Think of an economy with two agents: a borrower (Argentina), and a lender of last resort (LOLR or IMF). Argentina can borrow from private markets but cannot commit to repay. The IMF cares abut Argentina's welfare and can lend to Argentina. Yet, the IMF prefers to minimize its lending and favors private market lending. More details on the IMF's problem below.

Let b denote private markets debt, and ℓ denote the IMF loan. I'm assuming all debt is one period. Endowment y is exogenous, and z collects all exogenous (fundamental) variables. Additionally, there can be multiple equilibria in private markets, and a sunspot s determines the equilibrium in such cases.

Timing is as follows. Debt values from previous periods–b and ℓ–are known. Endowment z and sunspot s are realized. Then, IMF moves first and determines loan size ℓ. Argentina moves next, and decides borrowing from private markets b.

I'm allowing from the IMF to move first as a way for them to internalize their role as a LOLR. [But there is a lot of time-consistency issues here. So the IMF moving first can be interpreted as within-period commitment by the IMF.]

We solve the problem backwards. [I'm using value functions here, assuming Markov equilibria. But this is not an obvious decision]

Argentina's problem is given as

$$v^{nd}(b,\ell,s,z) = \max_{c,n} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}\left[\max\left\{ v^{nd}(b',\ell',s',z'), v^{d}(b',\ell',s',z') \right\} | z,s \right] \right\} \tag{1}$$

s.t.

$$c + b + R^{\ell}(\cdot)\ell = y(z) + n + \ell'$$
$$b' = R(n,\ell',s,z)n$$
$$\ell' \leq \hat{\boldsymbol{\ell}}'(b,\ell,s,z) \qquad (\to \text{IMF policy}) \tag{2}$$

where R^{ℓ} is the rate that the IMF charges (a parameter or a choice). Let $\boldsymbol{b}'(b,\ell,s,z)$, $\boldsymbol{\ell}'(b,\ell,s,z)$, and $\boldsymbol{d}(b,\ell,s,z)$ be the optimal debt issuance and default policies.

IMF's problem

$$\xi(b, \ell, s, z, d) = \min_{\hat{\ell}'} \left\{ \ell' + \beta^{\ell} \mathbb{E}[\xi(b', \ell', s', z', d') | s, z] \right\} \tag{3}$$

$$\text{s.t.}$$

$$v^{nd}(b, \ell, s, z) \geq v_{min}$$

$$\hat{\ell}' \in [0,\ \ell_{max}]$$

$$b' = \boldsymbol{b}'(b, \ell, s, z),\ \ \ell' = \boldsymbol{\ell}'(b, \ell, s, z), \quad \text{and} \quad d = \boldsymbol{d}(b, \ell, s, z)$$

where d is the default state of Argentina (in default or not), and β^{ℓ} is the IMF's discount factor.

Comments on the IMF problem

1) There are two main constraints to the IMF. First, they need to ensure a minimum utility level to Argentina: v_{min}. Second, the IMF faces a limit on how much they can spend: ℓ_{max}.

   - I expect a lot of inaction from this problem. Often, Argentina would be far from v_{min} and thus the IMF would achieve the utility constraint at no cost.
   - The IMF's policy is $\hat{\boldsymbol{\ell}}'(\cdot)$ is actually a credit line, and Argentina may decide not to use all of it. That's why we have $\ell' \leq \boldsymbol{\ell}'(\cdot)$ in Argentina's problem.

2) We are using a dual approach for the IMF. I don't know what the IMF's objective should be, but we assume that they want to deliver it at a minimum cost.

3) The IMF internalizes the optimal responses of Argentina—last line in equation (3). But Argentina also internalizes the policy that the IMF will have (this and next period).

4) We should extend the IMF problem to describe utility constraint when Argentina is in default.

We could take different approaches. One approach is to cook-up some policy $\hat{\boldsymbol{\ell}}'(\cdot)$, and compute outcomes for Argentina under different cases of the IMF policy. A second approach is to actually solve for the two problems together and obtain a sense of optimal policy response by a LOLR. The second approach is conceptually more challenging: we need to think about many details that no one solved before. The second approach would also be computationally more challenging. But I'm really curious to know what the second approach yields. Maybe on an easier problem?