Lec6

Wednesday, January 12, 2022 9:18 AM

Monty Hall Problem:

There are 3 doors, 1 with a Car Prize and 2 with goats.

Assume we want the Car. You Pick a door, then out of the two other doors,

Monty opens one of them to show a goat behind it, and gives you a chance to switch,

Is it better for you to switch?

Assume $P(Car) = \frac{1}{3}$ for all doors and between two goat doors, Monty would choose with

equal Probability. Answer is yes.

Answer:

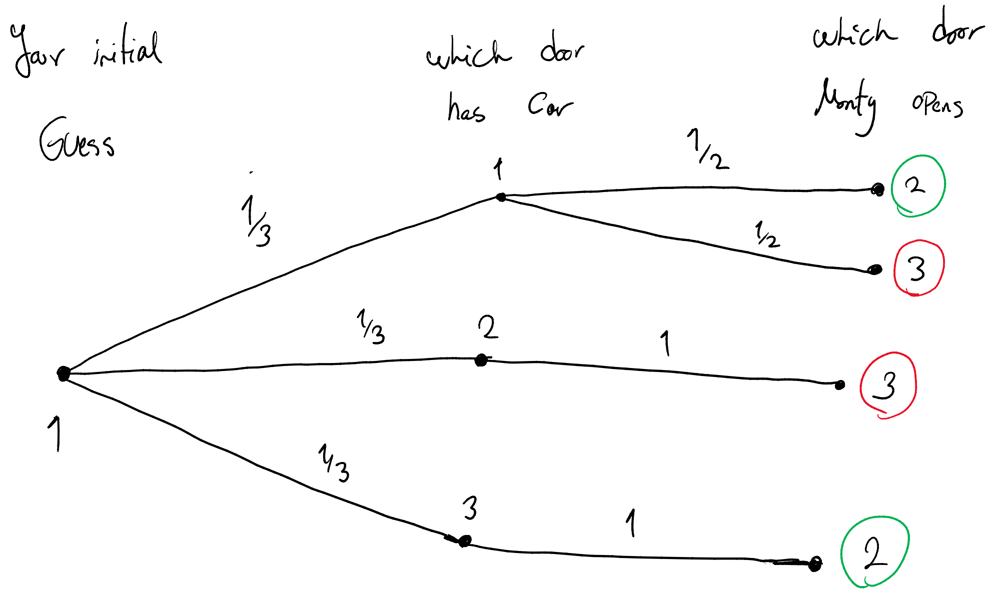
The Problem is the Same no matter which door we choose at first, So let's just

assume that it's door 1.

Note: Our evidence should be that one of the other doors has a goat

And the fact that Monty opens that door.

Probability Diagram:



2 is green

3 is red

$P(\text{Success on switch} \mid \text{Monty opens door } 2) = \frac{2}{3}$

$P(\text{Success on switch} \mid \text{Monty opens door } 3) = \frac{2}{3}$

(Renormalized)

Simply: Your inital guess is wrong $2/3$ of the time,

So you better switch.

Using LoTP: (Conditioning on things we wish we knew):

S: Success (assuming switch)

D_j: Door j has the Car

Assume we've chosen door 1, it's ok since the Problem is symmetrical.

Answer:

$$P(S) = P(S|D_1)P(D_1) + P(S|D_2)P(D_2) + P(S|D_3)P(P_3)$$

$$= 0 + 1 \times \frac{1}{3} + 1 \times \frac{1}{3} = \frac{2}{3}$$

Simpson's Paradox:

	Heart	Bandaid
Success	70	10
Fail	20	0

Dr. Hibbert

	Heart	Bandaid
Success	2	81
Fail	8	9

Dr. Nick

80%

On each given Procedure, the Success rate of Dr. Hibbert is higher, but

83%

Unconditionally Dr Nick's Success rate is higher.

How does this happen?

Because:

$$\frac{70}{70+20} + \frac{10}{10+0} \neq \frac{70+10}{70+20+10}$$

That's not how sums work!

Mathematically, Assume you've had a medical Procedure, our events are:

A: Successful Procedure

B: You went to Dr. Nick

C: You had heart Surgery

Then, given the Data:

① $P(A|B,C) < P(A|B^c,C)$

② $P(A|B,C^c) < P(A|B^c,C^c)$

C is Called Confounder / Control. Cause it's Something you want to Control for.

But

③ $P(A|B) > P(A|B^c)$

You Can't get from ① and ② to ③, or even ③'s Complement.

Since

$$P(A|B) = \overbrace{P(A|B,C)}^{< P(A|B^c,C)} \underbrace{P(C|B)} + \overbrace{P(A|B,C^c)}^{< P(A|B^c,C^c)} \underbrace{P(C^c|B)}$$

these things Ruin it.

Note: When you're getting the intersection with the evidence (A given B and C), You should set the Priors in your LoTP as $P(C|B)$ since we

Know C already happened. (Same as: $P((A|B)|C)$)