COMP.3040 Homework 4

June 23, 2019

Problem 3.1.

(a)

$q_1 0$
$\sqcup q_2 \sqcup$
$\sqcup \sqcup q_{accept}$

(c)

$q_1 000$
$\sqcup q_2 00$
$\sqcup x q_3 0$
$\sqcup x 0 q_4 \sqcup$
$\sqcup x 0 \sqcup q_{reject}$

(d)

$q_1 000000$	$\sqcup x q_5 0 x 0 x \sqcup$
$\sqcup q_2 00000$	$\sqcup q_5 x 0 x 0 x \sqcup$
$\sqcup x q_3 0000$	$q_5 \sqcup x 0 x 0 x \sqcup$
$\sqcup x 0 q_4 000$	$\sqcup q_2 x 0 x 0 x \sqcup$
$\sqcup x 0 x q_3 00$	$\sqcup x q_2 0 x 0 x \sqcup$
$\sqcup x 0 x 0 q_4 0$	$\sqcup x x q_3 x 0 x \sqcup$
$\sqcup x 0 x 0 x q_3 \sqcup$	$\sqcup x x x q_3 0 x \sqcup$
$\sqcup x 0 x 0 q_5 x \sqcup$	$\sqcup x x x 0 q_4 x \sqcup$
$\sqcup x 0 x q_5 0 x \sqcup$	$\sqcup x x x 0 x q_4 \sqcup$
$\sqcup x 0 q_5 x 0 x \sqcup$	$\sqcup x x x 0 x \sqcup q_{reject}$

Input string 00000000:

$q_1 00000000$
$\sqcup q_2 0000000$
$\sqcup x q_3 000000$
$\sqcup x 0 q_4 00000$
$\sqcup x 0 x q_3 0000$
$\sqcup x 0 x 0 q_4 000$
$\sqcup x 0 x 0 x q_3 00$
$\sqcup x 0 x 0 x 0 q_4 0$
$\sqcup x 0 x 0 x 0 x q_3 \sqcup$
$\sqcup x 0 x 0 x 0 q_5 x \sqcup$

$\sqcup x 0 x 0 x q_5 0 x \sqcup$
$\sqcup x 0 x 0 q_5 x 0 x \sqcup$
$\sqcup x 0 x q_5 0 x 0 x \sqcup$
$\sqcup x 0 q_5 x 0 x 0 x \sqcup$
$\sqcup x q_5 0 x 0 x 0 x \sqcup$
$\sqcup q_5 x 0 x 0 x 0 x \sqcup$
$q_5 \sqcup x 0 x 0 x 0 x \sqcup$
$\sqcup q_2 x 0 x 0 x 0 x \sqcup$
$\sqcup x q_2 0 x 0 x 0 x \sqcup$
$\sqcup x x q_3 x 0 x 0 x \sqcup$

$\sqcup x x x q_3 0 x 0 x \sqcup$
$\sqcup x x x 0 q_4 x 0 x \sqcup$
$\sqcup x x x x x q_4 0 x \sqcup$
$\sqcup x x x 0 x x q_3 x \sqcup$
$\sqcup x x x 0 x x x q_5 \sqcup$
$\sqcup x x x 0 x x q_5 x \sqcup$
$\sqcup x x x 0 x q_5 x x \sqcup$
$\sqcup x x x 0 q_5 x x x \sqcup$
$\sqcup x x x q_5 0 x x x \sqcup$
$\sqcup x x q_5 x 0 x x x \sqcup$

$\sqcup x q_5 x x 0 x x x \sqcup$
$\sqcup q_5 x x x 0 x x x \sqcup$
$q_5 \sqcup x x x 0 x x x \sqcup$
$\sqcup q_2 x x x 0 x x x \sqcup$
$\sqcup x q_2 x x 0 x x x \sqcup$
$\sqcup x x q_2 x 0 x x x \sqcup$
$\sqcup x x x q_2 0 x x x \sqcup$
$\sqcup x x x x q_3 x x x \sqcup$
$\sqcup x x x x x q_3 x x \sqcup$
$\sqcup x x x x x x q_3 x \sqcup$
$\sqcup x x x x x x x q_3 \sqcup$
$\sqcup x x x x x x x \sqcup q_{reject}$

Problem 3.2.

