Optimization with Metaheuristics

CSE480 & CSE591- Week 3&4 - Local Search Fundamentals

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YEDİTEPE UNIVERSITY

Reminder for Metaheuristics

Definition

A metaheuristic is a general algorithm that can produce approximate solutions for a class of different optimization problems.

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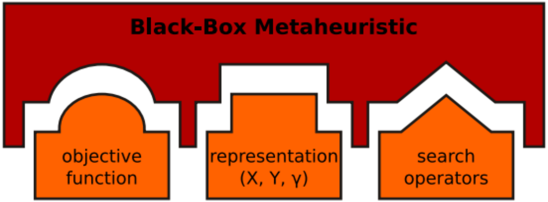
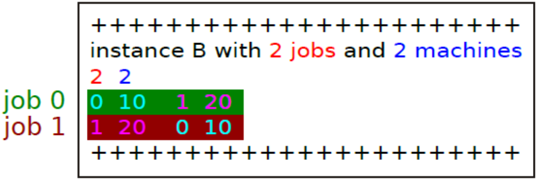
Why are metaheuristics general methods?



Random Sampling

- Random Sampling: Very first optimization algorithm.
- Recall JSSP: a Gantt chart for m machines and n jobs as an integer string of length m $\times$ n.



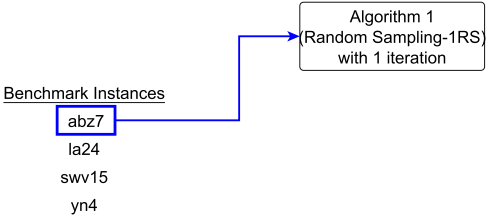
1. Put each of the numbers from 0 to $n-1$ exactly m times in an integer array of length $m \times n$ (so we have a valid point $x_0 \in \mathcal{X}$), then
2. Randomly shuffle the values like a deck of cards (so we get a random valid point $x \in \mathcal{X}$), and
3. Apply the representation mapping γ to get a Gantt chart $y = \gamma(x)$, $y \in \mathcal{Y}$.

Experiment and Analysis

Experimental Study

Benchmark Instances
abz7
la24
swv15
yn4

Experimental Study



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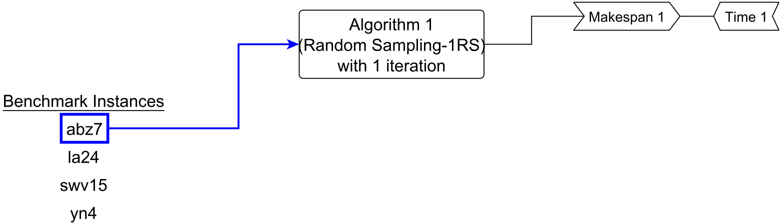

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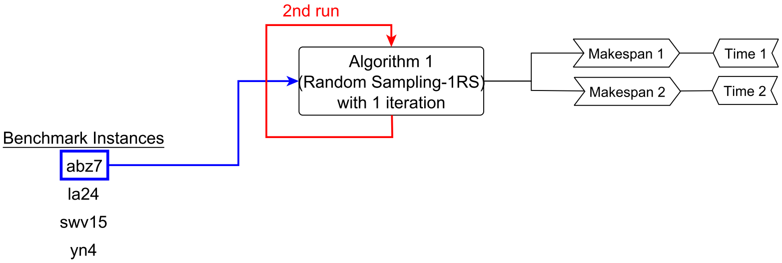

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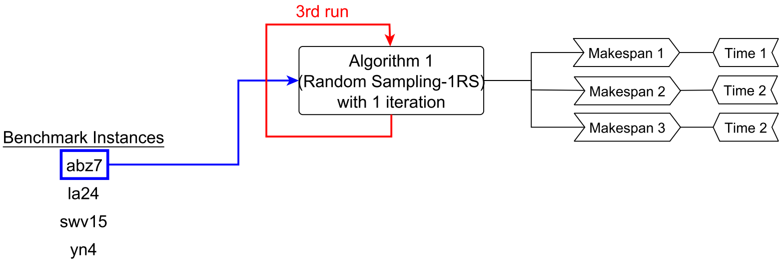

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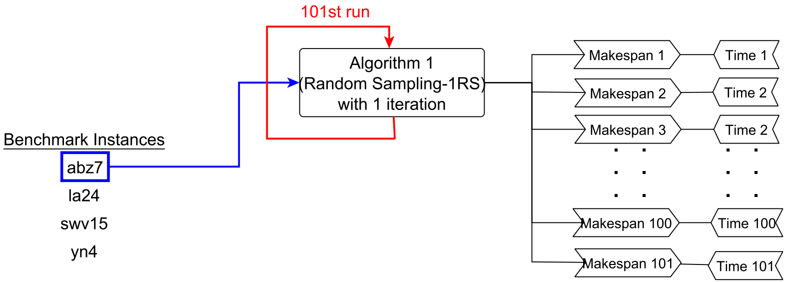

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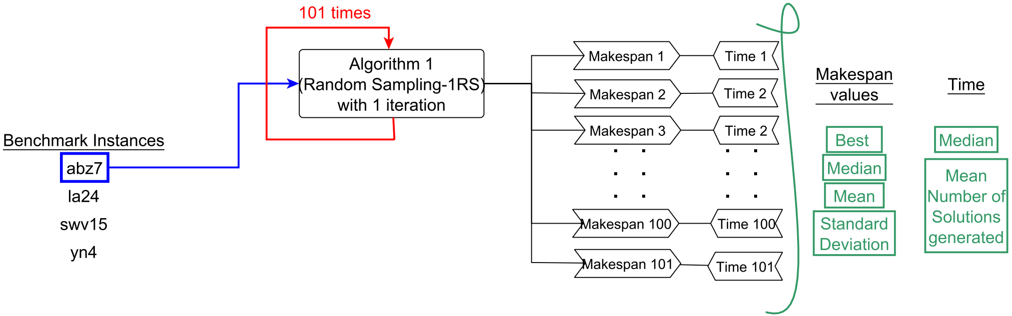

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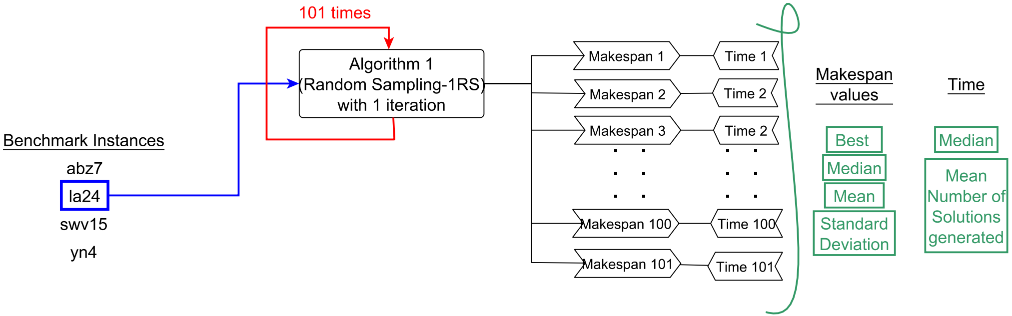

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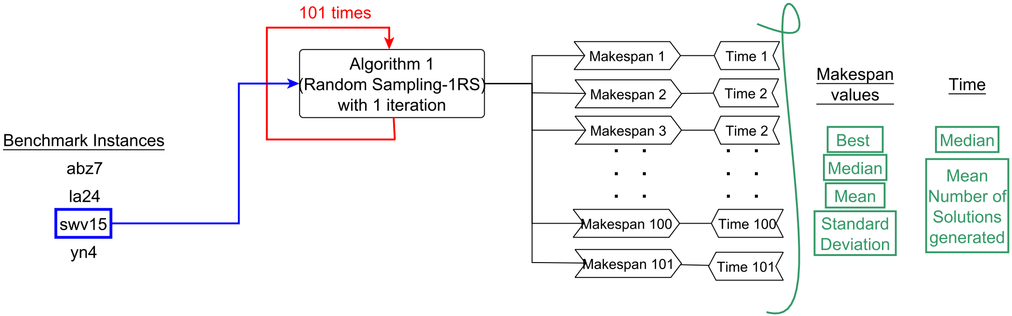

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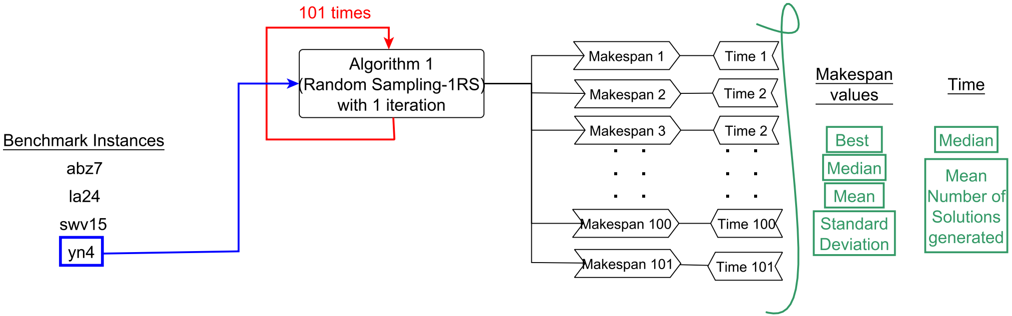

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`abz7`	1'131	1'334	1'326	106	0s	1
`la24`	1'487	1'842	1'814	165	0s	1
`swv15`	5'935	6'600	6'563	346	0s	1
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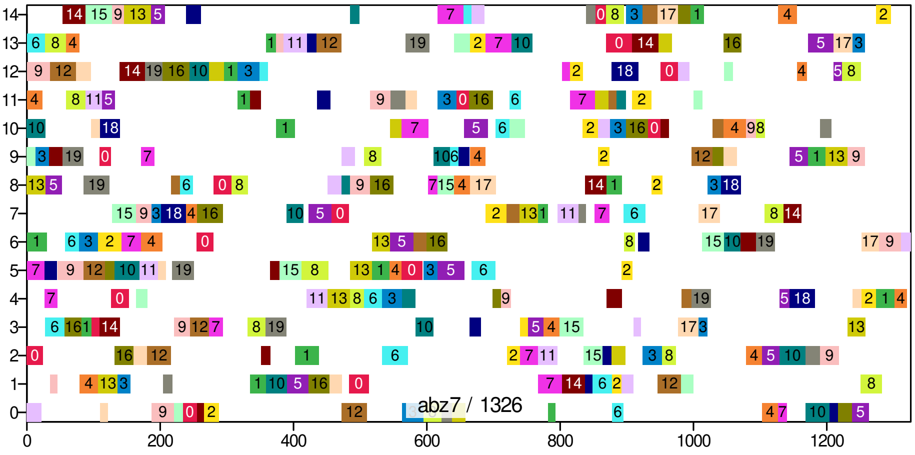
Median solution for abz7



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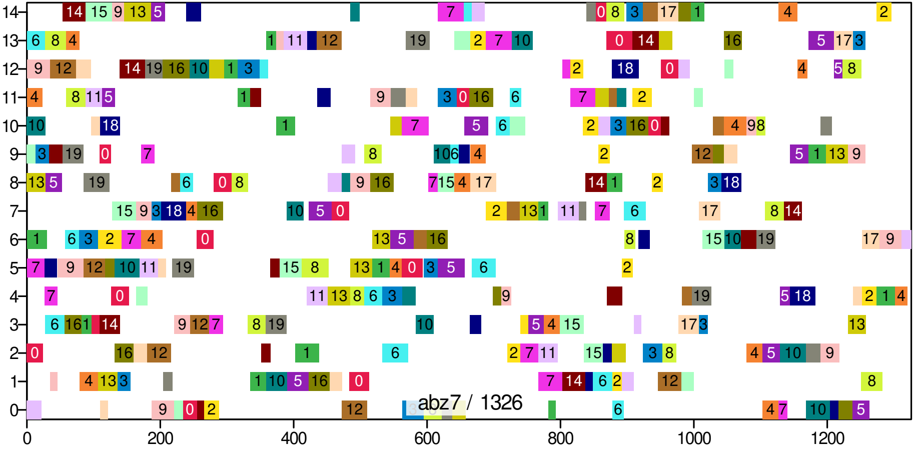

