

# STATE ESTIMATION OF VOLTAGE AND PHASE-SHIFT TRANSFORMER TAP SETTINGS

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**Abstract** - Traditionally, state estimation algorithms have treated each transformer tap setting (voltage transformer turns ratio or phase-shift transformer angle) as a fixed parameter of the network, even though the real-time measurement may be in error or non-existent. Such a strategy can lead to misdirecting residuals in adjacent valid measurements when the modeled transformer setting is incorrect. Ultimately, a network solution is derived which does not match actual real-time conditions.

A new transformer tap estimation technique is presented which incorporates the function directly into the state estimation algorithm. The procedure provides for turns ratio and phase angle measurements and treats each transformer tap setting as an independent state variable. Test results for an actual 300-bus network demonstrate the tap estimation capability.

**Keywords** - Transformer Tap Estimation, State Estimation.

## 1. INTRODUCTION

State estimation is the process which forms the basis of analyzing network security within an energy management system. State estimation processing produces a model of the power system which most likely represents actual network conditions. Since network conditions are constantly changing, state estimation is generally applied on a cyclic basis, creating a new network model when changes have become significant.

For real-time operations analysis, each newly constructed model can be used in numerous analyses, such as:

- analysis of impending security hazards
- approval of planned network outages
- detection and identification of measurement errors
- determination of generation penalties due to inherent transmission losses
- determination of remedial actions to relieve network security violations
- optimization of system operations to achieve improved economy, losses, or emissions

An archived set of state estimated models can

furthermore be used in a study mode framework for:

- reconstruction of past operating conditions
- analysis of alternate operating configurations
- attributing losses to their origin, e.g. interchange transactions and independent power producers

Because of the significance of the applications which use the state estimation model, it is imperative that the model be an accurate representation of the actual network condition. One limitation that has reduced the accuracy of first-generation state estimation algorithms is their treatment of transformer tap positions as fixed network parameters. It has been standard practice for voltage transformer turns ratios and phase transformer shift angles to take on their measured values or, if unmeasured, manually specified values.

Problems arise when a tap measurement is in error or an unmeasured tap is unknown. The estimated voltages, power flows, and power injections may deviate significantly from actual conditions in the region of an incorrectly modeled transformer. Also, misrepresentation of the transformer tap may lead to inadvertent bad data rejection of an accurate measurement which appears anomalous due to its inconsistency with the erroneous tap position. These model inaccuracies are then carried forward to the multitude of applications which rely upon an accurate model of the network.

There has been earlier documented work in the area of voltage transformer tap estimation within the framework of state estimation [1,2,3]. These efforts identified several situations which can result in the use of incorrect transformer tap values:

- automatic tap changing under load (TCUL) transformers with no tap telemetry
- unreported or incorrectly reported field change of tap position not under supervisory control
- incorrect tap position telemetry on a controllable or TCUL transformer

The benefits of tap estimation have also been documented and include the following:

- the effects of automatic and manual tap changes are immediately reflected into the state estimation model
- measured tap positions can be verified
- the overall quality of state estimation models is improved
- a mechanism for improved checkout and maintenance of the network security database is provided

The techniques employed by these earlier tap es-

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timization procedures, however, have several limitations because of their approach. First, they rely upon the existence of a presumed accurate voltage or reactive flow measurement at the transformer terminal. Also, the adjunct nature of these algorithms can result in extended solution times and the possibility of convergence problems. Finally, estimation of phase-shifting transformer taps has not been addressed.

The new method of transformer tap estimation presented in this paper overcomes these limitations because of the direct algorithmic approach. The method incorporates the tap position as an additional independent variable in the state vector. The method is applied to both voltage and phase-shifting transformers. The problem of discrete tap positions is addressed, and an appropriate integer variable technique is presented. Finally, the method is general and is compatible with State Estimation techniques which are based on network measurements and the Jacobian matrix.

Because of the simultaneous solution of taps with complex bus voltages, the method is not predicated on the presence of transformer tap position, terminal voltage, or reactive flow measurements, although the method exploits such measurements if available. Also, the method allows for bad data detection of anomalous tap measurements.

Feasibility of the method is demonstrated on a working state estimator in a utility Energy Management System, with initial results reported. Examples which illustrate the advantages of direct tap estimation are presented.

## 2. PREVIOUS TAP ESTIMATION ALGORITHMS

There have been two procedures for transformer tap estimation documented in the literature [1,2]. Fletcher and Stadlin [1] proposed voltage transformer tap estimation by modifying the turns ratio estimate between iterations of the state estimator solution. The turns ratio was revised to effectively minimize the residual of the transformer reactive flow measurement (which is closely coupled to the turns ratio). The method could be applied only if a reactive flow measurement was available, and the resultant tap estimate was subject to the flow measurement's accuracy. The method increased the state estimator iteration count. The method had other limitations as well, such as the requirement that reactive flow not be strongly coupled with another transformer whose tap was being estimated.

Implementation experience with the former method was reported by Smith [3]. He concludes that although the method was helpful in network model and telemetry checkout, there was insufficient accuracy in the tap estimates produced to use them with confidence.

