ← a new Node
        Parent-Node[s] ← node;  Action[s] ← action;  State[s] ← result
        Path-Cost[s] ← Path-Cost[node] + Step-Cost(node, action, s)
        Depth[s] ← Depth[node] + 1
        add s to successors
    return successors
```

Search strategies

A strategy is defined by picking the order of node expansion

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)

Search Algorithm Properties

Complete: Guaranteed to find a solution if one exists?

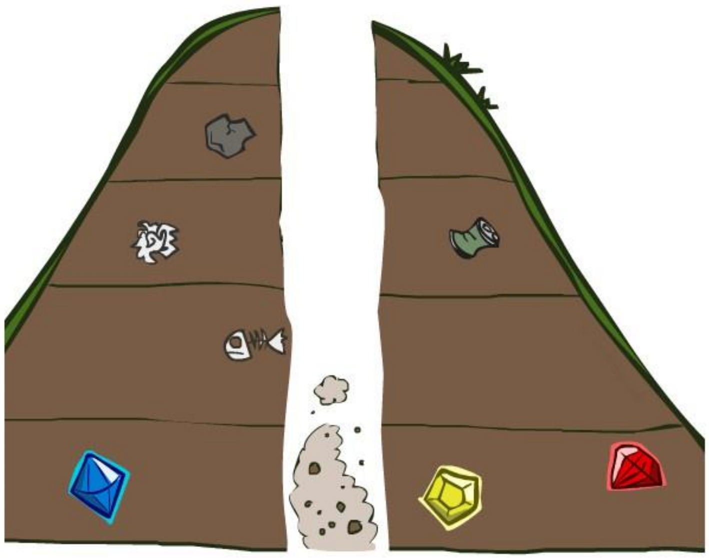
Optimal: Guaranteed to find the least cost path?

Time complexity?

- #nodes generated during search

Space complexity?

- Max #nodes stored in memory



Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

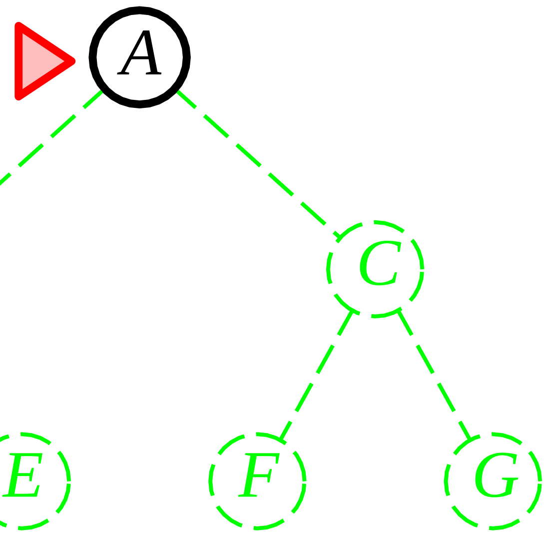
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end

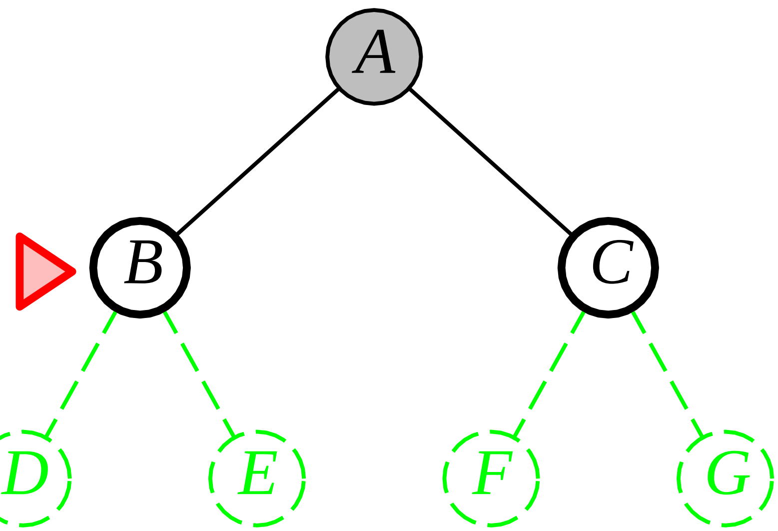


Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end



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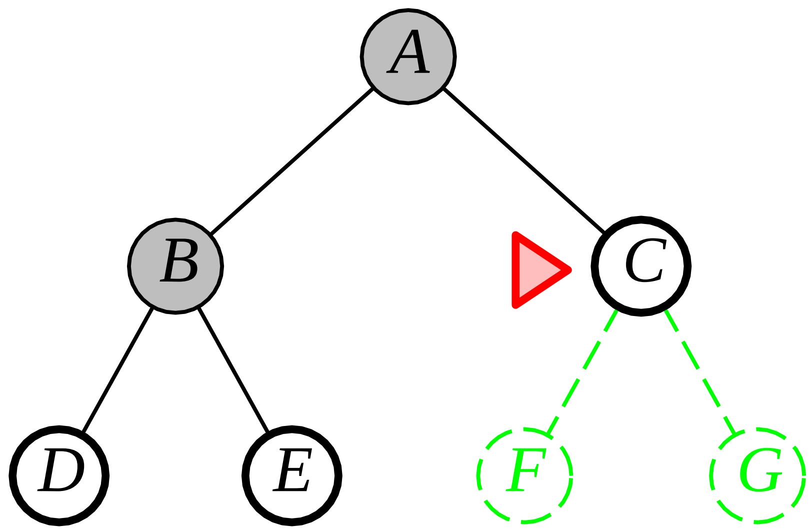

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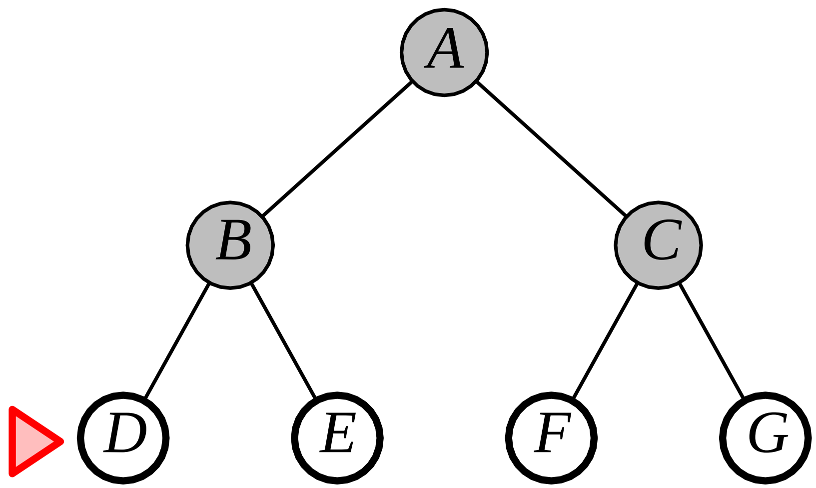


Properties of breadth-first search

