Partiel Maths

$A = \{ (x, y, z) \in \mathbb{R}^3 \mid x + y = z \}$

$(0,0,0) \in A$ car $0 + 0 = 0$

Soient $a(x_1, y_1, z_1) \in A$ et $b = (x_2, y_2, z_2) \in A$

alors $x_1 + y_1 = z_1$

et $x_2 + y_2 = z_2$

donc $(x_1 + x_2) + (y_1 + y_2) = z_1 + z_2$

donc $a + b \in A$

Soit $\lambda \in \mathbb{R}$ et $a(x, y, z) \in A$

alors $x + y = z$

donc $\lambda x + \lambda y = \lambda z$

donc $\lambda . a \in A$

A est donc un sev

$A = \{ (x, y, z) \in \mathbb{R}^3 \mid x + y = z \}$

$\cancel{(-y, y, 0)}$ $\cancel{\begin{cases} x + y = 0 \\ z = 0 \end{cases}}$

$\cancel{\begin{cases} x = -y \end{cases}}$

$\cancel{(x, y, z) = (-y, y, 0)}$

$\cancel{(x, y, z) = y(-1, 1, 0)}$

~~$(-1, 1, 0)$ est une base de A~~

$(x, y, z) = (x, y, x + y)$

$= (x, 0, x) + (0, y, y)$

$= x(1, 0, 1) + (0, 1, 1)$

$\Rightarrow$ Base de A

↳ car libre et génératrice

$B = \{(x,y) \in \mathbb{R}^2 \mid (x+y)^2 = 1\}$

$0 \notin B$ car $(0+0)^2 \neq 1$

$\boxed{\text{donc } B \text{ n'est pas un sev}}$

Exercice 2

$C = \{(x,y,z) \in \mathbb{R}^3 \mid x+y=0 \text{ et } y+z=0\}$

$(-y, y, -y)$ $\quad \begin{cases} x+y=0 \\ y+z=0 \end{cases} \quad \begin{cases} x=-y \\ y=-z \end{cases}$

$(-1, 1, -1)$ est une base de C

$(x,y,z) = (-y, y, -y)$

$(x,y,z) = y \cdot (-1, 1, -1)$

Sa dimension est $\mathbb{R}^3 \Rightarrow 1$

Exercice 3

~~1. $\{(1,0,2), (4,0,?)\}$~~ ~~$\{(1,0,2), (0,0,0), (2,0,4)\}$~~

~~2. $\{(0,1,2)\}$~~

$\boxed{1. \{(0,1,0), (1,0,0), (0,0,1)\}, (0,4,8)\}}$

$\boxed{2. \{\cancel{(0,1)}\ (1,0)\}}$

Exercice 4

1. $\dim(\mathbb{R}^3) = 3$ et nb de vect $= 4$
donc impossible d'être libre

2. $u_1(0,1,1)$ $u_2(1,0,1)$ $u_3(1,1,0)$

Soient $\lambda_1 = (0,1,1) \in \mathbb{R}^3 + \lambda_2(1,0,1) + \lambda_3(1,1,0) \in \mathbb{R}^3$

$$\begin{cases} \lambda_2 + \lambda_3 = 0 \\ \lambda_1 + \lambda_3 = 0 \\ \lambda_1 + \lambda_2 = 0 \end{cases} \qquad \begin{cases} \lambda_2 = -\lambda_3 \\ \lambda_1 = -\lambda_3 \\ \lambda_1 = -\lambda_2 \end{cases}$$

$$\begin{cases} -2\lambda_1 = 0 \\ \cancel{0} + \lambda_3 = 0 \\ \cancel{0} + \lambda_2 = 0 \end{cases}$$

$\boxed{\text{Donc F libre}}$

$$\begin{cases} \lambda_2 + \lambda_3 = x \\ \lambda_1 + \lambda_3 = y \\ \lambda_1 + \lambda_2 = z \end{cases} \qquad \begin{cases} z - \lambda_1 + y - \lambda_1 = x \\ \lambda_3 = y - \lambda_1 \\ \lambda_2 = z - \lambda_1 \end{cases}$$

$$\begin{cases} \frac{z+y-x}{2} = \lambda_1 \\ \frac{z+y-x}{2} + \lambda_3 = y \\ \frac{z+y-x}{2} + \lambda_2 = z \end{cases} \qquad \begin{cases} \lambda_1 = \frac{z+y-x}{2} \\ \lambda_3 = y - \left(\frac{z+y-x}{2}\right) \\ \lambda_2 = z - \left(\frac{z+y-x}{2}\right) \end{cases}$$

$$(x,y,z) = \frac{z+y-x}{2} \cdot (0,1,1) + \frac{z+x}{-2} \cdot (1,0,1) + \frac{y+x}{-2} \cdot (1,1,0)$$

3. $u_1(1,1,2)$ $u_2(2,1,-1)$ $u_3(1,0,-3)$

Soient $\lambda_1(1,1,2)+\lambda_2(2,1,-1)+\lambda_3(1,0,-3) \in \mathbb{R}^3$

$$\begin{cases} \lambda_1 + 2\lambda_2 + \lambda_3 = 0 \\ \lambda_1 + \lambda_2 = 0 \\ 2\lambda_1 - \lambda_2 - 3\lambda_3 = 0 \end{cases} \qquad \begin{cases} \lambda_1 + 2\lambda_2 + \lambda_1 = 0 \\ \lambda_1 = -\lambda_2 \\ \cancel{2}\lambda_1 = \cancel{2}\lambda_3 \end{cases}$$

$$\begin{cases} \boxed{0 = 0} \\ \lambda_1 = -\lambda_2 \\ \lambda_1 = \lambda_3 \end{cases}$$

$\boxed{\text{Par ma pas libre}}$

Exercice 5

Soient $\lambda_1(1,0,0)+\lambda_2(1,1,0)+\lambda_3(1,1,1) \in \mathbb{R}^3$

$$\begin{cases} \lambda_1 + \lambda_2 + \lambda_3 = x \\ \lambda_2 + \lambda_3 = y \\ \lambda_3 = z \end{cases} \qquad \begin{cases} \lambda_1 = x - \cancel{(y-z)} - \cancel{z} \\ \lambda_2 = y - z \\ \lambda_3 = z \end{cases}$$

$$(x,y,z) = x - y.(1,0,0) + y - z\ (1,1,0) + z.(1,1,1)$$

$$\begin{cases} \lambda_1 + \cancel{\lambda_2} + \cancel{\lambda_3} = 0 \\ \lambda_2 + \cancel{\lambda_3} = 0 \\ \cancel{\lambda_3} = 0 \end{cases}$$

$\boxed{\text{donc F libre}}$

$$\boxed{\begin{aligned} (1,2,3) &= 1-2.(1,0,0) + 2-3.(1,1,0) + 3(1,1,1) \\ &= \cancel{2}.(\vec{u_1}) - 1.(\vec{u_2}) + 3(\vec{u_3}) \\ &\ \ -1 \end{aligned}}$$

Exercice 6

$$f \quad \mathbb{R}^2 \to \mathbb{R}_2$$

$$(x, y) \to (x + y, x + y)$$

Soient $a(x_1, y_1)$ et $b(x_2, y_2) \in f$

$$f(a + b) = f(x_1 + x_2, y_1 + y_2)$$

$$= f(\{x_1 + x_2\} + \{y_1 + y_2\}, \{x_1 + x_2\} + \{y_1 + y_2\})$$

$$= f(x_1 + y_1, x_1 + y_1) + (x_2 + y_2, x_2 + y_2)$$

$$= f(a) + f(b)$$

$$\boxed{= f(a + b)}$$

De plus

Soient $\lambda \in \mathbb{R}$ et $a(x, y) \in F$
alors

$$\lambda . f(a) = (\lambda x + \lambda y, \lambda x + \lambda y)$$

donc

$\boxed{f \text{ est une app lin}}$.

3. $u_1(1,1,2)$ $u_2(2,1,-1)$ $u_3(1,0,-3)$

Soient $\lambda_1(1,1,2)+\lambda_2(2,1,-1)+\lambda_3(1,0,-3) \in \mathbb{R}^3$

$$\begin{cases} \lambda_1+2\lambda_2+\lambda_3=0 \\ \lambda_1+\lambda_2=0 \\ 2\lambda_1-\lambda_2-3\lambda_3=0 \end{cases} \qquad \begin{cases} \lambda_1+2\lambda_1+\lambda_1=0 \\ \lambda_1=-\lambda_2 \\ \cancel{2}\lambda_1=\cancel{2}\lambda_3 \end{cases}$$

$$\begin{cases} \boxed{0=0} \\ \lambda_1=-\lambda_2 \\ \lambda_1=\lambda_3 \end{cases}$$

$\boxed{\text{Par mai pas libre}}$

Exercice 5

Soient $\lambda_1(1,0,0)+\lambda_2(1,1,0)+\lambda_3(1,1,1) \in \mathbb{R}^3$

$$\begin{cases} \lambda_1+\lambda_2+\lambda_3=x \\ \lambda_2+\lambda_3=y \\ \lambda_3=z \end{cases} \qquad \begin{cases} \lambda_1=x-\cancel{(y-z)}-\cancel{z} \\ \lambda_2=y-z \\ \lambda_3=z \end{cases}$$

$$(x,y,z)=x-y.(1,0,0)+y-z\ (1,1,0)+z.(1,1,1)$$

$$\begin{cases} \lambda_1+\cancel{\lambda_2}+\cancel{\lambda_3}=0 \\ \lambda_2+\cancel{\lambda_3}=0 \\ \cancel{\lambda_3}=0 \end{cases}$$

$\boxed{\text{donc F libre}}$

$$(1,2,3)=1-2.(1,0,0)+2-3.(1,1,0)+3(1,1,1)$$
$$=\cancel{2}.(\vec{u_1})-1.(\vec{u_2})+3(\vec{u_3})$$
$$-1$$