A different approach was presented by Mukherjee et al [2]. The approach does not affect the basic state estimator solution, but instead introduces a tap correction in a post-processing step. The correction attempts to revise the tap to minimize the residual of a voltage measurement at one of the transformer terminals. The approach is predicated on the existence of a suitable voltage measurement and is limited by its accuracy. A key feature of the approach is that it seeks only to improve the tap positions of the network model which is passed on to subsequent security calculations; internal to the state estimator solution the effects of incorrect taps continue unabated.

It should be noted that the historical methods for tap estimation are subject to problems when the input tap measurement is grossly in error. In such a

case it is likely that one of the accurate transformer measurements will be incorrectly rejected as anomalous, leading to compounding of the tap error.

## 3. REVIEW OF STATE ESTIMATION

A number of approaches, formulations, and techniques for state estimation have been advanced since its beginning in 1968 [4,6]. This section briefly reviews the basis of some of those techniques. Section 7 defines the nomenclature used.

The classical approach is sometimes called the normal equations method. This method allows for different weights to be assigned to power system measurements, where the weights correspond to the reciprocal of the measurement variance. The state vector  $\mathbf{x}$  contains the voltage and angle at each bus. The function  $\mathbf{h}(\mathbf{x})$  is the vector of network equations which express the physical quantities being measured in terms of the network variables  $\mathbf{x}$ . The problem is defined as finding a voltage solution  $\mathbf{x}$  which minimizes an objective function  $f(\mathbf{x})$  which is given by the sum of weighted square errors:

$$f(\mathbf{x}) = (\mathbf{z} - \mathbf{h}(\mathbf{x}))^T \mathbf{W} (\mathbf{z} - \mathbf{h}(\mathbf{x})) \quad (1)$$

The solution which optimizes  $f(\mathbf{x})$  is determined by iteratively solving the normal equations:

$$\{\mathbf{H}^T(\mathbf{x}) \mathbf{W} \mathbf{H}(\mathbf{x})\} \Delta \mathbf{x} = \mathbf{H}^T(\mathbf{x}) \mathbf{W} (\mathbf{z} - \mathbf{h}(\mathbf{x})) \quad (2)$$

The bracketed quantity on the right-hand-side of (2) is the vector of residuals, and the bracketed quantity on the left-hand-side of (2) is called the gain matrix. The triangular factors of the gain matrix are typically reevaluated every few iterations. Equation 2 is solved each iteration for the correction to the state vector  $\Delta \mathbf{x}$  via forward and backward substitution on the triangular factors. It is noted that the Jacobian matrix  $\mathbf{H}(\mathbf{x})$  is comprised of the partial derivatives of measured physical quantities with respect to the state variables.

A variation of the normal equations method includes equality constraints for zero-injection buses. This variant can be solved using the method of Lagrange multipliers. The state vector is expanded to include the Lagrange multipliers  $\lambda$  as additional variables. Equation 2 is also changed. The Jacobian matrix is partitioned into submatrices, one block of which refers to injections at the zero-injection buses  $\mathbf{C}(\mathbf{x})$ , while the other block refers to all other measurements  $\mathbf{G}(\mathbf{x})$ . The solution which minimizes the sum of squared errors while maintaining the zero-injection equality constraints is given by:

$$\begin{bmatrix} \mathbf{G}^T(\mathbf{x}) \mathbf{W} \mathbf{G}(\mathbf{x}) & \mathbf{C}^T(\mathbf{x}) \\ \mathbf{C}(\mathbf{x}) & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{G}(\mathbf{x}) \mathbf{W} \mathbf{z} \\ \Delta c \end{bmatrix} \quad (3)$$

Other formulations and variations have been implemented, including orthogonal transformation, the hybrid method, and Hachtel's augmented matrix method, as summarized in [5]. Each of these techniques is a Newton solution approach involving the Jacobian matrix for determination of iterative direction of state variable change. As such, the transformer tap estimation methodology described in this paper may be adapted to these state estimation techniques as well.

## 4. METHODOLOGY

This section describes the methodology used to estimate transformer taps. The technique involves the conversion of one or more transformer tap positions from model parameters into independent solution vari-

ables. The turns ratios and phase-shift angles are appended to the solution vector  $\mathbf{x}$ . Each transformer tap to be estimated requires a new element in the solution vector.

The addition of the tap variables also introduces new elements in the Jacobian matrix  $H(\mathbf{x})$ . Furthermore, if there is a measurement associated with a tap variable, additional elements are appended to the measurement vector  $\mathbf{z}$ , the function  $h(\mathbf{x})$ , and the weighting factor matrix  $W$ . Tap measurements also generate additional Jacobian elements. The formulas for the relevant Jacobian elements are presented in Section 4.1.

The presence of tap variables also modifies the state estimation algorithm. Initially, transformer taps are modeled as continuous variables. A solution is developed which yields a "best-fit" value for the estimated turns ratios and phase-shift angles. The process of developing the "best-fit" solution is the classical approach, which allows for the possibility of bad-data detection. Therefore, any measurements (including tap measurements) may be rejected as anomalous during solution development.

