Self-Fulfilling Debt Crises with Long Stagnations

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We assess the quantitative relevance of expectations-driven sovereign debt crises, focusing on the southern European crisis of the early 2010s and the Argentine default of 2001. The source of multiplicity is the one proposed by Calvo. Crucial for multiplicity is an output process characterized by long periods of either high growth or stagnation, which we estimate using data for these countries. Our analysis suggests that expectations-driven debt crises are quantitatively relevant but state dependent, as they occur only during periods of stagnation. Expectations and how they respond to policy are the major factors explaining default rates and credit spread differences between Spain and Argentina.

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I. Introduction

How important are expectations in triggering sovereign debt crises? In this paper, we explore the quantitative implications of a model of sovereign debt crises that exhibits state-dependent multiplicity. The mechanism we consider to generate multiplicity is the one proposed by Calvo (1988), in which high interest rates induce high default probabilities that in turn justify the high rates. We build on Ayres et al. (2018), who argue that the mechanism in Calvo (1988) is of interest when the fundamental uncertainty is bimodal, with both good and bad times.[1] Our analysis suggests that the mechanism is quantitatively relevant, especially during periods of economic stagnation.

Our analysis of self-fulfilling equilibria in sovereign debt markets is motivated by two main episodes. The first one is the European sovereign debt crisis of the early 2010s, when spreads on Italian and Spanish public debt exceeded 5% after being close to zero from the introduction of the euro until April 2009. Spreads were considerably higher in Portugal and especially in Ireland and Greece. The crisis receded substantially after Mario Draghi's "whatever it takes" speech on July 26, 2012.[2] Shortly afterward, Draghi's words were confirmed by the commitment of the European Central Bank (ECB) to purchase sovereign debt bonds though the Outright Monetary Transactions (OMT) program. Two features of this episode point toward multiple equilibria. First, the abrupt movements in spreads were only weakly correlated with the usual fundamentals in sovereign debt models.[3] Second, spreads declined after the policy announcement, without the OMT bond-buying scheme being actually used. The potential self-fulfilling nature of these events in summer 2012 was explicitly used by Draghi to justify the policy.

The second episode is the 1998–2002 Argentine crisis. Back in 1993, Argentina had regained access to international capital markets, but the average country spread on dollar-denominated bonds from 1993 to 1998 was close to 7%. The average ratio of debt to gross domestic product (GDP) was 35% for those years, and the average yearly growth rate of GDP was around 5%. But a recession had started by the end of 1998, and the Argentine

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[1] See Lorenzoni and Werning (2019), who also discuss multiplicity as in Calvo (1988) using a bimodal endowment distribution.

[2] Mario Draghi was president of the ECB from 2011 to 2019. For his full speech, see https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html.

[3] See De Grauwe and Ji (2013) on the poor correlation between spreads and fundamentals during the European sovereign debt crisis.

government defaulted at the end of 2001, with a debt-to-GDP ratio slightly below 50%. Note that a 7% spread on a 35% debt-to-GDP ratio amounts to almost 2.5% of GDP of extra interest payments per year. Accumulated over that 6-year period, this represents 15% of GDP, almost one-third of the debt-to-GDP ratio of Argentina in 2001. Two questions arise. First, had Argentina faced lower interest rates, would Argentina have defaulted? Second, would Argentina have defaulted if the International Monetary Fund (IMF) had played the same role of lender of last resort that the ECB played for Spain and Italy? The answers to both questions are likely to be no.

These arguments are in line with the reasoning by Mussa (2002), chief economist of the IMF from 1991 to 2001: “Relatively straightforward calculations indicate that at interest rate spreads of 500 to 600 basis points, Argentina’s debt dynamics could have been sustainable under achievable degrees of budget discipline, whereas at persistent spreads of 1,000 or more, the situation would be virtually hopeless” (34).

Our paper provides a quantitative exploration of the sovereign debt crisis episodes in Argentina and Spain and of the role of policies like the one the ECB adopted. The main contribution is to argue that the mechanism that generates multiplicity in Calvo (1988) is quantitatively relevant, as hinted in Mussa (2002).

An essential assumption for multiplicity is a bimodal output growth process, with persistent good and bad times. We modify an otherwise standard sovereign default model to incorporate an endowment growth process that follows a Markov chain, featuring persistent high- and low-growth regimes. To calibrate the model, we estimate the output process for a set of countries that have recently been exposed to sovereign debt crises. We show that the model features self-fulfilling debt crises similar to those experienced by Argentina and Spain.

The model features equilibria in which interest rates can be high or low depending on expectations. That is, a sunspot realization can induce discrete jumps in the interest rates faced by the borrower, even with no change in fundamentals. These discrete jumps in rates can happen only if fundamentals are weak. It is only in times of persistently low growth that spreads can be high because of expectations. In the high-growth regime, the region of multiplicity is either empty or negligibly small. Thus, the multiplicity we compute is state dependent: expectations can trigger a crisis only during a persistent stagnation.

The schedule of interest rates faced by the borrower can also exhibit discrete jumps because of fundamentals due to the bimodal growth process. Thus, interest rate jumps are not necessarily a sign of a bad expectations draw. The interest rate jumps due to either fundamentals or expectations can induce responses by the sovereign that can be interpreted as endogenous austerity. In this case, the sovereign optimally refrains from increasing debt in order to avoid the costs associated with those jumps.

But those discrete interest rate jumps can also induce responses that resemble gambling for redemption of the type in Conesa and Kehoe (2017), where the borrower increases debt levels. Both endogenous austerity and gambling for redemption are featured in the equilibrium simulations discussed in the paper.

A policy intervention by a large creditor that is willing to offer the low interest rate schedule can neutralize the effects of the sunspot. In this sense, the effective probability of the sunspot needs to take into account the probability of such a policy intervention. In other words, if the policy intervention happens with probability 1, the effective probability of the high interest rate schedule should be zero. We provide a historical discussion of the likelihood of a policy intervention by either the ECB in Spain or the IMF in Argentina in order to calibrate the sunspot probability for these two countries.

A key assumption we make is that expectations are more pessimistic for Argentina, meaning that high interest rates are selected more frequently for Argentina than for Spain. As mentioned above, this choice is empirically based. In spite of doubts about whether the ECB would act as a lender of last resort, the events of summer 2012 made it clear that there was willingness to do so. We therefore calibrate the probability of a bad sunspot realization to be very small for Spain. On the other hand, the internal regulations of the IMF—together with the recent failure of large packages in the Asian and Russian crises—limited its willingness to act as a lender of last resort. Thus, we calibrate the probability of a bad sunspot realization to be larger for Argentina. We also provide robustness exercises and explore the implications of assuming either more optimistic expectations for Argentina or more pessimistic expectations for Spain.

We show that the Argentine calibration generates equilibrium paths that resemble the events of the 2001 debt crisis, including the observed jump in credit spreads. While endogenous austerity is present in the calibration for Argentina, the optimal policy eventually switches to gambling for redemption, thus generating high spreads in equilibrium. We follow Chatterjee and Eyigungor (2012) and construct an event case study for 1997–2001 by selecting output shocks to match the observed Argentine GDP growth. The model can account for the path of Argentina's spreads in the run-up to the crisis, and the sunspot plays a key role: absent a bad sunspot realization, Argentina would have avoided default.

The calibration for Spain features only endogenous austerity. That is, the threat of high spreads triggers endogenous austerity so that the high spreads are not observed along the equilibrium path. Thus, our model captures the austerity measures implemented by Spain but only in response to off-equilibrium high spreads. Spain's calibration features only endogenous austerity because of the low probability of the bad sunspot: expectations-driven high rates are a rare event, and thus it is optimal to avoid default costs by restraining borrowing in those rare cases. While we argue that a low sunspot probability reflects ECB policy after summer 2012, a time-varying probability could be a more accurate description and would help explain Spain's spreads over time. We opt for a model with fewer parameters with a time-invariant low sunspot probability.

A main finding of our paper is that changes in expectations—as measured by the sunspot probability of selecting an interest rate schedule—have a large quantitative effect on model outcomes. If we assume optimistic expectations for Argentina, default rates and credit spreads decline drastically. That is, without any change in fundamentals but only in beliefs, Argentina mutates from a serial defaulter to essentially a nondefaulter. Similarly, assuming pessimistic expectations for Spain results in high default rates and credit spreads, even if there is no change in fundamentals. Thus, our model suggests that the difference in expectations is a major factor explaining differences in default rates and credit spreads between Argentina and Spain. This result is more interesting—and policy relevant—when we connect the probability of the sunspot to the role of policy of a lender of last resort. A large creditor that can coordinate beliefs has a remarkable effect on the model outcomes: either low spreads and low default probabilities through endogenous austerity in the presence of policy intervention or high spreads and likely default through gambling for redemption in the absence of such intervention.

The presence of a large creditor, which has a dramatic effect in our economy, has no effect in the standard sovereign default model studied in Aguiar and Gopinath (2006), Arellano (2008), or Chatterjee and Eyigungor (2012). Because of the assumptions on the timing of moves and actions of borrower and creditors, those models assume away multiple equilibria (Ayres et al. 2018). Therefore, as long as loans are correctly priced according to the default probabilities (so that the creditor breaks even), a lender of last resort has no effect on equilibrium allocations. The standard model can match the high spreads leading up to Argentina's default, as in Chatterjee and Eyigungor (2012). The standard model would also be able to match the low spreads and low default probability of Spain, with appropriately calibrated—most likely higher—default costs. If multiplicity is indeed a feature of the actual economies, as we argue it can be, the effect of expectations on spreads will be captured in the standard model by the calibrated default cost. It is standard practice to calibrate the exogenous default costs so as to match average spreads in different countries. Thus, if different countries faced different lender of last resort policies, the calibrated default costs in the standard model would not be policy invariant.

The central result of this paper—that expectations-driven sovereign debt crises are empirically plausible—can contribute to the assessment of the role of policy in sovereign debt crises. In our quantitative exercises, it is when fundamentals are weak that a lender of last resort may be called in, not because of the weak fundamentals in themselves but because they create a role for expectations. Of course, the role of the lender of last resort in periods of stagnation will have effects on the economy beyond those periods in which interest rates could be high because of expectations.

The rest of the paper proceeds as follows. We next discuss related literature. In section II, we analyze a simple two-period model in which we can derive analytical expressions. We use this two-period model to highlight the role of the bimodal distribution in generating multiplicity as well as to formally establish the connection between policy and expectations-driven equilibria. In section III, we present the quantitative model. In section IV, we describe the calibration procedure, including the estimation of the endowment process and the calibration of the sunspot probabilities. Section V contains the discussion of the model results and robustness exercises. In section VI, we review the historical roles of the ECB in Spain and the IMF in Argentina. Section VII contains concluding remarks.

Related literature.—Our model follows the quantitative sovereign debt crises literature that grew out of the work of Eaton and Gersovitz (1981) and was further developed by Aguiar and Gopinath (2006) and Arellano (2008).[4] In these models, a sovereign borrower faces a stochastic endowment and issues noncontingent debt to a large number of risk-neutral lenders. There is no commitment to repay. Timing and choice of actions are important: the assumptions in Aguiar and Gopinath (2006) or Arellano (2008) are that the borrower moves first and chooses the level of noncontingent debt at maturity. We make two main changes to the standard setup. First, we assume that the borrower chooses current debt rather than debt at maturity. This assumption is key to generating multiplicity. When the borrower chooses debt at maturity, it is implicitly choosing the default probability and therefore also the interest rate on the debt. By contrast, when the borrower chooses current debt, default may be likely if interest rates are high or unlikely if interest rates are low.[5]

One fragility of the multiplicity mechanism in Calvo (1988) is that for commonly used distributions of the endowment process, the high-rate schedule is downward sloping, which means that the interest rates that the country faces decrease with the level of debt. That is not the case if the

[4] Other related works include Cole and Kehoe (2000), Chatterjee and Eyigungor (2012), Aguiar et al. (2014), Corsetti and Dedola (2014), Conesa and Kehoe (2017), Roch and Uhlig (2018), Bocola and Dovis (2019), Lorenzoni and Werning (2019), and Aguiar and Amador (2020), among others.

[5] Ayres et al. (2018) show that the timing of moves is also key. In particular, when lenders are first movers, there is multiplicity regardless of whether the borrower chooses current or future debt.

endowment is drawn from a bimodal distribution with good and bad times, as shown in Ayres et al. (2018) and Lorenzoni and Werning (2019). We depart from the standard setup in assuming that the endowment growth process follows such a bimodal distribution. This second change is empirically founded: we estimate a Markov-switching regime for the growth rate of output for Argentina, Brazil, Italy, Portugal, and Spain, and in all cases, we estimate processes that alternate between persistent high and persistent low growth. We view this feature of the endowment process as reflecting the likelihood of relatively long periods of stagnation in a way that is consistent with the evidence in Kahn and Rich (2007).

The paper closest to ours in its motivation is Lorenzoni and Werning (2019). They also consider the mechanism in Calvo (1988) but exploit a different source of multiplicity, one due to debt dilution with long maturities. In their environment, multiplicity arises because of the feedback between future and current bond prices: low bond prices tomorrow translate into low bond prices today, while high bond prices tomorrow translate into high prices today.

Lorenzoni and Werning (2019) make our same assumption on the actions of the borrower, which is also the assumption in Calvo (1988): the borrower chooses the funds it needs to raise rather than the face value of debt. Lorenzoni and Werning (2019) spell out the microfoundations for the assumption by allowing the government to reissue debt within the period infinitely many times. In the limit, a government that decides sequentially chooses the funds it wants to raise rather than the face value of debt. Evidence on lack of commitment to the preannounced issuance is presented in Brenner, Galai, and Sade (2009) and, more recently, in Monteiro and Fourakis (2023).[6]

In the context of self-fulfilling rollover crises, as in Cole and Kehoe (2000), Aguiar et al. (2022) also find, in a model calibrated to Mexico, that simulated moments are sensitive to expectations. We see our papers as complementary, as we explore a different source of multiplicity, the one proposed by Calvo (1988).

Bocola and Dovis (2019) allow for maturity choice in the context of self-fulfilling rollover crises and argue that expectations played a minor role during the European debt crisis. In their environment, governments prefer shorter maturities because of debt dilution incentives, while they prefer longer maturities because of rollover risk. Because maturities were reduced during the European debt crisis, their model suggests that rollover risk was limited during this period. While the source of multiplicity

[6] Brenner, Galai, and Sade (2009) survey treasuries and central banks around the world and report that more than half of the countries that answered the survey (30 out of 48) claim to have some discretion on how much to issue, regardless of whether a target was announced. Monteiro and Fourakis (2023) also show that the amounts issued of short-term debt for Portugal rarely meet the preannounced target and that the deviations are large.

in our model is different, our quantitative exercise also implies a limited role for expectations in the case of Spain: jumps in spreads due to bad expectations do not occur in equilibrium, only endogenous austerity. Another important difference is that we treat the sunspot probability as a fixed parameter, while Bocola and Dovis (2019) allow for a stochastic process. Our comparative statics exercises show that the probability of the sunspot has a substantial impact on outcomes, thus suggesting that stochastic changes in this probability may account for spreads during the European debt crisis of the early 2010s, an issue that may be worth pursuing in future research.

