Politecnico di Torino

B.Sc. in Electronic and Communications Engineering

Final Project

Simulation of a communication system

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Abstract

This project presents a complete simulation of a communication system involving two audio signals: *Imagine* by John Lennon and *Mamma Mia* by ABBA. The work is divided into two main parts: signal recovery and signal transmission.

In the first part, the goal was to clean the songs from a strong narrowband disturbance centered around 5567.5 Hz. A fourth-order Bessel low-pass filter and a custom digital notch filter were designed and evaluated. The notch filter, based on complex conjugate poles and zeros, showed superior performance in terms of Signal-to-Interference Ratio (SIR) and signal preservation.

The second part focused on transmitting both songs over a shared channel with a constrained baseband. Amplitude modulation was used to spectrally separate the signals, and different receiver-side filters were tested. The study compares the performance of Bessel and custom low-pass filters under both joint and individual transmission scenarios. Results show that Bessel filters generally outperformed custom filters, and that higher carrier frequencies provided better separation and SIR, especially when transmitting songs individually. Overall, the project demonstrates the value of frequency-aware filter design in audio signal processing and communications.

Introduction

In real-world communication systems, transmitted signals often face challenges such as narrowband interference and limited channel bandwidth. Audio signals, in particular, are sensitive to spectral distortions, making their recovery and transmission a relevant problem in digital signal processing.

This project explores the problem of recovering and transmitting two corrupted audio signals. The first part focuses on disturbance removal, using frequency-domain analysis to identify a narrowband interference near 5.567 kHz. Two filtering strategies were implemented: a standard fourth-order Bessel low-pass filter and a custom digital notch filter with two complex conjugate poles and zeros. These filters were evaluated in terms of Signal-to-Interference Ratio (SIR), allowing a quantitative comparison of their effectiveness.

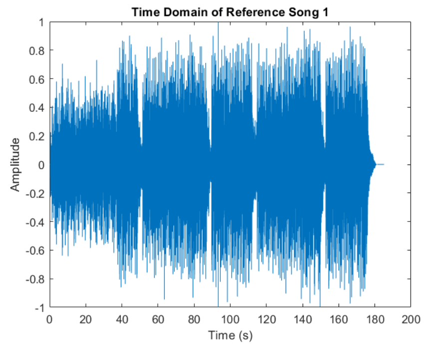
The second part addresses the challenge of transmitting both songs over a shared channel already occupied at low frequencies. Amplitude modulation was applied to move the audio signals into non-overlapping frequency bands. Different carrier frequencies were tested, and both Bessel and custom low-pass filters were used at the receiver side. Each configuration was evaluated based on the resulting SIR, and results were compared between joint and single-song transmission.

The entire simulation was implemented in MATLAB, and all filter designs were visualized and validated using time-domain plots, frequency spectra, and z-plane analyses. The project concludes with a comparison of all methods, highlighting the advantages and limitations of each approach under different scenarios.

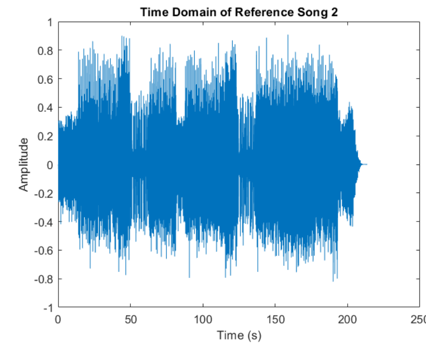
Part 0: ANALYZE THE AUDIO SIGNAL

The song *Imagine* is referred to as **Song one**, and *Mamma Mia* is referred to as **Song two** in the following analysis. The reference songs, provided as `.mp3` files, were loaded into MATLAB. Their time-domain waveforms, frequency-domain spectra, and corresponding power spectra are illustrated in Figure 1.

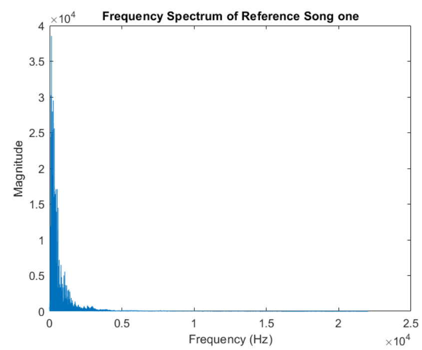
Song one has an occupied bandwidth of **961.30 Hz**, while Song two has an occupied bandwidth of **7589.85 Hz**. These bandwidth values correspond to the frequency ranges that contain 95% of the total power of each song. The calculation was performed using the functions `cumsum`, and `find` to compute the cumulative of the total power and determine the frequency range containing 95% of the signal energy.



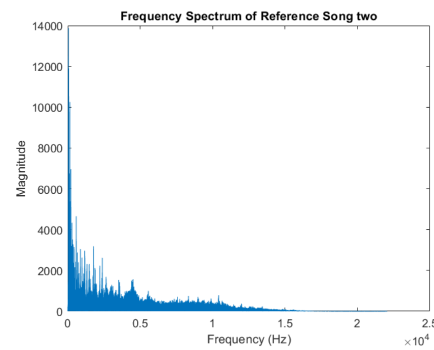
(a) Time domain of Song one



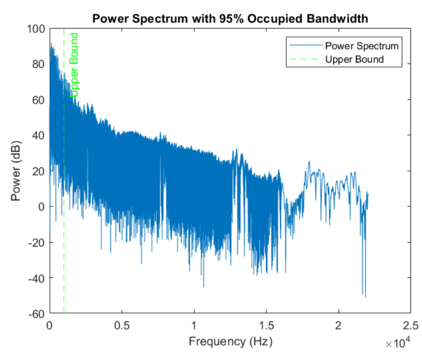
(b) Time domain of Song two



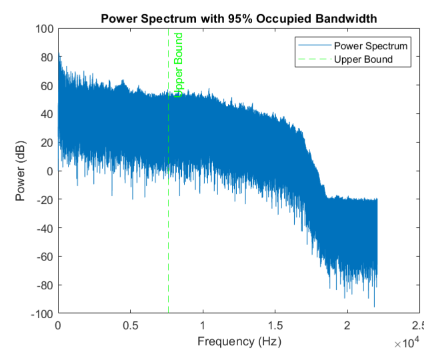
(c) Frequency spectrum of Song one



(d) Frequency spectrum of Song two



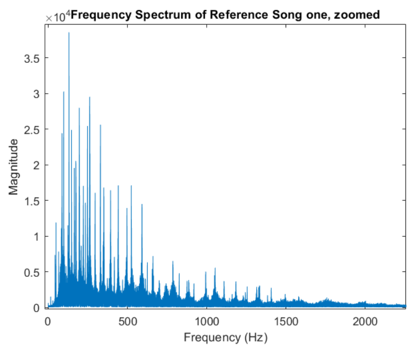
(e) Power spectrum of Song one (OBW: 961.30 Hz)



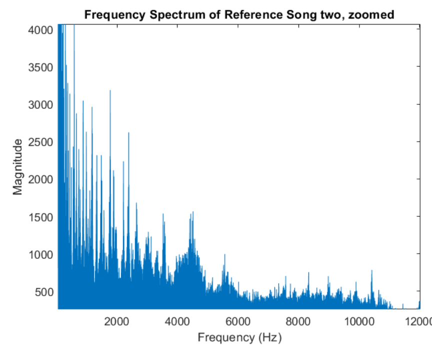
(f) Power spectrum of Song two (OBW: 7589.85 Hz)

Figure 1: Time-domain signals, frequency spectra, and power spectra of the reference songs. (a)–(b) show the time-domain waveforms, (c)–(d) the frequency spectra, and (e)–(f) the power spectra in dB with 95% occupied bandwidths.

By zooming in on the frequency domain of the two songs, as shown in Figure 2, it is evident that most of Song one's energy is concentrated at lower frequencies, while Song two exhibits more significant high-frequency components.



