Objective Function

The optimization function for this problem is given as follows:

$$\min \sum_{t=1}^{t_{end}} \left(c_{\text{buy}}(t) \cdot P_{\text{buy}}(t) + c_{\text{lcos}}(t) \cdot P_{\text{charge}}(t) - c_{\text{sell}}(t) \cdot P_{\text{sell}}(t) \right) \tag{1}$$

Decision Variables and Parameters

$P_{\text{PV}}(t)$ Photovoltaic system power output in kWh at time t,
$P_{\text{buy}}(t)$ Power bought from the grid in kWh at time t,
$P_{\text{sell}}(t)$ Power sold to the grid in kWh at time t,
$P_{\text{charge}}(t)$ Power charged to the battery in kWh at time t,
$P_{\text{discharge}}(t)$ Power discharged from the battery in kWh at time t,
$P_{\text{b-capacity}}(t)$ Capacity of the battery in kwh at time t,
$D(t)$ Power demand in kWh at time t.
η_{charge} Efficiency of charging.

Constraints

A) Linear Constraints:
The first equation can be energy balance:

$$P_{\text{PV}} + P_{\text{discharge}} + P_{\text{buy}} = D + P_{\text{sell}} + \frac{P_{\text{charge}}}{\eta_{\text{charge}}} + P_{\text{sell}}, \quad \forall t \tag{2}$$

The second constraint is the battery capacity calculation.

$$P_{\text{b-capacity}}(t) = P_{\text{b-capacity}}(t-1) + P_{\text{charge}}(t-1) - P_{\text{discharge}}(t-1) \tag{3}$$

The rest of the inequalities are:
Battery constraints:

$$\begin{aligned} 0 \le P_{\text{b-capacity}} &\le 160, \quad \forall t \\ 0 \le P_{\text{charge}} &\le 100, \quad \forall t \\ 0 \le P_{\text{discharge}} &\le 100, \quad \forall t \end{aligned} \tag{4}$$

Grid constraints:

$$\begin{aligned} 0 \le P_{\text{buy}} &\le 700, \quad \forall t \\ 0 \le P_{\text{sell}} &\le 700, \quad \forall t \end{aligned} \tag{5}$$

Figure 1 displays the outcomes of the aforementioned problem formulated using `pyomo` and tackled utilizing the `glpk` algorithm. As evident in this figure, around the 10th time slot, there is a notable surge in both the selling and purchasing of electricity, a situation not viable in a real-world context. To address this concern, I incorporated an extra constraint aimed at restricting grid purchases when photovoltaic (PV) power generation exceeds the energy demand.

$$P_{\text{buy}} = \{0, \quad \forall \{P_{\text{PV}}(t), D(t)\} \quad | \quad P_{\text{PV}}(t) \ge D(t)\} \tag{6}$$

Given the aforementioned constraints, the predicament has been successfully addressed. Figure 2 delineates the emerging trends. The aggregate cost over the planning horizon, factoring in the optimization, amounts to €1423, as opposed to €3213 in the scenario of exclusive reliance on grid purchases. This translates to a 55.8% reduction in costs.

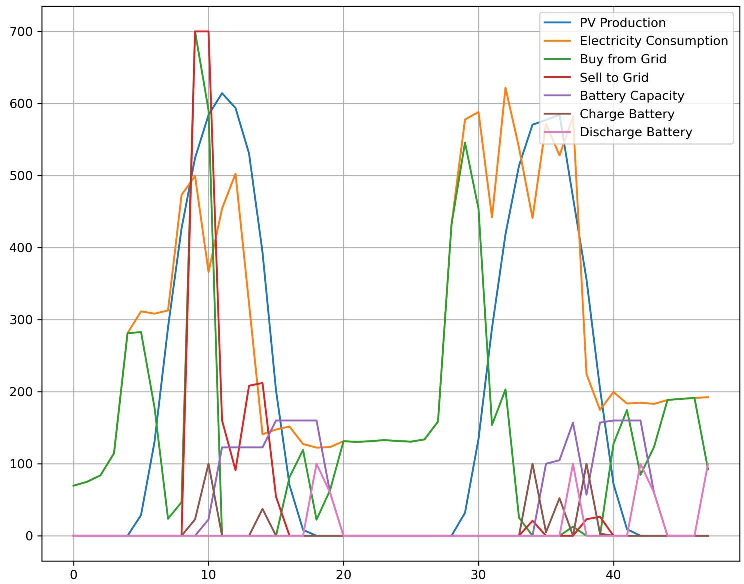

Figure 1: The trend of power generation and consumption for different source over the 48-hour horizon.