II. A Two-Period Model

Here, we illustrate the main mechanisms of the model in a simple two-period case. The economy is populated by a representative agent that draws utility from consumption in each period and by a continuum of risk-neutral foreign lenders. The initial wealth of the agent is denoted by ω. The endowment in the second period is distributed according to

$$y_2 = \begin{cases} y^{\mathrm{l}} & \text{with probability } p, \\ y^{\mathrm{h}} & \text{with probability } (1-p), \end{cases}$$

in which $y^{\mathrm{l}} < y^{\mathrm{h}}$.[7]

The representative agent preferences are given by $u(c_1) + \beta\mathbb{E}u(c_2)$, where u is strictly increasing and strictly concave and satisfies standard Inada conditions. We assume that the initial wealth and the discount factor β are low enough that the agent will want to borrow. In period 1, the borrower moves first and issues a noncontingent debt level b. Lenders respond with a gross interest rate R. We denote by $R(b)$ the interest rate schedule faced by the borrower. In period 2, after observing the endowment y_2, the borrower decides whether to pay the debt or to default. In case of repayment, the borrower consumes the endowment net of debt repayment, $c_2 = y_2 - Rb$. In case of default, there is a penalty expressed as a drop in output to $y^{\mathrm{d}} < y^{\mathrm{l}}$. In addition, the borrower must repay a fraction κ of the debt. Thus, consumption following default is given by $c_2 = y^{\mathrm{d}} - \kappa b$. The agent defaults if the cost of repayment is larger than the benefit:

$$\underbrace{(R-\kappa)b}_{\text{cost of repayment}} > \underbrace{y_2 - y^{\mathrm{d}}}_{\text{benefit of repayment}}. \tag{1}$$

In the first period, given initial wealth ω and an interest rate schedule $R(b)$, the borrower solves the following problem:

[7] The discrete distribution will help make clear the main mechanisms for multiplicity. We owe this to an insightful discussion by Fernando Alvarez.

$$
\begin{aligned}
V(\omega) &= \max_b \{u(c_1) + \beta \mathbb{E} u(c_2)\} \\
\text{subject to } c_1 &= \omega + b, \qquad (2) \\
c_2 &= \max\{y_2 - R(b)b, y^{\mathrm{d}} - \kappa b\}.
\end{aligned}
$$

The borrower is subject to a maximum debt level constraint, $b \leq \bar{B}$.

The assumption that the borrower moves first by choosing a level of debt and that lenders move next with an interest rate schedule is standard. We depart from the literature as in Aguiar and Gopinath (2006) and Arellano (2008) in that we assume that the borrower chooses current debt b rather than debt at maturity, Rb. We follow Calvo (1988) in making this assumption. The risk-neutral lenders will be willing to lend to the agent as long as the expected return is the same as the risk-free rate R^*; that is,

$$
\begin{aligned}
R^* = h(R; b) \equiv {} & \left[1 - \Pr\left(y_2 - y^{\mathrm{d}} < (R - \kappa)b\right)\right]R \\
& + \Pr\left(y_2 - y^{\mathrm{d}} < (R - \kappa)b\right)\kappa, \qquad (3)
\end{aligned}
$$

in which $h(R; b)$ is the expected return to the lender when the interest rate is R. Given a value for b, the expected return for lenders can be written as

$$
h(R; b) = \begin{cases} R & \text{if } R \leq \dfrac{y^{\mathrm{l}} - y^{\mathrm{d}}}{b} + \kappa, \\[2ex] R(1-p) + p\kappa & \text{if } \dfrac{y^{\mathrm{l}} - y^{\mathrm{d}}}{b} + \kappa < R \leq \dfrac{y^{\mathrm{h}} - y^{\mathrm{d}}}{b} + \kappa, \\[2ex] \kappa & \text{if } R > \dfrac{y^{\mathrm{h}} - y^{\mathrm{d}}}{b} + \kappa. \end{cases} \qquad (4)
$$

In figure 1, we plot the expected return as a function of the interest rate R for three levels of debt together with the risk-free rate, R^*. Notice that for low levels of R, the expected return is equal to R, since debt is repaid with probability 1. In this region, as R increases, the expected return increases one to one. Eventually, R will be high enough that the borrower will default in the low output state, which happens with probability p. At this point, the expected return jumps down. As R increases, the expected return increases at a lower rate, $1 - p$, since repayment happens only in the high output state. Finally, for high enough R, default will happen with probability 1, and the expected return will be the recovery rate κ. A higher level of debt decreases the expected return uniformly, shifting the curves downward.

For low levels of debt, there is only one solution to equation (3), with $R = R^*$. For intermediate levels of debt, there are two solutions: one

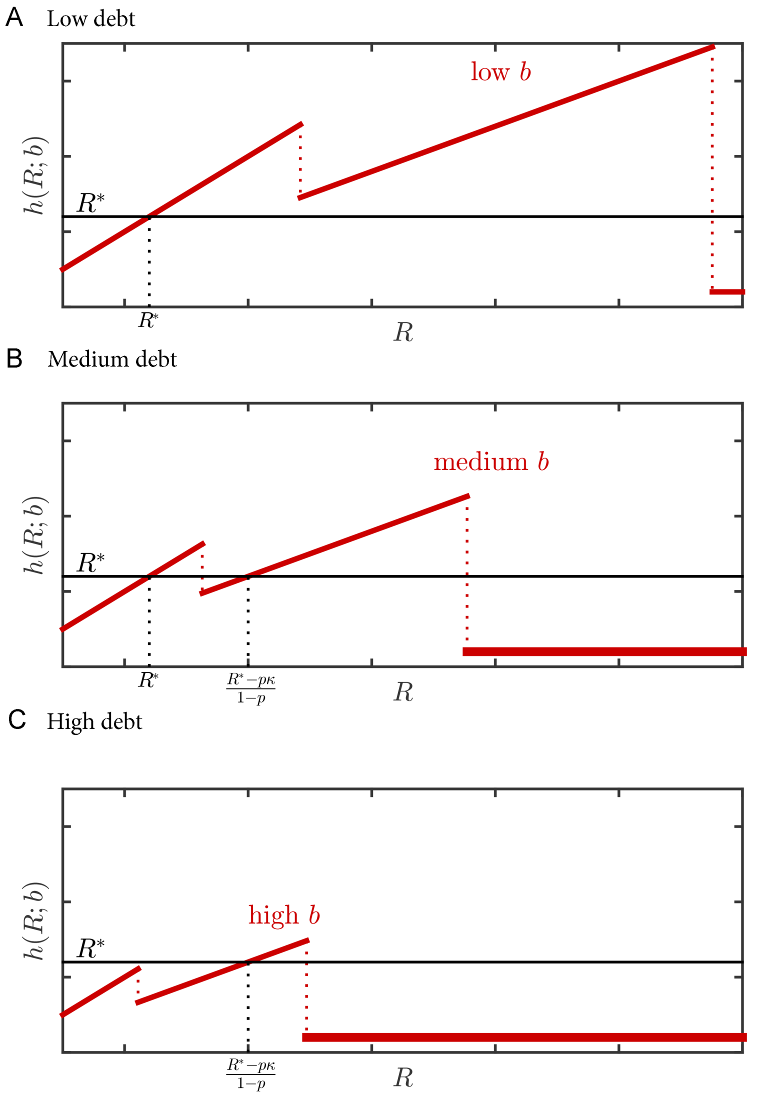

FIG. 1.—Expected return function for different levels of debt.

solution has $R = R^*$, associated with a zero probability of default, and the other has $R = (R^* - p\kappa)/(1 - p)$, associated with a probability of default equal to p. For higher levels of debt, the only solution is the high rate $R = (R^* - p\kappa)/(1 - p)$. Finally, for even higher debt, there is no solution.

We can now define the following correspondence relating debt levels to interest rates:

$$\mathcal{R}(b) = \begin{cases} R^* & \text{if } b \leq \dfrac{y^{\mathrm{l}} - y^{\mathrm{d}}}{R^* - \kappa}, \\ \dfrac{R^* - p\kappa}{1-p} & \text{if } (1-p)\dfrac{y^{\mathrm{l}} - y^{\mathrm{d}}}{R^* - \kappa} < b \leq (1-p)\dfrac{y^{\mathrm{h}} - y^{\mathrm{d}}}{R^* - \kappa}, \\ \infty & \text{if } b > (1-p)\dfrac{y^{\mathrm{h}} - y^{\mathrm{d}}}{R^* - \kappa}. \end{cases} \quad (5)$$

An *equilibrium* is an interest rate schedule $R(b)$ and a debt policy function $b(\omega)$ such that given the schedule, the debt policy function solves the problem of the borrower in equation (2), and the schedule $R(b)$ is a selection of the correspondence $\mathcal{R}(b)$.

The correspondence $\mathcal{R}(b)$ is plotted in figure 2 (dashed line). For all debt levels below $b_1 \equiv (1-p)((y^{\mathrm{l}} - y^{\mathrm{d}})/(R^* - \kappa))$, there is only one interest rate, the risk-free rate. For debt levels between b_1 and $b_2 \equiv (y^{\mathrm{l}} - y^{\mathrm{d}})/(R^* - \kappa)$, there are two possible interest rates, the risk-free rate

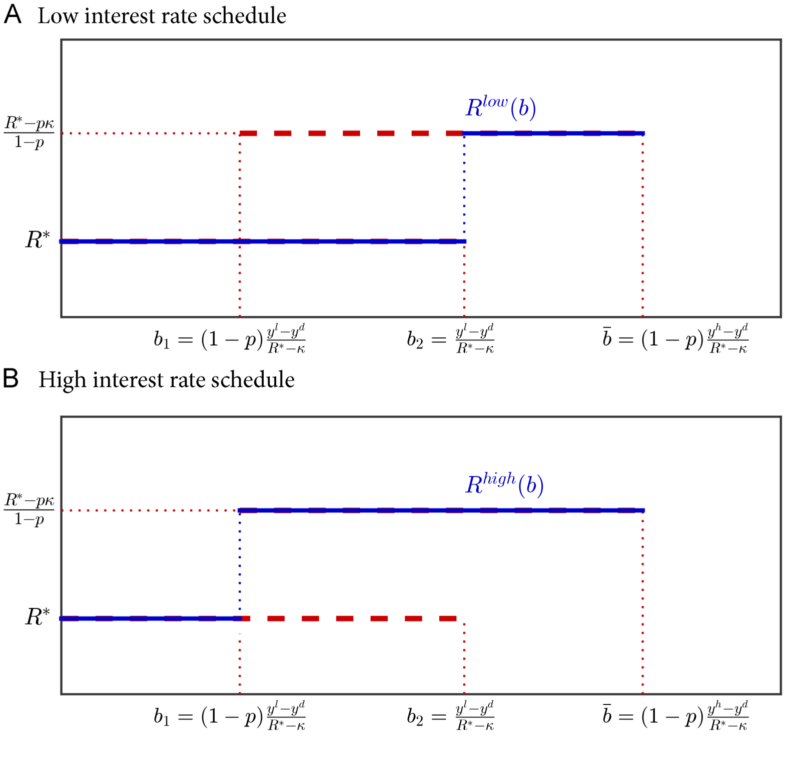

FIG. 2.—Interest rate schedules.

and a high rate. For debt levels between b_2 and $\bar{b} \equiv (1-p)((y^{h}-y^{d})/(R^{*}-\kappa))$, there is again only one interest rate, the high rate. There are multiple interest rate schedules that can be selected from this correspondence. We focus on two of those schedules: a low interest rate schedule, $R^{\text{low}}(b)$ (solid line in fig. 2*A*), and a high interest rate schedule, $R^{\text{high}}(b)$ (solid line in fig. 2*B*).[8]

We think of b_1 as the debt level above which interest rates jump because of expectations, since alternative expectations could sustain low interest rates. We think of b_2 as the debt level above which interest rates jump because of fundamentals, since no expectations could sustain lower interest rates. We think of $\bar{b}$ as an endogenous borrowing limit, since any debt issued above this level implies a default probability of 1.

Whether spreads are low or high has implications for the level of debt that can be raised. The region of multiplicity happens for intermediate levels of debt, between b_1 and b_2. If debt is sufficiently low, interest rates can only be low, whereas if debt is sufficiently high, rates can only be high. It is for intermediate levels of debt that interest rates can be either high or low depending on expectations.

As an illustration, figure 3 shows the optimal debt policy as a function of the initial wealth for the high and low interest rate schedules.[9] For high levels of wealth, the optimal choice of debt is below b_1, and thus the schedule does not matter. As wealth declines, the optimal amount of debt is higher, as indicated by the downward-sloping segment in the bottom right of figure 3. Eventually, for low enough wealth, the optimal amount of debt is equal to b_1. At that point, the schedules matter. For the high interest rate schedule, the borrower chooses to keep debt levels at b_1 in order to avoid the discrete jump in interest rates on the whole level of debt. Eventually, for low enough wealth, the marginal utility of consumption in the first period is high enough that the borrower chooses to increase its debt level discretely. This discrete jump shows that the borrower has incentives to avoid at least part of the multiplicity region between b_1 and b_2. As wealth decreases even more, debt levels keep increasing until they reach the endogenous borrowing limit, $\bar{b}$. When the borrower faces the low interest rate schedule, borrowing keeps on increasing as wealth declines until it reaches the level b_2. At this point, there is a choice to keep it constant for lower levels of wealth. Eventually, there is also a discrete jump, and debt levels continue to increase until they reach the endogenous borrowing limit.

[8] Notice that it is always the case that $b_1 < b_2$ and $b_1 < \bar{b}$. However, while fig. 2 represents the case in which $b_2 < \bar{b}$, there are parameter values such that the opposite is true. We chose to plot the case in which $b_2 < \bar{b}$ because this is the case in the quantitative model in section III.

[9] The example assumes $y^{l} = 11.5$, $y^{h} = 19.5$, $y^{d} = 6.5$, $\kappa = 0.2$, $p = 0.55$, $R^{*} = 1.5$, $\beta = 0.2$, and $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 5.5$.

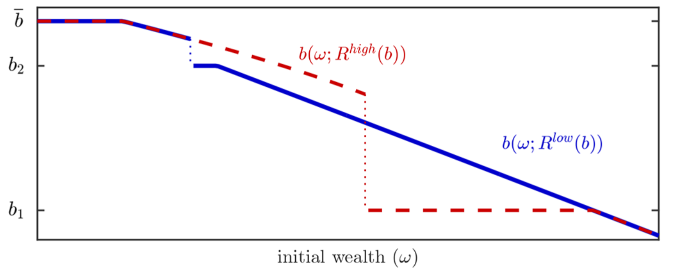

FIG. 3.—Debt policy function.

The choice of keeping debt levels constant as wealth decreases happens in our model because of the discrete jumps in interest rates. These jumps can be induced by either expectations (as depicted in the flat region of the dashed line in the bottom right of fig. 3) or weak fundamentals (as shown in the flat region of the solid line in the top left of fig. 3). These flat regions correspond to a form of endogenous austerity, where the borrower adjusts consumption to avoid the high rates. The jumps in the debt policy function indicate decisions by the borrower to end endogenous austerity, increase borrowing discretely, and accept a higher rate. They resemble the gambling for redemption described in Conesa and Kehoe (2017). As we show, the quantitative model of section III exhibits both endogenous austerity and gambling for redemption.

A. *The Probability of the Low Endowment State*

We start the comparative statics by considering alternative probabilities of the low endowment state, p. Figure 4 plots the interest rate correspondence $\mathcal{R}(b)$ for two values of p. The higher p is, the higher the interest rate that the borrower faces if default happens in the low endowment state.

