Homework 3
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CS 5200: Database Management System
Homework 3

Problem 1 (5 points). List all non-trivial functional dependencies that hold in the current state of the relation:

A_1	A_2	A_3
1	x	i
1	x	ii
2	x	i
2	x	iii

Each FD should be minimal: that is to say, removing any attribute(s) from the left no longer functionally determines the right.

Solution:

- $A_1 \to A_2$ (Valid and minimal)
- $A_1 \to A_3$ (Invalid)
- $A_2 \to A_1$ (Invalid)
- $A_2 \to A_3$ (Invalid)
- $A_3 \to A_1$ (Invalid)
- $A_3 \to A_2$ (Valid and minimal)
- $(A_1, A_2) \to A_3$ (Invalid)
- $(A_1, A_3) \to A_2$ (Not minimal, because $A_1 \to A_2$ already holds)
- $(A_2, A_3) \to A_1$ (Invalid)

Final Answer: The only valid non-trivial functional dependency is:

$$A_1 \to A_2$$

Problem 2 (10 points). List all non-trivial functional dependencies that do NOT hold given the current state of this relation:

	A_4	A_5	A_6
t_1	1	y	iii
t_2	4	y	iii
t_3	5	z	iii

With each FD you list, provide a pair of tuples that invalidate the FD. As shown in the table above, refer to the first tuple as t_1, the second as t_2, and the third as t_3.

Solution:

- $A_4 \to A_5$ (Invalidated by t_2, t_3)
- $A_5 \to A_4$ (Invalidated by t_1, t_2)
- $A_6 \to A_4$ (Invalidated by t_1, t_3)
- $A_6 \to A_5$ (Invalidated by t_1, t_3)
- $(A_4, A_6) \to A_5$ (Invalidated by t_2, t_3)
- $(A_5, A_6) \to A_4$ (Invalidated by t_1, t_2)

Final Answer: The following functional dependencies do not hold with counterexamples as shown above.

$$A_4 \to A_5, \quad A_5 \to A_4, \quad A_6 \to A_4, \quad A_6 \to A_5, \quad (A_4, A_6) \to A_5, \quad (A_5, A_6) \to A_4$$

Problem 3 (15 points). Consider a relational schema BAR(M, N, O, P) that has the following functional dependencies (FDs): O → P , O → M , N → O.

- What are the candidate key(s) of BAR?
- What is the highest normal form BAR is in? (You must justify your response.)
- If BAR violates 3NF, provide a decomposition that satisfies the FDs (remember to include all primary/foreign keys).

Solution:

Candidate Key

- To determine the candidate key, given:
- $N \to O$
- $O \to P$
- $O \to M$
- Since N determines O, and O determines both P and M, conclude:

$$N \to M, O, P$$

- Thus, the candidate key is:

$$\{N\}$$

Highest Normal Form

- 1NF: The schema is in 1NF because all attributes are atomic.
- 2NF: Since there is only one candidate key N, and all attributes fully depend on it, the schema satisfies 2NF.
- 3NF: The schema does not satisfy 3NF because O is not a candidate key but determines P and M.
- Thus, the highest normal form is:

$$\text{2NF}$$

3NF Decomposition

- To decompose into 3NF, need to create two relations:

$$BAR_1(N, O) \quad \text{(Primary Key: } N\text{, Foreign Key: } O\text{)}$$

$$BAR_2(O, M, P) \quad \text{(Primary Key: } O\text{)}$$

- This decomposition ensures that all functional dependencies are preserved and each relation satisfies 3NF.

Problem 4 (15 points). Consider a relational schema BAZ(Q, R, S, T) that has the following functional dependencies (FDs): R → S, T → Q.

- What are the candidate key(s) of BAZ?
- What is the highest normal form BAZ is in? (You must justify your response.)
- If BAZ violates 3NF, provide a decomposition that satisfies the FDs (remember to include all primary/foreign keys).

Solution:

Candidate Key

- To determine the candidate key, given:
- $R \to S$
- $T \to Q$
- Since neither R nor T alone can determine all attributes, we conclude:

$$(T, R) \to Q, R, S, T$$

- Thus, the candidate key is:

$$\{T, R\}$$

Highest Normal Form

- **1NF**: The schema is in 1NF because all attributes are atomic.
- **2NF**: The schema does not satisfy 2NF because S depends only on R, which is part of the candidate key $\{T, R\}$, causing partial dependency.
- Thus, the highest normal form is:

1NF

3NF Decomposition

- To decompose into 3NF, we create three relations:

$$BAZ_1(R, S) \quad \text{(Primary Key: } R\text{, Foreign Key: } R \to BAZ_3\text{)}$$
$$BAZ_2(T, Q) \quad \text{(Primary Key: } T\text{, Foreign Key: } T \to BAZ_3\text{)}$$
$$BAZ_3(R, T) \quad \text{(Primary Key: } R, T\text{)}$$

- This decomposition ensures that all functional dependencies are preserved and each relation satisfies 3NF.