

Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)

Problem 1

a)

$$X_n(w) = e^{-nw} \quad \text{for } w > 0 \quad w = 0 \rightarrow X(0) = x_{1(0)} = x_{n(0)}$$

$$-nw \leq 1 \rightarrow 2^n w = e^{-nw} \leq e^{-w}, e^{-w}, \dots, e^{-nw}. \lim_{n \rightarrow \infty} X_n(w) =$$

$$P(-nw \leq 1) = 1 - e^{-w} \quad \lim_{n \rightarrow \infty} e^{-nw} = 0 \quad \text{for all } w \in \mathbb{R}_{>0}$$

$$\lim_{n \rightarrow \infty} X_n(w) = 0 \quad \text{for all } w \in \mathbb{R}_{>0}$$

آنکه $X_n(w)$ باشد بعزماتیم احتمال و توزیع نیز

$\lim E\{1_{\{X_n(w) \leq 1\}}\} = ?$ اخواهید بگویید که میان مربعات این رسم میکنیم

$$E\{1_{\{X_n(w) \leq 1\}}\} = E\{1_{\{X_n(w) \leq 1\}}\} = \int_0^1 e^{-rw} dr = \frac{1}{r} e^{-rw} \Big|_0^1 = \frac{1}{r} (1 - e^{-rw})$$

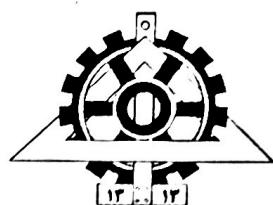
$$\rightarrow \lim_{n \rightarrow \infty} E\{1_{\{X_n(w) \leq 1\}}\} = \lim_{n \rightarrow \infty} \left(\frac{1 - e^{-rw}}{r} \right) = 0 \quad X_n \xrightarrow{n \rightarrow \infty} 0$$

Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



$$b) x_n(\omega) = \sin(\omega + \frac{1}{n}) \text{ for } \omega = 0 \rightarrow x_{n(0)} = \sin(\frac{1}{n})$$

$$\left\{ \begin{array}{l} n \rightarrow \infty \Rightarrow \omega + \frac{1}{n} \rightarrow 0 \\ \sin(\omega + \frac{1}{n}) \rightarrow \sin(\frac{1}{n}) \end{array} \right.$$

$$\lim_{n \rightarrow \infty} x_{n(0)} = \lim_{n \rightarrow \infty} \sin(\frac{1}{n}) = 0 \quad \text{با این نتیجه} \rightarrow x_n(\omega) = 0$$

$$\sin(\omega + \frac{1}{n}) = \sin \omega \cos \frac{1}{n} + \sin \frac{1}{n} \cos \omega \quad \lim_{n \rightarrow \infty} x_n(\omega) = \lim_{n \rightarrow \infty} (\sin \omega \cos \frac{1}{n} + \sin \frac{1}{n} \cos \omega) = \sin(\omega)$$

دستوراتی حل - - - - - نیز کمال متوالی ادعای نسبتی نیز نیز نسبتی . بس در اینجا
نهایتی بر معنی همچنانی بس نهایتی تعریف شده ، نهایتی در اصل و نهایتی در نظریه
نیز شبهی شود . بررسی نهایتی به معنی مطابق مربوط خواهد بود .

$$E \{ |x_n - x|^2 \} = E \{ (\sin(\omega + \frac{1}{n}) - \sin(\omega))^2 \}$$

$$= E \left\{ (\sin \omega \cos \frac{1}{n} + \sin \frac{1}{n} \cos \omega - \sin \omega)^2 \right\} = E \left\{ \sin^2 \omega (\cos \frac{1}{n})^2 + \sin^2 \frac{1}{n} \cos^2 \omega \right\}$$

$$= E \left\{ \sin^2 \frac{1}{n} \cos^2 \omega + 2 \sin \frac{1}{n} \cos \omega \sin \omega (\cos \frac{1}{n} - 1) + \sin^2 \omega (\cos \frac{1}{n} - 1)^2 \right\}$$

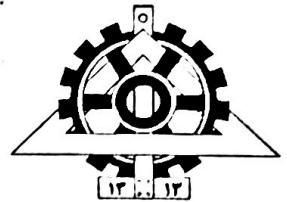
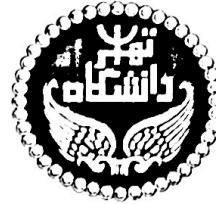
$$= E \underbrace{\left\{ \sin^2 \frac{1}{n} \cos^2 \omega + \sin^2 \frac{1}{n} \sin^2 \omega (\cos \frac{1}{n} - 1) + \sin^2 \omega (\cos \frac{1}{n} - 1)^2 \right\}}_{E_1} + E \underbrace{\left\{ 2 \sin \frac{1}{n} \cos \omega \sin \omega (\cos \frac{1}{n} - 1) \right\}}_{E_2} + E \underbrace{\left\{ \sin^2 \omega (\cos \frac{1}{n} - 1)^2 \right\}}_{E_3}$$

Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



$$= E_1 + E_\epsilon + E_\tau$$

$$= \sin^2 \frac{1}{n} \int_{-\pi}^{\pi} \cos^2 w dw + \sin^2 \frac{1}{n} (\cos \frac{1}{n} - 1) \int_{-\pi}^{\pi} \sin w dw + (\cos \frac{1}{n} - 1)^2$$

$$\int_{-\pi}^{\pi} \sin^2(w) dw$$

$$= \sin^2 \frac{1}{n} \left[\frac{1}{\tau} + \frac{\sin \tau}{\tau} \right] + \sin^2 \frac{1}{n} (\cos \frac{1}{n} - 1) \left[\frac{1}{\tau} - \frac{\cos \tau}{\tau} \right]$$

$$+ (\cos \frac{1}{n} - 1)^2 \left[\frac{1}{\tau} + \frac{\sin \tau}{\tau} \right]$$

$$\lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} \left\{ \left| \frac{1}{\tau} + \frac{\sin \tau}{\tau} \right| \right\} = \lim_{n \rightarrow \infty} \sin^2 \frac{1}{n} \left(\frac{1}{\tau} + \frac{\sin \tau}{\tau} \right) +$$

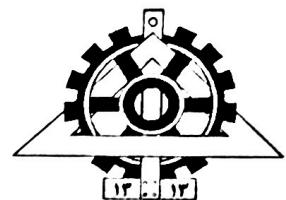
$$\lim_{n \rightarrow \infty} \sin^2 \frac{1}{n} (\cos \frac{1}{n} - 1) \left[\frac{1}{\tau} - \frac{\cos \tau}{\tau} \right] + \lim_{n \rightarrow \infty} (\cos \frac{1}{n} - 1)^2$$

$$\left[\frac{1}{\tau} + \frac{\sin \tau}{\tau} \right] = 0 \rightarrow \lambda_n \xrightarrow{n \rightarrow \infty} \lambda \checkmark$$

$$(c) \lambda_n(w) = \cos^n(w), \quad n \geq 1, \quad w = 0 \rightarrow \lambda_n(0) = 1 \quad \begin{matrix} \uparrow \\ \text{if } w \neq 0 \\ \downarrow \end{matrix}$$

$$\forall w \in \mathbb{R} \rightarrow \lambda_n(w) = \cos^n(w) \quad \cos 1 \leq \cos^n(w) \leq 1 \rightarrow \cos \leq \lambda_n(w) \leq 1$$

$$\lim_{n \rightarrow \infty} \lambda_n(w) = \lim_{n \rightarrow \infty} \cos^n(w) = \lambda(w) \quad \begin{matrix} \uparrow \\ \text{if } w \neq 0 \\ \downarrow \end{matrix}$$



Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)

$$P\{0 \leq w \leq 1\} = 1 \quad \text{لکه } w_{(n)} = \dots \text{ for all } w \in (0, 1) \quad w_{(n)} \xrightarrow{m.s} X$$

نیو جک. پس از اینکه سری مذکور را با خود می‌دانیم، پس نیو جک را می‌دانیم. فرض کنید سری مذکور را با سری مذکور مرتبط نماییم. فرض کنید سری مذکور را با سری مذکور مرتبط نماییم.

$$E\{|w_{(n)}|^r\} = E\{|w_n|^r\} = E\{c_n^{rn}(w)\}$$

$$= \int_0^1 c_n^{rn}(w) dw = \frac{1}{r+1} c_n^{r+1}(w) \Big|_0^1 = \frac{1}{r+1} [c_{n-1}^{r+1}]$$

$$\lim_{n \rightarrow \infty} E\{|w_{(n)}|^r\}$$

$$= \lim_{n \rightarrow \infty} \frac{c_n^{r+1}}{r+1} = \lim_{n \rightarrow \infty} E\{|w_n|^r\} \xrightarrow{m.s} Y$$

Problem 2

$n = 1, 2, \dots$

$$\{x_n\}_{n=1}^{\infty} \rightarrow \text{ind. } x_i \sim \text{uniform}(0, 1) \quad Y_n = n(1 - \max(x_1, \dots, x_n))$$

$$Y \sim \exp(\lambda) = f_Y(y) = e^{-y} \lambda \quad \text{prove: } y_n \xrightarrow{d} y \xrightarrow{n \rightarrow \infty} F_Y(y)$$

$$F_{Y_n}(y) = P(Y_n \leq y) = P(n(1 - \max(x_1, \dots, x_n)) \leq y) = P(\max(x_1, \dots, x_n) \geq 1 - \frac{y}{n}) \rightarrow F_Y(y)$$

$$\text{CDF} \rightarrow \text{PDF} \quad \text{برای } y > 0 \quad f_{Y_n}(y) = \frac{1}{n} n(1 - \max(x_1, \dots, x_n))^{n-1} \quad \text{برای } y < 0 \quad f_{Y_n}(y) = 0$$

