# LOLR Model and Sunspot Results

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# Economic Environment

- State variables in period $t$: private debt $b_t$, LOLR debt $l_t$, growth state $g_t \in \{g_L, g_H\}$, and transitory shock $\varepsilon_t$.
- Output in repayment: $y_t = g_t \exp(\sigma_\varepsilon \varepsilon_t)$.
- Output in default: $y_t^D = \phi_{g_t} y_t$ with state-dependent output loss.
- Growth follows a Markov chain $P_g$; $\varepsilon_t$ is iid with invariant probabilities $\pi_\varepsilon$.
- International lenders are risk-neutral with gross risk-free rate $1 + r^*$.

# Government Problem in Repayment

Given current $(b, l, g, \varepsilon)$ and no default, the sovereign chooses next debt $(b', l')$:

$$V^{ND}(b, l, g, \varepsilon) = \max_{b', l'} \left\{ u(c) \; + \; \beta g^{1-\gamma} \mathbb{E}\Big[ \max\{ V^{ND}(b', l', g', \varepsilon'), V^{D}(b', l', g', \varepsilon') \} \Big] \right\}.$$

Consumption in repayment is

$$c = y - b - l + n(b', l', g) + n_l(l', g),$$

where

$$n(b', l', g) = \frac{g\, b'}{R(b', l', g)}, \qquad n_l(l', g) = \frac{g\, l'}{R_l^{ND}}.$$

# Government Problem in Default

In default, private market issuance is shut down and the sovereign only chooses LOLR debt:

$$V^D(b, l, g, \varepsilon) = \max_{l'} \Big\{ u\Big( y^D - l + \frac{g\, l'}{R_l^D} \Big) \; + \; \beta g^{1-\gamma}\, \mathbb{E}\Big[ \theta V^D(\tilde{b}, l', g', \varepsilon') + (1-\theta) \max\{ V^{ND}(\kappa \tilde{b}, l', g', \varepsilon'), V^D(\tilde{b}, l', g', \varepsilon') \} \Big] \Big\},$$

with transformed debt states

$$\tilde{b} = \frac{b}{g}, \qquad \kappa \tilde{b} \text{ after restructuring on re-entry.}$$

# How $Q$ and $X$ Are Built

At current state $(b, \ell, g, \varepsilon)$:

$$d(b, \ell, g, \varepsilon) = \mathbf{1}\{V^{ND}(b, \ell, g, \varepsilon) < V^{D}(b, \ell, g, \varepsilon)\}.$$

Bond price is the payoff decomposition

$$Q(b, \ell, g, \varepsilon) = (1 - d) + d\, X(b, \ell, g, \varepsilon),$$

where:

- if no default $(d = 0)$, lenders receive 1;
- if default $(d = 1)$, lenders receive continuation value $X$.

Continuation value satisfies

$$X(b, \ell, g, \varepsilon) = \frac{1}{1 + r^*}\, \mathbb{E}\Big[\theta X' + (1 - \theta)\big((1 - e')X' + e'\, \kappa\, Q'\big) \mid g\Big],$$

with $e' = \mathbf{1}\{V^{ND}(\kappa b', \ell', g', \varepsilon') \geq V^{D}(b', \ell', g', \varepsilon')\}$.

So in the code, $(Q, X)$ are obtained jointly as a fixed point given current policy rules and the default/re-entry decisions.

# Pricing, Constraints, and LOLR Policy

- Debt price recursion implies $Q = (1-d) + dX$, with default indicator $d = \mathbf{1}\{V^{ND} < V^{D}\}$.
- Gross market schedule and implied private issuance:

$$R(b', l', g) = \frac{1+r^*}{\mathbb{E}[Q']}, \qquad n(b', l', g) = \frac{g\, b'}{R(b', l', g)}.$$

- Endogenous default probability schedule:

$$p^{def}(b', l', g) = \mathbb{E}[d'].$$

- Feasible repayment choices must satisfy:

$$p^{def}(b', l', g) \leq \bar{p}_u, \quad n(b', l', g) \leq \bar{n}_g, \quad l' \leq \bar{l}^{ND}.$$

- In default, LOLR borrowing is capped by $l' \leq \bar{l}^{D}$.

# Numerical Solution in the Code

The solver in `1period_lolr/src/solver_lolr.jl` iterates:

- Solve default value $V^D$ and default-state LOLR policy $l'_D$.
- Update default decision $d$ and re-entry decision $e = 1 - d$.
- Solve bond-price fixed point for continuation value $X$ and price $Q$.
- Compute schedules $(R, n, p^{def})$.
- Update repayment value $V^{ND}$ under feasibility constraints.
- Recover optimal policy indexes $(b', l')$ for simulation.

