4.2 Graph Representations

4.2a How then do we go about the representation of a graph in familiar fashions? To begin with, we consider the simpler case of the undirected graph which is necessarily connected in a symmetrical fashion.

4.2b From a symmetrical fashion, it becomes quite familiar to those with memories of their multiplication table that rows and columns would need to match up to determine if a connection between components exists.

4.2c As such, it can be stated that a representation for an undirected graph can be given by an adjacency matrix consideration as follows below

$$\begin{bmatrix} a_{0,0} & a_{0,1} & \dots & a_{0,m} \\ a_{1,0} & a_{1,1} & \dots & a_{1,m} \\ \dots & \dots & \dots & \dots \\ a_{n,0} & a_{n,1} & \dots & a_{n,m} \end{bmatrix}$$

The problem with which is that this necessarily takes space equivalent to $O(N^2)$, which is wasteful for those graphs without many edges but containing many nodes. In this representation, the first row and the first column are considered as indicating specific nodes in a symmetric fashion for n equal to m, and the inner spaces of this conforming to the connections between edges, not including self edges for now.

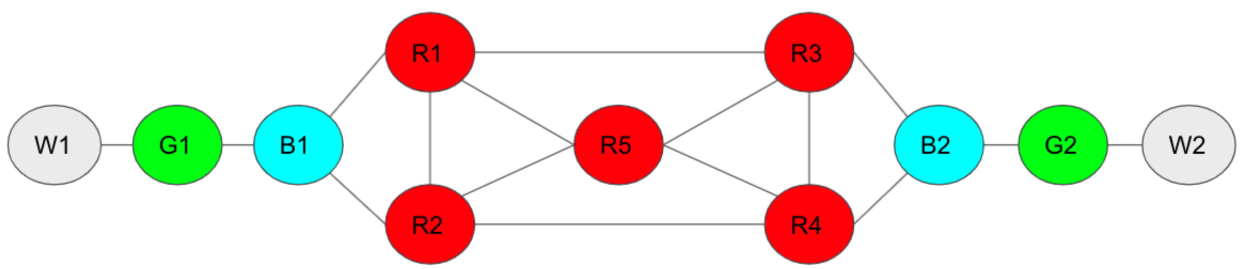
4.2d The next format to consider is that of the adjacency list, wherein a list of Nodes contains within each node of a list of Edges with which it connects other stated nodes. This representation allows a total data structure necessarily on the size of O(E). Again, we will notice the symmetry consideration in undirected graphs, where an edge in U will necessarily appear in an edge in V. With these considerations in mind, we now turn to the first required problem below, which we will consider in the remainder of this section.

P13 Required Problem : Mandrake the Magician has finished another of their shows, and is in the process of packing up their equipment. However, one of their tiger's has escaped their enclosure and they must rush off to help immediately. They turn to Mongo, their trusted assistant, and gives them the task of creating the necessary links of colored scarves for the next show. They quickly instruct Mongo that the scarves must be linked in such a way that all red scarves have at most 4 other scarves connected to them, all blue scarves must have 3, all green scarves must have 2, and white scarves may only be connected to one other scarf at any given time. If Mongo is to make the connection of scarves with the least possible number of edges showing, prove how Mongo will do so with 5 red scarves, 2 blue scarves, 2 green scarves, and 2 white scarves. Make sure to state what type of graph you are constructing in this case and why the differences matter. Also state how this is to be done in such a way that all scarves are as connected as they possibly can be for this set up.

S13a Solution : To create the densest graph is a rather interesting proposal. For this it is necessary to consider how connections are considered, and from that the first part of the answer is rather clear. From the definitions provided, undirected graphs constitute two such connections between each node, such that in this case a single undirected edge is really worth considering as a directed edge to and from each node. Based on this we should choose an undirected graph for the graph type.

S13b To now consider the rest of our graph, we need to consider our maximum number of edges, in this case the ideal being given by 5 R * 4 + 2B * 3 + 2G * 2 + 2W * 1, where the letters have stood in for the types of scarves and the numbers designate the types of nodes. As such we are left to consider our densest possible graph as one that must constitute at least 32 edges with the consideration that our undirected graphs count twice meaning we should arrive at a total of 16 such edges. Due to the reduction in number of visible edges through the use of undirected edges, it is not possible to arrive at a maximal minima of such connections for a directed graph as compared to an undirected graph in this case.

S13c Now we may begin to construct our graph based on the stated limitations. From the consideration that the White scarves may only be connected to one other scarf, we realize that we must have these as the end cap of our links of scarves. Similarly, the green must serve as an intermediary node, wherein they must connect from the white to the rest of the graph. From here, we recognize that in order to achieve our ideal set up in as compact a nature as possible, we wish to have our blue nodes connect from the greens to two red nodes each. This then leads to an interesting compaction of edges within our set as seen below.



S13d In this we can see that we have created a highly connected graph internally, and a sparsely connected graph externally. This gives the desired chain of links that Mandrake wanted while ensuring that the graph is in as minimal number of shown edges as possible. Additionally all nodes currently have as many connections as they are stated to have at their maximal consideration.