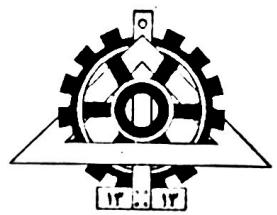
$$F_{Y_n}(w) = P(W \leq w) = P(\max(x_1, \dots, x_n) \leq w) = P(X_1 \leq w, X_2 \leq w, \dots, X_n \leq w)$$

Homework

Stochastic Process - Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



$$= P(X \leq w) P(X_2 \leq w) \dots P(X_n \leq w) = [F_X(w)]^n \quad F_w(w) = F_X^n(w)$$

$$f_w(w) = \frac{d}{dw} F_w(w) = n F_X^{n-1}(w) f_X(w)$$

$$F_{J_n}(y) = P(w \leq 1 - y_n) = 1 - P(w < 1 - y_n)$$

$$\Rightarrow F_{J_n}(y) = 1 \rightarrow F_X(1 - \frac{y}{n}) \quad \text{so} \quad F_X(w) = \begin{cases} 0 & w < 0 \\ x & 0 < w \leq 1 \\ 1 & w > 1 \end{cases}$$

$$F_X(1 - \frac{y}{n}) \rightarrow \text{بروکس بزرگتر} \quad \text{از} \quad F_{J_n}(y) \quad \text{برای} \quad y > 0$$

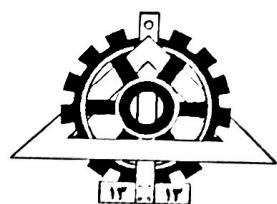
$$\text{از} \quad F_{J_n}(y) = 1 - (1 - \frac{y}{n})^n \quad \lim_{n \rightarrow \infty} F_{J_n}(y) = \lim_{n \rightarrow \infty} \left[1 - \left(1 - \frac{y}{n}\right)^n \right] = 1 - e^{-y} \quad \text{برای} \quad y > 0$$

$$\rightarrow \lim_{n \rightarrow \infty} F_{J_n}(y) = (1 - e^{-y}) | v(y) \quad F_y(y) = \int_{-\infty}^y f_y(y) dy$$

$$= \int_{-\infty}^y e^{-y} v(y) dy = \int_{-\infty}^y e^{-y} dy$$

$$= 1 - e^{-y} \quad y > 0 \rightarrow F_y(y) = (1 - e^{-y}) | v(y) \quad \lim_{n \rightarrow \infty} F_{J_n}(y) = F_y(y)$$

$$\rightarrow y_n \xrightarrow{d} y$$



Problem 5

ay

$$f_{X_n}(x) = \left(1 - \frac{1}{n}\right) \frac{1}{6\sqrt{n}} \exp\left\{-\frac{1}{6\sqrt{n}} \left(x - \frac{n-1}{n} \delta\right)^2\right\}$$

$$m < \infty \Rightarrow \lim_{n \rightarrow \infty} E\{(x_n - x_1)^m\} = 0? \quad E\{(x_{n+m})^m\} = E\{x_n^m\} \cdot m! E\{x_n^m\}$$

$$= E \{x n^{\tau}\} - r \varepsilon \{n\} \varepsilon \{n\} + E \{n^{\tau}\}$$

بِهِوَاللَّهِمَ صَاهِرٌ فَوْلَادِيْ حَسَابُكَ لَنِيْ هِيْ هُوَتَرَسْ سِيْ سِنْتَ مِيْ شَدَّ لَنَا اَذَابِنْ خَيْشْ هَدَمِيْ سِيرَلَهْ.

$$\lim_{n \rightarrow \infty} E\{(x_n - \mu)^r\} = \lim_{n \rightarrow \infty} E\{x_n^r\} < \lim E\{x_n\} E\{X\}$$

$$\lim_{n \rightarrow \infty} E\{x^n\} = \lim_{n \rightarrow \infty} E\{x^{n^k}\} \leq E\{x\} \lim_{n \rightarrow \infty} E\{x^n\} + E\{x^n\}$$

لَا مُنْبَرِّعَادِنِي نِرْمَلِ بِهِيَاهِنْ كَوْ اِلَانْزْ لَهْ اِسْتْ بِسْ دَارِمْ:

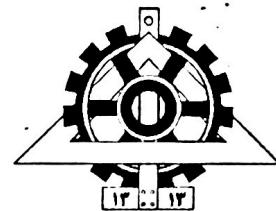
$$\lim F \{ |u_{n-k}|^r \} = r f^r - r f^r + r f^r = r f^r \neq 0$$

$$\lim_{n \rightarrow \infty} E \{ |x_n - x_1|^p \} = r_1^p \quad f.$$

b) $\lim_{n \rightarrow \infty} P(|x_{n+1} - x_n| > \epsilon) = 0$? or $\lim_{n \rightarrow \infty} P(|x_{n+1} - x_n|) =$ نحویں بوسیج میں مربعات نہیں.