(a) Zoomed frequency domain of Song one

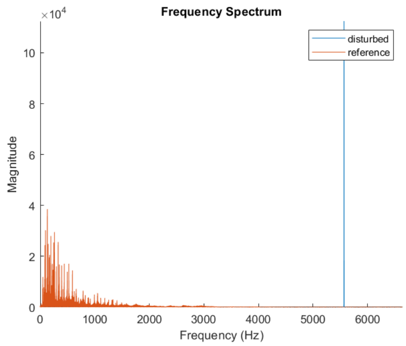


(b) Zoomed frequency domain of Song two

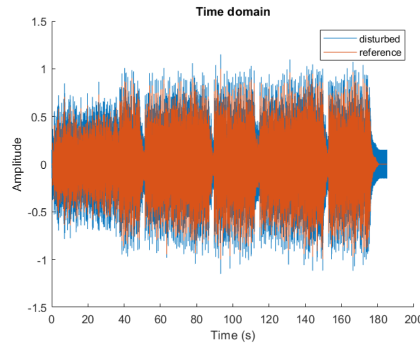
Figure 2: Zoomed-in frequency spectra of the two songs. Most of Song one's components lie below 1000 Hz, whereas Song two has significant content extending up to 6000 Hz.

Part 1: Analyze the Disturbance

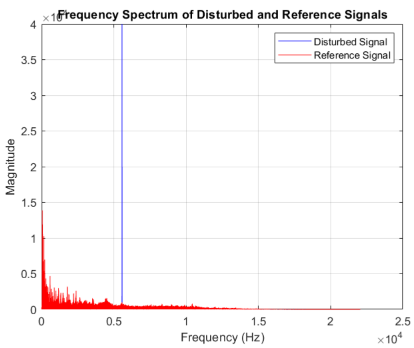
First, the provided `.mat` files of the songs are loaded. Then, their time domain waveforms and the frequency spectrum are plotted as shown in Figure 3 and Figure 3 for the two songs respectively. As shown in the Figures, a strong disturbance component appears around 5567.5 Hz for both of the songs. In contrast, the majority of the two songs' useful components are concentrated in the lower frequency range. This separation between the musical content and the interference forms the basis for filter design in the next section.



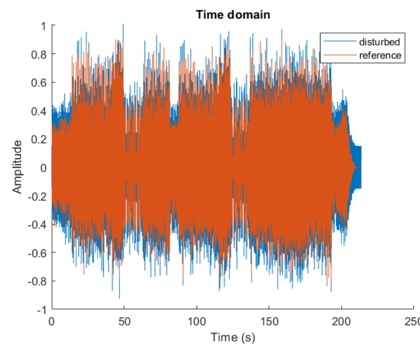
(a) Frequency-domain of song one



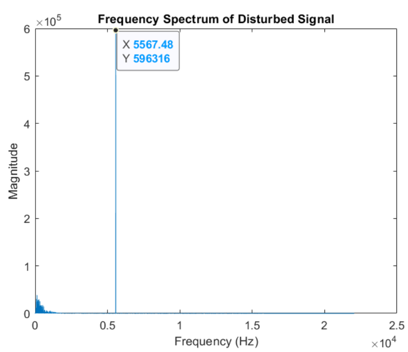
(b) Time-domain of song one



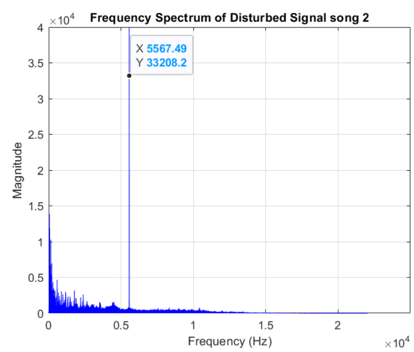
(c) Frequency-domain of song two



(d) Time-domain of song two



(e) Zoom-in on disturbance frequency for song one (around 5568 Hz)



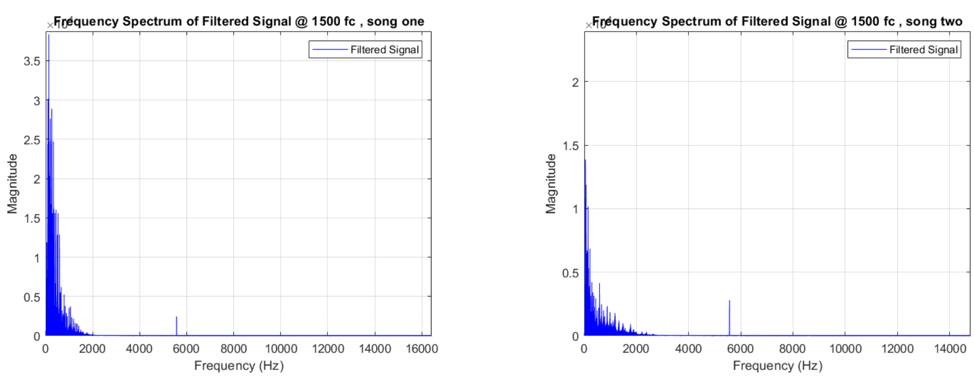
(f) Zoom-in on disturbance frequency for song two (around 5568 Hz)

Figure 3: Unfiltered disturbed songs and their reference signals in both time and frequency domains, including disturbance zoom at 5567.48 Hz.

1.2 Bessel Filter Design

A fourth-order Bessel low-pass filter was designed using the bilinear transformation method to attenuate the high-frequency disturbance. Due to the strong nature of the interference and the inherently gentle roll-off of Bessel filter, a single-stage implementation was insufficient to completely attenuate the noise while keeping the song component unchanged. Therefore it is necessary to keep reducing the cut off frequency in order to attenuate completely the noise. An example of the filtered signal after the filter with fc = 1500 hz is shown in Figure 4.

The filter however will result in delay. To address this, the `finddelay` function in MATLAB was used to estimate the delay between the filtered and reference signals. The `circshift` function was then applied to realign the filtered signal before calculating the Signal-to-Interference Ratio (SIR). For instance at 1500 Hz the SIR is 16.12 dB for song one and 3.53 dB for song two. The reason that the SIR for such filter is very low for the song two is that this song has a considerable portion of components at high frequencies which are filtered out by the bessel filter with cut off frequency of 1500 Hz. Therefore, it is true that the noise is also reduced but many of the components has also been erased.

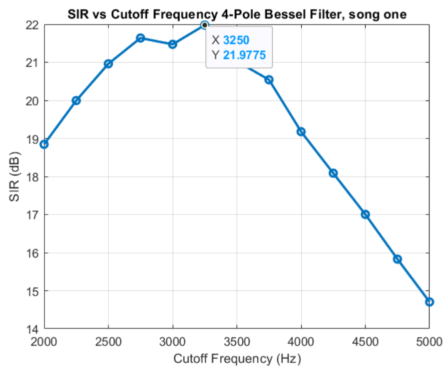


(a) filtered song one @ 1500 Hz frequency domain

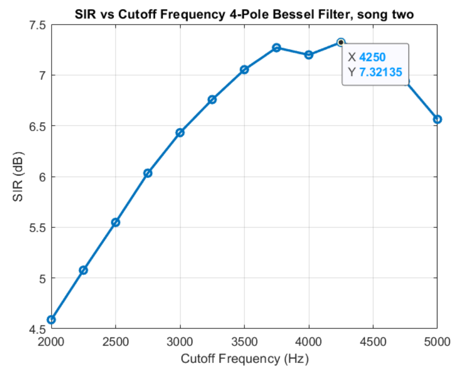
(b) filtered song two at 1500 Hz frequency domain

Figure 4: Frequency spectrum of filtered signals of both songs at 1500 Hz

To determine the optimal cutoff frequency, the filter was evaluated using a loop-based SIR optimization. The range for the cut off frequency was chosen between 2000 to 5000 Hz with points 250 Hz apart. 2000 Hz is a good starting point for the loop because almost all of the song components were in below 1500 hz frequencies as was shown in Figure 2. The resulting plot showing SIR evaluations vs cut off frequency is shown in Figure 5. The best performance was achieved with a cutoff frequency of 3250 Hz, yielding an SIR of 21.98 dB for song one and a cutoff frequency of 4250 Hz, yielding an SIR of 7.32 dB for song two.