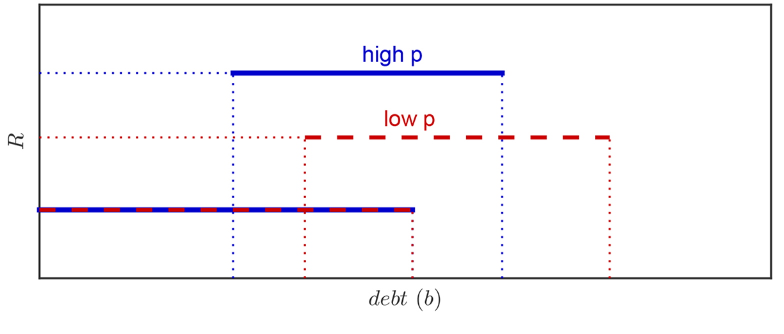

FIG. 4.—Interest rate correspondence for different p.

The higher the interest rate is, the lower the minimum debt level at which the borrower defaults in the low state. It follows that a higher p is associated with a higher interest rate and a larger region of multiplicity.

We relate the parameter p to the parameters in the full quantitative model in section III, which features a two-state Markov process in the growth rates of output. The probability of switching to or remaining at the low-growth regime is the analog of the value of p in this two-period model. In the estimations described in section III, we show that low output growth states are persistent, which means a higher p during stagnations. High output growth states are also very persistent, which means a lower p during expansions. Consequently, in the quantitative model, stagnations come with larger regions of multiplicity and higher interest rates.

B. *The Role of the Bimodal Distribution*

In order to highlight the essential role of the bimodal distribution, we consider a generalization in which the endowment in the second period is drawn from a bimodal normal distribution, $y_2 \sim pN(y^{\mathrm{l}}, \sigma^2) + (1-p)N(y^{\mathrm{h}}, \sigma^2)$. For different values of σ, figure 5A shows the expected return

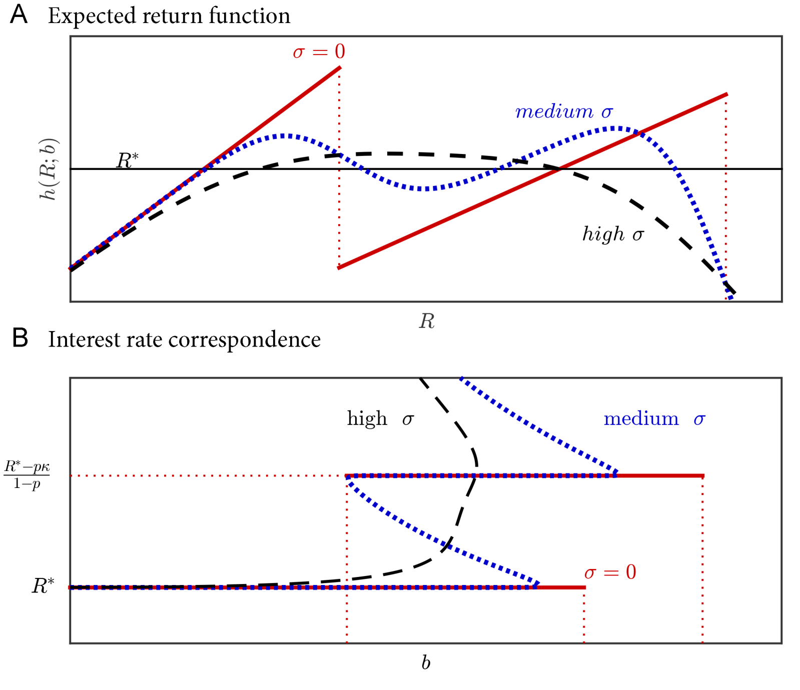

FIG. 5.—Varying standard deviation of endowment shock, σ.

function $h(R; b)$, and figure 5*B* shows the implied interest rate correspondence $\mathcal{R}(b)$. The case with $\sigma = 0$ (solid line) is the one analyzed before, in which there are two solutions to the arbitrage condition in equation (3). For strictly positive small levels of σ (dotted line), there are now four solutions to equation (3). However, the two solutions on the downward-sloping part of the expected return function are such that the expected return increases when the interest rate decreases. It follows that those solutions are also on the downward-slopping parts of the interest rates schedules depicted in figure 5*B*. Furthermore, not only the interest rate but also total future debt payments, $R(b)b$, go down when the level of current debt, b, goes up. This also implies that the agent would never choose to be in a decreasing part of the schedule. For these reasons and others, discussed in detail in Ayres et al. (2015) and Lorenzoni and Werning (2019), we rule out solutions along these parts of the interest rate schedule. For higher values of σ (dashed line), there are two solutions to equation (3), but one can be ruled out. Therefore, to have multiple admissible equilibria, we need to have relatively low levels of σ. We show that this is the case in the estimations of section III.

The endowment levels y^l and y^h are also important for multiplicity. As y^l approaches y^h, multiplicity disappears as the endowment distribution converges to the unimodal case. A similar rationale will apply to the quantitative results of section III.

C. The Role of Policy

It is straightforward to show that a large creditor, playing the role of a lender of last resort, can select the low interest rate equilibrium of the deterministic model described above. The role of the large creditor is to be willing to lend according to the low interest rate schedule up to the borrowing limit $\bar{B}$. In equilibrium, the large creditor may not need to lend, since individual creditors make zero profits under the low interest rate schedule and are thus willing to lend at those rates. To see this formally, we start by introducing a sunspot variable, s, which selects the high or low interest rate schedule, and assume that there is a large creditor that can intervene and offer to lend according to the low interest rate schedule. The intervention occurs with some exogenous probability, π.

The sunspot variable is realized in the beginning of period 1 and selects either the high or the low interest rate schedule. The sunspot variable takes two values,

$$s = \begin{cases} s_{\mathrm{B}} \text{ with probability } p_{\mathrm{B}}, \\ s_{\mathrm{G}} \text{ with probability } (1 - p_{\mathrm{B}}), \end{cases}$$

with s_B selecting the bad schedule (high rate) and s_G selecting the good schedule (low rate). Thus, for debt levels between b_1 and $b_2 \equiv (y^l - y^d)/(R^* - \kappa)$, $R = R^{\mathrm{low}}(b)$ when $s = s_G$ and $R = R^{\mathrm{high}}(b)$ when $s = s_B$.

The large creditor may intervene to provide funds according to the low interest rate schedule up to the maximum, $\bar{B}$. We assume that the intervention occurs with an exogenous probability $\pi \in [0, 1]$. This encompasses the case in which $\pi = 1$ and the intervention is certain, as appears to have been the case in the euro area in summer 2012. For simplicity, we assume that this shock is independent of the sunspot shock.

The timing of events in period 1 is now as follows: (1) the sunspot variable and the intervention shock are realized; (2) the borrower issues a noncontingent debt level b; (3) lenders respond with a gross interest rate R. The timing of events in the second period is as before.

Consider now an intervention with arbitrary probability π. If the intervention shock is realized, then the high-rate schedule offered by the private agents is redundant: the country would never borrow at the high rates. The only possible equilibrium outcome is the one exhibiting the low-rate schedule. Since, by definition, lenders make zero profits under the low-rate schedule, they are willing to lend at those rates. The large creditor does not have to lend in equilibrium; the promise of an intervention is all that is needed.

The previous discussion makes clear that if the intervention shock realizes, then the outcome of the sunspot is inessential. The effective probability of the high-rate schedule is $p_B \times (1 - \pi)$. This means that the economy with the intervention shock is identical to an economy without the intervention shock but with a lower sunspot probability.

III. A Quantitative Model of Self-Fulfilling Debt Crises

We calibrate an infinite-horizon model to evaluate the quantitative role of multiplicity in triggering sovereign debt crises. We estimate the endowment process in the model, using data on GDP growth for a set of developed economies as well as developing economies that were exposed to debt crisis episodes. The calibrated model generates self-fulfilling debt crises that can explain the events in Argentina in 2001 and can also shed light on the events in Spain in the 2010s.

A. *Model*

We expand the two-period model of section II to an infinite-horizon framework with output growth and long-term debt. We assume an endowment economy, where output growth follows a two-state Markov process. At the beginning of each period, the sovereign chooses whether to default on the total stock of outstanding debt. Upon repayment, the sovereign chooses consumption and finances it with the current endowment and new bond issuance net of repayment. Upon default, the sovereign is excluded from financial markets and loses a fraction of the endowment. While in default, the sovereign may be given the chance to reenter financial markets. If the sovereign reenters, it recovers the totality of the endowment and must honor a fraction of the defaulted debt. Specific details follow below.

Time is discrete, runs forever, and is indexed by $t = 0, 1, 2, \ldots$. We assume a small open economy that receives a stochastic endowment, Y_t, every period. The preferences of the sovereign are standard:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{6}$$

where C_t denotes consumption. The utility function is consistent with balanced growth, which allows us to detrend the model as shown below.

1. Endowment Process

The endowment grows over time as the result of a persistent and a transitory shock. In each period t, the endowment Y_t is given by

$$Y_t = \Gamma_t e^{\sigma \epsilon_t}, \tag{7}$$

$$\Gamma_t = g_t \Gamma_{t-1}, \tag{8}$$

where ϵ_t is independently and identically distributed (i.i.d.), $\epsilon_t \sim \mathcal{N}(0, 1)$, and g_t follows a two-state Markov process. Thus, g_t is the current trend growth, and Γ_t is the accumulated growth up to period t. We assume that g_t can be either high or low—$g_t \in \{g_{\mathrm{H}}, g_{\mathrm{L}}\}$—representing times of either fast growth or stagnation. The bimodal nature of g_t is empirically plausible and crucial for expectations to play a role in the model.

2. Debt Contract

The sovereign issues long-term bonds that promise a geometrically decreasing sequence of future payments, governed by the rate $\delta \in [0, 1]$, as in Hatchondo and Martinez (2009). A new debt issuance N_t in period t promises the following sequence of payments, starting from $t + 1$:

$$N_t R_t,\ (1-\delta) N_t R_t,\ (1-\delta)^2 N_t R_t,\ \ldots .$$

The value for R_t is determined on the date of issuance, t, and remains constant over the duration of the bond. For $\delta = 1$, R_t is the gross return on a one-period bond, while for $\delta = 0$, R_t is the net interest rate on a consol.

Let B_t denote total payments due at time t because of previous issuance. Thus, we have

$$\begin{aligned} B_t &= N_{t-1}R_{t-1} + (1-\delta)N_{t-2}R_{t-2} + (1-\delta)^2 N_{t-3}R_{t-3} + \cdots \\ &= \sum_{j=1}^{\infty}(1-\delta)^{j-1}N_{t-j}R_{t-j}. \end{aligned} \tag{9}$$

This debt contract formulation is convenient because it allows us to write debt payments B_t recursively as

$$B_t = (1-\delta)B_{t-1} + R_{t-1}N_{t-1}. \tag{10}$$

We refer to B_t as the debt service in period t.

Let Q be the price of a bond with a payment of $R_t = R$. We compute the interest rate ρ of such a bond so that the present value of promised cash flows equals its price, Q. That is, the rate ρ satisfies

$$Q = \sum_{j=1}^{\infty}\frac{(1-\delta)^{j-1}R}{(1+\rho)^j}. \tag{11}$$

Note that the rate ρ is pinned down by the ratio of Q and R. Previous work has typically normalized R and let the price Q be determined upon issuance in equilibrium. Instead, we normalize the price to $Q = 1$ upon issuance and let R be determined in equilibrium. This is a normalization only upon issuance, but the price of the bond can fluctuate thereon, as we discuss below. Then, solving for ρ in (11) yields

$$\rho = R - \delta. \tag{12}$$

When not in default, the resource constraints are

$$C_t + B_t = Y_t + N_t, \tag{13}$$

where, given our normalization, new issuance N_t has a price of 1.

We make the same assumptions on the timing of moves and actions of the borrower as in the two-period model. In particular, we assume that the borrower moves first and chooses current debt issuance N_t. Lenders move next and offer a schedule $\mathcal{R}(N_t; B_t, \Gamma_{t-1}, g_t, s_t)$, where s_t is a sunspot variable that selects a particular interest rate when more than one is consistent with an equilibrium.

3. Sunspot

The sunspot s_t is the key variable in our model that captures the role of expectations in selecting the equilibrium schedule. In light of the results of section II, we allow the sunspot to take two values: $s_t \in \{s_G, s_B\}$. When more than one schedule is possible, the $s_t = s_G$ value selects the good schedule (low rate), and $s_t = s_B$ selects the bad schedule (high rate). We allow the sunspot to follow a two-state Markov process. However, in our numerical exercises, we will assume that s_t is i.i.d. and that the bad sunspot occurs with probability p_B. The value of p_B captures how pessimistic or optimistic the beliefs of lenders are on average. It is also a function of the lender-of-last-resort policy. As we show in the quantitative results of section V, the value of p_B has substantial effects on the interest rate schedule as well as on the behavior of spreads and debt issuance.

4. Default Cost and Reentry

Default entails two costs for the sovereign. First, the sovereign remains temporarily excluded from financial markets. Second, a fraction of the endowment, $1 - \phi_t$, is lost. As is customary in the literature (see Arellano 2008), we allow for the fraction ϕ_t to depend on the exogenous state $g_t \in \{g_L, g_H\}$. Thus, when the sovereign is in default, the resource constraints are

$$C_t = \phi_t Y_t, \tag{14}$$

with $\phi_t = \phi(g_t)$.

While in default, the option to reenter credit markets happens with probability $1 - \theta$. If the sovereign chooses to reenter, the output loss, $1 - \phi_t$, is lifted, and the sovereign regains access to international financial markets. In addition, debt servicing resumes, but a fraction $1 - \kappa$ of payments is forgone. Thus, lenders recover a fraction κ of the outstanding debt. A sovereign that defaulted in a period with promised debt service B will then face a debt service κB upon reentry, and future debt payments will evolve as in equation (10).

5. Values of Default and No Default

Let $V^{\mathrm{nd}}(B, \Gamma_-, g, s, \epsilon)$ and $V^{\mathrm{d}}(B, \Gamma_-, g, s, \epsilon)$ be the maximal attainable values of no default and default, respectively, to a sovereign that starts this period with debt service B, accumulated trend growth Γ_-, current growth g, sunspot s, and output shock ϵ. The value of no default is

$$
\begin{aligned}
V^{\mathrm{nd}}(B,\Gamma_-,g,s,\epsilon) \;=\; & \max_{N,C}\left\{\frac{C^{1-\gamma}}{1-\gamma}+\beta\mathbb{E}\left[\max\left\{V^{\mathrm{nd}}(B',\Gamma,g',s',\epsilon'),\,V^{\mathrm{d}}(B',\Gamma,g',s',\epsilon')\right\}\middle|\Gamma_-,g,s\right]\right\} \\
& \text{subject to} \\
& C + B = Y + N, \\
& B' = (1-\delta)B + \mathcal{R}(N,B,\Gamma_-,g,s)N, \\
& Y = \Gamma e^{\sigma\epsilon},\; \Gamma = g\Gamma_-, \\
& N \le \bar{N}(B,\Gamma_-,g,s).
\end{aligned} \tag{15}
$$

The borrowing limit $\bar{N}(\cdot)$ is important in our environment. Since the borrower receives a unit of consumption for every unit of debt issued, default could always be postponed by issuing more debt. This possibility is ruled out by imposing a maximum amount of debt. In practice, we set $\bar{N}(\cdot)$ so that the probability of default next period is never larger than 65%.[10]

The value of default is

$$
\begin{aligned}
V^{\mathrm{d}}(B,\Gamma_-,g,s,\epsilon) \;=\; & \frac{C^{1-\gamma}}{1-\gamma}+\beta\mathbb{E}\left[\theta V^{\mathrm{d}}(B',\Gamma,g',s',\epsilon')\right. \\
& \left. +\,(1-\theta)\max\left\{V^{\mathrm{nd}}(\kappa B',\Gamma,g',s',\epsilon'),\,V^{\mathrm{d}}(B',\Gamma,g',s',\epsilon')\right\}\middle|\Gamma_-,g,s\right] \\
& \text{subject to} \\
& C = \phi(g)Y, \\
& B' = B, \\
& Y = \Gamma e^{\sigma\epsilon},\; \Gamma = g\Gamma_-,
\end{aligned} \tag{16}
$$

where $1-\phi$ is the fraction of output lost upon default. While in default, debt service is suspended. If the sovereign has the opportunity to reenter financial markets and decides to do so, only a fraction κ of the debt service is resumed next period.