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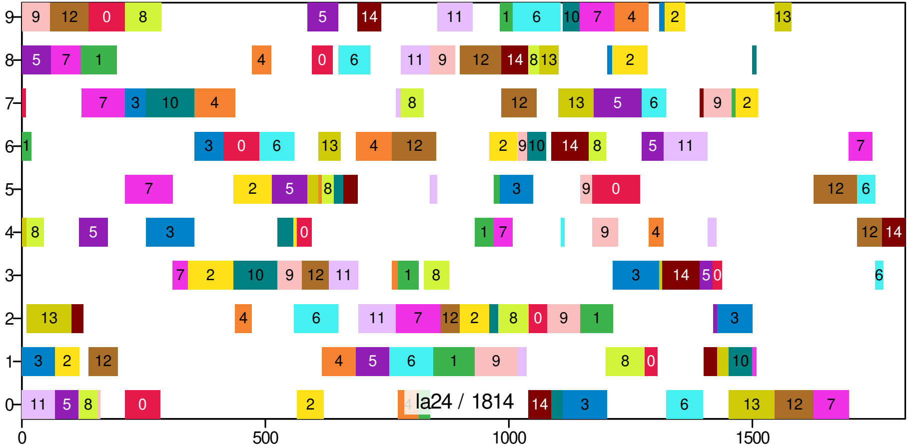
Median solution for `la24`



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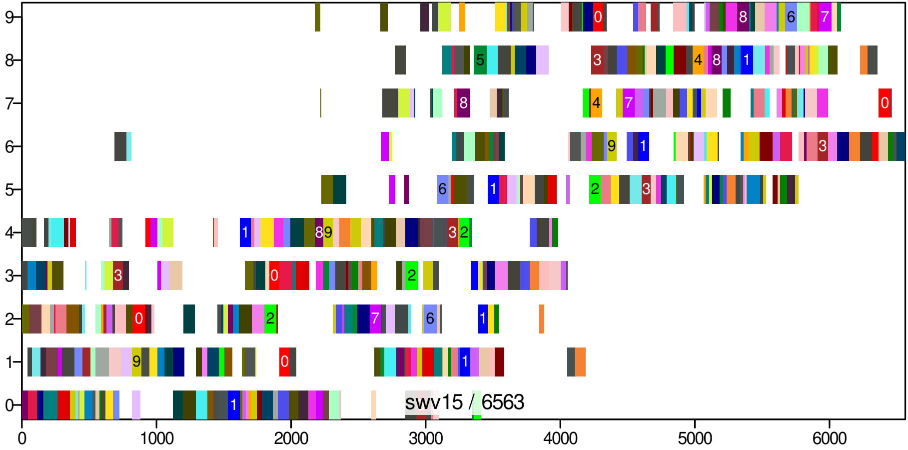

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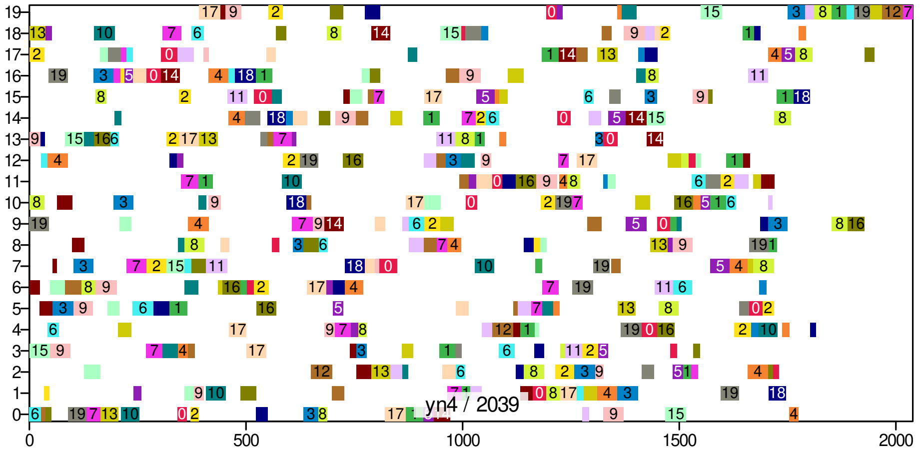
Median solution for `yn4`



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- The results are not good, there is lots of white space $\equiv$ wasted time. That was expected: Our solutions are random.
- Notice 1. We can create and test the schedules very very fast (much faster than 3min).
- Notice 2. There is a high variance in the results due to randomness.

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Exploit Variance: Random Sampling

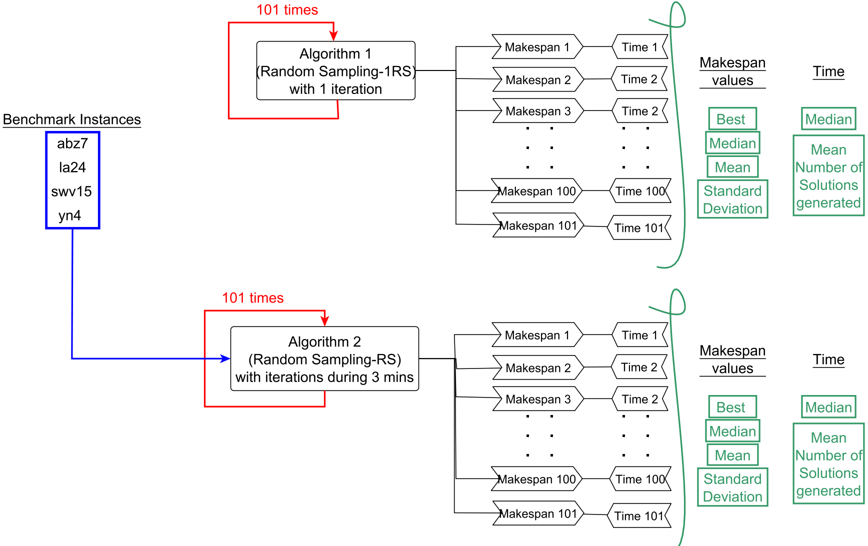
We can keep generating schedules until the 3 minutes are up and keep the best one.

New idea: The Random sampling algorithm (also called random search) repeats creating random solutions until the computational budget is exhausted.
It works as follows:

- create new random candidate solution y (via random sampling from the search space)
- remember best solution ever encountered
- repeat until 3 min are exhausted

Experiment and Analysis 2

Experimental Study



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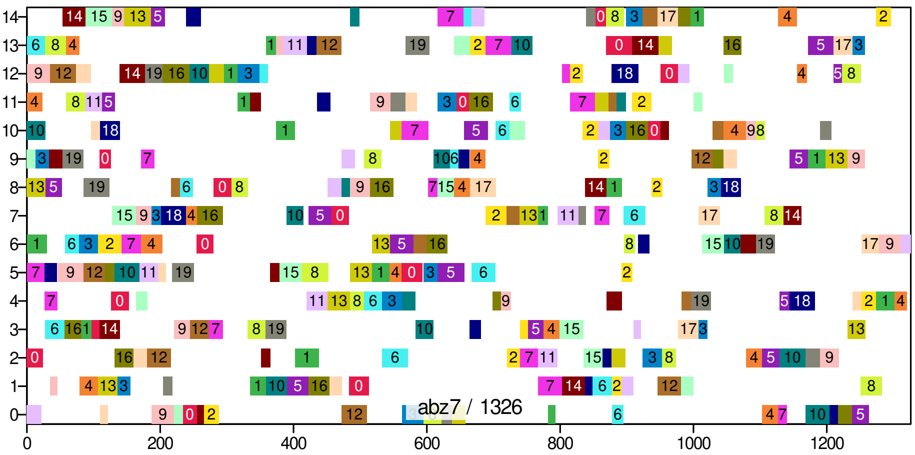
	`rs`	**895**	**947**	**949**	**12**	85s	6'512'505
`la24`	`1rs`	1487	1842	1814	165	0s	1
	`rs`	**1153**	**1206**	**1208**	**15**	82s	15'902'911
`swv15`	`1rs`	5935	6600	6563	346	0s	1
	`rs`	**4988**	**5166**	**5172**	**50**	87s	5'559'124
`yn4`	`1rs`	1754	2036	2039	125	0s	1
	`rs`	**1460**	**1498**	**1499**	**15**	76s	4'814'914

So what do we get?

`1rs`: median result of single random sample algorithm



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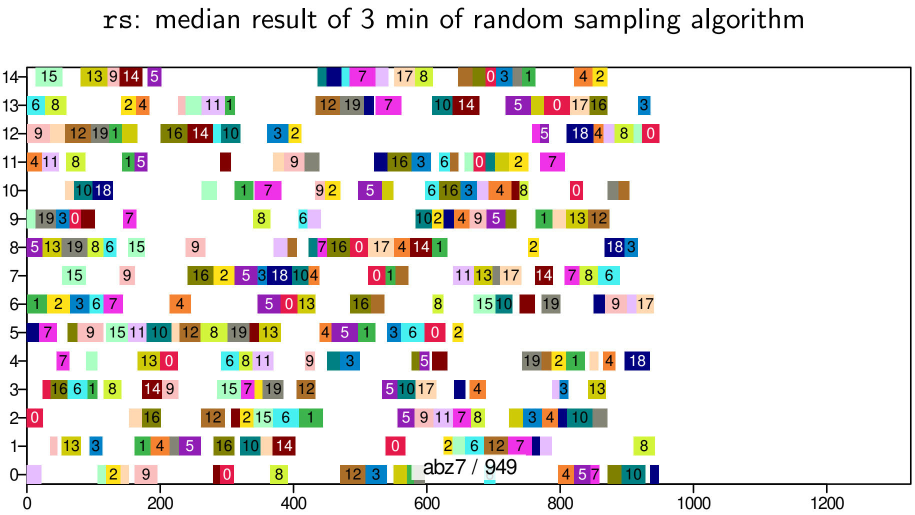

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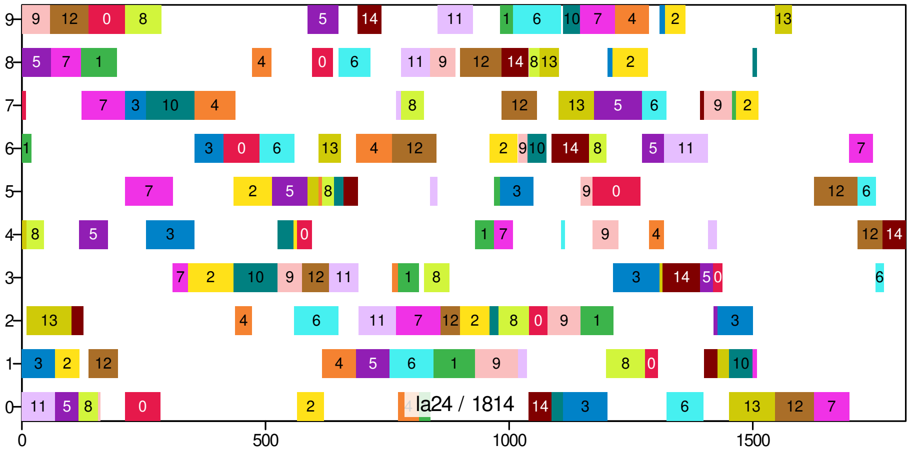

