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Homework: Motor control II

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The solutions for these exercises should be handed in before **July 14, 2023 at 10:15 am** as a pdf through the Moodle interface. Please post your questions regarding the exercise on the Moodle forum.

LQG motor control for viscous flow fields

A common experimental paradigm to probe how humans learn motor skills is a robotic artificial environment, in which humans grab a controller that introduces additional perturbation forces that alter their movement. A common perturbation force f is a viscous flow field in two dimensions that makes the movement veer off to the side in proportion to the velocity $v \in \mathbb{R}^2$:

$$f = f_0 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} v \, .$$

Here, f_0 controls the strength of the perturbation force.
The goal of the present exercise is to use the framework of linear-quadratic-Gaussian control to investigate how mismatches between the internal model and the actual behavior of the environment are reflected in the movements. To this end, you will get to simulate a time-discrete dynamical system of the form $x_{t+1} = Ax_t + Bu_t + \xi$. $x = (x_1, x_2, v_1, v_2) \in \mathbb{R}^4$ is the state vector. x_1, x_2 denote the hand position in two dimensions, and v_1, v_2 the hand velocities in those two dimensions. $u \in \mathbb{R}^2$ is the control force, and ξ is Gaussian noise with covariance matrix $Q = \sigma_{dyn}^2 \mathbb{I}_4$ ($\mathbb{I}_n$ being the $n \times n$ identity matrix).
The matrices A and B can be derived from an forward-Euler approximation of Newton's law of motion ($\dot{v} = f/m$, $\dot{x} = v$, $m = 1$ being the "mass" of the hand):

$$A_0 = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & -f_0 \Delta t/m \\ 0 & 0 & f_0 \Delta t/m & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \Delta t/m & 0 \\ 0 & \Delta t/m \end{pmatrix}$$

Note that the perturbation force can be removed by setting $f_0 = 0$. We assume that the position of the "hand" can be observed (but not the velocity), but the observation is noisy: $(y_1, y_2) = (x_1, x_2) + \xi = Cx + \zeta$, where ζ is 2-dimensional Gaussian noise with covariance matrix $\sigma_{obs}^2 \mathbb{I}_2$. Cast into the format of LQG control, the matrix C is given by

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} .$$

To force the movement to end at location (x_1, x_2) at time $t = 1$, we choose a final cost for the movement $x(t = 1)^T F x(t = 1)$, with a very large $F = 10^8 \cdot \mathbb{I}_4$. For the matrix that scales the control cost, choose $N = \mathbb{I}_2$. For simplicity, we ignore units.

Question 2

1. Implement the Kalman filter and the linear quadratic regulator. Feel free to use the provided code skeleton `lqr_skeleton.ipynb`. Run a simulated trajectory without perturbation ($f_0 = 0$). Use $x = (0, -10, 0, 0)$ as starting point, $\Delta t = 0.001$ and $\sigma_{dyn}^2 = \sigma_{obs}^2 = 0.1$. Initialize the Kalman filter with the correct location and velocity, but with a very large initial estimate for the covariance matrix (What happens when you initialize the Kalman filter wrongly? Why?). Plot the true position and the estimated position as a function of time. Vary the two noise levels (i.e., σ_{obs} and σ_{dyn}). How much noise can you deal with until the quality of the internal state estimate declines heavily?

2. Plot x_1 and x_2 both as a function of time and against each other. Plot the time course of the velocities v_1, v_2. Discuss the velocity profile. Is this a good model for human motor trajectories (see lecture)? Also, have a look at the control force. Anything that strikes you?

3. Now introduce the perturbation force ($f_0 = 1$) to the dynamics, but leave untouched the matrix A that is used in the internal model, i.e., in the Kalman filter and the regulator. This corresponds to a situation where a human subject now feels an unexpected control force and has not yet learned a new control law. Plots the trajectory and discuss how it is changed.

4. Assume that the system has learned, i.e., use the internal model that uses the correct dynamics including the perturbation force. Plot the trajectories and discuss how they are altered.

5. Now remove the perturbation force, but don't tell the internal model. Again, plot and discuss the trajectories.