Let $\mathbf{C}(B,\Gamma_-,g,s,\epsilon)$ and $\mathbf{N}'(B,\Gamma_-,g,s,\epsilon)$ denote the optimal consumption and debt issuance policies when not in default, and let $\mathbf{B}'(B,\Gamma_-,g,s,\epsilon)$ denote the implied debt service next period. Similarly, let $\mathbf{D}(B,\Gamma_-,g,s,\epsilon)$ denote the optimal default policy and $\mathbf{E}(B,\Gamma_-,g,s,\epsilon)$ denote the optimal reentry policy while in default. The optimal policies for $\mathbf{D}(\cdot)$ and $\mathbf{E}(\cdot)$ are given by

$$
\mathbf{D}(B,\Gamma_-,g,s,\epsilon) \;=\; \begin{cases} 0 & \text{if } V^{\mathrm{nd}}(B,\Gamma_-,g,s,\epsilon) \ge V^{\mathrm{d}}(B,\Gamma_-,g,s,\epsilon), \\ 1 & \text{otherwise}, \end{cases} \tag{17}
$$

$$
\mathbf{E}(B,\Gamma_-,g,s,\epsilon) \;=\; \begin{cases} 1 & \text{if } V^{\mathrm{nd}}(\kappa B,\Gamma_-,g,s,\epsilon) \ge V^{\mathrm{d}}(B,\Gamma_-,g,s,\epsilon), \\ 0 & \text{otherwise}\,. \end{cases} \tag{18}
$$

[10] Similar formulations for a borrowing limit can be found in Chatterjee and Eyigungor (2015) and Hatchondo, Martinez, and Sosa-Padilla (2016).

6. Pricing of Debt

We assume a continuum of risk-neutral lenders with deep pockets that discount future payments at rate r^*. Let $Z = (\Gamma_-, g, s, \epsilon)$ collect all the exogenous terms in the economy. Let $\mathcal{Q}(B, R, Z)$ be the beginning-of-period (before default decisions) value of a bond that promises R upon issuance, and let $\mathcal{X}(B, R, Z)$ be the value of such a bond in default. Then, $\boldsymbol{\mathcal{Q}}(\cdot)$ and $\boldsymbol{\mathcal{X}}(\cdot)$ are given by

$$\mathcal{Q}(B, R, Z) = [1 - \mathbf{D}(B, Z)]\left[R + \frac{1-\delta}{1+r^*}\mathbb{E}[\mathcal{Q}(\mathbf{B}'(B, Z), R, Z')|\Gamma_-, g, s]\right] + \mathbf{D}(B, Z)\mathcal{X}(B, R, Z), \quad (19)$$

$$\mathcal{X}(B, R, Z) = \frac{1}{1+r^*}\mathbb{E}[\theta\mathcal{X}(B, R, Z') + (1-\theta)[1 - \mathbf{E}(B, Z')]\mathcal{X}(B, R, Z') + (1-\theta)\mathbf{E}(B, Z')\mathcal{Q}(\kappa B, \kappa R, Z')|\Gamma_-, g, s], \quad (20)$$

where equation (20) incorporates that B remains constant during default episodes.

The value of the bond under no default $\boldsymbol{\mathcal{Q}}(\cdot)$ is standard: if the sovereign does not default ($\mathbf{D}(\cdot) = 0$), R is paid this period, and the remaining fraction $1 - \delta$ has a next period value given by $\boldsymbol{\mathcal{Q}}(\cdot)$, evaluated at the next period debt service $\mathbf{B}'(\cdot)$. Notice that if there is no default, R remains unchanged. The value of the bond in default $\boldsymbol{\mathcal{X}}(\cdot)$ reflects the two possible cases the sovereign will face next period. In the first case, the sovereign remains in default either because it does not have the chance to reenter financial markets or because it decides not to do so ($\mathbf{E}(\cdot) = 0$). In this case, the value of the bond is still given by the function $\boldsymbol{\mathcal{X}}(\cdot)$. In the second case, the sovereign—with probability $1 - \theta$—has the chance to reenter financial markets and decides to do so ($\mathbf{E}(\cdot) = 1$). In this case, the lender recovers only a fraction κ of R, but the price of the bond also reflects that the sovereign has to repay only a fraction κ of its former liabilities, which is why the price $\boldsymbol{\mathcal{Q}}(\cdot)$ is evaluated at $\kappa B'$ and κR at the end of equation (20).

Bond prices satisfy $\mathcal{Q}(B, R, Z) = R\mathcal{Q}(B, 1, Z)$ and $\mathcal{X}(B, R, Z) = R\mathcal{X}(B, 1, Z)$.[11] This is an intuitive result: a bond with an arbitrary payment R pays, in every state of nature, R times what a bond with a payment of 1 does. Since lenders are risk neutral, the price of any bond is thus a multiple of the price of a bond with $R = 1$. Thus, with a slight abuse

[11] Formally, it can be shown that for any $\lambda > 0$, eqq. (19) and (20) admit a solution with $\mathcal{Q}(B, \lambda R, Z) = \lambda\mathcal{Q}(B, R, Z)$ and $\mathcal{X}(B, \lambda R, Z) = \lambda\mathcal{X}(B, R, Z)$. The normalization of the price comes from setting $\lambda = 1/R$. See app. C (apps. A–F are available online) for a formal proof.

of notation, let $\mathcal{Q}(B, Z) = \mathcal{Q}(B, 1, Z)$ be the price of a bond with $R = 1$. Consequently, the schedule $\mathcal{R}(N, B, \Gamma_{-}, g, s)$ must satisfy

$$1 = \frac{\mathcal{R}(N, B, \Gamma_{-}, g, s)}{1 + r^{*}} \mathbb{E}[\mathcal{Q}(B', \Gamma, g', s', \epsilon') | \Gamma_{-}, g, s], \tag{21}$$

$$B' = (1 - \delta)B + \mathcal{R}(N, B, \Gamma_{-}, g, s)N. \tag{22}$$

By our normalization, the bond price on issuance—the left-hand side of equation (21)—is 1. Then, for each level of issuance N, $\mathcal{R}(\cdot)$ adjusts endogenously so that the sovereign receives one unit of consumption for each newly issued bond.

Equations (21) and (22) are the analog to equation (3) in the simple two-period model and can be used to discuss the intuition behind the multiplicity results. If lenders coordinate on a high $\mathcal{R}$, this leads to—given an issuance N—a higher B' via equation (22). In turn, a higher B' implies a higher probability of defaulting in the future. That means a lower (expected) $\mathcal{Q}$ next period, which justifies the higher $\mathcal{R}$ in equation (21).[12]

The sunspot captures the role of expectations in selecting the equilibrium interest rate schedule in equations (21) and (22). For the same reasons as discussed in section II, the relevant parts of the interest rate schedule are those increasing on issuance. The two increasing interest rate schedules are selected using the same approach as in section II. The high-rate schedule corresponds to the highest interest rates for each level of debt, and the low-rate schedule corresponds to the lowest rate.

7. Equilibrium

We can now formally define an equilibrium for this economy.

Definition 1 (Equilibrium). An equilibrium is a set of value functions $\{V^{\mathrm{d}}(B, \Gamma_{-}, g, s, \epsilon), V^{\mathrm{nd}}(B, \Gamma_{-}, g, s, \epsilon)\}$; policy functions $\mathbf{C}(B, \Gamma_{-}, g, s, \epsilon)$, $\mathbf{N}(B, \Gamma_{-}, g, s, \epsilon)$; default and reentry functions $\mathbf{D}(B, \Gamma_{-}, g, s, \epsilon)$, $\mathbf{E}(B, \Gamma_{-}, g, s, \epsilon)$; and pricing functions $\mathcal{Q}(B, \Gamma_{-}, g, s, \epsilon)$, $\mathcal{X}(B, \Gamma_{-}, g, s, \epsilon)$, and $\mathcal{R}(N, B, \Gamma_{-}, g, s)$ such that

i. the policy functions solve the sovereign's problem in equation (15) and achieve value $V^{\mathrm{nd}}(B, \Gamma_{-}, g, s, \epsilon)$;
ii. the value function $V^{\mathrm{d}}(B, \Gamma_{-}, g, s, \epsilon)$ satisfies equation (16);

[12] Equations (21) and (22) show that the actions available to the borrower also matter. If the borrower could directly choose next period debt service B', eq. (21) would determine $\mathcal{R}(\cdot)$, and a unique corresponding issuance N would then arise from eq. (22). See Ayres et al. (2018) for a detailed discussion on how timing and actions lead to multiple equilibria in models of default.

iii. the default and reentry policies, $\mathbf{D}(B, \Gamma_{-}, g, s, \epsilon)$ and $\mathbf{E}(B, \Gamma_{-}, g, s, \epsilon)$, are as in equations (17) and (18);
iv. functions $\mathcal{Q}(B, \Gamma_{-}, g, s, \epsilon)$ and $\mathcal{X}(B, \Gamma_{-}, g, s, \epsilon)$ satisfy equations (19) and (20); and
v. the schedule $\mathcal{R}(N, B, \Gamma_{-}, g, s)$ satisfies equation (21), with B' and N as in equation (22).

B. Model Normalization

Since the endowment process has a trend, the state variables in the model are nonstationary. For computational purposes, we normalize all nonstationary variables by trend growth Γ_{-}. This requires showing homogeneity properties of the equilibrium functions, similar to Aguiar and Gopinath (2006). We leave the detailed derivation to appendix C and proceed to present the detrended model.

For any variable X_t, we denote $x_t = X_t/\Gamma_{t-1}$ as the detrended value. Let $v^{\mathrm{nd}}(b, g, s, \epsilon)$ and $v^{\mathrm{d}}(b, g, s, \epsilon)$ be the detrended values of no default and default, respectively. The value of no default is

$$
\begin{aligned}
v^{\mathrm{nd}}(b, g, s, \epsilon) &= \max_{c,n}\left\{\frac{c^{1-\gamma}}{1-\gamma} + \beta g^{1-\gamma}\mathbb{E}\left[\max\left\{v^{\mathrm{nd}}(b', g', s', \epsilon'), v^{\mathrm{d}}(b', g', s', \epsilon')\right\}|g, s\right]\right\} \\
&\text{subject to} \\
c + b &= y + n, \\
gb' &= (1-\delta)b + R(n, b, g, s, \epsilon)n, \\
y &= ge^{\sigma\epsilon}, \\
n &\le \bar{n}(b, g, s),
\end{aligned} \tag{23}
$$

where $R(\cdot)$ is the schedule for the detrended variables, as discussed below.

Similarly, the value of default becomes

$$
\begin{aligned}
v^{\mathrm{d}}(b, g, s, \epsilon) &= \frac{c^{1-\gamma}}{1-\gamma} + \beta g^{1-\gamma}\mathbb{E}\,[\,\theta v^{\mathrm{d}}(b', g', s', \epsilon') \\
&\quad + (1-\theta)\max\left\{v^{\mathrm{nd}}(\kappa b', g', s', \epsilon'), v^{\mathrm{d}}(b', g', s', \epsilon')\right\}|g, s], \\
c &= \phi(g)y, \\
gb' &= b, \\
y &= ge^{\sigma\epsilon}.
\end{aligned} \tag{24}
$$

The schedule offered by foreign lenders is given by

$$1 = \frac{R(n, b, g, s)}{1 + r^*} \mathbb{E}[Q(b', g', s', \epsilon')|g, s], \tag{25}$$

$$gb' = (1 - \delta) b + R(n, b, g, s) n, \tag{26}$$

with the prices $Q(\cdot)$ and $X(\cdot)$ satisfying

$$Q(b, z) = [1 - \mathbf{d}(b, z)]\left[1 + \frac{1 - \delta}{1 + r^*}\mathbb{E}[Q(\mathbf{b}'(b, z), z')|z]\right] + \mathbf{d}(b, z) X(b, z), \tag{27}$$

$$X(b, z) = \frac{1}{1 + r^*}\mathbb{E}[\theta X(b', z') + (1 - \theta)\{[1 - \mathbf{e}(b', z')]X(b', z') + \mathbf{e}(b', z')\kappa Q(\kappa b', z')\}|z], \tag{28}$$

where $z = (g, s, \epsilon)$ collects the stationary exogenous states, $\mathbf{b}'(\cdot)$ in equation (27) is the next period payment implied by the optimal issuance policies in equation (23), and $b' = b/g$ in equation (28). The default and reentry policies, $\mathbf{d}(\cdot)$ and $\mathbf{e}(\cdot)$, are given by

$$\mathbf{d}(b, g, s, \epsilon) = \begin{cases} 0 & \text{if } v^{\mathrm{nd}}(b, g, s, \epsilon) \geq v^{\mathrm{d}}(b, g, s, \epsilon), \\ 1 & \text{otherwise,} \end{cases} \tag{29}$$

$$\mathbf{e}(b, g, s, \epsilon) = \begin{cases} 1 & \text{if } v^{\mathrm{nd}}(b, g, s, \epsilon) \geq v^{\mathrm{d}}(b, g, s, \epsilon), \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

IV. Calibration

We consider two calibrations of the model, one for the case of Spain during the European sovereign debt crisis and one for Argentina during the default episode of 2001. We calibrate all parameters using standard values in the literature or by targeting empirical moments, including the sunspot probability.

A. *Sunspot Probability*

We assume that the sunspot is i.i.d. The bad sunspot occurs with probability p_{B}. To assign values of p_{B} to the calibration of Spain and Argentina, we leverage on the connection between the probability of the bad sunspot and the probability of a policy intervention, highlighted in section II.C. More specifically, as we discuss in section VI, the ECB ended up acting as a lender of last resort in European bond markets, while Argentina did not have such support from the IMF. Therefore, we calibrate the probability of the bad sunspot to be very low in Spain, $p_{\mathrm{B}} = 1\%$, and to be relatively high in Argentina, $p_{\mathrm{B}} = 25\%$.

We explore the extent to which changing the value of p_B alters the equilibria of the model. In particular, we compute equilibria for both economies, assuming the alternative value of the sunspot probability—that is, switching to optimistic expectations for Argentina and to pessimistic expectations for Spain. If expectations did not matter, the outcome should be invariant to p_B. As we show below, the sunspot does matter substantially in both cases.

It is worth emphasizing that we explored numerically a wide range of values between 1% and 50% for p_B. The results showed that there is a threshold—around 5% for Argentina and 15% for Spain—such that for all values below the threshold, the results are very similar to the case of 1%, while for the values above the threshold, the results are very similar to the case of 25%.