(b)

$q_1 1 \# 1$
$x q_3 \# 1$
$x \# q_5 1$
$x q_6 \# x$

$q_7 x \# x$
$x q_1 \# x$
$x \# q_8 x$
$x \# x q_8 \sqcup$
$x \# x \sqcup q_{accept}$

(c)

$q_1 1 \# \# 1$
$x q_3 \# \# 1$
$x \# q_5 \# 1$
$x \# \# q_{reject} 1$

(d)

$q_1 10 \# 11$
$x q_3 0 \# 11$
$x 0 q_3 \# 11$
$x 0 \# q_5 11$
$x 0 q_6 \# x 1$
$x q_7 0 \# x 1$

$q_7 x 0 \# x 1$
$x q_1 0 \# x 1$
$x x q_2 \# x 1$
$x x \# q_4 x 1$
$x x \# x q_4 1$
$x x \# x 1 q_{reject}$

(e)

$q_1 10\#10$ $xq_7 0\#x0$ $xx\#xq_4 0$ $xx\#q_8 xx$
$xq_3 0\#10$ $q_7 x0\#x0$ $xx\#q_6 xx$ $xx\#xq_8 x$
$x0q_3\#10$ $xq_1 0\#x0$ $xxq_6\#xx$ $xx\#xxq_8\sqcup$
$x0\#q_5 10$ $xxq_2\#x0$ $xq_7 x\#xx$ $xx\#xx \sqcup q_{accept}$
$x0q_6\#x0$ $xx\#q_4 x0$ $xxq_1\#xx$

Input string 01100#01100:

$q_1 01100\#01100$ $xx100\#q_5 x1100$ $xxx0q_7 0\#xxx00$ $xxxxx\#q_4 xxxx0$
$xq_2 1100\#01100$ $xx100\#xq_5 1100$ $xxxq_7 00\#xxx00$ $xxxxx\#xq_4 xxx0$
$x1q_2 100\#01100$ $xx100\#q_6 xx100$ $xxq_7 x00\#xxx00$ $xxxxx\#xxq_4 xx0$
$x11q_2 00\#01100$ $xx100q_6\#xx100$ $xxxq_2 00\#xxx00$ $xxxxx\#xxxq_4 x0$
$x110q_2 0\#01100$ $xx10q_7 0\#xx100$ $xxxxq_2 0\#xxx00$ $xxxxx\#xxxxq_4 0$
$x1100q_2\#01100$ $xx1q_7 00\#xx100$ $xxxx0q_2\#xxx00$ $xxxxx\#xxxq_6 xx$
$x1100\#q_4 01100$ $xxq_7 100\#xx100$ $xxxx0\#q_4 xxx00$ $xxxxx\#xxq_6 xxx$
$x1100q_6\#x1100$ $xq_7 x100\#xx100$ $xxxx0\#xq_4 xx00$ $xxxxx\#xq_6 xxxx$
$x110q_7 0\#x1100$ $xxq_1 100\#xx100$ $xxxx0\#xxq_4 x00$ $xxxxx\#q_6 xxxxx$
$x11q_7 00\#x1100$ $xxxq_3 00\#xx100$ $xxxx0\#xxxq_4 00$ $xxxxxq_6\#xxxxx$
$x1q_7 100\#x1100$ $xxx0q_3 0\#xx100$ $xxxx0\#xxq_6 xx0$ $xxxxq_7 x\#xxxxx$
$xq_7 1100\#x1100$ $xxx00q_3\#xx100$ $xxxx0\#xq_6 xxx0$ $xxxxxq_1\#xxxxx$
$q_7 x1100\#x1100$ $xxx00\#q_5 xx100$ $xxxx0\#q_6 xxxx0$ $xxxxx\#q_8 xxxxx$
$xq_1 1100\#x1100$ $xxx00\#xq_5 x100$ $xxxx0q_6\#xxxx0$ $xxxxx\#xq_8 xxxx$
$xxq_3 100\#x1100$ $xxx00\#xxq_5 100$ $xxxxq_7 0\#xxxx0$ $xxxxx\#xxq_8 xxx$
$xx1q_3 00\#x1100$ $xxx00\#xq_6 xx00$ $xxxq_7 x0\#xxxx0$ $xxxxx\#xxxq_8 xx$
$xx10q_3 0\#x1100$ $xxx00\#q_6 xxx00$ $xxxxq_1 0\#xxxx0$ $xxxxx\#xxxxq_8 x$
$xx100q_3\#x1100$ $xxx00q_6\#xxx00$ $xxxxxq_2\#xxxx0$ $xxxxx\#xxxxxq_8\sqcup$
$xxxxx\#xxxxx \sqcup q_{accept}$

