Lec2

Tuesday, January 4, 2022 7:19 PM

Homework Hints:

① Don't lose Common Sense.

* You will encounter Counter-intuitive stuff, but you shouldn't abandon Common Sense.

② Do check your answers:

esp. By doing Simple and extreme Cases.

[check what you Can Count Manually, Check the ∞ limit]

③ Label People, objects, etc. If you have n People, label them $1, \dots, n$.

Example:

Q: 10 People, divide into two groups of 6 and 4.

A: $\binom{10}{4} = \binom{10}{6}$

We Pick the 4 (or the 6) first, then the remaining 6 (or 4) we pick the 4 (or the 6) first, then the remaining 6 (or 4) is the 2nd team.

Q: 10 People, two teams of 5.

A: $\frac{\binom{10}{5}}{2}$. we chose 5, remaining 5 are the second team.

However, this is symmetrical with regards to which team is the one we picked first, which means we're double counting at first.

[we don't say there's any difference between the two teams, that's where the symmetry comes from]

Self Note: In Probability, doing nothing and not being able to do anything are different things, there's 1 way to do the former, but 0 ways of doing the latter.

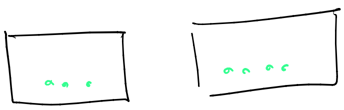
Problem: # ways of Picking K objects out of n, with replacement where order doesn't matter.

Answer: $\binom{n+K-1}{K}$

trivial ($K=0$): $\binom{n-1}{0} = 1 \Rightarrow$ Just don't choose anything. one way.

trivial ($K=1$): $\binom{n}{1} = n \Rightarrow$ Choosing only once, n items to choose from.

Simplest non-trivial ($n=2$): $\binom{K+1}{K} = \binom{K+1}{1} = K+1$



item 1 item 2

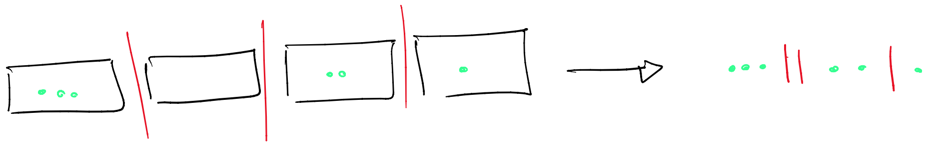
$\hookrightarrow$ # items in one box defines the other box as well $\in \underbrace{\{0, 1, \dots, K\}}_{K+1}$

each Dot represents one time that the item was picked.

$\Rightarrow$ Equivalently, how many ways are there to put K indistinguishable particles into n distinguishable Boxes.

[This is a case of truly indistinguishable counting problem, similar to what happens with Bosons. (Even God can't tell the difference)]

Visual Proof:



We'll have k •'s, $n-1$ |'s.

Since • and | are different $\Rightarrow \binom{n+k-1}{k} = \binom{n+k-1}{n-1}$

Story Proof: Proof By InterPretation.

① : $\binom{n}{k} = \binom{n}{n-k}$.

Thinking about what it means instead of manipulating factorials.

② $n\binom{n-1}{k-1} = k\binom{n}{k}$.

Pick k People out of n, with 1 as the President.

First way. Chose who's the President : n

then Pick the rest $k-1$ out of $n-1$ Remaining : $\binom{n-1}{k-1}$

Second way. Chose all k members : $\binom{n}{k}$

then chose the President out of these k : k

3 $\binom{m+n}{k} = \sum_{j=0}^{k} \binom{m}{j}\binom{n}{k-j}$ Vandermonde identity.

ways of Picking k People out of $m+n$ is equivalent to

Having two groups of m and n, then Picking j People out of the m and $k-j$ out of the n and Summing

The number of all Possible ways together.

Non-Naïve Def. of Probability:

A Probability Space consists of a Sample Space S and P, a function

which takes an event $A \subseteq S$ as input and returns $P(A) \in [0,1]$..

Such that

① $\underbrace{P(\emptyset) = 0}_{\text{impossible events don't happen}}$, $\underbrace{P(S) = 1}_{\text{Something happens (even nothing)}}$

② $P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n)$ if all A_i's are disjoint.