(a) song one SIR vs fc evaluation loop

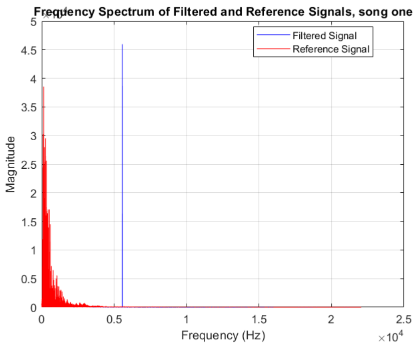


(b) song one SIR vs fc evaluation loop

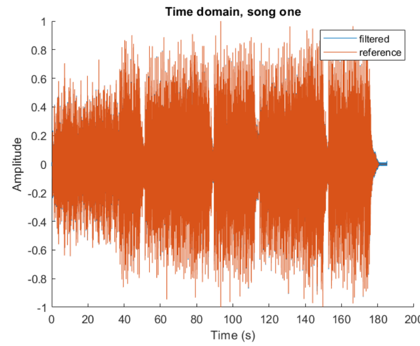
Figure 5: Signal-to-Interference Ratio (SIR) versus cutoff frequency (f_c) for song one and song two

Regarding the song two, as many of its components are near the noise frequency, therefore maximum SIR that can be achieved using a low pass filter like bessel, is 7.32 dB and as it is shown in Figure 6, still a lot of the noise component is present and can be seen in the frequency spectrum.

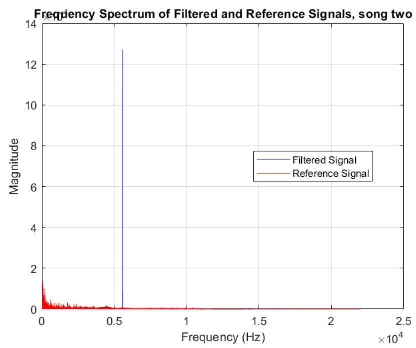
Therefore, a higher SIR does not always guarantee better perceptual quality. In this case, even at maximum SIR, noticeable residual noise remains around 5.5 kHz, also for the song one, as shown in Figure 6. In these cases, lowering the cutoff frequency to for instance 1500 Hz, below the SIR-optimal point, results in less SIR of 16.82 dB for song one and 3.53 dB for song two but much more suppression of high-frequency noise—leading to a cleaner output signal.



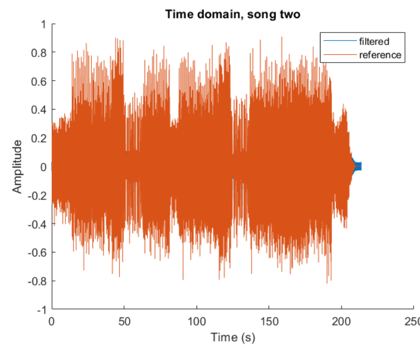
(a) Frequency spectrum of filtered and reference signals of song one



(b) Time domain of filtered and reference signals of song one



(c) Frequency spectrum of filtered and reference signals of song two

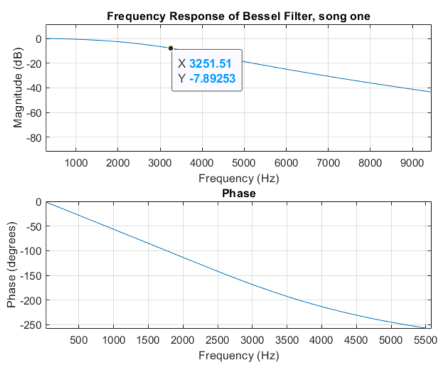


(d) Time domain of filtered and reference signals of song two

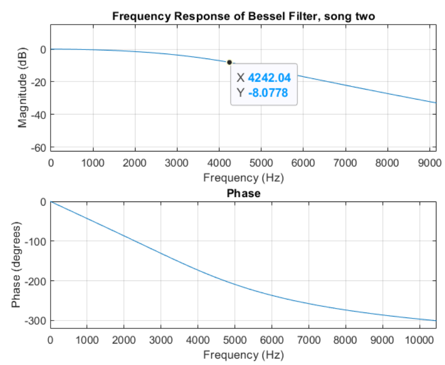
Figure 6: comparison between filtered signals with 4th order bessel filter at optimized frequencies for both songs and their reference signals in frequency and time domain

The transfer function of the 4^{th}-order Bessel filter for both of the songs is shown in Figure 7, plotted using `freqz` with 1024 frequency points. The attenuation at the specified cut-off frequency for each filter was found to be approximately –7.89 dB instead of the expected –3 dB.

This discrepancy arises from two main factors. First, the bilinear transform introduces frequency warping, which may shift the effective cut-off frequency in the digital domain. Second, numerical instability in the filter design at relatively high analog frequencies (e.g., $2\pi \cdot 3250$ rad/s) can cause the computed filter coefficients to become poorly scaled, triggering MATLAB warnings such as `Matrix is close to singular or badly scaled. Results may be inaccurate.` As a result, the frequency response deviates from the ideal analog behavior.Therefore, the observed deviation is attributed to the limitations of digital approximation of high-frequency analog Bessel filters using bilinear transformation.



(a) Bessel filter transfer function amplitute and phase for song one



(b) Bessel filter transfer function amplitute and phase for song two

Figure 7: Transfer function of the 4th-order Bessel low-pass filter with cut-off frequency $f_c = 3250$ Hz for song one and $f_c = 4250$ for song two.

1.3 Designing customized filter

In this section, a custom digital notch filter is designed to suppress a narrowband disturbance located at **5567.48 Hz**, which was identified through spectral analysis of the disturbed signals for both songs.

Filter Design Theory

The filter uses:

- Two complex conjugate **zeros** placed on the unit circle at the disturbance frequency to cancel it.
- Two complex conjugate **poles** placed slightly inside the unit circle at the same angle to maintain filter stability and control the notch bandwidth.

The transfer function of the filter is given by:

$$H(z) = \frac{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}{1 - 2r\cos(\omega_0)z^{-1} + r^2 z^{-2}}$$

where:

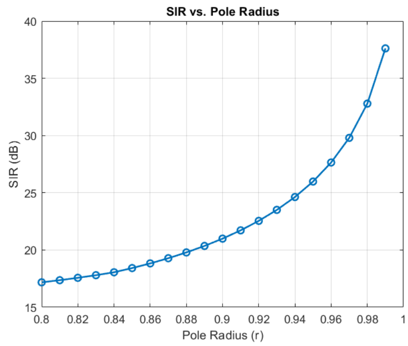
- $\omega_0 = 2\pi \cdot \frac{5567.48}{44100}$ is the normalized digital frequency,
- $r \in (0, 1)$ is the pole radius that controls the notch sharpness.

Implementation and Optimization

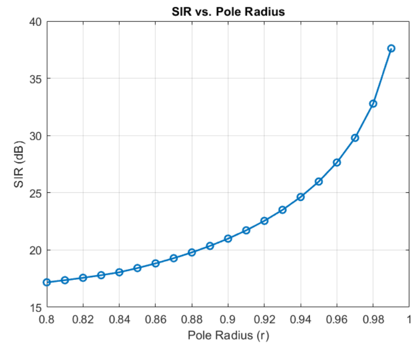
As the disturbance frequency is precisely known in this scenario, it may seem advantageous to make the notch filter as narrow as possible by selecting the highest feasible pole radius. To explore this idea, the filter was implemented and the pole radius r was swept over the range `[0.80, 0.99]` to identify the value that maximized the Signal-to-Interference Ratio (SIR). For both songs, the SIR curves showed a consistently increasing trend as r approached 0.99, suggesting that the optimal value — when considering only two decimal digits — is indeed $r = 0.99$.

However, in real-world applications, the disturbance frequency is often not perfectly constant due to variability, measurement errors, or environmental effects. Therefore, using a very narrow notch (i.e., a very high r) may fail to suppress slightly shifted or spread interference. A slightly lower pole radius provides a wider notch, which introduces tolerance around the target frequency and enhances robustness.

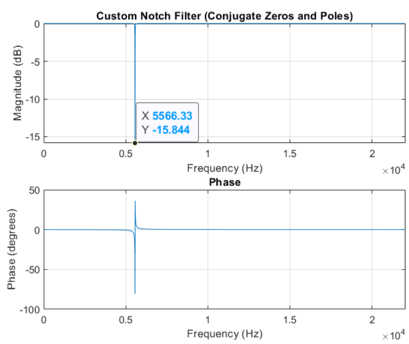
The sweep confirmed that the best performance was achieved at $r = 0.99$ for both songs, with maximum SIR values of 37.61 dB for song one and 25.89 dB for the second song. These results, along with the optimization process, are illustrated in Figure 8.



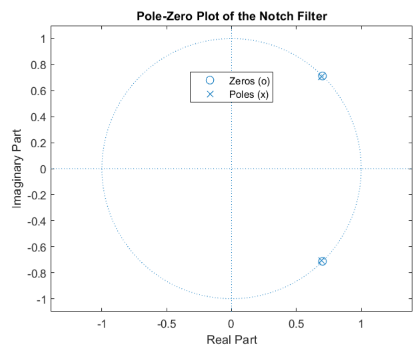
(a) SIR vs. pole radius for song one, radius swept from 0.80 to 0.99.



(b) SIR vs. pole radius for song two, radius swept from 0.80 to 0.99



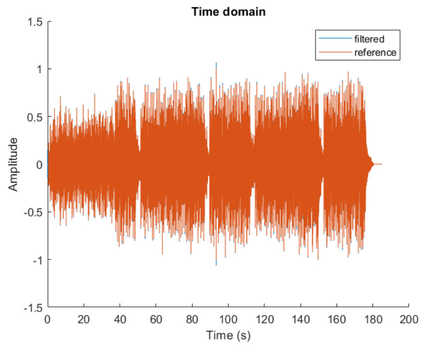
(c) The transfer function of the filter



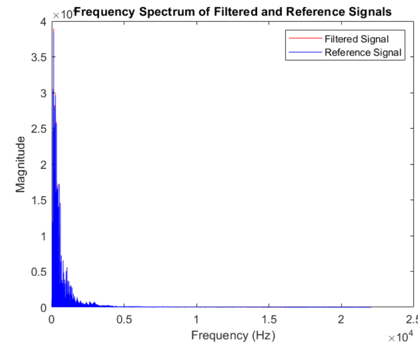
(d) Pole-zero plot of the custom notch filter. Zeros (circles) are placed on the unit circle at the disturbance frequency, and poles (crosses) are placed just inside the unit circle

Figure 8: Analysis and filter design for both songs

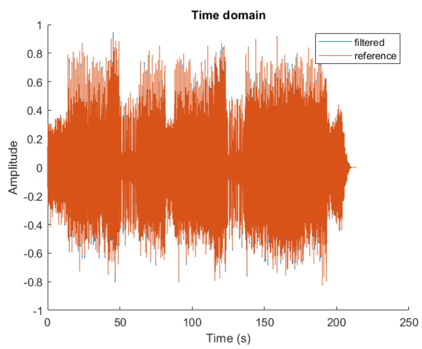
The frequency spectrum and time domain signals of the filtered songs vs their reference are shown in Figure 9. They almost identical which approves the notch filter implementation.



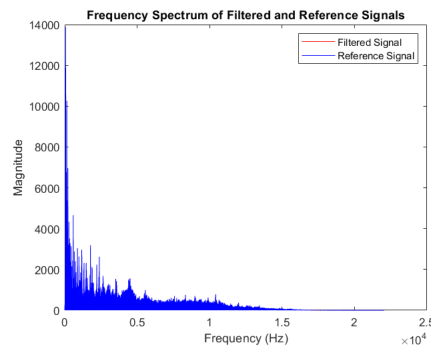
(a) time domain of notch filtered vs reference song one



(b) frequency domain of notch filtered vs reference song one



(c) time domain of notch filtered vs reference song two



(d) frequency domain of notch filtered vs reference song two

Figure 9: The resulting signals after implementing the notch filter, which are almost identical with the reference signals of the songs

1.4 Discussion

Compared to the Bessel filter used in Part 1.2, the custom notch filter demonstrates clear advantages:

- **Higher frequency selectivity**, which is particularly well-suited for removing a narrowband disturbance located at a known frequency.
- **Improved SIR and audio quality**, due to the minimal impact on nearby frequency components. This was especially evident for the second song, which contains significant high-frequency content near the

disturbance frequency. In this case, the notch filter effectively suppressed the noise while preserving musical detail.

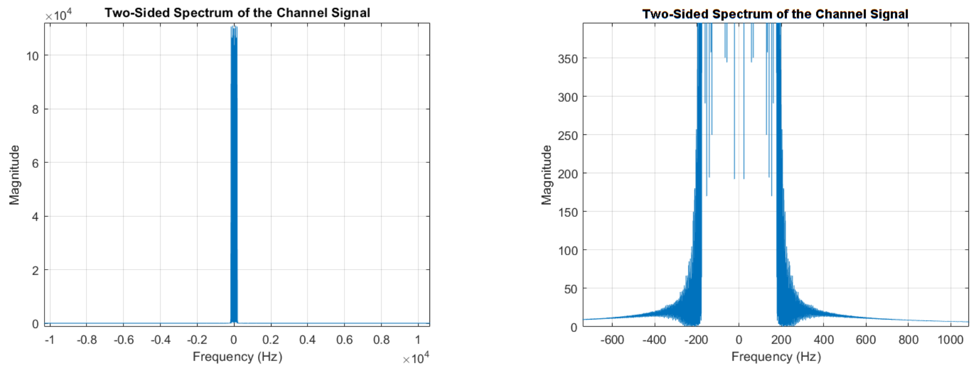
- For song one, whose spectral content is concentrated in the lower frequencies, far from the disturbance at 5.567 kHz, the Bessel filter performed reasonably well. However, the notch filter still achieved noticeably better results, improving the SIR by more than 10 dB compared to the Bessel design.

In general, notch filters are ideal when the disturbance is concentrated at specific, well-defined frequencies. They are particularly effective when the noise overlaps with or is near the useful frequency content of the signal, as they allow for precise removal without affecting other components. On the other hand, low-pass filters such as the Bessel design are more appropriate when the desired signal is concentrated at low frequencies and the noise lies significantly higher in the spectrum. In such cases, the broader cutoff of a low-pass filter can suppress the noise effectively without impacting the main content.

Part2 : SHARE THE CHANNEL

2.1 Analyze the channel

In this part, the songs are transmitted over a channel that has an already occupied baseband with a bandwidth of approximately 1000 Hz. To avoid interference, the signal must be spectrally shifted outside this range. The frequency spectrum of the channel is shown in Figure 10.



(a) Full two-sided spectrum of the channel signal.

(b) Zoomed-in view of the baseband region, showing strong spectral content up to 1000 Hz.

Figure 10: Spectral analysis of the given channel signal. The baseband is visibly occupied up to 1000 Hz, indicating that modulation should shift the signal into a higher-frequency band.

Attempting to transmit the songs directly in the baseband, without modulation, results in very poor performance due to spectral overlap with the already occupied channel. The measured SIR values were only 0.13 dB for *Imagine* and 0.04 dB for *Mamma Mia*, confirming that baseband transmission is not feasible in this scenario.

2.2 Multiplexing Two Songs Using Amplitude Modulation

In this part, both songs were transmitted simultaneously over the shared channel by applying amplitude modulation. The objective was to shift each song in frequency domain to a distinct frequency band in order to avoid both the baseband disturbance and mutual spectral overlap.

