# ANALYZIS OF EGNOS IONOSPHERIC MODEL



### Introduction to Integrity Integrity – the aim of the EGNOS analyzis

"Integrity is the measure of the trust that can be placed in the correctness of the information supplied by a navigation system. Integrity includes the ability of the system to provide timely warnings to users when the system should not be used for navigation" - This definition was adapted from the 2008 US Federal Navigation Plan - Navipedia



### Introduction

### Definition of performace key parameter

Measure of navigation output Accuracy: deviating from truth.

Ability of a system to provide Integrity: timely warnings when the system should not be used for navigation. Integrity risk is the probability of an undetected, threatening navigation system problem.

**Availability:** Fraction of time navigation system is usable as determined by compliance with accuracy, integrity and continuity requirements.

**Continuity:** Likelihood that the navigation supports accuracy and integrity requirements for the duration of intended operation. Continuity risk is the probability of a detected but unscheduled navigation interruption after initiation of an operation.





High Precision, Low Accuracy



Low Precision, Low Accuracy



High Precision, High Accuracy



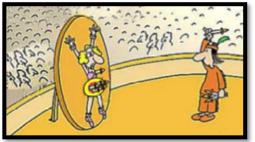
Low Precision, High Accuracy

Accuracy



### Introduction System performance

System performance:
 Accuracy, Integrity, Availability, Continuity



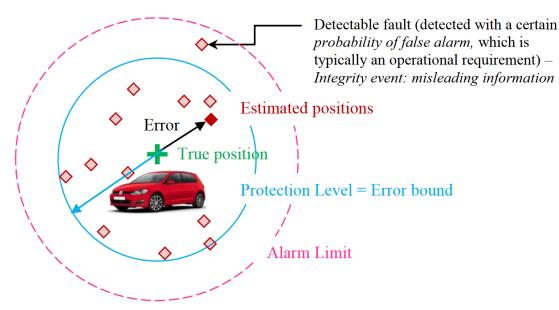
High accuracy, poor integrity

Integrity monitoring/mechanisms



Poor accuracy, high integrity

Detectable fault, but hazardous condition – Integrity event: hazardous misleading information



The concepts of Protection Level, Alarm Limit and integrity events.



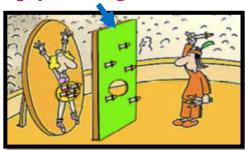
### Introduction Stanford diagram

System performance:
 Accuracy, Integrity, Availability, Continuity

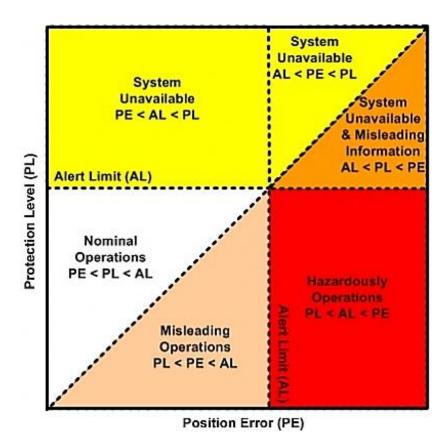


High accuracy, poor integrity

Integrity monitoring/mechanisms



Poor accuracy, high integrity



**AL: Alert Limit** 

PL: Protection Level

PE: Position Estimation

**HMI: Hazardously** 

Misleading Information



### **EGNOS**

### RTCA's protection level calculation

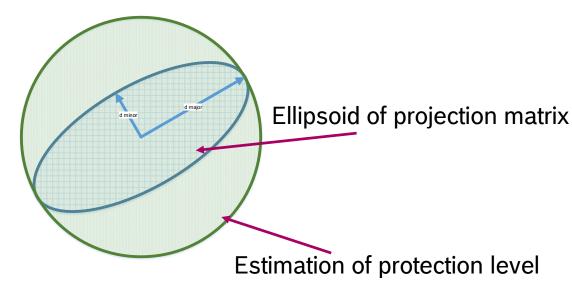
### Horizontal protection level

$$HPL = K_H d_{major}$$

 $K_H = 6.0 - position probability 10^{-7}$ 

Variance of the residual after application of ionospehric error

$$\sigma_{i}^{2} = \sigma_{i,flt}^{2} + \sigma_{i,UIRE}^{2} + \sigma_{i,air}^{2} + \sigma_{i,tropo}^{2}$$