The "best-fit" solution is then modified by imposing an integer constraint on the tap variables. The tap variables are revised to their nearest feasible discrete tap positions and then removed from the solution vector  $\mathbf{x}$  (in effect, the tap variables are converted back into network parameters). The state equations are solved again in a post-processing step to allow for changes in the bus voltage profile resulting from the discretization of the taps. Initial experience has shown that only one additional iteration is needed to achieve a convergent solution.

#### 4.1 JACOBIAN MATRIX FORMULATION

The Jacobian matrix is constructed by the state estimator for use in solving the state equations. This matrix consists of a sparse array of partial derivatives of the measured quantities with respect to the state variables.

The tap estimation technique, which incorporates transformer tap positions as independent state variables and provides for transformer tap measurements, requires the calculation of additional Jacobian matrix elements. These elements include the partial derivatives of the measured quantities with respect to the newly-defined state variables (i.e., turns ratios and phase-shift angles) and, in the case of transformer tap measurements, the partial derivatives of turns ratios and phase-shift angles with respect to bus voltage magnitudes and angles.

The equations used to calculate these additional Jacobian elements are derived from the standard power

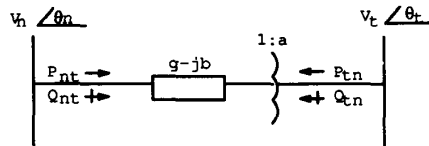


Figure 4-1. Voltage Transformer Model

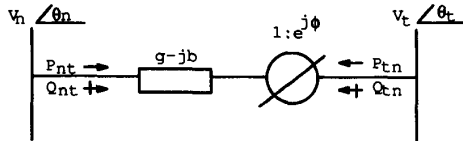


Figure 4-2. Phase-Shift Transformer Model

flow equations and are based on the transformer models shown in Figures 4-1 and 4-2.

For estimated voltage transformer tap positions, the additional Jacobian matrix elements are calculated as follows:

Active power flow or injection measurement:

$$\frac{\partial P_{tn}}{\partial a} = \frac{\partial P_t}{\partial a} = -\frac{V_t}{a^2} \left( \frac{2g}{a} - g V_n \cos \theta_{tn} + b V_n \sin \theta_{tn} \right) \quad (4)$$

$$\frac{\partial P_{nt}}{\partial a} = \frac{\partial P_n}{\partial a} = -\frac{V_t V_n}{a^2} \left( g \cos \theta_{nt} - b \sin \theta_{nt} \right) \quad (5)$$

Reactive flow or injection measurement:

$$\frac{\partial Q_{tn}}{\partial a} = \frac{\partial Q_t}{\partial a} = -\frac{V_t}{a^2} \left( \frac{2b}{a} - b V_n \cos \theta_{tn} - b V_n \sin \theta_{tn} \right) \quad (6)$$

$$\frac{\partial Q_{nt}}{\partial a} = \frac{\partial Q_n}{\partial a} = \frac{V_t V_n}{a^2} \left( b \cos \theta_{nt} + g \sin \theta_{nt} \right) \quad (7)$$

Voltage magnitude measurement:

$$\frac{\partial V_t}{\partial a} = \frac{V_t}{a} \quad (8)$$

Turns ratio measurement:

$$\frac{\partial a}{\partial a} = 1 \quad (9)$$

$$\frac{\partial a}{\partial V_t} = \frac{a}{V_t} \quad (10)$$

For estimated phase-shift transformer tap positions, the additional Jacobian matrix elements are calculated as follows:

Active power flow or injection measurement:

$$\frac{\partial P_{tn}}{\partial \phi} = \frac{\partial P_t}{\partial \phi} = V_t V_n \left( g \sin \phi \cos \theta_{tn} - g \cos \phi \sin \theta_{tn} - b \cos \phi \cos \theta_{tn} - b \sin \phi \sin \theta_{tn} \right) \quad (11)$$

$$\frac{\partial P_{nt}}{\partial \phi} = \frac{\partial P_n}{\partial \phi} = V_t V_n \left( g \sin \phi \cos \theta_{nt} + g \cos \phi \sin \theta_{nt} + b \cos \phi \cos \theta_{nt} - b \sin \phi \sin \theta_{nt} \right) \quad (12)$$

Reactive power flow or injection measurement:

$$\frac{\partial Q_{tn}}{\partial \phi} = \frac{\partial Q_t}{\partial \phi} = V_t V_n \left( g \cos \phi \cos \theta_{tn} + g \sin \phi \sin \theta_{tn} + b \sin \phi \cos \theta_{tn} - b \cos \phi \sin \theta_{tn} \right) \quad (13)$$

$$\frac{\partial Q_{nt}}{\partial \phi} = \frac{\partial Q_n}{\partial \phi} = \quad (14)$$

$$V_n \left( -g \cos \phi \cos \theta_{nt} + g \sin \phi \sin \theta_{nt} + b \sin \phi \cos \theta_{nt} + b \cos \phi \sin \theta_{nt} \right)$$