Complete??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.

Uniform-cost search

Expand least-cost unexpanded node

Implementation:

fringe = queue ordered by path cost, lowest first

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq c$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/c \rceil})$
where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/c \rceil})$

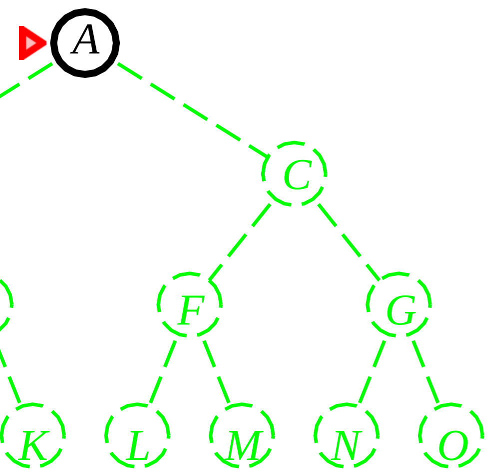
Optimal?? Yes—nodes expanded in increasing order of $g(n)$

Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front



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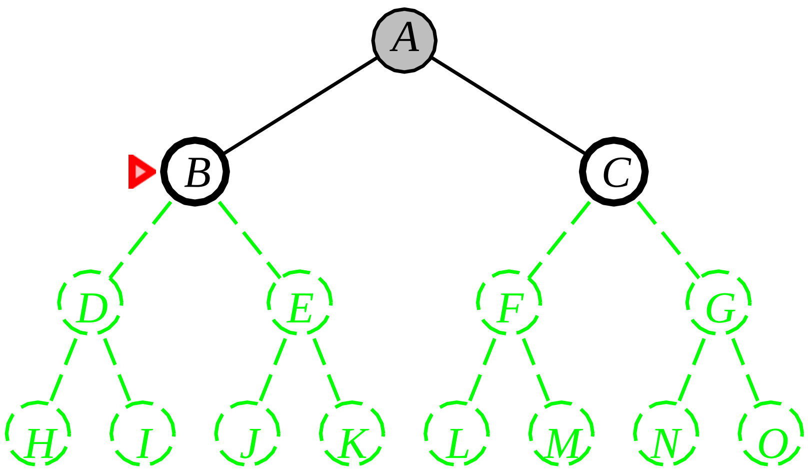

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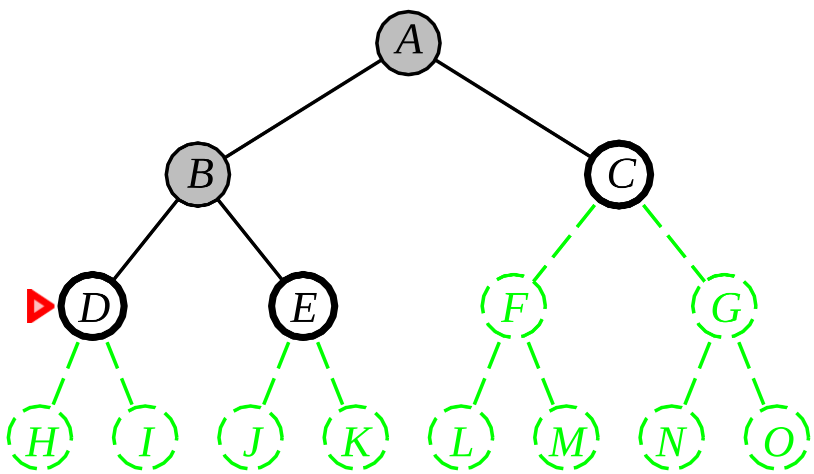

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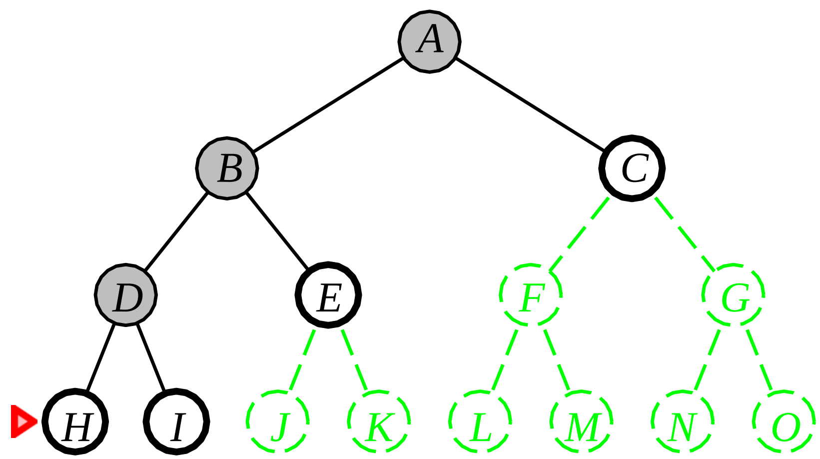

Depth-first search

Expand deepest unexpanded node

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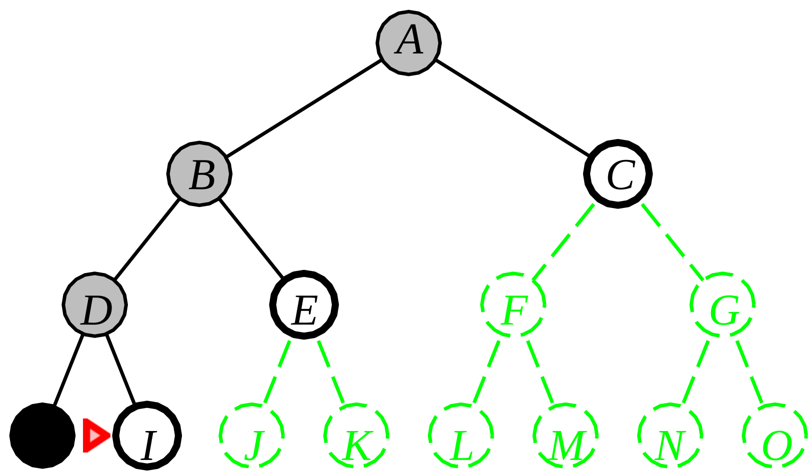

Pearson

Depth-first search

Expand deepest unexpanded node

Implementation:

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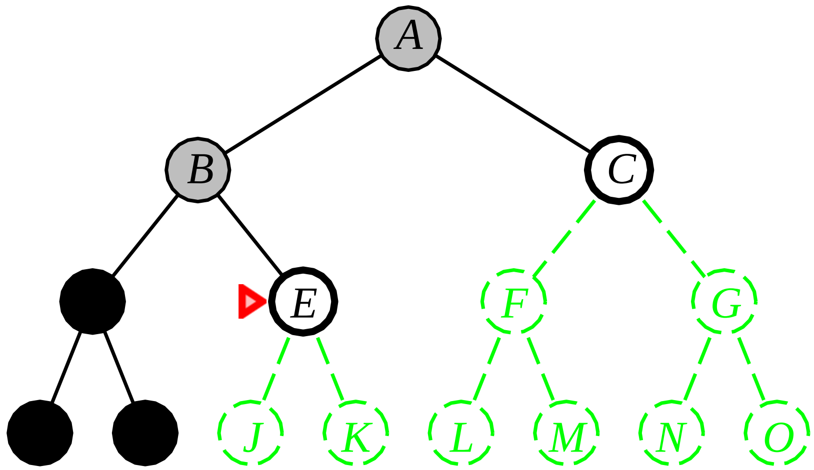