Inverse cummulative function of 2D Gaussian

$$r = F^{-1}(p) = \sqrt{-2\ln(1-p)}$$

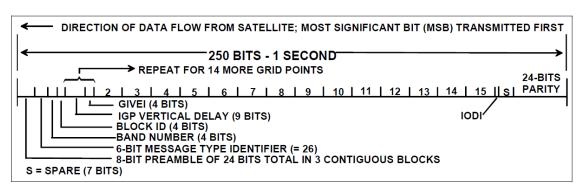
### **EGNOS**

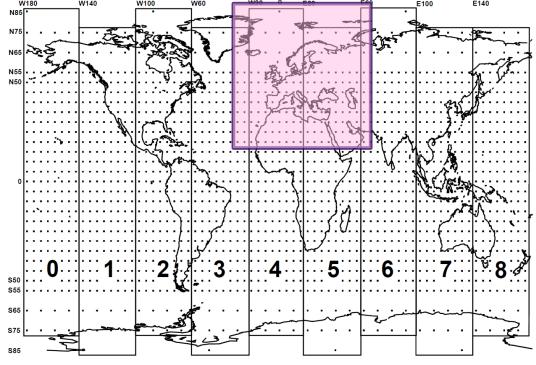
### Ionsopheric Grid Point and Message Formats

### **IGP Mask Message Format**



### **Ionospheric Delay Corrections Message Format**





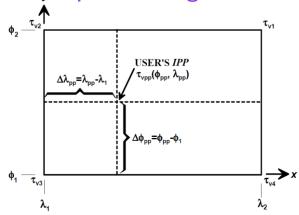


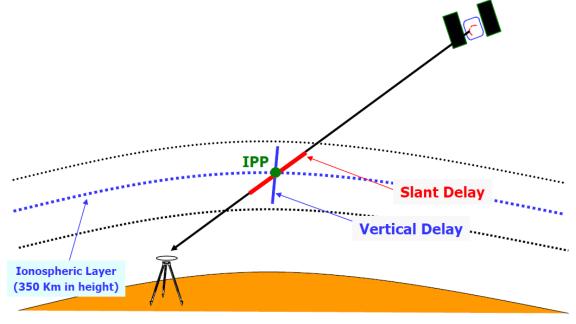
### **EGNOS**

### Ionospheric Variance

IONOSPHERIC PIERCE POINTS (IPP)

4 point interpolation algorithm definition







### GINA – Global Integrated Navigation Algorithm

### **EGNOS**

Cmake based C++ project

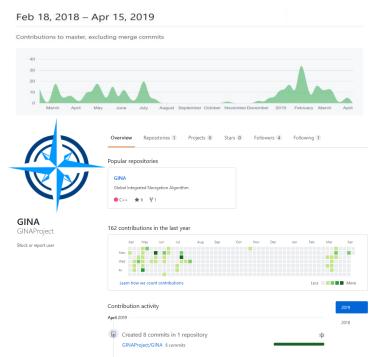
### Github Repo

Evaulation done by Octova/Matlab EGNOS is the latest module in GINA It contains ~4000 Line of Code