Voltage angle measurement:

$$\frac{\partial \theta_t}{\partial \phi} = 1 \quad (15)$$

$$\frac{\partial \theta_n}{\partial \phi} = -1 \quad (16)$$

Phase-shift angle measurement:

$$\frac{\partial \phi}{\partial \phi} = 1 \quad (17)$$

$$\frac{\partial \phi}{\partial \theta_t} = 1 \quad (18)$$

$$\frac{\partial \phi}{\partial \theta_n} = -1 \quad (19)$$

#### 4.2 IMPLEMENTATION

The transformer tap estimation technique was incorporated into an existing state estimation program currently in operation at Niagara Mohawk Power Corporation. The state estimator was installed in an Energy Management System security subsystem which includes contingency analysis and load flow, as well as other network analysis functions. The EMS and the security functions were developed by Stagg Systems Inc.

Several changes in support of tap estimation were made to the network security subsystem. It was necessary to provide for input of tap measurements as well as for detection and identification of bad tap measurements. A display was added to permit more convenient display of estimated tap positions and to compare tap estimates with measurements.

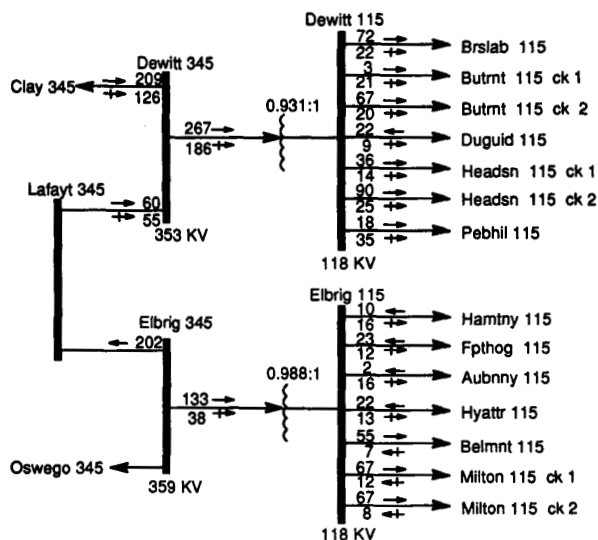


Figure 5-1. Measurements in the Vicinity of the Dewitt and Elbridge Transformers

#### 5. TEST RESULTS

Tap estimation was tested in an environment where some transformers had telemetered tap positions and some did not. The motivation was to validate the taps of telemetered transformers and to estimate the taps where telemetry was not available.

The initial tests reported here were designed to determine the effects of tap error on a network model with real-world measurements. The measurements were telemetered from remote terminal units operating in the field. The tests consisted of state-estimated solutions of scenarios with and without tap error (tap error was introduced in the measurement set in addition to the random "background" measurement error which is always present). The results demonstrate the potential importance of tap estimation, particularly when the taps are in error.

The tap estimation capability was implemented on an existing state estimator in operation at Niagara Mohawk. The state estimator was applied to a 300-bus internal network. The tests report results for voltage transformers.

##### 5.1 SINGLE TRANSFORMER CASE

A simple illustration of tap estimation is provided by imposing tap error on a single transformer at the Dewitt station. Figure 5-1 gives the network one-line diagram in the vicinity of the transformer. The power system measurements are also shown.

Table 5-1 describes the tap estimation scenarios. Cases 1 and 3, which have tap estimation enabled, introduce a turns ratio variable and a turns ratio measurement into the state estimator equations. Even though a gross error is introduced in the turns

Table 5-1. Description of Scenarios for Estimation of Dewitt Transformer Tap

Case	Tap Estimation	Measured Tap	Estimated Tap
1	On	0.931	0.931
2	Off	1.051	N/A
3	On	1.051	0.931

ratio (in cases 2 and 3), bad data detection was disabled and consequently no consideration was given to identifying and eliminating the anomalous tap. Case 2 had tap estimation disabled and used the conventional state estimator formulation (with the bad turns ratio represented as a model parameter).

Table 5-1 confirms that tap estimation is capable of estimating the Dewitt turns ratio at its telemetered value of 0.931. Case 1, which relies on the original measurement set, estimates the tap with no residual. Case 3 also estimates the tap to its originally telemetered value, notwithstanding the gross error imposed on the tap measurement (measurement redundancy in the area can be seen in Figure 5-1).

Detailed results are presented in Table 5-2. The table lists the measurements located near the transformer. The measurements consist of voltage, turns ratio, and branch MW and MVAR values (the transformers' terminals are zero-injection buses). The table gives the residuals associated with the measurements for cases 1, 2, and 3 (a residual is defined as the difference between a measurement and its state estimated value).

The residuals for case 1, which used the original measurement set with tap estimation enabled, are

relatively small in magnitude. It can be inferred that the measurements are relatively consistent in the neighborhood of the transformer.

