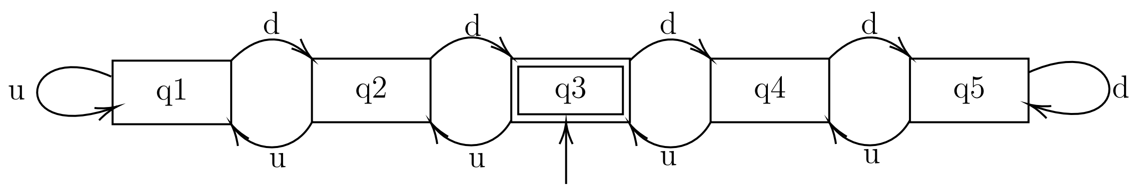
COMP.3040 Homework 1

June 16, 2019

Problem 1.3.



Problem 1.4.

(a) $L_1 = \{\text{w} \mid \text{w has at least three a's } \} = \{\ \{\varepsilon, a, aa, aaa\}, \{a, b\}, \delta_a, \varepsilon, aaa\}$

$\delta_a =$

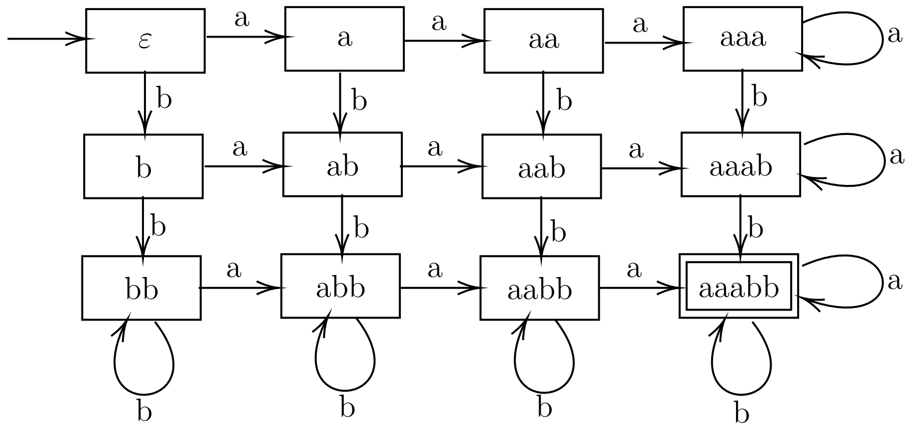
	a	b
ε	a	ε
a	aa	a
aa	aaa	aa
aaa	aaa	aaa

$L_2 = \{\text{w} \mid \text{w has at least two b's } \} = \{\ \{\varepsilon, b, bb\}, \{a, b\}, \delta_b, \varepsilon, bb\}$

$\delta_b =$

	b	a
ε	b	ε
b	bb	b
bb	bb	bb

Combining L_1 and L_2:



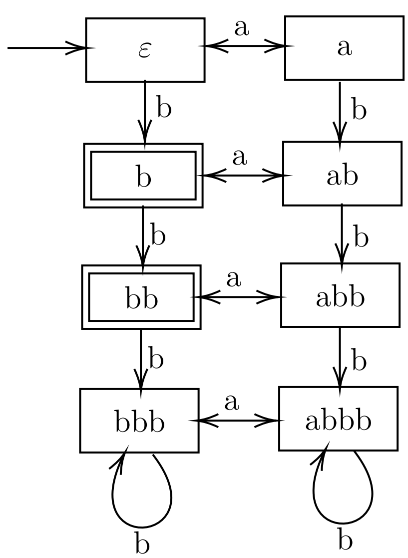
(c) $L_1 = \{$w $|$ w has even number of a's $\} = \{\ \{\varepsilon, a\}, \{a, b\}, \delta_a, \varepsilon, \varepsilon\}$

$$\delta_a = \begin{array}{c|c|c} & a & b \\ \hline \varepsilon & a & \varepsilon \\ \hline a & \varepsilon & a \end{array}$$

$L_2 = \{$w $|$ w has one or two b's $\} = \{\ \{\varepsilon, b, bb, bbb\}, \{a, b\}, \delta_b, \varepsilon, \{b, bb\}\}$

$$\delta_b = \begin{array}{c|cc} & a & b \\ \hline \varepsilon & \varepsilon & b \\ \hline b & b & bb \\ \hline bb & bb & bbb \\ \hline bbb & bbb & bbb \end{array}$$

Combining L_1 and L_2



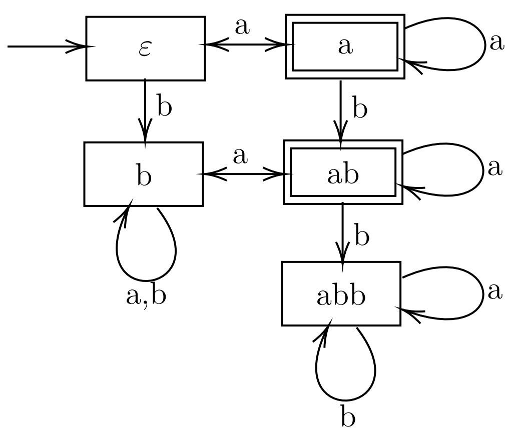
(e) $L_1 = \{$w $|$ w starts with an a $\} = \{\ \{\varepsilon, a, b\}, \{a, b\}, \delta_a, \varepsilon, a\}$

$$\delta_a = \begin{array}{c|cc} & a & b \\ \hline \varepsilon & a & b \\ \hline a & a & a \\ \hline b & b & b \end{array}$$

