1 A two period model:

The economy is populated by a representative agent that draws utility from consumption in each period, and by a continuum of risk-neutral foreign lenders.

The representative agent preferences are given by

$$u(c_1) + \beta \mathbb{E} u(c_2),$$

where u is strictly increasing, strictly concave and satisfies standard Inada conditions.

The initial wealth of the agent is denoted by ε, which we take it to be arbitrarily small. The endowment in the second period is distributed according to

$$y_2 = \begin{cases} y^l, & \text{with probability } p \\ y^h, & \text{with probability } (1-p) \end{cases}$$

in which $y^l < y^h$. As the initial wealth is low, the agent will want to borrow.

1.1 The "catalytic" lender of last resort.

We assume there is an international non-for profit institution (the European Central Bank or the International Monetary Fund) that is willing to lend an amount b^o at an interest rate R^o. We will focus only on cases in which $R^o \geq R^*$, where R^* is an international risk free rate.

We will study how the set of equilibria depends on these two policy instruments, (b^o, R^o). We denote this international institution as LOLR

1.2 Private Lenders

Assume first that $R^o = R^*$.

In period one, the borrower moves first and issues a non-contingent debt level b. Lenders respond with an interest rate R. As $R = R^*$, without loss of generality, the country would first borrow from the LOLR up to the maximum, and then will borrow the rest from private agents.

We denote by $R(b, b^o)$ the interest rate schedule faced by the borrower. In period two, after observing the endowment y_2, the borrower decides whether to pay the debt or to default. In case of repayment, the borrower consumes the endowment net of debt repayment, $c_2 = y_2 - [R^* b^o + R(b - b^o)]$. In case of default, consumption is $c_2 = y^d$. The agent defaults if the cost of repayment is larger than the benefit:

$$\underbrace{[R^* b^o + R(b - b^o)]}_{\text{cost of repayment}} > \underbrace{y_2 - y^d}_{\text{benefit of repayment}}.$$

In the first period, given initial wealth ε and an interest rate schedule

$R(b, b^o)$, the borrower solves the following problem

$$V(\omega) = \max_b \left\{ u(c_1) + \beta \mathbb{E} u(c_2) \right\}, \tag{1}$$

$$\text{subject to } \; c_1 = \omega + b,$$

$$c_2 = \max \left\{ y_2 - [R^* b^o + R(b - b^o)], y^d \right\},$$

and subject to a maximum debt level constraint, $b \leq \overline{B}$.

The assumption that the borrower moves first by choosing a level of debt and that lenders move next with an interest rate schedule is standard. We depart from the literature, as in Aguiar+Gopinath:2006 and Arellano:2008, in that we assume that the borrower chooses current debt b, rather than debt at maturity, Rb. The risk-neutral lenders will be willing to lend to the agent as long as the expected return is the same as the risk-free rate R^*, that is,

$$R^* = h(R; b, b^o) \equiv \left[1 - \Pr\left(y_2 - y^d < [R^* b^o + R(b - b^o)]\right)\right] R, \tag{2}$$

in which $h(R; b, b^o)$ is the expected return to the lender when the interest rate is R. Given a value for b, the expected return for lenders can be written as

$$h(R; b) = \begin{cases} R, & \text{if } [R^* b^o + R(b - b^o)] \leq y^l - y^d \\ R\ (1 - p), & \text{if } y^l - y^d < [R^* b^o + R(b - b^o)] \leq y^h - y^d \\ 0, & \text{if } [R^* b^o + R(b - b^o)] > y^h - y^d. \end{cases} \tag{3}$$

1.2.1 The case $b^o = 0$.

In Figure **??**, we plot the expected return as a function of the interest rate R, for three levels of debt, together with the risk-free rate R^*. Notice that for low levels of R, the expected return is equal to R since debt is repaid with probability one. In this region, as R increases, the expected return increases one to one. Eventually, R will be high enough that the borrower will default in the low output state, which happens with probability p. At this point, the expected return jumps down. As R increases, the expected return increases at a lower rate, $(1 - p)$, since repayment happens only in the high output state. Finally, for high enough R, default will happen with probability one and the expected return will be zero. A higher level of debt decreases the expected return, uniformly, shifting the curves downwards.