B. *Common Parameters*

A period in the model is 1 year. We set the annual risk-free rate $r^* = 3.5\%$. The discount factor is set to $\beta = 0.75$, as is standard in the literature, so that the borrower is impatient enough that borrowing and default are equilibrium outcomes. Arellano (2008) and Chatterjee and Eyigungor (2015) use quarterly discount factors of 0.95 and 0.94, respectively, which imply annual discount factors of 0.82 and 0.77, close to the value we chose.[13] The borrower's risk aversion coefficient is $\gamma = 3$. This value is between the ones used by Arellano (2008), $\gamma = 2$, and Bianchi, Hatchondo, and Martinez (2018), $\gamma = 3.3$. Finally, the recovery rate is set to 75% ($\kappa = 0.75$), in line with the estimates in Cruces and Trebesch (2013).[14]

C. *Country-Specific Parameters*

We use data to discipline the parameters that govern the output process and the value of δ, which pins down the average maturity of the debt. For Argentina, we set the average maturity of debt equal to 2.5 years, $\delta = 0.4$, close to the average maturity before the default in 2001. For Spain, we set the average maturity of debt equal to 6.7 years, $\delta = 0.15$, also close to the average value before Spain's debt crisis.

The endowment process in equations (7) and (8) is a regime-switching process characterized by five parameters $\{g_L, g_H, \sigma, p_L, p_H\}$. To calibrate

[13] Aguiar and Gopinath (2006) use a quarterly discount factor of 0.8, which corresponds to 0.41 annually. On the other hand, Aguiar et al. (2016) use annual discount factors between 0.84 and 0.89.

[14] The haircut estimates in Cruces and Trebesch (2013) vary from 16% to 40%, corresponding to values of κ between 0.84 and 0.6. We perform a sensitivity analysis for different values of κ in app. sec. F.5.

these parameters, we estimate the endowment process using the filter proposed by Kim (1994).[15] We use annual GDP per capita data from the Conference Board Total Economy Database for the period 1980–2017 and estimate the process separately for five countries: Argentina, Brazil, Italy, Portugal, and Spain.[16] We start in 1980 to avoid the high growth rates of the period of convergence in the 1960s and 1970s. We assume bounded uniform priors for the five parameters and explore the posterior distribution using a Metropolis-Hastings Markov chain Monte Carlo algorithm. Table 1 shows the estimates for all countries. As in the model, the estimation assumes the same standard deviation of shocks, σ, across the two growth states.[17]

Two things are worth mentioning regarding the estimates in table 1. First, they show clear evidence of a bimodal distribution for output growth in all countries. The average across countries of the difference between g_H and g_L is 6 percentage points, more than three times the standard deviation of the transitory shock. The difference between g_H and g_L is a key ingredient for expectations to play a role.[18] Second, both the low- and high-growth states are persistent (between 60% and 80% persistence). As we show below, this relatively high persistence for the low-growth state generates high but plausible interest rates in the low-growth state.

Table 1 also suggests differences between the southern European and the South American countries. The most notable one is that the difference between g_H and g_L is substantially smaller in Europe. Another difference is that the volatility of the disturbance appears to be higher in South America.

When comparing within regions, there is substantial homogeneity in Europe. We therefore chose to use the point estimates for Spain: $p_L = 0.629$, $p_H = 0.838$, $\ln(g_L) = -0.018$, $\ln(g_H) = 0.033$, and $\sigma = 0.017$. In contrast, in South America there are larger differences between Brazil and Argentina. Most notably, Argentina exhibits a larger difference between g_H and g_L as well as a larger shock volatility σ, while the persistence of the high-growth state p_H is lower. These differences most likely reflect the fact that the data from Argentina incorporate several episodes of default, so

[15] We do not directly use the filter in Hamilton (1989) because output growth has a moving average component. We use the filter in Kim (1994) instead. See app. A for details.

[16] In studying the European experience, we exclude Ireland and Greece in order to concentrate on a relatively homogeneous group of countries.

[17] In app. A, we also present estimates of the process allowing for state-dependent σ. We estimate similar values of σ across states, except in the case of Argentina, whose estimates are significantly affected by the 2001 default.

[18] To reduce the dimensionality of the state, the output innovation in the model is i.i.d. One potential concern is that the estimation detects two different regimes as an approximation to a one-regime but persistent process for the growth rate of output. This is not the case: we repeated the estimation assuming an AR(1) process for the innovation and also found a statistically and economically significant difference in the growth rates across regimes.

TABLE 1
Prior and Posterior Distributions

	$\ln(g_L)$	$\ln(g_H)$	p_L	p_H	σ
	A. Prior Distribution				
	$U(-.1$ to $.1)$	$U(-.1$ to $.1)$	$U(.1$ to $1.0)$	$U(.1$ to $1.0)$	$U(10^{-3}$ to $.5)$
	B. Posterior Distribution (Mean and 5th–95th Percentile Interval)				
Italy	−.017	.022	.646	.843	.016
	(−.037 to −.008)	(.018 to .028)	(.050 to .990)	(.627 to .990)	(.012 to .023)
Portugal	−.002	.048	.805	.720	.019
	(−.011 to .003)	(.041 to .057)	(.516 to .990)	(.454 to .939)	(.014 to .025)
Spain	−.018	.033	.629	.838	.017
	(−.026 to −.010)	(.026 to .039)	(.308 to .990)	(.653 to .990)	(.013 to .025)
Argentina	−.040	.060	.620	.581	.033
	(−.049 to −.022)	(.051 to .078)	(.346 to .877)	(.358 to .781)	(.025 to .044)
Brazil	−.033	.029	.589	.793	.019
	(−.071 to −.022)	(.025 to .032)	(.103 to .860)	(.627 to .923)	(.014 to .025)

Note.—For each country, we estimate an output process as $\Delta \ln y_t = \ln g_t + \sigma(\epsilon_t - \epsilon_{t-1})$, in which $\epsilon_t \sim N(0, 1)$ and $g_t \in \{g_L, g_H\}$, with $\Pr(g_{t+1} = g_L | g_t = g_L) = p_L$ and $\Pr(g_{t+1} = g_H | g_t = g_H) = p_H$. The table reports the mean and the interval between the 5th and 95th percentiles of the posterior distributions of each of the parameters for each country. The table also reports the prior distributions we used, which were chosen to be the same across countries. For each country, we use data on GDP per capita in 2016 US dollars (converted to 2016 price levels with updated 2011 purchasing power parity) between 1980 and 2017 from the Conference Board Total Economy Database as the measure of y_t. See app. A for a description of the estimation.

the data represent a combination of the true structural parameters and default costs, which are calibrated separately. In particular, Argentina's estimates are significantly affected by how unusual the years around the 2001 default were: GDP dropped by 20% during 1998–2002 and grew at a stunning 8% a year during 2003–8.[19] Thus, we think it is reasonable to consider a more conservative calibration for Argentina's process.[20] Specifically, we set a calibration for Argentina with $p_L = 0.60$, $p_H = 0.75$, $\ln(g_L) = -0.04$, $\ln(g_H) = 0.04$, and $\sigma = 0.023$. These values, with the exception of g_H, are within the 95% confidence interval of the posterior distributions of Argentina. Because we chose a higher value of p_H, the lower g_H brings the average growth closer to the one observed for Argentina. Table 2 contains the parameter values that we use in the quantitative analysis.

When comparing Argentina and Spain, the two calibrations capture the fact that South American countries have lower average debt maturity,

[19] To put these numbers in perspective, Argentina's GDP performance in this period was remarkably close to the experience of the United States during the Great Depression, in which GDP dropped by 27% during 1929–33 and recovered at a 7% annual rate during 1934–39.

[20] We obtain estimates for Argentina that are closer to those for Brazil when we use data for 1965–2000, i.e., when we use the same number of observations but omit the 2001 crisis. For this period, the estimated difference between g_H and g_L is 4.7 percentage points.

TABLE 2
Benchmark Calibration and Targeted Moments

Description/Moments	(1)	(2)	(3)
	A. Parameters That Are Common across Regions		
	Parameter	Value	
Discount factor	β	.75	
Risk aversion	γ	3.0	
Risk-free rate	R^*	1.035	
Reentry probability	$1-\theta$	.10	
Fraction of debt recovered after default	κ	.75	
	B. Parameters That Vary across Regions		
	Parameter	Argentina	Spain
Inverse of average maturity	δ	.40	.15
Probability of remaining in low growth	p_{L}	.600	.629
Probability of remaining in high growth	p_{H}	.750	.838
Low-growth rate	$\ln(g_{L})$	−.040	−.018
High-growth rate	$\ln(g_{H})$	.040	.033
Standard deviation of transitory shock	σ	.023	.017
Default cost in high-growth state	$\phi(g_{H})$	.90	.945
Default cost in low-growth state	$\phi(g_{L})$	.97	.935
Probability of bad sunspot	$p_{B} = 1 - p_{G}$	.25	.01
	C. Targeted Moments and Model Counterparts		
	Target	Model	
Argentina:			
Average face value of debt (% of GDP)	53	52	
Spread in interest rate schedule for $b = n =$ 19% and $g = g_{H}$ (%)	7.1	7.0	
Spain:			
Average market value of debt (% of GDP)	89	88	
Spread in interest rate schedule for $b = n =$ 15% and $g = g_{H}$ (%)	0	0	

deeper recessions, less persistent periods of high growth, and more volatile output growth and fluctuations overall.

Finally, given the choice of all other parameters, described above, we use data on debt levels and spreads over the risk-free rate to calibrate the costs of default for each country. As is standard in the quantitative sovereign default literature, following Arellano (2008), default costs are asymmetric. We choose the values so that the average stock of debt in the simulations is close to the debt position in the years before entering the crisis period and the spreads are close to the ones observed before the economy transitions from high to low growth.

Regarding debt levels, for Spain, we use the measure of net external debt from the Banco de España, which averaged 89% of GDP between 2008 and

2012.[21] For Argentina, most foreign assets were reserves held at the central bank, which were earmarked to back the monetary base at the time because of the currency board. Therefore, we decided not to subtract these reserves from the gross measure of the debt. We use Argentina's gross external debt from the World Bank's International Debt Statistics, which averaged 54% of GDP between 1997 and 2001 (see app. B for details on data sources and computations).

As pointed out by Dias, Richmond, and Wright (2014), existing measures of debt for Argentina and Spain are not comparable. The key difference is that while the data for Spain are calculated using market prices, the data for Argentina correspond to face values of the outstanding bonds. To get around these discrepancies between countries, we measure both the market value and the face value of debt in the model. The market value is Qb, with Q defined in equation (27). The face value is the undiscounted sum of principal payments due in the future, $F_t \equiv N_{t-1} + (1-\delta) N_{t-2} + (1-\delta)^2 N_{t-3} + \cdots$. We denote its detrended value by f_t. We then use the market value in the model to calibrate the case of Spain and the face value in the model to calibrate the case of Argentina.

In terms of spreads, we aimed to replicate the lower spreads observed in Argentina and Spain during their final periods of high growth before entering a recession. In the case of Argentina, spreads were at 7% by the end of 1998, while they were essentially zero for Spain in 2007. In our model, we target these levels of spreads in the high-growth interest rate schedules for Argentina and Spain, as depicted in figures 6 and 8, respectively, for their corresponding levels of debt service in 1998 and 2007.[22] The calibrated default costs along with the data targets and their corresponding model counterparts are presented in table 2.

In section V as well as in appendix section F.5, we perform sensitivity analysis for many of the parameters described above and show that the conclusions are reasonably robust to alternative parameter values.

D. Model Computation

The computation of the model has to take into account that there are multiple interest rate schedules satisfying equations (25) and (26). That is, the

[21] The recent work in Bocola, Bornstein, and Dovis (2019) discusses alternative debt measures that could predict credit spreads in European economies. We discuss a calibration with such alternative debt measures for Spain in app. sec. F.2.

[22] For the calibration, we assume the levels of debt service and issuance to be the same. In Argentina, they were approximately 19% of GDP, while in Spain, we estimate them to be between 10% and 15% of GDP. See app. B for more details on data sources and computations. Note that by targeting the levels of spreads in the schedules, we are not imposing that they are selected in equilibrium.

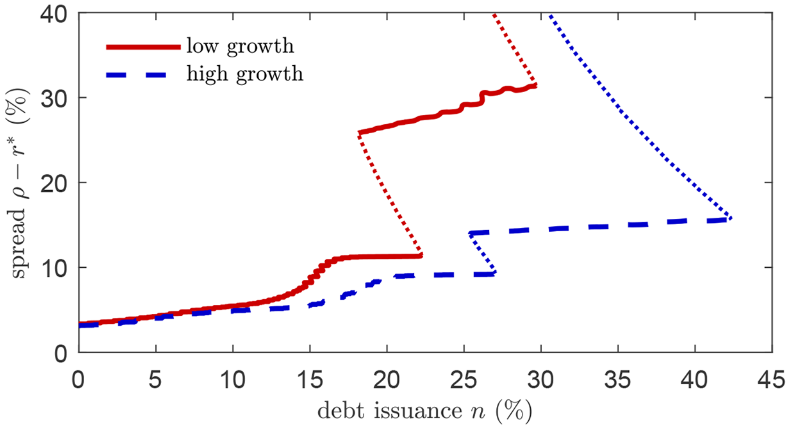

FIG. 6.—Interest rate spreads for Argentina ($b = 20\%$).

model computation must select schedules out of several possible equilibrium schedules, ruling out decreasing parts of the schedule (Ayres et al. 2015). The infinite-horizon nature of the model introduces an additional complexity because the selection of schedules affects value functions, which in turn affect the schedules. To overcome this issue, we develop an algorithm that iterates only on the value function and that, in each iteration, computes the respective interest rate correspondence, selecting the high and low interest rate schedules as a function of the sunspot. Appendix D contains more details on the algorithm used to compute the equilibrium.

5. Quantitative Results

We first show that for both calibrations, there are multiple interest rate schedules for the values for the debt service that Argentina faced in 2001 and Spain faced in 2012. This suggests that expectations may have played a role in triggering debt crises in both economies. A second set of results uses the simulation of the calibrated economies. We assess the relevance of the sunspot for the moments of the simulated economies—specifically, for spreads, default rates, and debt levels. A final set of results consists of an event case study of Argentina's default in 2001. We simulate the economy for 1997–2001 and compare the model-implied spreads with the ones observed in the data, assessing the role of the sunspot shock in generating the spike in spreads in 2001 and the subsequent default.

Recall that as a benchmark, we set $p_{\mathrm{B}} = 1\%$ for Spain, reflecting optimistic expectations, while we set $p_{\mathrm{B}} = 25\%$ for Argentina, reflecting pessimistic expectations. Beyond expectations, there are three differences between the calibrations for Argentina and Spain, all of them disciplined by data: the endowment process, the default costs, and the maturity of the debt. As it turns out, the difference in expectations is a major driver of differences in default probabilities and spreads between Spain and Argentina.

In what follows, we start by discussing the schedules for the two calibrated economies and then proceed to discuss both simulated economies. We end this section with Argentina's event case study.

A. *Multiplicity of Interest Rate Schedules*

1. Argentina

Here, we explore the calibration for Argentina, focusing on the sovereign debt crisis of 2001. By January 2001, sovereign debt spreads in Argentina were roughly 8%. From 1994 to 1998, the Argentine economy grew at high rates, close to 4% a year. By the end of 1998, a recession started, lasting until 2002. This can be interpreted as a regime switch from the high- to the low-growth state. Argentina defaulted on its debt in December 2001, after a couple of months with spreads that averaged around 30%.

In figure 6, we plot the interest rate schedules for the low- and high-growth states for debt service levels of 20% of GDP, close to what Argentina had in 2001 at the onset of the crisis.[23] The horizontal axis shows the new debt issuance, n, while the vertical axis shows the corresponding interest rate spreads.[24] For each growth state, the sunspot realization determines whether the high or low interest rate schedule is selected in the region of multiplicity. The dashed line corresponds to the high-growth state, while the solid line corresponds to the low-growth state. As discussed above, Argentina's calibration was chosen to match a 7% spread in the high-growth schedule, similar to what the country faced during the high-growth years before the crisis.