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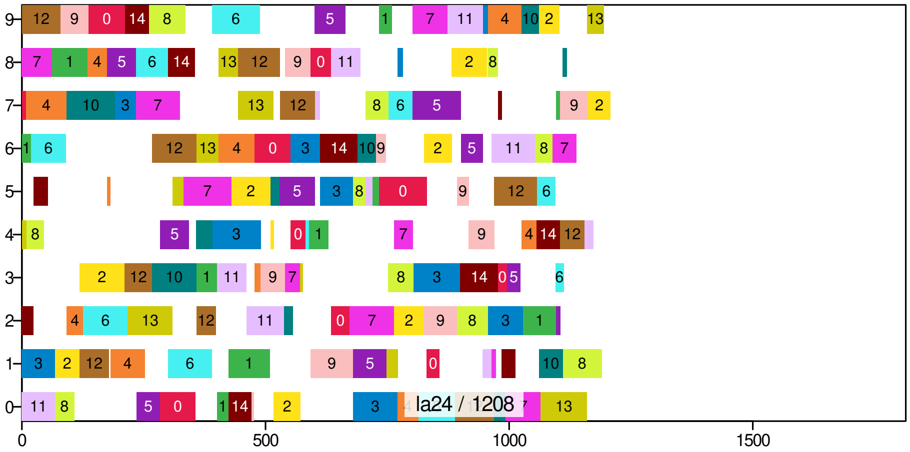
rs: median result of 3 min of random sampling algorithm



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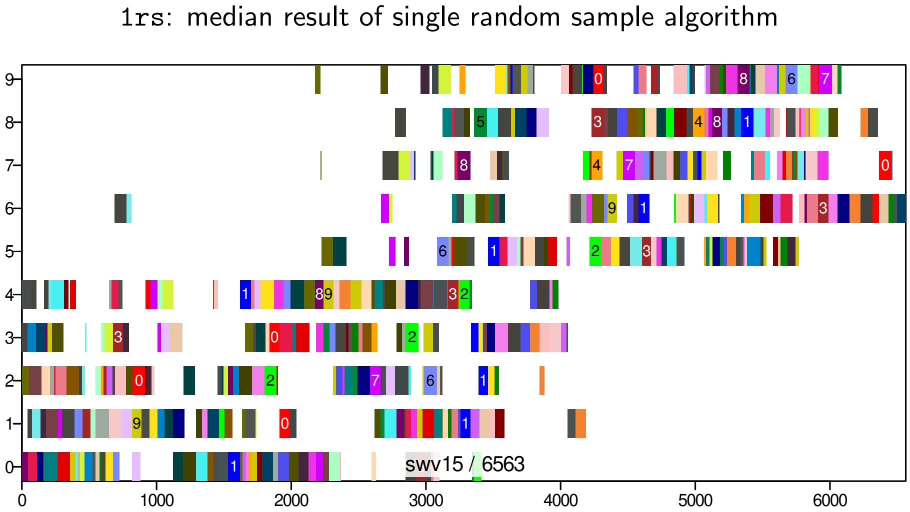

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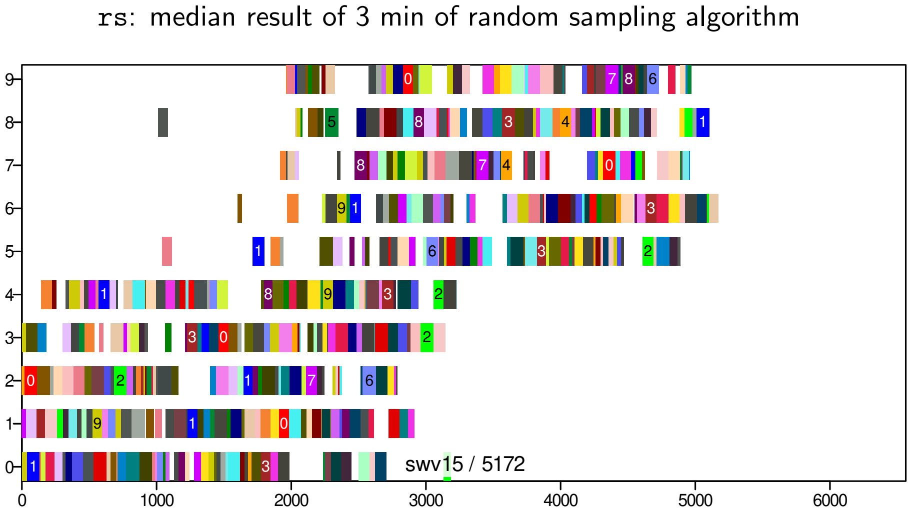

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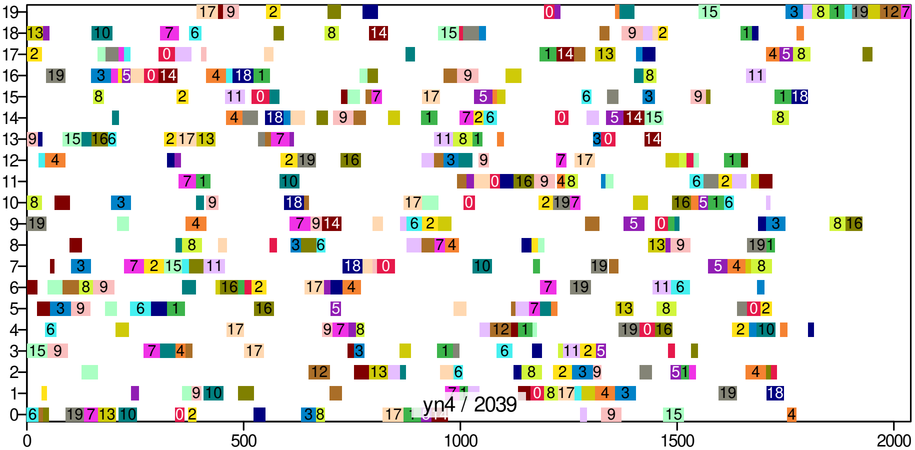

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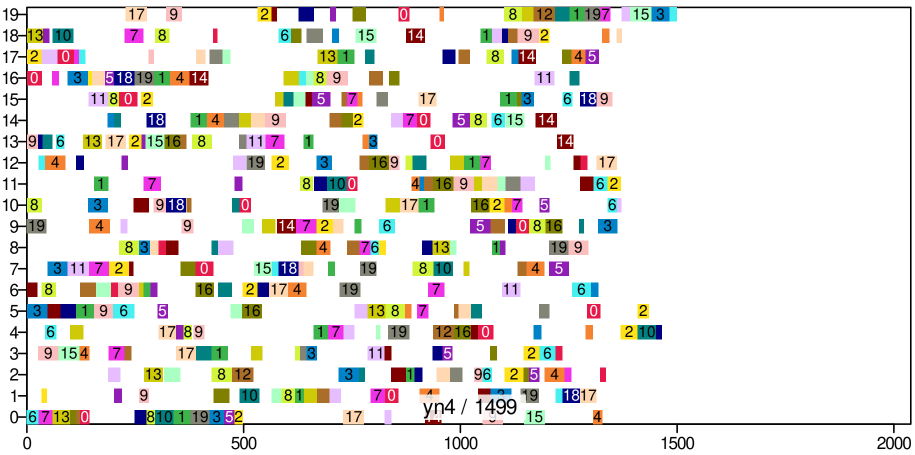

Progress over Time

What progress does the algorithm make over time?

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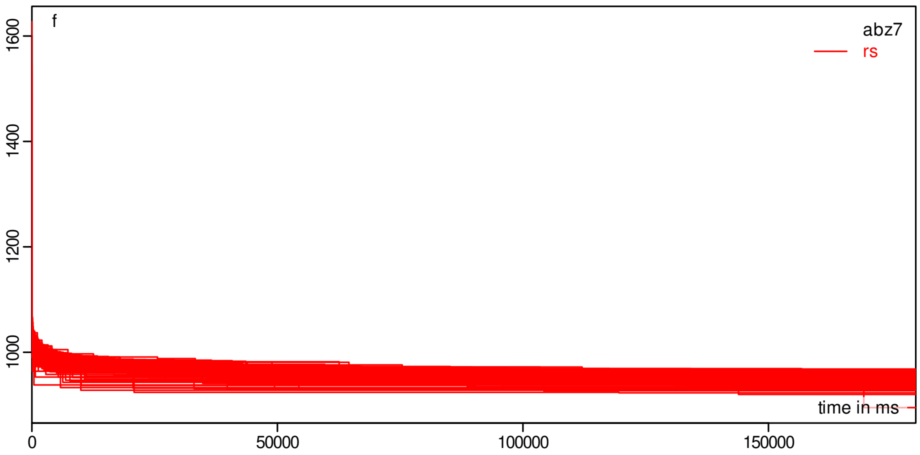

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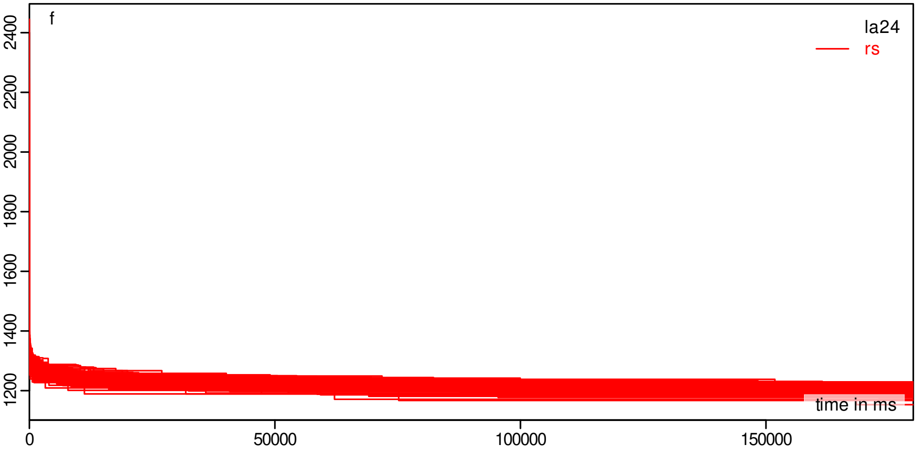

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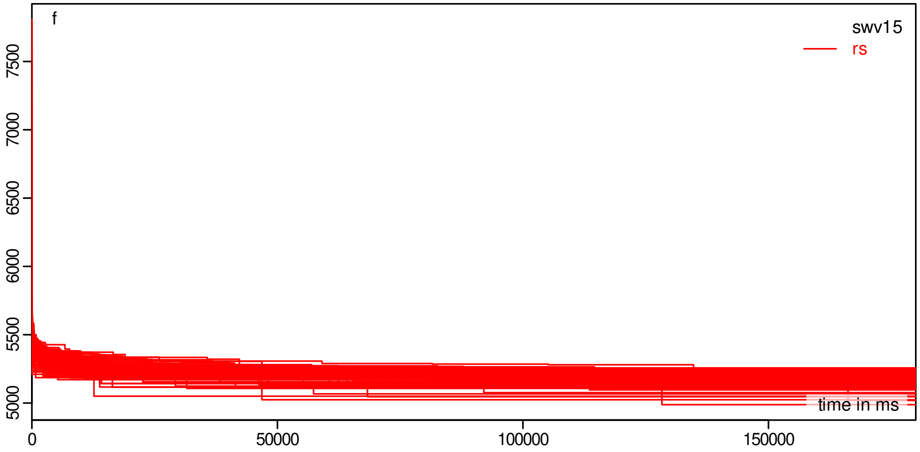