$$\text{Max } |K_{n,k}| = P(|Z_{n,k} - \mu| > \varepsilon) = P(|Z_{n,k} - \mu|^2 > \varepsilon^2) \leq \varepsilon^2 \frac{\mathbb{E}[|Z_{n,k} - \mu|^2]}{\varepsilon^2} \xrightarrow[n \rightarrow \infty]{\text{lim}}$$

$$\lim_{n \rightarrow \infty} P(|x_n - x| > \varepsilon) \leq \frac{\tau \delta^\tau}{\varepsilon^\tau} \longrightarrow 0 \quad \text{as } n \rightarrow \infty$$



Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)

$$c) \lim_{n \rightarrow \infty} F_{X_n}(u) \rightarrow F_x(u) \xrightarrow{d_{\text{weak}}} \lim_{n \rightarrow \infty} f_{X_n}(u) \rightarrow f_x(u)$$

$$\lim_{n \rightarrow \infty} F_{X_n}(u) = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n} \right) \frac{1}{\delta(\sqrt{n})} \exp \left(\frac{-1}{\tau_{f_x}} \left(u - \frac{n-1}{n} \delta \right)^2 \right)$$

$$+ \frac{1}{n} \delta \exp(-\delta u) U(u) = \frac{1}{\delta(\sqrt{n})} \exp \left\{ \frac{(u - \tau_x)^2}{\tau_{f_x}} \right\} \sim N(\mu, \sigma^2)$$

$$X_n \xrightarrow{d} u$$

Problem V $F_{X_n}(u) = \frac{e^{nu}}{1 + e^{nu}}$ $u = -\infty \rightarrow F_{X_n}(u) = 0 \rightarrow F_x(u) = 0$

a)

$$F_x(u) = \lim_{n \rightarrow \infty} \frac{e^{nu}}{1 + e^{nu}} = \lim_{n \rightarrow \infty} \frac{e^{nu}}{e^{nu} + 1} = 1 \quad F_x(u) = 1$$

که عدد ۱ بزرگتر از $F_x(u)$ است. زیرا x مقدار ای انتشار نماینده است. $F_x(u)$ نزدیک $F_{X_n}(u)$ است. از این نظر $F_x(u)$ متفاوت با $F_{X_n}(u)$ است.

$$\text{if } x < 0 \quad \lim_{n \rightarrow \infty} F_{X_n}(u) = F_x(u) = 0$$

b) $\lim_{n \rightarrow \infty} (F_{X_{n+k}}(u) - F_x(u)) \rightarrow ? \quad \forall k > 0 \quad \lim_{n \rightarrow \infty} (F_{X_{n+k}}(u) - F_x(u))$

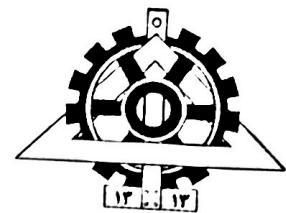
$$= \lim_{n \rightarrow \infty} \left[\frac{e^{(n+k)u}}{1 + e^{(n+k)u}} - \frac{e^{nu}}{1 + e^{nu}} \right] = \lim_{n \rightarrow \infty} \left[\frac{e^{(n+k)u} (1 + e^{nu}) - e^{nu} (1 + e^{(n+k)u})}{(1 + e^{(n+k)u})(1 + e^{nu})} \right]$$

Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



$$= \lim_{n \rightarrow \infty} \frac{e^{nu_k} e^{nu_{k+1}} \cdots e^{nu_n} e^{nu_1}}{(1 + e^{nu_k}) (1 + e^{nu_{k+1}}) \cdots (1 + e^{nu_n})} = \lim_{n \rightarrow \infty} \frac{e^{nu_1 + nu_2 + \cdots + nu_n}}{(1 + e^{nu_1}) (1 + e^{nu_2}) \cdots (1 + e^{nu_n})}$$

$$= \lim_{n \rightarrow \infty} \frac{e^{nu_1} e^{nu_2} \cdots e^{nu_n}}{(1 + e^{nu_1}) (1 + e^{nu_2}) \cdots (1 + e^{nu_n})} = \dots$$

$$\lim_{n \rightarrow \infty} (F_{X_{n+k}(u)} - F_{X_n(u)}) = \forall k \exists: \exists n \quad d_n$$

نحوه ای در نظر بگیرید که بر قرار است

$X: F_{X_n(u)} = 1 \quad \forall n$. متغیر تصادفی X را به این شرط معرفی می‌کنیم که باشد.