The size of the multiplicity region is state dependent, as figure 6 shows. In the high-growth state (dashed line), the region of multiplicity is small, so the schedules are similar under the good and bad sunspot. By contrast, in the low-growth state (solid line), interest rates can be either low or high depending on the sunspot for a larger set of issuance levels. In the low-growth state, there is multiplicity of schedules for issuance between 18% and 22% of output. For these levels of debt issuance, the low spread is around 12%. But there is also a high spread close to 30%. When expectations are good, the borrower can issue up to 22% of trend GDP at a spread of 12% or lower. Borrowing 22% when the debt service is 20%, as

[23] Debt service in 2000 for Argentina equaled 19.6% of GDP according to World Bank's International Debt Statistics. See app. B for more details on data sources and computations.

[24] Interest spreads are $\rho - r^*$, where ρ is the interest rate on new issuance, as defined in eq. (12).

in figure 6, implies a deficit of 2%. On the other hand, the country can issue only up to 18% with low spreads if expectations are bad. This means that a surplus of 2% of trend GDP is needed under the bad sunspot in order to maintain the lower spreads. Thus, under bad expectations, a large surplus adjustment is needed in order to obtain a low spread.

A noticeable feature of figure 6 is that the high spreads resemble the ones for Argentina around the 2001 debt crisis. Spreads in Argentina went up to nearly 30% during the last 2 months of 2001. This number was not targeted in our calibration.

In this model, expectations play a significant role when fundamentals—the growth rate of the economy—are weak. The reason for this state-dependent role of expectations is that in periods of low growth, the probability of observing low growth in the future is high because of the high persistence of the growth shock. Thus, if the borrower is expected to default in the low-growth state, which has a sizable probability, the interest rate must be high. This high interest rate will, in turn, induce the borrower to default in the low-growth state, which confirms the expectations. This will happen even for low debt levels. If the borrower is not expected to default in the low-growth state, however, the interest rates will be relatively low, and the borrower will be able to issue a larger amount of debt without risking default next period. This translates into a large region of multiplicity, in which for intermediate levels of debt, interest rates can be either high or low depending on expectations.

In contrast, persistence implies that in periods of high growth, the probability of switching to low growth is low. Thus, even if the borrower is expected to default in the low-growth state, the interest rate consistent with these expectations will be low, since the probability of the low-growth state is low. Expectations will be confirmed, meaning that the borrower will default in the low-growth state but only when debt levels are relatively large. If the borrower is not expected to default in the low-growth state, however, the interest rates are only marginally lower, and the debt levels such that the borrower will not default are not much higher. This translates into a small region of multiplicity.

The probability of switching to the low-growth state in the infinite-horizon model is the analog of the probability of the low endowment in the two-period model of section II. When that probability is low, the region of multiplicity is small, whereas when that probability is high, the region of multiplicity is large. In the infinite-horizon model, the probabilities are functions of the state. In low-growth states, the probability of future low growth is high, and the region of multiplicity is large. In high-growth states, that probability is low, and the region of multiplicity is small.

In what follows, we show how the schedules change as we vary debt service, b, and key parameters of the model. We restrict the analysis to the schedules for interest rate spreads in the low-growth state, since that is when multiplicity is more prevalent, as discussed above. Given the focus of our analysis, we are particularly interested in the effects of changing the probability of the sunspot, the debt service, and the average maturity. A full set of robustness results is presented in appendix section F.5.

The role of the sunspot.—We compute the schedule setting the probability of the bad sunspot to $p_B = 1\%$ rather than $p_B = 25\%$, as we used for the benchmark.[25] The schedule for the case in which $p_B = 1\%$ when the growth rate is low is depicted in figure 7*A* (dashed line) together with the one corresponding to the benchmark (solid line).

The effect of the change in p_B is striking. If the probability of the bad sunspot had been very low, the country would have faced zero spreads even if it issued new debt up to almost 35% of trend GDP. In our narrative, this implies that with optimistic expectations, Argentina could have been able to borrow to service the debt plus a few extra points of GDP and still be well within the debt choices that essentially rule out default, which explains the zero spread.

The role of the inherited debt.—We now show the effects of the inherited debt service b in figure 7*B*. The solid line, like the one in figure 6, corresponds to the benchmark. The dashed line corresponds to a debt service of $b = 15\%$. As the figure shows, it is possible to roll over a debt service of 15% with a single low interest rate. Only if the government attempts to borrow another 8% of output, for a total of 23%, will multiplicity potentially matter.

The role of average maturity.—To assess the role of the average maturity, δ, figure 7*C* reproduces the schedule for the benchmark calibration together with the ones for either higher or lower average maturity. As before, we show only the schedules for the low-growth state, where multiplicity is quantitatively relevant. We change the average maturity but keep total debt constant so that the service of the debt, b, changes accordingly. We also plot the cases of maturity of 6.7 years ($\delta = 0.15$) and a low maturity of 1.7 years ($\delta = 0.6$) together with the case of the benchmark average maturity of 2.5 years, plotted in figure 6

As the maturity increases, the incentives to dilute the debt are stronger, which leads to higher spreads today. On the other hand, given a total level of debt, longer maturity implies lower current debt service and thus a smaller effect of an increase in current rates on the future debt obligations. As figure 7*C* shows, longer maturity implies higher rates for low issuance levels. Yet as longer maturity implies lower debt service, the issuance needed to service the debt is lower and away from the multiplicity region. For example,

[25] As mentioned above, we have solved the model for several values of the probability of the bad sunspot, p_B, between 0% and 50%. The solution essentially depends on its value being above or below a threshold close to 10%. For values below the threshold, the results are very similar to the case of $p_B = 1\%$, while for values above the threshold, the results are very similar to $p_B = 25\%$.

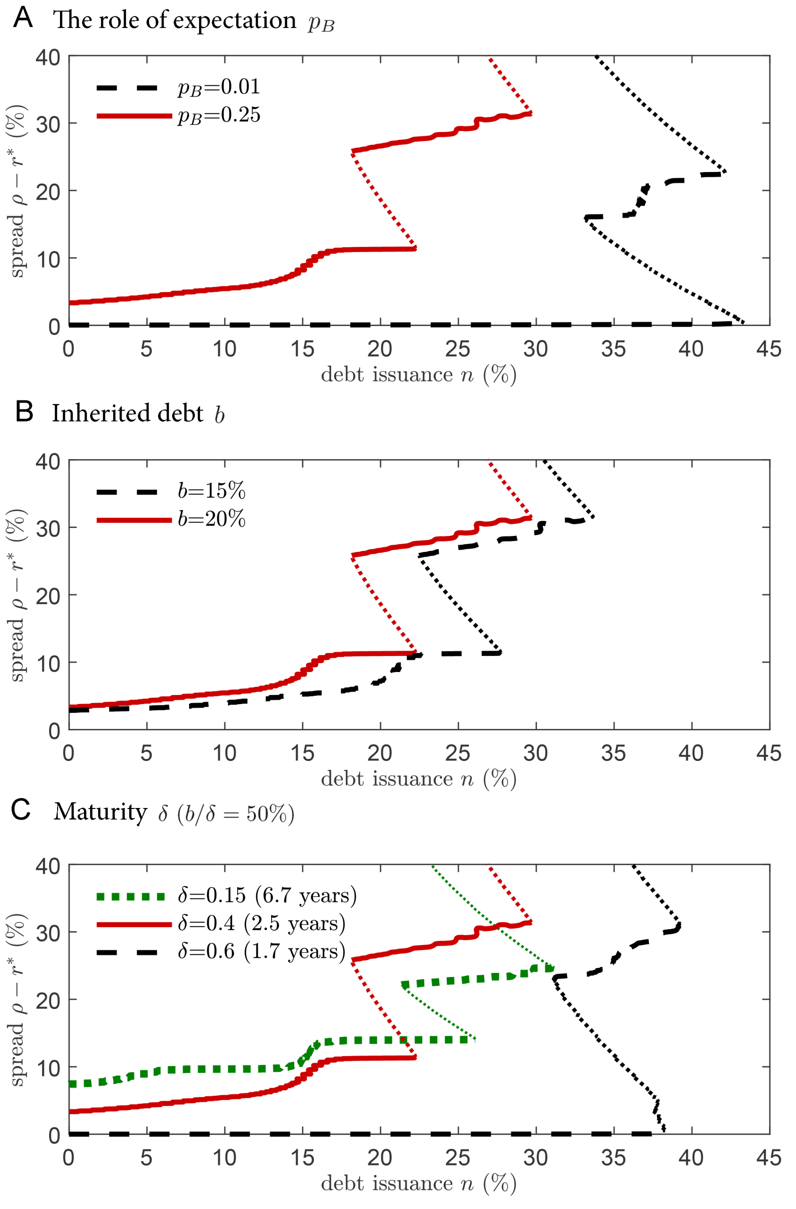

FIG. 7.—Interest rate spreads for Argentina in low-growth state ($b = 20\%$): comparative statics.

with a 50% debt-to-output ratio and $\delta = 0.15$, only 7.5% of output needs to be serviced every period, a value comfortably below the multiplicity region. Thus, while interest rates would have been higher, a higher maturity could have kept Argentina away from a crisis, even with positive but mild deficits.

The case of a shorter maturity of 1.7 years ($\delta = 0.60$) is quite different. Spreads are zero, even for debt issuance as large as 30% of output. This is quite remarkable, since it means that longer maturities do not necessarily translate into better financing conditions. The interpretation of this result is that for the short maturity, debt dilution incentives are minor (see Chatterjee and Eyigungor 2012; Aguiar et al. 2019). This effect, combined with Argentina's low debt level, implies that there is almost no default in equilibrium. One could be tempted to conclude that the short maturity has attractive features from a policy standpoint. However, the debt dilution effects are counterfactually large in this class of models; see Aguiar and Amador (2020) and chapter 7 in Aguiar and Amador (2021). Thus, we believe that these implications ought to be taken with caution.

2. Spain

We now explore the calibration for Spain. Before the crisis, Spain was in the high-growth regime, and the spread was essentially zero. Following the 2009–10 recession, growth was dismal, which we interpret as the low-growth regime. Spreads rose steadily past 6% by July 2012, when the ECB announced a policy intervention. Spreads then fell consistently and reached 1% by 2014.

In figure 8, we plot the interest rate schedules for both the low- and the high-growth states. In contrast to Argentina, we do not have direct data on Spain's debt services due each year. However, we can impute debt service using data on short and long debt, interest payments, and the maturity structure (see app. B for details). For 2011, we obtain an average debt service relative to GDP of $b = 15\%$, the value we use to plot Spain's schedules. As discussed above, the calibration for Spain was chosen to match a spread of essentially zero in the schedule, corresponding to the high-growth state.

For the low-growth state, for debt issuance levels between 17% and 22% of output, interest rates can be either low or high depending on the sunspot, as figure 8 shows. When expectations are good, the borrower can issue new debt up to 23% of trend GDP under the lower interest rate spread. In contrast, when expectations are bad, the borrower can issue only up to 17% of trend GDP under the lower spreads. This means that given a debt service of 15%, there is multiplicity of spreads for deficits of 2%–8%. The deficits Spain experienced in the years before the debt crisis belong to those intervals.

Figure 8 supports a narrative regarding the debt crisis in Spain consistent with multiple equilibria. By 2009, Spain had entered the low-growth

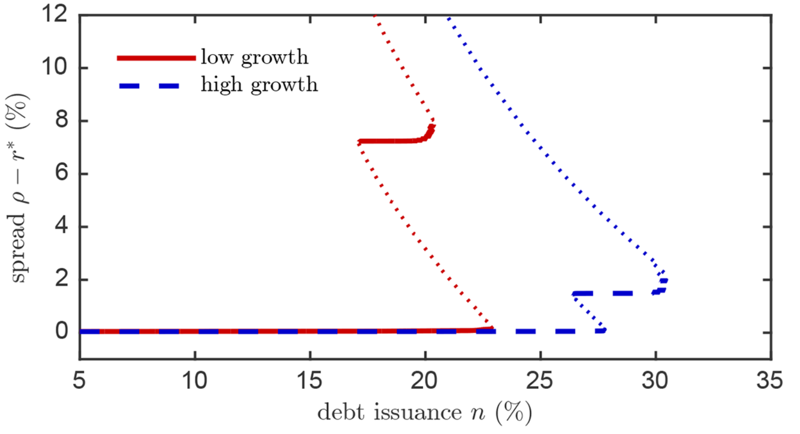

FIG. 8.—Interest rate correspondence for Spain ($b = 15\%$).

regime, so the relevant schedule is the solid line in figure 8. While expectations were good, Spain could run deficits with a low spread. Eventually, the bad sunspot was realized, and spreads increased. As figure 8 shows, the difference between the high and low interest rate spreads in the multiplicity region—which has not been calibrated—is about 7%. This difference in spreads is very similar to the maximum spread observed in Spain (slightly above 6%).

The narrative for Spain is only partially confirmed by the analysis of the simulated economy, as we discuss below. Endogenous austerity plays a key role in avoiding high spreads, so the economy does not borrow at expectations-driven high rates in equilibrium.

The role of the sunspot.—We now explore the effect of increasing the sunspot probability from $p_B = 1\%$ to $p_B = 25\%$, as in the benchmark for Argentina. Figure 9*A* plots the schedule under the low-growth state for both values of p_B.

As with Argentina, the value of p_B substantially changes the interest rate schedules. With more pessimistic expectations, spreads increase sharply, and Spain's capacity to borrow is substantially reduced. We show that this change in schedules carries its effect on equilibrium outcomes: had Spain faced the same value of p_B as Argentina, spreads and default rates would have been close to Argentina's.