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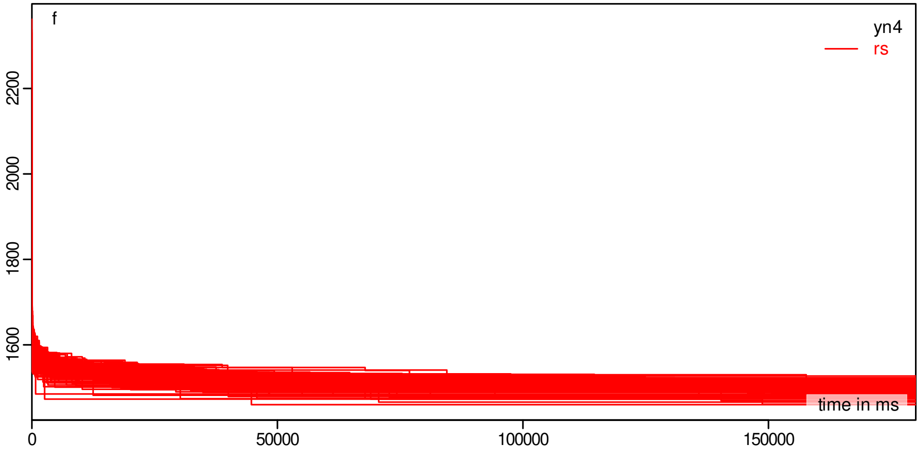

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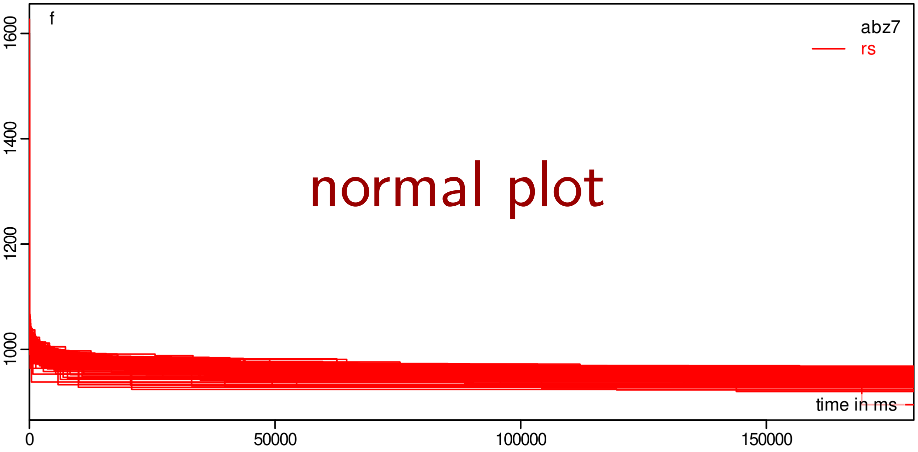

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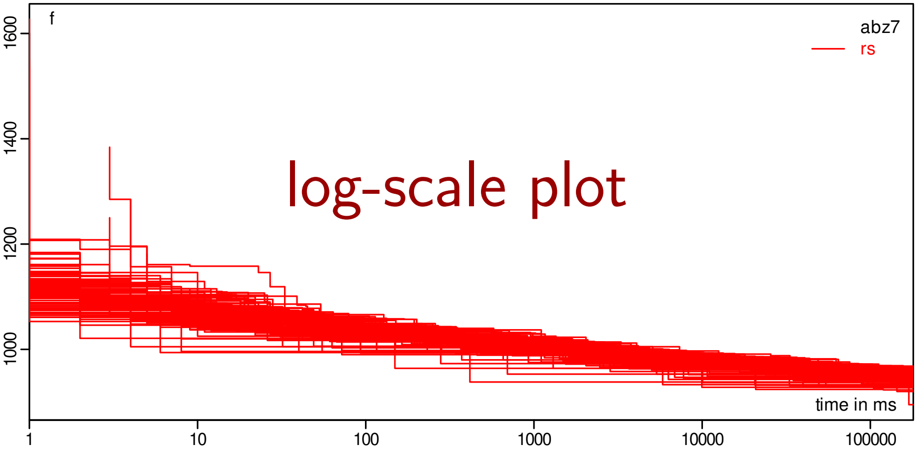

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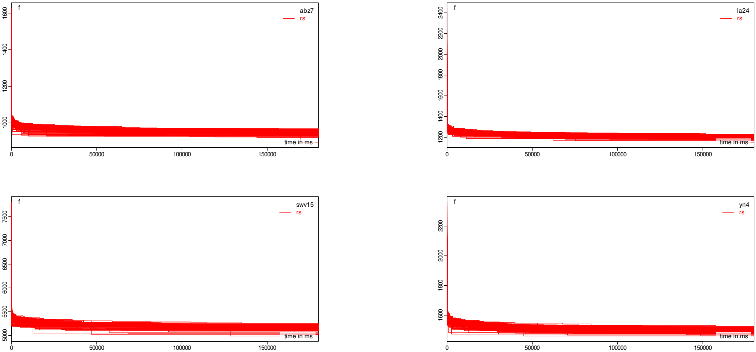

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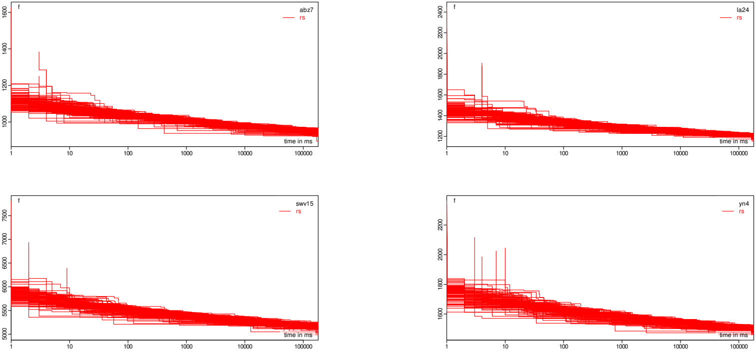

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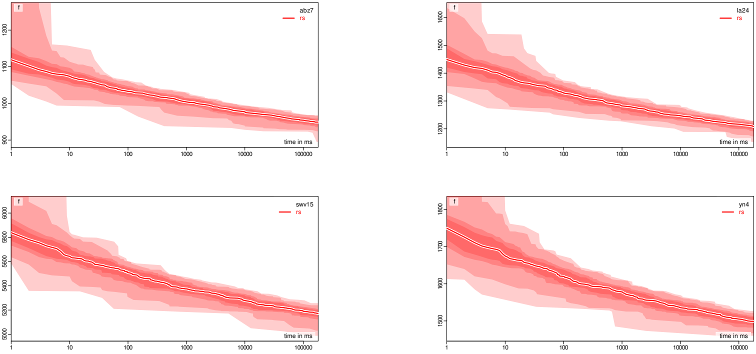
- This holds for runtime, but also for improvements of algorithms.



Information from Good Solutions

- Our first algorithm, random sampling, was not very efficient.
- It does not make any use of the information it “sees” during the optimization process.
- Each search step consists of creating an entirely new, entirely random candidate solution.
- Each search step is thus independent of all prior steps.
- Is this a good idea?
- Probably not.
- In almost all practical scenarios, good solutions are somewhat similar to other good solutions.
- In other words, every good solution we see is some useful information.

Basic Idea

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- Instead of generating a completely random new candidate solution in each step. . .
- . . . why can't we try to iteratively improve the best solution we have seen so far?

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  4. go back to 2. (until the time is up)

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- The idea is that if we have a good candidate solution, then there may exist similar solutions which are better.
- We hope to find one of them and then continue trying to do the same from there.

Ingredient: Unary Search Operator

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- How can we implement this for our JSSP scenario?
- Easy: Just swap two (different) job IDs in the string!
- Since the numbers of occurrences of the IDs will not change, the new strings will be valid.

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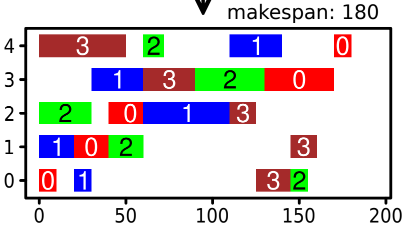

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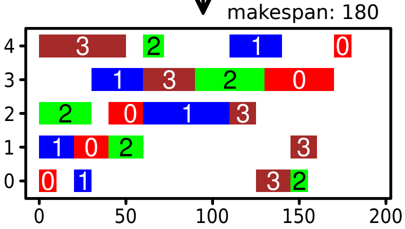

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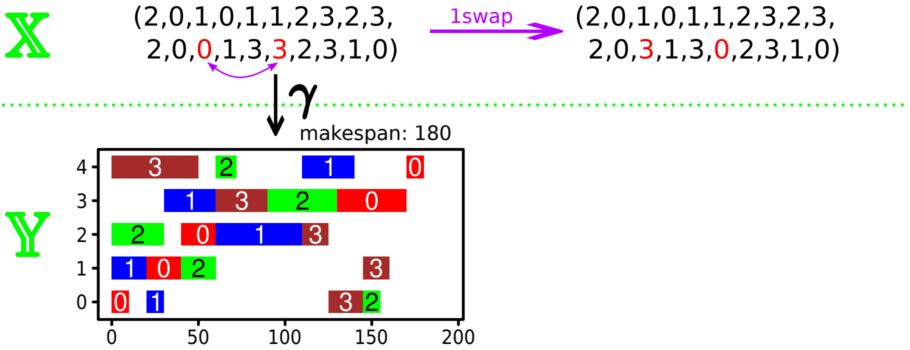

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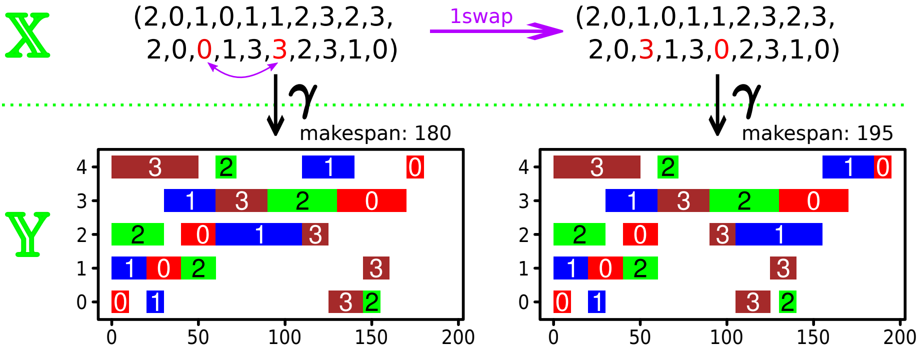

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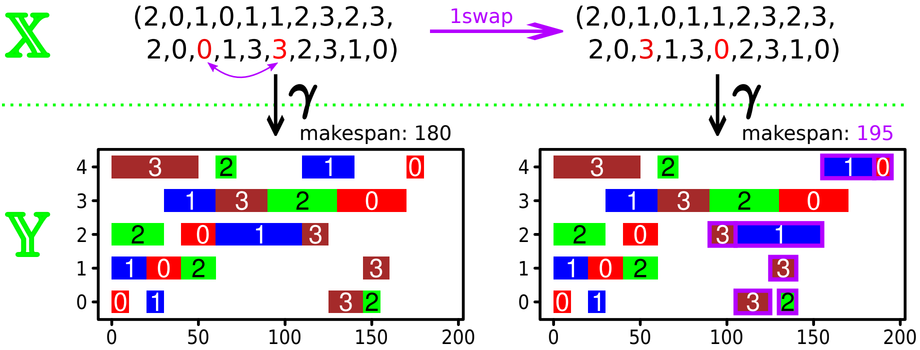

