Self-Fulfilling Debt Crises with Long Stagnations*

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Abstract

We assess the quantitative relevance of expectations-driven sovereign debt crises, focusing on the southern European crisis of the early 2010s and the Argentine default of 2001. The source of multiplicity is the one in Calvo (1988). Crucial for multiplicity is an output process characterized by long periods of either high growth or stagnation, which we estimate using data for these countries. We find that expectations-driven debt crises are quantitatively relevant but state dependent, as they occur only during periods of stagnation. Expectations, and how they respond to policy, are the major factors explaining default rates and credit spread differences between Spain and Argentina.

Keywords: Self-fulfilling debt crises, sovereign default, multiplicity, stagnations.

JEL codes: E44, F34.

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1 Introduction

How important are expectations in triggering sovereign debt crises? In this paper, we explore the quantitative implications of a model of sovereign debt crises that exhibits state-dependent multiplicity. The mechanism we consider to generate multiplicity is the one proposed by Calvo (1988), in which high interest rates induce high default probabilities that in turn justify the high rates. We build on Ayres et al. (2018), who argue that the mechanism in Calvo (1988) is of interest when the fundamental uncertainty is bimodal, with both good and bad times.[1] We show that the mechanism is quantitatively relevant, especially during periods of economic stagnation.

Our analysis of self-fulfilling equilibria in sovereign debt markets is motivated by two main episodes. The first one is the European sovereign debt crisis of the early 2010s, when spreads on Italian and Spanish public debt exceeded 5% after being close to zero from the introduction of the euro until April 2009. Spreads were considerably higher in Portugal and especially in Ireland and Greece. The crisis receded substantially after Mario Draghi's "*whatever it takes*" speech on July 26th of 2012. Shortly afterwards, Draghi's words were confirmed by the European Central Bank's (ECB) commitment to purchase sovereign debt bonds though the Outright Monetary Transactions (OMT) program.[2] Two features of this episode point towards multiple equilibria. First, the abrupt movements in spreads were only weakly correlated with the usual fundamentals in sovereign debt models.[3] Second, spreads declined after the policy announcement, without the OMT bond-buying scheme being actually used. The potential self-fulfilling nature of these events in the summer of 2012 was explicitly used by Draghi to justify the policy.

The second episode is the 1998–2002 Argentine crisis. Back in 1993, Argentina had regained access to international capital markets, but the average country spread on dollar-denominated bonds for the period 1993–1998 was close to 7%. The average debt-to-GDP ratio was 35% for those years, and the average yearly growth rate of GDP was around 5%. But a recession had started by the end of 1998, and the Argentine government defaulted at the end of 2001 with a debt-to-GDP ratio slightly below 50%. Note that a 7% spread on a 35% debt-to-GDP ratio amounts to almost 2.5% of GDP of extra interest payments per year. Accumulated over that six year period, this represents 15% of GDP, almost one-third of the debt-to-GDP ratio of Argentina in 2001. Two questions arise. First, had Argentina faced lower interest rates, would Argentina have defaulted? Second, would Argentina have defaulted if the IMF had played the same role of

[1]See Lorenzoni and Werning (2019), who also discuss multiplicity as in Calvo (1988) using a bimodal endowment distribution.

[2]Mario Draghi was president of the ECB from 2011 to 2019. His full speech is available at https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html.

[3]See De Grauwe and Ji (2013) on the poor correlation between spreads and fundamentals during the European sovereign debt crisis.

lender of last resort that the ECB played for Spain and Italy? Our analysis implies that the answers to both questions are likely to be no.

These arguments are in line with the reasoning in Mussa (2002): "*(...)Relatively straightforward calculations indicate that at interest rate spreads of 500 to 600 basis points, Argentina's debt dynamics could have been sustainable under achievable degrees of budget discipline, whereas at persistent spreads of 1000 or more, the situation would be virtually hopeless.*" Michael Mussa was the Chief Economist of the International Monetary Fund from 1991 to 2001.

Our paper provides a quantitative exploration of the sovereign debt crisis episodes in Argentina and Spain, and of the role of policies like to one the ECB adopted. The main contribution is to show that the mechanism that generates multiplicity in Calvo (1988) is quantitatively relevant, as hinted in Mussa (2002).

A key assumption for multiplicity is a bimodal output growth process, with persistent good and bad times. We modify an otherwise standard sovereign default model to incorporate an endowment growth process that follows a Markov chain, featuring persistent high- and low-growth regimes. To calibrate the model, we estimate the output process for a set of countries that have recently been exposed to sovereign debt crises. We show that the model features self-fulfilling debt crises similar to those experienced by Argentina and Spain.

The model features equilibria in which interest rates can be high or low, depending on expectations. That is, a sunspot realization can induce discrete jumps in the interest rates faced by the borrower, even with no change in fundamentals. These discrete jumps in rates can happen only if fundamentals are weak. It is only in times of persistently low growth that spreads can be high because of expectations. In the high-growth regime, the region of multiplicity is either empty or negligibly small. Thus, the multiplicity we compute is state dependent: expectations can trigger a crisis only during a persistent stagnation.

The schedule of interest rates faced by the borrower can also exhibit discrete jumps because of fundamentals, due to the bimodal growth process. Thus, interest rate jumps are not necessarily a sign of a bad-expectations draw. The interest rate jumps, due to either fundamentals or expectations, can induce responses by the sovereign that can be interpreted as endogenous austerity. In this case, the sovereign optimally refrains from increasing debt in order to avoid the costs associated with those jumps. But those discrete interest rate jumps can also induce responses that resemble gambling for redemption of the type in Conesa and Kehoe (2017), where the borrower increases debt levels. Both endogenous austerity and gambling for redemption are featured in the equilibrium simulations discussed in the paper.

A policy intervention by a large creditor that is willing to offer the low interest rate schedule can neutralize the effects of the sunspot. In this sense, the effective probability of the sunspot needs to take into account the probability of such a policy intervention. In other words, if the policy intervention happens with probability one, the effective probability of the high interest rate schedule should be zero. We provide a historical discussion of the likelihood of a policy intervention by either the ECB in Spain or the International Monetary Fund (IMF) in Argentina in order to calibrate the sunspot probability for these two countries.

A key assumption we make is that expectations are more pessimistic for Argentina, meaning that high interest rates are selected more frequently than for Spain. As mentioned above, this choice is empirically based. In spite of possible doubts about whether the ECB would act as a lender of last resort, the events of the summer 2012 made it clear that there was willingness to do so. We therefore calibrate the probability of a bad sunspot realization to be very small for Spain. On the other hand, the internal regulations of the IMF—together with the recent failure of large packages in the Asian and Russian crises—imposed restrictions on their willingness to act as a lender of last resort. Thus, we calibrate the probability of a bad sunspot realization to be larger for Argentina. We also provide robustness exercises and explore the implications of assuming either more optimistic expectations for Argentina or more pessimistic expectations for Spain.

We show that the Argentinean calibration generates equilibrium paths that resemble the events of the 2001 debt crisis, including the observed jump in credit spreads. While endogenous austerity is present in the calibration for Argentina, the optimal policy eventually switches to gambling for redemption, thus generating high spreads in equilibrium. We follow Chatterjee and Eyigungor (2012) and construct an event case study for the years 1997-2001 by selecting output shocks to match the observed Argentine GDP growth. The model can account for the path of Argentina's spreads towards the crisis, and the sunspot plays a key role: absent a bad sunspot realization, Argentina would have avoided default. Thus, we find that the combination of low growth and expectations is essential in accounting for Argentina's 2001 default.

The calibration for Spain features only endogenous austerity. That is, the threat of high spreads triggers endogenous austerity, so that the high spreads are not observed along the equilibrium path. Thus, our model captures the austerity measures implemented by Spain, but only in response to off-equilibrium high spreads. Spain's calibration features only endogenous austerity because of the low bad sunspot probability: expectations-driven high rates are a rare event, and thus it is optimal to avoid default costs by restraining borrowing in those rare cases. While we argue that a low sunspot probability reflects ECB policy after the summer of 2012, a time varying probability could be a more accurate description and would help explain Spain's spreads over time. We opt for a model with fewer parameters with a time-invariant, low sunspot probability.

A main finding of our paper is that changes in expectations, as measured by the sunspot probability of selecting an interest rate schedule, have a large quantitative effect on model outcomes. If we assume optimistic expectations for Argentina, default rates and credit spreads decline drastically. That is, without any change in fundamentals, but only in beliefs, Argentina mutates from a serial defaulter to essentially a non-defaulter. Similarly, assuming pessimistic expectations for Spain results in high default rates and credit spreads, even if there is no change in fundamentals. Thus, our model suggests that the difference in expectations is a major factor explaining differences in default rates and credit spreads between Argentina and Spain. This result is more interesting, and policy relevant, when we connect the probability of the sunspot to the role of policy of a lender of last resort. A large creditor that can coordinate beliefs has a remarkable effect on the model outcomes: either low spreads and low default probabilities, through endogenous austerity in the presence of policy intervention, or high spreads and likely default, through gambling for redemption in the absence of such intervention.

The presence of a large creditor, which has a dramatic effect in our economy, has no effect in the standard sovereign default model. The canonical models of Aguiar and Gopinath (2006) and Arellano (2008), or Chatterjee and Eyigungor (2012), rule out multiplicity, and therefore a lender of last resort has no effect on equilibrium allocations as long as loans are correctly priced according to the default probabilities (so that the creditor breaks even). The standard model can match the high spreads leading up to Argentina's default, as in Chatterjee and Eyigungor (2012). The standard model would also be able to match the low spreads and rare default in Spain, with appropriately calibrated default costs. However, if multiplicity is indeed a feature of the actual economy, as we argue it is, the default costs in the standard model, used to target average spreads, would not be policy invariant.

The central result of this paper–that expectations-driven sovereign debt crises are empirically plausible–can contribute to the assessment of the role of policy in sovereign debt crises. In our quantitative exercises, it is when fundamentals are weak that a lender of last resort may be called in—not because fundamentals are weak but because the weak fundamentals create conditions for a role of expectations. Of course, the role of the lender of last resort in periods of stagnation will have effects on the economy beyond those periods in which interest rates could be high because of expectations.

The rest of the paper proceeds as follows. We next discuss related literature. In Section 2, we analyze a simple two-period model in which we can derive analytical expressions. We use this two-period model to highlight the role of the bimodal distribution in generating multiplicity, as well as to formally establish the connection between policy and expectations-driven equilibria. In Section 3, we present the quantitative model. In Section 4, we describe the calibration procedure, including the estimation of the endowment process and the calibration of the sunspot probabilities. Section 5 contains the discussion of the model results and robustness exercises. In Section 6, we review the historical roles of the ECB in Spain and the IMF in Argentina. Section 7 contains concluding remarks.

Related literature Our model follows the quantitative sovereign debt crises literature that grew out of the work of Eaton and Gersovitz (1981) and was further developed by Aguiar and Gopinath (2006) and Arellano (2008).[4] In these models, a sovereign borrower faces a stochastic endowment and issues non-contingent debt to a large number of risk-neutral lenders. There is no commitment to repay. Timing and choice of actions are important: the assumptions in Aguiar and Gopinath (2006) or Arellano (2008) are that the borrower moves first and chooses the level of non-contingent debt at maturity. We make two main changes to the standard setup. First, we assume that the borrower chooses current debt rather than debt at maturity. This assumption is key to generating multiplicity. When the borrower chooses debt at maturity, it is implicitly choosing the default probability and therefore also the interest rate on the debt. By contrast, when the borrower chooses current debt, default may be likely if interest rates are high or unlikely if interest rates are low.[5]

One fragility of the multiplicity mechanism in Calvo (1988) is that for commonly used distributions of the endowment process, the high-rate schedule is downward sloping, which means that the interest rates that the country faces decrease with the level of debt. That is not the case if the endowment is drawn from a bimodal distribution with good and bad times, as shown in Ayres et al. (2018) and Lorenzoni and Werning (2019). We depart from the standard setup in assuming that the endowment growth process follows such a bimodal distribution. This change is empirically founded: we estimate a Markov-switching regime for the growth rate of output for Argentina, Brazil, Italy, Portugal, and Spain, and, in all cases, we estimate processes that alternate between persistent high and persistent low growth. We view this feature of the endowment process as reflecting the likelihood of relatively long periods of stagnation in a way that is consistent with the evidence in Kahn and Rich (2007).

The paper closest to ours in its motivation is Lorenzoni and Werning (2019). They also consider the mechanism in Calvo (1988) but exploit a different source of multiplicity due to debt dilution with long maturities. In their environment, multiplicity arises because of the feedback between future and current bond prices: low bond prices tomorrow translate into low bond prices today, while high bond prices tomorrow translate into high prices today.

Lorenzoni and Werning (2019) make our same assumption on the actions of the borrower, which is also the assumption in Calvo (1988): the borrower chooses the funds it needs to raise rather than the face value of debt. Lorenzoni and Werning (2019) spell

[4]Other related literature includes Aguiar and Amador (2020), Aguiar et al. (2014), Bocola and Dovis (2019), Cole and Kehoe (2000), Conesa and Kehoe (2017), Corsetti and Dedola (2014), Lorenzoni and Werning (2019), and Roch and Uhlig (2018), among others.

[5]Ayres et al. (2018) show that the timing of moves is also key. In particular, when lenders are first movers, there is multiplicity regardless of whether the borrower chooses current or future debt.

out the micro foundations for the assumption by allowing the government to reissue debt within the period infinitely many times. In the limit, a government that decides sequentially chooses the funds it wants to raise rather than the face value of debt. Evidence on on lack of commitment to the preannounced issuance is presented in Brenner et al. (2009) and more recently in Monteiro and Fourakis (2023).[6]

In the context of self-fulfilling rollover crises as in Cole and Kehoe (2000), Aguiar et al. (2021) also find, in a model calibrated to Mexico, that simulated moments are sensitive to expectations. We see our papers as complementary, as we explore a different source of multiplicity, the one proposed by Calvo (1988).

Bocola and Dovis (2019) allow for maturity choice in the context of self-fulfilling rollover crises, and argue that expectations played a minor role during the European debt crisis. In their environment, governments prefer shorter maturities because of debt dilution incentives, while they prefer longer maturities because of rollover risk. Because maturities were reduced during the European debt crisis, their model suggests that rollover risk was limited during this period. While the source of multiplicity in our model is different, our quantitative exercise also implies a limited role for expectations in the case of Spain: jumps in spreads due to bad expectations do not occur in equilibrium, only endogenous austerity. Another important difference is that we treat the sunspot probability as a fixed parameter, while Bocola and Dovis (2019) allow for a stochastic process. Our comparative statics exercises show that the probability of the sunspot has a substantial impact on outcomes, thus suggesting that stochastic changes in this probability may account for spreads during the European debt crisis of the early 2010s, an issue that may be worth pursuing in future research.

2 A two-period model

Here, we illustrate the main mechanisms of the model in a simple two-period case. The economy is populated by a representative agent that draws utility from consumption in each period and by a continuum of risk-neutral foreign lenders. The initial wealth of the agent is denoted by ω. The endowment in the second period is distributed according to

$$y_2 = \begin{cases} y^l, & \text{with probability } p, \\ y^h, & \text{with probability } (1-p), \end{cases}$$

[6] Brenner et al. (2009) survey treasuries and central banks around the world and report that more than half of the countries that answered the survey (30 out of 48) claim to have some discretion on how much to issue, regardless of whether a target was announced. Monteiro and Fourakis (2023) also show that the amounts issued of short-term debt for Portugal rarely meet the preannounced target, and the deviations are large.

in which $y^l < y^h$.[7]

The representative agent preferences are given by $u(c_1) + \beta \mathbb{E} u(c_2)$, where u is strictly increasing, strictly concave, and satisfies standard Inada conditions. We assume that the initial wealth and the discount factor β are low enough that the agent will want to borrow. In period one, the borrower moves first and issues a non-contingent debt level b. Lenders respond with a gross interest rate R. We denote by $R(b)$ the interest rate schedule faced by the borrower. In period two, after observing the endowment y_2, the borrower decides whether to pay the debt or to default. In case of repayment, the borrower consumes the endowment net of debt repayment, $c_2 = y_2 - Rb$. In case of default, there is a penalty expressed as a drop in output to $y^d < y^l$. In addition, the borrower must repay a fraction κ of the debt. Thus, consumption following default is given by $c_2 = y^d - \kappa b$. The agent defaults if the cost of repayment is larger than the benefit:

$$\underbrace{(R - \kappa)\, b}_{\text{cost of repayment}} \quad > \quad \underbrace{y_2 - y^d}_{\text{benefit of repayment}} \quad . \tag{1}$$

In the first period, given initial wealth ω and an interest rate schedule $R(b)$, the borrower solves the following problem:

$$V(\omega) = \max_b \{u(c_1) + \beta \mathbb{E} u(c_2)\}, \tag{2}$$

$$\text{subject to} \quad c_1 = \omega + b,$$

$$c_2 = \max\left\{y_2 - R(b)b, y^d - \kappa b\right\}.$$

The borrower is subject to a maximum debt level constraint, $b \leq \overline{B}$.

The assumption that the borrower moves first by choosing a level of debt and that lenders move next with an interest rate schedule is standard. We depart from the literature as in Aguiar and Gopinath (2006) and Arellano (2008) in that we assume that the borrower chooses current debt b rather than debt at maturity, Rb. We follow Calvo (1988) in making this assumption. The risk-neutral lenders will be willing to lend to the agent as long as the expected return is the same as the risk-free rate R^*; that is,

$$R^* = h(R; b) \equiv \left[1 - \Pr\left(y_2 - y^d < (R - \kappa)\, b\right)\right] R + \Pr\left(y_2 - y^d < (R - \kappa)\, b\right) \kappa, \tag{3}$$

in which $h(R; b)$ is the expected return to the lender when the interest rate is R. Given

[7] The discrete distribution will help make clear the main mechanisms for multiplicity. We owe this to an insightful discussion by Fernando Alvarez.

a value for b, the expected return for lenders can be written as

$$h(R;b) = \begin{cases} R, & \text{if } R \leq \frac{y^l - y^d}{b} + \kappa, \\ R(1-p) + p\kappa, & \text{if } \frac{y^l - y^d}{b} + \kappa < R \leq \frac{y^h - y^d}{b} + \kappa, \\ \kappa, & \text{if } R > \frac{y^h - y^d}{b} + \kappa. \end{cases} \tag{4}$$

In Figure 1, we plot the expected return as a function of the interest rate R for three levels of debt, together with the risk-free rate, R^*. Notice that for low levels of R, the expected return is equal to R, since debt is repaid with probability one. In this region, as R increases, the expected return increases one to one. Eventually, R will be high enough that the borrower will default in the low output state, which happens with probability p. At this point, the expected return jumps down. As R increases, the expected return increases at a lower rate, $(1-p)$, since repayment happens only in the high output state. Finally, for high enough R, default will happen with probability one, and the expected return will be the recovery rate κ. A higher level of debt decreases the expected return uniformly, shifting the curves downward.

For low levels of debt, there is only one solution to equation (3), with $R = R^*$. For intermediate levels of debt, there are two solutions: one solution has $R = R^*$, associated with a zero probability of default, and the other has $R = (R^* - p\kappa)/(1-p)$, associated with a probability of default equal to p. For higher levels of debt, the only solution is the high rate $R = (R^* - p\kappa)/(1-p)$. Finally, for even higher debt, there is no solution.

We can now define the following correspondence relating debt levels to interest rates:

$$\mathcal{R}(b) = \begin{cases} R^*, & \text{if } b \leq \frac{y^l - y^d}{R^* - \kappa}, \\ \frac{R^* - p\kappa}{1-p}, & \text{if } (1-p)\frac{y^l - y^d}{R^* - \kappa} < b \leq (1-p)\frac{y^h - y^d}{R^* - \kappa}, \\ \infty, & \text{if } b > (1-p)\frac{y^h - y^d}{R^* - \kappa}. \end{cases} \tag{5}$$

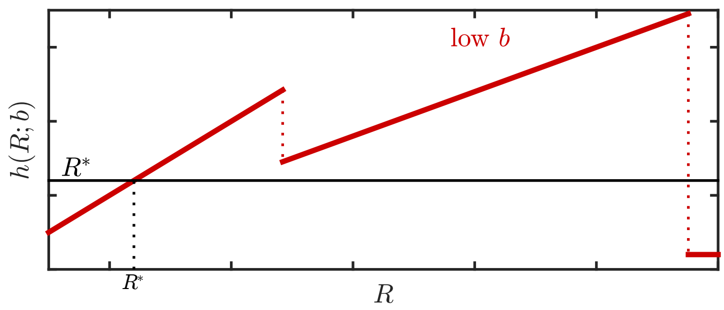
An *equilibrium* is an interest rate schedule $R(b)$ and a debt policy function $b(\omega)$ such that given the schedule, the debt policy function solves the problem of the borrower in equation (2), and the schedule $R(b)$ is a selection of the correspondence $\mathcal{R}(b)$.

The correspondence $\mathcal{R}(b)$ is plotted (red dashed line) in Figure 2. For all debt levels below $b_1 \equiv (1-p)\frac{y^l - y^d}{R^* - \kappa}$, there is only one interest rate, the risk-free rate. For debt levels between b_1 and $b_2 \equiv \frac{y^l - y^d}{R^* - \kappa}$, there are two possible interest rates, the risk-free rate and a high rate. For debt levels between b_2 and $\bar{b} \equiv (1-p)\frac{y^h - y^d}{R^* - \kappa}$, there is again only one interest rate, the high rate. There are multiple interest rate schedules that can be selected from this correspondence. We focus on two of those schedules: a low interest rate schedule, $R^{low}(b)$ in Figure 2a (blue solid line), and a high interest rate schedule, $R^{high}(b)$ in Figure 2b (blue solid line).[8]

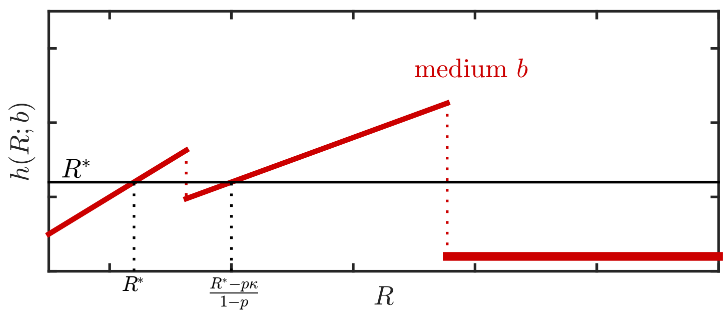
[8]Notice that it is always the case that $b_1 < b_2$ and $b_1 < \bar{b}$. However, while Figure 2 represents the

Figure 1: Expected return function for different levels of debt

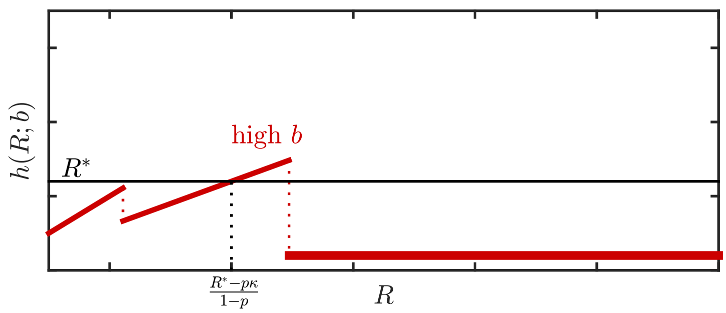
(a) low debt



(b) medium debt



(c) high debt



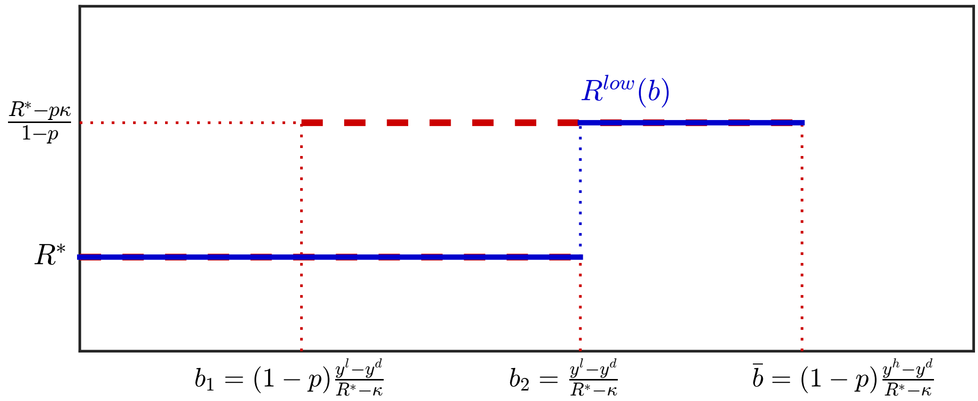
We think of b_1 as the debt level above which interest rates jump because of expectations, since alternative expectations could sustain low interest rates. We think of b_2 as the debt level above which interest rates jump because of fundamentals, since no expectations could sustain lower interest rates. We think of $\bar{b}$ as an endogenous borrowing limit, since any debt issued above this level implies a default probability of one.

Whether spreads are low or high has implications for the level of debt that can be raised. The region of multiplicity happens for intermediate levels of debt, between b_1 and b_2. If debt is sufficiently low, interest rates can only be low, whereas if debt is sufficiently high, rates can only be high. It is for intermediate levels of debt that interest rates can

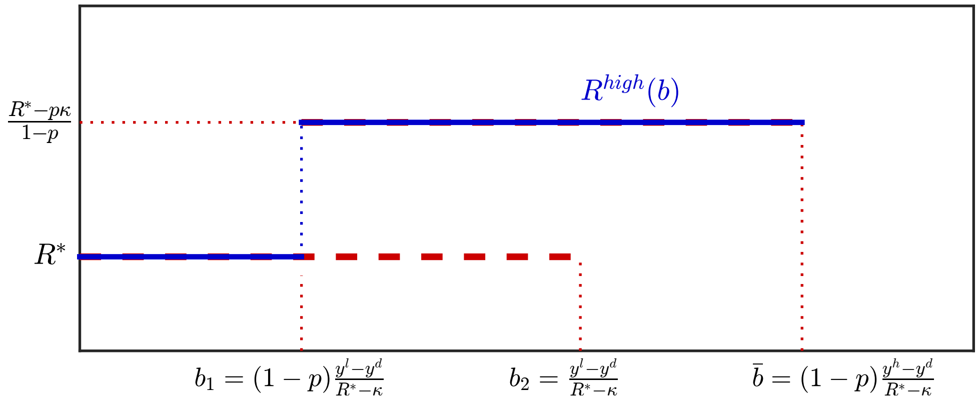
case in which $b_2 < \bar{b}$, there are parameter values such that the opposite is true. We chose to plot the case in which $b_2 < \bar{b}$ because this is the case in the quantitative model in Section 3.

Figure 2: Interest rate schedules

(a) low interest rate schedule



(b) high interest rate schedule



be either high or low, depending on expectations.