For low levels of debt, there is only one solution to equation (2), with $R = R^*$. For intermediate levels of debt, there are two solutions: one solution has $R = R^*$ associated with zero probability of default, the other has $R = R^*/(1-p)$ associated with probability of default equal to p. For higher levels of debt, the only solution is the high rate $R = R^*/(1 - p)$. Finally, for even higher debt, there is no solution. There are multiple solutions only for intermediate levels of debt.

We can now define the following correspondence relating debt levels to interest rates,

$$\mathcal{R}(b) = \begin{cases} R^*, & \text{if } b \le \frac{y^l - y^d}{R^*} \\ \frac{R^*}{1-p}, & \text{if } \frac{y^l - y^d}{R^*/(1-p)} < b \le \frac{y^h - y^d}{R^*/(1-p)} \\ \infty, & \text{if } b > \frac{y^h - y^d}{R^*/(1-p)} \end{cases} \tag{4}$$

An equilibrium is an interest rate schedule $R(b)$ and a debt policy function $b(\omega)$ such that, given the schedule, the debt policy function solves the problem of the borrower in (1), and the schedule $R(b)$ is a selection of the correspondence $\mathcal{R}(b)$.

The correspondence $\mathcal{R}(b)$ is plotted in Figure **??**. For all debt levels below $b_1 \equiv \frac{y^l - y^d}{R^*/(1-p)}$, there is only one interest rate, the risk-free rate. For debt levels between b_1 and $b_2 \equiv \frac{y^l - y^d}{R^*}$, there are two possible interest rates, the risk-free rate and a high rate. For debt levels between b_2 and $\bar{b} \equiv \frac{y^h - y^d}{R^*/(1-p)}$ there is again only one interest rate, the high rate. There are multiple interest rate schedules that can be selected from this correspondence. We focus on two of those schedules: a low interest rate schedule, $R^{low}(b)$ in Figure **??**, and a high interest rate schedule, $R^{high}(b)$ in Figure **??**. We think of b_1 as the debt level above which interest rates jump because of expectations, since alternative expectations could sustain low interest rates. We think of b_2 as the debt level above which interest rates jump because of fundamentals, since no expectations could sustain lower interest rates.

Whether spreads are low or high has implications for the level of debt that can be raised. The region of multiplicity happens for intermediate levels of debt, between b_1 and b_2. If debt is sufficiently low, interest rates can only be low, while if debt is sufficiently high, rates can only be high. It is for intermediate levels of debt that interest rates can be either high or low depending on expectations.

Figure **??** shows the optimal debt policy as a function of the initial wealth for the high and low interest rate schedules. For high levels of wealth, the optimal choice of debt is below b_1 regardless of which schedule the borrower is facing. As wealth decreases, the schedule matters. For the high interest rate schedule, the borrower chooses to keep debt levels at b_1 in order to avoid the discrete jump in interest rates on the whole level of debt. Eventually, for low enough wealth, the marginal utility in the first period is high enough that the borrower chooses to increase its debt level discretely. This discrete jump shows that the borrower has incentives to avoid at least part of the multiplicity region between b_1 and b_2. As wealth decreases even more, debt levels keep increasing until they reach the borrowing limit $\bar{b}$. When facing the low interest rate schedule, borrowing keeps on increasing as wealth declines until it reaches the level b_2. At this point, there is a choice to keep it constant for lower levels of wealth. Eventually, there is also a discrete jump, and debt continues to increase until they reach the borrowing limit.