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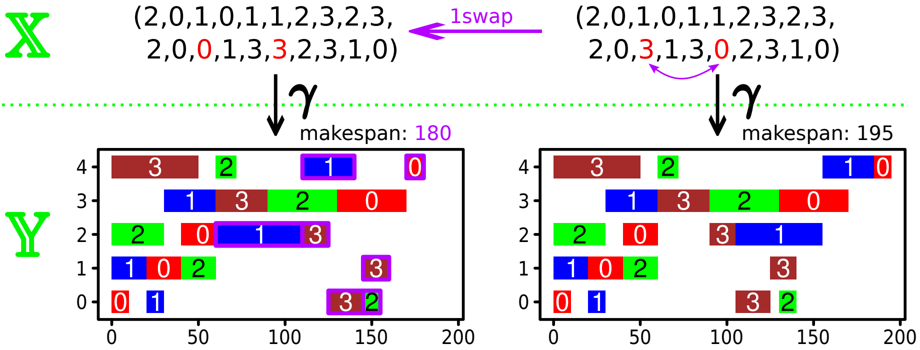

Experiment and Analysis

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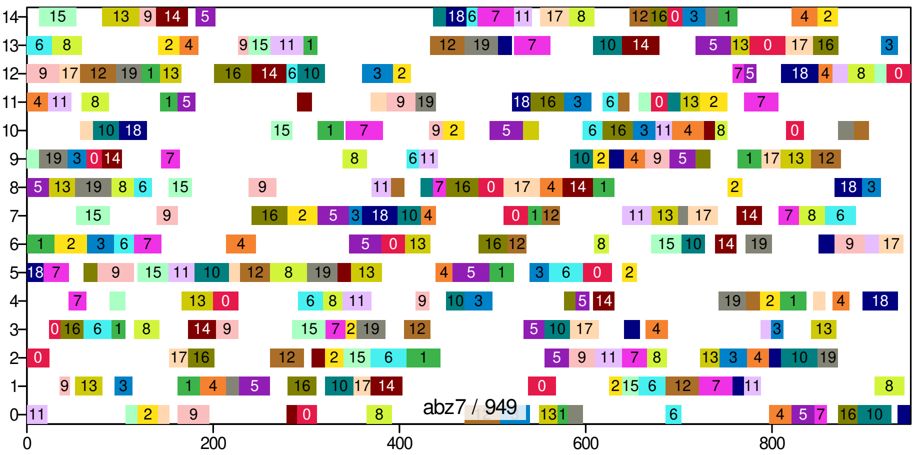

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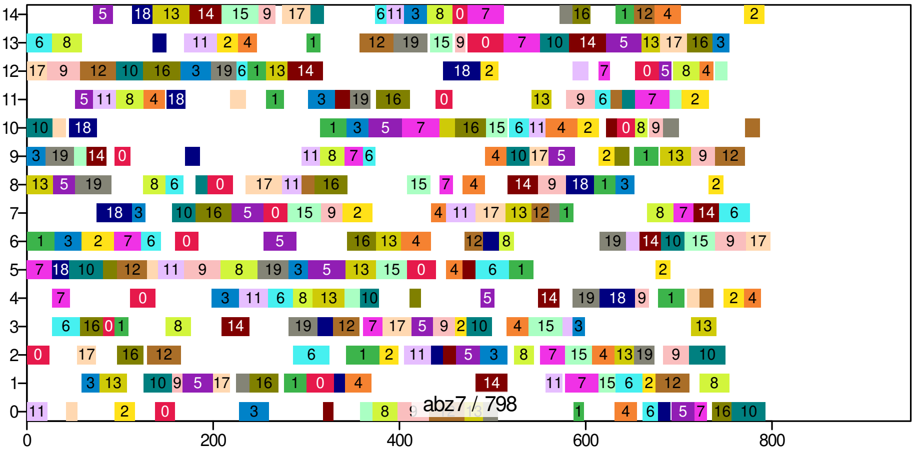

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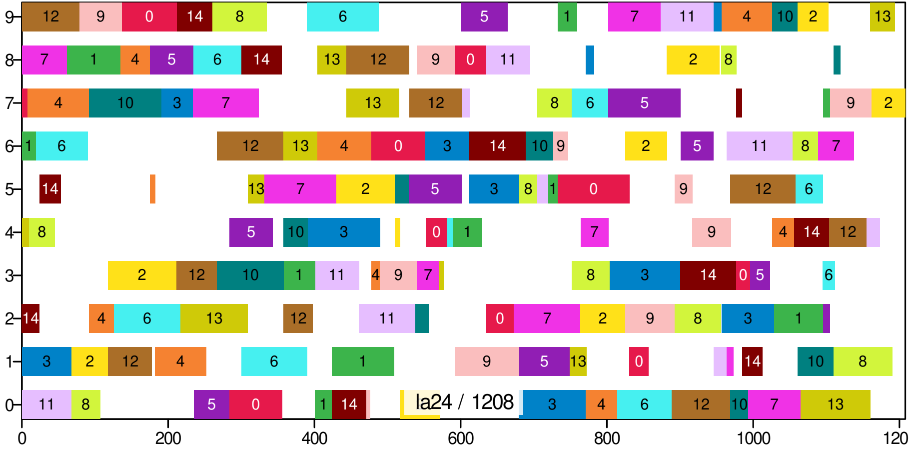

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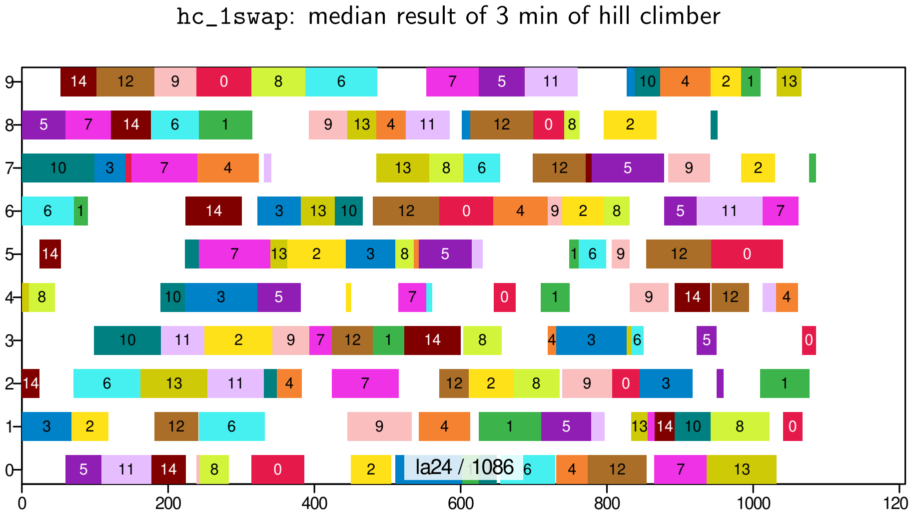

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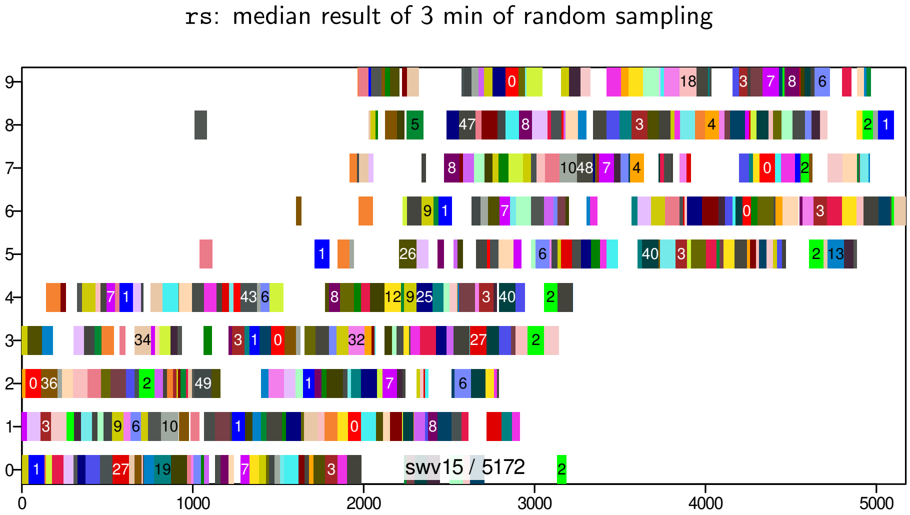

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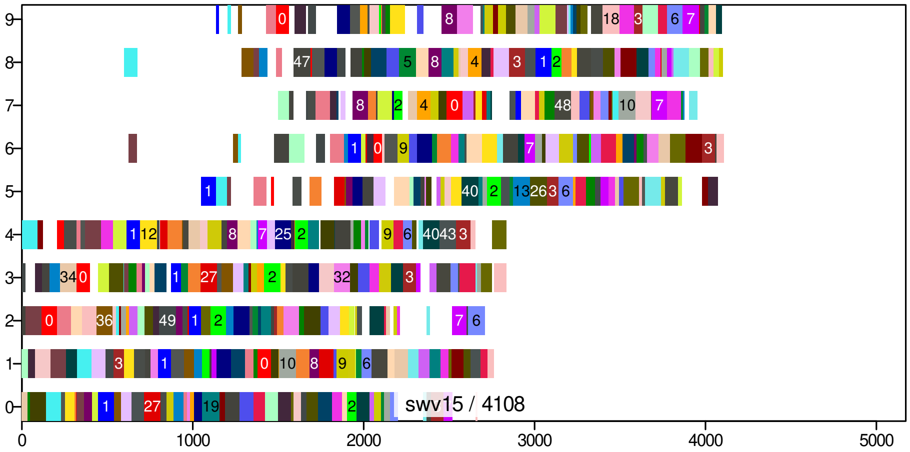

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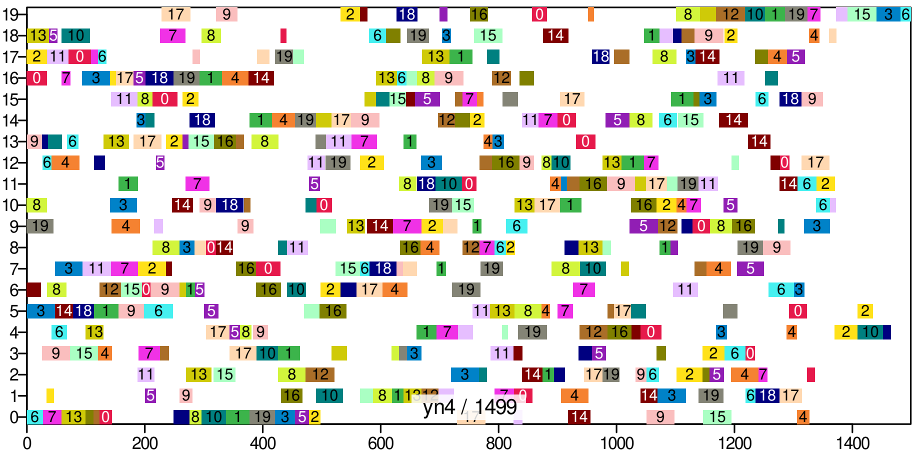

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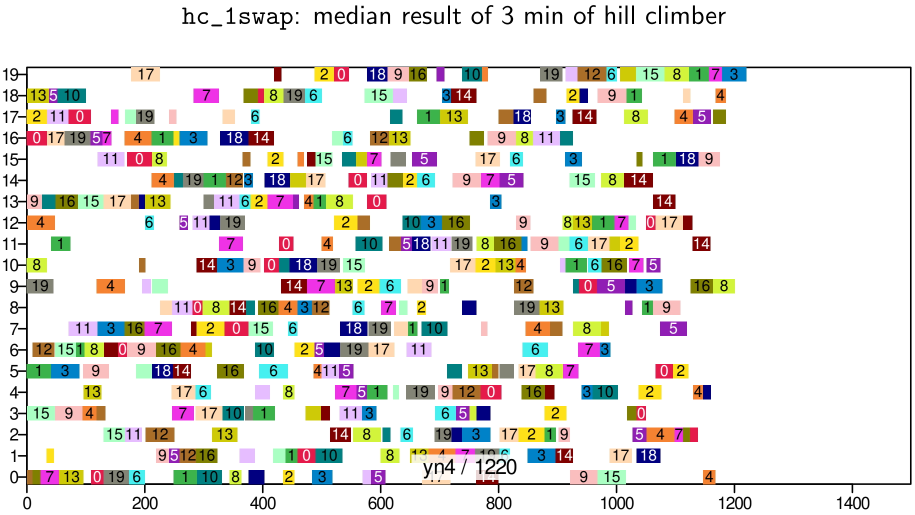

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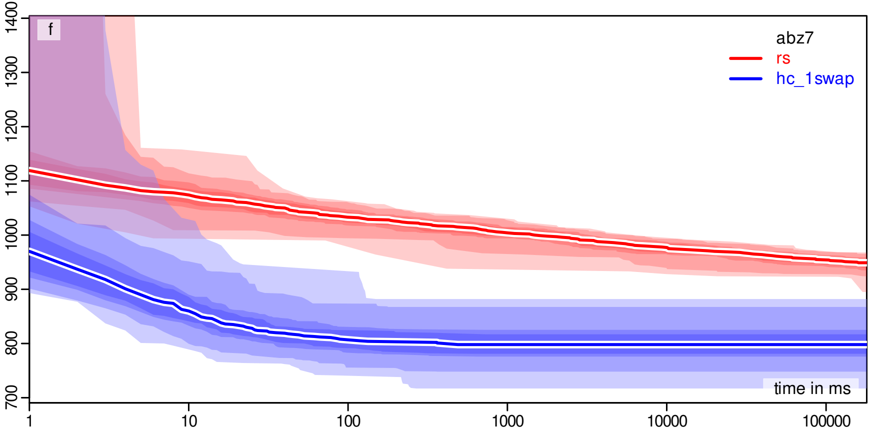

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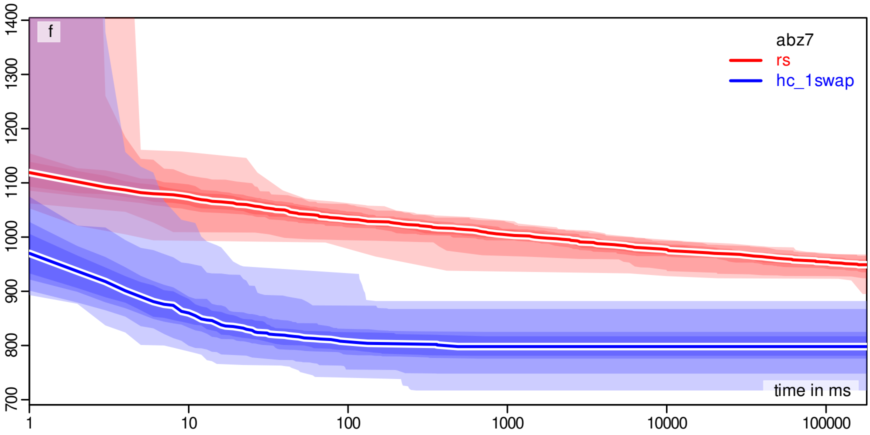

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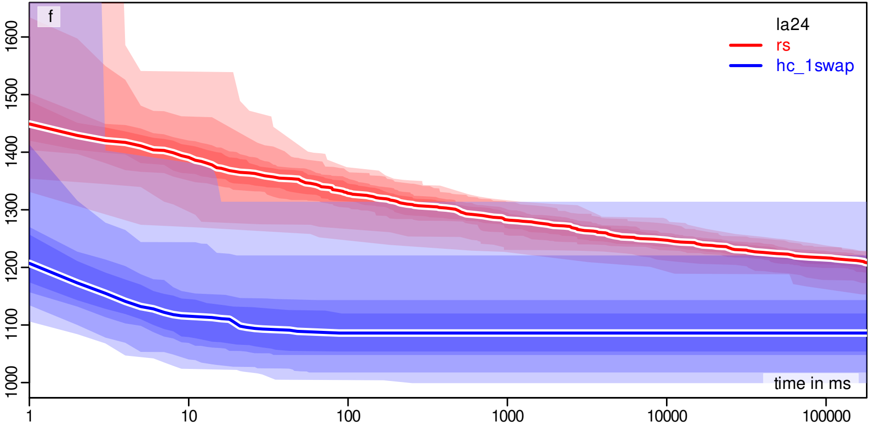

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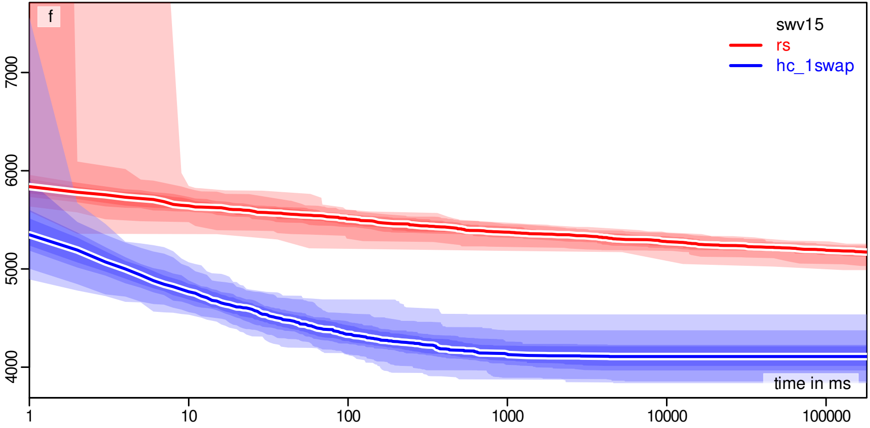

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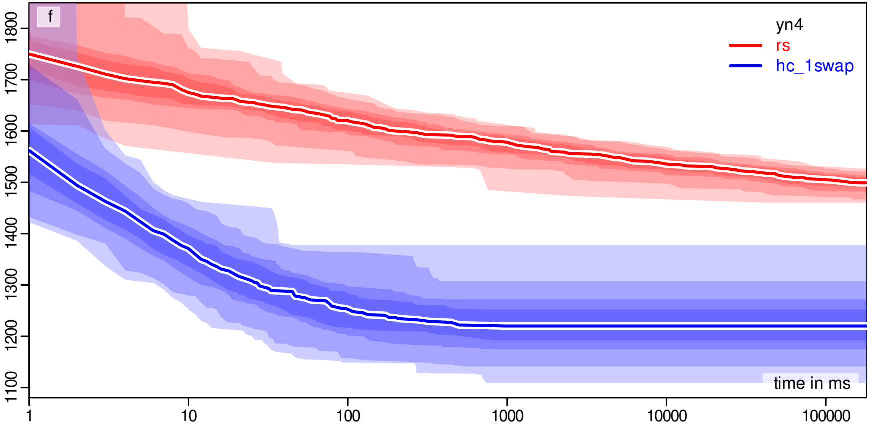

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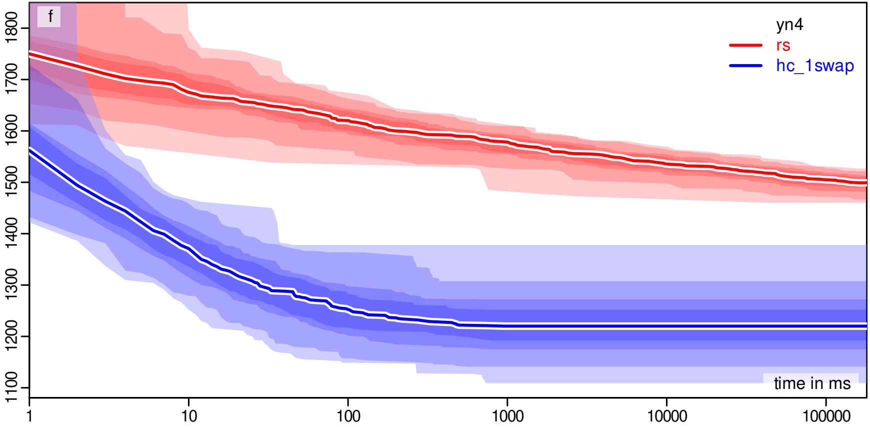

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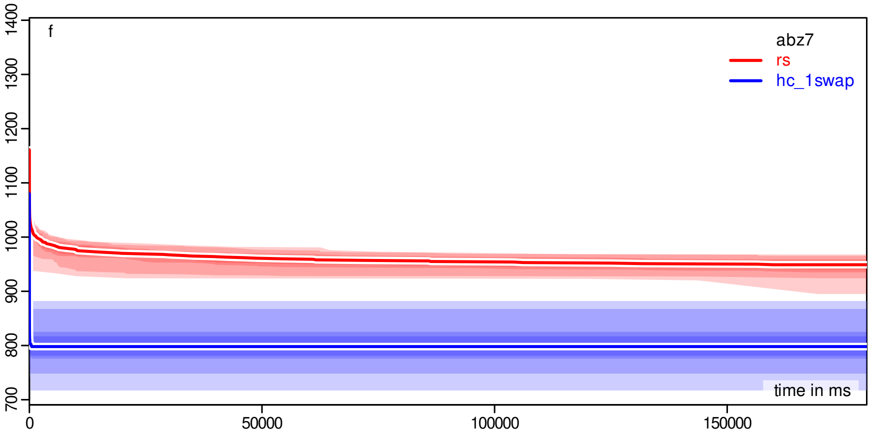

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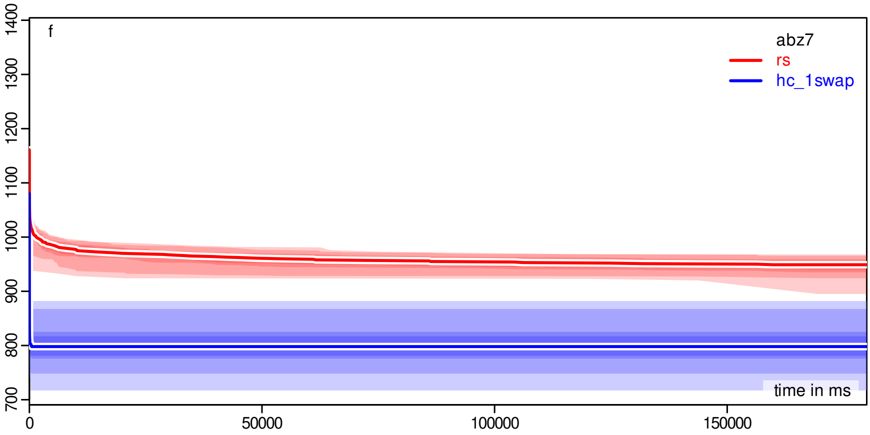