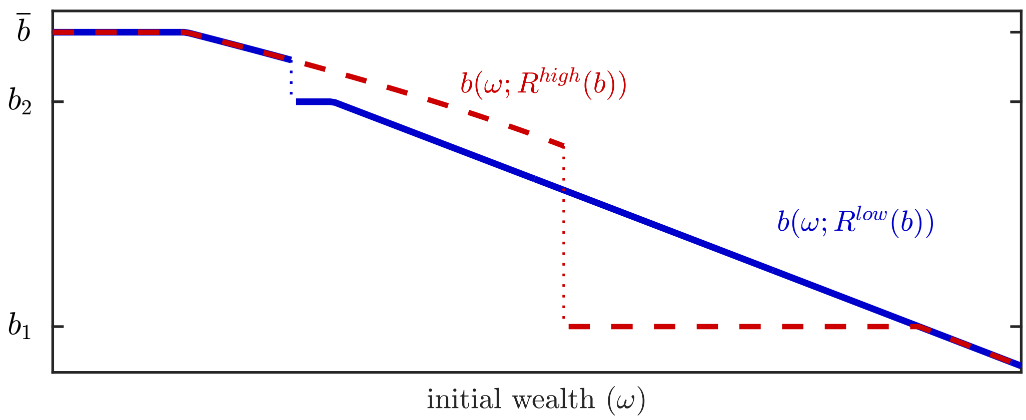
As an illustration, Figure 3 shows the optimal debt policy as a function of the initial wealth for the high and low interest rate schedules.[9] For high levels of wealth, the optimal choice of debt is below b_1, and thus the schedule does not matter. As wealth declines, the optimal amount of debt is higher, as indicated by the downward-slopping segment at the bottom right of the figure. Eventually, for low enough wealth, the optimal amount of debt is equal to b_1. At that point, the schedules matter. For the high interest rate schedule, the borrower chooses to keep debt levels at b_1 in order to avoid the discrete jump in interest rates on the whole level of debt. Eventually, for low enough wealth, the marginal utility of consumption in the first period is high enough that the borrower chooses to increase its debt level discretely. This discrete jump shows that the borrower has incentives to avoid at least part of the multiplicity region between b_1 and b_2. As wealth decreases even more, debt levels keep increasing until they reach the endogenous borrowing limit, $\bar{b}$. When the borrower faces the low interest rate schedule, borrowing

[9] The example assumes $y^l = 11.5$, $y^h = 19.5$, $y^d = 6.5$, $\kappa = 0.2$, $p = 0.55$, $R^* = 1.5$, $\beta = 0.2$, and $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 5.5$.

keeps on increasing as wealth declines until it reaches the level b_2. At this point, there is a choice to keep it constant for lower levels of wealth. Eventually, there is also a discrete jump, and debt levels continue to increase until they reach the endogenous borrowing limit.

The choice of keeping debt levels constant as wealth decreases happens in our model because of the discrete jumps in interest rates. These jumps can be induced by either expectations, as depicted in the flat region of the red dashed line at the bottom right of Figure 3, or weak fundamentals, as shown in the flat region of the blue solid line at the top left of Figure 3. These flat regions correspond to a form of endogenous austerity, where the borrower adjusts consumption to avoid the high rates. The jumps in the debt policy function indicate decisions by the borrower to end endogenous austerity, increase borrowing discretely, and accept a higher rate. They resemble the gambling for redemption described in Conesa and Kehoe (2017). As we show, the quantitative model of Section 3 exhibits both endogenous austerity and gambling for redemption.

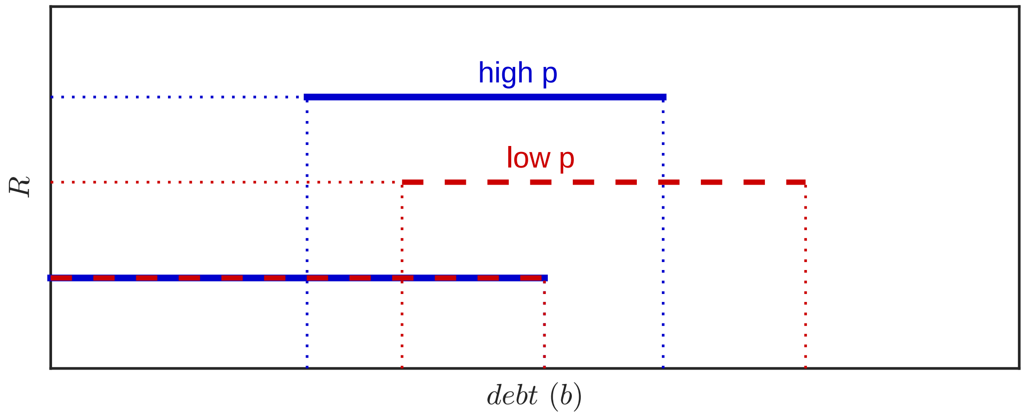
Figure 3: Debt policy function



The probability of the low endowment state We start the comparative statics by considering alternative probabilities of the low endowment state, p. Figure 4 plots the interest rate correspondence $\mathcal{R}(b)$ for two values of p. The higher p is, the higher is the interest rate that the borrower faces if default happens in the low endowment state. The higher the interest rate, the lower is the minimum debt level at which the borrower defaults in the low state. It follows that a higher p is associated with a higher interest rate and a larger region of multiplicity.

We relate the parameter p to the parameters in the full quantitative model in the next section, which features a two-state Markov process in the growth rates of output. The probability of switching to, or remaining at, the low-growth regime is the analog of the value of p in this two-period model. In the estimations described in Section 3, we show that low output growth states are persistent, which means a higher p during

Figure 4: Interest rate correspondence for different p



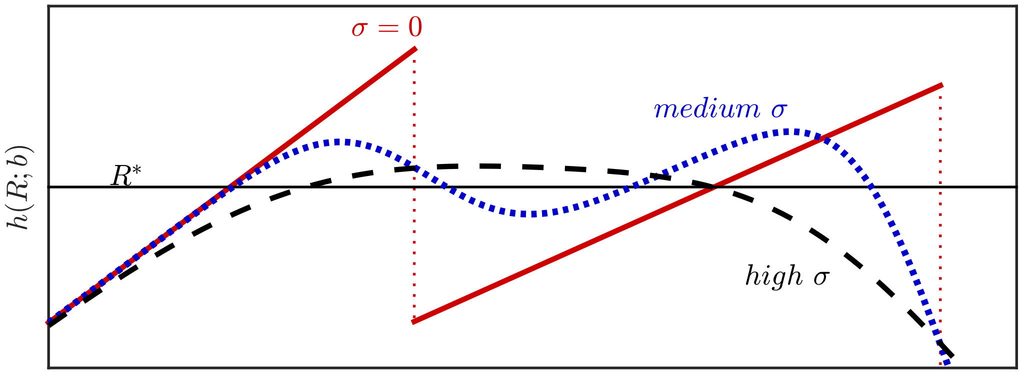
stagnations. High output growth states are also very persistent, which means a lower p during expansions. Consequently, in the quantitative model, stagnations come with larger regions of multiplicity and higher interest rates.

The role of the bimodal distribution In order to highlight the essential role of the bimodal distribution, we consider a generalization in which the endowment in the second period is drawn from a bimodal normal distribution, $y_2 \sim pN(y^l, \sigma^2) + (1 - p)N(y^h, \sigma^2)$. Figure 5 shows, for different values of σ, the expected return function $h(R; b)$ in Figure 5a and the implied interest rate correspondence $\mathcal{R}(b)$ in Figure 5b. The case with $\sigma = 0$ (red solid line) is the one analyzed before, in which there are two solutions to the arbitrage condition in equation (3). For strictly positive small levels of σ (blue dotted line), there are now four solutions to equation (3). However, the two solutions on the downward-sloping part of the expected return function are such that the expected return increases when the interest rate decreases. It follows that those solutions are also on the downward-slopping parts of the interest rates schedules depicted in panel 5b. Furthermore, not only the interest rate but also total future debt payments, $R(b)b$, go down when the level of current debt, b, goes up. This also implies that the agent would never choose to be in a decreasing part of the schedule. For these reasons and others, discussed in detail in Ayres et al. (2015) and Lorenzoni and Werning (2019), we rule out solutions along these parts of the interest rate schedule. For higher values of σ (black dashed line), there are two solutions to equation (3), but one can be ruled out. Therefore, to have multiple admissible equilibria, we need to have relatively low levels of σ. We show this is the case in the estimations of Section 3.

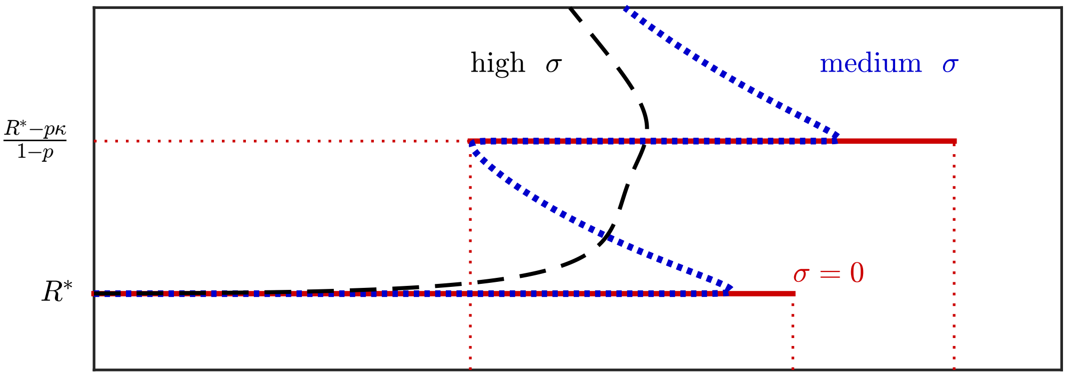
The endowment levels y^l and y^h are also important for multiplicity. As y^l approaches y^h, multiplicity disappears as the endowment distribution converges to the unimodal case. A similar rationale will apply to the quantitative results of Section 3.

Figure 5: Varying the standard deviation of endowment shock, σ

(a) expected return function



(b) interest rate correspondence



2.1 The role of policy

It is straightforward to show that a large creditor, playing the role of a lender of last resort, can select the low interest rate equilibrium of the deterministic model described above. The role of the large creditor is to be willing to lend according to the low interest rate schedule, up to the borrowing limit $\bar{B}$. In equilibrium, the large creditor may not need to lend, since individual creditors make zero profits under the low interest rate schedule and are thus willing to lending at those rates. To see this formally, we start by introducing a sunspot variable, s, which selects the high or low interest rate schedule, and assume there is a large creditor that can intervene, offering to lend according to the low interest rate schedule. The intervention occurs with some exogenous probability, π.

The sunspot variable is realized in the beginning of period one and selects either the high or the low interest rate schedule. The sunspot variable takes two values,

$$s = \begin{cases} s_B, & \text{with probability } p_B, \\ s_G, & \text{with probability } (1 - p_B), \end{cases}$$

with s_B selecting the bad schedule (high-rate) and s_G selecting the good schedule (low-rate). Thus, for debt levels between b_1 and $b_2 \equiv \frac{y^l - y^d}{R^* - \kappa}$, $R = R^{low}(b)$ when $s = s_G$ and $R = R^{high}(b)$ when $s = s_B$.

The large creditor may intervene to provide funds according to the low interest rate schedule up to the maximum $\bar{B}$. We assume the intervention occurs with an exogenous probability $\pi \in [0, 1]$. This encompasses the case in which $\pi = 1$, so the intervention is certain, as appears to have been the case in the euro area in the summer of 2012. For simplicity, we assume this shock is independent of the sunspot shock.

The timing of events in period one is now as follows: (1) the sunspot variable and the intervention shock are realized; (2) the borrower issues a non-contingent debt level b; (3) lenders respond with a gross interest rate R. The timing of events in the second period is as before.

Consider now an intervention with arbitrary probability π. If the intervention shock is realized, then the high-rate schedule offered by the private agents is redundant: the country would never borrow at the high rates. The only possible equilibrium outcome is the one exhibiting the low-rate schedule. Since, by definition, lenders make zero profits under the low-rate schedule, they are willing to lend at those rates. The large creditor does not have to lend in equilibrium; the promise of an intervention is all that is needed.

The previous discussion makes clear that if the intervention shock realizes, then the outcome of the sunspot is inessential. The effective probability of the high-rate schedule is $p_B \times (1 - \pi)$. This means that the economy with the intervention shock is identical to an economy without the intervention shock but with a lower sunspot probability.

3 A quantitative model of self-fulfilling debt crises

We calibrate an infinite-horizon model to evaluate the quantitative role of multiplicity in triggering sovereign debt crises. We estimate the endowment process in the model, using data on GDP growth for a set of developed as well as developing economies that were exposed to debt crisis episodes. The calibrated model generates self-fulfilling debt crises that can explain the events in Argentina in 2001 and can also shed light on the events in Spain in the 2010s.

3.1 Model

We expand the two-period model of Section 2 to an infinite-horizon framework with output growth and long-term debt. We assume an endowment economy, where output growth follows a two-state Markov process. At the beginning of each period, the sovereign chooses whether to default or not on the total stock of outstanding debt. Upon repayment, the sovereign chooses consumption, financed with current endowment and new bond issuance net of repayment. Upon default, the sovereign is excluded from financial markets and loses a fraction of the endowment. While in default, the sovereign may be given the chance to re-enter financial markets. If the decision is to re-enter, the sovereign recovers the totality of the endowment and must honor a fraction of the defaulted debt. Specific details follow below.

Time is discrete, runs forever, and is indexed by $t = 0, 1, 2, \ldots$. We assume a small open economy that receives a stochastic endowment, Y_t, every period. The preferences of the sovereign are standard:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{6}$$

where C_t denotes consumption. The utility function is consistent with balanced growth, which allows us to detrend the model as shown below.

Endowment process.—The endowment grows over time as the result of a persistent and a transitory shock. In each period t, the endowment Y_t is given by

$$Y_t = \Gamma_t e^{\sigma \epsilon_t}, \tag{7}$$

$$\Gamma_t = g_t \Gamma_{t-1}, \tag{8}$$

where ϵ_t is *i.i.d.*, $\epsilon_t \sim \mathcal{N}(0, 1)$, and g_t follows a two-state Markov process. Thus, g_t is the current trend growth, and Γ_t is the accumulated growth up to period t. We assume that g_t can be either high or low—$g_t \in \{g_H, g_L\}$—representing times of either fast growth or stagnation. The bimodal nature of g_t is empirically plausible and crucial for expectations to play a role in the model.

Debt contract.—The sovereign issues long-term bonds that promise a geometrically decreasing sequence of future payments, governed by the rate $\delta \in [0, 1]$, as in Hatchondo and Martinez (2009). A new debt issuance N_t in period t promises the following sequence of payments, starting from $t + 1$:

$$N_t R_t, \quad (1-\delta) N_t R_t, \quad (1-\delta)^2 N_t R_t, \ldots.$$

The value for R_t is determined on the date of issuance, t, and remains constant over the duration of the bond. For $\delta = 1$, R_t is the gross return on a one-period bond, while for

$\delta = 0$, R_t is the net interest rate on a consol.

Let B_t denote total payments due at time t because of previous issuance. Thus, we have

$$B_t = N_{t-1}R_{t-1} + (1-\delta)N_{t-2}R_{t-2} + (1-\delta)^2 N_{t-3}R_{t-3} + \dots$$
$$= \sum_{j=1}^{\infty} (1-\delta)^{j-1} N_{t-j} R_{t-j}. \tag{9}$$

This debt contract formulation is convenient because it allows to write debt payments B_t recursively as

$$B_t = (1-\delta)B_{t-1} + R_{t-1}N_{t-1}. \tag{10}$$

We refer to B_t as the debt service in period t.

Let Q be the price of a bond with a payment of $R_t = R$. We compute the interest rate ρ of such a bond so that the present value of promised cash flows equals its price, Q. That is, the rate ρ satisfies

$$Q = \sum_{j=1}^{\infty} \frac{(1-\delta)^{j-1} R}{(1+\rho)^j}. \tag{11}$$

Note that the rate ρ is pinned down by the ratio of Q and R. Previous work has typically normalized R and let the price Q be determined upon issuance in equilibrium. Instead, we normalize the price to $Q = 1$ upon issuance and let R be determined in equilibrium. This, together with the assumption that the borrower chooses the current debt issuance, allows for the multiplicity in R as discussed in the two-period model of Section 2. This is a normalization only upon issuance, but the price of the bond can fluctuate thereon, as we discuss below. Then, solving for ρ in (11) yields

$$\rho = R - \delta. \tag{12}$$

When not in default, the resource constraints are

$$C_t + B_t = Y_t + N_t, \tag{13}$$

where, given our normalization, new issuance N_t has a price of one.

We make the same assumptions on the timing of moves and actions of the borrower as in the two-period model. In particular, we assume that the borrower moves first and chooses current debt issuance N_t. Lenders move next and offer a schedule $\mathcal{R}(N_t; B_t, \Gamma_{t-1}, g_t, s_t)$, where s_t is a sunspot variable that selects a particular interest rate when more than one is consistent with an equilibrium.

Sunspot.—The sunspot s_t is the key variable in our model that captures the role of expectations in selecting the equilibrium schedule. In light of the results of Section 2, we allow the sunspot to take two values: $s_t \in \{s_G, s_B\}$. When more than one schedule is possible, $s_t = s_G$ value selects the good schedule (low rate), and $s_t = s_B$ selects the bad schedule (high rate). We allow the sunspot to follow a two-state Markov process. However, in our numerical exercises, we will assume that s_t is *i.i.d.* and that the bad sunspot occurs with probability p_B. The value of p_B captures how pessimistic or optimistic the beliefs of lenders are on average. It is also a function of policy of a lender of last resort. As we show in the quantitative results of Section 5, the value of p_B has substantial effects on the interest rate schedule as well as behavior of spreads and debt issuance.

Default cost and re-entry.—Default entails two costs for the sovereign. First, the sovereign remains temporarily excluded from financial markets. Second, a fraction of the endowment, $1 - \phi_t$, is lost. As is customary in the literature (see Arellano, 2008), we allow for the fraction ϕ_t to depend on the exogenous state $g_t \in \{g_L, g_H\}$. Thus, when the sovereign is in default, the resource constraints are

$$C_t = \phi_t Y_t, \tag{14}$$

with $\phi_t = \phi(g_t)$.

While in default, the option to re-enter credit markets happens with probability $1 - \theta$. If the sovereign chooses to re-enter, the output loss, $1 - \phi_t$, is lifted, and the sovereign regains access to international financial markets. In addition, debt servicing resumes, but a fraction $1 - \kappa$ of payments is forgone. Thus, lenders recover a fraction κ of the outstanding debt. A sovereign that defaulted in a period with promised debt service B will then face a debt service κB upon re-entry, and future debt payments will evolve as in equation (10).

Values of default and no-default.—Let $V^{nd}(B, \Gamma_-, g, s, \epsilon)$ and $V^{d}(B, \Gamma_-, g, s, \epsilon)$ be the maximal attainable values of no-default and default, respectively, to a sovereign that starts this period with debt service B, accumulated trend growth Γ_-, current growth g, sunspot s, and output shock ϵ. The value of no-default is

$$
\begin{aligned}
V^{nd}(B,\Gamma_-,g,s,\epsilon) = \max_{N,C} \Bigg\{ & \frac{C^{1-\gamma}}{1-\gamma} \\
& + \beta \mathbb{E}\left[\max\left\{ V^{nd}(B',\Gamma,g',s',\epsilon'), V^{d}(B',\Gamma,g',s',\epsilon') \right\} | \Gamma_-, g, s \right] \Bigg\}
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
s.t. & \\
& C + B = Y + N, \\
& B' = (1-\delta)B + \mathcal{R}(N,B,\Gamma_-,g,s)N, \\
& Y = \Gamma e^{\sigma\epsilon}, \quad \Gamma = g\Gamma_-, \\
& N \leq \bar{N}(B,\Gamma_-,g,s).
\end{aligned}
$$

The borrowing limit $\bar{N}(\cdot)$ is important in our environment. Since the borrower receives a unit of consumption for every unit of debt issued, default could always be postponed by issuing more debt. This possibility is ruled out by imposing a maximum amount of debt. In practice, we set $\bar{N}(\cdot)$ so that the probability of default next period is never larger than 65%.[10]

The value of default is

$$
\begin{aligned}
V^{d}(B,\Gamma_-,g,s,\epsilon) = \frac{C^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} \Bigg[& \theta V^{d}(B',\Gamma,g',s',\epsilon') \\
& + (1-\theta) \max\left\{ V^{nd}(\kappa B',\Gamma,g',s',\epsilon'), V^{d}(B',\Gamma,g',s',\epsilon') \right\} | \Gamma_-, g, s \Bigg]
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
s.t. & \\
& C = \phi(g) Y, \\
& B' = B, \\
& Y = \Gamma e^{\sigma\epsilon}, \quad \Gamma = g\Gamma_-,
\end{aligned}
$$

where $1-\phi$ is the fraction of output lost upon default. While in default, debt service is suspended. If the sovereign has the possibility to re-enter financial markets and decides to do so, only a fraction κ of the debt service is resumed next period.

Let $\mathbf{C}(B,\Gamma_-,g,s,\epsilon)$ and $\mathbf{N}'(B,\Gamma_-,g,s,\epsilon)$ denote the optimal consumption and debt issuance policies when not in default, and $\mathbf{B}'(B,\Gamma_-,g,s,\epsilon)$ be the implied debt service next period. Similarly, let $\mathbf{D}(B,\Gamma_-,g,s,\epsilon)$ denote the optimal default policy and $\mathbf{E}(B,\Gamma_-,g,s,\epsilon)$ denote the optimal re-entry policy while in default. The optimal policies

[10] Similar formulations for a borrowing limit can be found in Chatterjee and Eyigungor (2015) and Hatchondo et al. (2016).

for $\mathbf{D}(\cdot)$ and $\mathbf{E}(\cdot)$ are given by

$$\mathbf{D}(B,\Gamma_-,g,s,\epsilon)=\begin{cases}0 & \text{if } V^{nd}(B,\Gamma_-,g,s,\epsilon)\geq V^{d}(B,\Gamma_-,g,s,\epsilon),\\ 1 & \text{otherwise.}\end{cases} \tag{17}$$

$$\mathbf{E}(B,\Gamma_-,g,s,\epsilon)=\begin{cases}1 & \text{if } V^{nd}(\kappa B,\Gamma_-,g,s,\epsilon)\geq V^{d}(B,\Gamma_-,g,s,\epsilon),\\ 0 & \text{otherwise.}\end{cases} \tag{18}$$

Pricing of debt.—We assume a continuum of risk-neutral lenders with deep pockets that discount future payments at rate r^*. Let $Z=(\Gamma_-,g,s,\epsilon)$ collect all the exogenous terms in the economy. Let $\mathcal{Q}(B,R,Z)$ be the beginning-of-period (before default decisions) value of a bond that promises R upon issuance, and let $\mathcal{X}(B,R,Z)$ be the value of such a bond in default. Then, $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$ are given by

$$\begin{aligned}\mathcal{Q}(B,R,Z)=&\,[1-\mathbf{D}(B,Z)]\left[R+\frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B,Z),R,Z'\right)|\Gamma_-,g,s\Big]\right] \qquad (19)\\ &+\mathbf{D}(B,Z)\mathcal{X}(B,R,Z),\\ \mathcal{X}(B,R,Z)=&\,\frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(B,R,Z')+(1-\theta)\left[1-\mathbf{E}(B,Z')\right]\mathcal{X}(B,R,Z') \qquad (20)\\ &+(1-\theta)\mathbf{E}(B,Z')\mathcal{Q}(\kappa B,\kappa R,Z')|\Gamma_-,g,s\Bigg],\end{aligned}$$

where equation (20) incorporates that B remains constant during default episodes.

The value of the bond under no default $\mathcal{Q}(\cdot)$ is standard: if the sovereign doesn't default ($\mathbf{D}(\cdot)=0$), R is paid this period, and the remaining fraction $1-\delta$ has a next-period value given by $\mathcal{Q}(\cdot)$—evaluated at the next period debt service $\mathbf{B}'(\cdot)$. Notice that if there is no default, R remains unchanged. The value of the bond in default $\mathcal{X}(\cdot)$ reflects the two possible cases the sovereign will face next period. In the first case, the sovereign remains in default, either because it doesn't have the chance to re-enter financial markets or because it decides not to do so ($\mathbf{E}(\cdot)=0$). In this case, the value of the bond is still given by the function $\mathcal{X}(\cdot)$. In the second case, the sovereign—with probability $1-\theta$—has the chance to re-enter financial markets and decides to do so ($\mathbf{E}(\cdot)=1$). In this case, the lender recovers only a fraction κ of R, but the price of the bond also reflects that the sovereign has to repay only a fraction κ of its former liabilities, which is why the price $\mathcal{Q}(\cdot)$ is evaluated at $\kappa B'$ and κR at the end of equation (20).

Bond prices satisfy $\mathcal{Q}(B,R,Z)=R\mathcal{Q}(B,1,Z)$ and $\mathcal{X}(B,R,Z)=R\mathcal{X}(B,1,Z)$.[11] This is an intuitive result: a bond with an arbitrary payment R pays, in every state of nature,

[11] Formally, it can be shown that for any $\lambda>0$, equations (19) and (20) admit a solution with $\mathcal{Q}(B,\lambda R,Z)=\lambda\mathcal{Q}(B,R,Z)$ and $\mathcal{X}(B,\lambda R,Z)=\lambda\mathcal{X}(B,R,Z)$. The normalization of the price comes from setting $\lambda=1/R$. See Appendix C for a formal proof.

R times what a bond with a payment of 1 does. Since lenders are risk neutral, the price of any bond is thus a multiple of the price of a bond with $R = 1$. Thus, with a slight abuse of notation, let $\mathcal{Q}(B, Z) = \mathcal{Q}(B, 1, Z)$ be the price of a bond with $R = 1$. Consequently, the schedule $\mathcal{R}(N, B, \Gamma_-, g, s)$ must satisfy

$$1 = \frac{\mathcal{R}(N, B, \Gamma_-, g, s)}{1 + r^*} \mathbb{E}\left[\mathcal{Q}(B', \Gamma, g', s', \epsilon') | \Gamma_-, g, s\right], \tag{21}$$

$$B' = (1 - \delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N. \tag{22}$$

By our normalization, the bond price on issuance—the left-hand-side of equation (21)—is one. Then, for each level of issuance N, $\mathcal{R}(\cdot)$ adjusts endogenously so that the sovereign receives one unit of consumption for each newly issued bond.

Equations (21) and (22) are the analog to equation (3) in the simple two-period model and can be used to discuss the intuition behind the multiplicity results. If lenders coordinate on a high $\mathcal{R}$, this leads—given an issuance N—to a higher B' via equation (22). In turn, a higher B' implies a higher probability of defaulting in the future. That means a lower (expected) $\mathcal{Q}$ next period, which justifies the higher $\mathcal{R}$ in equation (21).[12]

The sunspot captures the role of expectations in selecting the equilibrium interest rate schedule in equations (21)–(22). For the same reasons as discussed in Section 2, the relevant parts of the interest rate schedule are those increasing on issuance. The two increasing interest rate schedules are selected using the same approach as in Section 2. The high-rate schedule corresponds to the highest interest rates for each level of debt, and the low-rate schedule corresponds to the lowest rate.

Equilibrium.—We can now formally define an equilibrium for this economy.