$L_2 = \{$w $|$ w has at most one b $\} = \{\ \{\varepsilon, b, bb\}, \{a, b\}, \delta_b, \varepsilon, \{\varepsilon, b\}\}$

$$\delta_b = \begin{array}{c|cc} & a & b \\ \hline \varepsilon & \varepsilon & b \\ \hline b & b & bb \\ \hline bb & bb & bb \end{array}$$

Combining L_1 and L_2



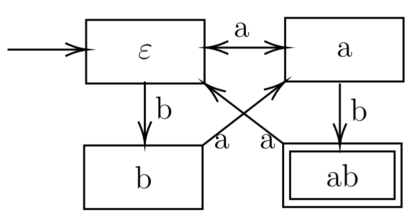
(f) $L_1 = \{$w $|$ w has odd number of a's $\} = \{\ \{\varepsilon, a\}, \{a, b\}, \delta_a, \varepsilon, a\}$

$$\delta_a = \begin{array}{c|cc} & a & b \\ \hline \varepsilon & a & \varepsilon \\ \hline a & \varepsilon & a \end{array}$$

$L_2 = \{$w $|$ w ends with b $\} = \{\ \{\varepsilon, b\}, \{a, b\}, \delta_b, \varepsilon, b\}$

$$\delta_b = \begin{array}{c|cc} & a & b \\ \hline \varepsilon & \varepsilon & b \\ \hline b & \varepsilon & b \end{array}$$

Combining L_1 and L_2



(g) $L_1 = \{$w $|$ w has even length $\} = \{\ \{\varepsilon, a, b, ab\}, \{a, b\}, \delta_a, \varepsilon, \{\varepsilon, ab\}\}$

$\delta_a =$

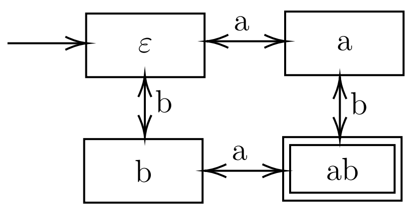
	a	b
ε	a	b
a	ε	ab
b	ab	ε
ab	b	a

$L_2 = \{$w | w has odd number of a's $\} = \{\ \{\varepsilon, a\}, \{a, b\}, \delta_b, \varepsilon, a\}$

$\delta_b =$

	a	b
ε	a	ε
a	ε	a

Combining L_1 and L_2



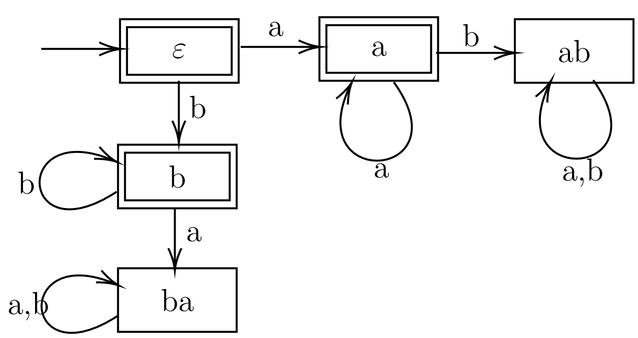
Problem 1.5.

(c) L $= \{\ \{\varepsilon, a, b, ab, ba\}, \{a, b\}, \delta, \varepsilon, \{a, b\}\}$

$\delta =$

	a	b
ε	a	b
a	a	ab
b	b	ba
ab	ab	ab
ba	ba	ba

State diagram:

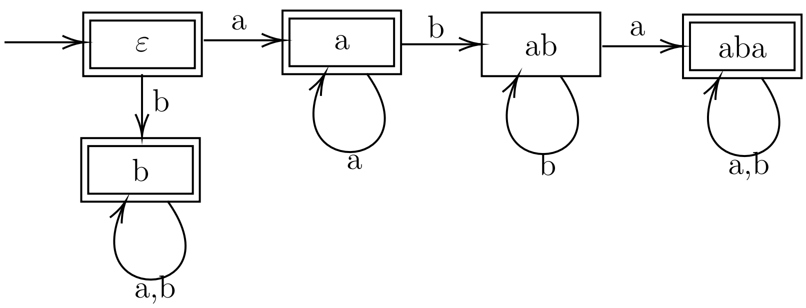


(d) L $= \{\ \{\varepsilon, a, b, ab, aba\}, \{a, b\}, \delta, \varepsilon, \{\varepsilon, a, b, aba\}\}$