In contrast, the residuals for case 2 are substantially larger than for case 1. The use of the incorrect turns ratio as a model parameter has the effect of increasing the high-side voltage while de-

Table 5-2. Measured and Estimated Quantities at Dewitt for Single Transformer Tap Estimation

Measurement Type	Measurement Location	Measured Value	State Estimated Residuals		
			Case 1	Case 2	Case 3
KV	Dewitt 345	353	1	-3	1
Line P	Dewitt 345 - Clay 345	-209	4	-1	4
Q	Dewitt 345 - Clay 345	-126	-1	-81	-3
Line P	Dewitt 345 - Lafayette 345	-60	-6	-9	-6
Q	Dewitt 345 - Lafayette 345	-55	2	-68	4
Xlmr P	Dewitt 345 - Dewitt 115	267	1	8	1
Q	Dewitt 345 - Dewitt 115	186	3	153	3
Tap	Dewitt 345 - Dewitt 115	0.931 (case 1)	0.000		
		1.051 (cases 2,3)			0.120
KV	Dewitt 115	117	-1	6	-1
Line P	Dewitt 115 - Brslab 115	72	2	3	2
Q	Dewitt 115 - Brslab 115	22	-1	23	-2
Line P	Dewitt 115 - Butmt 115 ck 1	3	-1	-1	-1
Q	Dewitt 115 - Butmt 115 ck 1	12	0	22	0
Line P	Dewitt 115 - Butmt 115 ck 2	67	-3	1	-3
Q	Dewitt 115 - Butmt 115 ck 2	20	-1	23	-1
Line P	Dewitt 115 - Duguid 115	-22	1	5	1
Q	Dewitt 115 - Duguid 115	9	1	23	1
Line P	Dewitt 115 - Headsn 115 ck 1	36	2	1	2
Q	Dewitt 115 - Headsn 115 ck 1	14	0	11	0
Line P	Dewitt 115 - Headsn 115 ck 2	90	-2	-5	-2
Q	Dewitt 115 - Headsn 115 ck 2	25	0	6	0
Line P	Dewitt 115 - Pebhill 115	18	-1	2	-1
Q	Dewitt 115 - Pebhill 115	35	0	33	0

creasing the low-side voltage, with dramatic increases in the reactive flow errors (especially MVAR flow through the transformer).

The potential consequence of tap error such as in case 2 is increased risk of rejecting a good measurement. Although bad data detection was not enabled for this scenario, it is not hard to imagine how any of the 345 KV reactive measurements might have been erroneously rejected due to the high residuals, most likely degrading the solution accuracy even further.

The effects of estimating the Dewitt tap in the presence of a bad turns ratio measurement are seen in Case 3. Table 5-2 gives a residual of 0.120 for the turns ratio, indicating an estimate of 0.931 (which matches the originally telemetered value). The other residuals are comparable to case 1, which used the original tap value. The results of cases 1 and 3 are not identical because they are based on slightly different measurement sets (the difference being the tap measurement).

It is clear from the results shown that tap estimation in case 3 corrects the voltage and reactive flow problems which are so pronounced in case 2. Thus, tap estimation can potentially reduce the effects of a gross tap measurement (or parametric) error even in the absence of bad data detection.

## 5.2 MULTI-TRANSFORMER CASE

Another set of test cases was developed using the two transformers shown in Figure 5-1 with the same measurement set. The test introduced tap error on

Table 5-3. Description of Scenarios for Multi-Transformer Tap Estimation

Case	Tap Estimation	Dewitt Tap Measure	Dewitt Tap Estimate	Elbridge Tap Measure	Elbridge Tap Estimate	Bad Data
4	On	0.931	0.931	0.988	0.994	(none)
5	Off	1.051	N/A	1.054	N/A	Marcy-Masena Line Q
6	On	1.051	0.937	1.054	0.994	Dewitt Tap, Elbridge Tap

both devices. This time, the state estimator bad data detection capability was enabled in the scenarios. Table 5-3 describes the scenarios, which parallel those presented in the preceding section.

Case 4 used the original telemetry and also estimated the taps at Elbridge and Dewitt. Case 5 used modified telemetry by introducing tap measurement error, but did not enable tap estimation. Case 6 used the same measurements as case 5, but with tap estimation enabled.

Summary results given in Table 5-3 show that slightly different taps were estimated relative to the measured values. In case 4, the Dewitt tap estimate matched its measured value but the Elbridge tap estimate was one tap position above its measurement. Upon investigation, it was discovered that the original

Table 5-4(a). Measured and Estimated Quantities at Dewitt for Multi-Transformer Tap Estimation