Definition 1 (Equilibrium)**.** *An equilibrium is a set of value functions* $\{V^d(B, \Gamma_-, g, s, \epsilon)$, $V^{nd}(B, \Gamma_-, g, s, \epsilon)\}$*; policy functions* $\mathbf{C}(B, \Gamma_-, g, s, \epsilon)$, $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$*; default and re-entry functions* $\mathbf{D}(B, \Gamma_-, g, s, \epsilon)$, $\mathbf{E}(B, \Gamma_-, g, s, \epsilon)$*; and pricing functions* $\mathcal{Q}(B, \Gamma_-, g, s, \epsilon)$, $\mathcal{X}(B, \Gamma_-, g, s, \epsilon)$ *and* $\mathcal{R}(N, B, \Gamma_-, g, s)$ *such that*

(i) the policy functions solve the sovereign's problem in equation (15) *and achieve value* $V^{nd}(B, \Gamma_-, g, s, \epsilon)$*;*

(ii) the value function $V^d(B, \Gamma_-, g, s, \epsilon)$ *satisfies equation* (16)*;*

(iii) the default and re-entry policies, $\mathbf{D}(B, \Gamma_-, g, s, \epsilon)$, $\mathbf{E}(B, \Gamma_-, g, s, \epsilon)$*, are as in equations* (17) *and* (18)*;*

(iv) functions $\mathcal{Q}(B, \Gamma_-, g, s, \epsilon)$ *and* $\mathcal{X}(B, \Gamma_-, g, s, \epsilon)$ *satisfy equations* (19) *and* (20)*;*

(v) the schedule $\mathcal{R}(N, B, \Gamma_-, g, s)$ *satisfies equation* (21) *with* B' *and* N *as in equation* (22)*.*

[12] Equations (21) and (22) show that the actions available to the borrower also matter. If the borrower could directly choose next-period debt service B', equation (21) would determine $\mathcal{R}(\cdot)$, and a unique corresponding issuance N would then arise from equation (22). See Ayres et al. (2018) for a detailed discussion on how timing and actions lead to multiple equilibria in models of default.

3.2 Model normalization

Since the endowment process has a trend, the state variables in the model are nonstationary. For computational purposes, we normalize all non-stationary variables by trend growth Γ_{-}. This requires showing homogeneity properties of the equilibrium functions, similarly to Aguiar and Gopinath (2006). We leave the detailed derivation to Appendix C and proceed to present the detrended model.

For any variable X_t, we denote $x_t = X_t/\Gamma_{t-1}$ as the detrended value. Let $v^{nd}(b, g, s, \epsilon)$ and $v^d(b, g, s, \epsilon)$ be the detrended values of no-default and default, respectively. The value of no default is

$$
\begin{aligned}
v^{nd}(b, g, s, \epsilon) = \max_{c,n} & \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta g^{1-\gamma}\mathbb{E}\left[\max\left\{v^{nd}(b', g', s', \epsilon'), v^d(b', g', s', \epsilon')\right\} | g, s\right] \right\}, \\
s.t.& \\
& c + b = y + n, \\
& gb' = (1-\delta)b + R(n, b, g, s, \epsilon)n, \\
& y = ge^{\sigma\epsilon}, \\
& n \leq \bar{n}(b, g, s),
\end{aligned} \tag{23}
$$

where $R(\cdot)$ is the schedule for the detrended variables, as discussed below.

Similarly, the value of default becomes

$$
\begin{aligned}
v^d(b, g, s, \epsilon) = & \frac{c^{1-\gamma}}{1-\gamma} + \beta g^{1-\gamma}\mathbb{E}\Bigg[\theta v^d(b', g', s', \epsilon') \\
& + (1-\theta)\max\left\{v^{nd}(\kappa b', g', s', \epsilon'), v^d(b', g', s', \epsilon')\right\} | g, s\Bigg], \\
& c = \phi(g)y, \\
& gb' = b, \\
& y = ge^{\sigma\epsilon}.
\end{aligned} \tag{24}
$$

The schedule offered by foreign lenders is given by

$$
1 = \frac{R(n, b, g, s)}{1+r^*}\mathbb{E}\left[Q(b', g', s', \epsilon')|g, s\right], \tag{25}
$$

$$
gb' = (1-\delta)b + R(n, b, g, s)n, \tag{26}
$$

with the prices $Q(\cdot)$ and $X(\cdot)$ satisfying

$$Q(b,z) = [1-\mathbf{d}(b,z)]\left[1+\frac{1-\delta}{1+r^*}\mathbb{E}\left[Q\left(\mathbf{b}'(b,z),z'\right)|z\right]\right] + \mathbf{d}(b,z)X(b,z), \tag{27}$$

$$X(b,z) = \frac{1}{1+r^*}\mathbb{E}\left[\theta X\left(b',z'\right) + (1-\theta)\left\{\left[1-\mathbf{e}\left(b',z'\right)\right]X\left(b',z'\right) + \mathbf{e}\left(b',z'\right)\kappa Q\left(\kappa b',z'\right)\right\}|z\right], \tag{28}$$

where $z = (g,s,\epsilon)$ collects the stationary exogenous states, $\mathbf{b}'(\cdot)$ in equation (27) is the next period payment implied by the optimal issuance policies in equation (23), and $b' = b/g$ in equation (28). The default and re-entry policies, $\mathbf{d}(\cdot)$ and $\mathbf{e}(\cdot)$, are given by

$$\mathbf{d}(b,g,s,\epsilon) = \begin{cases} 0 & \text{if } v^{nd}(b,g,s,\epsilon) \geq v^{d}(b,g,s,\epsilon), \\ 1 & \text{otherwise;} \end{cases} \tag{29}$$

$$\mathbf{e}(b,g,s,\epsilon) = \begin{cases} 1 & \text{if } v^{nd}(b,g,s,\epsilon) \geq v^{d}(b,g,s,\epsilon), \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

4 Calibration

We consider two calibrations of the model, one for the case of Spain during the European sovereign debt crisis, and one for Argentina during the default episode of 2001. We calibrate all parameters using standard values in the literature or by targeting empirical moments, including the sunspot probability.

Sunspot probability

We assume the sunspot is *i.i.d.* The bad sunspot occurs with probability p_B. To assign values of p_B to the calibration of Spain and Argentina, we leverage on the connection between the probability of the bad sunspot and the probability of a policy intervention highlighted in Section 2.1. More specifically, as we discuss in Section 6, the ECB ended up acting as a lender of last resort in European bond markets, while Argentina did not have such support from the IMF. Therefore, we calibrate the probability of the bad sunspot in Spain to be very low, $p_B = 1\%$, and to be relatively high in Argentina, $p_B = 25\%$.

We explore the extent to which changing the value of p_B alters the equilibria of the model. In particular, we compute equilibria for both economies assuming the alternative value of the sunspot probability—that is, switching to optimistic expectations for Argentina and to pessimistic expectations for Spain. If expectations did not matter, the outcome should be invariant to p_B. As we show below, the sunspot does matter substantially in both cases.

It is worth emphasizing that we explored numerically a wide range of values between 1% and 50% for p_B. The results showed that there is a threshold, around 5% for Argentina and 15% for Spain, such that for all values below the threshold, the results are very similar to the case of 1%, while for the values above the threshold, the results are very similar to the case of 25%.

Common parameters

A period in the model is one year. We set the annual risk-free rate $r^* = 3.5\%$. The discount factor is set to $\beta = 0.75$, as is standard in the literature, so that the borrower is impatient enough that borrowing and default are equilibrium outcomes. Arellano (2008) and Chatterjee and Eyigungor (2015) use quarterly discount factors of 0.95 and 0.94, respectively, which imply annual discount factors of 0.82 and 0.77, close to the value we chose.[13] The borrower's risk aversion coefficient is $\gamma = 3$. This value is between the ones used by Arellano (2008), $\gamma = 2$, and Bianchi et al. (2018), $\gamma = 3.3$. Finally, the recovery rate is set to 75% ($\kappa = 0.75$), in line with the estimates in Cruces and Trebesch (2013).[14]

Country-specific parameters

We use data to discipline the parameters that govern the output process and the value of δ, which pins down the average maturity of the debt. For Argentina, we set the average maturity of debt equal to 2.5 years, $\delta = 0.4$, close to the average maturity before the default in 2001. For Spain, we set the average maturity of debt equal to 6.7 years, $\delta = 0.15$, also close to the average value before Spain's debt crisis.

The endowment process in equations (7) and (8) is a regime-switching process characterized by five parameters $\{g_L, g_H, \sigma, p_L, p_H\}$. To calibrate these parameters, we estimate the endowment process using the filter proposed in Kim (1994).[15] We use annual GDP per capita data from The Conference Board Total Economy Database for the period 1980 to 2017 and estimate the process separately for five countries: Argentina, Brazil, Italy, Portugal, and Spain.[16] We start in 1980 to avoid the high growth rates of the period of convergence in the 1960s and 1970s. We assume bounded uniform priors for the five parameters and explore the posterior distribution using a Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm. Table 1 shows the estimates for all countries. As in the model, the estimation assumes the same standard deviation of shocks, σ, across the two growth states.[17]

Two things are worth mentioning regarding the estimates in Table 1. First, the estimates show clear evidence of a bimodal distribution for output growth in all countries. The average across countries of the difference between g_H and g_L is 6 percentage

[13] Aguiar and Gopinath (2006) use a quarterly discount factor of 0.8, which corresponds to 0.41 annually. On the other hand, Aguiar et al. (2016) use annual discount factors between 0.84 and 0.89.

[14] The haircut estimates in Cruces and Trebesch (2013) vary from 16% to 40%, corresponding to values of κ between 0.84 and 0.6. We perform sensitivity analysis for different values of κ in Appendix F.5.

[15] We do not directly use the filter in Hamilton (1989) because output growth has a moving average component. We use the filter in Kim (1994) instead. See Appendix A for details.

[16] In studying the European experience, we do not include Ireland and Greece in order to concentrate on a relatively homogeneous group of countries.

[17] In Appendix A, we also present estimates of the process allowing for state-dependent σ. We estimate similar values of σ across states, expect in the case of Argentina, whose estimates are significantly affected by the 2001 default.

Table 1: Prior and posterior distributions

	$\ln(g_L)$	$\ln(g_H)$	p_L	p_H	σ
	Prior distribution				
	$U[-0.1, 0.1]$	$U[-0.1, 0.1]$	$U[0.1, 1.0]$	$U[0.1, 1.0]$	$U[10^{-3}, 0.5]$
Countries	**Posterior distribution** (mean, and 5th to 95th percentile intervals)				
Italy	-0.017	0.022	0.646	0.843	0.016
	[-0.037,-0.008]	[0.018,0.028]	[0.050,0.990]	[0.627,0.990]	[0.012,0.023]
Portugal	-0.002	0.048	0.805	0.720	0.019
	[-0.011,0.003]	[0.041,0.057]	[0.516,0.990]	[0.454,0.939]	[0.014,0.025]
Spain	-0.018	0.033	0.629	0.838	0.017
	[-0.026,-0.010]	[0.026,0.039]	[0.308,0.990]	[0.653,0.990]	[0.013,0.025]
Argentina	-0.040	0.060	0.620	0.581	0.033
	[-0.049,-0.022]	[0.051,0.078]	[0.346,0.877]	[0.358,0.781]	[0.025,0.044]
Brazil	-0.033	0.029	0.589	0.793	0.019
	[-0.071,-0.022]	[0.025,0.032]	[0.103,0.860]	[0.627,0.923]	[0.014,0.025]

Note: For each country, we estimate an output process as: $\Delta \ln y_t = \ln g_t + \sigma(\epsilon_t - \epsilon_{t-1})$, in which $\epsilon_t \sim N(0,1)$ and $g_t \in \{g_L, g_H\}$, with $\Pr(g_{t+1} = g_L | g_t = g_L) = p_L$ and $\Pr(g_{t+1} = g_H | g_t = g_H) = p_H$. The table reports the mean and the interval between the 5th and 95th percentiles of the posterior distributions of each of the parameters for each country. The table also reports the prior distributions we used, which were chosen to be the same across countries. For each country, we use data on GDP per capita in 2016 US$ (converted to 2016 price level with updated 2011 PPPs) between 1980 and 2017 from The Conference Board Total Economy Database as the measure of y_t. See Appendix A for a description of the estimation.

points, more than three times the standard deviation of the transitory shock. The difference between g_H and g_L is a key ingredient for expectations to play a role.[18] Second, both the low- and high-growth states are persistent (between 60% and 80% persistence). As we show below, this relatively high persistence for the low-growth state generates high but plausible interest rates in the low-growth state.

Table 1 also suggests differences between the southern European and the South American countries. The most notable one is that the difference between g_H and g_L is substantially smaller in Europe. Another difference is that the volatility of the disturbance appears to be higher in South America.

When comparing within regions, there is substantial homogeneity in Europe. We therefore chose to use the point estimates for Spain, $p_L = 0.629$, $p_H = 0.838$, $\ln(g_L) = -0.018$, $\ln(g_H) = 0.033$, and $\sigma = 0.017$. In contrast, in South America there are larger differences between Brazil and Argentina. Most notably, Argentina exhibits a larger difference between g_H and g_L, as well as a larger shock volatility σ, while the persistence of the high-growth state p_H is lower. These differences most likely reflect the fact that the data from Argentina incorporate several episodes of default, so the data represent a combination of the true structural parameters and default costs, which are calibrated separately. In particular, Argentina's estimates are significantly affected by how unusual the years around the 2001 default were: GDP dropped by 20% during 1998-2002 and grew at a stunning 8% a year during 2003-2008.[19] Thus, we think it is reasonable to consider a more conservative calibration for Argentina's process.[20] Specifically, we set a calibration for Argentina with $p_L = 0.60$, $p_H = 0.75$, $\ln(g_L) = -0.04$, $\ln(g_H) = 0.04$, and $\sigma = 0.023$. These values, with the exception of g_H, are within the 95% confidence interval of the posterior distributions of Argentina. Because we chose a higher value of p_H, the lower g_H brings the average growth closer to the one observed for Argentina. Table 2 contains the parameter values that we use in the quantitative analysis.

When comparing Argentina and Spain, the two calibrations capture the fact that South American countries have lower average debt maturity, deeper recessions, less persistent periods of high growth, and more volatile output growth and fluctuations overall.

Finally, given the choice of all other parameters, described above, we use data on debt levels and spreads over the risk-free rate to calibrate the costs of default for each

[18]To reduce the dimensionality of the state, the output innovation in the model is *i.i.d.* One potential concern is that the estimation detects two different regimes as an approximation to a one-regime but persistent process for the growth rate of output. This is not the case: we repeated the estimation assuming an AR(1) process for the innovation, and also found a statistically and economically significant difference in the growth rates across regimes.

[19]To put these numbers in perspective, Argentina's GDP performance in this period was remarkably close to the experience of U.S. during the Great Depression, during which GDP dropped by 27% during 1929-1933 and recovered at a 7% annual rate during 1934-1939.

[20]We obtain estimates for Argentina that are closer to those for Brazil when we use data for 1965-2000—that is, when we use the same number of observations but omit the 2001 crisis. For this period, the estimated difference between g_H and g_L is 4.7 percentage points.

Table 2: Benchmark calibration and targeted moments

A. Parameters that are common across regions

Description	**Parameter**	**Value**
discount factor	β	0.75
risk aversion	γ	3.0
risk-free rate	R^*	1.035
re-entry probability	$1-\theta$	0.10
fraction of debt recovered after default	κ	0.75

B. Parameters that vary across regions

Description	**Parameter**	**Argentina**	**Spain**
inverse of average maturity	δ	0.40	0.15
probability of remaining in low growth	p_L	0.600	0.629
probability of remaining in high growth	p_H	0.750	0.838
low-growth rate	$\ln(g_L)$	-0.040	-0.018
high-growth rate	$\ln(g_H)$	0.040	0.033
standard deviation of transitory shock	σ	0.023	0.017
default cost in high-growth state	$\phi(g_H)$	0.90	0.945
default cost in low-growth state	$\phi(g_L)$	0.97	0.935
probability of bad sunspot	$p_B = 1 - p_G$	0.25	0.01

C. Targeted moments and model counterparts

Moments	**Target**	**Model**
Argentina		
average face value of debt (% of GDP)	53	52
spread in the interest rate schedule for $b = n = 19\%$ and $g = g_H$ (%)	7.1	7.0
Spain		
average market value of debt (% of GDP)	89	88
spread in the interest rate schedule for $b = n = 15\%$ and $g = g_H$ (%)	0	0

country. As is standard in the quantitative sovereign default literature, following Arellano (2008), default costs are asymmetric. We choose the values so that the average stock of debt in the simulations is close to the debt position the years before entering the crisis period and the spreads are close to the ones observed before the economy transitions from high to low growth.

Regarding debt levels, for Spain, we use the measure of net external debt from the Banco de España, which averaged 89% of GDP between 2008 and 2012.[21] For Argentina, most foreign assets were reserves held at the central bank, which were earmarked to back the monetary base at the time, due to the currency board. Therefore, we decided not to subtract these reserves from the gross measure of the debt. We use Argentina's gross external debt from the World Bank's International Debt Statistics, which averaged 54% of GDP between 1997 and 2001. See Appendix B for details on data sources and computations.

As pointed out by Dias et al. (2014), existing measures of debt for Argentina and Spain are not comparable. The key difference is that while the data for Spain are calculated using market prices, the data for Argentina correspond to face values of the outstanding bonds. To get around these discrepancies between countries, we measure both the market value and the face value of debt in the model. The market value is Qb, with Q defined in equation (27). The face value is the undiscounted sum of principal payments due in the future, $F_t \equiv N_{t-1} + (1-\delta)N_{t-2} + (1-\delta)^2 N_{t-3} + ...$ We denote its detrended value by f_t. We then use the market value in the model to calibrate the case of Spain and the face value in the model to calibrate the case of Argentina.

In terms of spreads, we aimed to replicate the lower spreads observed in Argentina and Spain during their final periods of high growth before entering recession. In the case of Argentina, spreads were at 7 percent by the end of 1998, while they were essentially zero for Spain in 2007. In our model, we target these levels of spreads in the high-growth interest-rate schedules for Argentina and Spain, as depicted in Figures 6 and 8, respectively, for their corresponding levels of debt service in 1998 and 2007.[22] The calibrated default costs, along with the data targets and their corresponding model counterparts, are presented in Table 2.

In Section 5, as well as in the Appendix F.5, we perform sensitivity analysis for many of the parameters described above and show that the conclusions are reasonably robust to alternative parameter values.

[21]The recent work in Bocola et al. (2019) discusses alternative debt measures that could predict credit spreads in European economies. We discuss a calibration with such alternative debt measures for Spain in Appendix F.2.

[22]For the calibration, we assume the levels of debt service and issuance to be the same. In Argentina, they were approximately 19% of GDP, while in Spain, we estimate them to be between 10% and 15% of GDP. See Appendix B for more details on data sources and computations. Note that by targeting the levels of spreads in the schedules, we are not imposing that they are selected in equilibrium.

Model computation The computation of the model has to take into account that there are multiple interest rate schedules satisfying equations (25) and (26). That is, the model computation must select schedules out of several possible equilibrium schedules, ruling out decreasing parts of the schedule (Ayres et al., 2015). The infinite-horizon nature of the model introduces an additional complexity because the selection of schedules affects value functions which in turn affect the schedules. To overcome this issue, we develop an algorithm that iterates only on the value function and that, in each iteration, computes the respective interest rate correspondence, selecting the high and low interest rate schedules as a function of the sunspot. Appendix D contains more details on the algorithm used to compute the equilibrium.

5 Quantitative results

We first show that for both calibrations, there are multiple interest rate schedules for the values for the debt service that Argentina faced in 2001 and Spain faced in 2012. This suggests that expectations may have played a role in triggering debt crises in both economies. A second set of results uses the simulation of the calibrated economies. We assess the relevance of the sunspot for the moments of the simulated economies—specifically, for spreads, default rates, and debt levels. A final set of results consists of an event case study of Argentina's default in 2001. We simulate the economy for the years 1997–2001 and compare the model-implied spreads with the ones observed in the data, assessing the role of the sunspot shock in generating the spike in spreads in 2001 and the subsequent default.

Recall that, as a benchmark, we set $p_B = 1\%$ for Spain, reflecting optimistic expectations, while we set $p_B = 25\%$ for Argentina, reflecting pessimistic expectations. Beyond expectations, there are three differences between the Argentina and Spain calibrations, all of them disciplined by data: the endowment process, the default costs, and the maturity of the debt. As it turns out, the difference in expectations is a major driver of differences in default probabilities and spreads between Spain and Argentina.

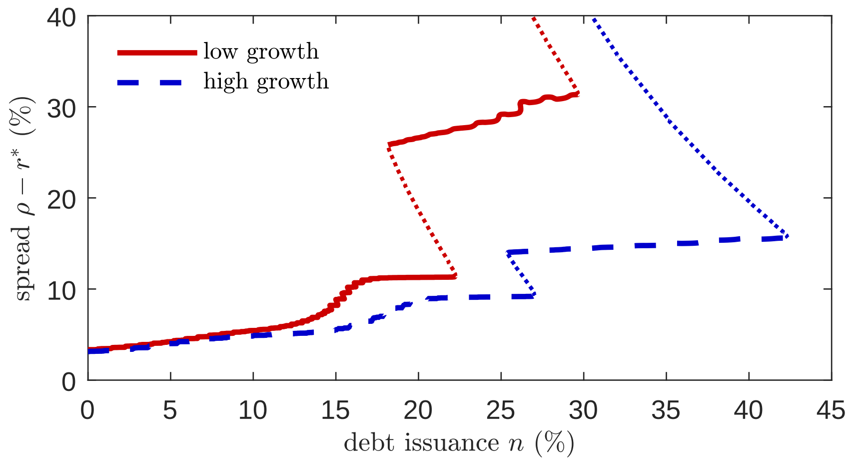
In what follows, we start by discussing the schedules for the two calibrated economies and then proceed to discuss both simulated economies. We end this section with Argentina's event case study.

5.1 Multiplicity of interest rate schedules

5.1.1 Argentina

Here, we explore the calibration for Argentina, focusing on the sovereign debt crisis of 2001. By January 2001, sovereign debt spreads in Argentina were roughly 8%. From 1994 to 1998, the Argentine economy grew at high rates, close to 4% a year. By the end

Figure 6: Interest rate spreads for Argentina ($b = 20\%$)



of 1998, a recession started, lasting until 2002. This can be interpreted as a regime switch from the high- to the low-growth state. Argentina defaulted on its debt in December 2001, after a couple of months with spreads that averaged around 30%.

In Figure 6, we plot the interest rate schedules for the low- and high-growth states for debt service levels of 20% of GDP, close to what Argentina had in 2001 at the onset of the crisis.[23] The horizontal axis has the new debt issuance, n, while the vertical axis has the corresponding interest rate spreads.[24] For each growth state, the sunspot realization determines whether the high or low interest rate schedule is selected in the region of multiplicity. The (blue) dashed line corresponds to the high-growth state, while the (red) solid line corresponds to the low-growth state. As discussed above, Argentina's calibration was chosen to match a 7% spread in the high-growth schedule, similar to what the country faced during the high-growth years before the crisis.

The size of the multiplicity region is state-dependent, as Figure 6 shows. In the high-growth state (dashed blue line), the region of multiplicity is small, so that the schedules are similar under the good and bad sunspot. By contrast, in the low-growth state (solid red line), interest rates can be either low or high, depending on the sunspot, for a larger set of issuance levels. In the low-growth state, there is multiplicity of schedules for issuance between 18% and 22% of output. For these levels of debt issuance, the low spread is around 12%. But there is also a high spread close to 30%. When expectations are good, the borrower can issue up to 22% of trend GDP at a spread of 12% or lower. Borrowing 22% when the debt service is 20%, as in Figure 6, implies a deficit of 2%. On the other hand, the country can issue only up to 18% with low spreads if expectations are bad. This means that a surplus of 2% of trend GDP is needed under the bad sunspot in order to maintain the lower spreads. Thus, under bad expectations, a large surplus adjustment

[23] Debt service in 2000 for Argentina equaled 19.6% of GDP, according to World Bank's International Debt Statistics. See Appendix B for more details on data sources and computations.

[24] Interest spreads are $\rho - r^*$, where ρ is the interest rate on new issuance as defined in equation (12).

is needed in order to obtain a low spread.

A noticeable feature of Figure 6 is that the high spreads resemble the ones for Argentina around the 2001 debt crisis. Spreads in Argentina went up to nearly 30% during the last two months of 2001. This number was not targeted in our calibration.

In this model, expectations play a significant role when fundamentals—the growth rate of the economy—are weak. The reason for this state-dependent role of expectations is that in periods of low growth, the probability of observing low growth in the future is high, due to the high persistence of the growth shock. Thus, if the borrower is expected to default in the low-growth state, which has a sizable probability, the interest rate must be high. This high interest rate will, in turn, induce the borrower to default in the low-growth state, confirming the expectations. This will happen even for low debt levels. If the borrower is not expected to default in the low-growth state, however, the interest rates will be relatively low, and the borrower will be able to issue a larger amount of debt without risking default next period. This translates into a large region of multiplicity, in which for intermediate levels of debt, interest rates can be either high or low, depending on expectations.

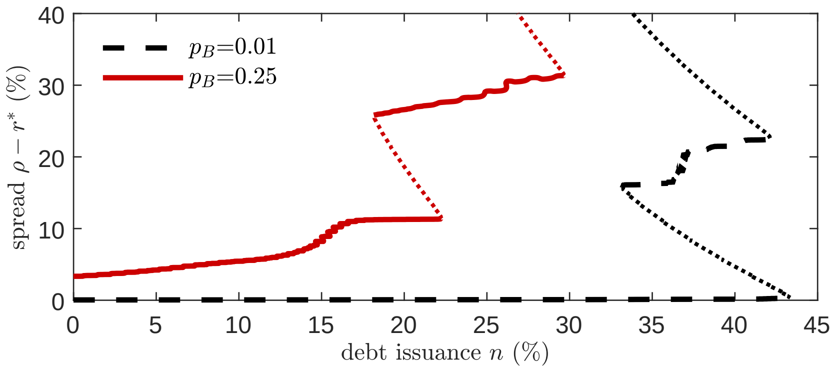
In contrast, persistence implies that in periods of high growth, the probability of switching to low growth is low. Thus, even if the borrower is expected to default in the low-growth state, the interest rate consistent with these expectations will be low, since the probability of the low-growth state is low. Expectations will be confirmed, meaning that the borrower will default in the low-growth state, but only when debt levels are relatively large. If the borrower is not expected to default in the low-growth state, however, the interest rates are only marginally lower, and the debt levels are such that the borrower will not default are not much higher. This translates into a small region of multiplicity.

The probability of switching to the low-growth state in the infinite-horizon model is the analog of the probability of the low endowment in the two-period model of Section 2. When that probability is low, the region of multiplicity is small, whereas when that probability is high, the region of multiplicity is large. In the infinite-horizon model, the probabilities are functions of the state. In low-growth states, the probability of future low growth is high, and the region of multiplicity is large. In high-growth states, that probability is low, and the region of multiplicity is small.