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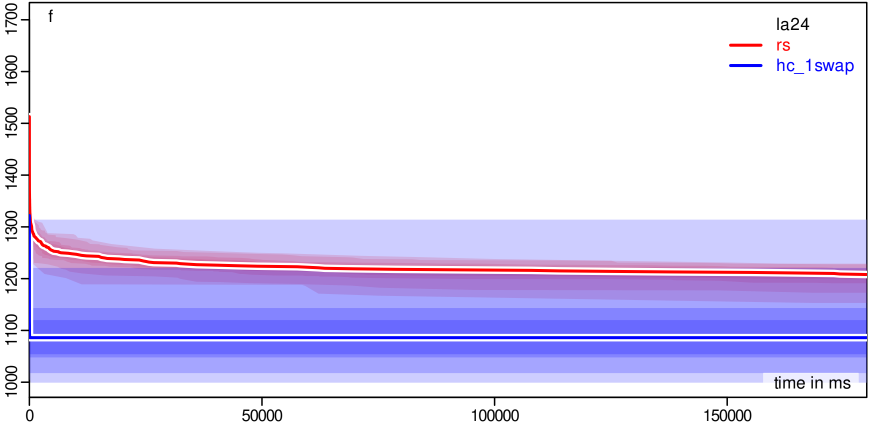

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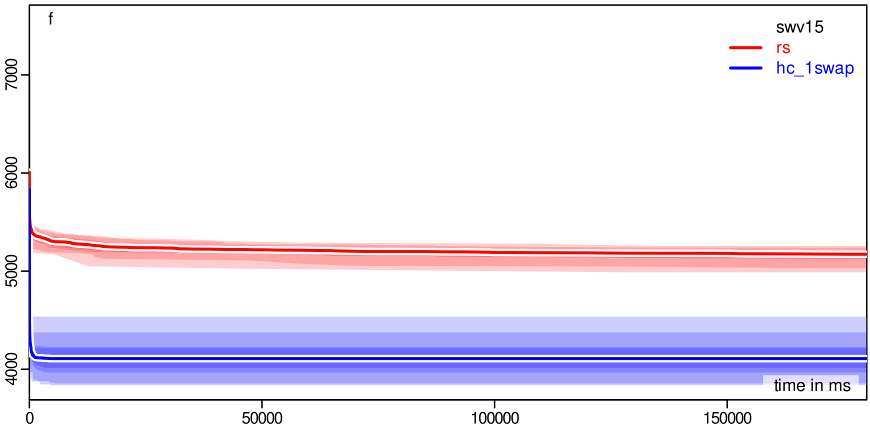

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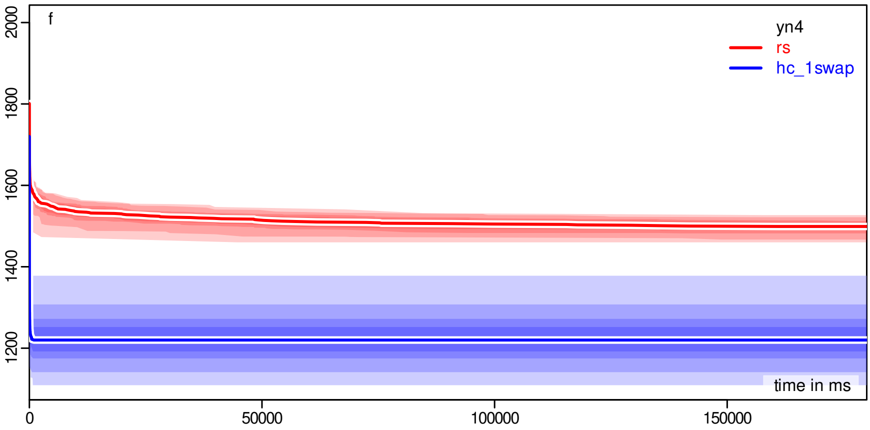

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- We have three minutes but after about 1 second, our `hc_1swap` algorithm stops improving!

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- This is called Premature Convergence.[8 9]

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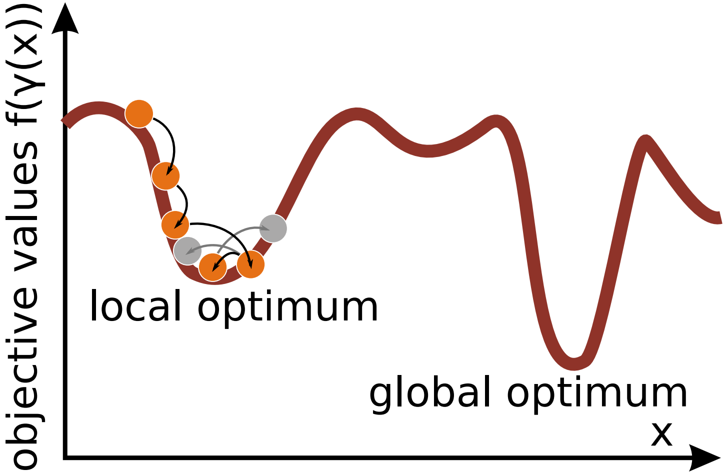

Improved Algorithm Concept 1

Stochastic Hill Climber with Restarts

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- Of course, we will always remember the overall best solution we ever had (in another variable).

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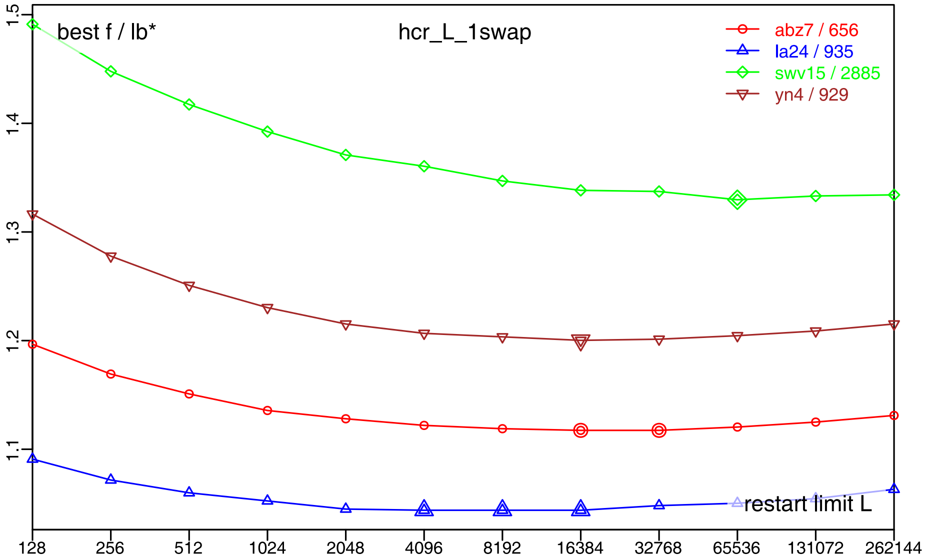

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- $L = 2^{14} = 16'384$ seems to be a reasonable choice.

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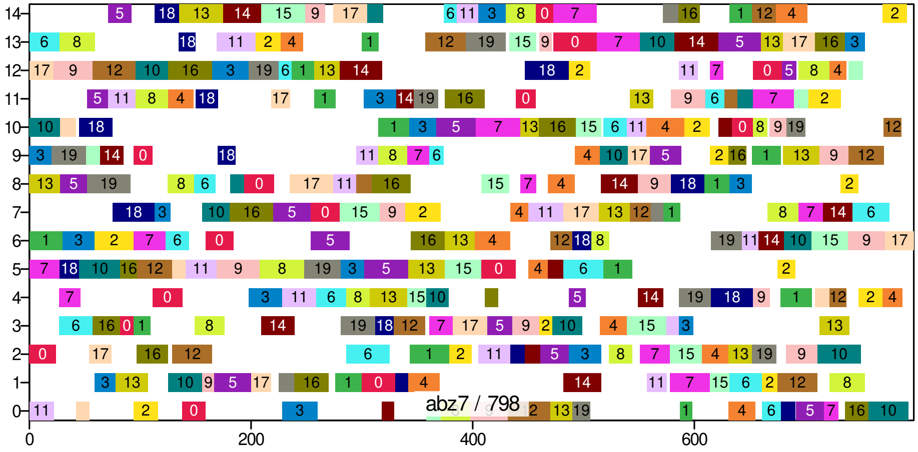
	hc_1swap	717	800	798	28	0s	16'978
	hcr_16384_1swap	**714**	**732**	**733**	**6**	91s	18'423'530
la24	rs	1153	1206	1208	15	82s	15'902'911
	hc_1swap	999	1095	1086	56	0s	6'612
	hcr_16384_1swap	**953**	**976**	**976**	**7**	80s	34'437'999
swv15	rs	4988	5166	5172	50	87s	5'559'124
	hc_1swap	3837	4108	4108	137	1s	104'598
	hcr_16384_1swap	**3752**	**3859**	**3861**	**42**	92s	11'756'497
yn4	rs	1460	1498	1499	15	76s	4'814'914
	hc_1swap	1109	1222	1220	48	0s	31'789
	hcr_16384_1swap	**1081**	**1115**	**1115**	**11**	91s	14'804'358

So what do we get?

hc_1swap: median result of 3 min of hill climber



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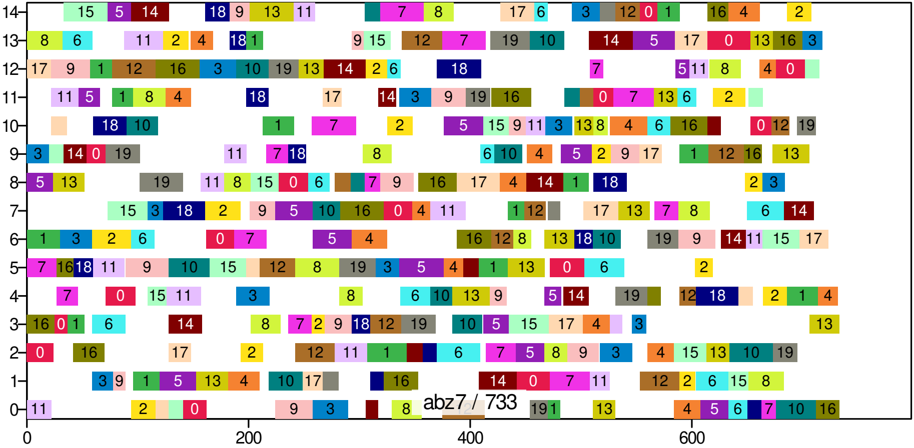
`hcr_16384_1swap`: median result of 3 min of hill climber which restarts after $L = 16'384$ search steps without improvement



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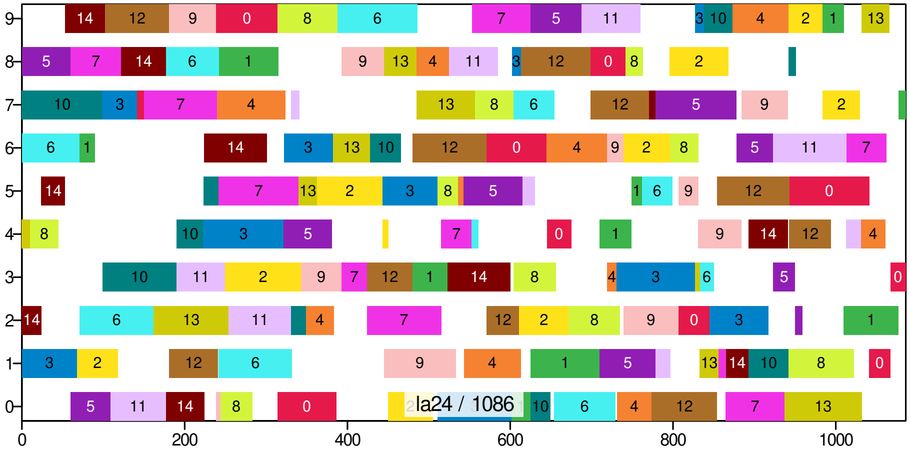