Simulation in `simulation_lolr.jl` then generates moments, default episodes, and debt distributions.

# Sunspot Outputs Used in This Section

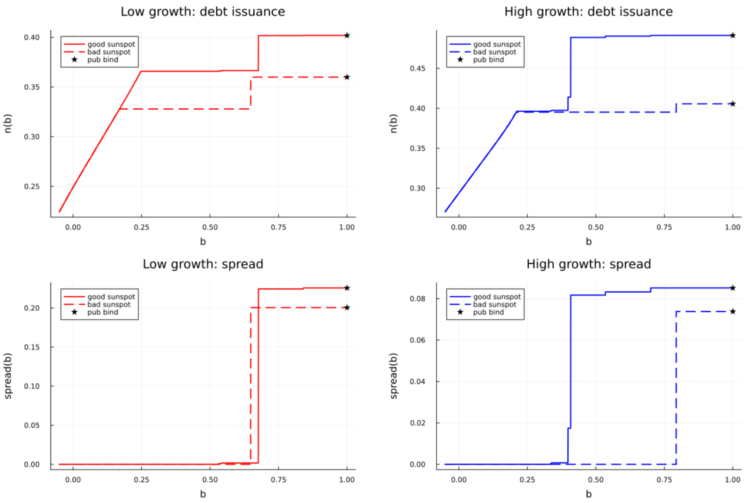
- Moments table file: `../1period_sunspot/result/sunspot_moments_pb.tex`.
- Policy figure file: `../1period_sunspot/result/sunspot_policy_2x2.png`.
- The table compares $P_b = 25\%$ vs. $P_b = 1\%$ for:
  - First moments.
  - Low-growth-state moments.
  - High-growth-state moments.

# Sunspot Moments Table

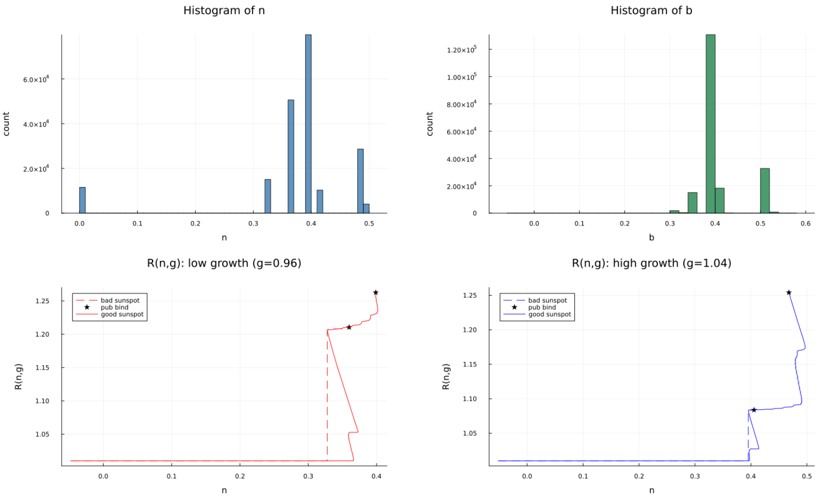
Table: Sunspot moments for alternative bad-sunspot probabilities (P_b)

| Moment | P_b=25% | P_b=1% |
|---|---|---|
| *First moments* | | |
| avg(spread) | 0.0156 | 0.0665 |
| avg(qb/y) | 0.3945 | 0.5363 |
| avg(f/y) | 0.4058 | 0.5797 |
| avg(n/y) | 0.3945 | 0.5363 |
| avg(b/y) | 0.3944 | 0.5288 |
| avg(tb/y) | -0.0001 | -0.0075 |
| default rate | 0.0467 | 0.1402 |
| *Low-growth state* | | |
| avg(spread) | 0.0000 | 0.0016 |
| avg(qb/y) | 0.3712 | 0.4511 |
| avg(f/y) | 0.3749 | 0.4563 |
| avg(n/y) | 0.3712 | 0.4511 |
| avg(b/y) | 0.3947 | 0.4530 |
| avg(tb/y) | 0.0234 | 0.0019 |
| default rate | 0.1207 | 0.3497 |
| *High-growth state* | | |
| avg(spread) | 0.0239 | 0.0838 |
| avg(qb/y) | 0.4069 | 0.5589 |
| avg(f/y) | 0.4222 | 0.6125 |
| avg(n/y) | 0.4069 | 0.5589 |
| avg(b/y) | 0.3943 | 0.5489 |
| avg(tb/y) | -0.0126 | -0.0100 |
| default rate | 0.0004 | 0.0066 |

