Stochastic Processes

University of Tehran

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Homework 6

Due : 1403/9/17

Problem 1

Let $X(t)$ be a random process with $\mathbb{E}\{X(t)\} = 1$ and $R_X(t_1, t_2) = 1 + e^{-\left(|t_1|+|t_2|\right)}\delta(t_1 - t_2)$.

(a) Is $X(t)$ mean ergodic?

(b) Is $X(t)$ correlation ergodic?

Problem 2

Let $\{A_n\}_{n=-\infty}^{+\infty}$ be a sequence of i.i.d binary random variables with pmf $P\{A_k = 1\} = P\{A_k = -1\} = 0.5$. Assume poisson points with uniform density λ are distributed in the interval $(-\infty, +\infty)$ and the stochastic process $X(t)$ is defined as

$$X(t) = A_i \qquad t_i \leq t < t_{i+1}$$

where t_i is the time of occurence of ith poisson point. In other words at ith poisson point at $t = t_i$ the process $X(t)$ changes to A_i and is constant between two consecutive poisson points.

(a) Compute the mean and autocorrelation function of $X(t)$.

(b) Is $X(t)$ an independent increment process?

(c) Is $X(t)$ a Markov process?

(d) Is $X(t)$ a SSS process?

(e) Is $X(t)$ a mean ergodic process?

Problem 3

Let $X(t)$ be a zero-mean, real-valued stationary Gaussian random process. For the parts (b), (c), and (d) suppose that $R_X(\tau) = e^{-|\tau|}$.

(a) For a general zero-mean, real-valued normal vector $\underline{X} = [X_1, X_2, X_3, X_4]^T$ prove that:

$$\mathbb{E}\{X_1X_2X_3X_4\} = \mathbb{E}\{X_1X_2\}\mathbb{E}\{X_3X_4\} + \mathbb{E}\{X_1X_3\}\mathbb{E}\{X_2X_4\} + \mathbb{E}\{X_1X_4\}\mathbb{E}\{X_2X_3\}$$

(b) Is $X(t)$ mean ergodic?

(c) Is $X(t)$ correlation ergodic?

(d) Let the process $Y(t)$ be defined as $Y(t) + \frac{d}{dt}Y(t) = X(t), \forall t$. Find the mean and the autocorrelation function of $Y(t)$.

(e) Let the process $Y(t)$ be defined as $Y(t) + \frac{d}{dt}Y(t) = X(t), t \geq 0, Y(0) = 0$. Find the mean and the autocorrelation function of $Y(t)$.

Problem 4

Let $X(t)$ be a Wiener process with parameter N_0. Is $X(t)$ mean ergodic?

Problem 5

Let $\{X_n\}_{n=-\infty}^{\infty}$ be a strictly stationary and ergodic random process. Define $Y_n = \frac{1}{M+1}X_n$, where M is a Poisson R.V. with parameter $\lambda = 1$, and independent of X_n's.

(a) In what senses Y_n is stationary?

(b) In what senses Y_n is ergodic?