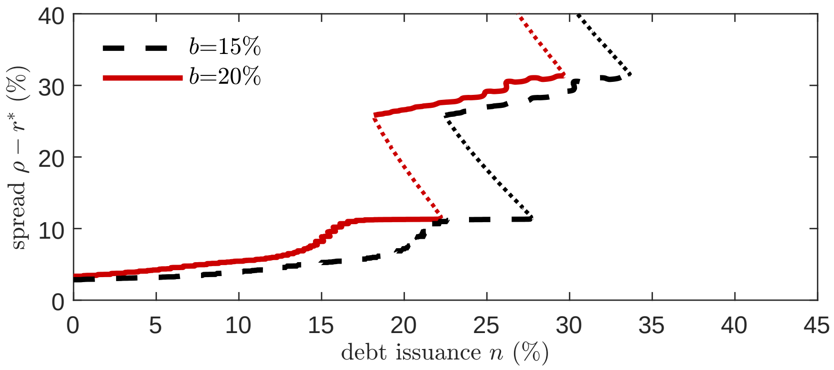
In what follows, we show how the schedules change as we vary debt service, b, and key parameters of the model. We restrict the analysis to the schedules for interest rate spreads in the low-growth state, since that is when multiplicity is more prevalent, as discussed above. Given the focus of our analysis, we are particularly interested in the effects of changing the probability of the sunspot, the debt service, and the average maturity. A full set of robustness results is presented in Appendix F.5.

Figure 7: Interest rate spreads for Argentina in the low growth state ($b = 20\%$): Comparative statics

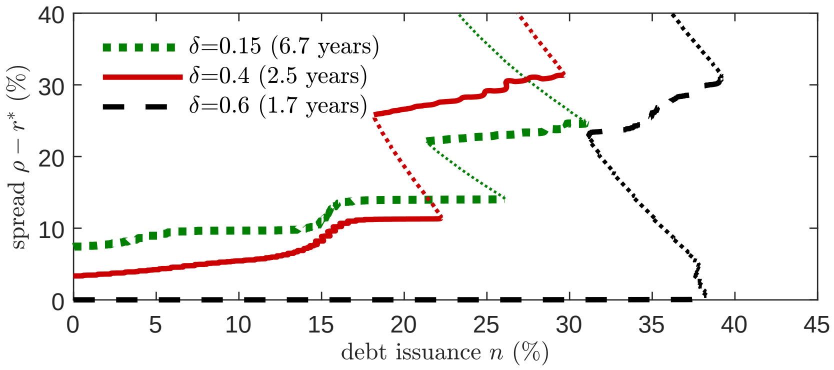
(a) The role of expectation p_B



(b) Inherited debt b



(c) Maturity δ ($b/\delta = 50\%$)



The role of the sunspot We compute the schedule setting the probability of the bad sunspot to $p_B = 1\%$, rather than $p_B = 25\%$ as we used for the benchmark.[25] The schedule for this case, when the growth rate is low, is depicted in Figure 7a (black dashed line), together with the one corresponding to the benchmark (red solid line).

The effect of the change in p_B is striking. If the probability of the bad sunspot had been very low, the country would have faced zero spreads even if it issued new debt up to almost 35% of trend GDP. In our narrative, this implies that, with optimistic expectations, Argentina could have been able to borrow to service the debt, plus a few extra points of GDP, and still be well within the debt choices that essentially rule out default, which explains the zero spread.

The role of the inherited debt We now show the effects of the inherited debt service b in Figure 7b. The solid red line is the same one depicted in Figure 6, corresponding to the benchmark. The black dashed line corresponds to a debt service of $b = 15\%$. As the figure shows, it is possible to roll over a debt service of 15% with a single low interest rate. Only if the government attempts to borrow another 8% of output, for a total of 23%, will multiplicity potentially matter.

The role of average maturity To assess the role of the average maturity, δ, Figure 7c reproduces the schedule for the benchmark calibration, together with the ones for either higher or lower average maturity. As before, we show only the schedules for the low-growth state where multiplicity is quantitatively relevant. We change the average maturity but keep total debt constant, so that the service of the debt, b, changes accordingly. Together with the case of the benchmark average maturity of two and a half years, plotted in Figure 6, we also plot the cases of maturity of 6.7 years ($\delta = 0.15$) and a low maturity of 1.7 years ($\delta = 0.6$).

As the maturity increases, the incentives to dilute the debt are stronger, which leads to higher spreads today. On the other hand, given a total level of debt, longer maturity implies lower current debt service and thus a smaller effect of an increase in current rates on the future debt obligations. As Figure 7c shows, longer maturity implies higher rates for low issuance levels. Yet, as longer maturity implies lower debt service, the issuance needed to service the debt is lower and away from the multiplicity region. For example, with a 50% debt-to-output ratio and $\delta = 0.15$, only 7.5% of output needs to be serviced every period, a value comfortably below the multiplicity region. Thus, while interest rates would have been higher, a higher maturity could have kept Argentina away from a crisis, even with positive but mild deficits.

[25] As mentioned above, we have solved the model for several values of the probability of the bad sunspot, p_B, between 0% and 50%. The solution essentially depends on its value being above or below a threshold close to 10%. For values below the threshold, the results are very similar to the case of $p_B = 1\%$, while for values above the threshold, the results are very similar to $p_B = 25\%$.

The case of a shorter maturity, of 1.7 years ($\delta = 0.60$), is quite different. Spreads are zero, even for debt issuance as large as 30% of output. This is quite remarkable, since it means that longer maturities do not necessarily translate into better financing conditions. The interpretation is that for the short maturity, debt dilution incentives are minor.[26] This effect, combined with Argentina's low debt level, implies that there is almost no default in equilibrium. One could be tempted to conclude that the short maturity has attractive features from a policy standpoint. However, the debt dilution effects are counterfactually large in this class of models—see Aguiar and Amador (2020) and chapter 7 in Aguiar and Amador (2021). Thus, we believe these implications ought to be taken with caution.

5.1.2 Spain

We now explore the calibration for Spain. Before the crisis, Spain was in the high-growth regime, and the spread was essentially zero. Following the 2009-10 recession, growth was dismal, which we interpret as the low-growth regime. Spreads rose steadily past 6% by July 2012, when the ECB announced a policy intervention. Spreads then fell consistently and reached 1% by 2014.

In Figure 8, we plot the interest rate schedules for both the low- and high-growth states. In contrast to Argentina, we don't have direct data on Spain's debt services due each year. However, we can impute debt service using data on short and long debt, interest payments, and the maturity structure.[27] For 2011 we obtain an average debt service relative to GDP of $b = 15\%$, the value we use to plot Spain's schedules. As discussed above, the Spain calibration was chosen to match a spread of essentially zero in the schedule corresponding to the high growth state.

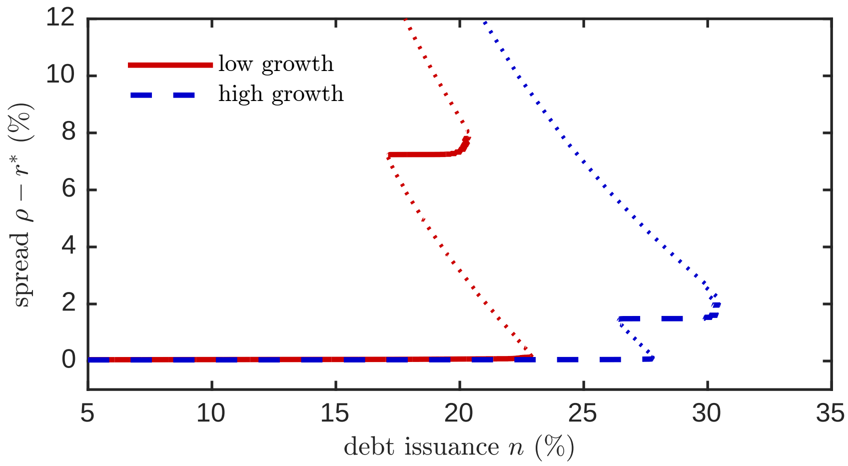
For the low-growth state, for debt issuance levels between 17% and 22% of output, interest rates can be either low or high, depending on the sunspot, as Figure 8 shows. When expectations are good, the borrower can issue new debt up to 23% of trend GDP under the lower interest rate spread. In contrast, when expectations are bad, the borrower can only issue up to 17% of trend GDP under the lower spreads. This means that, given a debt service of 15%, there is multiplicity of spreads for deficits of 2% to 8%. The deficits Spain experienced in the years before the debt crisis belong to those intervals.

Figure 8 supports a narrative regarding the debt crisis in Spain consistent with multiple equilibria. By 2009, Spain had entered the low-growth regime, so the relevant schedule is the red one in Figure 8. While expectations were good, Spain could run deficits with a low spread. Eventually, the bad sunspot realized and spreads jumped up. As Figure 8 shows, the difference between the high and low interest rate spreads in the multiplicity

[26] See Chatterjee and Eyigungor (2012) and Aguiar et al. (2019).

[27] See Appendix B for details.

Figure 8: Interest rate correspondence for Spain ($b = 15\%$)



region—which has not been calibrated—is about 7%. This difference in spreads is very similar to the maximum spread observed in Spain (slightly above 6%).

The narrative for Spain is only partially confirmed by the analysis of the simulated economy, as we discuss below. Endogenous austerity plays a key role in avoiding the high spreads, so that the economy does not borrow at expectations-driven high rates in equilibrium.

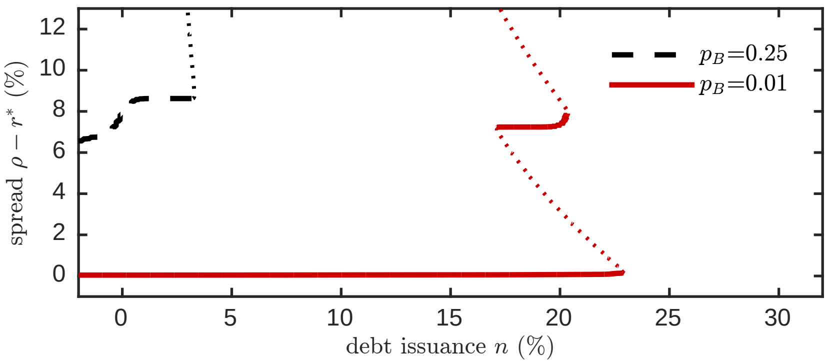
The role of the sunspot We now explore the effect of increasing the sunspot probability from $p_B = 1\%$ to $p_B = 25\%$, as in the benchmark for Argentina. Figure 9a plots the schedule under the low-growth state for both values of p_B.

As with Argentina, the value of p_B substantially changes the interest rate schedules. With more pessimistic expectations, spreads increase sharply, and Spain's capacity to borrow is substantially reduced. We show this change in schedules carries its effect on equilibrium outcomes: had Spain faced the same value of p_B as Argentina, spreads and default rates would have been close to Argentina's.

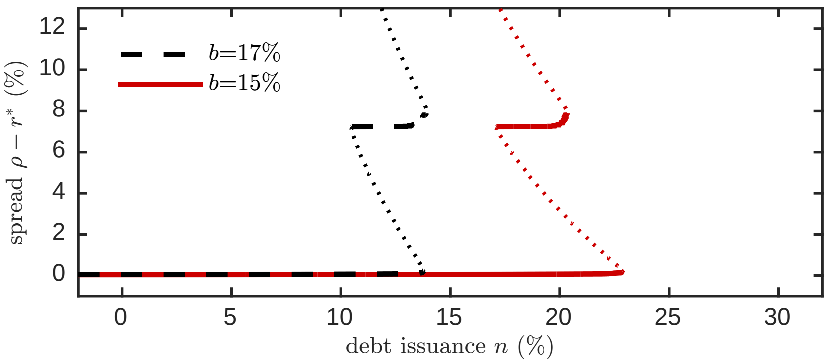
The role of the inherited debt The comparative statics with respect to the initial value of b is similar to the ones in the case of Argentina: the schedule shifts to the left as the debt services b increase. However, we think it's interesting to consider in detail the counterfactual in which, rather than starting with a debt services level of $b = 15\%$, as in 2012, Spain started with a debt services level of $b = 17\%$, as in the ergodic mean of the model (see Table 4 below). Thus, Figure 9b shows the low-growth schedule for a debt service of $b = 17\%$, together with the benchmark of $b = 15\%$ in Figure 8. For the case of $b = 17\%$, the region of multiplicity occurs for levels of debt issuance between $n = 11\%$ and $n = 15\%$ of output, which would involve current account surpluses, since 17% of output must be rolled over. These numbers imply that because the debt obligations were

Figure 9: Interest rate spreads for Spain in the low-growth state ($b = 15\%$): Comparative statics

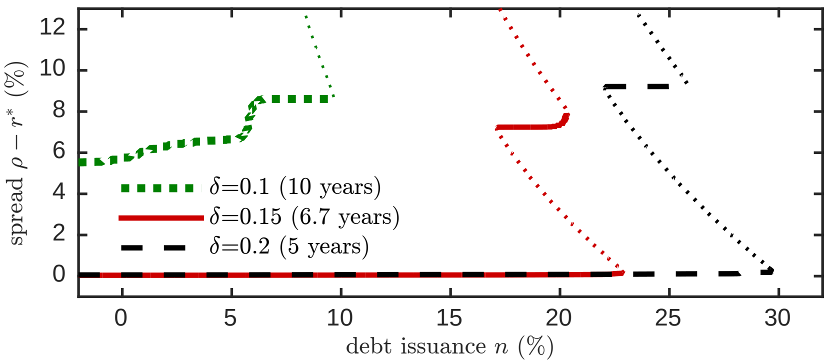
(a) The role of expectation p_B



(b) Inherited debt b



(c) Maturity δ ($b/\delta = 100\%$)



lower than the average of the invariant distribution, Spain managed to handle the crisis substantially better and run deficits while expectations were good.

The role of average maturity Figure 9c reports the schedules for different levels of debt maturity, δ. The intuition of the results is the same as before: shorter maturity ameliorates the debt-dilution problem and moves the multiplicity region further to the right.

5.2 The simulated economies

We now show the moments of the simulated economy for both calibrations, and discuss how they change as we vary the probability of the sunspot.[28]

Argentina Table 3 shows the moments for the simulated economy for Argentina. The first block lists unconditional first moments. The second and third blocks show first moments conditional on the growth state. Finally, the fourth block lists unconditional second moments. The columns represent two different parameterizations of the economy. The first column shows the moments for the benchmark calibration, and the second column shows the effect of reducing the probability of the bad sunspot to $p_B = 1\%$. All moments are computed during periods where the country is not in default.

The main takeaway from Table 3 is that the probability of the sunspot plays a crucial role in driving up average spreads, particularly in the low-growth state, and affects borrowing choices. As the sunspot probability goes down from 25% to 1%, default rates decrease from 5.3% to 0.5%, while spreads decrease from 16.2% to 0.3%. That is, without any changes in fundamentals, only in beliefs, a borrower mutates from a serial defaulter to virtually a non-defaulter.

Endogenous austerity and gambling for redemption play a key role in producing the simulated moments for Argentina. Both can be observed in the behavior of the policy function for new debt issuance (Figure 10, top panel) and the corresponding spreads (Figure 10, bottom panel). The left column shows results for the low-growth state, while the right panel shows results for the high-growth state.

Endogenous austerity is the reason why new debt issuance, n, declines as the outstanding debt services, b, increase. Even if the outstanding debt service is higher, the borrower reduces new issuance in order to avoid discrete jumps in interest rates, thus cutting down consumption sharply. These interest rate jumps depend on future debt services and therefore occur for lower levels of n as b increases—see Argentina's schedule

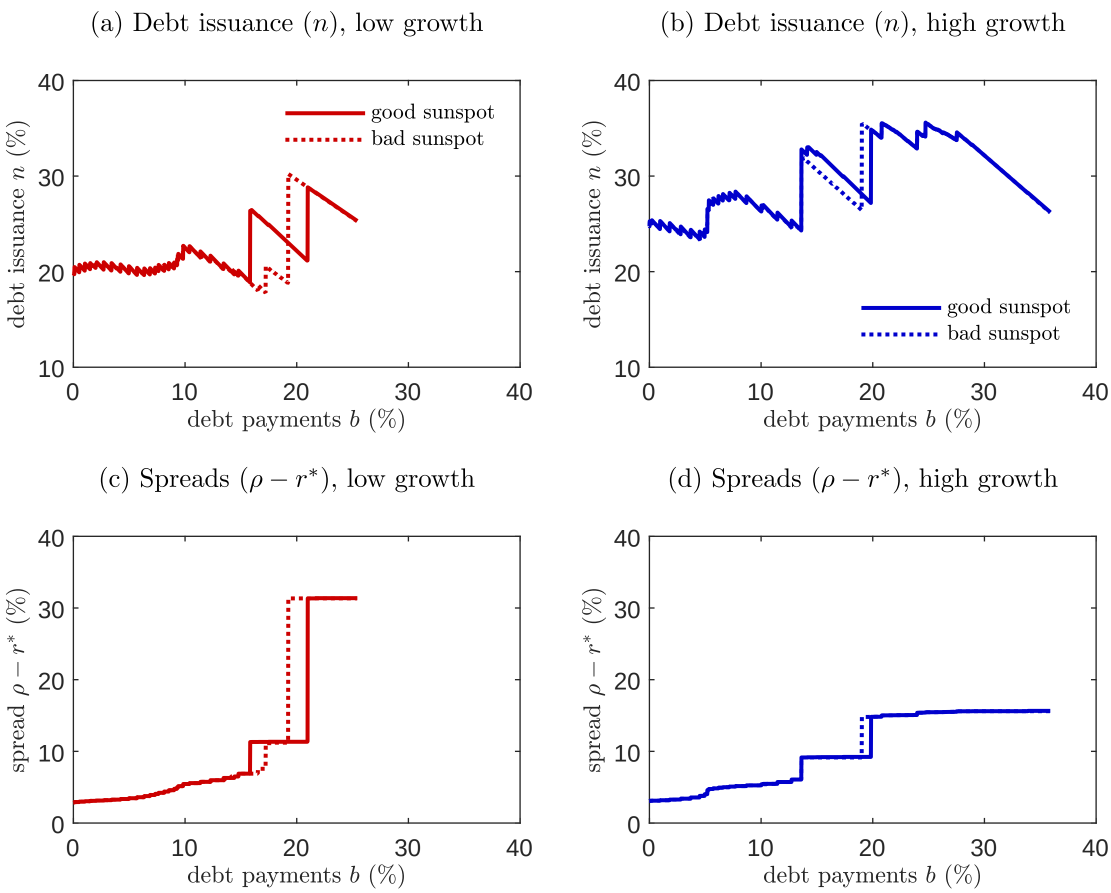
[28] This section reports outcomes across the ergodic distribution of the model. Appendices report outcomes conditional on a crisis episode. In particular, Appendix F.3 computes the model response to a bad sunspot realization and relates results to this section. Appendix F.4 compares crisis episodes in the model with the evidence in Paluszynski and Stefanidis (2023).

Table 3: Simulation moments: Argentina

	Benchmark ($p_B = 25\%$)	$p_B = 1\%$
First moments (%)		
avg($spread$)	16.2	0.3
avg(qb/y)	56	76
avg(f/y)	53	73
avg(n/y)	27	30
avg(b/y)	31	32
avg(tb/y)	4.4	2.1
default rate	5.3	0.5
Low-growth state		
avg($spread$)	27.6	0.3
avg(qb/y)	28	79
avg(f/y)	41	77
avg(n/y)	26	28
avg(b/y)	24	34
avg(tb/y)	-2	5.9
default rate	13.6	1.3
High-growth state		
avg($spread$)	15.5	0.2
avg(qb/y)	58	73
avg(f/y)	54	71
avg(n/y)	27	31
avg(b/y)	32	31
avg(tb/y)	4.8	-0.3
default rate	0	0
Second moments		
corr($spreads, y$)	-0.52	-0.29
std($spreads$) - p.p.	3.7	0.1
std(c)/std(y) - p.p.	2.4	1.6

Note: b denotes total debt service, qb denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

Figure 10: Policy functions and equilibrium spreads for Argentina



in Figure 7b. Gambling for redemption, instead, is the mechanism behind the sharp increases in new issuance n. The borrower chooses to further increase issuance, even at higher borrowing costs, in order to increase consumption.

The simulations for Argentina in the benchmark calibration ($p_B = 25\%$) visit the region of debt service levels where spreads are high because of expectations. Both endogenous austerity and gambling for redemption are at play there, resulting in high equilibrium spreads and default rates, in particular in periods of low growth.

Expectations play a key role: with more optimistic expectations ($p_B = 1\%$), endogenous austerity would have prevailed for Argentina, inducing low spreads and default probabilities, as the second column of Table 3 shows.

The sunspot also significantly affects second moments, as the last rows in Table 3 show. In particular, as p_B moves from 25% to 1%, the volatility of spreads decreases by a factor of 40, while the volatility of consumption decreases by one-third.

Spain The moments for the simulated economy for Spain are in Table 4, which is the analog of Table 3 for Argentina, for the optimistic benchmark ($p_B = 1\%$) and for the case of pessimistic expectations ($p_B = 25\%$).

In the low-growth state, the average debt service b is around 16% of GDP. Just rolling over the debt implies borrowing in the multiplicity region, as Figure 9b shows. Yet, as Table 4 also shows, spreads in the low-growth state are essentially zero, while default rates are high. The reason is that more than 99.3% of default episodes occur in high-growth to low-growth transitions. While in the low-growth state, endogenous austerity prevails, and spreads are low.

This can be seen in Figure 11, which shows debt issuance policies and the corresponding equilibrium spreads. The debt issuance policy function in the low (high) growth state is depicted in solid red (blue) for the good sunspot and dashed red (blue) for the bad sunspot. The bottom row in Figure 11 shows the corresponding equilibrium spreads. As discussed before, endogenous austerity is the reason why issuance, n, declines as debt service, b, increase.

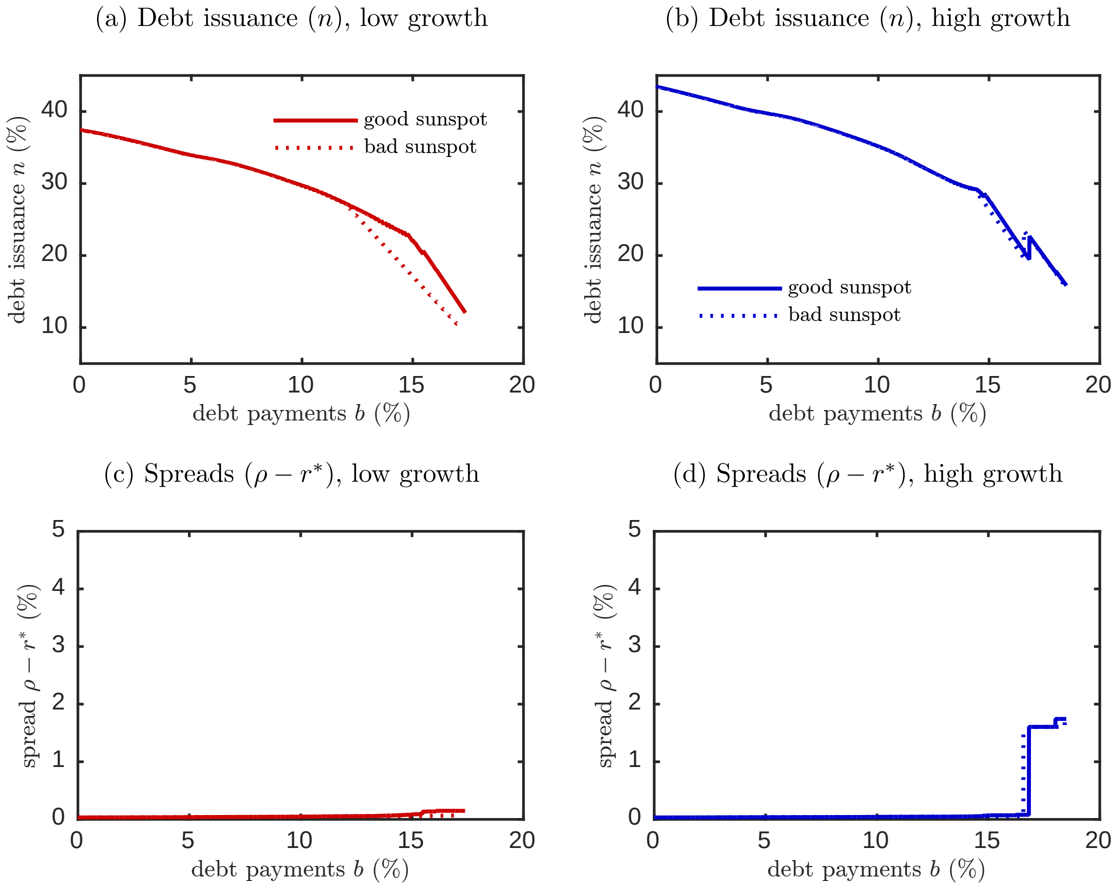
In the low-growth state, the policy function exhibits strong endogenous austerity, with issuance declining as debt service increases. The realization of the bad sunspot induces even stronger endogenous austerity. Overall, default rates are close to zero under low growth, and thus equilibrium spreads are low.

All this implies that while just rolling over debt service in the low-growth state leads to borrowing in the multiplicity region, that never happens in equilibrium. Endogenous austerity keeps the economy below the multiplicity region so that high spreads due to bad expectations do not happen in equilibrium in the low-growth state. Thus, while there is multiplicity of spreads for the debt service levels Spain faced in 2012 (Figure 8), the simulated economy does not give rise to those high spreads.

The actual experience of Spain, and other European countries, was to implement austerity measures in response to the observed high spreads. In the model, the high spreads triggering endogenous austerity happen off equilibrium, so they are not observed in the simulations. Thus, our model is only partially successful in explaining the events in those countries during the sovereign debt crisis period.

We conjecture that with reasonable changes to our model, Calvo-type multiplicity could still explain the jump in Spain's spreads in 2012. In particular, as we discuss in Section 6, the ECB announcement that it would intervene in sovereign debt markets was a significant policy change taken during the summer of 2012, and market intervention was far from granted the years before that. Thus, a time varying value for p_B could more accurately capture the historical likelihood of an ECB intervention. Stochastic changes in p_B, as in Bocola and Dovis (2019), may induce a rise in spreads as a result of expectations becoming more pessimistic over time. That is, a gradual increase in p_B could lead to spreads increasing to almost 7%, as observed for Spain during July 2012. We opt for a simpler model with fewer parameters and thus a time-invariant low sunspot probability.

Figure 11: Policy functions and equilibrium spreads for Spain



Sunspot probability and behavioral responses The probability of the bad sunspot, p_B, plays a crucial role in shaping the importance of endogenous austerity and gambling for redemption behaviors. Think of a borrower who happens to be in the region of multiplicity. If the bad sunspot occurs, the borrower has two options: strongly cut down issuance to avoid the high interest rates (endogenous austerity), or keep/increase issuance at higher interest rates in the hopes of high growth in the near future (gambling for redemption). The cost of endogenous austerity is the strong decline in consumption, while the benefit is to keep financial market access. The benefit of financial market access is particularly appealing if the expectations-driven high rates are a rare event. That is, endogenous austerity is more appealing when p_B is low. If the borrower believes instead that markets will often feature expectations-driven high rates, financial market access is not as appealing. That is, gambling for redemption is more appealing when p_B is high.

Our calibration for Spain assumes a low p_B, and thus Spain features endogenous austerity, while the calibration for Argentina assumes a high p_B, and thus Argentina exhibits gambling for redemption.

Table 4: Simulation moments: Spain

	Benchmark ($p_B = 1\%$)	$p_B = 25\%$
avg($spread$)	1.2	7.7
avg(qb/y)	88	53
avg(f/y)	86	52
avg(n/y)	18	12
avg(b/y)	17	14
avg(tb/y)	-1.4	1.6
default rate	5.2	6.1
Low-growth state		
avg($spread$)	0.1	9.7
avg(qb/y)	80	35
avg(f/y)	84	47
avg(n/y)	17	12
avg(b/y)	16	12
avg(tb/y)	-0.9	0.5
default rate	18	20
High-growth state		
avg($spread$)	1.4	7.5
avg(qb/y)	89	55
avg(f/y)	86	53
avg(n/y)	18	12
avg(b/y)	17	14
avg(tb/y)	-1.5	1.8
default rate	0	0
Second moments		
corr($spreads, y$)	0.46	-0.34
std($spreads$) - p.p.	0.7	1.5
std(c)/std(y) - p.p.	2.7	2.3

Note: b denotes total debt service, qb denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

Expectations and the role of policy As Section 2.1 discusses, in the presence of a lender of last resort, the effective probability of facing the high-rate schedule is $p_B \times (1-\pi)$, where π is the probability of a policy intervention. In that sense, a lower p_B can be understood as a higher probability of an intervention, π. Our results show that a higher probability of intervention has large effects on credit spreads and default rates, not only during a debt crisis but also on the ergodic distribution of the model. Indeed, for both Spain's and Argentina's calibrations, if p_B is low (probability of intervention is high) the spreads are virtually zero, while if p_B is high (probability of intervention is low) the spreads are high. Appendix F.1 quantifies the potential gains of a policy intervention by computing a Pareto frontier between lenders' and borrowers' utility for different sunspot realizations.

5.3 An event case study: Argentina's 2001 default

We now test the quantitative capacity of the model to account for the events leading up to Argentina's default in 2001.[29] The exercise is in the spirit of the ones in Arellano (2008) and Chatterjee and Eyigungor (2012). In particular, we ask whether the model can account for the path of interest rate spreads during the 1997–2001 period, and quantify the relevance of the sunspot. As we show, the model can account for the path of Argentina's spreads, and the sunspot plays a key role: absent a bad sunspot realization, Argentina would have avoided default.

We design the event case study as follows. We simulate the model for years 1997–2001 by selecting the output shocks (g_t and ϵ_t) to match observed Argentina's GDP growth. We assume Argentina was in a high-growth state during 1997–1998 and switched to the low-growth state during 1999–2001.[30] We pick the initial debt service level so that the debt service is close to 20.2% in 2000, as was the case for Argentina in that year. We then consider two possible cases for the sunspot realization: a "bad sunspot" case, in which the sunspot selects the high interest rate schedule in all years; and a "good sunspot" case, in which the sunspot selects the high interest rate schedule all years *except* in 2001. That is, the "bad sunspot" and the "good sunspot" cases differ only in their 2001 sunspot realization.[31] Figure 12 shows the model-implied interest rate spreads for both cases and compares them with Argentina's actual credit spreads.

There are two key findings in this exercise. First, for the years 1997–2000, the model-implied credit spreads are aligned with the data, an outcome that is not directly targeted by our exercise design. Second, the model accounts for the spike in spreads in 2001, but

[29] We are thankful to the referees that suggested this exercise.

[30] We set the sequence of growth shocks g_t as the most likely state implied by the Kim (1994) filter we used to estimate the output process. See Appendix E for more details on the computations.

[31] The economy is not in a multiplicity region during 1998-1999, and thus the sunspot is not relevant for the initial years.

only in the "bad sunspot" case. In contrast, the "good sunspot" case generates generates substantially lower spreads. Importantly, the model predicts default in the year 2002 only under the "bad sunspot" case, but not under the "good sunspot" case.[32] Furthermore, since Argentina experienced high growth starting in 2003, the model predicts no default in 2003 and in the high-growth years that followed. In this sense, the role of expectations was essential: Argentina would have avoided default altogether under more favorable expectations.

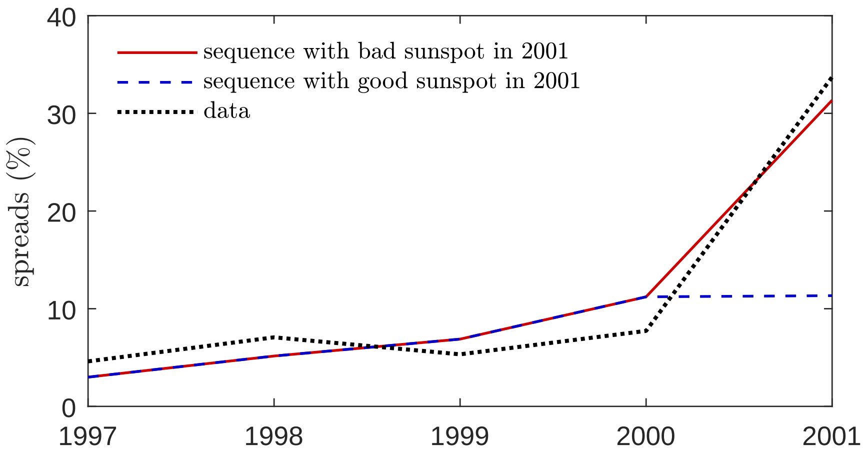
The exercise in this section is analogous to the one in Figure 3 of Chatterjee and Eyigungor (2012), which can also match the path of Argentina's output and spreads in the period before 2001.[33] Like in their case, our quantitative model relies on low output realizations (low-growth shocks in our case) to account for the interest rate spreads during Argentina's debt crisis. Unlike their case, the low output realizations are not enough in our exercise, and bad sunspot shocks are needed to generate the spike in spreads and the subsequent default. The difference in results reflects modeling differences such as the stochastic process for output and the assumptions on debt recovery. The results may also differ because of the choice of unobserved parameters to match credit spreads, especially those related to default costs. These may be masking the role of expectations, which are not allowed to play a role in their analysis.

The implications for policy are probably the most important difference between our analysis and the one in Chatterjee and Eyigungor (2012). As discussed in Section 2.1, a lender of last resort can coordinate expectations and eliminate the effect of the sunspot on spreads and default rates. Our analysis suggests that Argentina's 2001 default could have been avoided with an intervention of a lender of last resort that would coordinate the behavior of lenders in the low interest rate schedule. By contrast, in sovereign default models with no role for expectations, the presence of a lender of last resort has no effect on equilibrium spreads, as long as loans are correctly priced according to the default probabilities (so that the creditor breaks even). Relative to those models without a role for expectations, an advantage of our analysis is that we can match effects of policy intervention, while previous work could achieve this only by recalibrating the model parameters, such as changing default costs. To put it differently, by assuming away the possibility of multiple equilibria, the standard approach in the literature may be confounding exogenous default costs with policy interventions. As we discuss next in Section 6, support by a lender of last resort was a key difference between the experiences of Spain in 2012 and Argentina in 2001.

[32] Argentina defaulted on December 26th, 2001. In the model, default occurs in 2002 under the bad-sunspot case.

[33] See also Figure 5 in Arellano (2008) for a similar exercise.

Figure 12: Argentina: Spreads from 1997 to 2001



Note: End-of-period spreads. For 2001, we use spreads in the end of November. Model predicts default in 2002 for the bad sunspot sequence, but no default under the good sunspot sequence.

6 Policy and expectations: A historical discussion

We have argued that expectations can be a major driver in explaining default rates and credit spreads in sovereign debt crises, especially when growth rates are persistently low and debt service levels are moderately high. We have also established a theoretical link between expectations and the policy of a lender of last resort, which can help discipline the quantitative analysis of the role of expectations. In this section, we review the historical roles of the ECB during the European sovereign debt crisis and the IMF during the crisis in Argentina. We argue that, while the ECB eventually decided to act as a full fledged lender of last resort, the IMF was constrained—by its own rules and recent experience—in providing Argentina with enough financing to rule out undesirable outcomes. This discussion justifies our choice of a low probability of the bad sunspot for Spain ($p_B = 1\%$) and a relatively high probability for Argentina ($p_B = 25\%$) in our benchmark calibration. In what follows, we briefly describe the history of these two relationships—the one between Spain and the ECB and the one between Argentina and the IMF—in the years before to the two crises.

Spain and the ECB The founding treaties of the European Monetary Union were clear in their objective to limit the ability of the ECB to act as a lender of last resort in the sovereign debt markets of its member states. For example, Article 123 of the Treaty on the Functioning of the European Union specifically prohibits direct purchases of sovereign bonds by the ECB.[34] In spite of this limitation, spreads on sovereign debt of those member

[34] See Cochrane et al. (2024) for a thorough discussion of the evolution of the role of the ECB in preventing banking and sovereign debt crises.

countries were virtually zero for almost ten years following the introduction of the single currency in 1999. The increase in spreads in the early 2010s raised the question of whether those limits on the actions of the ECB were appropriate. The answer was a clear no, framed in Mario Draghi's "*whatever it takes*" speech of July 26th of 2012: "*Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.*" The intentions of the president of the ECB were supported by the launching of the OMT bond buying program a couple of months later.

An important argument in favor of a decisive role of the ECB in managing expectations through the euro-area sovereign debt crisis is the apparent impact of Draghi's speech. The speech coincided with the immediate reversion of the increasing trend of spreads. The fact that spreads declined after the policy announcement, and that the actual bond-buying scheme was not used, suggests that the policy announcement was enough to coordinate expectations in the good equilibrium.[35]

Mario Draghi's speech of July 2012 indeed represented a radical change in the way the ECB would intervene in sovereign-bond markets. Even though Draghi starts his magical "*whatever it takes*" sentence by asserting that the implied policies are within the ECB's mandate, whether they were was far from clear in the years preceding the summer of 2012. According to Paul De Grauwe, the ECB was reluctant to intervene in sovereign debt markets, even as rising spreads and likely default were threatening the monetary union:

> *The ECB has made it clear that it does not want to pursue its role of lender of last resort in the government bond market. This has forced the Eurozone members to create a surrogate institution (the European Financial Stability Facility or EFSF and the future European Stability mechanism or ESM). The problem with that institution is that it will never have the necessary credibility to stop the forces of contagion; it cannot guarantee that the cash will always be available to pay out sovereign bondholders.* (Paul De Grauwe, August 18, 2011, VOX column.[36])

The main opposition to having the ECB intervene in those markets came from Germany. Allegedly, during 2011 and the first half of 2012, the issue was addressed in several G7 and G20 meetings. In those meetings, Mario Monti, at that time prime minister of Italy, had made the proposal to have the ECB intervene, but it was rejected by Germany's Chancellor Angela Merkel. This position changed over the summer of 2012 as

[35] A notable article on the effects of Draghi's speech was published in the *Economist* in February 2021: "*The bond-buying scheme the ECB assembled to render Mr Draghi's promise credible was never used: his words were enough to calm the financial furies.*" The full article can be found here: https://www.economist.com/europe/2021/02/18/mario-draghi-begins-the-toughest-job-in-european-politics.

[36] Paul De Grauwe's VOX column can be found here: https://cepr.org/voxeu/columns/european-central-bank-lender-last-resort.

the European debt crisis unfolded.[37]

Given the sequence of events, it seems clear that Draghi's speech was meant to convince the markets that there had been a shift in the way the ECB was willing to intervene. The discussion suggests that before July of 2012, it was reasonable to believe that the probability of an intervention by the ECB, large enough to rule out bad outcomes, was well below one. After Draghi's speech and Merkel's confirmation a few weeks after the announcement, that probability became essentially one.

In the notation of Section 2.1, for any given probability of the bad sunspot realization in Spain, p_B, the events of the summer of 2012 made it clear that the probability of the intervention, π, was close to one. Thus, the probability of the high interest rate schedule occurring, $p_B(1-\pi)$, became very close to zero.

It is tempting to see the smooth increase and subsequent decrease of spreads as driven by time varying market perceptions of the probability of an ECB intervention, π. However, in the spirit of keeping parameters to a minimum, we refrained from following that route. Instead, given that by the summer of 2012 it became clear that the ECB was prepared to"*do whatever it takes*", and that Draghi insisted that "*believe me, it will be enough*," we choose to set the probability of the sunspot in the benchmark calibration for Spain to a negligible number, $p_B = 1\%$.

Argentina and the IMF The relationship between the IMF and Argentina has been brilliantly analyzed by Michael Mussa in his 2002 book *Argentina and the Fund: From Triumph to Tragedy* (Mussa, 2002). A notable feature of Argentina's default is that the ratio of debt to output was lower than 50% by the end of 2000. Regarding this, Mussa notes (page 16):"*it may be asked why a debt-to-GDP ratio of just above 40% was worrying for Argentina.*" In addressing this question, Mussa uses arguments that are close to the mechanism in our paper. He mentions:

> *Argentina was clearly vulnerable to changes in financial market sentiment. . . . If market sentiment ever shifted to an expectation of significant risk that Argentina might default, its market access would be cut off and that expectation would soon become self-fulfilling.* (Page 16)

Spreads on Argentinean debt during the year 2000 were close to 600 basis points. By mid-2001, the spreads were getting closer to 1500 basis points. The impact of these 900

[37] Peter Spiegel, managing editor for the *Financial Times*, reviews the historical circumstances that led to the shift in ECB policy in his article on May 15th, 2014. Spiegel explains, "*Mr Draghi's programme was unlikely to have quelled markets without Ms Merkel's acquiescence, which was given despite the public objections of the powerful German Bundesbank (with two members associated with the Bundesbank resigning from the ECB over the issue). . . . Ms Merkel's willingness to back OMT was a reflection of how deep the crisis had grown that summer. . . . Ms Merkel's ultimate embrace of Mr Draghi's programme capped a year-long shift of thinking in Berlin.*" See full article here: https://www.ft.com/content/b4e2e140-d9c3-11e3-920f-00144feabdc0 .

additional basis points on the service of the debt was gigantic. Calculations of this type are reminiscent of the coordination problems leading to multiple equilibria developed by Calvo (1988) and endorsed in our quantitative model.

Argentina had agreements with the IMF during most of the years between 1991 and 2001. However, the IMF's ability to do "*whatever it takes*" was severely limited by its own rules, and also by the previous programs that the IMF had signed with other countries. In particular, the IMF was under heavy criticism after two recent large programs, one with Russia and another one with Indonesia, had ended in major crises. This is explicitly mentioned by Mussa:

> *Another approach would have been a very big bailout that would have provided the Argentine government with guarantees of official support sufficient to cover its prospective financing requirements for several years. However, this would have meant a substantial escalation in the magnitude and duration of official financing packages, and there was no support for this among the major countries that provide the Fund resources.* (Page 35)

Mussa is referring to what became known as the Prague 2000 communiqué of the Fund's International Monetary and Finance committee, which discussed the late 90s large packages.[38] Bullet 21 of the communiqué mentions that "*the Committee notes that Fund resources are limited and that extraordinary access should be exceptional; further, neither creditors nor debtors should expect to be protected from adverse outcomes by official action.*" Such was the reaction to the criticism that the Fund faced after the already failed large programs in Russia and Indonesia.

In discussing the renewal and possible augmentation of the program by late 2000, Mussa mentions that an option that was then considered–and not adopted–was to terminate the program. He writes, "*some in the official community appeared to favor this approach. ... This included those who were deeply opposed to large packages of official support.*" (Page 30)

This discussion shows that the IMF was in no position to intervene in Argentina, in the way the ECB was ready to intervene in European bond markets. For this reason, we feel comfortable calibrating the probability of the bad sunspot, p_B, to 25% for Argentina.

7 Conclusion

In the model of sovereign debt crises of Calvo (1988), there are multiple interest rate schedules because expectations of high probabilities of default are self-confirming. In particular, if expectations of default are high, interest rates must be high, and high interest rates increase the burden of debt, inducing the borrower to default. The question

[38] The communiqué is available here: https://www.imf.org/en/News/Articles/2015/09/28/04/51/cm092400

that remains is whether the source of multiplicity is quantitatively relevant. In particular, we are interested in determining the role that it may have played in the sovereign debt crises of Argentina in 2001 and southern Europe in the early 2010s.

We argue that the mechanism in Calvo (1988) is quantitatively relevant and that key for multiplicity is a bimodal output process with persistent good and bad times. We estimate this output process for a set of countries that have recently been exposed to sovereign debt crises. We show that a sunspot realization can induce discrete jumps in interest rates even with no change in fundamentals. These expectations-driven jumps in interest rates can occur only during stagnations. Interest rate jumps, either because of expectations or because of fundamentals, can induce either endogenous austerity, in which the borrower refrains from borrowing to avoid the jump in rates, or gambling for redemption, in which the borrower increases debt beyond the jump in rates.

We consider two calibrations of the model, one targeted to Argentina and another to Spain. We show that the Argentine calibration generates equilibrium paths that resemble the events of the 2001 debt crisis, with credit spreads going up by magnitudes similar to the observed ones. In the calibration for Spain, expectations-driven high rates induce austerity measures resembling those observed in the early 2010s. As a result of this endogenous austerity, the high spreads remain off equilibrium for Spain. Endogenous austerity and gambling for redemption are central in explaining our findings.

A key takeaway of our paper is that expectations, as measured by the sunspot probability of selecting an interest rate schedule, have a large quantitative effect on model outcomes. Assuming optimistic expectations for Argentina results in substantially lower default rates and credit spreads, even with no change in fundamentals. Similarly, assuming pessimistic expectations for Spain results in substantially higher default rates and credit spreads. Thus, expectations are a major driver explaining default rates and credit spread differences between Spain and Argentina.

We argue that policy plays a key role in shaping expectations. A lender of last resort can eliminate expectations-driven debt crises, as the ECB did for Spain and the IMF did not do for Argentina. This key interaction between policy and expectations opens the question of how to optimally design policy interventions within the boundaries of institutional limitations. We leave this question for future research.

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A Endowment process estimation

In this appendix, we provide more details on the estimation of the GDP process used in the quantitative evaluation of the model. We use the filter in Kim (1994) to obtain the likelihood function and explore the posterior distribution using a Metropolis-Hastings MCMC algorithm with a random walk proposal density.

Let Y_t denote the country's GDP during year t, and let $\Delta y_t = \log(Y_t) - \log(Y_{t-1})$ denote GDP growth. The process in equations (7)–(8) implies

$$\Delta y_t = g_t + \sigma\left(\epsilon_t - \epsilon_{t-1}\right), \tag{A.1}$$

where $\epsilon_t \sim \mathcal{N}(0,1)$, and g_t follows a two-state Markov process with transition probabilities $p_g(g'|g)$. Denote g_L and g_H to the possible values of g: $g_t \in \{g_L, g_H\}$. The transition probability is fully summarized by the two parameters p_L and p_H, where $p_L = p_g\left(g_{t+1} = g_L | g_t = g_L\right)$ and $p_H = p_g\left(g_{t+1} = g_H | g_{it} = g_H\right)$.

Let $\theta = \{g_L, g_H, \sigma, p_L, p_H\}$ collect all parameters determining the process in equation (A.1). Let $\mathcal{Y}_t = \{\Delta y_0, \Delta y_1, \ldots, \Delta y_t\}$ collect all observations up to period t and $\mathcal{L}(\theta|\mathcal{Y}_T)$ be the likelihood of parameters θ, where T is the total number of observations. We construct the likelihood $\mathcal{L}(\theta|\mathcal{Y}_T)$ using the filter in Kim (1994).

We assume uniform priors on parameters, which bounds the possible space for θ (see Table 1). Additionally, we include the normalization of $g_L \leq g_H$ as part of our priors. Let $p(\theta)$ denote the prior selection. The posterior distribuion of parameters θ is then given by

$$\mathcal{P}(\theta|\mathcal{Y}_T) = \mathcal{L}(\theta|Y_T)p(\theta). \tag{A.2}$$

We explore the posterior $\mathcal{P}(\theta|\mathcal{Y}_T)$ using a Metropolis-Hastings MCMC algorithm with a random walk as a proposal density. In particular, the proposal density is a normal $\mathcal{N}\left(\theta_{n-1}, \bar{\sigma}\Sigma_\theta\right)$, where θ_{n-1} is the last draw of the chain, Σ_θ has $\theta^* = \arg\max_\theta \mathcal{P}\left(\theta|\mathcal{Y}_T\right)$ in its diagonal and zeros otherwise, and $\bar{\sigma}$ is selected so that the rejection rate in the chain is between 60% and 70%. We simulate 10 chains, each of length 125,000, and compute posteriors by pooling 1 out of every 10 draws from the last 100,000 observations in each chain. Table 1 contains all estimates.

Data are from the Conference Board Total Economy Database, and we used GDP per capita in 2016 U.S. dollars (converted to 2016 price level with updated 2011 PPPs) as our measure of output.

Different standard deviations across states We also estimate the endowment process for the case in which the standard deviation of the temporary shock depends on whether the economy is at the low-growth state, $g_t = g_L$, or the high-growth state,

Table A.1: Prior and posterior distributions: different σ across growth states

	$\ln(g_L)$	$\ln(g_H)$	p_L	p_H	σ_L	σ_H
	Prior distribution					
	$U[-0.1,0.1]$	$U[-0.1,0.1]$	$U[0.1,1.0]$	$U[0.1,1.0]$	$U[10^{-3},0.5]$	$U[10^{-3},0.5]$
Countries	**Posterior distribution** (mean, and 5th to 95th percentile intervals)					
Italy	-0.014	0.026	0.654	0.766	0.020	0.011
	[-0.022,-0.008]	[0.019,0.030]	[0.356,0.974]	[0.523,0.990]	[0.013,0.031]	[0.006,0.020]
Portugal	-0.002	0.048	0.790	0.703	0.020	0.018
	[-0.011,0.003]	[0.040,0.058]	[0.503,0.976]	[0.403,0.990]	[0.012,0.029]	[0.011,0.029]
Spain	-0.017	0.035	0.608	0.810	0.019	0.015
	[-0.025,-0.009]	[0.027,0.040]	[0.325,0.866]	[0.618,0.978]	[0.009,0.031]	[0.009,0.025]
Argentina	-0.027	0.072	0.776	0.570	0.046	0.013
	[-0.043,-0.023]	[0.057,0.076]	[0.562,0.916]	[0.344,0.783]	[0.034,0.075]	[0.008,0.024]
Brazil	-0.032	0.029	0.605	0.790	0.022	0.019
	[-0.071,-0.019]	[0.026,0.032]	[0.059,0.874]	[0.619,0.926]	[0.011,0.063]	[0.014,0.026]

Note: For each country, we estimate an output process as: $\Delta \ln y_t = \ln(g_t) + \sigma_t \epsilon_t - \sigma_{t-1}\epsilon_{t-1}$, in which $\epsilon_t \sim N(0,1)$ and $g_t \in \{g_L, g_H\}$, with $\Pr(g_{t+1} = g_L | g_t = g_L) = p_L$ and $\Pr(g_{t+1} = g_H | g_t = g_H) = p_H$. If $g_t = g_L$, then $\sigma_t = \sigma_L$, otherwise $\sigma_t = \sigma_H$. The table reports the mean and the interval between the 5th and 95th percentiles of the posterior distributions of each of the parameters for each country. The table also reports the prior distributions we used, which were chosen to be the same across countries. For each country, we use data on GDP per capita in 2016 US$ (converted to 2016 price level with updated 2011 PPPs) between 1980 and 2017 from the Conference Board Total Economy Database as the measure of y_t.

$g_t = g_H$. We denote the standard deviations in the low- and high-growth states by σ_L and σ_H, respectively. Table A.1 shows the estimates for all countries. First, note that the standard deviation posterior mean is similar across growth states, except for the case of Argentina, and that the mean of σ_L is larger than the mean of σ_H in all cases. In the case of Italy, the difference between σ is somewhat larger, but the values are still within the interval analyzed in Figures F.7c and F.8c, which are shown to not have significant implications for our results. Finally, Table A.1 also shows that the posterior distributions of the remaining parameters are very similar to their counterparts in Table 1.

B Data appendix

In this appendix, we describe the data sources we used to compute the series of debt levels and debt services for Argentina and Spain.

Argentina:

Debt level We use the data on gross external debt reported by the World Bank's International Debt Statistics (series code DT.DOD.DECT.GN.ZS). This measure corresponds to the principal of bonds issuance, at face value, summed over all outstanding bonds.

Debt service Debt service s computed as the sum of external short-term debt in the previous year and current debt service on external long-term debt. Data on short-term debt (series code DT.DOD.DSTC.ZS) and on debt service on external long-term debt (series code DT.TDS.DECT.CD) are from the World Bank's International Debt Statistics.

Data from the World Bank's International Debt Statistics are expressed as a percentage of GNI. We convert them to percentage of GDP using the series of the ratio of GNP to GDP for Argentina from the University of Pennsylvania, retrieved from the Federal Reserve Bank of St. Louis's FRED database (series code GNPGDPARA156NUPN).

Spain:

Debt level We use the Net External Debt reported by the Banco de España (series code BE_17_ 30.8).

Debt service Data on debt service for Spain are not readily available. We impute debt services as the sum of: interest payments due in the year, short-term debt maturing in the year, and the fraction of long-term debt that matures in the year.

- *Interest payments*: We use data on net investment income from the balance of payments. It is computed as investment payments (series code BE_17_5A.7) minus investment income (series code BE_17_5A.6), both from the Banco de España.

- *Short-term debt*: We use data on net short-term debt related to portfolio income excluding the Banco de España. It is computed as the liabilities (series code BE_17_27.8) minus assets (series code BE_17_22.8), both from the Banco de España.

- *Long-term debt maturing*: We compute the stock of long-term debt as the debt level minus the short-term debt, both as described above. We multiply this amount by $\delta = 0.15$ to estimate the principal payments on long-term debt.

We do not have data on average maturity for the net external debt of Spain. In turn, we use $\delta = 0.15$, which is the average maturity of Spain's central government debt of 6 to 7 years, as reported by the Tesoro Público de España.

C Model normalization

In this appendix, we provide more details on the normalization step. First, we show that the value functions and the interest rate schedule satisfy certain homogeneity properties. Second, we use these homogeneity properties to derive the stationary system presented in Section 3.2. The algorithm for solving this stationary system is discussed in Appendix D.

We proceed by guess and verify. In particular, we guess that equilibrium functions satisfy certain homogeneity properties and then use the equilibrium equations to verify that our guess is consistent with the equilibrium definition. That is, we do not claim that all possible equilibria satisfy these properties, but rather that the equilibrium admits a solution with these properties.

Proposition 1 (Homogeneity of equilibrium functions)**.** *For any $\lambda > 0$, the equilibrium functions admit a solution that satisfies the following homogeneity properties: (i) $V^{nd}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda^{1-\gamma}V^{nd}(B, \Gamma_-, g, s, \epsilon)$ and $V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda^{1-\gamma}V^d(B, \Gamma_-, g, s, \epsilon)$; (ii) $\mathbf{B}'(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda\mathbf{B}'(B, \Gamma_-, g, s, \epsilon)$; (iii) $\mathcal{Q}(B, \lambda R, \Gamma_-, g, s) = \lambda\mathcal{Q}(B, R, \Gamma_-, g, s)$ and $\mathcal{X}(B, \lambda R, \Gamma_-, g, s) = \lambda\mathcal{X}(B, R, \Gamma_-, g, s)$; (iv) $\mathcal{Q}(\lambda B, R, \lambda\Gamma_-, g, s) = \mathcal{Q}(B, R, \Gamma_-, g, s)$ and $\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s) = \mathcal{X}(B, R, \Gamma_-, g, s)$; and (v) $\mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s) = \mathcal{R}(N, B, \Gamma_-, g, s)$.*

Proof. Let's start with (i). From equation (16), the value of default $V^d(\cdot)$ can be written as

$$\begin{aligned}
V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon) &= \frac{[\phi(g)g\lambda\Gamma_- e^{\sigma\epsilon}]^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}\Bigg[\theta V^d(\lambda B, g\lambda\Gamma_-, g', s', \epsilon') \\
&+ (1-\theta)\max\left\{\begin{matrix} V^{nd}(\kappa\lambda B, g\lambda\Gamma_-, g', s', \epsilon'), \\ V^d(\lambda B, g\lambda\Gamma_-, g', s', \epsilon') \end{matrix}\right\} |\Gamma_-, g, s\Bigg] \\
&= \lambda^{1-\gamma}\frac{[\phi(g)g\Gamma_- e^{\sigma\epsilon}]^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}\Bigg[\theta\lambda^{1-\gamma}V^d(B, g\Gamma_-, g', s', \epsilon') \\
&+ (1-\theta)\max\left\{\begin{matrix} \lambda^{1-\gamma}V^{nd}(\kappa B, g\Gamma_-, g', s', \epsilon'), \\ \lambda^{1-\gamma}V^d(B, g\Gamma_-, g', s', \epsilon') \end{matrix}\right\} |\Gamma_-, g, s\Bigg] \\
&= \lambda^{1-\gamma}V^d(B, \Gamma_-, g, s, \epsilon),
\end{aligned}$$

where the second equality uses the guess for $V^{nd}(\cdot)$ and $V^d(\cdot)$, and the last equality verifies the guess for $V^d(\cdot)$.

Next, we verify the homogeneity property for the value of no default, $V^{nd}(\cdot)$. For this purpose, it's convenient to express the issuance limit $\bar{N}(B, \Gamma_-, g, s)$ in (15) as a set constraint. In particular, let $\mathbb{N}(B,\Gamma_-,g,s)$ be the set of issuance N such that next period default probability is below $\bar{p}$. Formally,

$$\mathbb{N}(B,\Gamma_-,g,s)=\begin{Bmatrix} N: & \mathbb{P}\left(V^{nd}(B',\Gamma,g',s',\epsilon')\leq V^d(B',\Gamma,g',s',\epsilon')\middle|\,\Gamma_-,g,s\right)\leq\bar{p}, \\ \text{s. to} & B'=(1-\delta)B+\mathcal{R}(N,B,\Gamma_-,g,s)N. \end{Bmatrix} \tag{C.1}$$

Thus, the set $\mathbb{N}(B,\Gamma_-,g,s)$ contains all possible issuance N such that the default probability next periods is below a number $\bar{p}$. Under our guess, the set $\mathbb{N}(\cdot)$ satisfies that if $N\in\mathbb{N}(B,\Gamma_-,g,s)$, then $\lambda N\in\mathbb{N}(\lambda B,\lambda\Gamma_-,g,s)$. To see this, note that, when the state is $(\lambda B,\lambda\Gamma_-,g,s)$, the probability of default after issuance λN is given as

$$\begin{aligned}
&\mathbb{P}\left(V^{nd}\begin{pmatrix}(1-\delta)\lambda B+\mathcal{R}(\lambda N,\lambda B,\lambda\Gamma_-,g,s)\lambda N,\\ \lambda\Gamma,g',s',\epsilon'\end{pmatrix}\leq V^d\begin{pmatrix}(1-\delta)\lambda B+\mathcal{R}(\lambda N,\lambda B,\lambda\Gamma_-,g,s)\lambda N,\\ \lambda\Gamma,g',s',\epsilon'\end{pmatrix}\middle|\,\Gamma_-,g,s\right)\\
&=\mathbb{P}\left(V^{nd}\begin{pmatrix}(1-\delta)\lambda B+\mathcal{R}(N,B,\Gamma_-,g,s)\lambda N,\\ \lambda\Gamma,g',s',\epsilon'\end{pmatrix}\leq V^d\begin{pmatrix}(1-\delta)\lambda B+\mathcal{R}(N,B,\Gamma_-,g,s)\lambda N,\\ \lambda\Gamma,g',s',\epsilon'\end{pmatrix}\middle|\,\Gamma_-,g,s\right)\\
&=\mathbb{P}\left(\lambda^{1-\gamma}V^{nd}\begin{pmatrix}(1-\delta)B+\mathcal{R}(N,B,\Gamma_-,g,s)N,\\ \Gamma,g',s',\epsilon'\end{pmatrix}\leq \lambda^{1-\gamma}V^d\begin{pmatrix}(1-\delta)B+\mathcal{R}(N,B,\Gamma_-,g,s)N,\\ \Gamma,g',s',\epsilon'\end{pmatrix}\middle|\,\Gamma_-,g,s\right)\\
&=\mathbb{P}\left(V^{nd}\begin{pmatrix}(1-\delta)B+\mathcal{R}(N,B,\Gamma_-,g,s)N,\\ \Gamma,g',s',\epsilon'\end{pmatrix}\leq V^d\begin{pmatrix}(1-\delta)B+\mathcal{R}(N,B,\Gamma_-,g,s)N,\\ \Gamma,g',s',\epsilon'\end{pmatrix}\middle|\,\Gamma_-,g,s\right)\\
&=\mathbb{P}\left(V^{nd}(B',\Gamma,g',s',\epsilon')\leq V^d(B',\Gamma,g',s',\epsilon')\middle|\,\Gamma_-,g,s\right)
\end{aligned}$$

where the second line uses our guess for $\mathcal{R}(\cdot)$ and the third line uses the guess for $V^{nd}(\cdot)$ and $V^d(\cdot)$. The last line shows that the probability of default after issuance λN under state $(\lambda B,\lambda\Gamma_-,g,s)$ is the same as the probability of default after issuance N under state (B,Γ_-,g,s). Thus, if $N\in\mathbb{N}(B,\Gamma_-,g,s)$, then $\lambda N\in\mathbb{N}(\lambda B,\lambda\Gamma_-,g,s)$.

Let $\mathcal{U}(N,B,\Gamma_-,g,s,\epsilon)$ be the value of no default when issuing N. That is

$$\begin{aligned}
\mathcal{U}(N,B,\Gamma_-,g,s,\epsilon)=&\left\{\frac{C^{1-\gamma}}{1-\gamma}+\beta\mathbb{E}\left[\max\left\{V^{nd}(B',\Gamma,g',s',\epsilon'),V^d(B',\Gamma,g',s',\epsilon')\right\}|\Gamma_-,g,s\right]\right\}\\
&s.t.\\
&\quad C+B=Y+N,\\
&\quad B'=(1-\delta)B+\mathcal{R}(N,B,\Gamma_-,g,s)N,\\
&\quad Y=\Gamma e^{\sigma\epsilon},\quad \Gamma=e^g\Gamma_-.
\end{aligned}\tag{C.2}$$

When defining $\mathcal{U}(\cdot)$, we imposed all constrains of $V^{nd}(\cdot)$ in equation (15) except for issuance limit $N\in\mathbb{N}(B,\Gamma_-,g,s)$. Thus, we have

$$V^{nd}(B,\Gamma_-,g,s,\epsilon)=\max_N\left\{\mathcal{U}(N,B,\Gamma_-,g,s,\epsilon)\ \text{ s. to }\ N\in\mathbb{N}(B,\Gamma_-,g,s)\right\}. \tag{C.3}$$

Notice that $\mathcal{U}(\lambda N, \lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda^{1-\gamma}\mathcal{U}(N, B, \Gamma_-, g, s, \epsilon)$. In particular,

$$
\begin{aligned}
&\mathcal{U}(\lambda N, \lambda B, \lambda\Gamma_-, g, s, \epsilon) = \frac{[g\lambda\Gamma_- e^{\sigma\epsilon} + \lambda N - \lambda B]^{1-\gamma}}{1-\gamma} \\
&+ \beta\mathbb{E}\left[\max\left\{\begin{matrix} V^{nd}\Big((1-\delta)\lambda B + \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s)\lambda N, \lambda g\Gamma_-, g', s', \epsilon'\Big) \\ V^{d}\Big((1-\delta)\lambda B + \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s)\lambda N, \lambda g\Gamma_-, g', s', \epsilon'\Big)\end{matrix}\right\} |\Gamma_-, g, s\right] \\
&= \lambda^{1-\gamma}\frac{[g\Gamma_- e^{\sigma\epsilon} + N - B]^{1-\gamma}}{1-\gamma} \\
&+ \beta\mathbb{E}\left[\max\left\{\begin{matrix} \lambda^{1-\gamma}V^{nd}\Big((1-\delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N, g\Gamma_-, g', s', \epsilon'\Big) \\ \lambda^{1-\gamma}V^{d}\Big((1-\delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N, g\Gamma_-, g', s', \epsilon'\Big)\end{matrix}\right\} |\Gamma_-, g, s\right] \\
&= \lambda^{1-\gamma}\mathcal{U}(N, B, \Gamma_-, g, s, \epsilon),
\end{aligned} \tag{C.4}
$$

where the second equality uses the guess on $V^d(\cdot)$, $V^{nd}(\cdot)$, and $\mathcal{R}(\cdot)$.

We use equation (C.3) to show that the issuance policy is linear in (B, Γ_-). Let $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$ be the optimal policy in equation (C.3). Then, under our guess, we have $\mathbf{N}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$. We proceed to prove this by contradiction. Assume that there is another policy, $\lambda\hat{N} \neq \lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$, which is feasible and delivers higher utility $\mathcal{U}$. That is, $\lambda\hat{N} \in \mathbb{N}(\lambda B, \lambda\Gamma_-, g, s)$ and $\mathcal{U}\left(\lambda\hat{N}, \lambda B, \lambda\Gamma_-, g, s, \epsilon\right) > \mathcal{U}\left(\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon), \lambda B, \lambda\Gamma_-, g, s, \epsilon\right)$. Using the homogeneity properties of the set $\mathbb{N}(\cdot)$, we know that if $\lambda\hat{N} \in \mathbb{N}(\lambda B, \lambda\Gamma_-, g, s)$, then $\hat{N} \in \mathbb{N}(B, \Gamma_-, g, s)$. Using the homogeneity of $\mathcal{U}$ in equation (C.4), we know that if $\mathcal{U}\left(\lambda\hat{N}, \lambda B, \lambda\Gamma_-, g, s, \epsilon\right) > \mathcal{U}\left(\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon)\lambda B, \lambda\Gamma_-, g, s, \epsilon\right)$, then $\mathcal{U}\left(\hat{N}, B, \lambda\Gamma_-, g, s, \epsilon\right) > \mathcal{U}\left(\mathbf{N}(B, \Gamma_-, g, s, \epsilon), B, \Gamma_-, g, s, \epsilon\right)$. This is a contradiction, because $\hat{N}$ would be feasible under state $(B, \Gamma_-, g, s, \epsilon)$ and deliver higher utility than $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$, contradicting that $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$ is an optimal policy. Then, we have that $\mathbf{N}(\lambda B, \lambda G_-, g, s, \epsilon) = \lambda\mathbf{N}(B, G_-, g, s, \epsilon)$.

Finally, we can show the homogeneity property of $V^{nd}(\cdot)$. In particular,

$$
\begin{aligned}
V^{nd}(\lambda B, \lambda G_-, g, s, \epsilon) &= \mathcal{U}(\mathbf{N}(\lambda B, \lambda G_-, g, s, \epsilon), \lambda B, \lambda G_-, g, s, \epsilon) \\
&= \mathcal{U}(\lambda\mathbf{N}(B, G_-, g, s, \epsilon), \lambda B, \lambda G_-, g, s, \epsilon) \\
&= \lambda^{1-\gamma}\mathcal{U}(\mathbf{N}(B, G_-, g, s, \epsilon), B, G_-, g, s, \epsilon) \\
&= \lambda^{1-\gamma}V^{nd}(B, \Gamma_-, g, s, \epsilon),
\end{aligned} \tag{C.5}
$$

where the second line uses the linearity of $\mathbf{N}(\cdot)$, the third line uses the homogeneity of $\mathcal{U}(\cdot)$, and the first and last lines use the definition of $\mathcal{U}(\cdot)$.

Let's move to (ii). We have that

$$
\mathbf{B}'(B, \Gamma_-, g, s, \epsilon) = (1-\delta)B + \mathcal{R}(\mathbf{N}(B, \Gamma_-, g, s, \epsilon), B, \Gamma_-, g, s, \epsilon)\mathbf{N}(B, \Gamma_-, g, s, \epsilon).
$$

Then,

$$
\begin{aligned}
\mathbf{B}'(\lambda B, \lambda\Gamma_-, g, s, \epsilon) &= (1-\delta)\lambda B + \mathcal{R}(\mathbf{N}(\lambda B, \lambda\Gamma_-, g, s, \epsilon), \lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathbf{N}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) \\
&= (1-\delta)\lambda B + \mathcal{R}(\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon), \lambda B, \lambda\Gamma_-, g, s, \epsilon)\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon) \\
&= (1-\delta)\lambda B + \mathcal{R}(\mathbf{N}(B, \Gamma_-, g, s, \epsilon), B, \Gamma_-, g, s, \epsilon)\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon) \\
&= \lambda\mathbf{B}'(B, \Gamma_-, g, s, \epsilon),
\end{aligned} \tag{C.6}
$$

where the second line uses the linearity of $\mathbf{N}(\cdot)$, the third line uses our guess for $\mathcal{R}(\cdot)$, and the fourth line confirms the linearity of $\mathbf{B}'(\cdot)$.

Let's move to (iii). When proving the homogeneity properties of the price functions $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$, we are essentially comparing two bonds—one that pays R and another that pays λR—while keeping the debt services B fixed. Thus, the state of the borrower doesn't change as we move λ.

From equation (19), we have

$$
\begin{aligned}
\mathcal{Q}(B, \lambda R, \Gamma_-, g, s, \epsilon) &= [1-\mathbf{D}(B, \Gamma_-, g, s, \epsilon)]\left[\lambda R + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), \lambda R, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(B, \Gamma_-, g, s, \epsilon)\mathcal{X}(B, \lambda R, \Gamma_-, g, s, \epsilon) \\
&= [1-\mathbf{D}(B, \Gamma_-, g, s, \epsilon)]\left[\lambda R + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\lambda\mathcal{Q}\left(\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), R, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(B, \Gamma_-, g, s, \epsilon)\lambda\mathcal{X}(B, \lambda R, \Gamma_-, g, s, \epsilon) \\
&= \lambda\mathcal{Q}(B, R, \Gamma_-, g, s, \epsilon),
\end{aligned} \tag{C.7}
$$

where the second line uses our guess for $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$, and the third line verifies the guess for $\mathcal{Q}(\cdot)$. Mutatis mutandis, the same procedure for $\mathcal{X}(\cdot)$ using equation (20) gives the following result

$$
\begin{aligned}
\mathcal{X}(B, \lambda R, \Gamma_-, g, s, \epsilon) &= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(B', \lambda R, \Gamma, g', s', \epsilon') + (1-\theta)\left[1-\mathbf{E}(B', \Gamma, g', s', \epsilon')\right]\mathcal{X}(B', \lambda R, \Gamma, g', s', \epsilon') \\
&+ (1-\theta)\mathbf{E}(B', \Gamma, g', s', \epsilon')\mathcal{Q}(\kappa B', \kappa\lambda R, \Gamma, g', s', \epsilon)|\Gamma_-, g, s\Bigg] \\
&= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\lambda\mathcal{X}(B', R, \Gamma, g', s', \epsilon') + (1-\theta)\left[1-\mathbf{E}(B', \Gamma, g', s', \epsilon')\right]\lambda\mathcal{X}(B', R, \Gamma, g', s', \epsilon') \\
&+ (1-\theta)\mathbf{E}(B', \Gamma, g', s', \epsilon')\lambda\mathcal{Q}(\kappa B', \kappa R, \Gamma, g', s', \epsilon)|\Gamma_-, g, s\Bigg] \\
&= \lambda\mathcal{X}(B, R, \Gamma_-, g, s, \epsilon).
\end{aligned} \tag{C.8}
$$

Let's move to (iv). We start by showing that default and re-entry policies satisfy $\mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \mathbf{D}(B, \Gamma_-, g, s, \epsilon)$ and $\mathbf{E}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \mathbf{E}(B, \Gamma_-, g, s, \epsilon)$. In particular, from equation (17) we have

$$
\begin{aligned}
\mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) &= \mathbb{I}\left\{V^{nd}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) < V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\right\} \\
&= \mathbb{I}\left\{\lambda^{1-\gamma}V^{nd}(B, \Gamma_-, g, s, \epsilon) < \lambda^{1-\gamma}V^d(B, \Gamma_-, g, s, \epsilon)\right\} \\
&= \mathbf{D}(B, \Gamma_-, g, s, \epsilon),
\end{aligned} \tag{C.9}
$$

where $\mathbb{I}\{\cdot\}$ is an indicator function. Similarly, using equation (18), we get

$$\begin{aligned}
\mathbf{E}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) &= \mathbb{I}\left\{V^{nd}(\kappa\lambda B, \lambda\Gamma_-, g, s, \epsilon) \geq V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\right\} \\
&= \mathbb{I}\left\{\lambda^{1-\gamma}V^{nd}(\kappa B, \Gamma_-, g, s, \epsilon) \geq \lambda^{1-\gamma}V^d(B, \Gamma_-, g, s, \epsilon)\right\} \\
&= \mathbf{E}(B, \Gamma_-, g, s, \epsilon). \qquad \text{(C.10)}
\end{aligned}$$

Then, using equation (19), we have

$$\begin{aligned}
\mathcal{Q}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) &= [1 - \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)]\left[1 + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(\lambda B, \lambda\Gamma_-, g, s, \epsilon), R, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) \\
&= [1 - \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)]\left[1 + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\lambda\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), R, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) \\
&= [1 - \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)]\left[1 + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), R, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathcal{X}(B, R, \Gamma_-, g, s, \epsilon) \\
&= [1 - \mathbf{D}(B, \Gamma_-, g, s, \epsilon)]\left[1 + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), R, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(B, \Gamma_-, g, s, \epsilon)\mathcal{X}(B, R, \Gamma_-, g, s, \epsilon) \\
&= \mathcal{Q}(B, R, \Gamma_-, g, s, \epsilon), \qquad \text{(C.11)}
\end{aligned}$$

where the second line uses the linearity of $\mathbf{B}'(\cdot)$, the third line uses our guess on $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$, the fourth line uses the homogeneity properties of $\mathbf{D}(\cdot)$ and $\mathbf{E}(\cdot)$, and the last line confirms our guess. Again, using equation (20), we can follow the same procedure for $\mathcal{X}(\cdot)$:

$$\begin{aligned}
\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) &= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(\lambda B, R, \lambda\Gamma, g', s', \epsilon') + (1-\theta)\left[1 - \mathbf{E}(\lambda B, \lambda\Gamma, g', s', \epsilon')\right]\mathcal{X}(\lambda B, R, \lambda\Gamma, g', s', \epsilon') \\
&+ (1-\theta)\mathbf{E}(\lambda B, \lambda\Gamma, g', s', \epsilon')\mathcal{Q}(\kappa\lambda B, \kappa R, \lambda\Gamma, g', s', \epsilon)|\Gamma_-, g, s\Bigg] \\
&= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(B, R, \Gamma, g', s', \epsilon') + (1-\theta)\left[1 - \mathbf{E}(B, \Gamma, g', s', \epsilon')\right]\mathcal{X}(B, R, \Gamma, g', s', \epsilon') \\
&+ (1-\theta)\mathbf{E}(B, \Gamma, g', s', \epsilon')\mathcal{Q}(\kappa B, \kappa R, \Gamma, g', s', \epsilon)|\Gamma_-, g, s\Bigg] \\
&= \mathcal{X}(B, R, \Gamma_-, g, s, \epsilon). \qquad \text{(C.12)}
\end{aligned}$$

Let's move to (v). Using equations (21) and (22), we can express $\mathcal{R}(\cdot)$ as

$$\begin{aligned}
\left(\mathcal{R}\left(\lambda N, \lambda B, \lambda\Gamma_-, g, s\right)\right)^{-1} &= \frac{\mathbb{E}\left[\mathcal{Q}\left((1-\delta)\lambda B, +\mathcal{R}\left(\lambda N, \lambda B, \lambda\Gamma_-, g, s\right)\lambda N, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\right]}{1+r^*} \\
&= \frac{\mathbb{E}\left[\mathcal{Q}\left((1-\delta)\lambda B, +\mathcal{R}\left(N, B, \Gamma_-, g, s\right)\lambda N, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\right]}{1+r^*} \\
&= \frac{\mathbb{E}\left[\mathcal{Q}\left((1-\delta)B, +\mathcal{R}\left(N, B, \Gamma_-, g, s\right)N, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\right]}{1+r^*} \\
&= \left(\mathcal{R}\left(N, B, \Gamma_-, g, s\right)\right)^{-1}, \qquad \text{(C.13)}
\end{aligned}$$

where the second line uses our guess for $\mathcal{R}(\cdot)$, the third line use the homogeneity of $\mathcal{Q}(\cdot)$, and the last line confirms our guess for $\mathcal{R}(\cdot)$. $\square$

The model normalization from Section 3.2 uses the homogeneity properties just derived. In particular, the detrended model from Section 3.2 comes from setting $\lambda = 1/\Gamma_-$ and using: (i) $v^{nd}(b,g,s,\epsilon) = V^{nd}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$ and $v^d(b,g,s,\epsilon) = V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$; (ii) $Q(b,g,s,\epsilon) = \mathcal{Q}(\lambda B, 1, \lambda\Gamma_-, g, s, \epsilon)$ and $X(b,g,s,\epsilon) = \mathcal{X}(\lambda B, 1, \lambda\Gamma_-, g, s, \epsilon)$; (iii) $R(n,b,g,s,\epsilon) = \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s, \epsilon)$; and (iv) $\mathbf{d}(b,g,s,\epsilon) = \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$ and $\mathbf{e}(b,g,s,\epsilon) = \mathbf{E}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$, where $b = B/\Gamma_-$ and $n = N/\Gamma_-$.

D Computational algorithm

In this appendix, we provide details about the computation of the detrended model of Section 3.2.

Step 0: We set an evenly spaced grid $\vec{b} = \{b_1, b_2, \ldots, b_{N_b}\}$ for debt service values, and a grid $\vec{\epsilon} = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{N_\epsilon}\}$ for the transitory shocks to endowment. Both are country-specific. For the grid $\vec{\epsilon}$, we use the discretization method in Tauchen (1986) with parameter 3.5, $N_\epsilon = 17$, and the standard deviations of the transitory shock reported in Table 2. For the debt service values $\vec{b}$ for Argentina, we set $N_b = 3000$, $b_1 = -0.05$, and $b_{N_b} = 0.5$. For Spain, we need a finer grid to deal with convergence issues due to the longer maturity of the debt. We set $N_b = 7350$, $b_1 = -0.01$, and $b_{N_b} = 0.3$. In both cases, b_1 and b_{N_b} were chosen so that the grid covers all values of b in our simulations.

Step 1: Guess a value for no default $v^{nd}(b,g,s,\epsilon)$ and an implied policy for debt services $\mathbf{b}'(b,g,s,\epsilon)$. We restrict debt service to be on the grid, $\mathbf{b}'(b,g,s,\epsilon) \in \vec{b}$.

Step 2: Given $v^{nd}(b,g,s,\epsilon)$, we compute the value of default $v^d(b,g,s,\epsilon)$, using equation (24). We can then compute the policies for default and re-entry, $\mathbf{d}(b,g,s,\epsilon)$ and $\mathbf{e}(b,g,s,\epsilon)$, as in equations (29) and (30). We also compute the maximum of both values, $w(b,g,s,\epsilon) = \max\left\{v^{nd}(b,g,s,\epsilon),\ v^d(b,g,s,\epsilon)\right\}$.

Step 3: Given $\mathbf{d}(b,g,s,\epsilon)$ and $\mathbf{e}(b,g,s,\epsilon)$, as well as the guess for debt service policy, $\mathbf{b}'(b,g,s,\epsilon)$, we jointly solve for prices $Q(b,g,s,\epsilon)$ and $X(b,g,s,\epsilon)$ using equations (27) and (28). We use linear interpolation for $Q(\cdot)$ and $X(\cdot)$ for values of b outside the grid—which may occur because of the recovery κ or the growth rate g, even if the debt service policy is on the grid $\vec{b}$.

Step 4: For each debt service $b_i \in \vec{b}$ on the grid, we compute the expected next-period price of a bond starting with debt service b_i: $Q_i^{\mathbb{E}}(g,s) = \mathbb{E}\left[Q(b_i, g', s', \epsilon')|g,s\right]$. Then, for each $b_i \in \vec{b}$, we use equation (25) to compute the implied rate consistent with $Q_i^{\mathbb{E}}(g,s)$: $R_i(g,s) = \frac{1+r^*}{Q_i^{\mathbb{E}}(g,s)}$. Then, we compute the implied issuance: $n_i(b,g,s) = \frac{b_i-(1-\delta)b}{R_i(g,s)}$. Note that the issuance depends on next-period debt services as well as current debt services.

The above steps yield, for each (b,g,s), a correspondence $\{R_i, n_i\}_i$ resembling the ones in Figures 6 and 8 (where we omitted the inputs of R_i and n_i to ease notation). In what follows, we restrict our attention to the pairs $\{R_i, n_i\}_i$ with implied default probabilities below 65% in the next period.[39]

For a given n, there may be multiple values of R in the correspondence, so the next step consists of restricting the set of pairs (R_i, n_i) in $\{R_i, n_i\}_i$ that are available to the borrower according to the sunspot realization. In the case of the good sunspot, we simply allow the borrower to choose any of the pairs in the correspondence $\{R_i, n_i\}_i$, because the borrower will always prefer the lowest rates. Therefore, the result is equivalent to computing the increasing interest rate schedule with the lowest rates from which the borrower can choose from.

For the bad sunspot, we first identify the increasing parts of the correspondence. Note that the vector $\vec{b}$ is increasing, so $j > i$ implies $b_j > b_i$, and then $Q_i^{\mathbb{E}} \geq Q_j^{\mathbb{E}}$, and thus $R_i \leq R_j$. We then consider that a pair (R_i, n_i) is in the increasing part of the correspondence if n_{i+1} is larger than n_{i-1}. Then, from the correspondence $\{R_i, n_i\}_i$, we identify the points $\hat{\imath}$ for which there exists $j > \hat{\imath}$ such that: $n_j < n_{\hat{\imath}}$, and n_j is an increasing part of the correspondence. Under the bad sunspot, we make all these $\hat{\imath}$ points of the correspondence unavailable to the borrower. Thus, this procedure results in an increasing interest rate schedule in which the highest rate is always selected for possible issuance n.

Step 5: For each state (b, g, s), we have a schedule $\{R_i, n_i\}_i$ associated with a next-period debt service b_i in the grid. Given the state (b, g, s, ϵ), we choose b_i to maximize utility as in equation (23). Let $\hat{\mathbf{b}}'(b, g, s, \epsilon)$ be the implied optimal policy. Let $\hat{v}^{nd}(b, g, s, \epsilon)$ be the implied value of no default, and $\hat{w}(b, g, s, \epsilon) = \max\left\{\hat{v}^{nd}(b, g, s, \epsilon),\ v^d(b, g, s, \epsilon)\right\}$ be the maximum of both values.

If $\max|w(b, g, s, \epsilon) - \hat{w}(b, g, s, \epsilon)| < 1e^{-5}$, the model converged. Otherwise, we update $v^{nd}(b, g, s, \epsilon)$ and go to **Step 2**.