$$F_{X_n}(u) = \dots$$

Problems

$$\{w_k\}_{k=1}^{\infty} \quad \text{independent Gaussian with } \mu_w = 0 \quad \text{Var}(w_k) = \frac{1}{\kappa} \quad \forall k.$$

$$u_k = \frac{1}{\varepsilon} (u_{k-1} + w_k) \quad \& \quad u_0 = \frac{1}{\varepsilon} u_0 + \frac{1}{\varepsilon} w_0 \rightarrow u_1 = \frac{1}{\varepsilon} w_1$$

$$u_k = \frac{1}{\varepsilon} u_0 + \frac{1}{\varepsilon} w_0 + \frac{1}{\varepsilon} w_1 + \cdots + \frac{1}{\varepsilon} w_k, \quad u_n = \frac{1}{\varepsilon} u_0 + w_0 + \frac{1}{\varepsilon} w_1 + \cdots + \frac{1}{\varepsilon} w_n$$

$$\rightarrow u_n = \sum_{i=1}^n \left(\frac{1}{\varepsilon} \right)^{n-i+1} w_i$$

چه برسی کند که این ماده می‌برداقت نتیجه داشته باشد این سوابع را انتشود و کوشش اسماهاد می‌کند

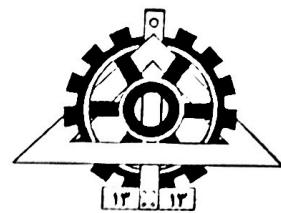
$$\text{m.s: } \lim_{n \rightarrow \infty} E \{ |u_{n+k} - u_n| \} \rightarrow 0!$$

Homework

Stochastic Process - Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



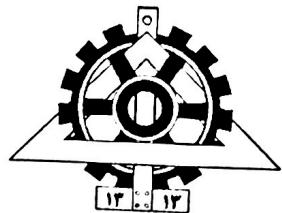
$$\begin{aligned}
 & E\{|w_{n+k} \cdot w_n|^2\} = E\left\{\left(\sum_{i=1}^{n+k} \left(\frac{1}{c}\right)^{n+k-i+1} w_i\right) \cdot \sum_{i=1}^n \left(\frac{1}{c}\right)^{n-i+1} w_i\right\} \\
 & = E\left\{\sum_{i=1}^{n+k} \sum_{j=1}^{n+k} \left(\frac{1}{c}\right)^{n+k-i+1} \left(\frac{1}{c}\right)^{n+k-j+1} w_i w_j\right\} + \\
 & E\left\{\sum_{i=1}^n \sum_{j=1}^n \left(\frac{1}{c}\right)^{n-i+1} \left(\frac{1}{c}\right)^{n-j+1} w_i w_j\right\} \\
 & - 2 E\left\{\sum_{i=1}^{n+k} \sum_{j=1}^n \left(\frac{1}{c}\right)^{n+k-i+1} \left(\frac{1}{c}\right)^{n-j+1} w_i w_j\right\} \\
 & = \sum_{i=1}^{n+k} \sum_{j=1}^{n+k} \left(\frac{1}{c}\right)^{n+k-i+1} \left(\frac{1}{c}\right)^{n+k-j+1} E\{w_i w_j\} \\
 & - 2 E\sum_{i=1}^n \sum_{j=1}^n \left(\frac{1}{c}\right)^{n-i+1} \left(\frac{1}{c}\right)^{n-j+1} w_i w_j \\
 & = \sum_{i=1}^{n+k} \sum_{j=1}^{n+k} \left(\frac{1}{c}\right)^{n+k-i+1} \left(\frac{1}{c}\right)^{n+k-j+1} E\{w_i w_j\} \\
 & + \sum_{i=1}^n \sum_{j=1}^n \left(\frac{1}{c}\right)^{n-i+1} \left(\frac{1}{c}\right)^{n-j+1} E\{w_i w_j\} \\
 & - 2 \sum_{i=1}^n \sum_{j=1}^n \left(\frac{1}{c}\right)^{n-i+1} \left(\frac{1}{c}\right)^{n-j+1} E\{w_i w_j\}
 \end{aligned}$$

Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



$$E\{w_i w_j\} = \sum E\{w_i w_j\} = \begin{cases} 0 & i \neq j \\ E\{w_i^2\} = 6^r & i = j \end{cases}$$

$$E = \left\{ |w_{n+k} - w_n|^r \right\} = \sum_{i=1}^{n+k} \left(\frac{1}{c} \right)^r |w_{n+k-i+1} - w_i|^r + \sum_{i=1}^n \left(\frac{1}{c} \right)^r |w_{n-i+1}|^r$$

$$\sum_{i=1}^n \left(\frac{1}{c} \right)^r |w_{n-i+1}|^r \rightarrow c = n+k \text{ and } c = n+1 \rightarrow \text{میتوانیم} \sum_{i=1}^n \left(\frac{1}{c} \right)^r |w_{n-i+1}|^r = E\{w_i w_j\}$$

$$\lim_{n \rightarrow \infty} E\{|w_{n+k} - w_n|^r\} = 6^r \lim_{n \rightarrow \infty} \left(\sum_{i=1}^{n+k} \left(\frac{1}{c} \right)^r |w_{n+k-i+1}|^r - \sum_{i=1}^n \left(\frac{1}{c} \right)^r |w_{n-i+1}|^r \right)$$

$$= 6^r \lim_{n \rightarrow \infty} \left[\left(\frac{1}{c} \right)^{n+k+1} \sum_{i=1}^{n+k} |w_i| - \left(\frac{1}{c} \right)^{n+1} \sum_{i=1}^n |w_i| \right] \neq 0 \rightarrow \text{نمایم}$$