To determine suitable carrier frequencies, the occupied bandwidths (OBW) of the two signals—analyzed in Part 0—were used. Moreover, the channel interference was found to occupy approximately 169.32 Hz in the positive frequency spectrum for 99 percent of its total power. Song one had a positive-side OBW of 961.30 Hz, while song two exhibited a significantly larger bandwidth of 7589.85 Hz.

Considering 1000 Hz frequency for the baseband and 2000 Hz for the song one for half of its frequency components, we put the carrier frequency for the first song at 3000 Hz. Then 2000 Hz for the other half of the song one and 8000 Hz for the half of the song two, we put the second carrier frequency for song two at 13000 Hz. The other half of the song two will add to this and will be 21000 Hz, which is less than the overal bandwidth of 22000 Hz. Based on this information, the following carrier frequencies were chosen:

- **Song 1** was modulated around a carrier frequency of **3000 Hz**, which provides adequate spacing from the baseband noise and accommodates its spectral width.
- **Song 2** was modulated around **13000 Hz**, far enough to prevent any overlap with the spectral region occupied by song one, while remaining below the Nyquist frequency.

The modulation was performed using the following equations:

$$x_1^{\text{mod}}(t) = x_1(t) \cdot \cos(2\pi f_{c1} t)$$

$$x_2^{\text{mod}}(t) = x_2(t) \cdot \cos(2\pi f_{c2} t)$$

where $x_1(t)$ and $x_2(t)$ are the baseband signals for song one and song two, and $f_{c1} = 3000\,\text{Hz}$, $f_{c2} = 13000\,\text{Hz}$ are the respective carrier frequencies.

The two modulated signals were then added together along with the baseband interference to form the final multiplexed signal:

$$x_{\text{tx}}(t) = x_1^{\text{mod}}(t) + x_2^{\text{mod}}(t) + x_{\text{channel}}(t)$$

This configuration ensures that each song occupies a distinct, non-overlapping frequency region. The frequency spectrum of the multiplexed signal is shown in Figure 11, clearly demonstrating the separated bands of the two modulated songs.

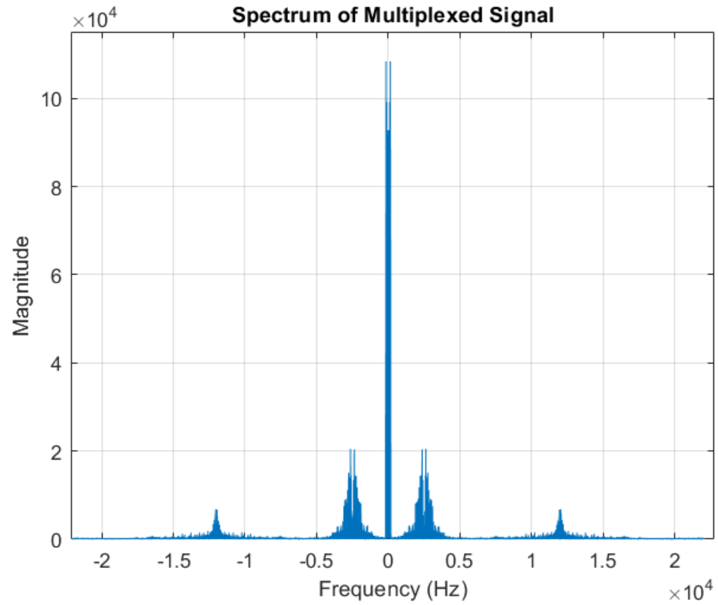


Figure 11: Frequency spectrum of the multiplexed signal. Song one is modulated at 3 kHz and song two at 13 kHz carrier frequency. The layout avoids overlap with the channel's baseband noise and ensures spectral separation between the two songs.

2.3 Receiving with Bessel Low-Pass Filter

In this part, the goal was to recover the two songs from the multiplexed transmitted signal using coherent demodulation followed by low-pass filtering. The transmitted signal is the result obtained from the previous part.

The demodulation process was performed by multiplying the received signal with a cosine function matching the modulation carrier, applied separately for each song. A factor of 2 is included in the multiplication to recover the original signal amplitude, as shown below:

$$x(t)\cos(2\pi f_c t) \cdot 2\cos(2\pi f_c t) = x(t) \cdot (1 + \cos(4\pi f_c t))$$

Therefore, the demodulated signal is computed as:

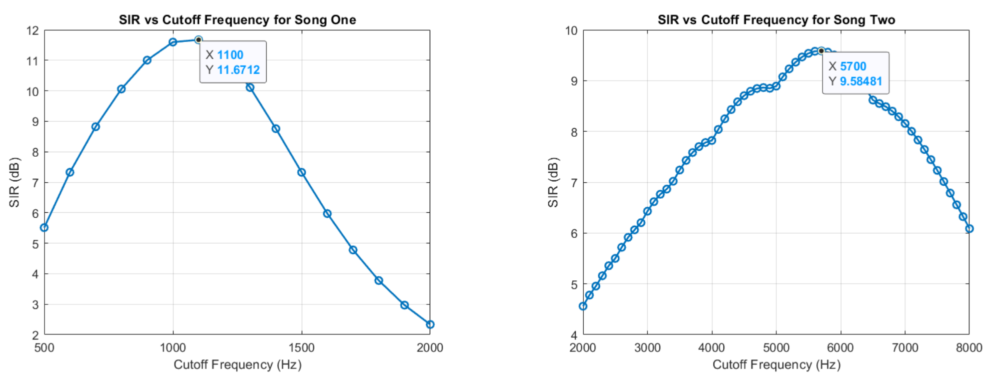
$$x_{\text{demod}}(t) = x_{\text{tx}}(t) \cdot 2\cos(2\pi f_c t)$$

where $f_{c1} = 3000\,\text{Hz}$ is the carrier frequency used for modulating Song one, and $f_{c2} = 13000\,\text{Hz}$ for Song two. This process produces a baseband copy of the original signal, along with a high-frequency component centered around $2f_c$, as illustrated in Figure 13.

To isolate the desired baseband content, a 4th-order Bessel low-pass filter was designed using the bilinear transform method. To maximize the SIR, a range of cutoff frequencies was tested. For instance, the range for Song one was from 500 Hz to 2000 Hz, and for Song two from 2000 Hz to 8000 Hz, with each point spaced by 100 Hz. This range was selected based on the occupied bandwidth of the songs. The optimal configuration was determined by evaluating the SIR for each value.

This resulted in an optimal cutoff frequency of **fc = 1100 Hz** for song one, with a corresponding SIR of **11.67 dB**, and **fc = 5700 Hz** for song two, with a corresponding SIR of **9.58 dB**. The optimization process for both songs is shown in Figure 12.

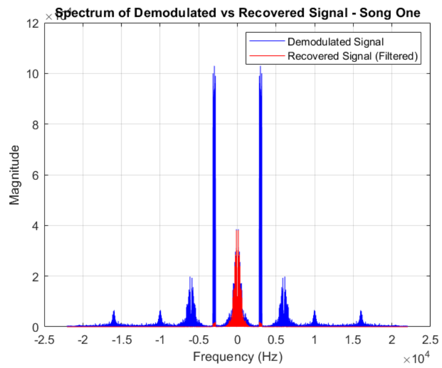
The frequency spectra of the recovered signals, compared against the demodulated signals, are shown in Figure 13 for this set of carrier frequencies.



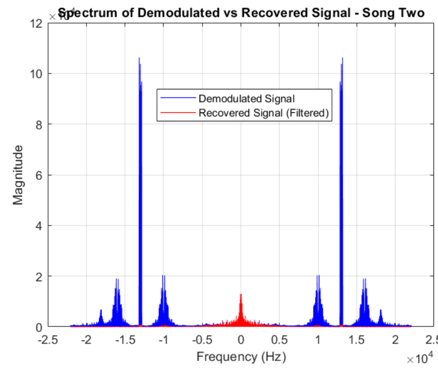
(a) SIR vs. cutoff frequency for song one. The best cutoff frequency is at 1100 Hz, yielding 11.67 dB SIR

(b) SIR vs. cutoff frequency for second song. The best cutoff frequency is at 5700 Hz, yielding 9.58 dB SIR

Figure 12: SIR optimization process for both songs using Bessel low-pass filters.