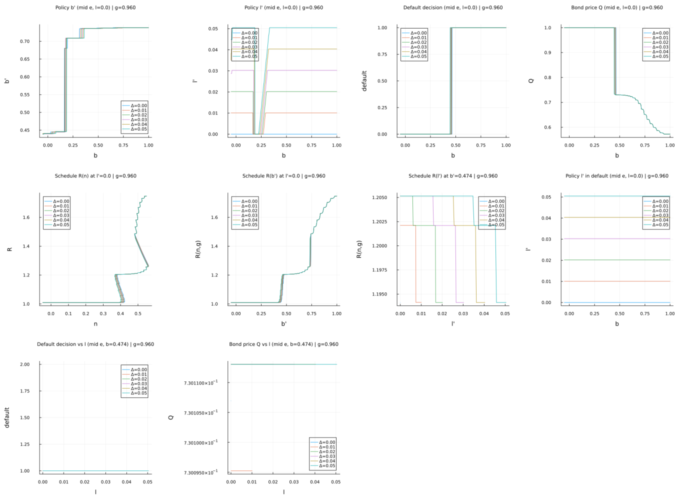
# Sunspot Optimal Policy: 2x2 Figure



# Sunspot n and R Figure



# LOLR Policies by Δ (Spread 0.06)



# LOLR Moments by $\Delta$ at Spread 0.06

Table: LOLR moments across $\Delta$ at fixed spread = 0.06

| Moment | $\Delta$=0.00 | $\Delta$=0.01 | $\Delta$=0.02 | $\Delta$=0.03 | $\Delta$=0.04 | $\Delta$=0.05 |
|---|---|---|---|---|---|---|
| *First moments* | | | | | | |
| avg(spread) | 0.0921 | 0.0923 | 0.0921 | 0.0921 | 0.0921 | 0.0921 |
| avg(qb/y) | 0.6701 | 0.6701 | 0.6701 | 0.6701 | 0.6701 | 0.6701 |
| avg(f/y) | 0.7386 | 0.7386 | 0.7386 | 0.7386 | 0.7386 | 0.7386 |
| avg(n/y) | 0.6701 | 0.6701 | 0.6701 | 0.6701 | 0.6701 | 0.6701 |
| avg(b/y) | 0.6786 | 0.6786 | 0.6786 | 0.6786 | 0.6786 | 0.6786 |
| avg(tb/y) | 0.0085 | 0.0088 | 0.0091 | 0.0093 | 0.0096 | 0.0099 |
| default rate | 0.1480 | 0.1480 | 0.1480 | 0.1480 | 0.1480 | 0.1480 |
| *Low-growth state* | | | | | | |
| avg(spread) | 0.2510 | 0.2510 | 0.2510 | 0.2510 | 0.2510 | 0.2510 |
| avg(qb/y) | 0.5833 | 0.5832 | 0.5833 | 0.5833 | 0.5833 | 0.5833 |
| avg(f/y) | 0.7355 | 0.7353 | 0.7355 | 0.7355 | 0.7355 | 0.7355 |
| avg(n/y) | 0.5833 | 0.5832 | 0.5833 | 0.5833 | 0.5833 | 0.5833 |
| avg(b/y) | 0.4220 | 0.4220 | 0.4220 | 0.4220 | 0.4220 | 0.4220 |
| avg(tb/y) | -0.1614 | -0.1601 | -0.1592 | -0.1581 | -0.1571 | -0.1560 |
| default rate | 0.3880 | 0.3880 | 0.3880 | 0.3880 | 0.3880 | 0.3880 |
| *High-growth state* | | | | | | |
| avg(spread) | 0.0921 | 0.0922 | 0.0921 | 0.0921 | 0.0921 | 0.0921 |
| avg(qb/y) | 0.6702 | 0.6701 | 0.6702 | 0.6702 | 0.6702 | 0.6702 |
| avg(f/y) | 0.7386 | 0.7386 | 0.7386 | 0.7386 | 0.7386 | 0.7386 |
| avg(n/y) | 0.6702 | 0.6701 | 0.6702 | 0.6702 | 0.6702 | 0.6702 |
| avg(b/y) | 0.6788 | 0.6788 | 0.6788 | 0.6788 | 0.6788 | 0.6788 |
| avg(tb/y) | 0.0086 | 0.0089 | 0.0091 | 0.0094 | 0.0097 | 0.0099 |
| default rate | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |