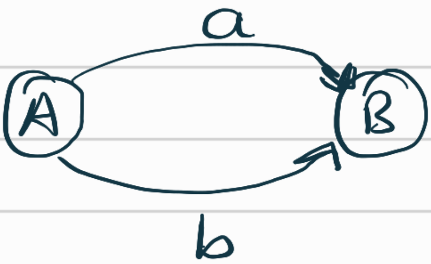
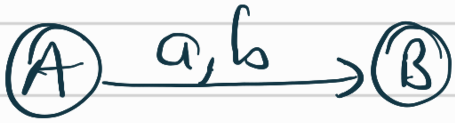
Conversion of Regular Expression to finite Automata

• a+b



or



• ab

• a^*

Example: Convert the following Regular Expressions to their equivalent finite automata

1) ba^*b
2) $(a+b)c$
3) $a(bc)^*$

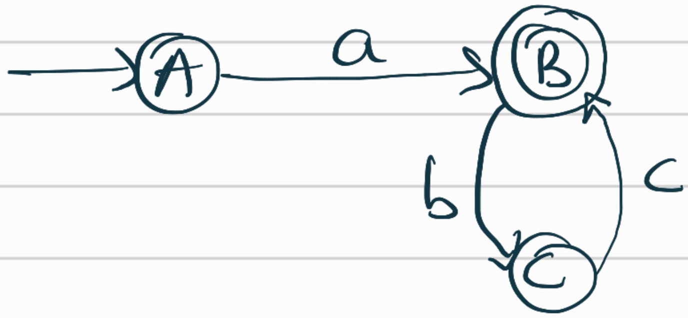
Solution

1) ba^*b

2) $(a+b)c$

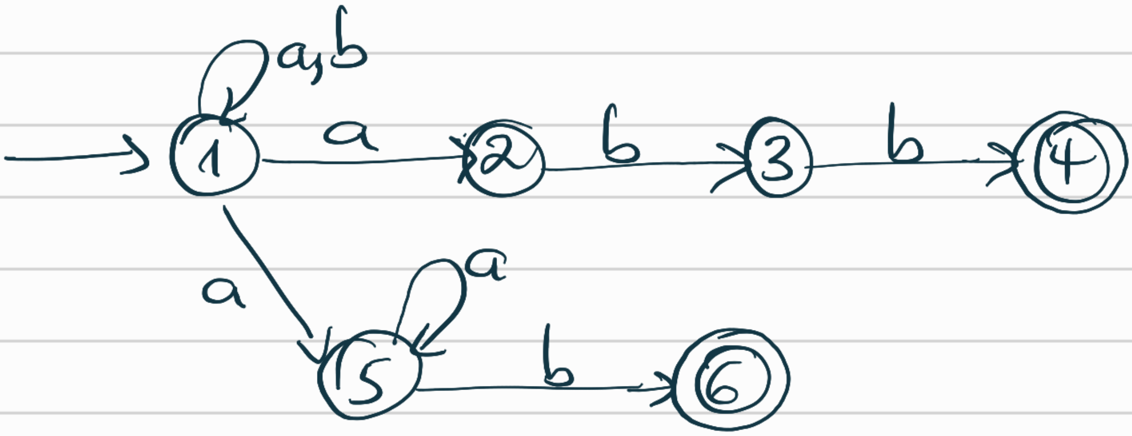
ac ✓
bc ✓

3) $a(bc)^*$



Example 2: Convert the following Regular Expression to its equivalent finite Automata

$(a|b)^*(abb|a^+b)$



Exercise: Convert the following Regular Expression to its equivalent finite automata

$10 + (0+11)0^*1$

Equivalence of two finite Automata

Steps

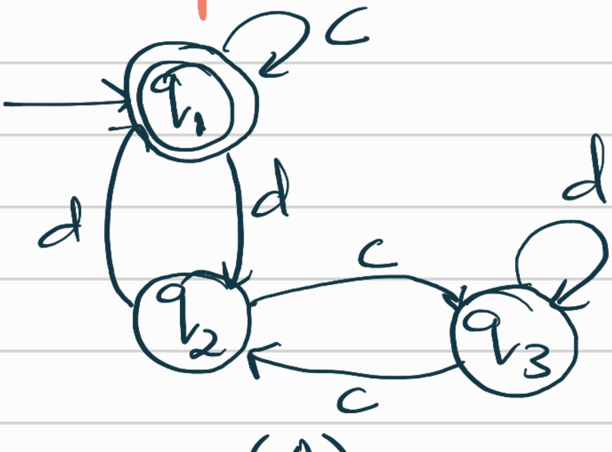
1) For any pair of states $\{q_i, q_j\}$ the transition for input $a \in \Sigma$ is defined by $\{q_a, q_b\}$ where $\delta(q_i, a) = q_a$ and $\delta(q_j, a) = q_b$

The two automata are not equivalent if

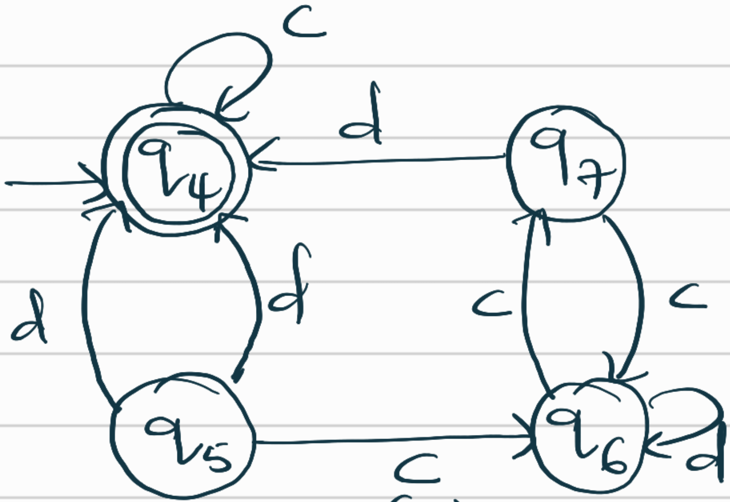
for a pair $\{q_a, q_b\}$ one is an INTERMEDIATE state and the other is a FINAL State.