حل ببرس لفیر اک ده اینجا هولو چیز بر راه است.

$$\lim_{n \rightarrow \infty} P\{|w_{n+k} - w_n|\}, \epsilon \rightarrow .$$

$$\lim_{n \rightarrow \infty} P\left(\sum_{i=1}^{n+k} \left(\frac{1}{c} \right)^{n+k-i+1} w_i - \sum_{i=1}^n \left(\frac{1}{c} \right)^{n-i+1} w_i, > \epsilon \right)$$

$$P\left(\sum_{i=1}^{n+k} \left(\frac{1}{c} \right)^{n+k-i+1} w_i - \sum_{i=1}^n \left(\frac{1}{c} \right)^{n-i+1} w_i, \epsilon \right)$$

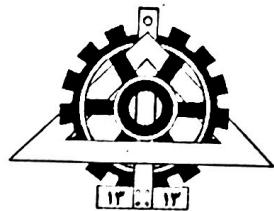
$$P = \left(\sum_{i=1}^n \left(\frac{1}{c} \right)^{n+1-i} \left(\frac{1}{c} \right)^k - 1 \right) w_i + \sum_{j=n+1}^{n+k} \left(\frac{1}{c} \right)^{n+k-j-1} \epsilon$$

Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



$$\rightarrow \lim_{n \rightarrow \infty} P\{X_{n+k} - X_n \leq e\} \neq 0 \rightarrow \text{همه راهی در این مسیر باشند}$$

برای همه راهی در توزیع صعودی باشند.

$$X_n = \sum_{i=1}^n \left(\frac{1}{2}\right)^{n-i+1} w_i$$

هر راهی مجموع متغیرهای مقادیر و سل هویتی را داشت. لذا این توزیع بنویسیم.

$$M_{Xn} = E \left\{ \sum_{i=1}^n \left(\frac{1}{2}\right)^{n-i+1} w_i \right\}_0 \text{ if } \Rightarrow Z = aX_1 + bX_2 + \dots + X_n$$

$$\rightarrow \text{Var}(Z) = a^2 \text{Var}(X_1) + b^2 \text{Var}(X_2) + \dots + Z \text{Var}(X_n)$$

$$\rightarrow \text{Var}(X_n) = 6^2 \sum_{i=1}^n \left(\left(\frac{1}{2}\right)^{n-i+1}\right)^2 = 8 = \sum_{i=0}^{n-1} r^{-2n} \cdot r^i$$

$$\text{Var}(X_n) = r^{-2n} \left(\frac{1 - r^n}{r} \right) 8^2 = \frac{-r^{-2n}}{r} (1 - r^n) 8^2$$

$$\lim_{n \rightarrow \infty} (\dots, \frac{-r^{-n}}{r} (1 - r^n) 8^2) \mid \lim_{n \rightarrow \infty} \text{Var}(X_n)$$

$$= \lim_{n \rightarrow \infty} -\frac{1}{r} (r^{-n} - 1) 8^2 = -\frac{8^2}{r} \cdot \lim_{n \rightarrow \infty} r^n \sim N(0, \frac{64}{r})$$

با این نتیجه و دادن $\frac{64}{r}$ به این معنی که راهی شود لذا

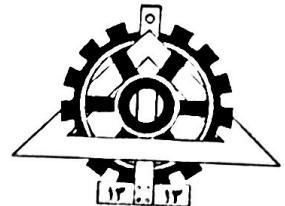
$$X_n \xrightarrow{d} X \quad \text{پس از}$$

Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



Problem 1

a) $\{\mathcal{Z}_k\}_{k=1}^{\infty}$ \rightarrow i.i.d. Rand. Variables $\mathcal{Z}_k \sim N(0,1)$

$$X_k = \alpha_0 + \alpha_1 X_{k-1} + \mathcal{Z}_k \quad X_0$$

$$X_1 = \alpha_0 + \mathcal{Z}_1 \quad X_2 = \alpha_0 + \alpha_1 \mathcal{Z}_0 + \mathcal{Z}_2 = \frac{1}{c} \alpha_0 + \frac{1}{c} \mathcal{Z}_1 + \mathcal{Z}_2$$

$$X_3 = \frac{1}{c} \alpha_0 + \mathcal{Z}_3 = \frac{1}{c} \alpha_0 + \frac{1}{c} \mathcal{Z}_1 + \frac{1}{c} \mathcal{Z}_2 + \mathcal{Z}_3$$

$$X_k = f(\alpha_0, \mathcal{Z}_1, \dots, \mathcal{Z}_k) = \frac{1}{c^k} \alpha_0 + \sum_{i=1}^k \left(\frac{1}{c}\right)^{k-i} \mathcal{Z}_i$$

$$\rightarrow u_k = \frac{1}{c^k} \alpha_0 + \sum_{i=1}^k \left(\frac{1}{c}\right)^{k-i} \mathcal{Z}_i$$

