STOCHASTIC PROCESSES

UNIVERSITY OF TEHRAN

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HOMEWORK 7

Problem 1. Let $X(t)=\sum_{k=-\infty}^{+\infty} A_K P\left(t-kT\right)$ where $A_k \in \left\{+1,-1\right\}$ is a sequence of i.i.d. random variables with $\Pr\left\{A_k=1\right\}=\Pr\left\{A_k=-1\right\}=0.5$ and $P(t)$ is a given (Energy Signal) pulse.

(a) Find the power spectral density (PSD) of $X(t)$.

(b) For the special case where $P(t)=\Pi\left(\frac{t-0.5T}{T}\right)$, plot the PSD of $X(t)$.

Problem 2. Let $x(t)$ be a zero-mean stationary Gaussian random process with PSD $S_x(f)=\Lambda(f)$. Let $x'(t)=\frac{d}{dt}x(t)$.

(a) Find the pdf of vector $\underline{X}=\left[x(t),x'(t),x(t-1)\right]^T$.

(b) Find the PSD of $x'(t)$ and cross power spectrum of $x(t)$ and $x'(t)$.

(c) Can τ be found such that $x'(t+\tau)$ and $x'(t)$ be independent random variables for all t?

Problem 3. Let $X(t)=A\cos\left(2\pi Yt+\Theta\right)$, where $A>0$ is a known constant, Y is a random variable with pdf $f_Y(y)$, and Θ is a uniform random variable on $[0,2\pi]$. Y and Θ are independent. Find the power spectrum of $X(t)$.

Problem 4. Let $X(t)$ be a zero-mean stationary random process with $R_X(\tau)=e^{-|\tau|}$. Define

$$Y(t)=\int_0^2 X(t-s)ds.$$

Find the power spectrum of $Y(t)$.

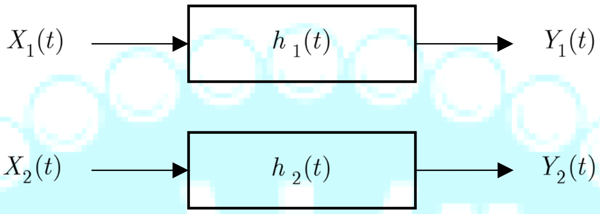
Problem 5. Let $X(t)$ be a stationary random process with

$$S_x(f)=\frac{4(\pi^2 f^2+1)}{(4\pi^2 f^2+1)(4\pi^2 f^2+9)}$$

a- Find the innovation process of $X(t)$.

b- Find a causal filter with impulse response $h(t)$, so that when $X(t)$ is passed through it the output $Y(t)$ has the autocorrelation function $R_y(\tau)=e^{-|\tau|}$.

Problem 6. Let $X_1(t)$ and $X_2(t)$ be two random processes with cross-correlation function $R_{X_1X_2}(t_1,t_2)$. Let $Y_1(t)$ and $Y_2(t)$ denote the outputs of two deterministic systems with known impulse responses $h_1(t)$ and $h_2(t)$ to the inputs $X_1(t)$ and $X_2(t)$ respectively, as depicted in the figure.



(a) Find $R_{X_1Y_2}(t_1,t_2)$, the cross-correlation function, and $S_{X_1Y_2}(f)$, the cross-spectral density of $X_1(t)$ and $Y_2(t)$. Simplify your answer.

(b) Find $R_{Y_1Y_2}(t_1,t_2)$ and $S_{Y_1Y_2}(f)$. Simplify your answer.

(c) Show that if $X_1(t)$ and $X_2(t)$ are jointly stationary then $Y_1(t)$ and $Y_2(t)$ are also jointly stationary.

(d) Assume that $\forall f,\ S_{Y_1Y_2}(f)=0$, where $h_1(t)$ and $h_2(t)$ are unknown. What can be inferred about $h_1(t)$ and $h_2(t)$? Articulate

Problem 7. Let C be a random variable uniformly distributed on the interval $[-2,3]$. Let the random process $X(t)=e^{-C}\cos(2\pi Ct)$ undergo an ideal low-pass filter with the cutoff frequency $f_0=1$. Denote the output process as $Y(t)$.

(a) Find the power spectral density of $X(t)$.

(b) Find the power of $Y(t)$.