The role of the inherited debt.—The comparative statics with respect to the initial value of b are similar to the ones in the case of Argentina: the schedule shifts to the left as the debt services b increase. However, we think it is interesting to consider in detail the counterfactual in which—rather than starting with a debt service level of $b = 15\%$, as in 2012—Spain started with a debt service level of $b = 17\%$, as in the ergodic mean of the model (see

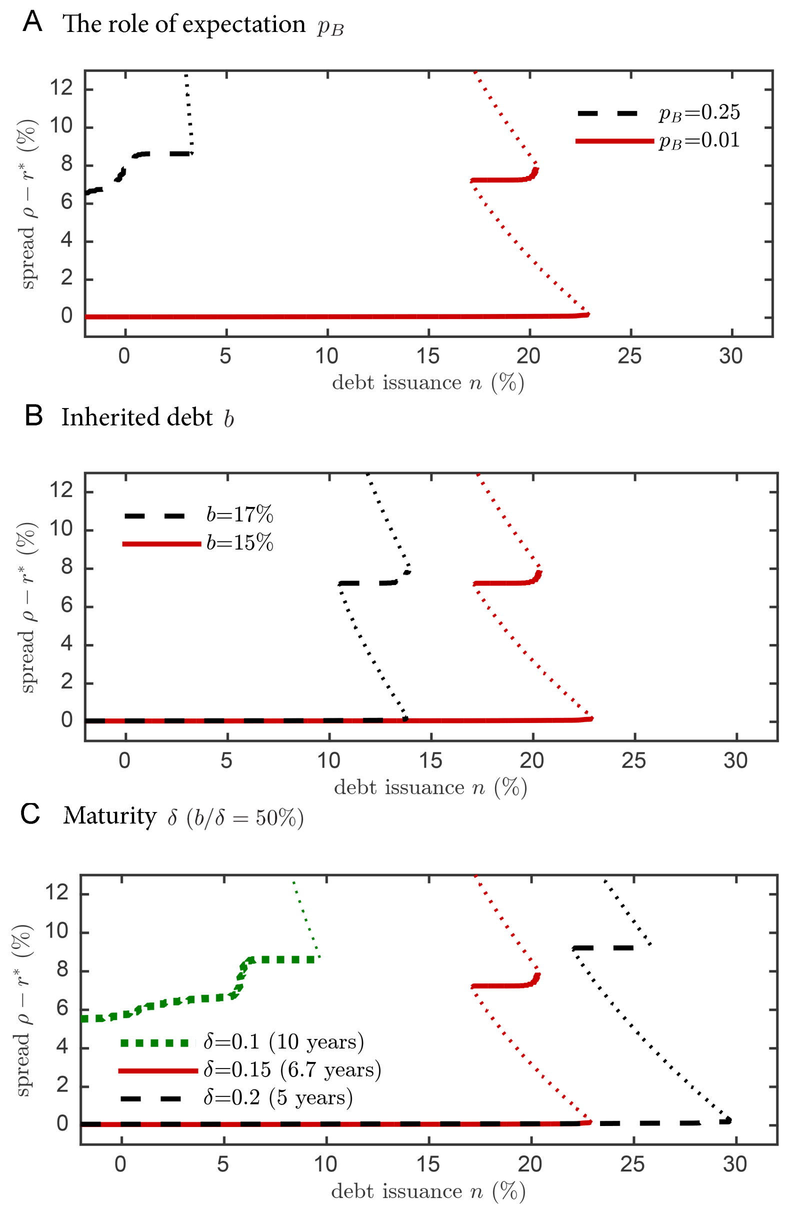

FIG. 9.—Interest rate spreads for Spain in low-growth state (b = 15%): comparative statics.

table 4). Thus, figure 9*B* shows the low-growth schedule for a debt service of $b = 17\%$ together with the benchmark of $b = 15\%$ in figure 8. For the case of $b = 17\%$, the region of multiplicity occurs for levels of debt issuance between $n = 11\%$ and $n = 15\%$ of output, which would involve current account surpluses, since 17% of output must be rolled over. These numbers imply that because the debt obligations were lower than the average of the invariant distribution, Spain managed to handle the crisis substantially better and run deficits while expectations were good.

The role of average maturity.—Figure 9*C* reports the schedules for different levels of debt maturity, δ. The intuition of the results is the same as before: shorter maturity ameliorates the debt dilution problem and moves the multiplicity region further to the right.

B. The Simulated Economies

We now show the moments of the simulated economy for both calibrations and discuss how they change as we vary the probability of the sunspot.[26]

1. Argentina

Table 3 shows the moments for the simulated economy for Argentina. Panel A lists unconditional first moments. Panels B and C show first moments conditional on the growth state. Finally, panel D lists unconditional second moments. The columns represent two different parameterizations of the economy. Column 1 shows the moments for the benchmark calibration, and column 2 shows the effect of reducing the probability of the bad sunspot to $p_{\mathrm{B}} = 1\%$. All moments are computed during periods when the country is not in default.

The main takeaway from table 3 is that the probability of the bad sunspot plays a crucial role in driving up average spreads, particularly in the low-growth state, and affects borrowing choices. As the bad sunspot probability goes down from 25% to 1%, default rates decrease from 5.3% to 0.5%, while spreads decrease from 16.2% to 0.3%. That is, without any changes in fundamentals, only in beliefs, a borrower mutates from a serial defaulter to virtually a nondefaulter.

Endogenous austerity and gambling for redemption play a key role in producing the simulated moments for Argentina. Both can be observed in the behavior of the policy function for new debt issuance (fig. 10*A*, 10*B*) and the corresponding spreads (fig. 10*C*, 10*D*). Figure 10*A* and 10*C* show results for

[26] This section reports outcomes across the ergodic distribution of the model. Appendix F reports outcomes conditional on a crisis episode. In particular, app. sec. F.3 computes the model response to a bad sunspot realization and relates results to this section. Appendix sec. F.4 compares crisis episodes in the model with the evidence in Paluszynski and Stefanidis (2023).

TABLE 3
SIMULATION MOMENTS: ARGENTINA

	Benchmark (p_B = 25%) (1)	p_B = 1% (2)
	A. First Moments (%)	
avg(spread)	16.2	.3
avg(qb/y)	56	76
avg(f/y)	53	73
avg(n/y)	27	30
avg(b/y)	31	32
avg(tb/y)	4.4	2.1
Default rate	5.3	.5
	B. Low-Growth State	
avg(spread)	27.6	.3
avg(qb/y)	28	79
avg(f/y)	41	77
avg(n/y)	26	28
avg(b/y)	24	34
avg(tb/y)	−2	5.9
Default rate	13.6	1.3
	C. High-Growth State	
avg(spread)	15.5	.2
avg(qb/y)	58	73
avg(f/y)	54	71
avg(n/y)	27	31
avg(b/y)	32	31
avg(tb/y)	4.8	−.3
Default rate	0	0
	D. Second Moments	
corr(spreads, y)	−.52	−.29
SD(spreads) (percentage points)	3.7	.1
SD(c)/SD(y) (percentage points)	2.4	1.6

NOTE.—b = total debt service; qb = market value of debt; f = face value of debt; n = debt issuance; tb = trade balance; y = output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

the low-growth state, while figure 10*B*, 10*D* show results for the high-growth state.

Endogenous austerity is the reason why new debt issuance, n, declines as the outstanding debt service, b, increases. Even if the outstanding debt service is higher, the borrower reduces new issuance in order to avoid discrete jumps in interest rates, thus cutting down consumption sharply. These interest rate jumps depend on future debt services and therefore occur for lower levels of n as b increases (see Argentina's schedule in fig. 7*B*). Gambling for redemption instead is the mechanism behind the sharp

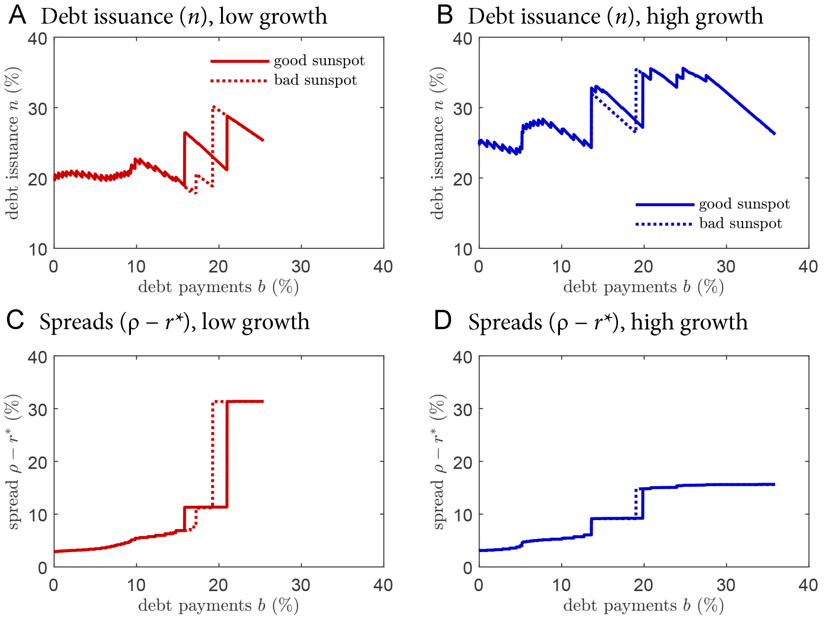

FIG. 10.—Policy functions and equilibrium spreads for Argentina.

increases in new issuance n. The borrower chooses to further increase issuance, even at higher borrowing costs, in order to increase consumption.

The simulations for Argentina in the benchmark calibration ($p_B = 25\%$) visit the region of debt service levels where spreads are high because of expectations. Both endogenous austerity and gambling for redemption are at play there, resulting in high equilibrium spreads and default rates, in particular, in periods of low growth.

Expectations play a key role: with more optimistic expectations ($p_B = 1\%$), endogenous austerity would have prevailed for Argentina, inducing low spreads and default probabilities, as column 2 in table 3 shows.

The sunspot also significantly affects second moments, as panel D in table 3 shows. In particular, as p_B moves from 25% to 1%, the volatility of spreads decreases by a factor of 40, while the volatility of consumption decreases by one-third.

2. Spain

The moments for the simulated economy for Spain are in table 4, which is the analog of table 3 for Argentina. It shows the moments for the optimistic benchmark ($p_B = 1\%$) and for the case of pessimistic expectations ($p_B = 25\%$).

TABLE 4
Simulation Moments: Spain

	Benchmark (p_B = 1%) (1)	p_B = 25% (2)
	A. First Moments (%)	
avg(spread)	1.2	7.7
avg(qb/y)	88	53
avg(f/y)	86	52
avg(n/y)	18	12
avg(b/y)	17	14
avg(tb/y)	−1.4	1.6
Default rate	5.2	6.1
	B. Low-Growth State	
avg(spread)	.1	9.7
avg(qb/y)	80	35
avg(f/y)	84	47
avg(n/y)	17	12
avg(b/y)	16	12
avg(tb/y)	−.9	.5
Default rate	18	20
	C. High-Growth State	
avg(spread)	1.4	7.5
avg(qb/y)	89	55
avg(f/y)	86	53
avg(n/y)	18	12
avg(b/y)	17	14
avg(tb/y)	−1.5	1.8
Default rate	0	0
	D. Second Moments	
corr(spreads, y)	.46	−.34
SD(spreads) (percentage points)	.7	1.5
SD(c)/SD(y) (percentage points)	2.7	2.3

Note.—b = total debt service; qb = market value of debt; f = face value of debt; n = debt issuance; tb = trade balance; y = output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

In the low-growth state, the average debt service b is around 16% of GDP. Just rolling over the debt implies borrowing in the multiplicity region, as figure 9B shows. Yet as table 4 also shows, spreads in the low-growth state are essentially zero, while default rates are high. The reason is that more than 99.3% of default episodes occur in transitions from high growth to low growth. While in the low-growth state, endogenous austerity prevails, and spreads are low. Interestingly, spreads are procyclical if the sunspot probability is low, but they are countercyclical if the sunspot probability is high.

This can be seen in figure 11, which shows debt issuance policies and the corresponding equilibrium spreads. The debt issuance policy function in the low-growth (high-growth) state is depicted by solid lines for the good sunspot and dotted lines for the bad sunspot. Figure 11*C* and 11*D* show the corresponding equilibrium spreads. As discussed before, endogenous austerity is the reason why issuance, n, declines as debt service, b, increases.

In the low-growth state, the policy function exhibits strong endogenous austerity, with issuance declining as debt service increases. The realization of the bad sunspot induces even stronger endogenous austerity. Overall, default rates are close to zero under low growth, and thus equilibrium spreads are low.

All this implies that while just rolling over debt service in the low-growth state leads to borrowing in the multiplicity region, that never happens in equilibrium. Endogenous austerity keeps the economy below the multiplicity region so that high spreads due to bad expectations do not happen in equilibrium in the low-growth state. Thus, while there is multiplicity of spreads for the debt service levels Spain faced in 2012 (fig. 8), the simulated economy does not give rise to those high spreads.

The actual experience of Spain and other European countries was to implement austerity measures in response to the observed high spreads. In the model, the high spreads triggering endogenous austerity happen off

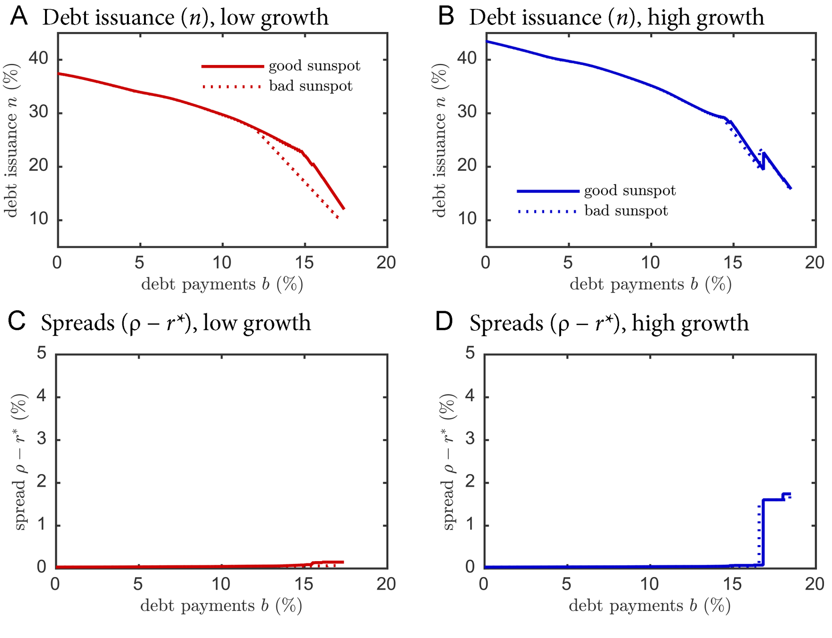

FIG. 11.—Policy functions and equilibrium spreads for Spain.

equilibrium, so they are not observed in the simulations. Thus, our model is only partially successful in explaining the events in those countries during the sovereign debt crisis period.

We conjecture that with reasonable changes to our model, Calvo-type multiplicity could still explain the jump in Spain's spreads in 2012. In particular, as we discuss in section VI, the ECB announcement that it would intervene in sovereign debt markets was a significant policy change taken during summer 2012, and market intervention was far from granted the years before that. Thus, a time-varying value for p_B could more accurately capture the historical likelihood of an ECB intervention. Stochastic changes in p_B (as in Bocola and Dovis 2019) may induce a rise in spreads as a result of expectations becoming more pessimistic over time. That is, a gradual increase in p_B could lead to spreads increasing to almost 7%, as observed for Spain during July 2012. We opt for a simpler model with fewer parameters and thus a time-invariant low sunspot probability.

3. Sunspot Probability and Behavioral Responses

The probability of the bad sunspot, p_B, plays a crucial role in shaping the importance of endogenous austerity and gambling for redemption behaviors. Think of a borrower who happens to be in the region of multiplicity. If the bad sunspot occurs, the borrower has two options: strongly cut down issuance to avoid the high interest rates (endogenous austerity) or keep/increase issuance at higher interest rates in the hopes of high growth in the near future (gambling for redemption). The cost of endogenous austerity is the strong decline in consumption, while the benefit is to keep financial market access. The benefit of financial market access is particularly appealing if expectations-driven high rates are a rare event. That is, endogenous austerity is more appealing when p_B is low. If the borrower believes instead that markets will often feature expectations-driven high rates, financial market access is not as appealing. That is, gambling for redemption is more appealing when p_B is high.

Our calibration for Spain assumes a low p_B, and thus Spain features endogenous austerity, while the calibration for Argentina assumes a high p_B, and thus Argentina exhibits gambling for redemption.

4. Expectations and the Role of Policy

As section II.A discusses, in the presence of a lender of last resort, the effective probability of facing the high-rate schedule is $p_B \times (1 - \pi)$, where π is the probability of a policy intervention. In that sense, a lower p_B can be understood as a higher probability of an intervention, π. Our results show that a higher probability of intervention has large effects on credit spreads and default rates not only during a debt crisis but also on the ergodic distribution of the model. Indeed, for both Spain's and Argentina's calibrations, if p_B is low (probability of intervention is high), the spreads are virtually zero, while if p_B is high (probability of intervention is low), the spreads are high. Appendix section F.1 quantifies the potential gains of a policy intervention by computing a Pareto frontier between lenders' and borrowers' utility for different sunspot realizations.

