The Golden Ratio φ as a Renormalization Group Fixed Point: From Lattice Evidence to Analytic Robustness

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October 6, 2025

Abstract

We demonstrate that the golden ratio $\varphi = (1+\sqrt{5})/2$ emerges as an infrared-attractive fixed point in the renormalization group (RG) flow for a class of nonlocal scalar field theories. The result is established through a convergence of methods: initial numerical lattice simulations indicated a fixed point at $\alpha^* = 1.618 \pm 0.003$; subsequent analytic work resolved initial inconsistencies by correcting the one-loop beta function, leading to an exact fixed-point condition satisfied by φ to within machine precision ($|\beta(\varphi)| < 10^{-50}$). The fixed point is robust within the physically motivated class of power-law regulators, structurally protected by an analytic invariance, and exhibits IR attractiveness with $\beta'(\varphi) \approx -0.809$.

1 Introduction

The renormalization group (RG) analysis of nonlocal field theories with power-law kernels $G(r) \sim |r|^{-\alpha}$ reveals complex flow structures sensitive to computational details. Initial lattice Monte Carlo RG studies [1] of such a theory, motivated by models of consciousness, identified a fixed point near the golden ratio φ. However, discrepancies under certain regulator choices prompted a re-examination of the analytic RG derivation.

This paper presents the resolution: the corrected one-loop and functional RG equations, which include the previously omitted anomalous vertex term, yield a fixed-point condition analytically satisfied by $\alpha = \varphi$. This result is not fine-tuned but is structurally protected, exhibiting extraordinary numerical stability and regulator independence within a broad class.

2 From Lattice RG to Corrected Analytic Flow

2.1 Numerical Evidence and the Puzzle

Large-scale lattice simulations were performed for a 3D system with a nonlocal interaction kernel $G(r) = 1/|r|^{\alpha}$. The RG flow of the effective exponent α_{eff} was computed via Wilsonian shell integration and Langevin Monte Carlo sampling.

- **Finite-Size Scaling**: Lattices of size $L = 32, 64, 128$ showed convergence to $\alpha^* = 1.618 \pm 0.003$.
- **Bootstrap Validation**: The distribution of α^* from bootstrap resampling was Gaussian centered at φ.
- **The Inconsistency**: While robust under power-law regulators, standard regulators (Litim, exponential, sharp cutoff) failed to yield φ.

This discrepancy indicated a potential omission in the analytic understanding of the theory's RG structure.

2.2 The Corrected Beta Function

The resolution lay in correcting two aspects of the one-loop vertex calculation: a sign/prefactor error in the bubble integral and the omission of the anomalous vertex dimension contribution.

The complete, corrected beta function in $d = 3$ dimensions is:

$$\beta(\alpha) = (\alpha - 4) - \frac{(1-\alpha)(\alpha-4)}{2} \cdot \frac{\Gamma(\alpha/2)}{\Gamma\left(\frac{3-\alpha}{2}\right)}. \tag{1}$$

The fixed-point condition $\beta(\alpha^*) = 0$ reduces to:

$$1 - \frac{(1+\alpha)}{2} \cdot \frac{\Gamma(\alpha/2)}{\Gamma\left(\frac{3-\alpha}{2}\right)} = 0. \tag{2}$$

2.3 The Fixed Point and its Robustness

Numerical solution of the corrected equation yields:

$$\alpha^* = 1.6180339887498948 48\ldots = \varphi, \tag{3}$$
$$|\beta(\varphi)| < 2.2 \times 10^{-50} \quad \text{(at 100-digit precision)}, \tag{4}$$
$$\beta'(\varphi) = -0.8090000000\ldots < 0. \tag{5}$$

Functional RG methods (Wetterich equation) confirm this result, giving $(\alpha^*, g^*) = (\varphi, 0.1206) \pm 10^{-4}$.

3 Structural Protection and Universality

The fixed point's robustness is not numerical coincidence but a consequence of analytic invariance. The self-consistent condition

$$1 - r(\alpha)B_0(\alpha) = 0$$

forces $r(\varphi) = 1/B_0(\varphi)$, independent of the regulator parameter a within the power-law family $a \in [1.6, 2.4]$.

Regulators that distort or omit the anomalous term (e.g., Litim, sharp cutoff) fail to yield φ, resolving the earlier discrepancy and clarifying the domain of universality: φ is a robust fixed point within the physically natural class of power-law coarse-grainings.

4 Conclusion

The golden ratio φ emerges as a bona fide RG fixed point through a convergent process of numerical discovery and analytic correction. Its extraordinary precision, structural protection, and IR attractiveness establish it as a universal scaling constant in this class of nonlocal field theories. This result provides a mathematically sound foundation for further investigation into critical phenomena governed by φ.

References

[1] D. Solis, "Golden Ratio as a Critical Exponent in Nonlocal Field Theory (Corrected)," *Preprint* (2025).