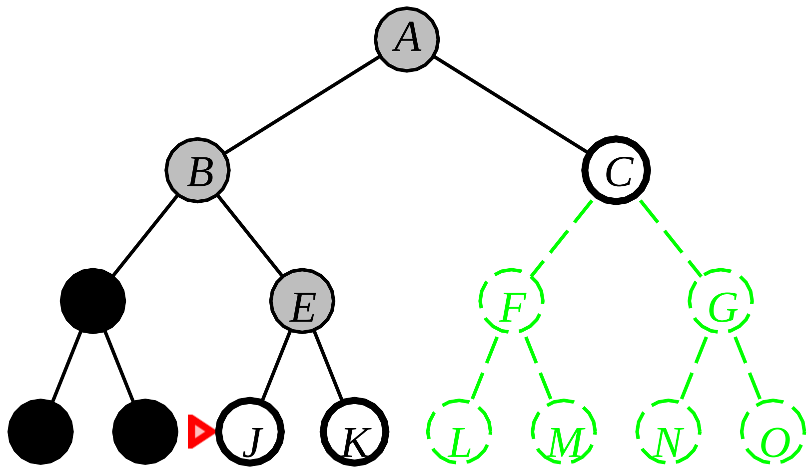


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front

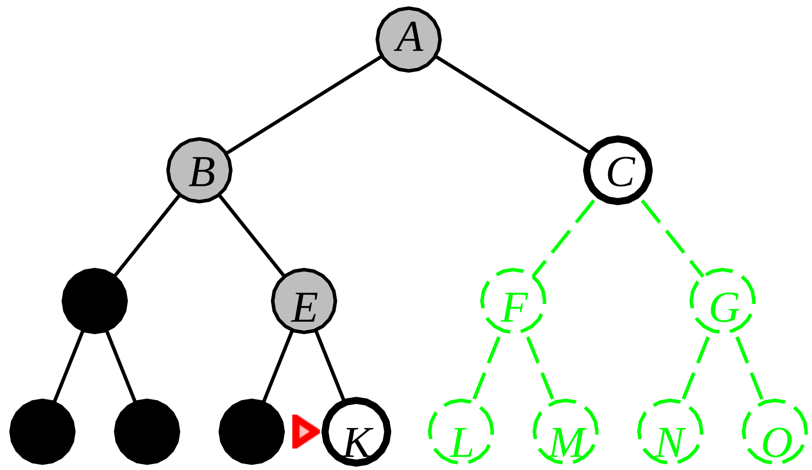


Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front



Depth-first search

Expand deepest unexpanded node

Implementation:

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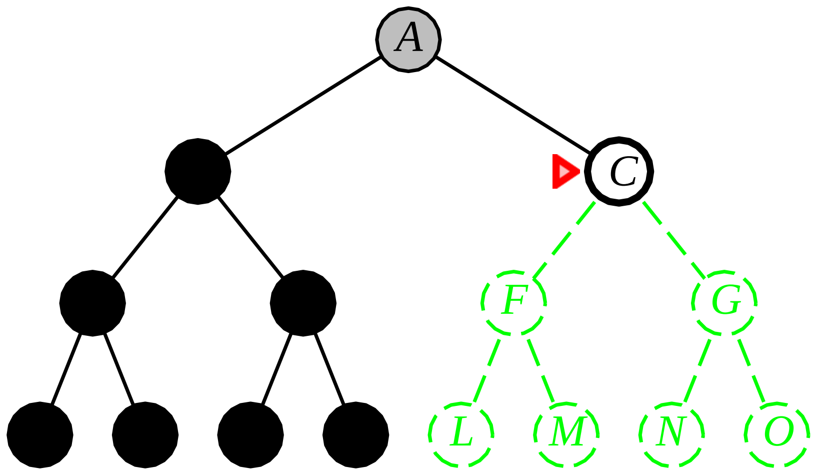

Depth-first search

Expand deepest unexpanded node

Implementation:

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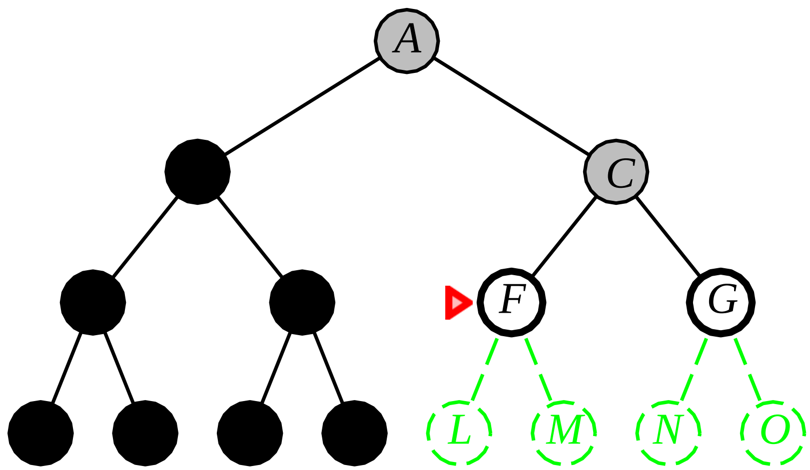

Depth-first search

Expand deepest unexpanded node

Implementation:

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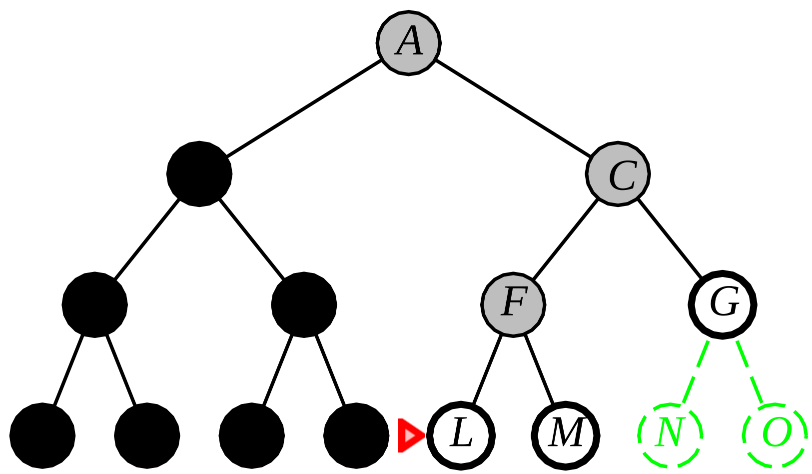

Depth-first search

Expand deepest unexpanded node

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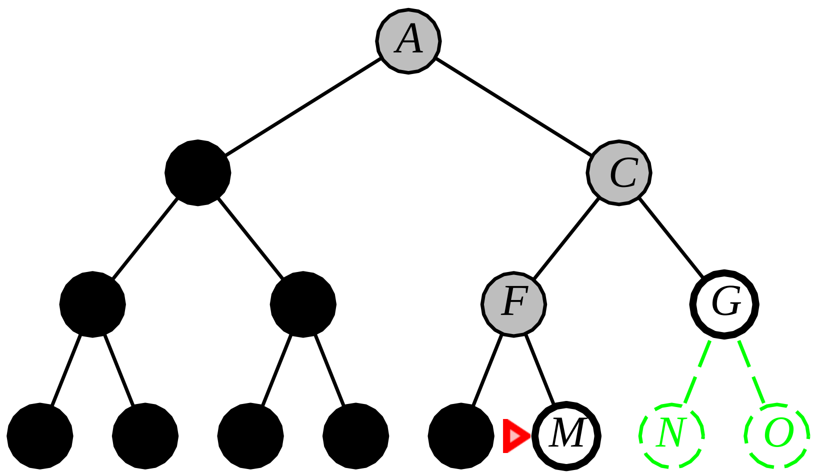


Properties of depth-first search

Complete??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

$\Rightarrow$ complete in finite spaces

Time??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No

Depth-limited search

= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors

Recursive implementation:

```
function Depth-Limited-Search(problem,limit) returns soln/fail/cutoff
    Recursive-DLS(Make-Node(Initial-State[problem]),problem,limit)

function Recursive-DLS(node,problem,limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if Goal-Test(problem,State[node]) then return node
   else if Depth[node] = limit then return cutoff
   else for each successor in Expand(node,problem) do
        result ← Recursive-DLS(successor,problem,limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result /= failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

```
function Iterative-Deepening-Search(problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← Depth-Limited-Search(problem, depth)
        if result /= cutoff then return result
    end
```

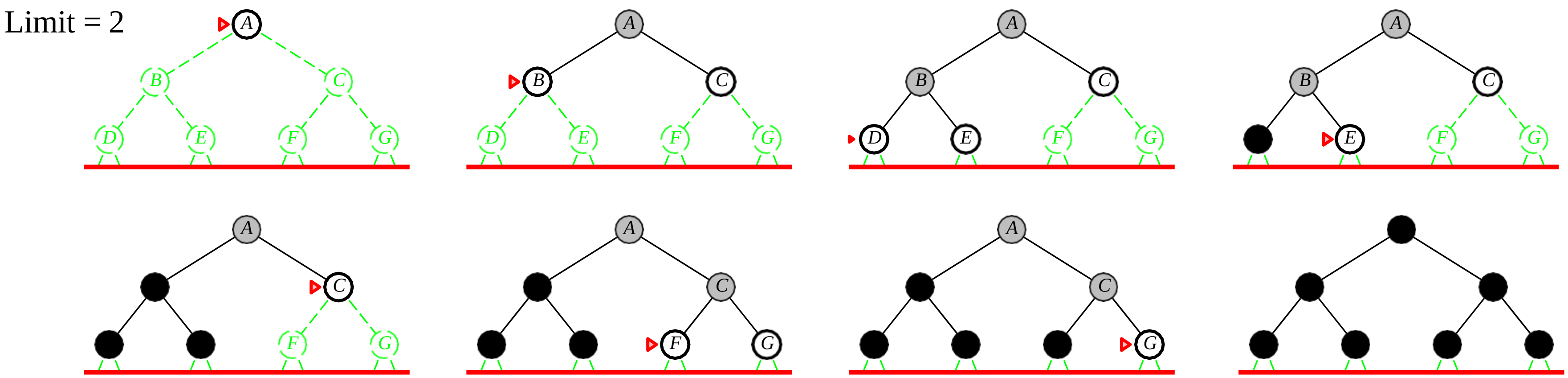
Iterative deepening search $l = 0$

Limit = 0

Iterative deepening search $l = 1$

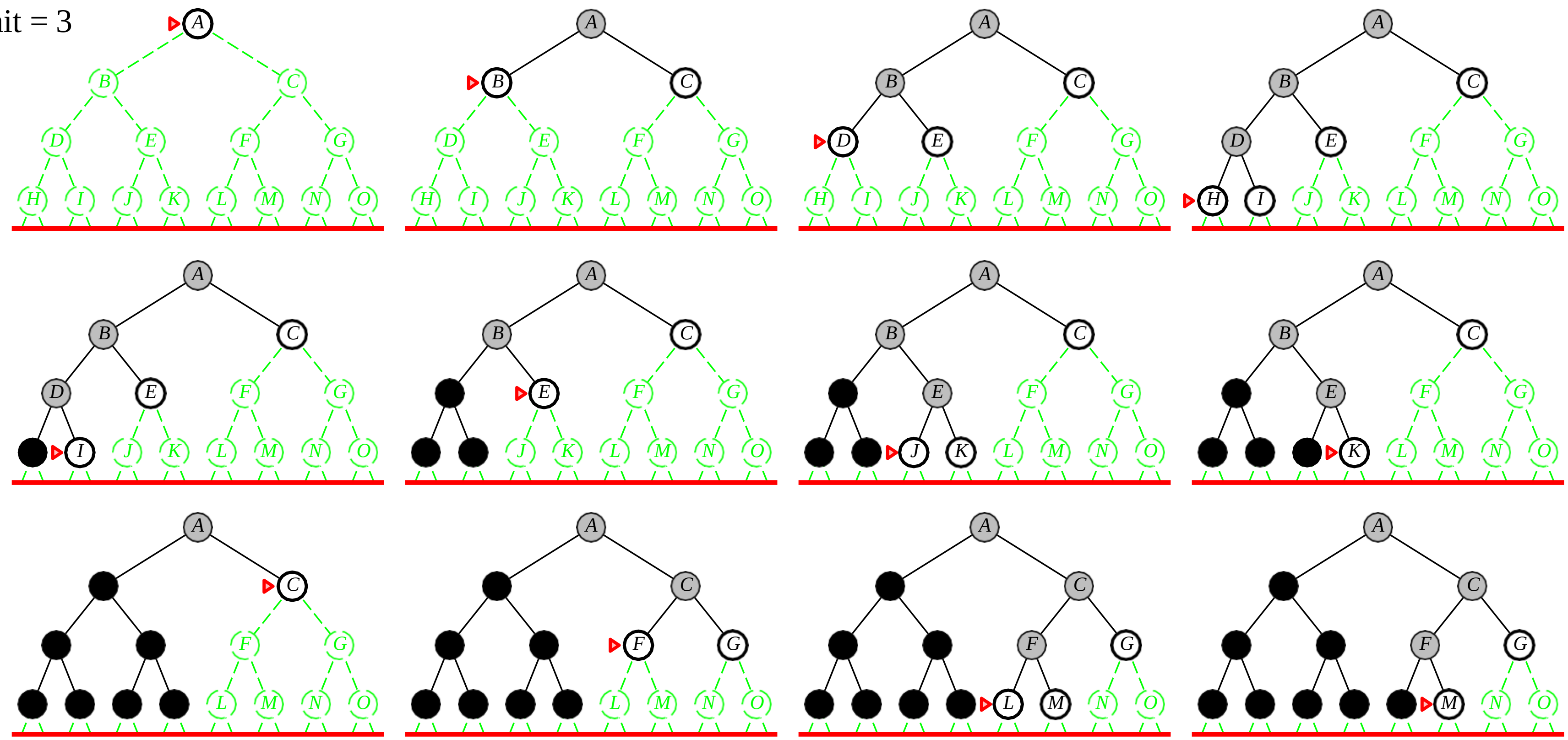
Limit = 1

Iterative deepening search $l = 2$



Iterative deepening search $l = 3$

Limit = 3



Properties of iterative deepening search

Complete??

Properties of iterative deepening search

Complete?? Yes

Time??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost = 1
Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

IDS does better because other nodes at depth d are not expanded

BFS can be modified to apply goal test when a node is generated

Summary of Uninformed Search algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[1]	Yes[1,2]	No	No	Yes[1]	Yes[1,4]
Optimal cost?	Yes[3]	Yes	No	No	Yes[3]	Yes[3,4]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

Figure 3.15 Evaluation of search algorithms. b is the branching factor; m is the maximum depth of the search tree; d is the depth of the shallowest solution, or is m when there is no solution; ℓ is the depth limit. Superscript caveats are as follows: [1] complete if b is finite, and the state space either has a solution or is finite. [2] complete if all action costs are $\geq \epsilon > 0$; [3] cost-optimal if action costs are all identical; [4] if both directions are breadth-first or uniform-cost.

Overview

- Informed Search
- Heuristics
- Greedy Search
- A* Search
- Graph Search

Informed Search
Best-first Search

Idea: Prioritize fringe nodes based on "desirability"

Use a cost estimate function f(n)

- Expand "best" node with lowest f value first
- Choice of f determines search strategy
- f(n) could include heuristic function h(n)

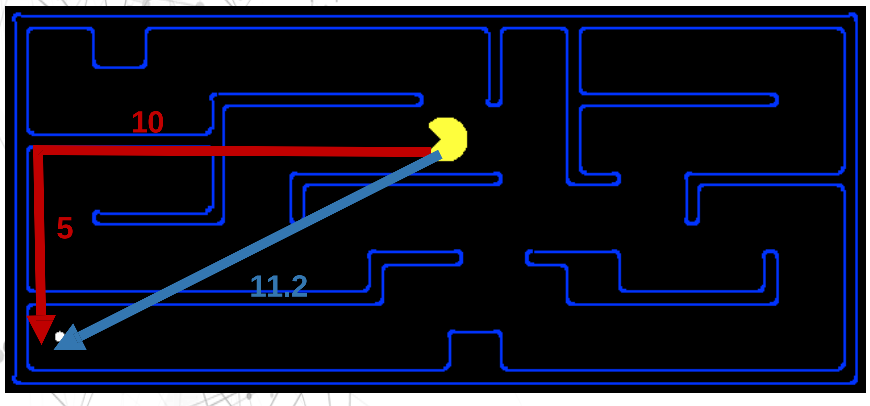
Two special cases:

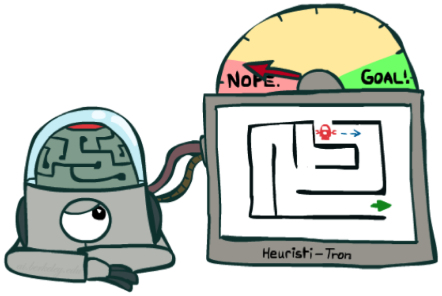
- Greedy Best-first Search
- A* Search

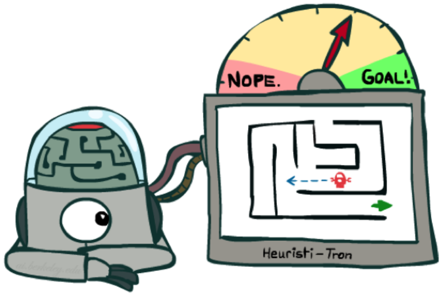
Search Heuristics

A search heuristic is:

- A function that *estimates* how close a state is to a goal
    - Zero for the goal state [h(goal) = 0]
- Has to be designed for each particular search problem
- Examples: Manhattan distance or Euclidean distance for paths

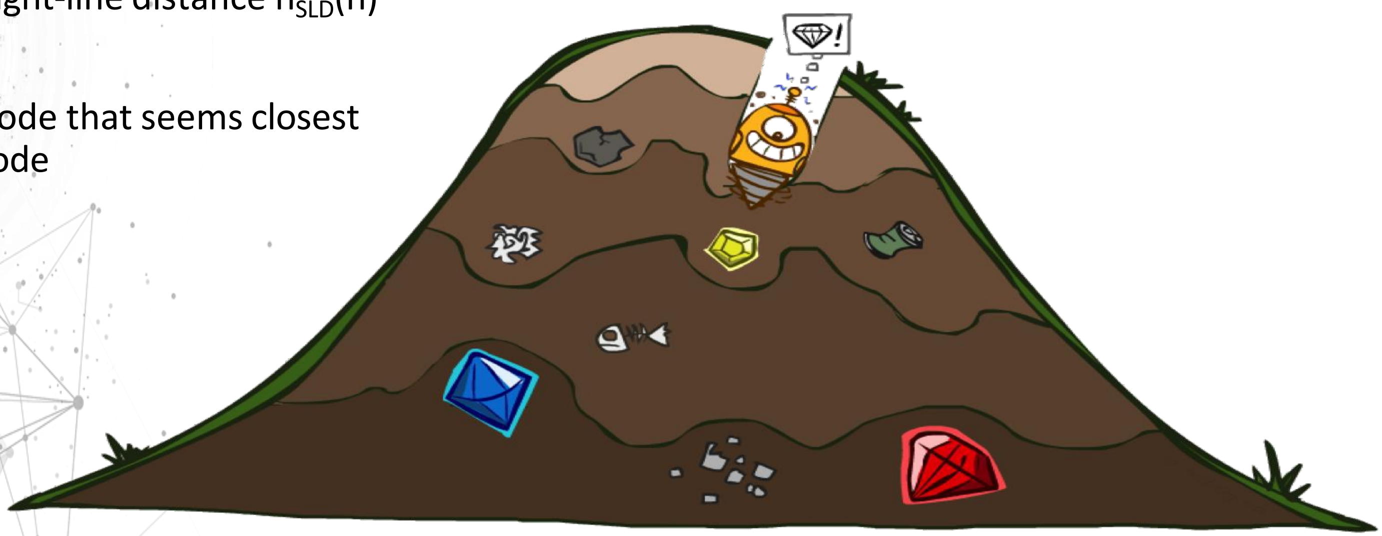






Greedy Best-First Search

- Use function f(n) = heuristic function h(n)
  - e.g. straight-line distance $h_{SLD}(n)$
- Expand the node that seems closest to the goal node



Graph search

```
function Graph-Search(problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem, State[node]) then return node
        if State[node] is not in closed then
            add State[node] to closed
            fringe ← InsertAll(Expand(node, problem), fringe)
    end
```

Review: Tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
   fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test[problem] applied to State(node) succeeds return node
        fringe ← InsertAll(Expand(node, problem), fringe)
```

A strategy is defined by picking the order of node expansion

Best-first search

Idea: use an evaluation function for each node
– estimate of "desirability"

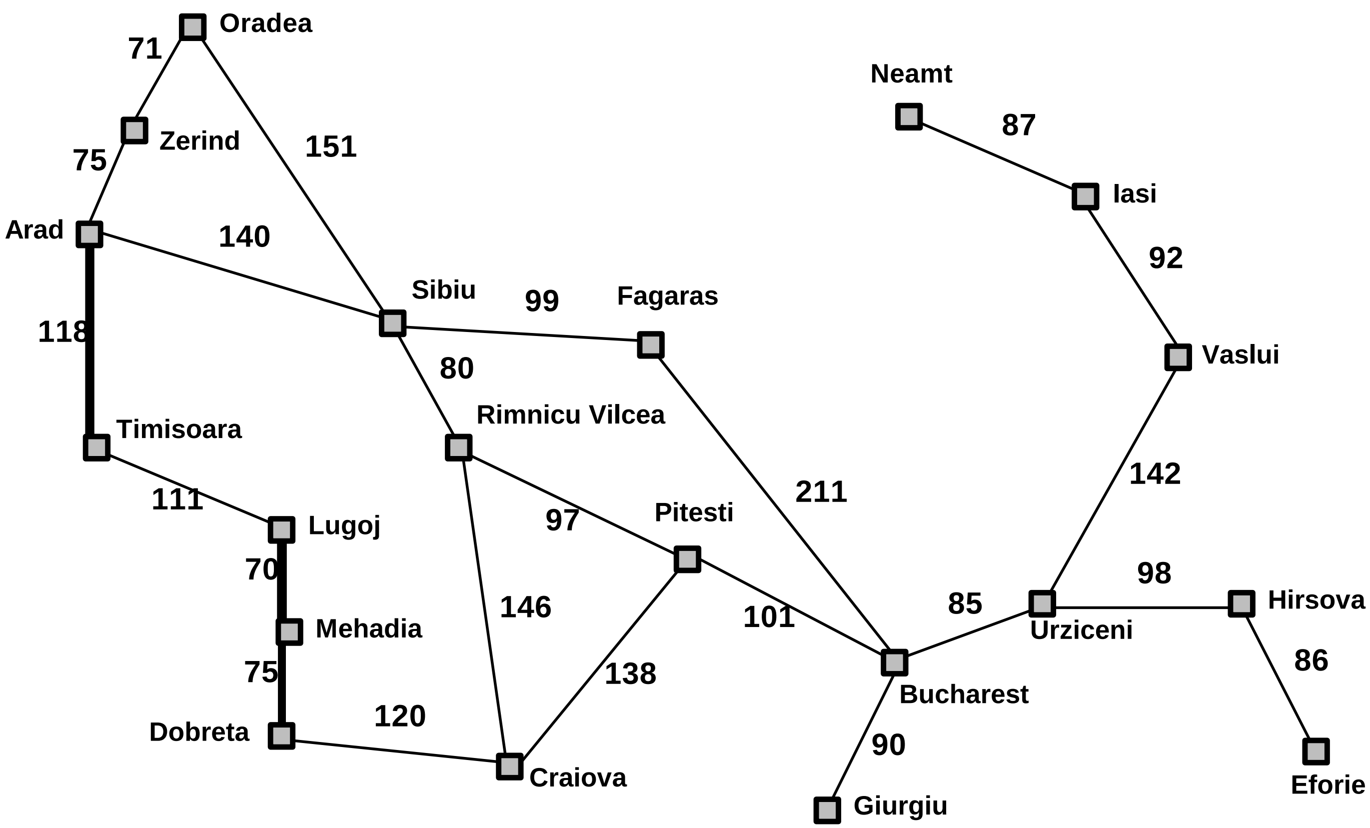
⇒ Expand most desirable unexpanded node

Implementation:

fringe is a queue sorted in decreasing order of desirability

Special cases:
greedy search
A* search

Romania with step costs in km



Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

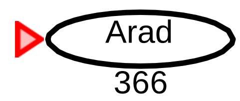
Greedy search

Evaluation function $h(n)$ (heuristic)
= estimate of cost from n to the closest goal

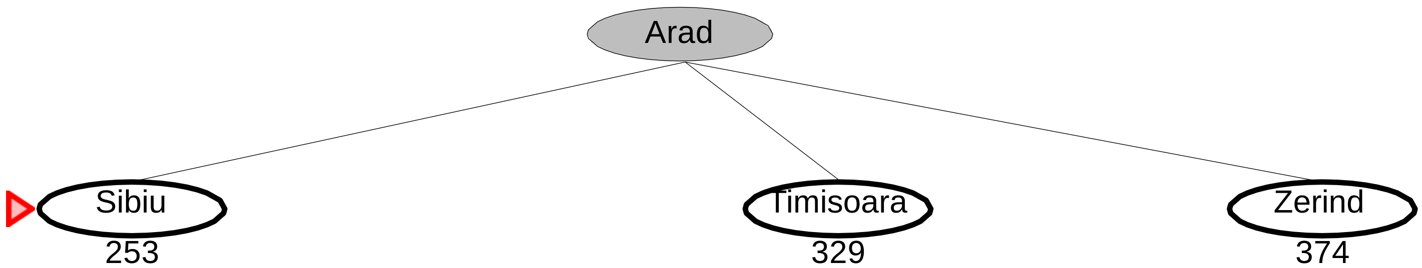
E.g., $h_{\mathrm{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that appears to be closest to goal

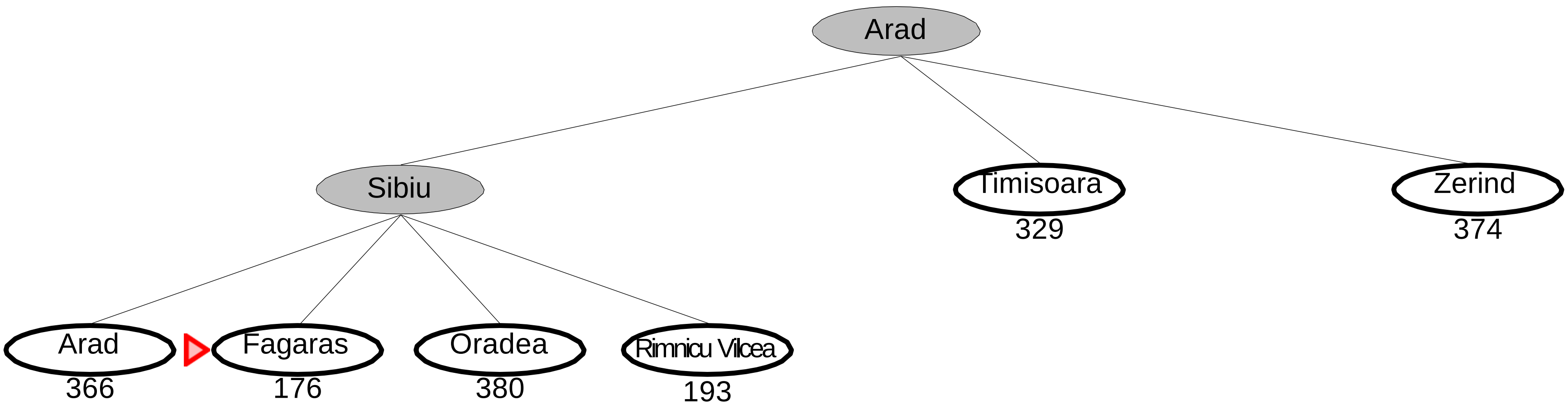
Greedy search example



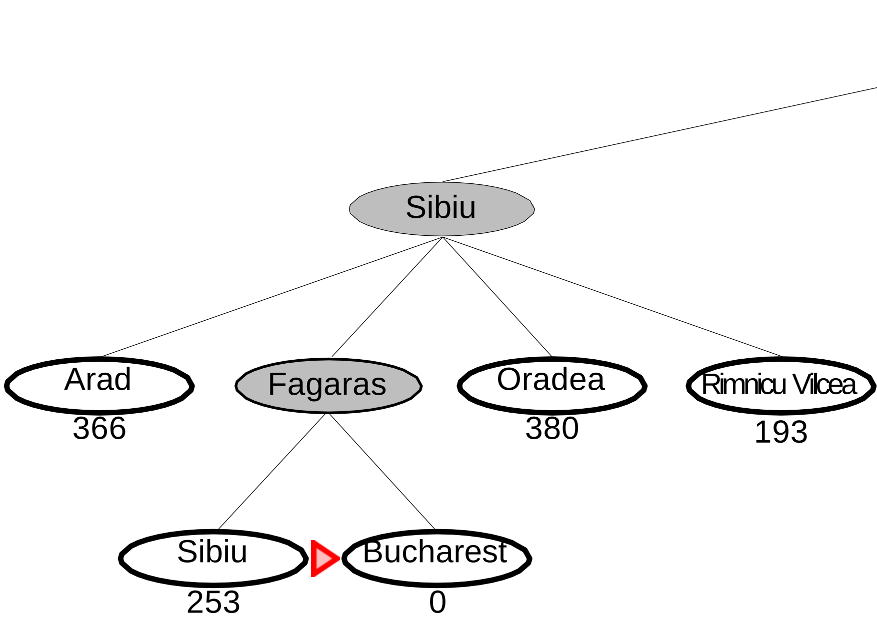
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g., with Oradea as goal,
Iasi → Neamt → Iasi → Neamt →
Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

Properties of greedy search

Complete?? No–can get stuck in loops,
e.g., Iasi → Neamt → Iasi → Neamt →
Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic

improvement Space?? $O(b^m)$—keeps all nodes in memory

Optimal??

Properties of greedy search

Complete?? No–can get stuck in loops,

e.g., Iasi → Neamt → Iasi →

Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic

improvement Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n

$h(n)$ = estimated cost to goal from n

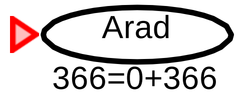
$f(n)$ = estimated total cost of path through n to goal

A* search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n. (Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

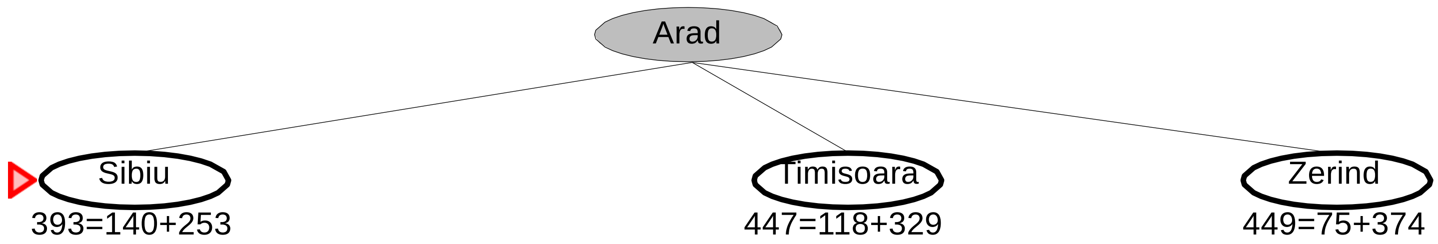
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road

distance Theorem: A* search is optimal

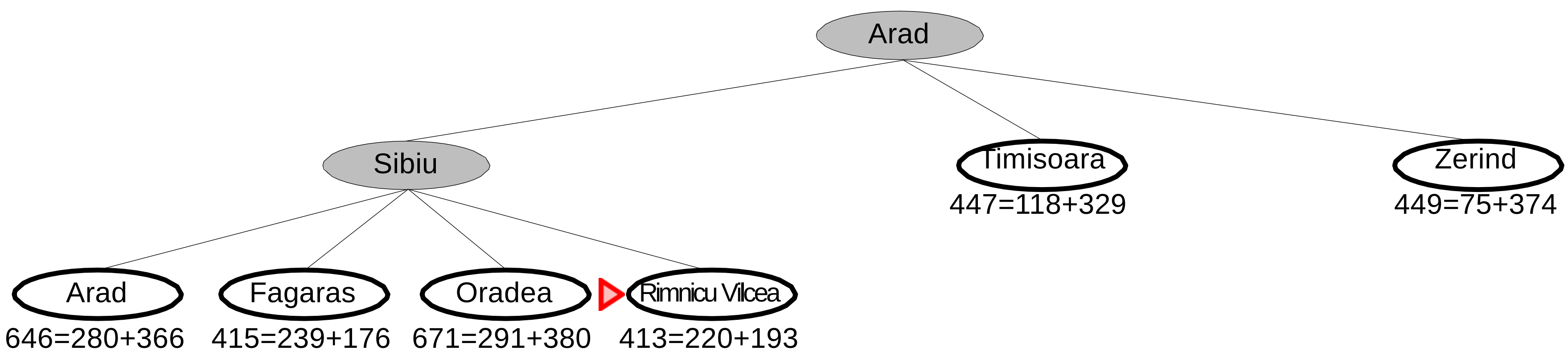
A* search example



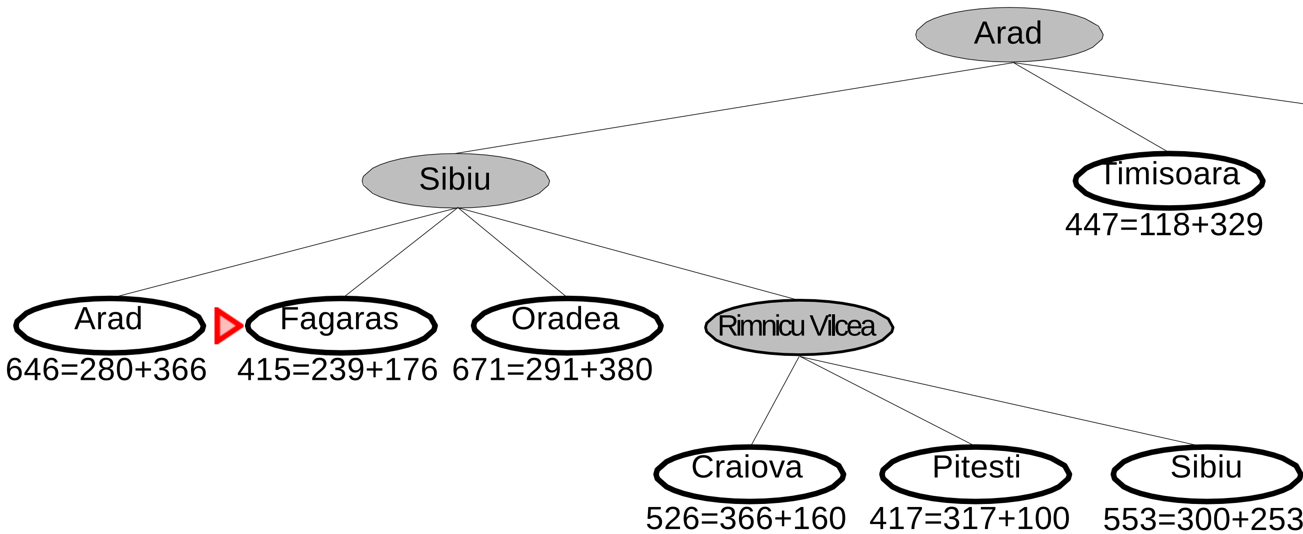
A* search example



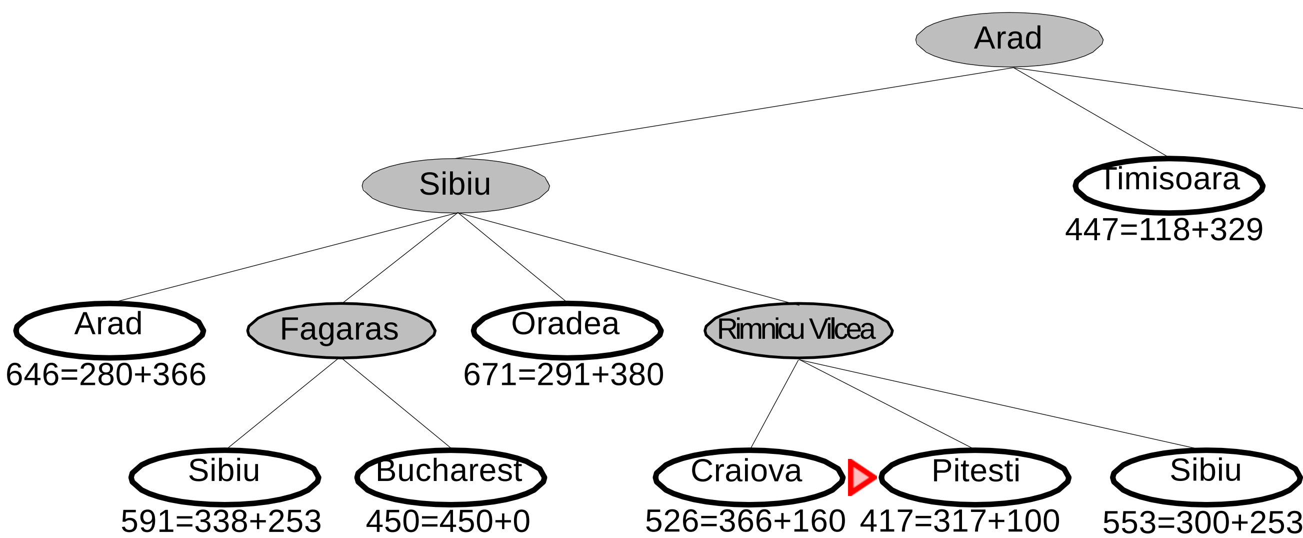
A* search example



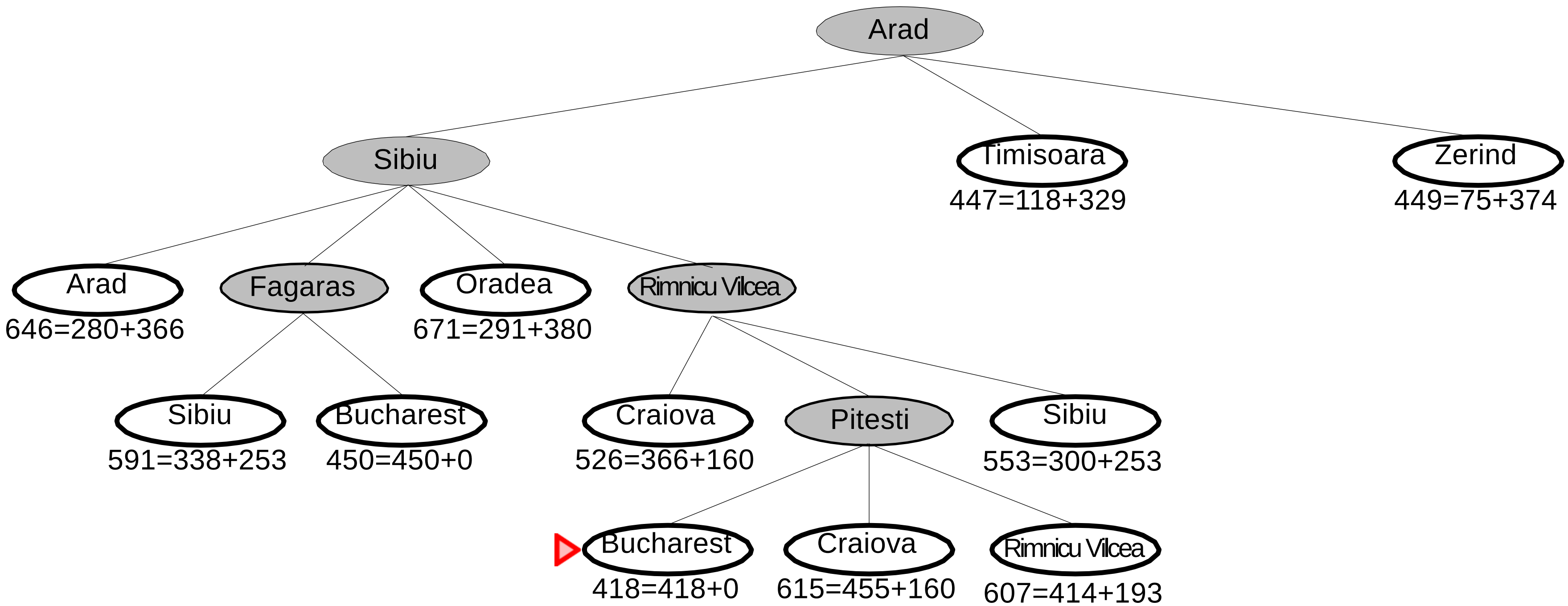
A* search example



A* search example

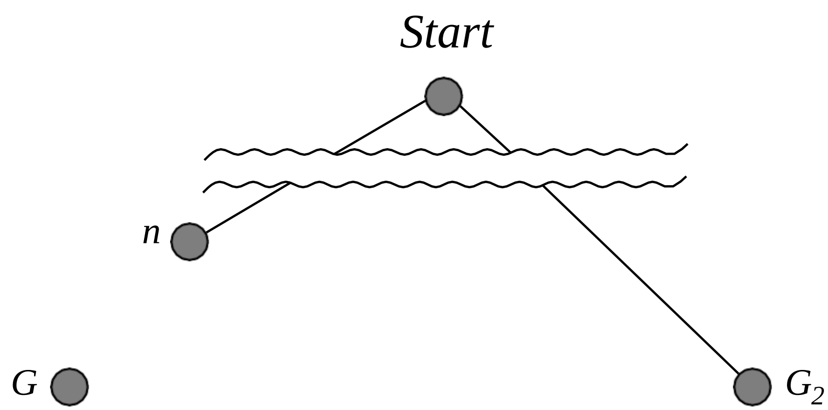


A* search example



Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



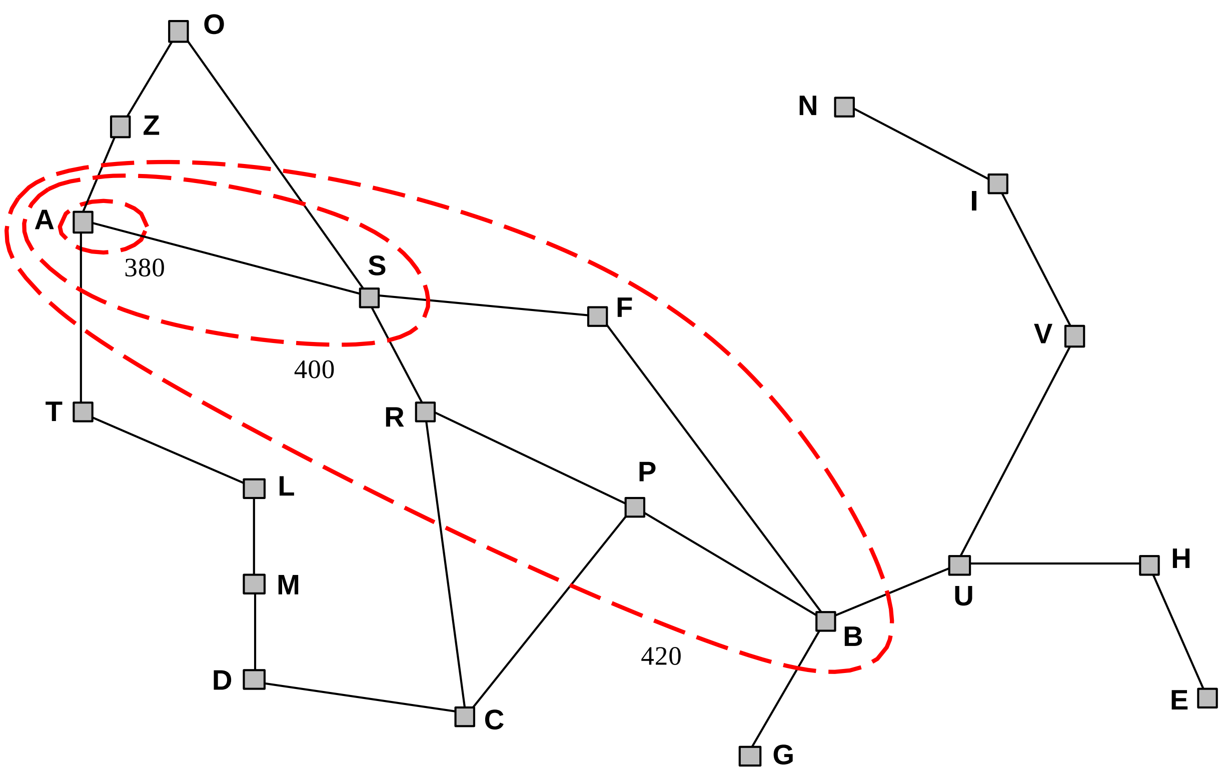
$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C*$

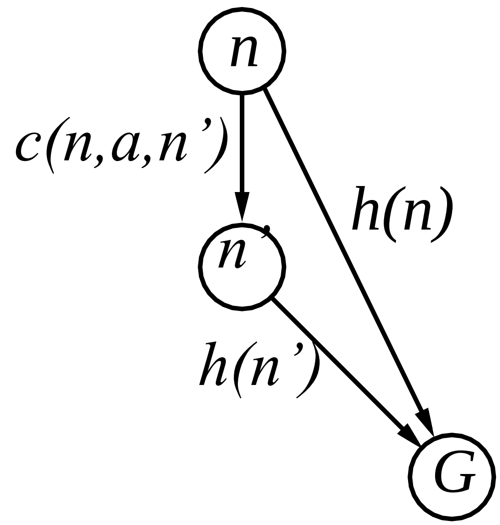
A*expands some nodes with $f(n) = C*$

A*expands no nodes with $f(n) > C*$

Proof of lemma: Consistency

A heuristic is consistent if

$$h(n) \leq c(n, a, n\ ) + h(n\ )$$



If h is consistent, we have

$$\begin{aligned} f(n\ ) &= g(n\ ) + h(n\ ) \\ &= g(n) + c(n, a, n\ ) + h(n\ ) \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

I.e., $f(n)$ is nondecreasing along any path.

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S)$ =??
$h_2(S)$ =??

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S)$ =?? 6
$h_2(S)$ =?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible), then
h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$*(h_1)$ = 539 nodes
A$*(h_2)$ = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$*(h_1)$ = 39,135 nodes
A$*(h_2)$ = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Summary

A problem consists of five parts: the **initial state**, a set of **actions**, a **transition model** describing the results of those actions, a set of **goal states**, and an **action cost function**.

Uninformed search methods have access only to the **problem definition**. Algorithms build a search tree in an attempt to find a solution.

Informed search methods have access to a **heuristic** function h(n) that estimates the cost of a solution from n.