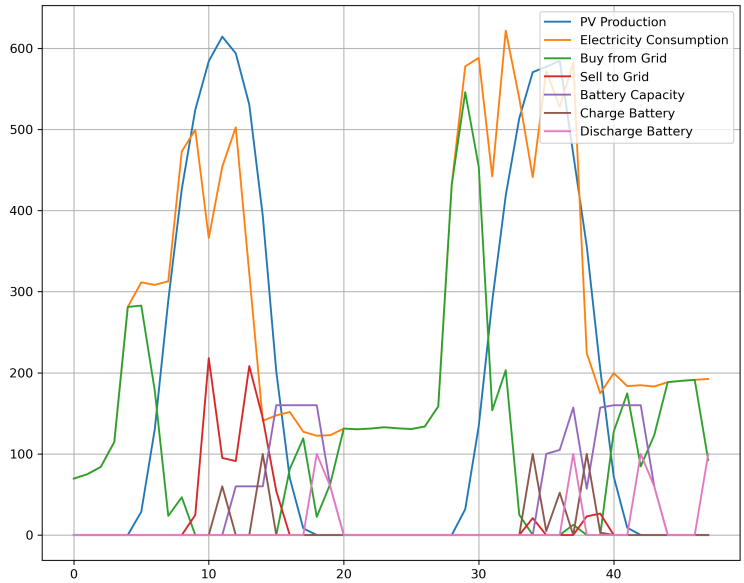

Figure 2: The trend of power generation and consumption after considering constraint 6.

B) Optional Constraint:
For this section, I added the following decision variables: $B_{\text{sell}}(t)$ Selling binary switch at time t,
$B_{\text{buy}}(t)$ Buying binary switch at time t,
$B_{\text{charge}}(t)$ Charging binary switch at time t,
$B_{\text{discharge}}(t)$ Discharging binary switch at time t.

Binary constraints:

$$\begin{aligned} B_{\text{sell}}(t) &= \{0,1\} \\ B_{\text{buy}}(t) &= \{0,1\} \\ B_{\text{charge}}(t) &= \{0,1\} \\ B_{\text{discharge}}(t) &= \{0,1\} \end{aligned} \tag{7}$$

In this section, Equations 2 and 3 can be modified with the new switching variables:
Energy balance:

$$P_{\text{PV}} + B_{\text{discharge}} \cdot P_{\text{discharge}} + B_{\text{buy}} \cdot P_{\text{buy}} = D + B_{\text{sell}} \cdot P_{\text{sell}} + B_{\text{charge}} \cdot \frac{P_{\text{charge}}}{\eta_{\text{charge}}} + P_{\text{sell}}, \quad \forall t \tag{8}$$

Battery capacity:

$$P_{\text{b-capacity}}(t) = P_{\text{b-capacity}}(t-1) + B_{\text{charge}}(t-1) \cdot P_{\text{charge}}(t-1) - B_{\text{charge}}(t-1) \cdot P_{\text{discharge}}(t-1) \tag{9}$$

Exclusivity constraints:

$$\begin{aligned} B_{\text{sell}}(t) &= 1 - B_{\text{buy}}(t) \\ B_{\text{charge}}(t) &= 1 - B_{\text{discharge}}(t) \end{aligned} \tag{10}$$

`glpk` is a solver for mixed-integer linear programming, and `ipopt` is an algorithm to solve only nonlinear programming. However, the current problem is mixed-integer non-linear (MINLP) and requires a solver that can work in such a space.

C) Optional Constraint:
This condition adds another set of constraints to the system by transforming Equations 4 and 5 from continuous to discrete space.
Battery constraints:

$$\begin{aligned} P_{\text{charge}} &\in \{0, 100\} \\ P_{\text{discharge}} &\in \{0, 100\} \end{aligned} \tag{11}$$

Grid constraints:

$$\begin{aligned} P_{\text{sell}} &\in \{0, 100, 200, \ldots, 700\} \\ P_{\text{buy}} &\in \{0, 100, 200, \ldots, 700\} \end{aligned} \tag{12}$$

Additional Potential Constraints

- Smoothing Constraints: Could be introduced to limit abrupt changes in energy flows, accounting for the physical limitations of the system.
- Regulatory Constraints: Incorporating any applicable regulatory constraints or market rules that may affect energy transactions in the optimization model.
- Battery Degradation: incorporating the life cycle of battery by changing its efficiency and penalty for the number of usage