Measurement Type	Measurement Location	Measured Value	State Estimated Residuals		
			Case 4	Case 5	Case 6
KV	Dewitt 345	353	2	-5	0
Line P	Dewitt 345 - Clay 345	-209	4	-3	4
Q	Dewitt 345 - Clay 345	-126	10	90	-4
Line P	Dewitt 345 - Lafayette 345	-60	-6	-4	-6
Q	Dewitt 345 - Lafayette 345	-55	-6	-42	0
Xlmr P	Dewitt 345 - Dewitt 115	267	1	6	0
Q	Dewitt 345 - Dewitt 115	186	0	136	7
Tap	Dewitt 345 - Dewitt 115	0.931 (case 4)	0.000		
		1.051 (cases 5,6)		N/A	0.114*
KV	Dewitt 115	117	-1	6	-1
Line P	Dewitt 115 - Brslab 115	72	-3	1	-2
Q	Dewitt 115 - Brslab 115	22	-1	22	0
Line P	Dewitt 115 - Butmt 115 ck 1	3	1	2	1
Q	Dewitt 115 - Butmt 115 ck 1	12	0	11	2
Line P	Dewitt 115 - Butmt 115 ck 2	67	2	5	2
Q	Dewitt 115 - Butmt 115 ck 2	20	0	22	1
Line P	Dewitt 115 - Duguid 115	-22	-1	-1	-1
Q	Dewitt 115 - Duguid 115	9	0	21	1
Line P	Dewitt 115 - Headsn 115 ck 1	36	-2	-4	-1
Q	Dewitt 115 - Headsn 115 ck 1	14	0	6	0
Line P	Dewitt 115 - Headsn 115 ck 2	90	-1	2	0
Q	Dewitt 115 - Headsn 115 ck 2	25	-1	32	1
Line P	Dewitt 115 - Pebhill 115	18	1	0	2
Q	Dewitt 115 - Pebhill 115	35	-2	10	-2

Table 5-4(b). Measured and Estimated Quantities at Elbridge for Multi-Transformer Tap Estimation

Measurement Type	Measurement Location	Measured Value	State Estimated Residuals		
			Case 4	Case 5	Case 6
KV	Elbrig 345	359	4	-2	3
Line P	Elbrig 345 - Lafayette 345	202	2	1	2
Xlmr P	Elbrig 345 - Elbrig 115	133	-6	-4	-6
Q	Elbrig 345 - Elbrig 115	38	5	23	1
Tap	Elbrig 345 - Elbrig 115	0.988 (case 4)	-0.006		
		1.051 (cases 5,6)		N/A	0.060*
KV	Elbrig 115	118	0	5	0
Line P	Elbrig 115 - Hampton 115	-10	1	2	1
Q	Elbrig 115 - Hampton 115	16	7	16	7
Line P	Elbrig 115 - Fpthog 115	-23	0	1	0
Q	Elbrig 115 - Fpthog 115	12	1	10	1
Line P	Elbrig 115 - Aubny 115	-2	6	8	6
Q	Elbrig 115 - Aubny 115	16	-2	16	-2
Line P	Elbrig 115 - Hyatt 115	-22	1	4	0
Q	Elbrig 115 - Hyatt 115	13	-2	12	-2
Line P	Elbrig 115 - Belmont 115	55	-6	-2	-6
Q	Elbrig 115 - Belmont 115	-7	-1	14	-1
Line P	Elbrig 115 - Milton 115 ck 1	67	-3	-2	-3
Q	Elbrig 115 - Milton 115 ck 1	-12	4	7	3
Line P	Elbrig 115 - Milton 115 ck 2	67	-3	2	-3
Q	Elbrig 115 - Milton 115 ck 2	-8	0	3	-1

Table 5-4(c). Measured and Estimated Quantities at Marcy and Massena for Multi-Transformer Tap Estimation

Measurement Type	Measurement Location	Measured Value	State Estimated Residuals		
			Case 4	Case 5	Case 6
KV	Marcy 765	765	6	-2	6
Line P	Marcy 765 - Massena 765	-605	-12	-15	-12
Q	Marcy 765 - Massena 765	-4	2	94*	6

\* indicates anomalous measurement rejected by the state estimator

Elbridge tap measurement was a bad telemetry point. It had been manually overridden using engineering judgment. The tap estimation capability verified that the overridden value was in fact reasonable, and it

was probably one position off. There were no bad data points detected.

Detailed results are presented in Table 5-4, which lists measurements and state estimated residuals for the cases. As expected, residuals are relatively small at the Dewitt and Elbridge transformer terminals for case 4.

Case 5 introduces gross tap errors for the Dewitt and Elbridge transformers. As can be seen from Table 5-4(a & b), voltage and reactive flow residuals tend to increase dramatically (as in the preceding section). However, the Dewitt residuals are larger than those at Elbridge, partly because the Dewitt transformer is carrying nearly five times the Elbridge MVAR flow.

An unexpected result in case 5 was that a reactive measurement in another part of the network was rejected as bad data. The incorrectly rejected measurement was a reactive flow on a 765 KV line from Marcy to Massena. Table 5-4(c) presents the residuals at the Marcy station. It is believed that the incorrect Dewitt/Elbridge taps led to incorrect measurement rejection at Marcy due to a combination of factors, including a difference in the density of measurements at Marcy relative to the transformer terminal buses, as well as possible pre-existing tap errors at the Massena station. This case is an illustration of improper bad data detection which is caused by or aggravated by model errors in the taps.

Case 6 used the same telemetry as case 5, but enabled tap estimation for the Dewitt/Elbridge transformers. Table 5-3 shows that, with bad data detection enabled, the state estimator correctly rejected both of the bad taps as anomalous measurements instead of the Marcy-Massena reactive flow measurement. The Dewitt/Elbridge turns ratios are each estimated at one tap position above their originally telemetered values.

