卫星 s 估计状态为

$$x_s = \begin{bmatrix} r_{ECI}^s \\ v_{ECI}^s \\ \kappa^s \end{bmatrix} \tag{1}$$

式中，r_{ECI}^s、v_{ECI}^s分别为卫星 s 在 ECI 坐标系下的位置和速度，κ^s表示卫星 s 关于轨道运动力模型参数（一般为光压参数、经验力模型参数等）；

卫星 s 估计参数x_s关于时间 t 的一阶微分方程为

$$\dot{x}_s = \begin{bmatrix} \dot{r}_{ECI}^s \\ \dot{v}_{ECI}^s \\ \dot{\kappa}^s \end{bmatrix} = \begin{bmatrix} v_{ECI}^s \\ a_{ECI}^s \\ 0 \end{bmatrix} \tag{2}$$

式中，a_{ECI}^s为卫星 s 的加速度，轨道运动力模型参数$\dot{\kappa}^s$表示为随机常数，

$$a_{ECI}^s = -\frac{GM_E}{r_s^3} + a_{geo} + a_{3rdbody} + a_{srp} + a_{rel} + a_{amt} + \epsilon \tag{3}$$

- r_s：卫星 s 与地球地心的距离值
- GM_E：地球引力常数
- a_{geo}：地球引力改正（EGM08 等地球重力场模型）
- $a_{3rdbody}$：三体引力
- a_{srp}：太阳光压
- a_{rel}：相对论效应
- a_{amt}：大气阻力
- ϵ：其他未模型化加速度

假设t_k时刻卫星 s 状态参数为$x_s(t_k)$，则在t_{k+1}时刻，卫星 s 状态参数为

$$x_s(t_{k+1}) = \int_{t_k}^{t_{k+1}} \dot{x}_s dt \tag{4}$$

对上式采用数值积分进行解算，常用数值积分方法有

- 各阶龙格-库塔方法（RK4, RK5, RK8 等）
- Adams-Bashforth/Adams-Moulton 预测-校正法
- Bulirsch-Stoer 方法
- 变阶变步长方法（VSVO56 等）
- 辛积分方法

t_k时刻到t_{k+1}时刻，卫星 s 状态参数的转移矩阵为$\Phi_s(t_k, t_{k+1})$，当积分时间间隔较小时，即有

$$\Phi_s(t_k, t_{k+1}) = \frac{\partial x_s(t_{k+1})}{\partial x_s(t_k)} \tag{5}$$

此时，$\frac{\partial x_s(t_{k+1})}{\partial x_s(t_k)}$可用数值微分的方法求解得到；

$\Phi_s(t_k, t_{k+1})$也可用如下变分方程进行数组积分计算得到，即

$$\frac{d\Phi_s(t_k, t)}{dt} = A\Phi_s(t_k, t) \tag{6}$$

$$A = \begin{bmatrix} 0 & I & 0 \\ \frac{\partial a_{ECI}^s(t)}{\partial r_{ECI}^s(t)} & \frac{\partial a_{ECI}^s(t)}{\partial v_{ECI}^s(t)} & \frac{\partial a_{ECI}^s(t)}{\partial \kappa(t)} \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$\Phi_s(t_k, t_{k+1}) = \int_{t_k}^{t_{k+1}} A\Phi_s(t_k, t)dt \tag{8}$$

数值积分方法同式（4）；

利用卡尔曼滤波器进行卫星状态估计，

在t_{k+1}时刻，地面基准站 b 观测到卫星 s，观测值为无电离层组合伪距和载波相位，即$\rho_b^s(t_{k+1})$和$\varphi_b^s(t_{k+1})$，地面基准站 b 的 ECEF 坐标为$r_{b,ECEF}$，首先将地面基准站 b 的 ECEF 坐标转换到 ECI 坐标系，即

$$r_{b,ECI}(t_{k+1}) = U(t_{k+1}) \cdot R_z(-dt_b \cdot \omega_e) \cdot r_{b,ECEF} \tag{9}$$