Input string 01101#01100:

$q_1 01101\#01100$	$xx1q_3 01\#x1100$	$xxx01\#q_5 xx100$	$xxxx1\#xxxq_4 00$
$xq_2 1101\#01100$	$xx10q_3 1\#x1100$	$xxx01\#xq_5 x100$	$xxxx1\#xxq_6 xx0$
$x1q_2 101\#01100$	$xx101q_3\#x1100$	$xxx01\#xxq_5 100$	$xxxx1\#xq_6 xxx0$
$x11q_2 01\#01100$	$xx101\#q_5 x1100$	$xxx01\#xq_6 xx00$	$xxxx1\#q_6 xxxx0$
$x110q_2 1\#01100$	$xx101\#xq_5 1100$	$xxx01\#q_6 xxx00$	$xxxx1q_6\#xxxx0$
$x1101q_2\#01100$	$xx101\#q_6 xx100$	$xxx01q_6\#xxx00$	$xxxxq_7 1\#xxxx0$
$x1101\#q_4 01100$	$xx101q_6\#xx100$	$xxx0q_7 1\#xxx00$	$xxxq_7 x1\#xxxx0$
$x1101q_6\#x1100$	$xx10q_7 1\#xx100$	$xxxq_7 01\#xxx00$	$xxxxq_1 1\#xxxx0$
$x110q_7 1\#x1100$	$xx1q_7 01\#xx100$	$xxq_7 x01\#xxx00$	$xxxxxq_3\#xxxx0$
$x11q_7 01\#x1100$	$xxq_7 101\#xx100$	$xxxq_1 01\#xxx00$	$xxxxx\#q_5 xxxx0$
$x1q_7 101\#x1100$	$xq_7 x101\#xx100$	$xxxxq_2 1\#xxx00$	$xxxxx\#xq_5 xxx0$
$xq_7 1101\#x1100$	$xxq_1 101\#xx100$	$xxxx1q_2\#xxx00$	$xxxxx\#xxq_5 xx0$
$q_7 x1101\#x1100$	$xxxq_3 01\#xx100$	$xxxx1\#q_4 xxx00$	$xxxxx\#xxxq_5 x0$
$xq_1 1101\#x1100$	$xxx0q_3 1\#xx100$	$xxxx1\#xq_4 xx00$	$xxxxx\#xxxxq_5 0$
$xxq_3 101\#x1100$	$xxx01q_3\#xx100$	$xxxx1\#xxq_4 x00$	$xxxxx\#xxxx0q_{reject}\sqcup$

Problem 3.8.