Table 5-4 shows the residuals for case 6 are comparable to those for case 4 in which the taps were at their originally measured positions. However, the solutions are not identical due to the differences in the measurement sets (with bad data rejection of the taps, the difference is precisely the presence or absence of the tap measurements).

In summary, the tap estimation procedure was applied with bad data detection to dual transformers, where the estimated taps were very close to the telemetered positions. Gross error was then introduced on both tap measurements. With tap estimation disabled (i.e., the incorrect taps represented model error), the state estimator rejected a valid measurement in another part of the network. Furthermore, the solution near the transformers suffered from relatively high voltage and MVAR residuals. With tap estimation enabled, the state estimator correctly identified and rejected both anomalous tap measurements. The resulting solution residuals were comparable to the case with the original tap measurements.

## 6. CONCLUSIONS

A new technique for estimating transformer tap positions has been presented. The technique, which incorporates tap positions as additional state variables, requires no special assumptions regarding voltage or reactive power flow measurements. Furthermore, the direct approach of the formulation retains the integrity of the state estimation algorithm. The tap estimation function was incorporated into an existing state estimation program operating on a real-

time Energy Management System. The effectiveness of the technique was illustrated using a real-world network consisting of 300 internal buses.

The test results illustrate the technique for a network containing transformer tap positions which are incorrectly modeled or inaccurately measured. The technique successfully improves the solution by reducing measurement residuals in the presence of both small and gross tap measurement error. In the case of gross tap measurement error, the tap position is correctly detected and rejected as bad data. Without tap estimation, gross errors in tap measurements can cause incorrect rejection of valid measurements.

## 7. NOMENCLATURE

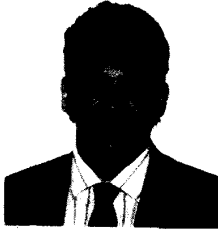
$n$	-	number of states	
$m$	-	number of measurements	
$k$	-	number of zero-injection measurements	
$\mathbf{x}$	-	state vector	( $n \times 1$ )
$\mathbf{h}(\mathbf{x})$	-	network equations	( $m \times 1$ )
$\mathbf{z}$	-	measurement set	( $m \times 1$ )
$\mathbf{W}$	-	weighting factor matrix	( $m \times m$ )
$f(\mathbf{x})$	-	objective function	(scalar)
$\mathbf{H}(\mathbf{x})$	-	Jacobian matrix $\partial \mathbf{h} / \partial \mathbf{x}$	( $m \times n$ )
$\mathbf{C}(\mathbf{x})$	-	partition of Jacobian $\mathbf{H}(\mathbf{x})$ referring to zero-injection measurements	( $k \times n$ )
$\mathbf{G}(\mathbf{x})$	-	partition of Jacobian $\mathbf{H}(\mathbf{x})$ referring to measurements other than zero-injections	( $m-k \times n$ )
$\lambda$	-	Lagrange multipliers	( $k \times 1$ )
$\Delta \mathbf{c}$	-	present injection at zero-injection buses	( $k \times 1$ )
$V_t$	-	tap bus voltage magnitude	
$V_n$	-	non-tap bus voltage magnitude	
$g$	-	transformer conductance	
$b$	-	transformer susceptance	
$a$	-	voltage transformer turns ratio	
$\theta_t$	-	tap bus voltage angle	
$\theta_n$	-	non-tap bus voltage angle	
$\theta_{tn}$	-	$\theta_t - \theta_n$	
$\theta_{nt}$	-	$\theta_n - \theta_t$	
$\phi$	-	phase-shift transformer angle	
$P_{tn}$	-	active power flow from tap bus to non-tap bus	
$P_{nt}$	-	active power flow from non-tap bus to tap bus	
$P_t$	-	active power injection at tap bus	
$P_n$	-	active power injection at non-tap bus	
$Q_{tn}$	-	reactive power flow from tap bus to non-tap bus	
$Q_{nt}$	-	reactive power flow from non-tap bus to tap bus	
$Q_t$	-	reactive power injection at tap bus	
$Q_n$	-	reactive power injection at non-tap bus	

## 8. REFERENCES

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#### AUTHOR BIOGRAPHIES



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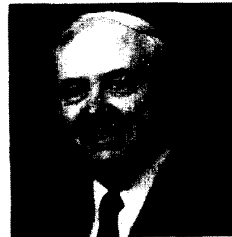
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## Discussion

**L. Radu** (Consolidated Edison, New York, NY): This paper presents an implementation in which taps have been included in the general WLS formulation as state variables. Such simultaneous state-and-parameter estimation is indeed preferable to the sequential state-and-parameter estimation, which appears to be the only other approach with which the authors' were familiar (see Section 2). However, as early as mid 70s such techniques have been proposed (see references A, B) and implemented in EMS Control Centers.

