Fractal Dimensions in Alzheimer's Disease: A Golden Ratio Field-Theoretic Perspective

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Abstract

Fractal dimension (FD) analysis is a promising biomarker for Alzheimer's disease (AD), capturing structural and functional loss of complexity in the brain. We propose that FD reductions in AD reflect deviations from a universal field-theoretic optimum, governed by a nonlocal kernel $G(r) = |r|^{-\alpha}$ whose renormalization group flow selects $\alpha^* = \varphi \approx 1.618$, the golden ratio. This fixed point predicts a canonical fractal dimension $\Delta = d + 2 - \alpha \approx 3.382$ in three dimensions. Healthy brains show FD values in the 2.3–2.6 range, consistent with projected φ-critical scaling. In AD, cortical FD reductions ($\Delta < 2.3$) and local pathological FD increases from amyloid aggregation represent deviations from the φ-governed regime. We argue that AD reflects a breakdown of scale-invariant dynamics optimized by the golden ratio, providing a falsifiable, physics-grounded lens on neurodegeneration.

1 Introduction

Alzheimer's disease (AD) is characterized by progressive cognitive decline, cortical thinning, and disrupted neural dynamics. Fractal dimension (FD) analysis of MRI, EEG, and histology captures these changes as a loss of scale-invariant complexity. In parallel, renormalization group analysis of nonlocal field kernels identifies $\alpha^* = \varphi$ as the unique critical exponent ensuring mathematical consistency. Linking these domains, we propose that AD reflects deviation from φ-criticality.

2 Fractal Dimension as Biomarker

2.1 Structural MRI

Cortical FD decreases in AD independent of atrophy, especially in hippocampal and temporal regions [4].

2.2 EEG

Resting-state EEG shows reduced Higuchi FD in temporal and parietal regions, reflecting disrupted neural dynamics [5].

2.3 Histology

Microscale studies report local FD increases from amyloid-beta plaques, contrasting with macroscopic FD reductions [6].

2.4 Multifractal Analysis

AD brains exhibit a broader spectrum of scaling exponents, consistent with deviation from a clean power-law [6].

3 Field-Theoretic Anchor

The nonlocal kernel $G(r) = |r|^{-\alpha}$ is renormalizable only at $\alpha^* = \varphi$. The canonical fractal dimension is:

$$\Delta = d + 2 - \alpha^* \approx 3.382 \quad (d = 3).$$

Healthy brains: $\Delta \approx 2.3$–2.6. AD brains: $\Delta < 2.3$ or local Δ increases. $\Rightarrow$ AD corresponds to departures from the φ-critical regime.

4 Interpretation in Information-Theoretic Terms

The golden ratio is the "most irrational" number, producing correlations that maximize novelty per scale decade while retaining coherence. This property ensures optimal information transfer across scales, aligning with predictive coding frameworks where neural systems minimize long-term surprise [1–3]. In AD, multifractal broadening and reduced FD signal a breakdown of φ-governed predictive efficiency, leading to decoherence in neural dynamics.

5 Falsifiability

The theory predicts $\alpha = 1.618 \pm 0.01$.

- If future neuroimaging yields $\alpha = 1.55 \pm 0.02$, the model is falsified.
- If $\alpha = 1.618 \pm 0.01$, the theory is validated with no free parameters.

6 Conclusion

Alzheimer's disease reflects deviations from φ-criticality, detectable as FD reductions and multifractal distortions. The golden ratio emerges as both a mathematical fixed point and a neurobiological benchmark, providing a falsifiable bridge between field theory and neurodegeneration research.

Table 1: Fractal Dimension Ranges in Healthy and AD Brains Across Modalities

Modality	**Healthy FD Range**	**AD FD Range**
MRI (Cortical)	2.3–2.6 [4]	¡2.3 [4]
EEG (Higuchi)	2.3–2.5 [5]	¡2.3 [5]
Histology (Amyloid Plaques)	2.3–2.6 [6]	¿2.6 (local) [6]

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