E Computation of Argentina's event case study

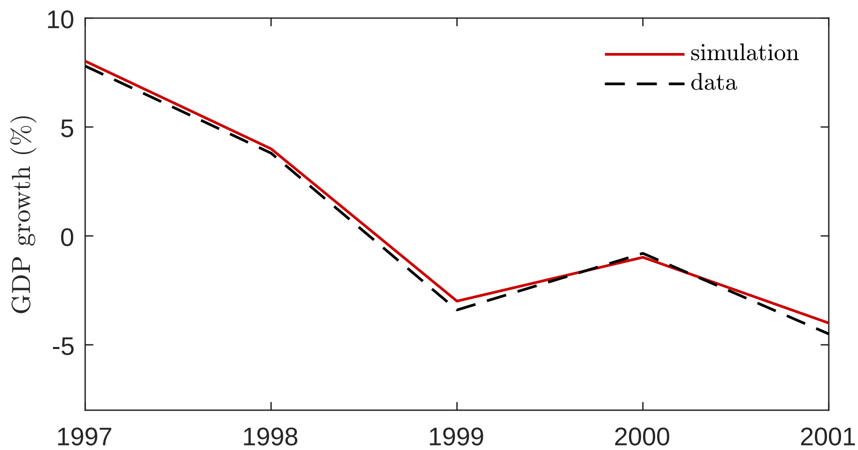
In this appendix, we provide more details on the event case study for Argentina's default discussed in Section 5.3.

We run two simulations using the policy functions of the benchmark model for Argentina. Each simulation is determined by a sequence of state variables $(b_t, g_t, s_t, \epsilon_t)$ between 1997 and 2001. To isolate the role of expectations, we choose the same sequence of endowment shock realizations (g_t, ϵ_t), as well as the same initial debt service b_{1997}, but different sunspot realizations. For both sequences, we assume expectations are bad, $s_t = s_B$, from 1997 to 2000. Then, in 2001, we compare the case of a good sunspot realization with the case of a bad sunspot.

[39] The default probability restriction is implemented by setting a utility penalty of $1e^{24}$ for the pairs violating the restriction.

The sequence of endowment shocks, g_t and ϵ_t, are chosen from their grids to match the series of GDP growth for Argentina in 1997–2001.[40] The stochastic process in equations (7)–(8) implies that output growth can be expressed as $\Delta y_t = g_t + \sigma(\epsilon_t - \epsilon_{t-1})$. We begin by setting the sequence of growth shocks g_t as implied by the Kim (1994) filter we used to estimate the output process. In turn, we set $g_{1997} = g_{1998} = g_H$ and $g_{1999} = g_{2000} = g_{2001} = g_L$.[41] Next, we select the sequence of transitory shocks. The initial transitory shock, ϵ_{1996}, is set to be equal to the mid-point in the grid $\vec{\epsilon}$. After that, the points ϵ_t are chosen from the grid to minimize the absolute difference between the GDP growth in the data and the growth implied by the grid point in each period, $g_t + \sigma(\epsilon_t - \epsilon_{t-1})$. Figure E.1 shows that the simulated series of output growth, the solid red line, is virtually identical to the series of GDP growth in the data, the dashed black line.

Figure E.1: Argentina: GDP growth from 1997 to 2001



We are left with the choice of the initial debt service, b_{1997}. We set the initial debt service such that the simulation-implied debt service in 2000 equals 20.2% of GDP, the value observed in the data.[42] Given this choice of b_{1997} and the above explained endowment shocks (g_t, ϵ_t), we compute the sequence of debt services implied by the policy function of the borrower, with $b_{t+1} = b'(b_t, g_t, s_t, \epsilon_t)$, and the corresponding interest rate spreads. Note that our assumptions about the sequence of the exogenous state variables and initial debt service imply that both simulations will have the same sequence of debt services up to 2001 and interest rate spreads up to 2000. What differs is the choice of debt issuance in 2001 due to the different sunspot realizations, which translates into different interest rate spreads for that year. The resulting series for interest rate spreads are illustrated in Figure 12.

[40] A perfect match may be unfeasible due to the state space discretization, described in Appendix D.

[41] The parameter values for g_L and g_H are reported in Table 2.

[42] See Appendix B for a description of the data. An exact match between the model and the data may not be possible due to the discretization of the state space and the policy functions. We achieve a debt service of 19.7% of GDP in 2000.

F Additional model results

F.1 Pareto frontier: measuring coordination inefficiency

In this appendix, we discuss the inefficiencies associated with the lack of coordination among lenders. That is, we quantify the welfare loss, for lender and borrowers, due to a bad sunspot realization. As we show, the inefficiency is rather large for Argentina's calibration, but substantially smaller for Spain's calibration.

For the value for the borrower, we take their value function when in no default $v^{nd}(b, g, s, \epsilon)$. For the value of the lender, we take the market value of the outstanding debt relative to the borrower's GDP. In particular, let $\mathcal{M}(b, g, s, \epsilon) = \frac{Q(b,g,s,\epsilon)\times b}{y}$ be the value to lenders, where $Q(b, g, s, \epsilon)$ denotes the value of outstanding bonds.[43] Figure F.1 below plots the three values discussed—$v^{nd}(b, \cdot)$, $Q(b, \cdot)$, and $\mathcal{M}(b, \cdot)$—as we vary debt services. b. and the sunspot realization, s. Figure F.2 shows the Pareto frontier, which has the borrower's value $v^{nd}(\cdot)$ on the x-axis, and the lender's value $\mathcal{M}(\cdot)$ on the y-axis. All plots are made for both calibrations, Argentina and Spain, and for the low-growth state where multiplicity is relevant.

We focus on Argentina's calibration, where the coordination problem is more prevalent. The value of the borrower is always lower under the bad sunspot, as it entails borrowing at the less favorable (higher rates) schedule. However, the effect of the sunspot on the bond price is not obvious. For lower debt-service values b, the bad sunspot induces lower issuance (endogenous austerity), which boosts the value of the already outstanding debt.[44] For larger values of debt service b, the sunspot actually triggers larger issuance (gambling for redemption), which depresses the value of already outstanding debt. In turn, the value of the lender, $\mathcal{M}(b, \cdot)$, also depends on the sunspot realization.

The sunspot can have a large quantitative effect. For instance, for a debt-service value of $b = 20\%$, as Argentina had in 2001, the value to the lenders is $\mathcal{M} = 15\%$ of output under the good sunspot (point G) and $\mathcal{M} = 17.5\%$ of output under the bad sunspot (point B). That is, a coordination failure (a bad sunspot realization) implies a loss for lenders of 2.5% of GDP. This is a sizable inefficiency.

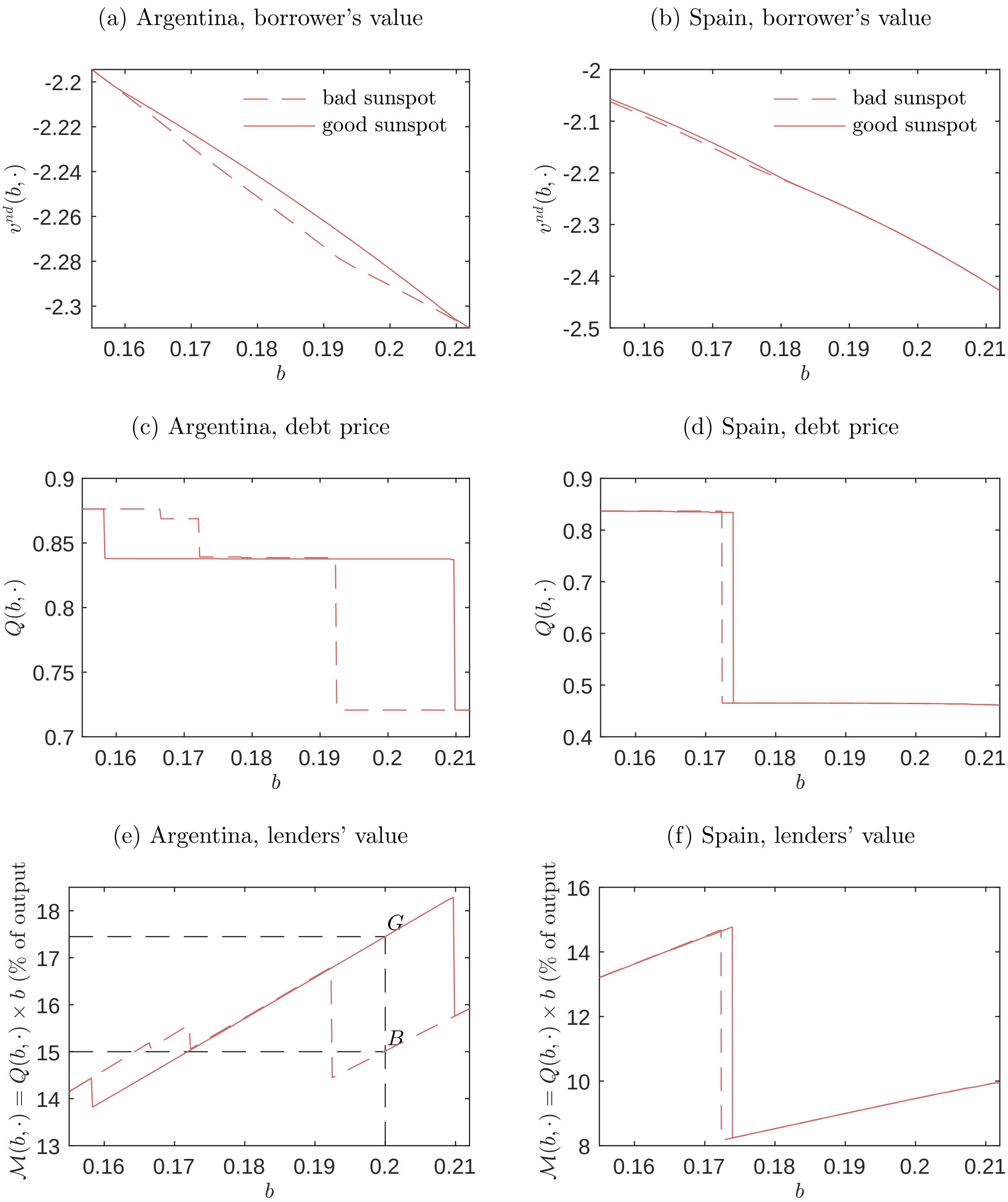
The Pareto frontier in Figure F.2 shows a similar outcome. For instance, compare points P_B and P_G. In both of these points, the borrower has a value of $v^{nd} = -2.29$. However, point P_B, under the bad sunspot, gives a value to lenders of $\mathcal{M} = 15\%$, while point P_G, under the good sunspot, gives lenders a value of $\mathcal{M} = 17.7\%$. That is, were lenders able to coordinate, the welfare gain to them would be 2.7% of output.[45]

[43] Note that $\mathcal{M}(\cdot)$ represents the value of outstanding debt at the beginning of the period; that is, this period's value of debt issued in the past. The value to new lenders is always the risk-free rate $1 + r^*$. See equations (21)-(22).

[44] See Figure 10 and discussion in Section 5.2.

[45] Note that the implied debt service levels in points P_G and P_B are *not* the same. In particular, given b, the borrower's value is lower under the bad sunspot. Thus, moving from the bad sunspot in P_B to the

Figure F.1: Pareto frontier: Policies



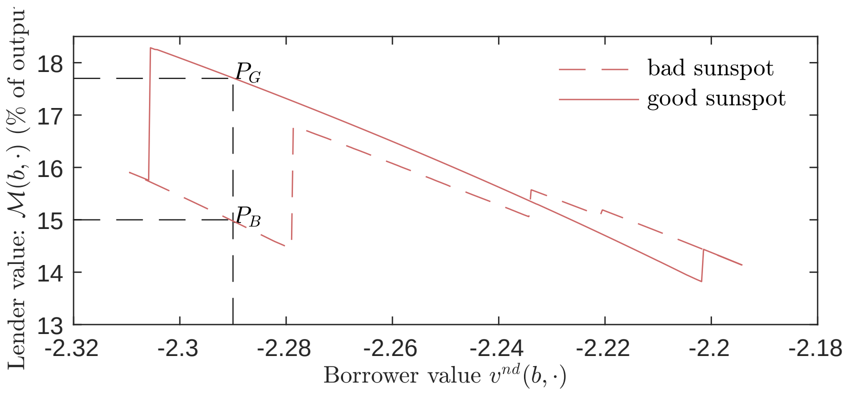
F.2 Spain's debt measure: alternative calibration

The recent work in Bocola et al. (2019) argues that calibrating sovereign default models to public debt moments—rather than external debt moments—can improve their quantitative performance. They use data for Spain, Italy, and Portugal for a period similar to the one we consider. In turn, in this appendix, we revisit our calibration targets for Spain. We do so in two ways. First, we discuss how our calibration target

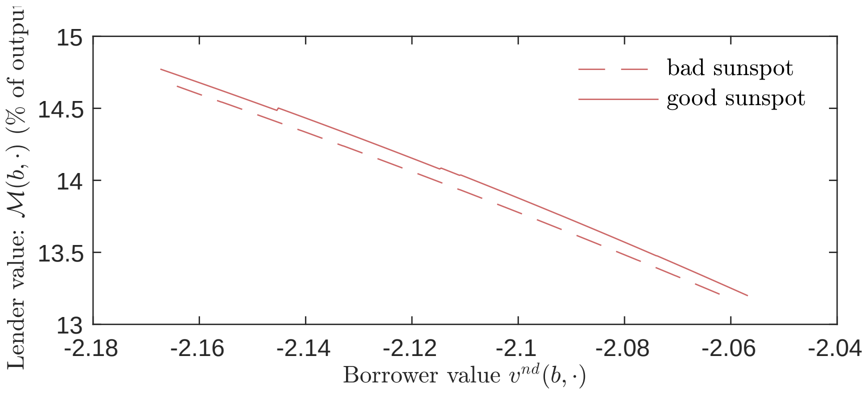
good sunspot in P_G, implies increasing debt service b in order to keep the borrower's utility v^{nd} constant.

Figure F.2: Pareto Frontier

(a) Argentina



(b) Spain



for Spain compares with the preferred debt measures in Bocola et al. (2019). Second, we show that our quantitative findings are robust to using a calibration that targets a different debt-to-GDP ratio.

Alternative debt measures Table 1 in Bocola et al. (2019) shows that once GDP is controlled for, net external debt NED—the measure of debt used in our benchmark calibration—explains 85% of the variation in spreads in Spain, roughly as much as the total public debt (TPD) debt measure, which they put forward as their preferred measure. Thus, our calibration target aligns with a debt measure that can account for the bulk of the variations in credit spreads.

Calibration targeting TPD We also consider an alternative *lower-debt* calibration for Spain, which targets a debt-to-GDP ratio of 70%, roughly the TPD value of Spain during 2008 and 2012 (see Figure 1 in Bocola et al., 2019). In particular, we keep all parameters unchanged except for default cost parameters, which we adjust to target the lower debt level.[46] As we show below, we find our main results are robust in the *lower-*

[46] The *lower-debt* calibration assumes default costs of $\phi(g_H) = 0.959$ and $\phi(g_L) = 0.953$, slightly above default costs in the benchmark calibration (see Table 2)

debt calibration, namely: Spain's interest rate schedule exhibits multiple equilibria in a relevant area of debt service, but it opts for endogenous austerity, and the probability of the sunspot realization, p_B, drastically affects outcomes for Spain.

Figure F.3 below shows the interest rate schedule for the *lower-debt* calibration in the low-growth state. The schedules are for debt service of $b = 12\%$ and $b = 11\%$, which are smaller than in our benchmark calibration and reflect the lower debt level targeted in this calibration. As in our benchmark calibration, the schedule exhibits multiplicity in a relevant area. For $b = 12\%$, the interest rate schedule exhibits multiplicity for debt issuance levels between roughly 8% and 10% of GDP. Thus, when expectations are good, the borrower needs a surplus of only 2% of GDP to avoid the high interest rates. In contrast, when expectations are bad, the borrower needs a surplus of 4% of GDP to avoid the high interest rates. Thus, to avoid high interest rates, the borrower run substantially larger surpluses.

As in our benchmark calibration, Spain's policy exhibits endogenous austerity, and it optimally decides to run such larger surpluses to avoid the high interest rates. Figure F.4 shows this: issuance is lower under the bad sunspot realization, so that credit spreads in equilibrium remain low.

Table F.1 shows how some key moments change with p_B under the *low-debt* calibration. As in our benchmark calibration, a change in p_B drastically changes outcomes for Spain: default rates increase from 0.4% to 5.9%, and credit spreads increase from 0.1%. to 8.0%. Thus, expectations are still a major driver explaining default rates and credit spreads in the *lower-debt* calibration.

In summary, our main results are robust to targeting TPD for Spain's calibration.

Figure F.3: Interest rate correspondence for southern Europe (low-growth state)

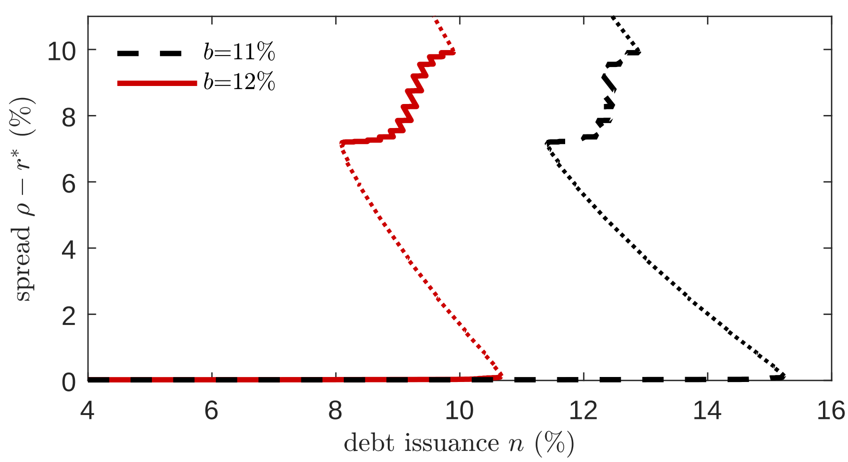
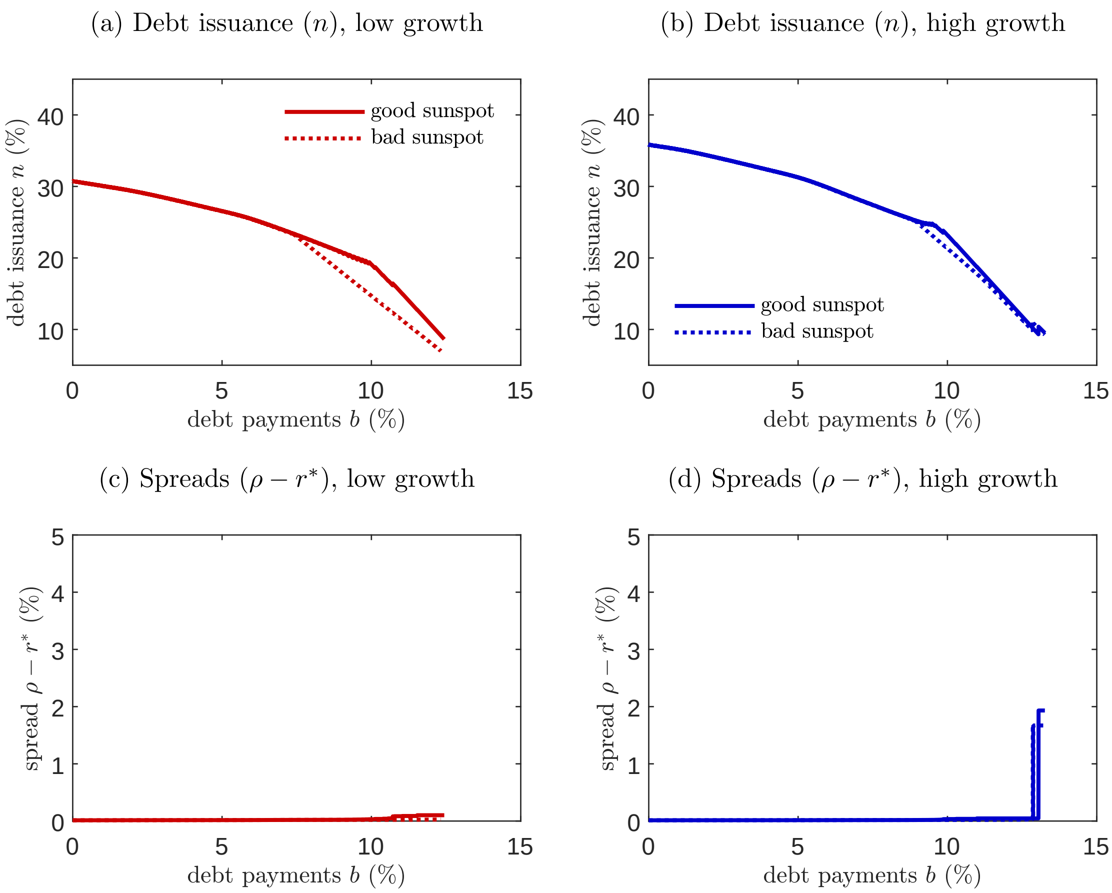


Table F.1: Simulation moments: Spain *low-debt* calibraiton

	Benchmark ($p_B = 1\%$)	$p_B = 25\%$
First moments (%)		
avg($spread$)	0.1	8
avg(qb/y)	68	38
avg(f/y)	66	38
avg(n/y)	11	9
avg(b/y)	12	10
avg(tb/y)	1.1	1.2
default rate	0.4	5.9
Low-growth state		
avg($spread$)	0.1	10.3
avg(qb/y)	70	22
avg(f/y)	68	34
avg(n/y)	9	9
avg(b/y)	13	9
avg(tb/y)	3.6	0.2
default rate	1.3	20.2
High-growth state		
avg($spread$)	0	7.7
avg(qb/y)	67	40
avg(f/y)	65	38
avg(n/y)	12	9
avg(b/y)	12	10
avg(tb/y)	0.1	1.3
default rate	0	0.1
Second moments		
corr($spreads, y$)	-0.81	-0.4
std($spreads$) - p.p.	0	1.5
std(c)/std(y) - p.p.	1.5	2

Note: b denotes total debt service, qb denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

Figure F.4: Policy functions and equilibrium spreads for Spain



F.3 Sunspot shock: Model impulse-response function

In this appendix, we discuss some key model responses after sunspot realization—that is, we compute model impulse-response functions (*irf*) to a sunspot shock.[47] We focus on the responses of credit spreads and debt issuance, which, as discussed in Section 5.2, highlight the difference between endogenous austerity and gambling for redemption behaviors. We begin by discussing how we compute an *irf* in a highly non-linear model like ours, and then compare how the *irf* relate to results discussed in Section 5.2.

Computing *irf* to a sunspot shock Computing an *irf* in a non-linear model requires some assumptions, two of which are particularly important: (1) the point in the state space where the shock occurs, as the response to a shock can drastically differ depending on the state variables; and (2) what happens after the initial shock, since, in non-linear environments, assuming no further shocks can have very different implications that averaging across many potential histories. These are a key difference with linear (or linearized) models, where the effect of the shock is independent of the state where it occurs, and averaging across potential paths equates to a not having further shocks. We

[47]We are thankful to a referee for suggesting the exercise in this appendix.

explain next how we address these two issues, which largely follows the "generalized impulse function" construction proposed in Koop et al. (1996).

We compute the model *irf* starting at point we deem as empirically relevant. For Argentina's calibration, we compute the *irf* starting at low-growth state and debt service of 20%, which reflect Argentina's situation in 2001. For Spain's calibration, we compute the *irf* starting at low-growth state and debt service of 15%, which reflect Spain's situation in 2012. This is the same rationale we we used to select the state for computing the schedules on Section 5.1.1 for Argentina and Section 5.1.2 for Spain. Importantly, these are also states where there is multiplicity.

Given this initial point at $t = 0$, we simulate the economy N times for $t = 1, \ldots, T$. The *irf* at time t is the average across the N simulations. We use $N = 50,000$ and $T = 3$. The initial point we selected is such that there is no default at $t = 0$. We select paths with no default when computing the *irf* for $t \geq 1$, as credit-spreads and issuance are not well defined during periods of market exclusion.

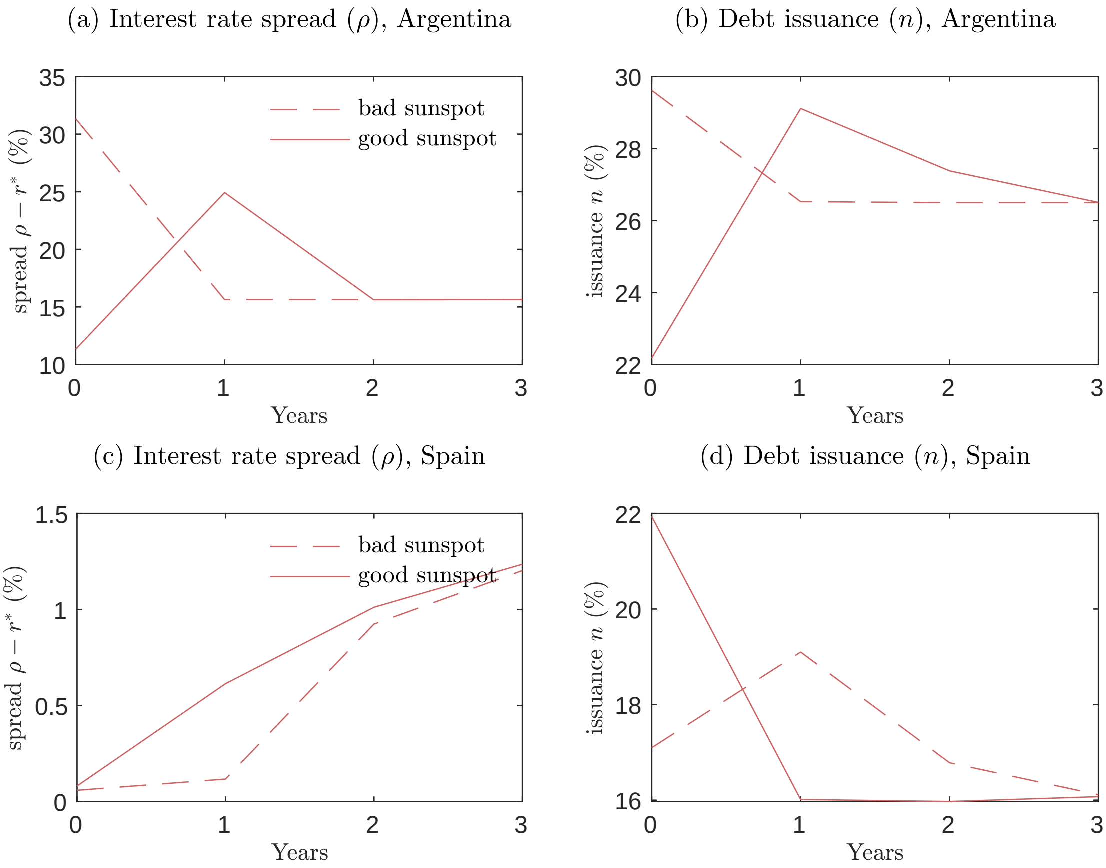
We compute the *irf* computation for two cases: first, a case in which we set the $t = 0$ sunspot realization to be the good case ($s_0 = s_G$), which we label "good sunspot" *irf*; and second, a case in which we set the $t = 0$ sunspot at the bad case ($s_0 = s_B$), which we label "bad sunspot" *irf*. For $t \geq 1$, we draw realization of the sunspot—as we do with other random variables variables in the model. We think of the comparison of the "good sunspot" *irf* and the "bad sunspot" *irf* as measuring the effects of a sunspot realization in the model.