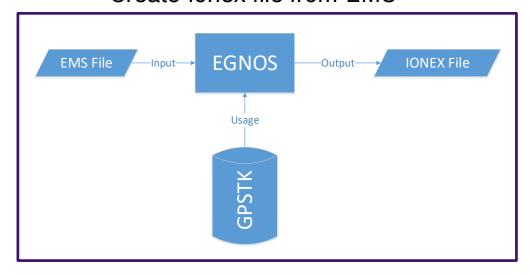




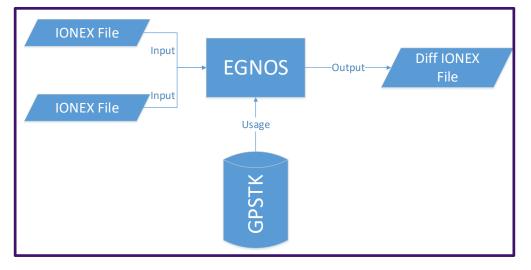
### **EGNOS - GINA**

### EGNOS Features - EMS Processing

### Create Ionex file from EMS



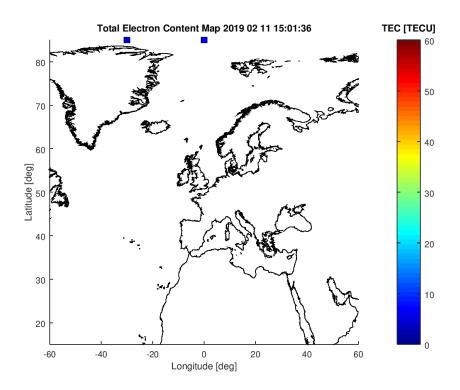
#### Difference of lonex files

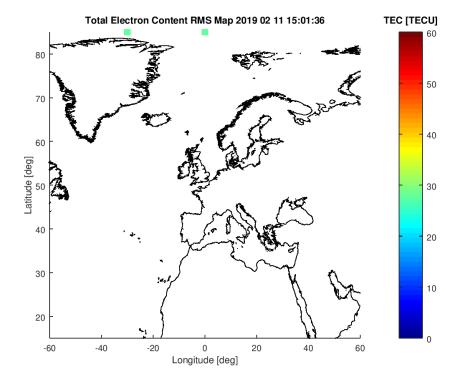




### **EGNOS - GINA**

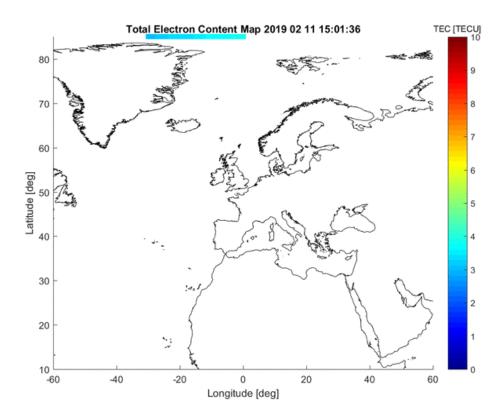
### Raw measurement

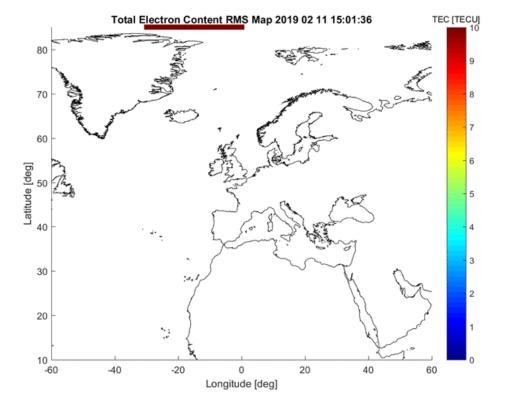






## EGNOS - GINA Interpolated IGPs

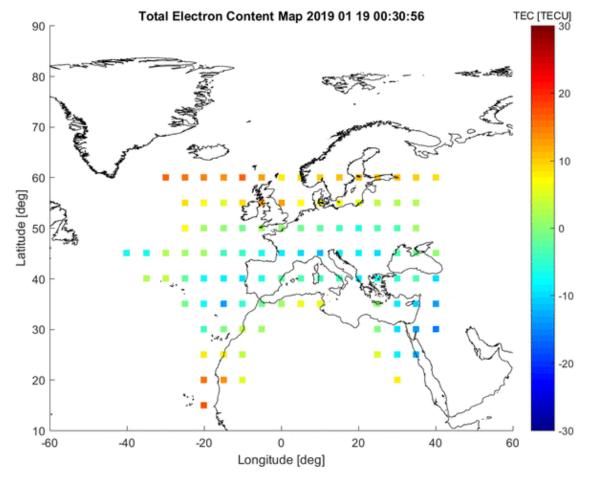






### EGNOS - GINA EMS - CODE's GIM

$$dTEC = TEC_1 - TEC_2$$
$$dRMS = \sqrt{RMS_1 + RMS_2}$$

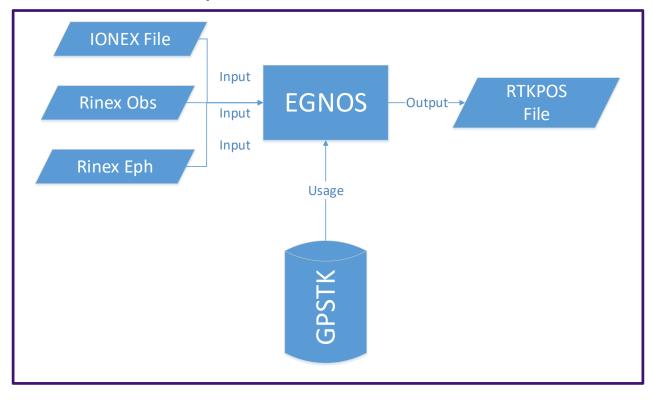




### **EGNOS - GINA**

### EGNOS Features - Navigation Engine with EGNOS Iono Model

### Calculate position with IONEX correction

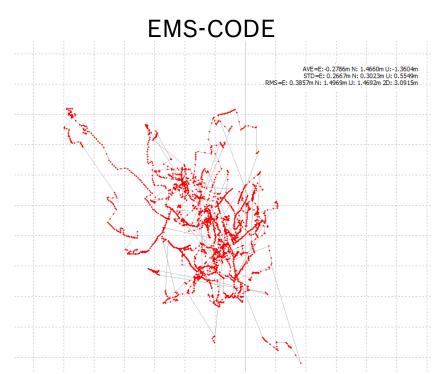




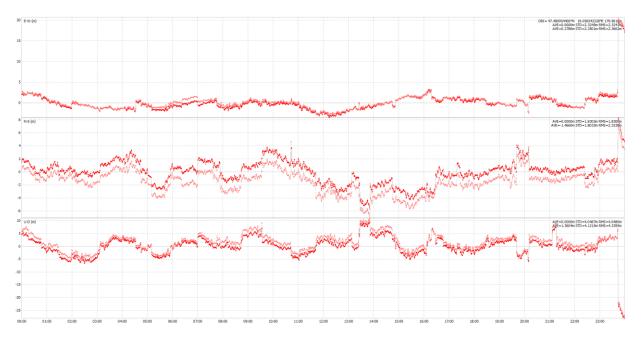
### **EGNOS - GINA**

### Navigation Engine with EGNOS Iono Model

### Position Difference Ground Track



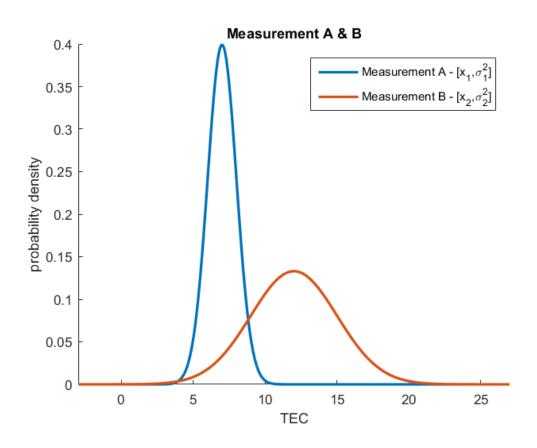
