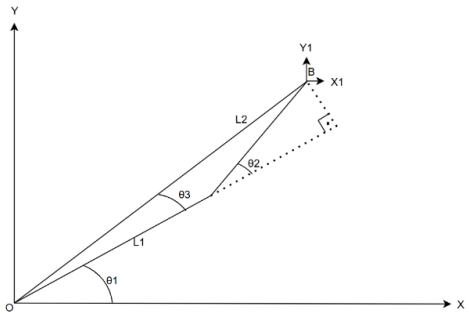
Kinematics Equations



Denklem 1 Forward Kinematics Position Equations

$$x_1 = L_1 * \cos(\theta_1) + L_2 * \cos(\theta_1 + \theta_2)$$
$$y_1 = L_1 * \sin(\theta_1) + L_2 * \sin(\theta_1 + \theta_2)$$

Denklem 2 Inverse Kinematics (IK) Equation for Joint Angle θ_2

$$OB^2 = x_1^2 + y_1^2 = (L_2 * \sin(\theta_2))^2 + (L_1 + L_2 * \cos(\theta_2))^2$$
$$x_1^2 + y_1^2 = L_2^2 * \sin(\theta_2)^2 + L_1^2 + L_2^2 * \cos(\theta_2)^2 + 2 * L_1 * L_2 * \cos(\theta_2)$$
$$x_1^2 + y_1^2 - L_1^2 + L_2^2 = 2 * L_1 * L_2 * \cos(\theta_2)$$
$$\cos(\theta_2) = \frac{x_1^2 + y_1^2 - L_1^2 + L_2^2}{2 * L_1 * L_2}$$
$$\theta_2 = \cos^{-1}\left(\frac{x_1^2 + y_1^2 - L_1^2 + L_2^2}{2 * L_1 * L_2}\right)$$

Denklem 3 Inverse Kinematics (IK) Equations for Joint Angle θ_1

$$\beta = \theta_1 + \theta_3 = \tan^{-1}(y_1 / x_1)$$
$$\cos(\theta_3) = \frac{L_1 + L_2 * \cos(\theta_2)}{\sqrt{x_1^2 + y_1^2}} = \frac{L_1 + L_2 * \frac{x_1^2 + y_1^2 - L_1^2 + L_2^2}{2 * L_1 * L_2}}{\sqrt{x_1^2 + y_1^2}}$$
$$\cos(\theta_3) = \frac{2 * L_1{}^2 + x_1^2 + y_1^2 - L_1^2 + L_2^2}{2 * L_1 * \sqrt{x_1^2 + y_1^2}}$$
$$\theta_3 = \cos^{-1}\left(\frac{2 * L_1{}^2 + x_1^2 + y_1^2 - L_1^2 + L_2^2}{2 * L_1 * \sqrt{x_1^2 + y_1^2}}\right)$$
$$\theta_1 = \tan^{-1}(y_1 / x_1) - \cos^{-1}\left(\frac{2 * L_1{}^2 + x_1^2 + y_1^2 - L_1^2 + L_2^2}{2 * L_1 * \sqrt{x_1^2 + y_1^2}}\right)$$

Dynamic Equations

$T = \sum_{1}^{n} \frac{1}{2} * m_n * v_n^2 + \frac{1}{2} * I_n * \dot{\theta}_n^{\,2}$: The Kinetic Energy Equation

$I_n = \frac{1}{12} * m_n * L_n^2$: Moment of İnertia

$P = \sum_{1}^{n} m_n * g * y_n$: The Potential Energy Equation

$y_n = \sum_{1}^{n} \frac{L_n}{2} * \left[\sin(\sum_{1}^{n} \theta_n) \right] + L_{n-1} * \left[\sin(\sum_{1}^{n-1} \theta_{n-1}) \right] + L_{n-2} * \left[\sin(\sum_{1}^{n-2} \theta_{n-1}) \right] + ... + L_1 * \left[\sin(\theta_1) \right]$: Center of Mass

$L = T - P$: Lagrange Equations

$\tau_n = \frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_n}) - \frac{\partial L}{\partial \theta_n}$: Torque Equation

$$T = \frac{1}{6} m_1 L_1^2 \dot{\theta}_1^2 + m_2 \left[\frac{1}{2} L_1^2 \dot{\theta}_1^2 + \frac{1}{2} L_1 L_2 \cos(\theta_2) \dot{\theta}_1^2 + \frac{1}{2} L_1 L_2 \cos(\theta_2) \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{6} L_2^2 \dot{\theta}_1^2 + \frac{1}{3} L_2^2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{6} L_2^2 \dot{\theta}_2^2 \right]$$

$$P = m_1 g \frac{L_1}{2} \sin(\theta_1) + m_2 g \left(L_1 \sin(\theta_1) + \frac{L_2}{2} \sin(\theta_1 + \theta_2) \right)$$

$$\tau_1 = \ddot{\theta}_1 \left(\frac{L_1^2 m_1}{3} + L_1^2 m_2 + \frac{L_2^2 m_2}{3} + L_1 L_2 m_2 \cos(\theta_2) \right) - \dot{\theta}_2 \left(L_1 L_2 \dot{\theta}_1 m_2 \sin(\theta_2) + \frac{L_1 L_2 \dot{\theta}_2 m_2 \sin(\theta_2)}{2} \right) +$$

$$+ \ddot{\theta}_2 \left(\frac{m_2 L_2^2}{3} + \frac{L_1 m_2 L_2 \cos(\theta_2)}{2} \right) + \frac{L_2 g m_2 \cos(\theta_1 + \theta_2)}{2} + \frac{L_1 g m_1 \cos(\theta_1)}{2} + L_1 g m_2 \cos(\theta_1)$$

$$\tau_2 = \frac{L_2 m_2 \left(3 L_1 \dot{\theta}_1^2 \sin(\theta_2) + 2 L_2 \ddot{\theta}_1 + 2 L_2 \ddot{\theta}_2 + 3 g \cos(\theta_1 + \theta_2) + 3 L_1 \ddot{\theta}_1 \cos(\theta_2) \right)}{6}$$