(a) Recovered vs. transmitted signal spectrum for song one.



(b) Recovered vs. transmitted signal spectrum for second song.

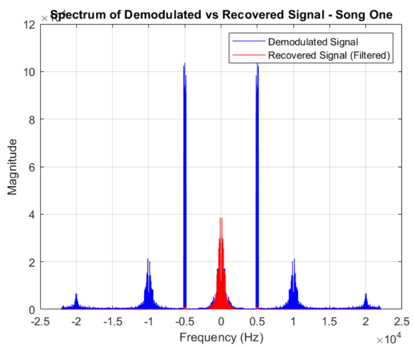
Figure 13: Frequency spectra comparing transmitted and recovered signals for both songs.

Then, increasing the carrier frequency for song two from 13 kHz to 15 kHz, and simultaneously increasing song one's carrier from 3 kHz to 5 kHz, resulted in significantly improved SIR performance. Specifically, the optimized SIR for song two reached **10.14 dB** at a Bessel filter cutoff frequency of **7200 Hz**.

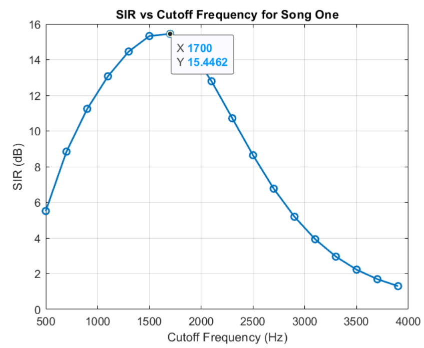
This improvement is due to the behavior of coherent demodulation: higher carrier frequencies push the mirrored unwanted frequency components further into the high-frequency domain. As a result, the Bessel filter can more effectively attenuate these undesired components after demodulation, improving the signal-to-interference ratio.

The same behavior is observed for song one. Increasing its carrier frequency to 5 kHz better separates the demodulated signal from the baseband interference introduced by the channel. This allows the filter to isolate the useful signal more cleanly, yielding an SIR of **15.45 dB** at an optimized cutoff frequency of **1700 Hz**.

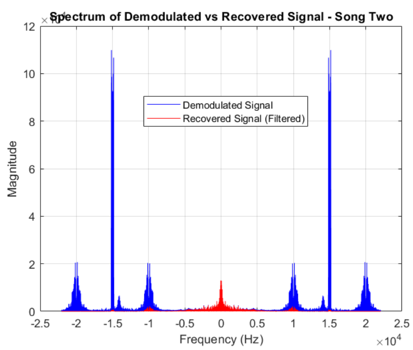
These findings are illustrated in Figure 14, which shows the frequency spectrum of the recovered signal compared to the demodulated signal and the variation of SIR with respect to cutoff frequency for both songs.



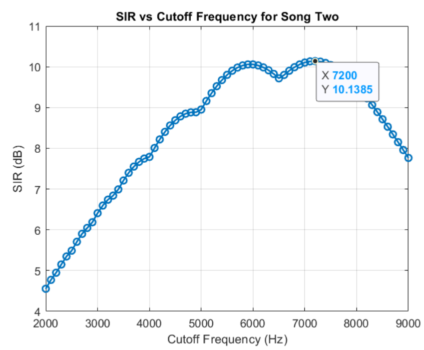
(a) Recovered vs. demodulated signal spectrum for song one.



(b) SIR vs. cutoff frequency for song one. Best performance at 1700 Hz.



(c) Recovered vs. Demodulated signal spectrum for song two.



(d) SIR vs. cutoff frequency for song two. Best performance at 7200 Hz.

Figure 14: Frequency-domain comparison and SIR optimization results after increasing carrier frequencies for both songs.

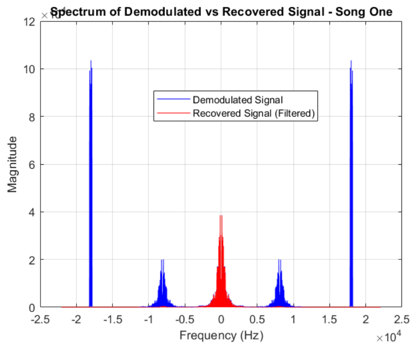
Finally, when considering only one song at a time being transmitted and received through the channel, the best practice is to choose the highest feasible carrier frequency. This pushes the demodulated baseband and interference components further into the high-frequency spectrum, allowing the low-pass filter to suppress them more effectively.

In order to choose appropriate carrier frequencies, the overall available bandwidth—limited by the Nyquist frequency to 22 kHz—must be taken into account. Also when demodulating, the For song one, a carrier frequency of **18 kHz** was selected, as the majority of its spectral content lies below 4 kHz.

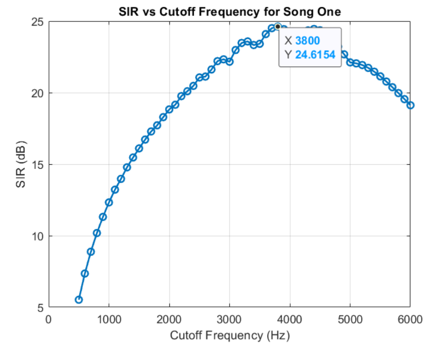
This configuration yielded a maximum SIR of **24.61 dB** at a Bessel filter cutoff frequency of **3800 Hz**.

For song two, whose significant frequency components are concentrated below 6 kHz, a carrier frequency of **16 kHz** was chosen. This placement ensured sufficient spectral separation and led to an SIR of **11.50 dB** at an optimal cutoff of **7900 Hz**.

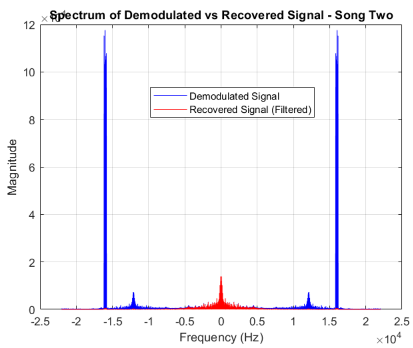
These results confirm that, when transmitting a single song in isolation, using a higher carrier frequency improves filter performance by pushing unwanted frequency components farther into the spectrum. The results of these single-signal transmission experiments are summarized in Figure 15 and Table 1.



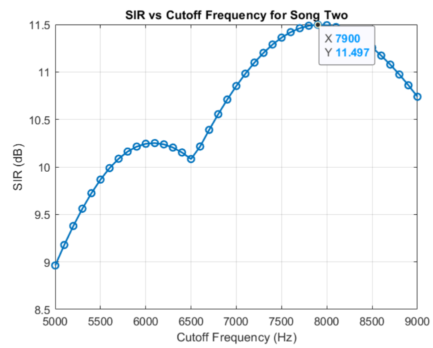
(a) Recovered vs. demodulated signal spectrum for song one.



(b) SIR vs. cutoff frequency for song one. Best performance at 3800 Hz, yielding SIR of 24.61 dB.



(c) Recovered vs. Demodulated signal spectrum for song two.



(d) SIR vs. cutoff frequency for song two. Best performance at 7900 Hz, yielding SIR of 11.5 dB.

Figure 15: Results when transmitting one song at the time, using bessel filter at the receiver.

Song	Carrier Frequency (Hz)	Cutoff Frequency (Hz)	SIR (dB)
Song 1	3000	1100	11.67
Song 2	13000	5700	9.58
Song 1	5000	1700	15.45
Song 2	15000	7200	10.14
Song 1 (single transmission)	18000	3800	24.61
Song 2 (single transmission)	16000	7900	11.5

Table 1: Summary of carrier frequencies, filter cutoff values, and SIR results for Part 2.3.