Model *irf* to a sunspot shock Figure F.5 shows the response of credit spreads and issuance under the "good sunspot" *irf* and the "bad sunspot" *irf*. We plot the *irf* both for Argentina (top row) and for Spain (bottom row).

Argentina exhibits a gambling for redemption behavior in "bad sunspot" *irf*. That is, relative to the "good sunspot" *irf*, Argentina issues more debt, and the credit spread consequently increases. In comparison, Spain exhibits an endogenous austerity behavior: issuance is lower under the "bad sunspot" *irf*, actually leading to lower credit spreads in equilibrium that under the "good sunspot" *irf*. Thus, the *irf* computations provide findings similar to those we discussed in Section 5.2.

The *irf* results show same results as previosuly discussed in Figure 10 for Argentina and Figure 11 for Spain. In the presence of multiplicity, a bad sunspot leads to higher debt issuance (gambling for redemption) for Argentina's calibration, and lower debt issuance (endogenous austerity) for Spain's calibration. Additionally, the *irf* shows the dynamic response a few years after the sunspot shock, although only for non-default periods. For Argentina, such non-default periods are largely high-growth periods, and the gambling for redemption strategy leads to a quick reversal of spreads. For Spain, such non-default periods contain both low-growth and high-growth periods, and spreads still increase on

Figure F.5: Impulse response functions



(a) Interest rate spread (ρ), Argentina
(b) Debt issuance (n), Argentina
(c) Interest rate spread (ρ), Spain
(d) Debt issuance (n), Spain

average, although less so under the bad sunspot *irf* because of the endogenous austerity behavior.

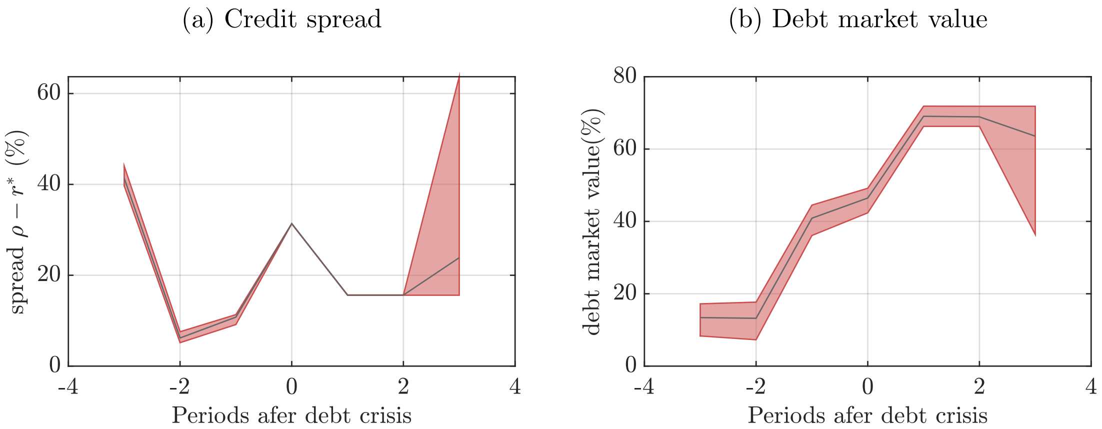
F.4 Debt crisis episodes: Model and evidence

In this appendix, we compare crisis episodes in the model with evidence. We follow the definition of a crisis episode in Paluszynski and Stefanidis (2023), and compare our model results with the evidence they present. As we show, the model predictions align with evidence.

Paluszynski and Stefanidis (2023) define a debt crisis period as a year t subject to two conditions: (1) credit spreads are above one standard deviation of their historical mean, and (2) credit spreads are higher in year t than in years $t-1$ and $t+1$. This last condition implies no default in period $t+1$. They find that in such crisis periods, the (market value of) debt-to-output starts increasing before the crisis period and keeps on increasing for a few years after it eventually goes down—hence their title, "Borrowing into debt crises." See their Figure 10.

We compute debt crisis episodes in the model following exactly the same definition proposed by Paluszynski and Stefanidis (2023). We simulate the model and identify

Figure F.6: Debt crisis episode: model



periods satisfying conditions (1) and (2) above as debt crisis episodes. We then report the average across all the identified debt crisis episodes for three periods before and after the crisis episode. Note that these debt crisis episodes may be induced either by sunspots and/or output shocks. We use Argentina's calibration, as Spain's calibration doesn't exhibit this type of crisis episode. Figure F.6 below shows the model behavior across debt crises periods—shadow areas are 10% confidence intervals, as reported in Paluszynski and Stefanidis (2023).

A debt crisis episode in the model resembles the empirical findings in Paluszynski and Stefanidis (2023). In particular, debt values start increasing before the crisis episode (labeled as $t = 0$) and keep on increasing a few years after, before eventually declining. Thus, the model predicts a behavior of debt around debt crisis episodes that is consistent with evidence.

A caveat must be raised: the sample in Paluszynski and Stefanidis (2023) includes OECD countries, so, for instance, Argentina is not in the sample. We have not found a comparable analysis for emerging economies.

F.5 Model robustness results

In this appendix, we show how the quantitative results of our model change with reasonable perturbations to the benchmark parameters in Table 2. Specifically, we show both the interest rate schedules in the low-growth state (Figures F.7 and F.8) and simulation results (Tables F.2 and F.3) for Argentina and Spain for different values of the parameters related to the persistence of the low- and high-growth states (p_L and p_H), maturity (δ), recovery rate (κ), standard deviation of the transitory shocks to endowment (σ), cost of default in the high-growth state ($\phi(g_H)$), and discount factor (β).

Maturity δ Columns (2) and (3) in Tables F.2 and F.3 report the simulation results for different values of the parameter δ for the cases of Argentina and Spain, respectively. Their respective interest rate schedules under the benchmark calibration are discussed in Section 5.1.1 (Figure 7c) and Section 5.1.2 (Figure 9c). For both Argentina and Spain, the simulation results show that a shorter maturity (higher δ) diminishes the debt dilution incentives, and the borrower is able to issue more debt at lower spreads, associated with lower default rates. In the case of Argentina, when δ moves from 0.15 to 0.60, the average spread decreases from 16.4% to 0.1%, while debt issuance increases from 14% to 35% of GDP. Similarly, for Spain, the average spread decreases from 7.8% to 1.1%, and debt issuance increases from 9% to 24% of GDP when δ increases from 0.10 to 0.20.

Persistence of the low growth-state p_L Figures F.7a and F.8a show the interest rate schedules for higher values of p_L (black dashed line) relative to the benchmark (red solid line). In line with the discussion in Section 2, higher values of p_L are associated with an increase in the high interest rates, and the borrower is able to issue less debt. The simulation results in Tables F.2 and F.3 show that, with higher p_L, the borrower chooses endogenous austerity when facing the higher spreads. In the case of Argentina, default rates decrease from 5.3% to 4.2% when p_L increases to 0.65 relative to the benchmark value of 0.60—see columns (1) and (4) in Tables F.2 and F.3.

Recovery rate κ Figures F.7b and F.8b show the interest rate schedules for recovery rates of 70% (black dashed line), lower than the benchmark of 75% (red solid line). In both cases, lower recovery rates increase the high interest rates and the borrower is able to issue less debt. But the changes are quantitatively small. Tables F.2 and F.3 show that the simulation results with lower values of κ are very similar to the benchmark case—see columns (1) and (5) in Tables F.2 and F.3.

Standard deviation of the transitory shocks to endowment σ Figures F.7c and F.8c show the interest rate schedules for higher values of σ (black dashed line) relative to the benchmark (red solid line). In this case, the variation in σ has different implications for the schedules and simulation results between Argentina and Spain. In the case of Argentina, the interest rate schedules shift to the left, so the borrower is able to issue less debt in equilibrium. This is reflected in column (6) in Table F.2, in which average issuance decreases to 24% of GDP in comparison to 27% in the benchmark case in column (1). In the case of Spain, the interest rate schedules shift to the right, so the borrower is able to actually issue more debt in equilibrium. This is reflected in column (6) in Table F.3, in which average issuance decreases to 19% of GDP in comparison to 18% in the benchmark case in column (1). The differences between Argentina and

Spain reflects two opposing effects of increasing σ. On the one hand, a higher σ reduces the value of being in default, since smoothing consumption through debt issuance is no longer an option, thus increasing possible debt issuances to the borrower when not in default. On the other hand, a higher σ makes bad draws more likely, increasing default probabilities and, thus, lowering possible debt issuances to the borrower. In the case of Argentina, in which the borrower adopts a gambling for redemption strategy, the lower value of repayment dominates, and the borrower is able to issue less debt. In the case of Spain, which adopts an austerity strategy, the lower value of default dominates, and the borrower is able to issue more debt. Overall, the quantitative implications of a change in σ that we analyzed are small, even though we chose values of σ above the 95th percentile of the posterior distribution for most countries in Table 1.

Cost of default in the high-growth state $\phi(g_H)$ Figures F.7d and F.8d show the interest rate schedules for higher values of $\phi(g_H)$ (black dashed line) relative to the benchmark (red solid line) for Argentina and Spain, respectively. Higher values of $\phi(g_H)$ represent a lower cost of default, which makes default relatively more attractive. As a result, for both Argentina and Spain, the interest rate schedules shift to the left, meaning that the borrower can issue less debt in equilibrium. By comparing the simulation results in Tables F.2 and F.3, the results with lower default costs in column (7) are very similar to the benchmark case in column (1).

Persistence of the high growth-state p_H Figures F.7e and F.8e show the interest rate schedules for lower values of p_H (black dashed line) relative to the benchmark (red solid line) for Argentina and Spain, respectively. A lower p_H means that the high-growth state is less persistent, so the economy is more likely to enter a stagnation period. This increases the high interest rates in the high-growth schedules, making the gambling for redemption strategy less attractive. With lower incentives to gamble for redemption, and therefore to default, the borrower is able to issue more debt at lower rates. This explains the schedules shifting to the right in both Figures F.7e and F.8e. In the case of Argentina, that actually pushes the borrower to end its gambling for redemption strategy and to adopt austerity instead, so the interest rates become much lower. This outcome is reflected in the simulation results in column (8) in Tables F.2 and F.3. In the case of Argentina, default rates decrease from 5.3% in the benchmark case in column (1) to 0.3% in column (8).

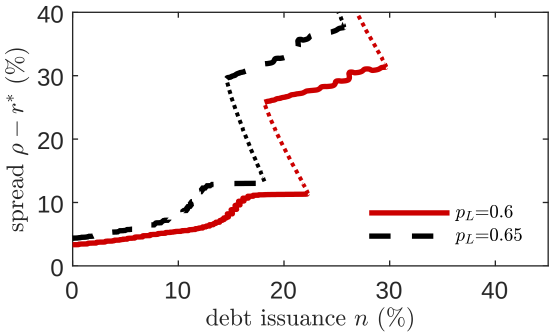
Discount factor β Figures F.7f and F.8f show the interest rate schedules for higher values of β (black dashed line) relative to the benchmark (red solid line) for Argentina and Spain, respectively. A higher value of β means that the borrower is more patient and therefore less likely to adopt a gambling for redemption strategy. Thus, for both for

Argentina and Spain, the interest rate schedules shifts to the right with higher β, which means that the borrower can issue more debt at lower rates. Tables F.2 and F.3 show that default rates are significantly lower with higher β relative to the benchmark—see columns (9) and (1) in Tables F.2 and F.3.

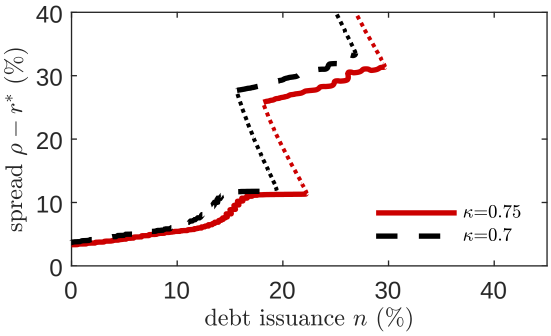
Difference between growth rates ($g_H - g_L$) Figures F.7g and F.8g show the interest rate schedules for different distances between the high- and low-growth rates, which we denote $\Delta^g \equiv g_H - g_L$. We pick g_L and g_H such that, as we vary Δ^g, the (unconditional) average growth rate of the economy remains the same. Therefore, a larger difference Δ^g implies a higher g_H and a lower g_L relative to the benchmark. For Argentina (Figure F.7g) we compare the benchmark with $\Delta^g = 8\%$ (red solid line), to a case with a larger difference in growth rates, $\Delta^g = 10\%$ (black dashed line). For Spain (Figure F.8g), we compare the benchmark with $\Delta^g = 5\%$ (red solid line), to a case with a lower difference in growth rates, $\Delta^g = 4\%$ (black dashed line). The plots show that a higher Δ^g shifts the schedule right for Argentina, while the opposite occurs for Spain. Accordingly, a larger Δ^g increases debt issuance for Argentina, while it actually lowers it for Spain–see columns (10) and (1) in Tables F.2 and F.3. The differences between Argentina and Spain reflect two opposing effects of increasing Δ^g. On the one hand, a larger Δ^g implies a more volatile output path as well as a lower g_L, both of which decrease the value of default, thus increasing possible debt issuances to the borrower. On the other hand, a larger Δ^g implies lower output realizations in the low-growth state, which lowers the value of repayment in low-growth periods, thus increasing default rates and lowering possible debt issuances. For Argentina, the reduction in the value of default dominates, and the economy is able to issue more debt. For Spain, the increase in the value of repayment dominates. Importantly, the overall changes are quantitatively small, and the plots show that the multiplicity in the interest rate schedules is robust to reasonable changes in Δ^g.

Figure F.7: Interest rate spreads for Argentina in the low growth state ($b = 20\%$): Comparative statics

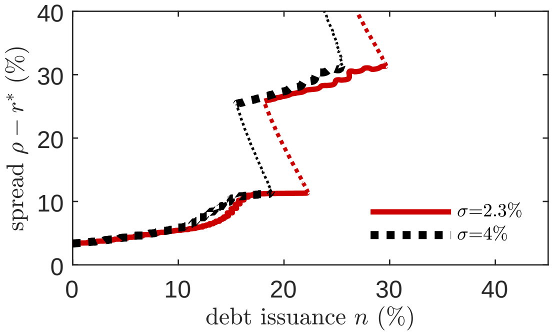
(a) Probability of remaining in low growth (p_L)



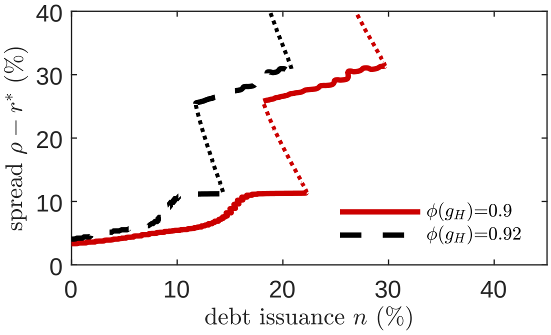
(b) Recovery rate (κ)



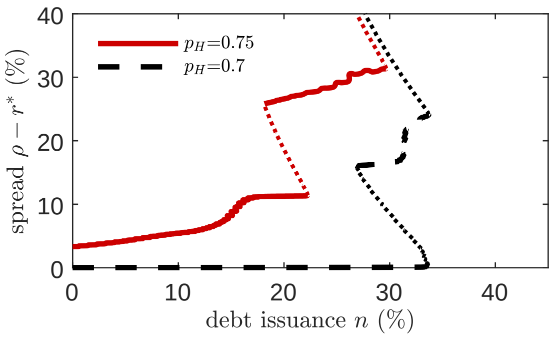
(c) St. deviation of transitory shock (σ)



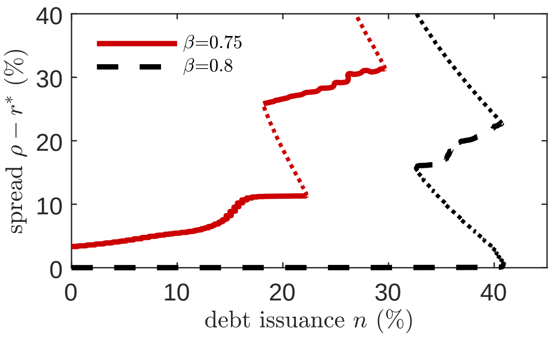
(d) Cost of default in high growth ($\phi(g_H)$)



(e) Probability of remaining in high growth (p_H)



(f) Discount factor (β)



(g) Difference between growth rates ($g_H - g_L$)

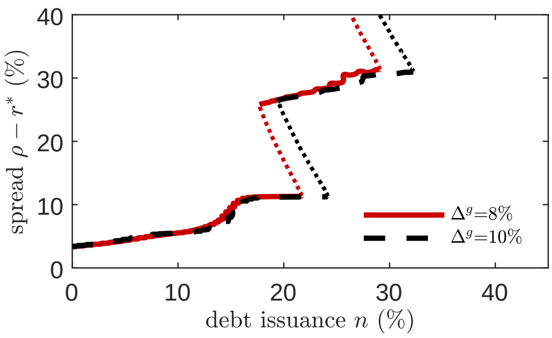
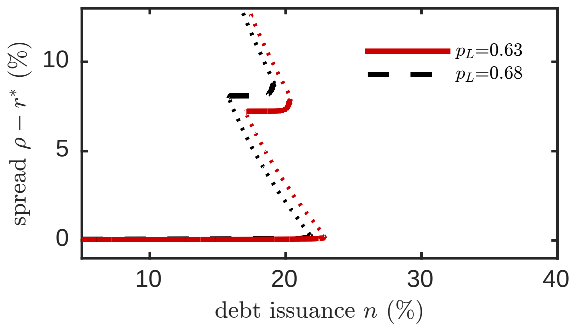
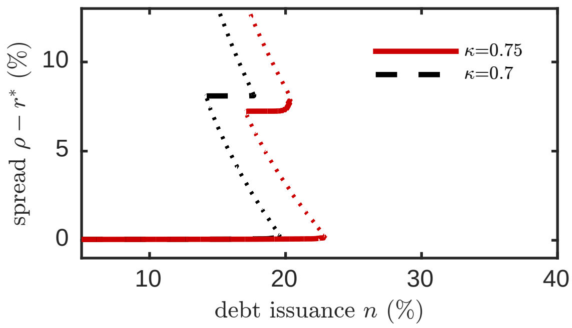


Figure F.8: Interest rate spreads for Spain in the low growth state ($b = 15\%$): Comparative statics

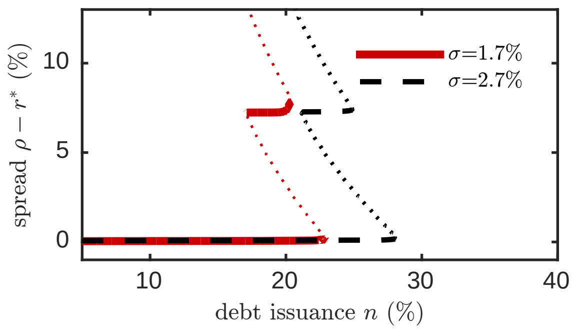
(a) Probability of remaining in low growth (p_L)



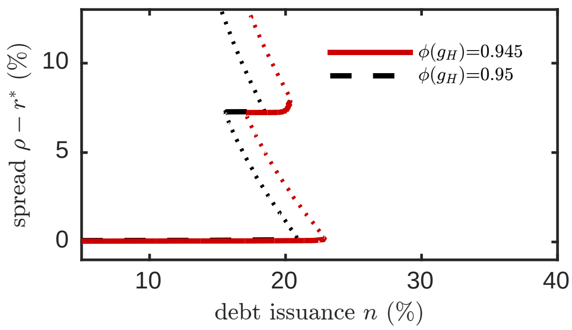
(b) Recovery rate (κ)



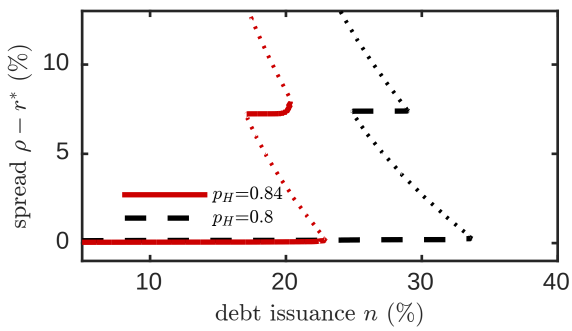
(c) St. deviation of transitory shock (σ)



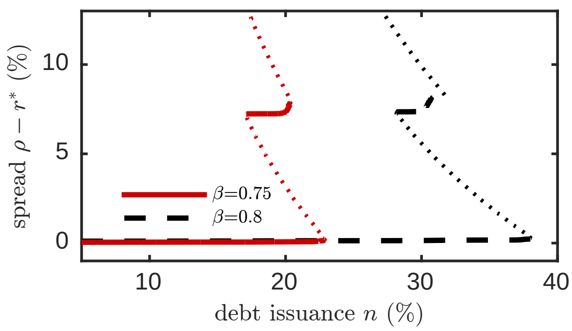
(d) Cost of default in high growth ($\phi(g_H)$)



(e) Probability of remaining in high growth (p_H)



(f) Discount factor (β)



(g) Difference between growth rates ($g_H - g_L$)

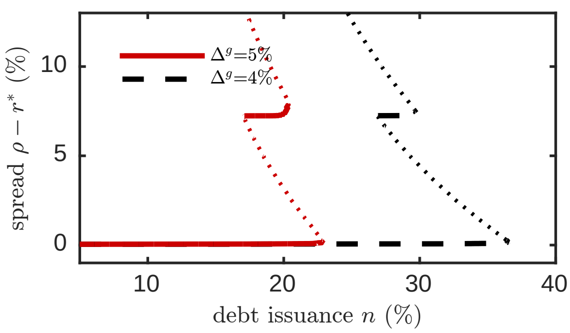


Table F.2: Simulation moments: Argentina

	Benchmark (1)	$\delta = 0.15$ (2)	$\delta = 0.6$ (3)	$p_L = 0.65$ (4)	$\kappa = 0.70$ (5)	$\sigma = 4\%$ (6)	$\phi(g_H) = 0.92$ (7)	$p_H = 0.7$ (8)	$\beta = 0.8$ (9)	$g_H - g_L = 10\%$ (10)
First moments (%)										
avg($spread$)	16.2	16.4	0.1	17.4	17.4	16.2	16	0.2	0.1	16
avg(qb/y)	56	51	60	50	55	46	43	65	73	62
avg(f/y)	53	52	58	50	49	47	43	63	70	56
avg(n/y)	27	14	35	26	26	24	22	25	29	29
avg(b/y)	31	18	37	31	30	28	25	27	31	33
avg(tb/y)	4.4	4.5	1.6	5	4.1	4	3.6	2	1.9	4.4
default rate	5.3	6	0.3	4.2	6.1	5.6	5.1	0.3	0.2	4.6
Low-growth state										
avg($spread$)	27.6	23.6	0.1	34.6	30.3	26.8	26.4	0.2	0.1	27.1
avg(qb/y)	28	11	63	15	32	19	18	68	76	36
avg(f/y)	41	43	61	37	39	38	34	65	73	43
avg(n/y)	26	17	33	24	25	23	21	23	26	28
avg(b/y)	24	15	39	23	24	23	20	29	32	25
avg(tb/y)	-2	-1.9	5.2	-1.7	-1.2	-0.6	-1	5.3	6	-3
default rate	13.6	15.3	0.7	10.1	15.6	14.4	13	0.7	0.6	12
High-growth state										
avg($spread$)	15.5	15.6	0.1	16.7	16.2	15.2	15.3	0.2	0.1	15.3
avg(qb/y)	58	55	59	51	57	49	45	63	71	64
avg(f/y)	54	53	57	51	50	48	43	61	68	57
avg(n/y)	27	13	37	26	26	24	22	27	30	29
avg(b/y)	32	19	36	31	30	29	26	27	30	34
avg(tb/y)	4.8	5.2	-0.7	5.3	4.6	4.4	3.9	-0.5	-0.6	4.8
default rate	0	0.1	0	0	0	0.1	0	0	0	0
Second moments										
corr($spreads, y$)	-0.52	-0.65	0.03	-0.51	-0.59	-0.43	-0.5	-0.21	-0.12	-0.55
std($spreads$) - p.p.	3.7	2.6	0.1	3.9	4.6	4.2	3.6	0.1	0	3.5
std(c)/std(y) - p.p.	2.4	2.1	1.6	2.4	2.2	1.6	2	1.6	1.7	2.3

Note: b denotes total debt service, qb denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

Table F.3: Simulation moments: Spain

	Benchmark (1)	$\delta = 0.1$ (2)	$\delta = 0.2$ (3)	$p_L = 0.68$ (4)	$\kappa = 0.70$ (5)	$\sigma = 2.7\%$ (6)	$\phi(g_H) = 0.95$ (7)	$p_H = 0.8$ (8)	$\beta = 0.8$ (9)	$g_H - g_L = 4\%$ (10)
First moments (%)										
avg($spread$)	1.2	7.8	1.1	1.2	1.1	1.1	1.1	1.2	0.2	0.8
avg(qb/y)	88	52	98	88	86	95	87	100	110	105
avg(f/y)	86	52	93	85	83	91	84	96	106	100
avg(n/y)	18	9	24	18	18	19	18	20	18	20
avg(b/y)	17	11	23	17	16	18	16	19	20	19
avg(tb/y)	-1.4	1.6	-0.9	-1.2	-1.9	-1.5	-1.6	-1.2	1.9	-0.4
default rate	5.2	6	4.7	5.3	5.1	5.1	5.4	5.4	0.4	4.2
Low-growth state										
avg($spread$)	0.1	9.4	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.1
avg(qb/y)	80	33	95	81	77	88	78	95	114	103
avg(f/y)	84	47	93	84	80	90	82	95	110	101
avg(n/y)	17	10	21	16	17	18	17	18	15	18
avg(b/y)	16	10	23	16	15	17	16	18	21	19
avg(tb/y)	-0.9	0.2	1.1	-0.1	-2	-0.3	-1	0.5	6	1.2
default rate	18	20	14.7	15.6	17	16.9	17.3	15.4	1.3	14
High-growth state										
avg($spread$)	1.4	7.5	1.4	1.4	1.4	1.3	1.3	1.5	0.2	1
avg(qb/y)	89	55	99	89	88	97	88	102	109	105
avg(f/y)	86	53	93	85	83	92	84	96	105	100
avg(n/y)	18	9	24	18	18	20	18	21	19	20
avg(b/y)	17	11	23	17	16	18	16	19	20	19
avg(tb/y)	-1.5	1.8	-1.4	-1.4	-1.9	-1.8	-1.7	-1.8	0.3	-0.8
default rate	0	0.1	0.4	0	0	0.1	0	0	0	0
Second moments										
corr($spreads, y$)	0.46	-0.36	0.27	0.49	0.46	0.28	0.45	0.42	-0.66	0.15
std($spreads$) - p.p.	0.7	1.3	1	0.7	0.7	0.7	0.7	0.9	0	0.8
std(c)/std(y) - p.p.	2.7	2.3	2.8	2.6	2.9	2.3	2.7	2.8	1.9	2.9

Note: b denotes total debt service, qb denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.