C. *An Event Case Study: Argentina's 2001 Default*

We now test the quantitative capacity of the model to account for the events leading up to Argentina's default in 2001.[27] The exercise is in the spirit of the ones in Arellano (2008) and Chatterjee and Eyigungor (2012). In particular, we ask whether the model can account for the path of interest rate spreads during the 1997–2001 period and quantify the relevance of the sunspot. As we show, the model can account for the path of Argentina's spreads, and the sunspot plays a key role: absent a bad sunspot realization, Argentina would have avoided default.

We design the event case study as follows. We simulate the model for 1997–2001 by selecting the output shocks (g_t and ϵ_t) to match observed Argentina's GDP growth. We assume that Argentina was in a high-growth state during 1997–1998 and switched to the low-growth state during 1999–2001.[28] We pick the initial debt service level so that the debt service is close to 20.2% in 2000, as was the case for Argentina in that year. We then consider two possible cases for the sunspot realization: a "bad sunspot" case, in which the sunspot selects the high interest rate schedule in all years, and a "good sunspot" case, in which the sunspot selects the high interest rate schedule in all years except in 2001. That is, the bad sunspot and the good sunspot cases differ only in their 2001 sunspot realization.[29] Figure 12 shows the model-implied interest rate spreads for both cases and compares them with Argentina's actual credit spreads.

There are two key findings in this exercise. First, for 1997–2000, the model-implied credit spreads are aligned with the data, an outcome that is not directly targeted by our exercise design. Second, the model accounts for the spike in spreads in 2001, but only in the "bad sunspot" case. In contrast, the "good sunspot" case generates substantially lower spreads. Importantly, the model predicts default in the year 2002 only under the "bad sunspot"

[27] We are thankful to the referees who suggested this exercise.

[28] We set the sequence of growth shocks g_t as the most likely state implied by the Kim (1994) filter we used to estimate the output process. See app. E for more details on the computations.

[29] The economy is not in a multiplicity region during 1998–99, and thus the sunspot is not relevant for the initial years.

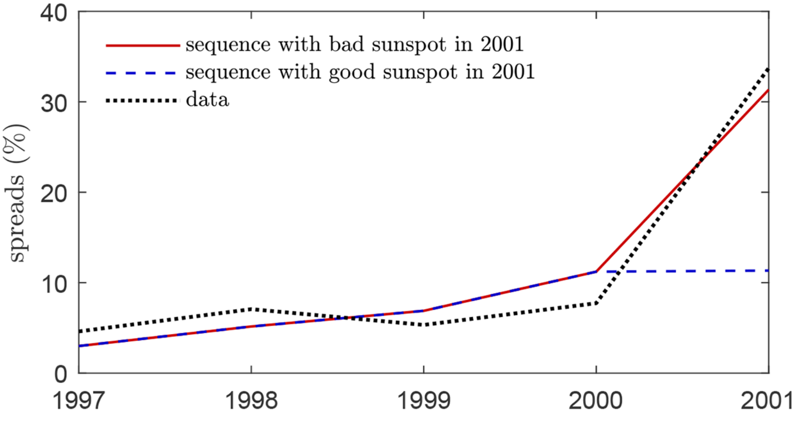

FIG. 12.—Argentina: end-of-period spreads from 1997 to 2001. For 2001, we use spreads at the end of November. The model predicts default in 2002 for the bad sunspot sequence but no default under the good sunspot sequence.

case, but not under the "good sunspot" case.[30] Furthermore, since Argentina experienced high growth starting in 2003, the model predicts no default in 2003 and in the high-growth years that followed. In this sense, the role of expectations was essential: Argentina would have avoided default altogether under more favorable expectations.

The exercise in this section is analogous to the one in figure 3 of Chatterjee and Eyigungor (2012), which can also match the path of Argentina's output and spreads in the period before 2001.[31] Like in their case, our quantitative model relies on low output realizations (low-growth shocks in our case) to account for the interest rate spreads during Argentina's debt crisis. Unlike their case, the low output realizations are not enough in our exercise, and bad sunspot shocks are needed to generate the spike in spreads and the subsequent default. The difference in results reflects modeling differences, such as the stochastic process for output and the assumptions on debt recovery. We conjecture that a main difference is the choice of unobserved parameters to match average credit spreads, especially those related to default costs. In models that assume away multiplicity, the calibration of default costs may be masking the role of expectations, which are not allowed to play a role.

The implications for policy are probably the most important difference between our analysis and the one in Chatterjee and Eyigungor (2012). As discussed in section II.C, a lender of last resort can coordinate expectations

[30] Argentina defaulted on December 26, 2001. In the model, default occurs in 2002 under the bad sunspot case.

[31] See also fig. 5 in Arellano (2008) for a similar exercise.

and eliminate the effect of the sunspot on spreads and default rates. Our analysis suggests that Argentina's 2001 default could have been avoided with an intervention of a lender of last resort that coordinated the behavior of lenders in the low interest rate schedule. By contrast, in sovereign default models with no role for expectations, the presence of a lender of last resort has no effect on equilibrium spreads, as long as loans are correctly priced according to the default probabilities (so that the creditor breaks even). To put it differently, by assuming away the possibility of multiple equilibria, the standard approach in the literature may be confounding exogenous default costs with policy interventions. As we discuss next in section VI, support by a lender of last resort was a key difference between the experiences of Spain in 2012 and Argentina in 2001.

VI. Policy and Expectations: A Historical Discussion

We have argued that expectations can be a major driver in explaining default rates and credit spreads in sovereign debt crises, especially when growth rates are persistently low and debt service levels are moderately high. We have also established a theoretical link between expectations and the policy of a lender of last resort, which can help discipline the quantitative analysis of the role of expectations. In this section, we review the historical roles of the ECB during the European sovereign debt crisis and the IMF during the crisis in Argentina. We argue that while the ECB eventually decided to act as a full-fledged lender of last resort, the IMF was constrained—by its own rules and recent experience—in providing Argentina with enough financing to rule out undesirable outcomes. This discussion justifies our choice of a low probability of the bad sunspot for Spain ($p_B = 1\%$) and a relatively high probability for Argentina ($p_B = 25\%$) in our benchmark calibration. In what follows, we briefly describe the history of these two relationships—the one between Spain and the ECB and the one between Argentina and the IMF—in the years before the two crises.

A. Spain and the ECB

The founding treaties of the European Economic and Monetary Union were clear in limiting the ability of the ECB to act as a lender of last resort in the sovereign debt markets of its member states. For example, Article 123 of the Treaty on the Functioning of the European Union specifically prohibits direct purchases of sovereign bonds by the ECB.[32] In spite of this limitation, spreads on sovereign debt of those member countries were virtually zero for almost 10 years following the introduction of the single

[32] See Cochrane et al. (2025) for a thorough discussion of the evolution of the role of the ECB in preventing banking and sovereign debt crises.

currency in 1999. The increase in spreads in the early 2010s raised the question of whether those limits on the actions of the ECB were appropriate. The answer was a clear no, framed in Draghi's "whatever it takes" speech of July 26, 2012: "Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough." The intentions of the president of the ECB were supported by the launching of the OMT bond-buying program a couple of months later.

An important argument in favor of a decisive role of the ECB in managing expectations through the euro area sovereign debt crisis is the apparent impact of Draghi's speech. The speech coincided with the immediate reversion of the increasing trend of spreads. The fact that spreads declined after the policy announcement and that the actual bond-buying scheme was not used suggests that the policy announcement was enough to coordinate expectations in the good equilibrium.[33]

Draghi's speech of July 2012 indeed represented a radical change in the way the ECB would intervene in sovereign bond markets. Draghi began his magical "whatever it takes" sentence by asserting that the implied policies were within the ECB's mandate. Whether those policies were indeed part of the mandate was far from clear in the years preceding summer 2012. According to De Grauwe (2011), the ECB was reluctant to intervene in sovereign debt markets, even as rising spreads and likely default were threatening the monetary union:

> The ECB has made it clear that it does not want to pursue its role of lender of last resort in the government bond market. This has forced the Eurozone members to create a surrogate institution (the European Financial Stability Facility or EFSF and the future European Stability mechanism or ESM). The problem with that institution is that it will never have the necessary credibility to stop the forces of contagion; it cannot guarantee that the cash will always be available to pay out sovereign bondholders.

The main opposition to having the ECB intervene in those markets came from Germany. Allegedly, during 2011 and the first half of 2012, the issue was addressed in several G7 and G20 meetings. In those meetings, Mario Monti, at that time prime minister of Italy, had made the proposal to have the ECB intervene, but it was rejected by the German chancellor, Angela Merkel. This position changed over summer 2012 as the European debt crisis unfolded.[34]

[33] A notable article on the effects of Draghi's speech in *The Economist* (2021) reads, "The bond-buying scheme the ECB assembled to render Mr Draghi's promise credible was never used: his words were enough to calm the financial furies."

[34] Peter Spiegel, managing editor for the *Financial Times*, reviews the historical circumstances that led to the shift in ECB policy: "Mr Draghi's programme was unlikely to have quelled

Given the sequence of events, it seems clear that Draghi's speech was meant to convince the markets that there had been a shift in the way the ECB was willing to intervene. The discussion suggests that before July 2012, it was reasonable to believe that the probability of an intervention by the ECB large enough to rule out bad outcomes was well below 1. After Draghi's speech and Merkel's confirmation a few weeks after the announcement, that probability became essentially 1.

In the notation of section II.C, for any given probability of the bad sunspot realization in Spain, p_B, the events of summer 2012 made it clear that the probability of the intervention, π, was close to 1. Thus, the probability of the high interest rate schedule occurring, $p_B(1-\pi)$, became very close to zero.

It is tempting to see the smooth increase and subsequent decrease of spreads as driven by time-varying market perceptions of the probability of an ECB intervention, π. However, in the spirit of keeping parameters to a minimum, we refrained from following that route. Instead, given that by summer 2012 it became clear that the ECB was prepared to "do whatever it takes" and that Draghi insisted that "believe me, it will be enough," we choose to set the probability of the sunspot in the benchmark calibration for Spain to a negligible number, $p_B = 1\%$.

B. Argentina and the IMF

The relationship between the IMF and Argentina has been brilliantly analyzed by Mussa (2002). A notable feature of Argentina's default is that the ratio of debt to output was lower than 50% by the end of 2000. Regarding this, Mussa (2002) notes, "It may be asked why a debt-to-GDP ratio of just above 40% was worrying for Argentina." In addressing this question, he uses arguments that are close to the mechanism in our paper:

> Argentina was clearly vulnerable to changes in financial market sentiment. . . . If market sentiment ever shifted to an expectation of significant risk that Argentina might default, its market access would be cut off and that expectation would soon become self-fulfilling. (16)

Spreads on Argentine debt during 2000 were close to 600 basis points. By mid-2001, the spreads were getting closer to 1,500 basis points. The impact of these 900 additional basis points on the service of the debt was gigantic. Calculations of this type are reminiscent of the coordination problems

markets without Ms Merkel's acquiescence, which was given despite the public objections of the powerful German Bundesbank (with two members associated with the Bundesbank resigning from the ECB over the issue). . . . Ms Merkel's willingness to back OMT was a reflection of how deep the crisis had grown that summer. . . . Ms Merkel's ultimate embrace of Mr Draghi's programme capped a year-long shift of thinking in Berlin" (Spiegel 2014).

leading to multiple equilibria developed by Calvo (1988) and endorsed in our quantitative model.

Argentina had agreements with the IMF during most of the years between 1991 and 2001. However, the IMF's ability to do "whatever it takes" was severely limited by its own rules and also by the previous programs that the IMF had signed with other countries. In particular, the IMF was under heavy criticism after two recent large programs—one with Russia and another one with Indonesia—had ended in major crises. This is explicitly mentioned by Mussa (2002):

> Another approach would have been a very big bailout that would have provided the Argentine government with guarantees of official support sufficient to cover its prospective financing requirements for several years. However, this would have meant a substantial escalation in the magnitude and duration of official financing packages, and there was no support for this among the major countries that provide the Fund resources. (35)

Mussa is referring to what became known as the Prague 2000 communiqué of the IMF's International Monetary and Finance Committee, which discussed the late 1990s large packages.[35] Bullet 21 of the communiqué mentions that "the Committee notes that Fund resources are limited and that extraordinary access should be exceptional; further, neither creditors nor debtors should expect to be protected from adverse outcomes by official action." Such was the reaction to the criticism that the IMF faced after the already failed large programs in Russia and Indonesia.

In discussing the renewal and possible augmentation of the program by late 2000, Mussa mentions that an option that was then considered—and not adopted—was to terminate the program. He writes, "Some in the official community appeared to favor this approach. . . . This included those who were deeply opposed to large packages of official support" (Mussa 2002, 30).

This discussion shows that the IMF was in no position to intervene in Argentina in the way the ECB was ready to intervene in European bond markets. For this reason, we feel comfortable calibrating the probability of the bad sunspot, p_B, to 25% for Argentina.

VII. Conclusion

In Calvo's (1988) model of sovereign debt crises, there are multiple interest rate schedules because expectations of high probabilities of default are self-confirming. In particular, if expectations of default are high, interest rates must be high, and high interest rates increase the burden of debt, inducing

[35] See https://www.imf.org/en/News/Articles/2015/09/28/04/51/cm092400.

the borrower to default. The question that remains is whether the source of multiplicity is quantitatively relevant. In particular, we are interested in determining the role that it may have played in the sovereign debt crises of Argentina in 2001 and southern Europe in the early 2010s.

We argue that the mechanism in Calvo (1988) is quantitatively relevant and that key for multiplicity is a bimodal output process with persistent good and bad times. We estimate this output process for a set of countries that have recently been exposed to sovereign debt crises. We show that a sunspot realization can induce discrete jumps in interest rates even with no change in fundamentals. These expectations-driven jumps in interest rates can occur only during stagnations. Interest rate jumps—either because of expectations or because of fundamentals—can induce either endogenous austerity, in which the borrower refrains from borrowing to avoid the jump in rates, or gambling for redemption, in which the borrower increases debt beyond the jump in rates.

We consider two calibrations of the model, one targeted to Argentina and another targeted to Spain. We show that the Argentine calibration generates equilibrium paths that resemble the events of the 2001 debt crisis, with credit spreads going up by magnitudes similar to the observed ones. In the calibration for Spain, expectations-driven high rates induce austerity measures resembling those observed in the early 2010s. As a result of this endogenous austerity, the high spreads remain off equilibrium for Spain. Endogenous austerity and gambling for redemption are central in explaining our findings.

A key takeaway of our paper is that expectations—as measured by the sunspot probability of selecting an interest rate schedule—have a large quantitative effect on model outcomes. Assuming optimistic expectations for Argentina results in substantially lower default rates and credit spreads, even with no change in fundamentals. Similarly, assuming pessimistic expectations for Spain results in substantially higher default rates and credit spreads. Thus, expectations are a major driver explaining default rates and credit spread differences between Spain and Argentina.

We argue that policy plays a key role in shaping expectations. A lender of last resort can eliminate expectations-driven debt crises, as the ECB did for Spain and the IMF did not do for Argentina. This key interaction between policy and expectations opens the question of how to optimally design policy interventions within the boundaries of institutional limitations. We leave this question for future research.

Data Availability

Code replicating the tables and figures in this article can be found in Ayres et al. (2025) in the Harvard Dataverse, https://doi.org/10.7910/DVN/GJBOC6.

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