While tap estimation is a useful feature for any State Estimator, it should not be viewed as an alternative to metering the tap positions. The tap estimation is successful provided good local metering redundancy (power flows, taps, voltages) is available. In practical implementations it is useful to have the capability of enabling tap estimation for individual transformers (e.g. to estimate fix taps which might not be correctly defined in the data base). Tap observability is an important aspect not considered in the paper. Again, if tap measurements are available the observability is assured. However, in case of telemetry failures it is important to determine if both voltage and tap are observable. One possible way to handle this is to associate each non measured tap with a reactive flow, injection or voltage adjacent at the transformer and ignore such measurement in the observability processing.

## References

- [A] Couch, Sullivan and Dembecki: "A State Estimator Oriented to a 5 GW Power System", IEEE PES Summer Meeting, Anaheim, 1974.
- [B] Couch, Sullivan and Dembecki: "Results from a Decoupled State Estimator with Transformer Ratio Estimation for a 5 GW Power System in the Presence of Bad Data", *Proc. 5th PSCC*, Cambridge, UK, September 1975.

Manuscript received June 3, 1991.

**P. A. Teixeira, S. R. Brammer, W. L. Rutz, W. C. Merritt, and J. L. Salmonsén:** The authors thank L. Radu for his interesting discussion and for calling to our attention the references cited.

The references were published in the mid-70's as conference level papers which unfortunately were not widely distributed. The papers describe a special-purpose state estimator which was applied to a 5 GW power system existing in Australia. Although the power system is relatively large, the network model size is only 33 buses. The model is so small that the authors could use a full-matrix formulation (no sparsity coding or optimal ordering was required).

The references do propose a direct method of transformer tap estimation, where the tap is converted from a model parameter to a variable. However, the references unfortunately do not provide any details of the mathematical formulation, such as the Jacobian matrix, constants vector, or discretization of tap variables. No results of tap estimation are provided, either. The formulation is restricted to on-load tap changing voltage transformers (the desired voltage at the regulated bus is used as a pseudo measurement). Fixed-tap, remotely controllable, and phase-shifting transformers are not addressed.

In a sense, our paper completes these early efforts in tap estimation by expanding the treatment beyond just on-load tap changing transformers, by providing the details needed to actually incorporate tap estimation in a practical state estimator, and by demonstrating the results that may be obtained. We agree that the direct method of tap estimation is an important contribution to state estimation technology

because the approach is fundamental, retains the integrity of the state estimation algorithm, requires no special assumptions, and has been demonstrated in a practical working EMS environment. This contribution will make possible direct tap estimation in many more EMS control centers, which we certainly welcome. In fact, since the initial publication of our paper at PICA we have received over 80 requests for copies of the paper (several requests are international and span five continents).

The fact that the method is practical is proven by field experience at Niagara Mohawk. Tap estimation was of assistance during the development stages of their security database. Tap estimation uncovered four instances of erroneous tap telemetry and/or tap conversion tables (the tables were used to convert telemetry values into turns ratios or phase-shift angles). It helped convince EMS personnel that they needed to dig to find the problem somewhere between the physical transformer and the network model database.

During the phase-in of security calculations at Niagara Mohawk, tap estimation contributed to the credibility of the state estimator and increased operator acceptance of state estimated results. Now that the security model has been fine-tuned, tap estimation most always estimates taps within one position of telemetry.

We agree with the discussor that it is useful to have the capability of enabling tap estimation for individual transformers. In fact, such a capability was provided at Niagara Mohawk. The feature was used in the reported scenario analysis to enable estimation of just the Dewitt and Elbridge transformer taps.

We also agree with the discussor that, in general, state estimation is not an alternative to good metering practice. Measurement redundancy is vital to achieving an accurate estimate of the network state.

However, we disagree that tap estimation can't substitute for a tap measurement. There can be many instances where tap estimation succeeds without an actual tap measurement. For example, voltage measurements at the transformer terminals or a reactive flow measurement through the transformer can compensate for the absence of a tap measurement.

There can even be cases where none of the just cited measurements is needed to estimate the tap because of sufficient measurement redundancy in adjacent network elements. Similarly, an active flow measurement through the transformer can compensate for the lack of a tap measurement in the case of phase-shifting transformers.

In actual field experience at Niagara Mohawk, there are a few foreign stations which provide tap measurements for transformers whose tap conversion tables remain unknown. Lack of a conversion table ordinarily renders the telemetry value useless. In such cases tap estimation has been found very helpful. This demonstrates that while a tap measurement is useful, it is not essential.

Unfortunately, tap observability could not be addressed in the initial paper due to length constraints. It is true that tap estimation adds state variables which, like all other state variables, require measurements to be observable. A general strategy similar to the one proposed by the discussor may be useful, but care must be used in associating state variables and network measurements.

A special case exists for determining observability of radial transformers. Radial voltage transformer taps are observable only if at least one of the following measurements is available:

- turns ratio
- low-side voltage
- low-side reactive injection
- reactive flow through transformer

In closing, we hope that we have satisfactorily responded to the discussion. Our goal in writing the paper was to cover the subject of tap estimation sufficiently for others to implement the technique. We expect this contribution to add to the state-of-the-art and result in tap estimation in more power control centers.

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