$\delta =$

	a	b
ε	a	b
a	a	ab
b	b	b
ab	aba	ab
aba	aba	aba

State diagram:

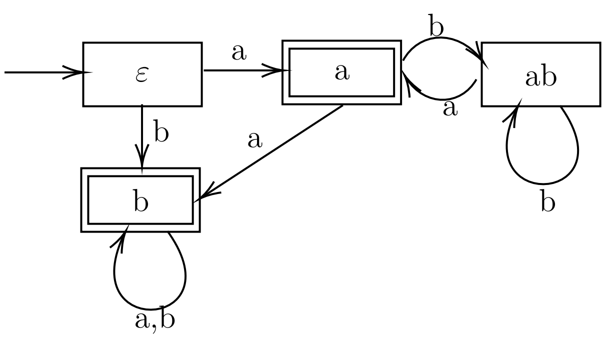


(e) L = $\{\ \{\varepsilon, a, b, ab\}, \{a, b\}, \delta, \varepsilon, \{a, b\}\}$

$\delta =$

	a	b
ε	a	b
a	b	ab
b	b	b
ab	a	ab

State diagram:

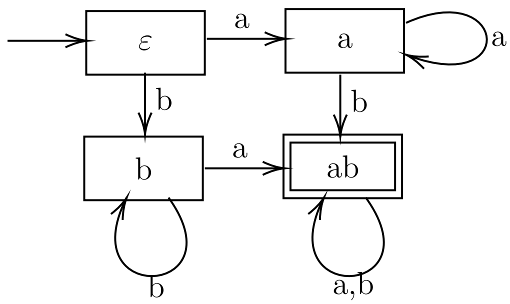


(f) L = $\{\ \{\varepsilon, a, b, ab\}, \{a, b\}, \delta, \varepsilon, \{ab\}\}$

$\delta =$

	a	b
ε	a	b
a	a	ab
b	ab	b
ab	ab	ab

State diagram:

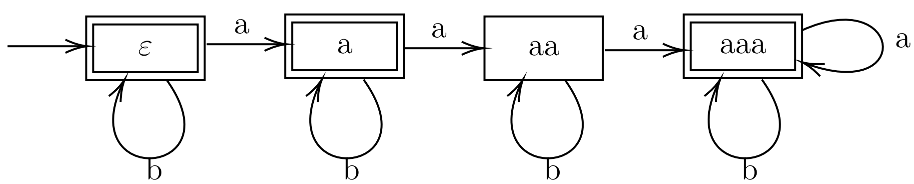


(g) L = $\{\ \{\varepsilon, a, aa, aaa\}, \{a, b\}, \delta, \varepsilon, \{\varepsilon, a, aaa\}\}$

$\delta =$

	a	b
ε	ε	a
a	a	aa
aa	aa	aaa
aaa	aaa	aaa

State diagram:

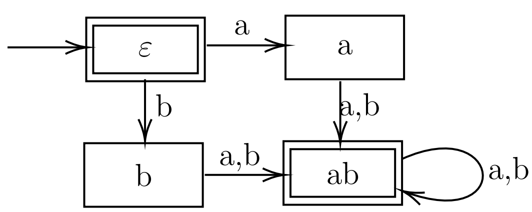


(h) L = $\{\ \{\varepsilon, a, b, ab\}, \{a, b\}, \delta, \varepsilon, \{\varepsilon, ab\}\}$

$\delta =$

	a	b
ε	a	b
a	ab	ab
b	ab	ab

State diagram:



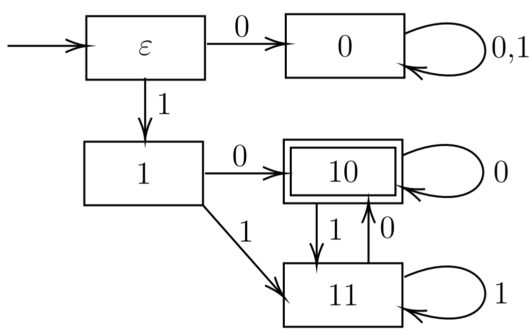
Problem 1.6.

(a) L = $\{\ \{\varepsilon, 0, 1, 10, 11\}, \{0, 1\}, \delta, \varepsilon, 10\}$

$\delta =$

	0	1
ε	0	1
0	0	0
1	10	11
10	10	11
11	10	11

State diagram:

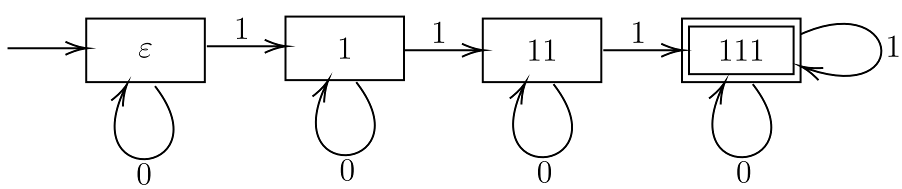


(b) $L = \{\, \{\varepsilon, 1, 11, 111\}, \{0, 1\}, \delta, \varepsilon, 111\}$

$\delta =$

	0	1
ε	ε	1
1	1	11
11	11	111
111	111	111

State diagram:

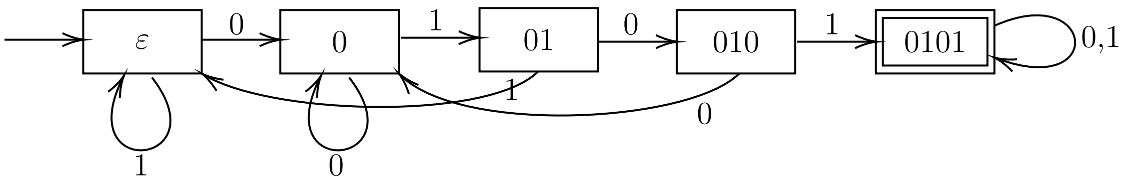


(c) $L = \{\, \{\varepsilon, 0, 01, 010, 0101\}, \{0, 1\}, \delta, \varepsilon, 0101\}$

$\delta =$

	0	1
ε	0	ε
0	0	01
01	010	ε
010	0	0101
0101	0101	0101

State diagram:

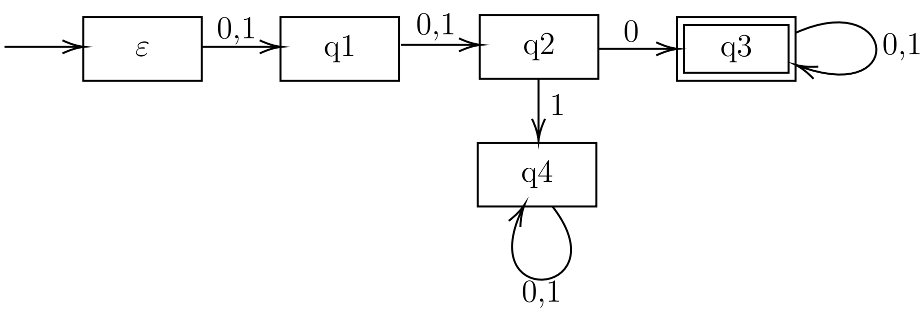


(d) L = $\{\ \{\varepsilon, q1, q2, q3, q4\}, \{0, 1\}, \delta, \varepsilon, q3\}$

$\delta =$

	0	1
ε	$q1$	$q1$
$q1$	$q2$	$q2$
$q2$	$q3$	$q4$
$q3$	$q3$	$q3$
$q4$	$q4$	$q4$

State diagram:

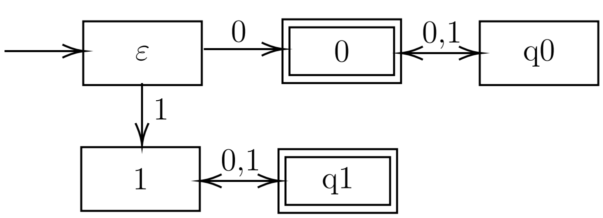


(e) L = $\{\ \{\varepsilon, 0, 1, q0, q1\}, \{0, 1\}, \delta, \varepsilon, \{0, q1\}\}$

$\delta =$

	0	1
ε	0	1
0	$q0$	$q0$
1	$q1$	$q1$
$q0$	0	0
$q1$	1	1

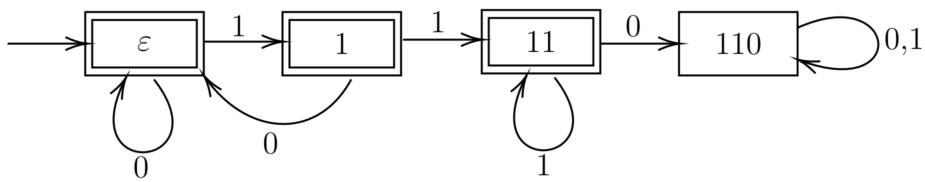
State diagram:



(f) L = $\{\ \{\varepsilon, 1, 11, 110\}, \{0, 1\}, \delta, \varepsilon, \{\varepsilon, 1, 11\}\}$

$\delta =$	0	1
ε	ε	1
1	ε	11
11	110	11
110	110	110

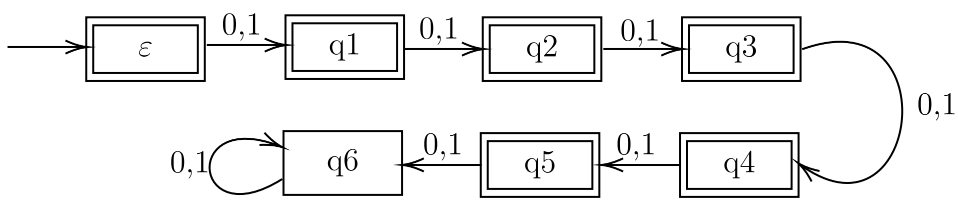
State diagram:



(g) L = $\{ \{\varepsilon, q1, q2, q3, q4, q5, q6\}, \{0,1\}, \delta, \varepsilon, \{\varepsilon, q1, q2, q3, q4, q5\}\}$

$\delta =$	0	1
ε	$q1$	$q1$
$q1$	$q2$	$q2$
$q2$	$q3$	$q3$
$q3$	$q4$	$q4$
$q4$	$q5$	$q5$
$q5$	$q6$	$q6$
$q6$	$q6$	$q6$

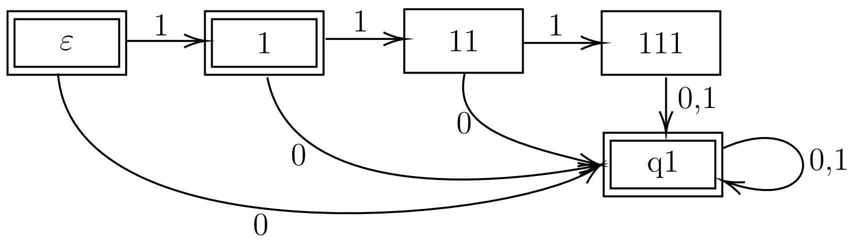
State diagram:



(h) L = $\{ \{\varepsilon, 1, 11, 111, q1\}, \{0,1\}, \delta, \varepsilon, \{\varepsilon, 1, q1\}\}$

$\delta =$	0	1
ε	$q1$	1
1	$q1$	11
11	$q1$	111
111	$q1$	$q1$
$q1$	$q1$	$q1$

State diagram:

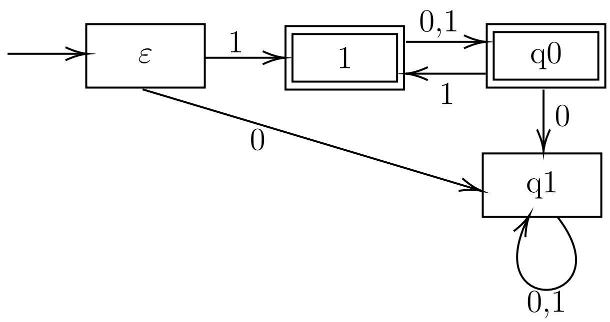


(i) L = $\{\ \{\varepsilon, q0, 1, q1\}, \{0,1\}, \delta, \varepsilon, \{q0, 1\}\}$

$\delta =$

	0	1
ε	$q1$	1
1	$q0$	$q0$
$q0$	$q1$	1
$q1$	$q1$	$q1$

State diagram:

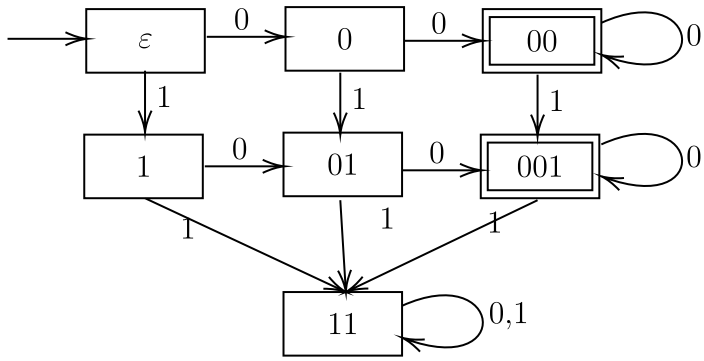


(j) L = $\{\ \{\varepsilon, 0, 00, 1, 01, 001, 11\}, \{0,1\}, \delta, \varepsilon, \{00, 001\}\}$

$\delta =$

	0	1
ε	0	1
0	00	01
00	00	001
1	01	11
01	001	11
001	001	11
11	11	11

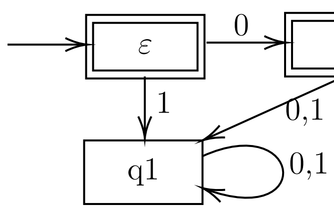
State diagram:



(k) $L = \{\ \{\varepsilon, 0, q1\}, \{0,1\}, \delta, \varepsilon, \{\varepsilon, 0\}\}$

$$\delta = \begin{array}{c|c|c} & 0 & 1 \\ \hline \varepsilon & 0 & q1 \\ \hline 0 & q1 & q1 \\ \hline q1 & q1 & q1 \end{array}$$

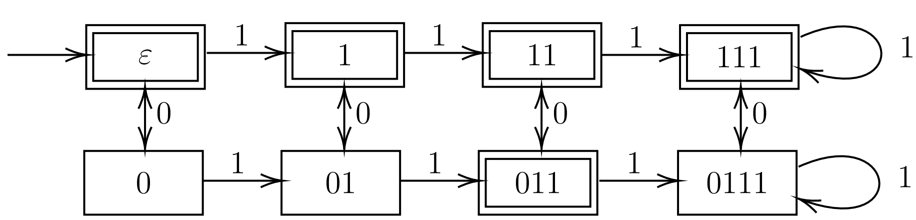
State diagram:



(l) $L = \{\ \{\varepsilon, 1, 11, 111, 0, 01, 011, 0111\}, \{0,1\}, \delta, \varepsilon, \{\varepsilon, 1, 11, 111, 011\}\}$

$$\delta = \begin{array}{c|c|c} & 0 & 1 \\ \hline \varepsilon & 0 & 1 \\ \hline 1 & 01 & 11 \\ \hline 11 & 011 & 111 \\ \hline 111 & 0111 & 111 \\ \hline 0 & \varepsilon & 01 \\ \hline 01 & 1 & 011 \\ \hline 011 & 11 & 0111 \\ \hline 0111 & 111 & 0111 \end{array}$$

State diagram:

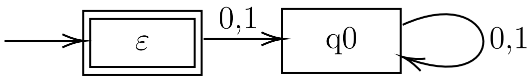


(m) $L = \{\ \{\varepsilon, q0\}, \{0,1\}, \delta, \varepsilon, \varepsilon\}$

$\delta =$

	0	1
ε	$q0$	$q0$
$q0$	$q0$	$q0$

State diagram:

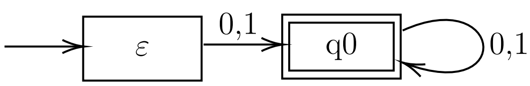


(n) $L = \{\ \{\varepsilon, q0\}, \{0,1\}, \delta, \varepsilon, q0\}$

$\delta =$

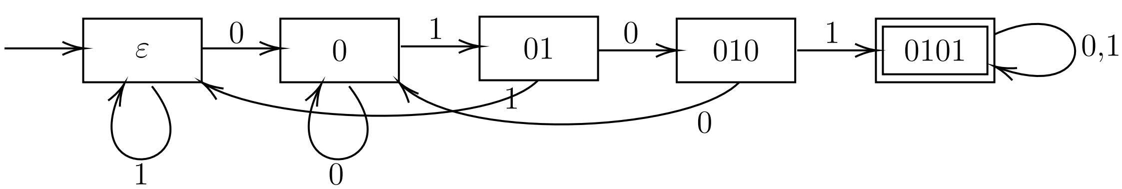
	0	1
ε	$q0$	$q0$
$q0$	$q0$	$q0$

State diagram:

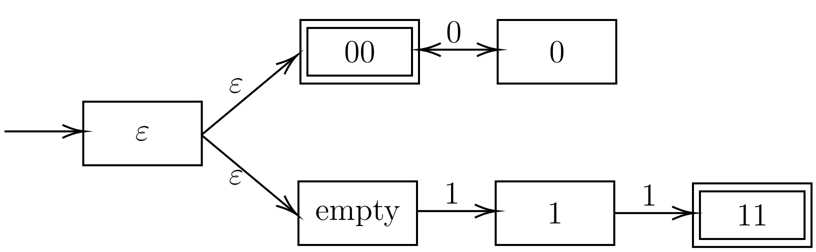


Problem 1.7.

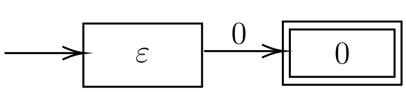
(b)



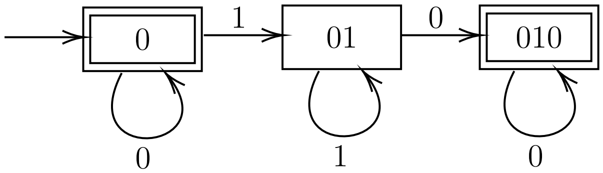
(c)



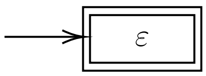
(d)



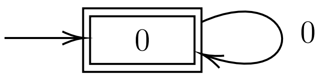
(e)



(g)

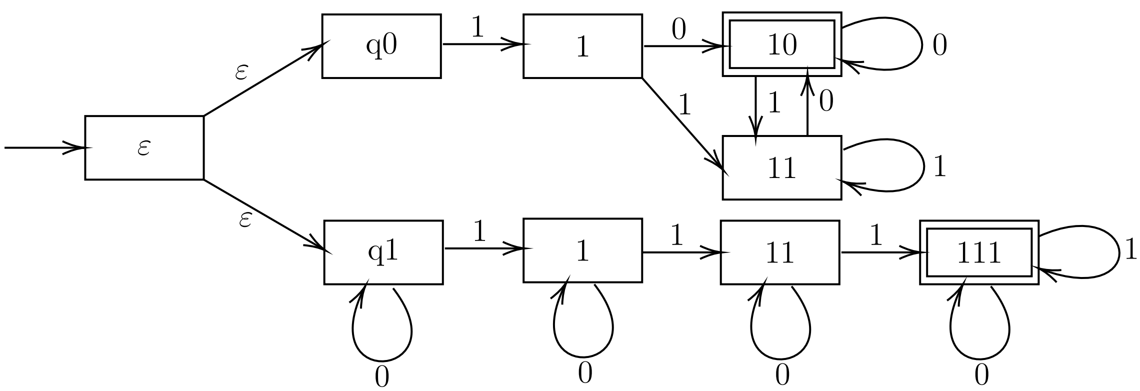


(h)

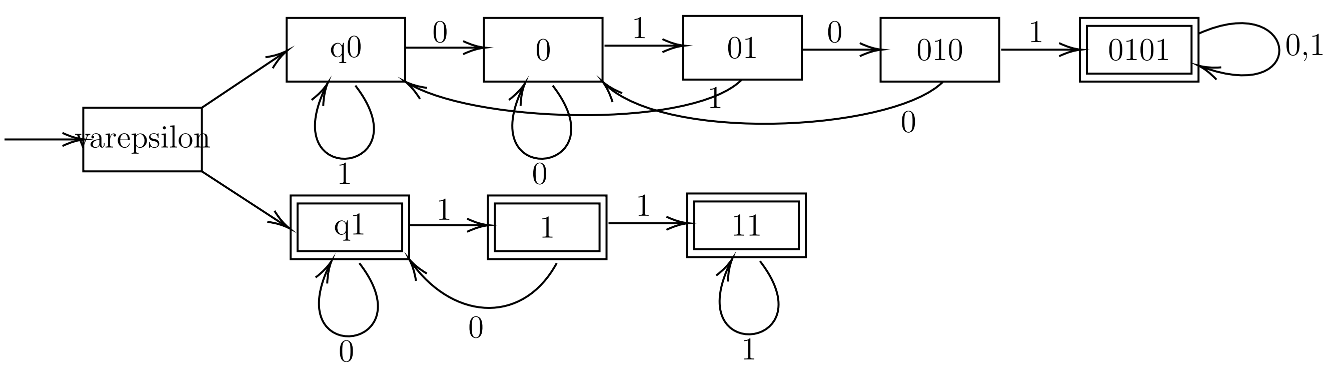


Problem 1.8.

(a)

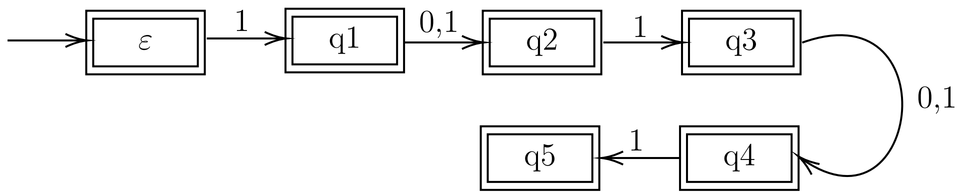


(b)



Problem 1.9.

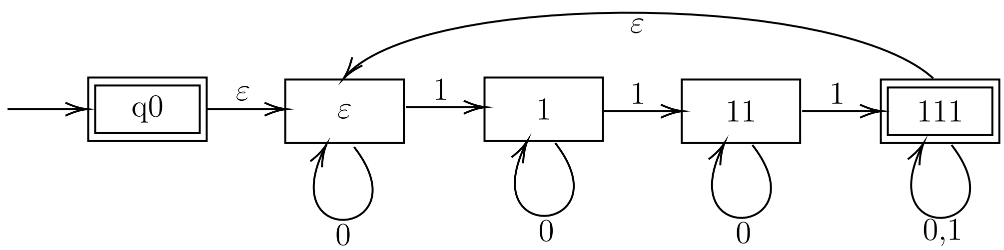
(a)



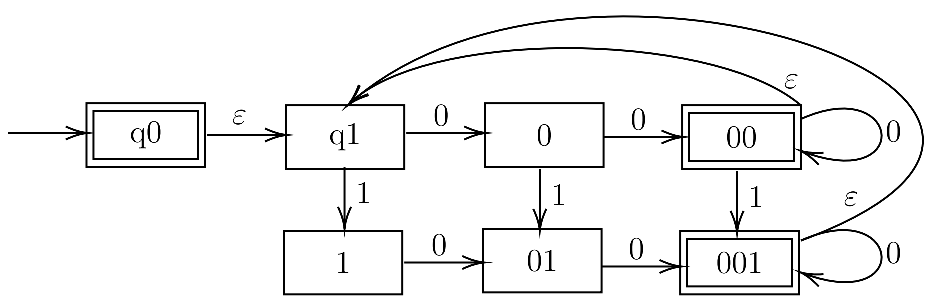
(b) Set cannot be empty while containing 1s. Set does not exist.

Problem 1.10.

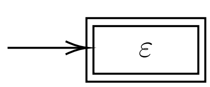
(a)



(b)



(c)



Problem 1.12.