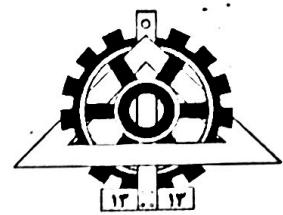
به معنی متغیر اسنجیس آمده باشید که مجموع باقی جهه بخوبی u_k نام نهاده شد و میتواند در این توزیع که از نمونه خواهد شد. u_k نام دارد. این مجموع مجموعه ای از متغیرهاست که مجموع آنها برابر باشد.

$$m_{u_k} = E\{u_k\} = \frac{1}{c^k} E\{\alpha_0\} + \sum_{i=1}^k \left(\frac{1}{c}\right)^{k-i} \underbrace{E\{\mathcal{Z}_i\}}_{=0} \rightarrow m_{u_k} = \frac{1}{c^k} E\{\alpha_0\}$$

$$\text{Var}(u_k) = \sigma_{u_k}^2 = E\{u_k^2\} - m_{u_k}^2$$

$$E(u_k^2) = E\left\{\left(\frac{1}{c}\right)^k \alpha_0 + \sum_{i=1}^k \left(\frac{1}{c}\right)^{k-i} \mathcal{Z}_i\right\}^2$$

خواهی برای اینجا $E(u_k)$ را بخواهید



Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)

$$= E \left\{ \left(\frac{1}{\varepsilon}\right)^k u_0 + \left(\frac{1}{\varepsilon}\right)^{k-1} z_1 + \left(\frac{1}{\varepsilon}\right)^{k-2} z_2 + \dots + z_k \right\}$$

$$\rightarrow E(x_k) = \left(\frac{1}{\varepsilon}\right)^k E(u_0) + \left(\frac{1}{\varepsilon}\right)^{k-1} E(z_1)$$

$$+ \dots + \left(\frac{1}{\varepsilon}\right) E(z_{k-1}) + \left(\frac{1}{\varepsilon}\right) E(z_k)$$

$$\rightarrow E(x_k) = \left(\frac{1}{\varepsilon}\right)^k E(u_0) + \left(\frac{1}{\varepsilon}\right)^{k-1} + \dots + \frac{1}{\varepsilon} + 1$$

$$E(x_k) = \left(\frac{1}{\varepsilon}\right)^k E(u_0) + \sum_{i=0}^{k-1} \left(\frac{1}{\varepsilon}\right)^i = \left(\frac{1}{\varepsilon}\right)^k E(u_0)$$

$$+ \frac{1 - \left(\frac{1}{\varepsilon}\right)^k}{\frac{\varepsilon}{\pi}} = \left(\frac{1}{\varepsilon}\right)^k E(u_0) + \frac{\varepsilon}{\pi} \cdot \frac{1}{\pi} \left(\frac{1}{\varepsilon}\right)^{k-1}$$

$$= \left(\frac{1}{\varepsilon}\right)^k E(u_0) + \frac{\varepsilon}{\pi} - \frac{1}{\pi} \left(\frac{1}{\varepsilon}\right)^{k-1} = \frac{\varepsilon}{\pi} - \frac{1}{\pi} \left(\frac{1}{\varepsilon}\right)^{k-1} + \left(\frac{1}{\varepsilon}\right)^k$$

$$\rightarrow E(x_k) = \frac{\varepsilon}{\pi} - \frac{1}{\pi} \left(\frac{1}{\varepsilon}\right)^{k-1} + \left(\frac{1}{\varepsilon}\right)^k E(u_0)$$

برای این مقدار نفع تقریبی می‌باشد.

$$m_k = E\{u_i\} = 0 \rightarrow E\{x_k\} = m_k$$

$k = 1, 2, \dots$

$$E(u_i) = \delta u_i = \frac{\varepsilon}{\pi} \rightarrow \delta u_k = \frac{\varepsilon}{\pi} \rightarrow x_k \sim N(0, \frac{\varepsilon}{\pi})$$

$$u_0 = \frac{1}{\varepsilon} u_0 + z_0 \rightarrow m_{u_0} = 0 \quad X_0 \sim N(0, \frac{\varepsilon}{\pi})$$

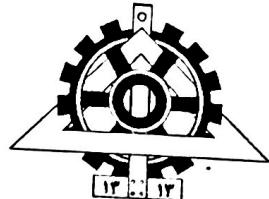
$$\text{Var}(u_0) = \delta u_0^2 = \frac{1}{\varepsilon} \delta u_0^2 + \delta z_0^2 = \frac{1}{\varepsilon} \times \frac{\varepsilon}{\pi} + 1$$