### Position with EMS and with CODE

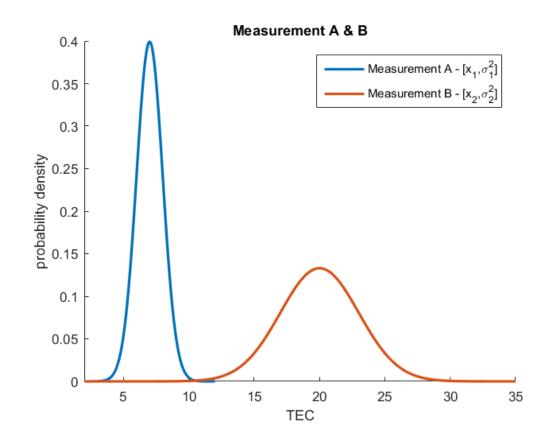




### Bayesian statistic

### Are the measurements consistent?







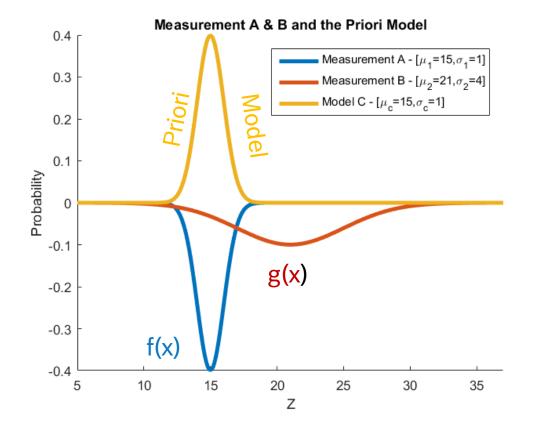
### Likelihood of the measurements Priori beleif is the Measurement A

$$P(A = x_1 \text{ and } B = x_2 | Priori \text{ is model } C) = ?$$

$$P(A = x_1 \text{ and } B = x_2 | Priori \text{ is model } A)$$

$$= P(A = x_1 | Model A)P(B = x_2 | Model A)$$

$$= \sum_{i} P(A = x_1 | R = x_i)P(R = x_i) \sum_{i} P(B = x_2 | R)$$





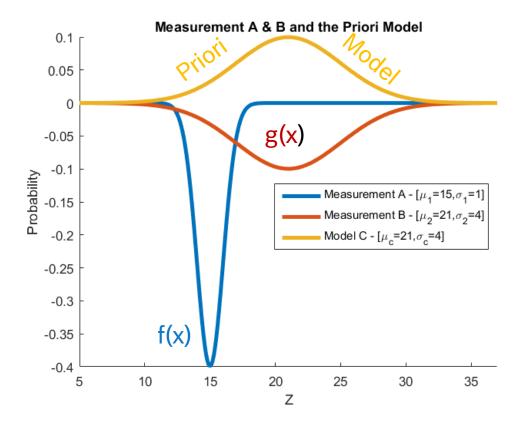
### Likelihood of the measurements Priori beleif is the Measurement B

$$P(A = x_1 \text{ and } B = x_2 | Priori \text{ is model } C) = ?$$

$$P(A = x_1 \text{ and } B = x_2 | Priori \text{ is model } B)$$

$$= P(A = x_1 | Model B) P(B = x_2 | Model B)$$

$$= \sum_{i} P(A = x_1 | R = x_i) P(R = x_i) \sum_{i} P(B = x_2 | R)$$





### Likelihood of the measurements

### Which Priori Model would give the maximum likelihood?

$$P(A = x_1 \text{ and } B = x_2 | Priori \text{ is model } C)$$