2.4 Receiving with a Custom Filter

In this final part, a custom low-pass filter was designed using two complex conjugate poles and two complex conjugate zeros, as required by the project specification. The filter structure was chosen to preserve the signal components in the passband while attenuating higher frequencies beyond a desired cutoff.

Filter Design

The digital filter was implemented in the following form:

$$H(z) = \frac{1 - 2r_z \cos(\omega_z) z^{-1} + r_z^2 z^{-2}}{1 - 2r_p \cos(\omega_p) z^{-1} + r_p^2 z^{-2}}$$

This structure ensures attenuation of high-frequency components while preserving the low-frequency content of the demodulated signal. During this part, both songs were transmitted together, with carrier frequencies of 5000 Hz for song one and 15000 Hz for song two.

Optimization Procedure

To optimize the custom filter design, two parameters were swept:

- The **pole radius** $r_p \in [0.50, 0.99]$, which controls the steepness of the filter's roll-off. Values closer to 1 provide a sharper transition but risk instability.
- The **zero angle** $\omega_z \in [\pi/3, \pi]$, corresponding to zero placement between mid-high and Nyquist frequencies. This controls where high-frequency attenuation begins.

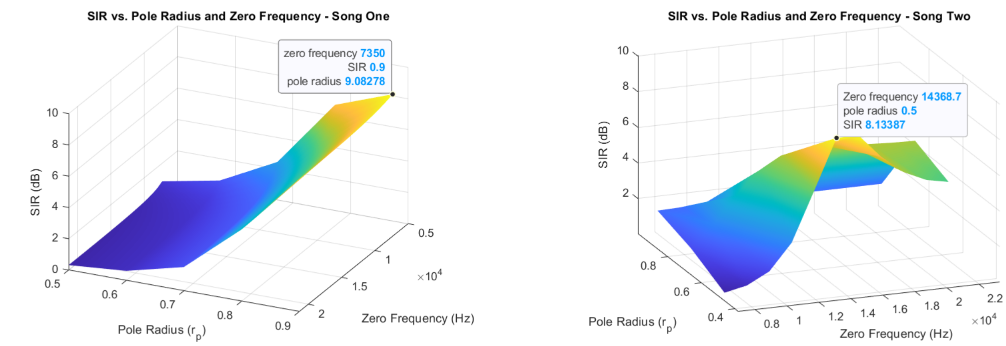
The other two parameters were held fixed to simplify the search:

- The **pole angle** $\omega_p = 0$, placing poles near the real axis to emphasize low-frequency content.
- The **zero radius** $r_z = 1$, placing the zeros on the unit circle for maximum attenuation at ω_z. This ensures the filter strongly suppresses frequencies above the desired cutoff while preserving the baseband.

The zero frequency corresponding to a given angle ω_z is calculated as $f_z = \frac{\omega_z}{2\pi} \cdot f_s$, where f_s is the sampling rate, which is 44100 Hz.

For each filter configuration, the demodulated signal was filtered, aligned with the reference signal, and the Signal-to-Interference Ratio (SIR) was computed. The highest SIR for song one was **9.08 dB**, achieved with a pole radius of **0.90** and a zero angle of **1.047 rad**, corresponding to a zero frequency of **7350 Hz**. For song two, the best result was an SIR of **8.13 dB**, obtained at a pole radius of **0.50** and a zero angle of **2.046 rad**, which corresponds to a zero frequency of **14,368.7 Hz**.

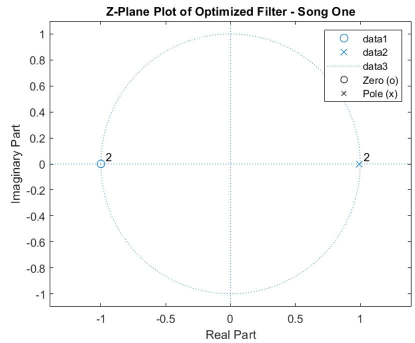
The 3D optimization surfaces showing SIR variations across pole radius and zero angle are presented in Figure 16. The corresponding filter structure and frequency-domain analysis are summarized in Figure 17.



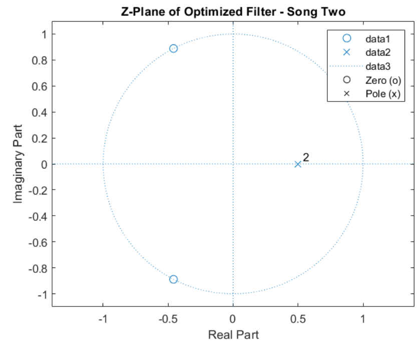
(a) SIR vs. pole radius and zero frequency for song one (best: 9.08 dB at $r_p = 0.90,\ f_z = 7350\,\text{Hz}$).

(b) SIR vs. pole radius and zero frequency for song two (best: 8.13 dB at $r_p = 0.50,\ f_z = 14,368.7\,\text{Hz}$).

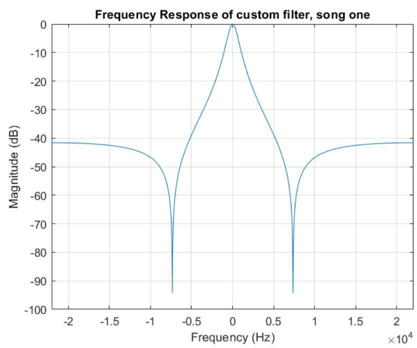
Figure 16: SIR optimization for the custom filter: sweeping pole radius and zero angle.



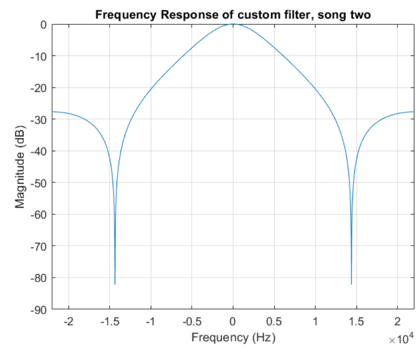
(a) Z-plane of the filter for song one.



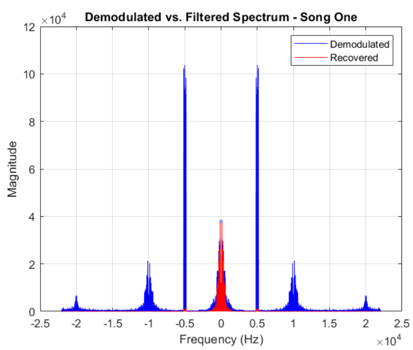
(b) Z-plane of the filter for song two.



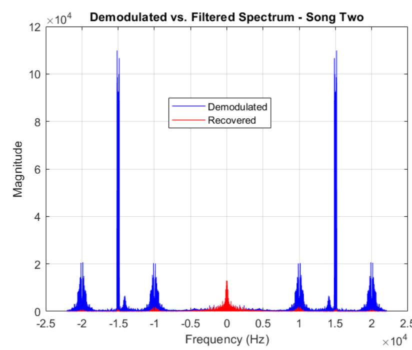
(c) Frequency response for song one.



(d) Frequency response for song two.



(e) Demodulated vs. filtered spectrum for song one.



(f) Demodulated vs. filtered spectrum for song two.

Figure 17: Filter structure and spectral results for the optimized custom filter, transmitting both songs together.

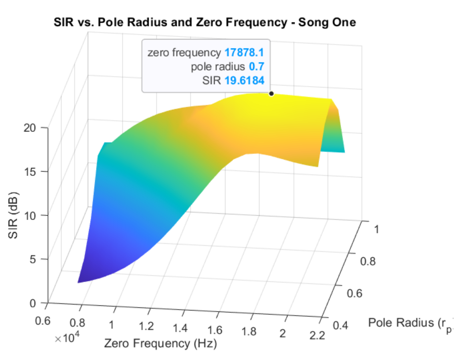
Transmitting One Song at a Time

As in the previous sections, each song was also transmitted independently over the channel, using higher carrier frequencies to minimize spectral overlap. Song one was transmitted at 18 kHz, and song two at 16 kHz. The filter design and optimization procedure remained the same as described earlier.

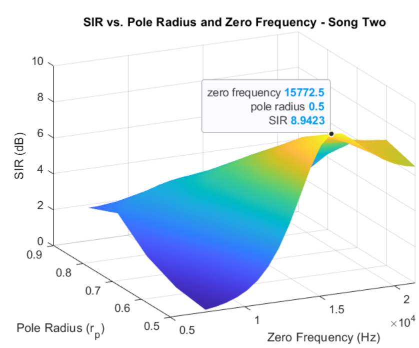
The zero frequency corresponding to a given angle ω_z is calculated as $f_z = \frac{\omega_z}{2\pi} \cdot f_s$, where f_s is the sampling rate.

For song one, the best SIR was **19.62 dB**, achieved with a pole radius of **0.70** and a zero angle of **2.55 rad**, which corresponds to a zero frequency of **17,878.1 Hz**. For song two, the highest SIR obtained was **8.94 dB**, corresponding to a pole radius of **0.50** and a zero angle of **2.247 rad**, which corresponds to a zero frequency of **15,772.5 Hz**.

The SIR optimization surfaces for both songs are shown in Figure 18. The corresponding filter structure and frequency-domain analysis are summarized in Figure 19.

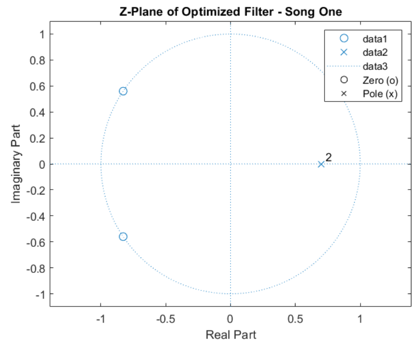


(a) SIR vs. pole radius and zero frequency for song one (best: 19.62 dB at $r_p = 0.70$, $f_z = 17,878.1$ Hz).

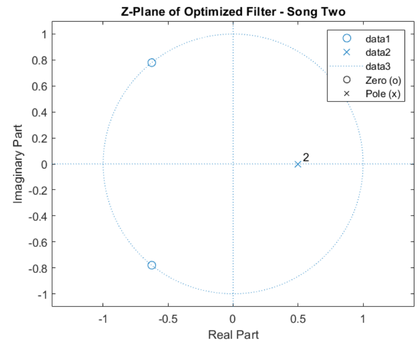


(b) SIR vs. pole radius and zero frequency for song two (best: 8.94 dB at $r_p = 0.50$, $f_z = 15,772.5$ Hz).

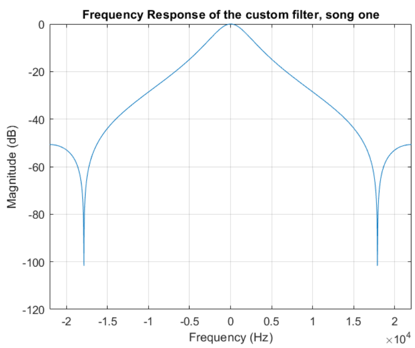
Figure 18: SIR optimization for the custom filter when transmitting each song independently.



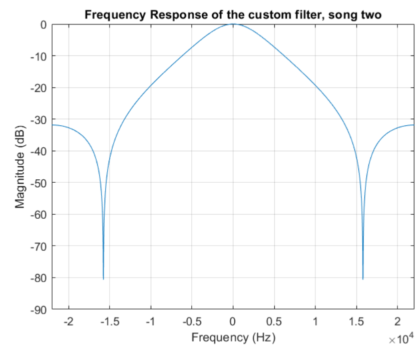
(a) Z-plane of the filter for song one.



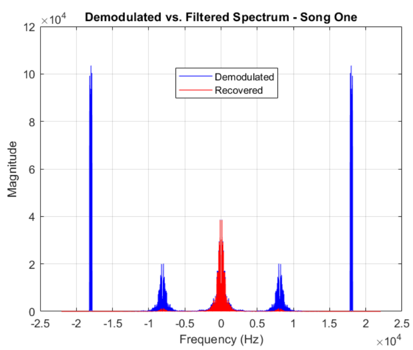
(b) Z-plane of the filter for song two.



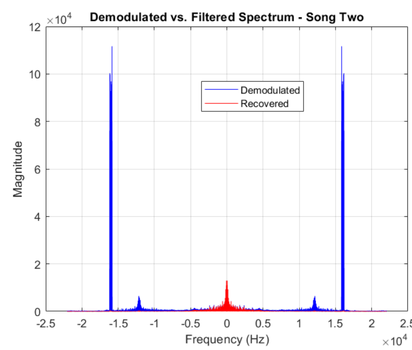
(c) Frequency response for song one.



(d) Frequency response for song two.



(e) Demodulated vs. filtered spectrum for song one.



(f) Demodulated vs. filtered spectrum for song two.

Figure 19: Filter structure and spectral results for the optimized custom filter, when transmitting each song at the time.

Song	Carrier Freq. (kHz)	Transmission	Pole Radius	Zero Angle (rad)	Zero Freq. (Hz)	SIR (dB)
Song 1	5.0	Together	0.89	1.047	7350.0	8.72
Song 2	15.0	Together	0.62	2.046	14,368.7	6.23
Song 1	18.0	Alone	0.70	2.550	17,878.1	19.62
Song 2	16.0	Alone	0.50	2.247	15,772.5	8.94

Table 2: Summary of optimized custom filter performance for Part 2.4. Each song was evaluated both independently and under joint transmission. Results include optimized pole radius, zero angle, computed zero frequency, and resulting SIR.

Discussion: Comparison of Part 2.3 and 2.4

Parts 2.3 and 2.4 focused on recovering signals transmitted over a shared channel, using different filter designs at the receiver: a Bessel low-pass filter in Part 2.3, and a custom second-order IIR low-pass filter with complex conjugate poles and zeros in Part 2.4.

Both filters were evaluated under two scenarios:

1. When both songs were transmitted simultaneously.
2. When each song was transmitted individually.

The Bessel filter generally provided smoother performance and higher SIR, especially under simultaneous transmission.

Song	Carrier Frequency (kHz)	Transmission	Filter Type	SIR (dB)
Song 1	5.0	Together	Bessel	15.45
Song 2	15.0	Together	Bessel	10.14
Song 1	18.0	Alone	Bessel	24.61
Song 2	16.0	Alone	Bessel	11.50
Song 1	5.0	Together	Custom	9.08
Song 2	15.0	Together	Custom	8.13
Song 1	18.0	Alone	Custom	19.62
Song 2	16.0	Alone	Custom	8.94

Table 3: Comparison of SIR results between Bessel and custom filters for different transmission conditions.

From this comparison, it is evident that Bessel filters outperform custom filters in most configurations. However, custom filters still offer competitive performance in simpler scenarios and provide more tunable filter structures. Moreover, the results confirm that transmitting songs individually leads to higher SIR due to reduced spectral overlap and inter-signal interference.

1 Conclusions

This project addressed the recovery and transmission of two corrupted audio signals using digital filters. In Part 1, a narrowband disturbance at 5567.5 Hz was removed using a Bessel low-pass filter and a custom notch filter. While the Bessel filter preserved low-frequency content, the notch filter achieved higher SIR due to its frequency selectivity.

In Part 2, amplitude modulation enabled both songs to be multiplexed over a shared channel. Bessel and custom low-pass filters were used for recovery. Bessel filters performed better overall. Moreover, significantly higher SIR was achieved when each song was transmitted individually, due to reduced spectral overlap.

This project highlights the importance of adaptive filter design in relation to signal characteristics, interference placement, and transmission strategy.

Code Availability

The accompanying ZIP file contains all MATLAB code used in this project. Each project section is implemented in a separate '.m' file. Two main scripts, 'part1.m' and 'part2.m', run the full pipelines for Parts 1 and 2, respectively.

Usage instructions and configurable flags (e.g., 'label', 'optimized') are described in detail in the included 'README.txt' file.