2) If the initial state is the final state of one automaton, then in the second automaton, Initial state must be final state for them to be equivalent.

Example:



(A)

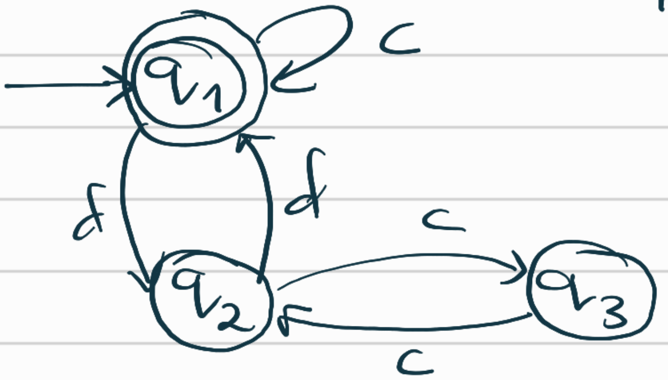


(B)

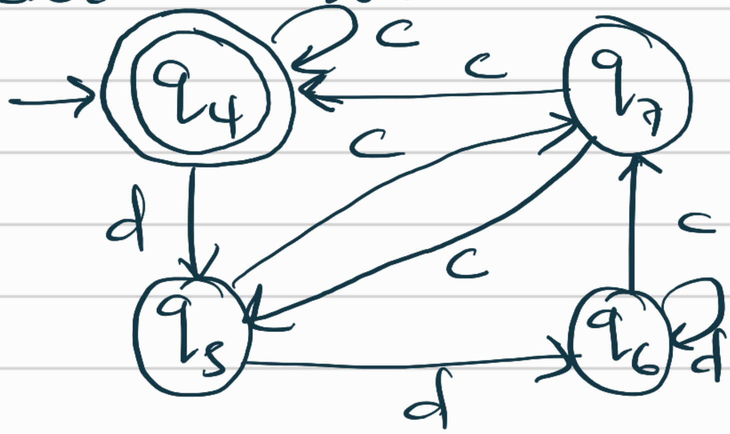
states	c	d
(q_1, q_4)	(q_1, q_4) FS FS	(q_2, q_5) IS IS
(q_2, q_5)	(q_3, q_6) IS IS	(q_1, q_4) FS FS
(q_3, q_6)	(q_2, q_7) IS IS	(q_3, q_6) IS IS
(q_2, q_7)	(q_3, q_6) IS IS	(q_1, q_4) FS FS

A and B are equivalent.

Exercise: Find out whether the following automata are equivalent or not



(A)



(B)

Pumping Lemma (for Regular Languages)

→ Pumping Lemma is used to prove that a language is NOT REGULAR

NB: It can not be used to prove that a language is Regular.

Lemma: If A is a RL, then A has a pumping length 'p' such that any string 'S' where $|S| \geq p$ may be divided into 3 parts $S = xyz$ such that the following conditions must be true:

1) $xy^iz \in A$ for every $i \geq 0$
2) $|y| > 0$
3) $|xy| \leq P$

Steps (we prove using Contradiction)

→ Assume that A is a Regular Language
→ It has to have a pumping length (say P)
→ All strings longer than P can be pumped $|S| \geq P$

→ Now find a string 'S' in A such that $|S| \geq P$
→ Divide S into xyz
→ Show that $xy^iz \notin A$ for some i
→ Then, consider all ways that S can be divided into xyz
→ Show that none of these can satisfy all the 3 pumping conditions at the same time.
→ S can not be pumped == CONTRADICTION

Example: Using pumping Lemma, prove that the language

$$A = \{a^n b^n \mid n \geq 0\} \text{ is Not Regular}$$

Solution

Assume that A is regular
Pumping length = P

$S = \underbrace{a^P b^P}_{x\ y\ z}$ ⇒ $S = aaaaaaabbbbbbb$

$P = 7$

Case 1: The y is in the 'a' part

$\underbrace{aa}_{x}\underbrace{aaa}_{y}\underbrace{aabbbbbbb}_{z}$

Case 2: The y is in the 'b' part

$\underbrace{aaaaaaabb}_{x}\underbrace{bbb}_{y}\underbrace{bb}_{z}$

Case 3: The y is in the 'a' and 'b' part

$$\underbrace{aaaaaaa}_{x}\underbrace{abb}_{y}\underbrace{bbbbb}_{z}$$

<u>Case 1:</u> $xy^iz \Rightarrow xy^2z$

aa aaaaaaaa abbbbbbb Does not lie in our Language

$11 \neq 7$

<u>Case 2:</u> $xy^iz \Rightarrow xy^2z$

aaaaaaabb bbbbbbbbb Does not lie in our Language

$7 \neq 11$

Case 3: $xy^iz \Rightarrow xy^2z$

aaaaa aabbaabb bbbbb Does not lie in our Language

- $|xy| \le p$, $p = 7$

Case 1: $|xy| = 6 < 7$ true

Case 2: $|xy| = 13 \nleq 7$ Not true

Case 3: $|xy| = 9 \nleq 7$ Not true

Exercise: Using pumping Lemma, prove that the Language

$A = \{yy \mid y \in \{0,1\}^*\}$ is NOT Regular

Hint: It is not regular bc you need memory

e.g <u>01</u><u>01</u>