The choice of keeping debt levels constant as wealth decreases is a form of endogenous austerity. This happens in our model because of the discrete jumps in interest rates induced by both expectations and

1.2.2 The case $b^o > 0$.

As we are interested in exploring policies that may rule out bad equilibria without the LOLR loosing money, we will focus only on the cases for which there is multiple equilibria, as depicted in Figure XXX. Notice that the good equilibrium implies that there is never default, so the equilibrium interest rate faced by the country satisfies $R = R^*$.

Consider now the case in which the policy parameter b^o is strictly positive. As before, we first study the function $h(R; b, b^o)$, which is the expected return to the lender when the interest rate is R.

As shown in Figure XXX, there are two thresholds that matter: the first is the value for R such that the country is indifferent between paying the debt and defaulting in the low income state. For lower values of this threshold, the country pays in both states of nature and the return is R. The second threshold is the - higher - value for R such that the country is indifferent between paying the debt and defaulting in the high income state. The country only defaults in the low income state if R is in between the thresholds, so the expected return is $(1 - p)R$. Finally, the country defaults in every state for interest rates higher that the second threshold, so the return is zero.

To understand the effect of the policy parameter b^o, we therefore only need to understand how it changes the two thresholds.

The first threshold is defined by the value for R^{T_1} equality

$$R^* b^o + R^{T_1}(b - b^o) = y^l - y^d,$$

which implies

$$R^{T_1} = \frac{y^l - y^d - R^* b^o}{(b - b^o)} = R^{T_1}(b, b^o).$$

Notice that

$$\frac{\partial R^{T_1}}{\partial b^0} = \frac{\left[y^l - y^d - bR^*\right]}{(b - b^o)^2}$$

As we assumed that b was such that the good - and the bad - equilibrium exists, then $\left[y^l - y^d - bR^*\right] > 0$, so the derivative is positive. Thus, as b^0 is increased, the threshold R^{T_1} moves to the rigth - i.e., it increases.

Thus, this "catalytic" policy has the same effect on the thresholds as a reduction in total debt. Thus, it moves the multiplicity region to the right. Therefore, there is a potential effect on the equilibrium. Imagine, for instance that the the original equilibrium had the country borrowing in the multiplicity region, at high spreads, when $b^0 = 0$. Using the language adopted above, in this equilibrium the country was "gambling for redemption", at high rates. Imagine, on the other hand, that the country was almost indiferent betwen this action and "endogenous austerity". Then, it follows that a relatively low value of "catalytic" lending can push the country towards "endogenous austerity" and low rates.

Conclusions:

1. The "whatever it takes" policy rules out multiple equilibria.

2. The "catalytic" approach moves the multiplicity region and therefore it can, well designed, affect the country's actions and therefore the equilibrium interest rates. It is conceivable that, as the new rates are lower, the country ends up borrowing even more that in the bad equilibrium with $b^0 = 0$.

A final point: Imagine now that - as it is in reality - an agreement with the LOLR implies a commitment to a maximum amount the country can borrow, but it is the choice of the country to tap on those finds or not.[1]

So far, I have assumed that $R^o = R^*$. Imagine now that $R^o = R^* + \delta$, for very small δ. Clearly, as the equilibrium is either $R = R^*$ or $R = \frac{R^*}{1-p}$, in the good equilibrium, the country would never tap on the funds of the LOLR, since it charges a higher rate. However, I conjecture that the commitment to lend at $R^o = R^* + \delta$ moves the return function $h(R; b, b^o)$ in a similar fashion, so it would still imply moving the multiplicity region to the right, indusing endogenous austerity, even though the funds are never used. One way to prove this conjecture would be by taking the limit when $\delta \to 0$.

[1] Incidentally, during 1997 and 1998, Argentina had an aggrement with the Fund, but did not tap on those, it chose to issue bonds in private markets. I also think - this must be checked - that Clinton's and the IMF support package for Mexico, that totalled about 30 billion I believe, was not totally tapped - and it weas returned completely in about a year.