Problem Set 6
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CS 5800: Algorithm
Problem Set 6

Exercises

Dantzig

Q1: What approach to solving systems of linear equations does Dantzig consider to be inefficient (and, by his own quote "very stupid")?

Dantzig thought that step-by-step descent along edges of the convex polyhedral set which involves moving from one vertex to an adjacent vertex while optimizing the objective function. He referred to this as a "very stupid" method because it solves the problem by "wandering along some path of outside edges" until reaching the optimal vertex. This approach is inherently inefficient as it does not leverage a direct pathway or interior progression towards the solution.

"the first idea that occurred to me is one that would occur to any trained mathematician, namely the idea of step by step descent (with respect to the objective function) along edges of the convex polyhedral set from one vertex to an adjacent one. I rejected this algorithm outright on intuitive grounds - it had to be inefficient because it proposed to solve the problem by wandering along some path of outside edges until the optimal vertex was reached."

As well, Dantzig mentioned and provided some supporting insight which is *"Today we know that before 1947 that four isolated papers had been published on special cases of the linear programming problem by Fourier (1824) [5], de la Vallde Poussin (1911) [6], Kantorovich (1939) [7] and Hitchcock (1941) [8]. All except Kantorovich's paper proposed as a solution method descent along the outside edges of the polyhedral set which is the way we describe the simplex method today. There is no evidence that these papers had any influence on each other."*

Q2: What quality of large problems does Dantzig attribute the success of the simplex method to?

Dangzig attribute the success of the simplex method to handle large problems to two critical qualities of real-world linear programming problems that:

Sparsity of Matrices

- Most practical linear programming problems have matrices that are highly sparse, meaning they contain very few non-zero coefficients. This sparsity reduces computational complexity and memory requirements during iterative computations.

Near-Triangular Structure of Bases

- By rearranging rows and columns, the bases encountered in the simplex method can often be transformed into nearly triangular forms. This structure allows efficient factorization and computation, preserving sparsity and minimizing operations.

"The success of solving linear programming therefore depends on a number of factors: (1) the power of computers, (2) extremely clever algorithms; but it depends most of all upon (3) a lot of good luck that the matrices of practical problems will be very very sparse and that their bases, after rearrangement, will be nearly triangular."

Dantzig also discussed the situation of the sparse matrices with instance *"To determine $s = arg\ min_j[\pi A_{.j} + \pi_0]$ requires forming the scalar product of two vectors π and $A_{.j}$ for each j. This "pricing out" operation it is called is usually very cheap because the vectors $A_{.j}$ are sparse, i.e., they typically have few non-zero coefficients 9(perhaps on the average 4 or 5 non-zeros). Nevertheless if the number of columns n is large, say several thousand, pricing can use up a lot of CPU time. (Parallel processors could be used very effectively for pricing by assigning subsets of the columns to different processors, [18].)"*

Akpan

Q3: What problem does this paper propose to solve?

Akpan in the paper addresses the problem of optimizing the allocation of raw materials in a bakery to maximize profit. It aims to determine the optimal production quantities of three types of bread that big loaf, giant loaf, and small loaf what using a linear programming model to allocate limited raw materials such as flour, sugar, yeast, salt, wheat gluten, and soybean oil efficiently.

"Sometimes many production companies are faced with problems of how to utilize the available resources in order to maximize profit; this is because the use of linear programming which brings a suitable quantitative approach of decision-making has not been fully applied."

Q4: What approach does this paper use?

Akpan use the Simplex Algorithm, a widely recognized method in linear programming, to solve the optimization problem. They first formulate the problem mathematically, defining an objective function to maximize profit and constraints based on the availability of raw materials. The problem is converted to its standard form and solved using TORA software.

"This work utilized the concept of Simplex algorithm; an aspect of linear programing to allocate raw materials to competing variables (big loaf, giant loaf and small loaf) in bakery for the purpose of profit maximization."

Pardeshi

Q5: What problem does this paper propose to solve?

Pardeshi in paper aims to analyze student learning habits during the COVID-19 pandemic by examining the time spent on learning with and without instructors in online and offline settings. It utilizes linear programming to determine how students allocate their learning hours under different conditions, factoring in constraints such as resource availability and learning preferences.

"Afterwards, it is also required to identify the feasible amount of time students will spend during online learning as per the availability of required resources."

"The main objective of this paper is to understand the student learning habits during pandemic. It has considered two situations: learning time given by students with instructors during online mode and offline mode. Similarly it also gives insights on learning time spent by students without instructor during online and offline mode."

Q6: What term, phrase, or concept were you unfamiliar with?

Slack variables, surplus variables, and artificial variables. While slack and surplus variables are common in linear programming, their practical use in educational contexts was initially unclear.

"Type 1: Slack variables demoted as S which are used to represent unused variables either in the form of resources used to optimize the given function.
Type 2: Surplus variables are indicated by the letter (-S) and are used to describe the amount by which a resource's solution value expresses it. These variables are also known as negative slack variables and, like slack variables in the objective function, have a zero coefficient.
Type 3 Artificial variables, denoted as (A) - These variables are used to generate an initial solution to the LP problem."