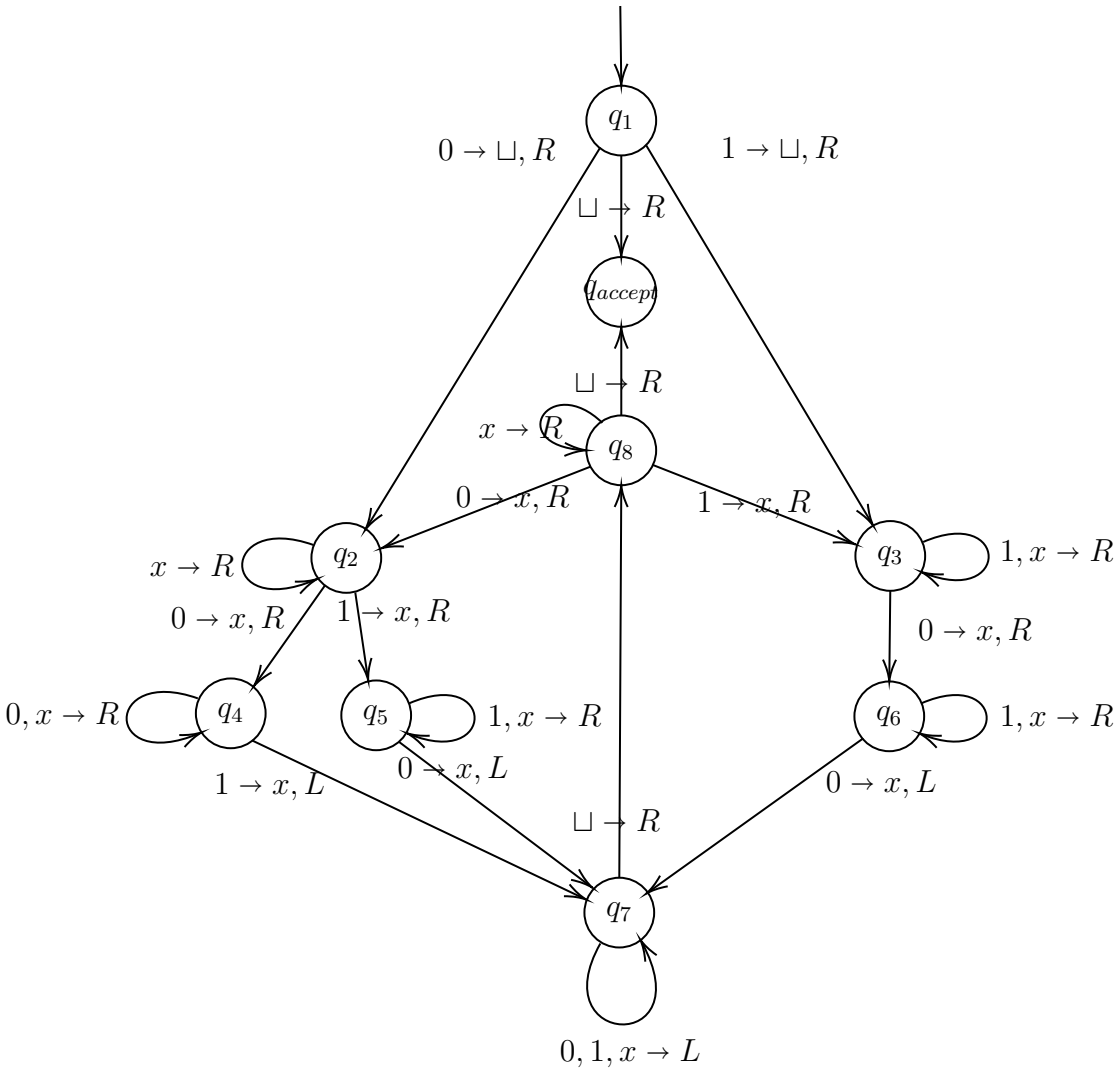
(b) 1. If first alphabet is $\sqcup$, input accepted.
2. If first alphabet is 0 write $\sqcup$, move right and jump to step 4.
3. If first alphabet is 1 write $\sqcup$, move right and jump to step 8.

4. Go right if read x. Repeat till read 0 or 1.
5. If read 0, write x, go right till read 1. Write x and go left.
6. If read 1, write x, go right till read 0. Write x and go left.
7. Go to step 11.

8. Go right if read x or 1. Repeat till read 0.
9. Write x. Go right if read x or 1. Repeat till read 0.
10. Write x and go left.

11. Go left till read $\sqcup$. Go right.
12. Go right if read x. Repeat till read 0, 1 or $\sqcup$.
13. If read $\sqcup$, input accepted.
14. If read 0, go to step 4.
15. If read 1, go to step 8.

State diagram:



Input string 010100:

$q_1010100$	$q_7 \sqcup xx100$	$\sqcup xxxq_7xx\sqcup$	$\sqcup xq_8xxxx\sqcup$
$\sqcup q_210100$	$\sqcup q_8xx100$	$\sqcup xxq_7xxx\sqcup$	$\sqcup xxq_8xxx\sqcup$
$\sqcup xq_50100$	$\sqcup xq_8x100$	$\sqcup xq_7xxxx\sqcup$	$\sqcup xxxq_8xx\sqcup$
$\sqcup xxq_7100$	$\sqcup xxq_8100$	$\sqcup q_7xxxxx\sqcup$	$\sqcup xxxxq_8x\sqcup$
$\sqcup xq_7x100$	$\sqcup xxxq_300$	$q_7 \sqcup xxxxx\sqcup$	$\sqcup xxxxxq_8\sqcup$
$\sqcup q_7xx100$	$\sqcup xxxxq_60$	$\sqcup q_8xxxxx\sqcup$	$\sqcup xxxxx \sqcup q_{accept}$

Input string 010101:

$q_1010101$	$\sqcup q_7xx101$	$\sqcup xq_8x101$	$\sqcup xxxxq_61$
$\sqcup q_210101$	$q_7 \sqcup xx101$	$\sqcup xxq_8101$	$\sqcup xxxxx \sqcup q_{reject}$
$\sqcup xq_50101$	$\sqcup q_8xx101$	$\sqcup xxxq_301$	

(c) 1. If first alphabet is $\sqcup$, input rejected.
2. If first alphabet is 0 write $\sqcup$, move right and jump to step 4.