式中，$U(t_{k+1})$为t_{k+1}时刻 ECEF 到 ECI 的转换矩阵，ω_e为地球自转角速度，dt_b为地面基准站 b 的接收机钟差；

卫星信号在t_{k+1}时刻传播至地面基准站接收机 b，则卫星 s 发射信号时刻为

$$t_{k+1}^s = t_{k+1} - \left(\frac{\rho_b^s(t_{k+1})}{c} + dt^s\right) \tag{10}$$

dt^s为卫星钟差，c 为光速，在t_{k+1}^s，卫星 s 的状态参数为

$$x_s(t_{k+1}^s) = \int_{t_{k+1}}^{t_{k+1}^s} \begin{bmatrix} v_{ECI}^s(t) \\ a_{ECI}^s(t) \\ 0 \end{bmatrix} dt \tag{11}$$

由于卫星信号传播时间一般在爱 70 毫秒左右，所以这里从t_{k+1}时刻反向积分至t_{k+1}^s时刻，以此降低计算量；

$$\rho_b^s(t_{k+1}) = \sqrt{\left(r_{b,ECI}(t_{k+1}) - x_s(t_{k+1}^s)\right)^T \cdot \left(r_{b,ECI}(t_{k+1}) - x_s(t_{k+1}^s)\right)} + T_b^s(t_{k+1}) + c \cdot (dt_b - dt^s) + \varepsilon_\rho \tag{12}$$

$$\varphi_b^s(t_{k+1}) = \sqrt{\left(r_{b,ECI}(t_{k+1}) - x_s(t_{k+1}^s)\right)^T \cdot \left(r_{b,ECI}(t_{k+1}) - x_s(t_{k+1}^s)\right)} + T_b^s(t_{k+1}) + c \cdot (dt_b - dt^s) + B_b^s(t_{k+1}) + \varepsilon_\varphi \tag{13}$$

式中，$T_b^s(t_{k+1})$为对流层延迟，$B_b^s(t_{k+1})$为无电离层组合模糊度，ε_ρ和ε_φ为伪距和载波相位观测误差；$T_b^s(t_{k+1})$可表示为天顶对流层延迟和映射函数的乘积，将天顶对流层延迟$Z_b^0(t_{k+1})$作为待估参数，地面基准站坐标$r_{b,ECI}(t_{k+1})$一般是已知的（由 IGS 周解提供）；

假设在t_{k+1}时刻,需要估计的卫星为s_1、s_2、s_3……、s_n,地面基准站为b_1、b_2、b_3……、b_m，卡尔曼滤波待估参数为

$$X = \begin{bmatrix} x_{s_1} \\ x_{s_2} \\ \vdots \\ x_{s_n} \\ dt^{s_1} \\ dt^{s_2} \\ \vdots \\ dt^{s_n} \\ dt_{b_1} \\ dt_{b_2} \\ \vdots \\ dt_{b_m} \\ B_{b_1}^{s_1} \\ B_{b_2}^{s_1} \\ \vdots \\ B_{b_m}^{s_1} \\ B_{b_1}^{s_2} \\ B_{b_1}^{s_2} \\ \vdots \\ B_{b_m}^{s_2} \\ \vdots \\ \vdots \\ Z_{b_1}^0 \\ Z_{b_2}^0 \\ \vdots \\ Z_{b_m}^0 \end{bmatrix} \tag{14}$$

伪距和载波相位$\rho_{b_j}^{s_i}(t_{k+1})$和$\varphi_{b_j}^{s_i}(t_{k+1})$，关于待估参数的雅可比矩阵为

$$H_{\rho,b_j}^{s_i} = \left[\cdots \quad \frac{\partial \rho_{b_j}^{s_i}(t_{k+1})}{\partial x_{s_i}\left(t_{k+1}^{s_i}\right)} \cdot \frac{\partial x_{s_i}\left(t_{k+1}^{s_i}\right)}{\partial x_{s_i}(t_{k+1})} \quad \cdots \quad \frac{\partial \rho_{b_j}^{s_i}(t_{k+1})}{\partial dt^{s_i}} \quad \cdots \quad \frac{\partial \rho_{b_j}^{s_i}(t_{k+1})}{\partial dt_{b_j}} \quad \cdots \quad \frac{\partial \rho_{b_j}^{s_i}(t_{k+1})}{\partial B_{b_j}^{s_i}} \quad \cdots \quad M_{b_j}^{s_i}\right] \tag{15}$$