Model C is a Gaussian with mean  $x_c$  and  $\sigma_c$ .

$$\begin{split} &P(A=x_1 \ and \ B=x_2|Model \ C) = P(A=x_1 \ |Model \ C)P(B=x_2 \ |Model \ C) \\ &= \sum_{i} P(A=x_1|R=x_i)P(R=x_i) \sum_{i} P(B=x_2|R=x_i)P(R=x_i) \\ &= \int_{-\infty}^{\infty} \left[ x,\sigma_1 \right] |_{x_1} [x_c,\sigma_c]|_{x} dx \int_{-\infty}^{\infty} [x,\sigma_2]|_{x_2} [x_c,\sigma_c]|_{x} dx \\ &= \int_{-\infty}^{\infty} \left[ x_1,\sigma_1 \right] |_{x_1} [x_c,\sigma_c]|_{x} dx \int_{-\infty}^{\infty} [x_2,\sigma_2]|_{x_1} [x_c,\sigma_c]|_{x} dx = \left[ x_1,\sqrt{\sigma_1^2+\sigma_c^2} \right] |_{x_c} \left[ x_2,\sqrt{\sigma_2^2+\sigma_c^2} \right] |_{x_c} \\ &= \left[ \sqrt{\frac{(\sigma_1^2+\sigma_c^2)(\sigma_2^2+\sigma_c^2)}{(\sigma_1^2+\sigma_c^2)+(\sigma_2^2+\sigma_c^2)}}, \frac{x_2(\sigma_1^2+\sigma_c^2)+x_1(\sigma_2^2+\sigma_c^2)}{(\sigma_1^2+\sigma_c^2)+(\sigma_2^2+\sigma_c^2)} \right] |_{x_c} \end{split}$$