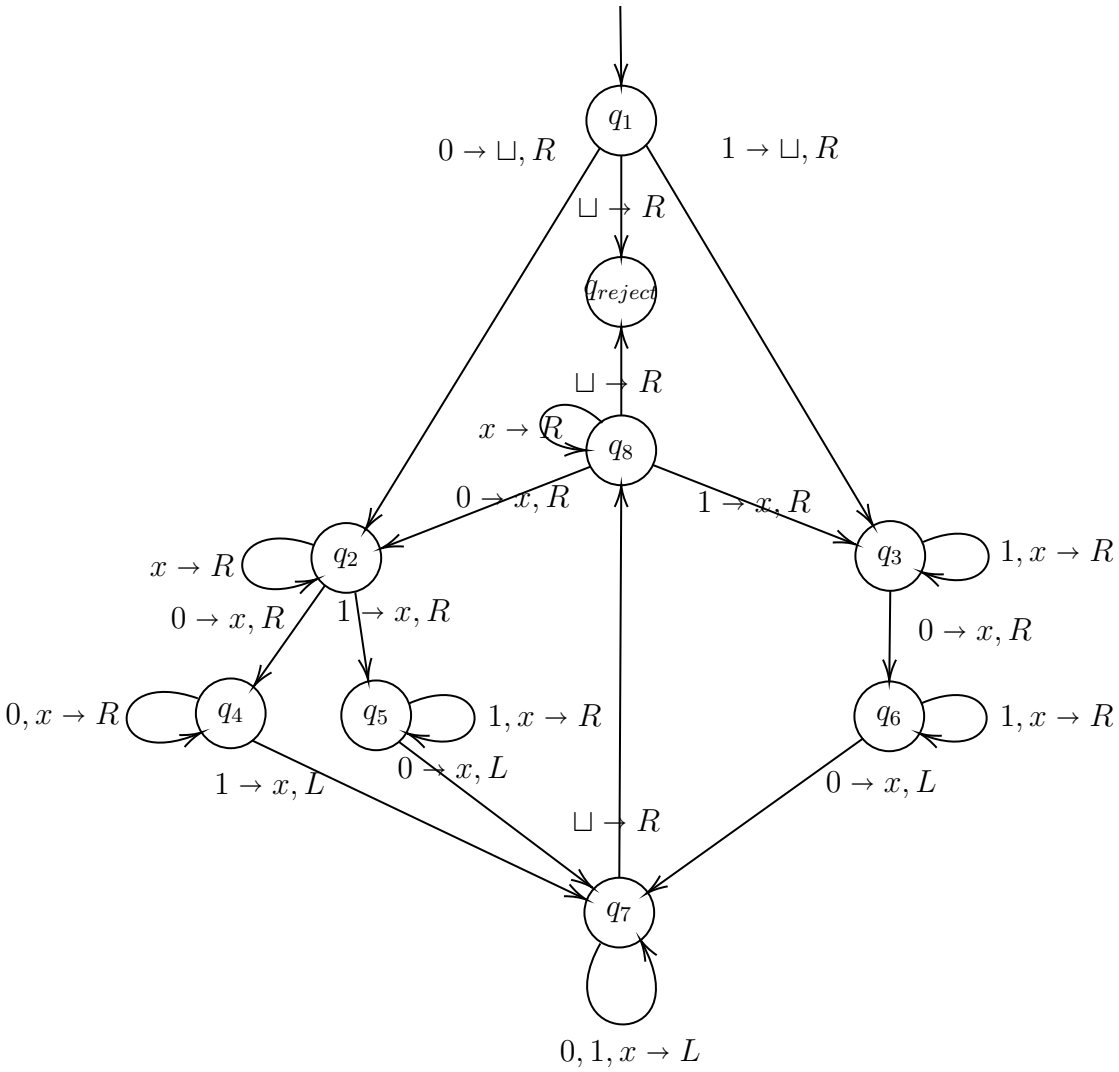
3. If first alphabet is 1 write ⊔, move right and jump to step 8.