$$H_{\varphi,b_j}^{s_i} = \left[\cdots \quad \frac{\partial \varphi_{b_j}^{s_i}(t_{k+1})}{\partial x_{s_i}\left(t_{k+1}^{s_i}\right)} \cdot \frac{\partial x_{s_i}\left(t_{k+1}^{s_i}\right)}{\partial x_{s_i}(t_{k+1})} \quad \cdots \quad \frac{\partial \varphi_{b_j}^{s_i}(t_{k+1})}{\partial dt^{s_i}} \quad \cdots \quad \frac{\partial \varphi_{b_j}^{s_i}(t_{k+1})}{\partial dt_{b_j}} \quad \cdots \quad \frac{\partial \varphi_{b_j}^{s_i}(t_{k+1})}{\partial B_{b_j}^{s_i}} \quad \cdots \quad M_{b_j}^{s_i}\right] \tag{16}$$

$$\frac{\partial \rho_{b_j}^{s_i}(t_{k+1})}{\partial x_{s_i}\left(t_{k+1}^{s_i}\right)} = \begin{bmatrix} \dfrac{x_s\left(t_{k+1}^{s_i}\right)}{\sqrt{\left(r_{b,ECI}(t_{k+1}) - x_{s_i}\left(t_{k+1}^{s_i}\right)\right)^T \cdot \left(r_{b,ECI}(t_{k+1}) - x_{s_i}\left(t_{k+1}^{s_i}\right)\right)}} \\ \dfrac{y_s\left(t_{k+1}^{s}\right)}{\sqrt{\left(r_{b,ECI}(t_{k+1}) - x_{s_i}\left(t_{k+1}^{s_i}\right)\right)^T \cdot \left(r_{b,ECI}(t_{k+1}) - x_{s_i}\left(t_{k+1}^{s_i}\right)\right)}} \\ \dfrac{z_s\left(t_{k+1}^{s}\right)}{\sqrt{\left(r_{b,ECI}(t_{k+1}) - x_{s_i}\left(t_{k+1}^{s_i}\right)\right)^T \cdot \left(r_{b,ECI}(t_{k+1}) - x_{s_i}\left(t_{k+1}^{s_i}\right)\right)}} \\ 0_{3\times 1} \\ 0_{k\times 1} \end{bmatrix} \tag{17}$$

$$\frac{\partial x_{s_i}\left(t_{k+1}^{s_i}\right)}{\partial x_{s_i}(t_{k+1})} = \Phi_{s_i}\left(t_{k+1}, t_{k+1}^{s_i}\right) \tag{18}$$

$$\frac{\partial \rho_{b_j}^{s_i}(t_{k+1})}{\partial B_{b_j}^{s_i}} = 0, \frac{\partial \varphi_{b_j}^{s_i}(t_{k+1})}{\partial B_{b_j}^{s_i}} = 1 \tag{19}$$

式中，$M_{b_j}^{s_i}$为对流层映射函数值；

将地面站接收机钟差和卫星钟差建模为白噪声或一阶马尔科夫随机过程，无电离层组合模糊度建模为随机常数，天顶对流层延迟建模为一阶马尔科夫随机过程，则在t_k至t_{k+1}时刻，卡尔曼滤波待估参数 X 的转移矩阵为

$$F(t_k, t_{k+1}) = \mathrm{diag}\left(\Phi_{s_1}(t_k, t_{k+1}), \Phi_{s_2}(t_k, t_{k+1}), \Phi_{s_3}(t_k, t_{k+1}), \ldots, \Phi_{s_n}(t_k, t_{k+1}), 1, 1, \ldots, 1\right) \tag{20}$$

式中，diag(*)表示对角矩阵化；

t_{k+1}时刻，待估参数 X 的协方差矩阵为

$$P(t_{k+1}) = F(t_k, t_{k+1}) \cdot P(t_k) \cdot F^T(t_k, t_{k+1}) + Q(t_{k+1}) \tag{21}$$

$Q(t_{k+1})$为系统噪声矩阵；

若将地面基准站b_R设置为参考钟，则有

$$\frac{\partial \rho_{b_R}^{s_i}(t_{k+1})}{\partial dt_{b_R}} = 0 \tag{22}$$

$$\frac{\partial \varphi_{b_R}^{s_i}(t_{k+1})}{\partial dt_{b_R}} = 0 \tag{23}$$