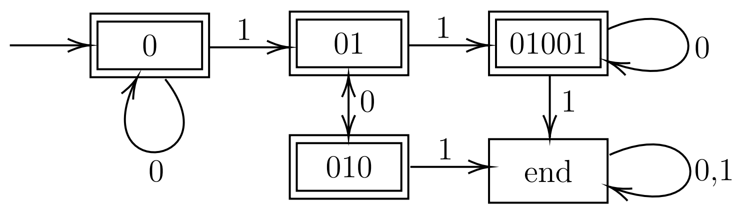
$D = \{\ \{\varepsilon, a, b, ba, bab\}, \{a, b\}, \delta, \varepsilon, \{b, ba\}\}$

$\delta =$

	a	b
ε	a	b
a	a	a
b	ba	b
ba	ba	bab
bab	bab	bab

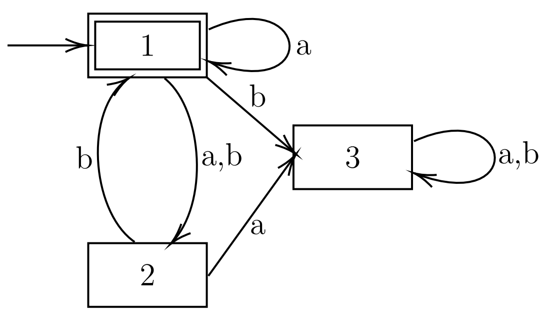
Regular expressio: D = b^+a^*

Problem 1.13.

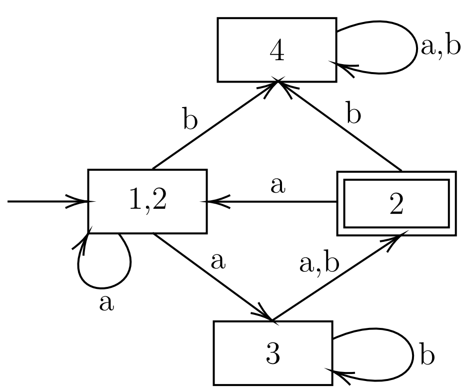


Problem 1.16.

(a)



(b)



Problem 1.17.

(a) $\{\ \{\varepsilon, 0, 01, 010, 00, 001\}, \{0, 1\}, \delta, \varepsilon, \{\varepsilon, 01, 010, 001\}\}$

$\delta =$

	0	1	ε
ε	0	–	ε
0	00	01	0
01	010, 0	–	ε, 01
010	0	–	ε, 010
00	–	001	00
001	0	–	ε, 001

(b) $\{\ \{\varepsilon, 0, 01, 010, 00, 001, end\}, \{0, 1\}, \delta, \varepsilon, \{\varepsilon, 01, 010, 001\}\}$

$\delta =$

	0	1
ε	0	*end*
0	00	01
01	010	*end*
010	0	01
00	*end*	001
001	0	*end*
end	*end*	*end*

Problem 1.18.

(a) $1\Sigma^*0$

(b) $\Sigma^*1\Sigma^*1\Sigma^*1\Sigma^*$

(c) $\Sigma^*0101\Sigma^*$

(d) $\Sigma\Sigma0\Sigma^*$

(e) $0(\Sigma\Sigma)^* \cup 1\Sigma(\Sigma\Sigma)^*$

(f) $1?(0^+1?)^*1? \cup 1^*$

(g) $\Sigma?\Sigma?\Sigma?\Sigma?\Sigma?$

(h) $\Sigma^4\Sigma^* \cup \Sigma? \cup 0\Sigma\Sigma \cup \Sigma0\Sigma \cup \Sigma\Sigma0$

(i) $(1\Sigma)^*1?$

(j) $00^+1? \cup 1?00^+ \cup 0^+1?0^+$

(k) $\varepsilon \cup 0$

(l) $(1^*01^*01^*)^* \cup 0^*10^*10^*$

(m) ε

(n) Σ^+

Problem 1.20.

(a) Member: a, b
Not member: $ba, bbaa$

(b) Member: $ab, abab$
Not member: ba, aa

(c) Member: a, b
Not member: ab, ba

(d) Member: ε, aaa
Not member: b, bb

(e) Member: $aba, abaa$
Not member: a, b

(f) Member: aba, bab
Not member: a, b

(g) Member: ab, b
Not member: a, aa

(h) Member: a, ba
Not member: b, ε

Problem 1.21.

(a) $a^*b(a^*ba^*ba^*)^*$

(b) $(\Sigma a^*b(bb)^*a)^* \cup \Sigma a^*b((bb)^*a\Sigma a^*b(bb)^*)^*$

Problem 1.22.

(a) $\{\ \{\varepsilon, q0, q1, a, b, q2, q3, -\}, \{/, \#, a, b\}, \delta, \varepsilon, \{q3\}\}$

$$\delta = \begin{array}{c|c|c|c|c}
 & / & \# & a & b \\
\hline
\varepsilon & q0 & - & - & - \\
\hline
q0 & - & q1 & - & - \\
\hline
q1 & - & - & a & b \\
\hline
a & - & q2 & a & b \\
\hline
b & - & q2 & a & b \\
\hline
q2 & q3 & - & - & - \\
\hline
q3 & - & - & - & - \\
\hline
- & - & - & - & -
\end{array}$$

(b) $/\#(a?b?)^*\#/$