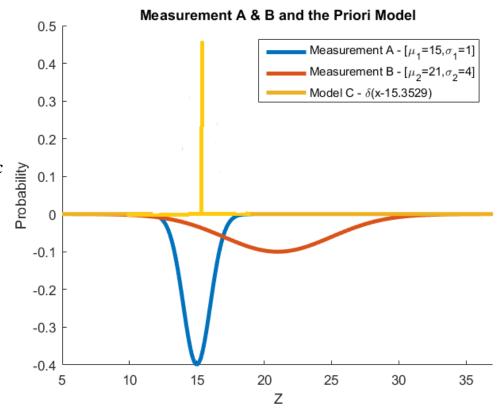
### Likelihood of the measurements

### Which Priori Model would give the maximum likelihood?

$$\min\left(\frac{({\sigma_1}^2 + {\sigma_c}^2)({\sigma_2}^2 + {\sigma_c}^2)}{({\sigma_1}^2 + {\sigma_c}^2) + ({\sigma_2}^2 + {\sigma_c}^2)}\right) \Rightarrow {\sigma_c}^2 = 0$$

$$\frac{x_2(\sigma_1^2 + \sigma_c^2) + x_1(\sigma_2^2 + \sigma_c^2)}{(\sigma_1^2 + \sigma_c^2) + (\sigma_2^2 + \sigma_c^2)} = \frac{x_2(\sigma_1^2) + x_1(\sigma_2^2)}{(\sigma_1^2) + (\sigma_2^2)} = x_c$$

$$\lim_{\sigma \to \infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} = \delta(x-\mu)$$





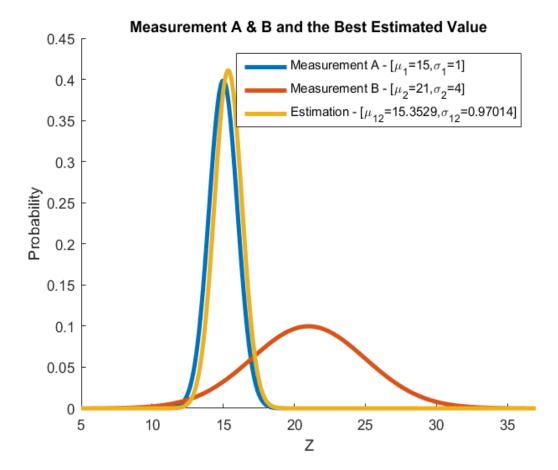
### Maximum likelihood

### But wouldn't we get the same result with a Kalman filter or LSQ?

$$P(Priori \ D \ at \ \mu | A = x_1 \ and \ B = x_2)$$

$$= \frac{\left[x_1, \sqrt{\sigma_1^2 + \sigma_2^2}\right]|_{x_2} [x_{12}, \sigma_{12}]|_{\mu}}{\int_{-\infty}^{\infty} \left[x_1, \sqrt{\sigma_1^2 + \sigma_2^2}\right]|_{x_2} [x_{12}, \sigma_{12}]|_{x} dx}$$

$$= \frac{\left[x_1, \sqrt{\sigma_1^2 + \sigma_2^2}\right]|_{x_2} [x_{12}, \sigma_{12}]|_{\mu}}{\left[x_1, \sqrt{\sigma_1^2 + \sigma_2^2}\right]|_{x_2} [x_{12}, \sigma_{12}]|_{\mu}} = [x_{12}, \sigma_{12}]|_{\mu}$$





### Next Steps Think Tank





# THANK YOU