Homework

Stochastic Process - Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)



$$u_r = \frac{1}{\epsilon} u_0 + \frac{1}{\epsilon} z_1 + z_r \rightarrow m_{u_r} =$$

$$\text{Var}(u_{r+1} - 6u_r) = \frac{1}{\epsilon^2} 6u_r + \frac{1}{\epsilon^2} 6z_r^2 + 6z_r^2 = \frac{12}{\epsilon^2}$$

میانگین مربعات فاصله از میانگین نوشتاری

$$u_r \sim N(0, \frac{\epsilon}{\epsilon})$$

b) $E\{x_{n+k} x_k\} = E\left\{\left(\frac{1}{\epsilon} u_{n+k-1} + z_{n+k}\right) x_k\right\} = \frac{1}{\epsilon} E\{u_{n+k} x_k\}$

$$+ E\{x_k z_{n+k}\} = \frac{1}{\epsilon} E\{x_{n+k-1} x_k\} = \frac{1}{\epsilon} E\left\{\left(\frac{1}{\epsilon} u_{n+k-1} + z_{n+k-1}\right) x_k\right\}$$

$$= \frac{1}{\epsilon} E\{u_{n+k-1} x_k\}$$

متناهی میانگین داریم و لوبینگلر صداب $\frac{1}{\epsilon}$ نیست مگر خواهد شد
متناهی میانگین داشت

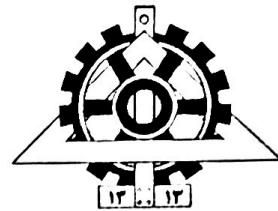
$$E\{u_{n+k} x_k\} = \left(\frac{1}{\epsilon}\right)^n E\{x_k\} = \left(\frac{1}{\epsilon}\right)^n \times \frac{\epsilon}{\epsilon} = \frac{1}{\epsilon} \left(\frac{1}{\epsilon}\right)^{n-1}$$

لذا میانگین بندیم.

Problem 5 در درس ابتداء کردنیم بهتر است در معنی احتمال راستی
که راه داشت بدهیم زیرا آن کسر است

$$u_n \xrightarrow{m.s} x. \quad \lim_{n \rightarrow \infty} E\{(u_n - x_i)^2\} \rightarrow 0$$

$$u_n \xrightarrow{P} x. \quad \forall \epsilon > 0. \quad \lim_{n \rightarrow \infty} P\{|u_n - x| > \epsilon\} \rightarrow 0$$



Homework

Stochastic Process – Fall 2024

Instructor: Dr. Ali Olfat

Erfan Panahi (Student Number: 810103084)

$$P(|x_{n-k}| > \varepsilon) = P(|x_{n-k}|^r > \varepsilon^r) \leq \frac{\sum |x_{n-k}|^r}{\varepsilon^r}$$

$$\lim_{n \rightarrow \infty} P(|x_{n-k}| > \varepsilon) = \lim_{n \rightarrow \infty} P(|x_{n-k}|^r > \varepsilon^r) \leq \lim_{n \rightarrow \infty} \frac{\sum |x_{n-k}|^r}{\varepsilon^r}.$$

$$= \lim_{n \rightarrow \infty} P(|x_{n-k}| > \varepsilon) \underset{\longrightarrow}{<} 0. \quad \checkmark$$

برای نشان دادن این مجموعه از اعداد ممکن است که ایران سرایل خواهد بود کیم.

$f_{n+k}(n) = 0$ for all $n > N$, $|x_n| > n$, and for some $N > 0$

$$\text{پس از: } \lim_{n \rightarrow \infty} P(|x_{n-k}| > \varepsilon) = 0 \text{ or } \lim_{n \rightarrow \infty} P(|x_{n+k-k}| > \varepsilon) = 0$$

We must show, $\lim_{n \rightarrow \infty} \sum |x_{n-k}| = 0$ or $\lim_{n \rightarrow \infty} \sum |x_{n+k-k}| = 0$.

فتن کسی: $P(|x_n| < \eta) = 0$ در اینجا $\eta = \sqrt{\varepsilon}$ است.

$$\begin{aligned} \sum |x_{n-k}|^r &= \int_{-\infty}^{+\infty} |x_{n-k}|^r \underset{\substack{\sum \\ |x_{n-k}| > \varepsilon}}{\geq} \int_{|x_{n-k}| > \varepsilon} |x_{n-k}|^r + \int_{|x_{n-k}| \leq \varepsilon} |x_{n-k}|^r \\ &\leq \int_{|x_{n-k}| > \varepsilon} |x_{n-k}|^r + \leq P(|x_{n-k}| > \varepsilon) \varepsilon^r + \varepsilon^r \rightarrow \varepsilon^r \end{aligned}$$

$$\varepsilon > 0 \rightarrow \lim_{n \rightarrow \infty} \sum |x_{n-k}|^r \rightarrow 0 \rightarrow \sum x_n \xrightarrow{n \rightarrow \infty} X \quad \text{QED} \quad \checkmark$$