Stochastic Processes

University of Tehran

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Homework 8

Due : 1403/10/16

Problem 1

Let $X(t)$ be a WSS process with zero mean and autocorrelation function $R_X(\tau) = \Lambda(\tau)$. Find the KL expansion of $X(t)$ for $t \in [0, 1]$.

Problem 2

Let $x(n)$ be a sequence of i.i.d. random variables with $Pr\{x(n) = 1\} = Pr\{x(n) = -1\} = 0.5$. Define the process $y(n) = 0.8y(n-1) + x(n)$.

(a) Find the PSD of $y(n)$.

(b) Prove that $y(n)$ is a Markov process.

(c) Let $z(n) = x(n-1) + x(n)$. Find the pdf of $z(n)$.

(d) Find the mean, the autocorrelation function, and the PSD of $z(n)$.

Problem 3

Consider the difference equation $y(n) = 0.3y(n-1) + x(n)$, where $x(n)$ is a stationary white noise with $R_X(m) = \delta(m)$.

(a) If the equation is valid for $-\infty < n < \infty$, find PSD and autocorrelation function of $y(n)$.

(b) If the differential equation is valid for $n \geq 0$ and $y(n) = 0$ for $n < 0$, find the autocorrelation function of $y(n)$.

Problem 4

Let $v(n)$ be a WSS white process with unit variance. The process $x(n)$ is related to $v(n)$ as:

$$\sum_{k=1}^{+\infty} (k+1)^2 3^{-k} x(n-k) = v(n)$$

(a) Prove that $x(n)$ is an $ARMA(N, M)$ process and find N and M.

(b) Does $x(n)$ have an $MA(\infty)$ model? If yes, find it. If no, why?

Problem 5

Let $x(t)$ be a zero-mean wide-sense stationary stochastic process with spectrum

$$S_X(f) = \begin{cases} 1 + \cos(20\pi f), & |f| \leqslant \frac{1}{20} \\ 0, & |f| > \frac{1}{20} \end{cases}$$

Consider the Continuous to Discrete (C/D) converter with sampling period of T depicted in the Figure 2, i.e., $y(n) = x(nT)$ for all $n \in \mathbb{Z}$.

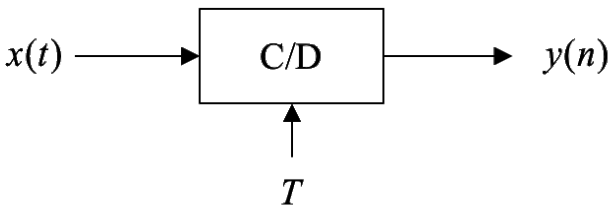

Figure 1

(a) Assume that the sampling period is $T = 10sec$. Determine $C_Y(n, m)$ and $S_Y(f)$. Is it possible to reconstruct $x(t)$ from $y(n)$ in mean-square sense? Show how the reconstruction can be achieved.

(b) Assume that the sampling period is $T = 20sec$. Determine $C_Y(n, m)$ and $S_Y(f)$.

Problem 6

Let $x(n)$ be a discrete stationary random process with the PSD:

$$S_X(f) = \frac{4}{5 - 4\cos(2\pi f)}$$

(a) Find the innovation process of $x(n)$.

(b) Has $x(n)$ an AR model? If yes, find it.

(c) Has $x(n)$ an MA model? If yes, find it.