4. Go right if read x. Repeat till read 0 or 1.
5. If read 0, write x, go right till read 1. Write x and go left.
6. If read 1, write x, go right till read 0. Write x and go left.
7. Go to step 11.

8. Go right if read x or 1. Repeat till read 0.
9. Write x. Go right if read x or 1. Repeat till read 0.
10. Write x and go left.

11. Go left till read ⊔. Go right.
12. Go right if read x. Repeat till read 0, 1 or ⊔.
13. If read ⊔, input rejected.
14. If read 0, go to step 4.
15. If read 1, go to step 8.

State diagram:



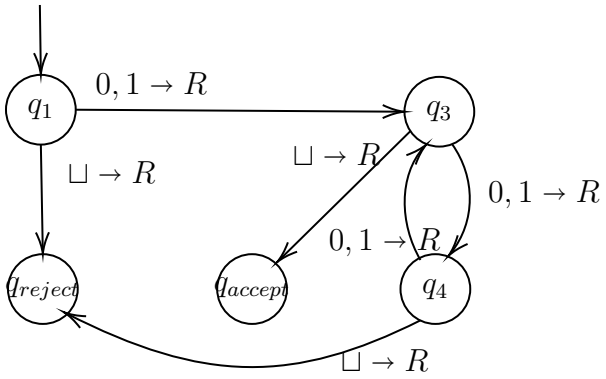
Input string 000111:

$q_1000111$	$\sqcup q_8x0x11$
$\sqcup q_200111$	$\sqcup xq_80x11$
$\sqcup xq_40111$	$\sqcup xxq_2x11$
$\sqcup x0q_4111$	$\sqcup xxxq_211$
$\sqcup xq_70x11$	$\sqcup xxxxq_51$
$\sqcup q_7x0x11$	$\sqcup xxxx1q_{accept}$
$q_7 \sqcup x0x11$	

Input string 000110:

$q_1000110$	$q_7 \sqcup x0x10$	$\sqcup xxxq_7xx$	$\sqcup xq_8xxxx$
$\sqcup q_200110$	$\sqcup q_8x0x10$	$\sqcup xxq_7xxx$	$\sqcup xxq_8xxx$
$\sqcup xq_40110$	$\sqcup xq_80x10$	$\sqcup xq_7xxxx$	$\sqcup xxxq_8xx$
$\sqcup x0q_4110$	$\sqcup xxq_2x10$	$\sqcup q_7xxxxx$	$\sqcup xxxxq_8x$
$\sqcup xq_70x10$	$\sqcup xxxq_210$	$q_7 \sqcup xxxxx$	$\sqcup xxxxxq_8\sqcup$
$\sqcup q_7x0x10$	$\sqcup xxxxq_50$	$\sqcup q_8xxxxx$	$\sqcup xxxxx \sqcup q_{reject}$

Modified M2 for detecting odd number of 0s:



Input string 000:

q_1000

$0q_300$

$00q_40$

$000q_3\sqcup$

$000 \sqcup q_{accept}$

Input string 0000:

q_10000

$0q_3000$

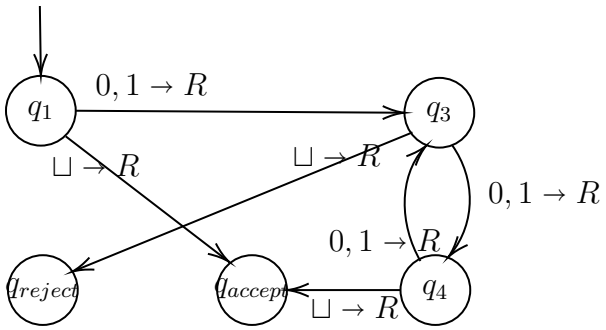
$00q_400$

$000q_30$

$0000q_4\sqcup$

$0000 \sqcup q_{reject}$

Modified M2 for detecting even number of 0s:



Input string 000:

q_1000

$0q_300$

$00q_40$

$000q_3\sqcup$

$000 \sqcup q_{reject}$

Input string 0000:

q_10000

$0q_3000$

$00q_400$

$000q_30$

$0000q_4\sqcup$

$0000 \sqcup q_{accept}$