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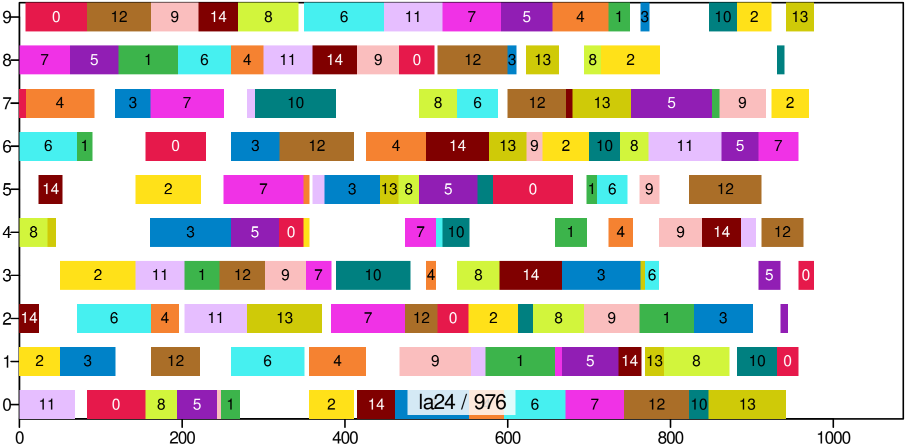

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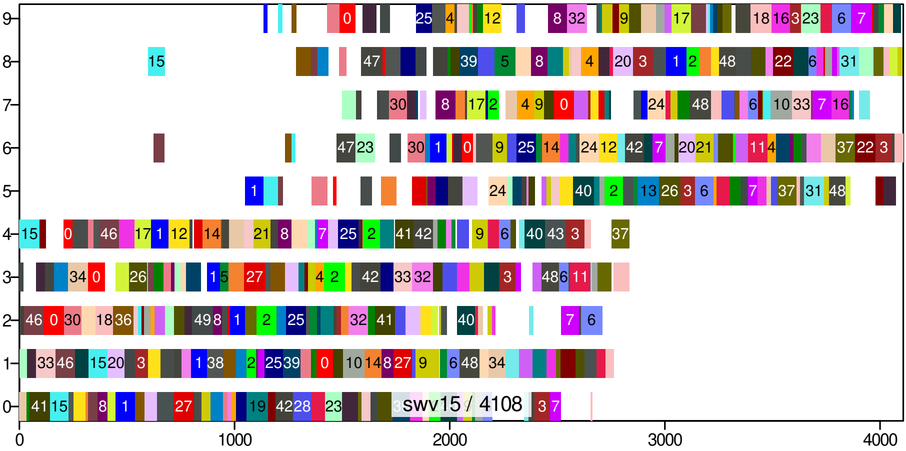

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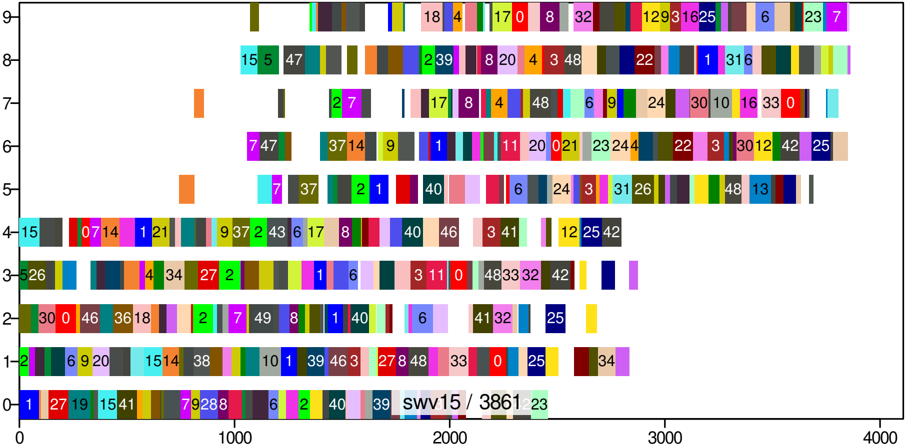

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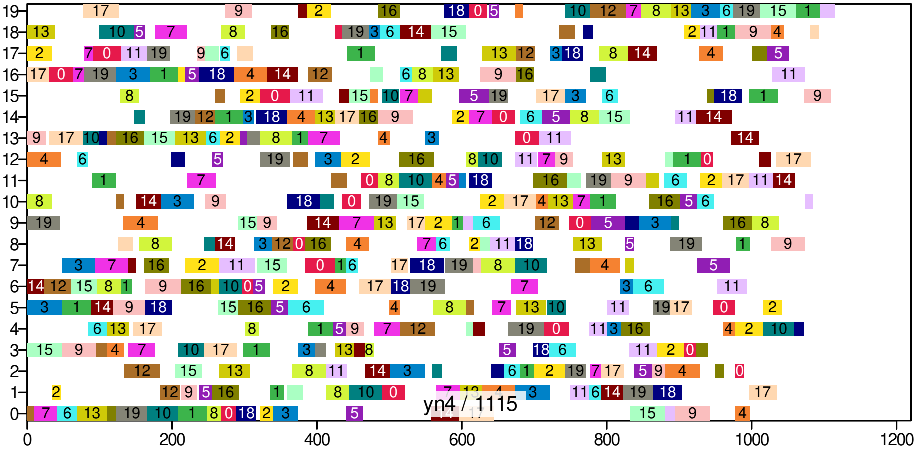

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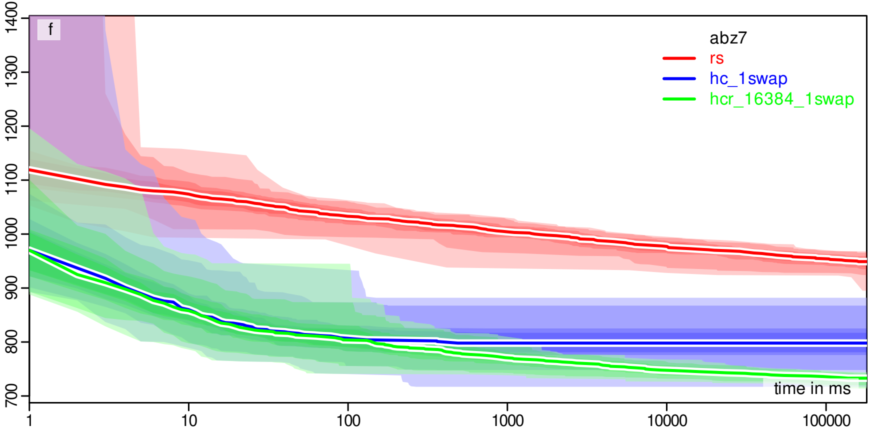

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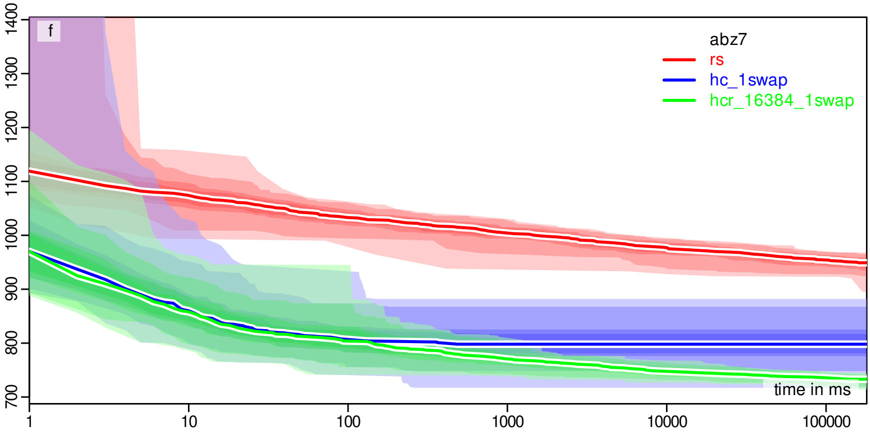

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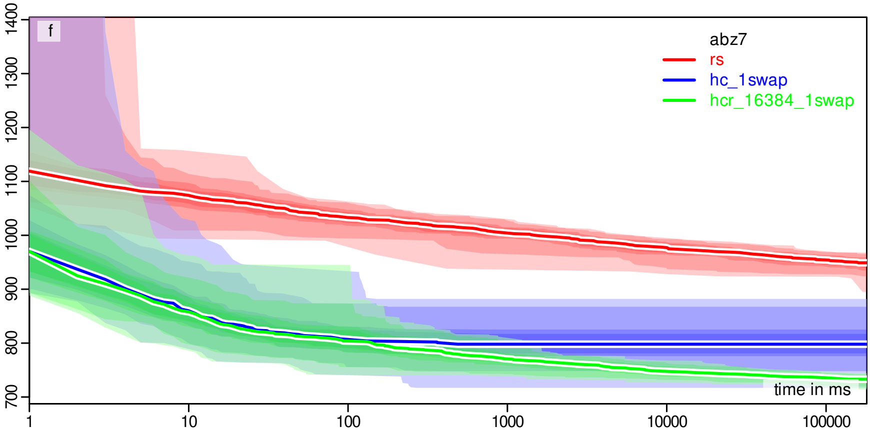

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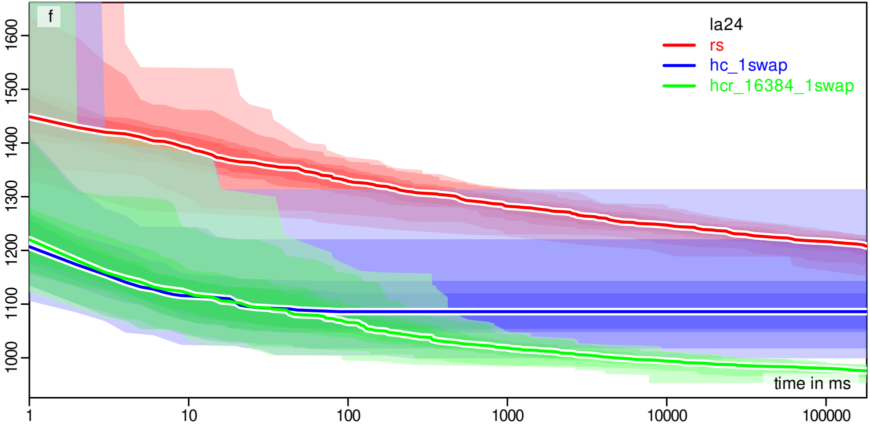

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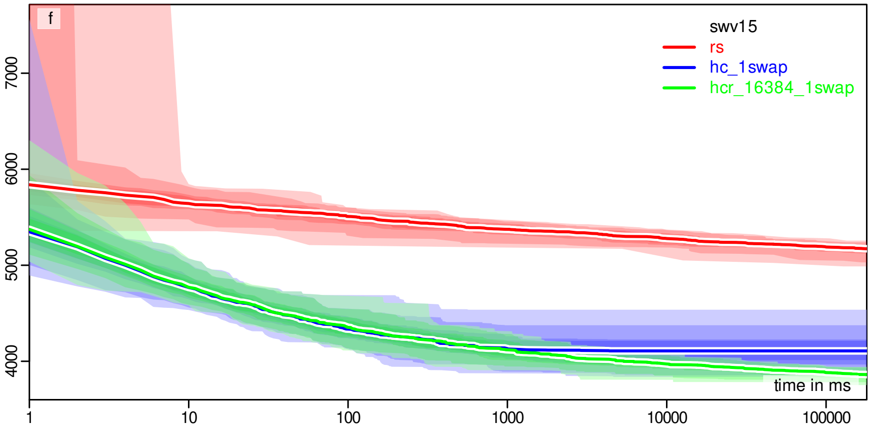

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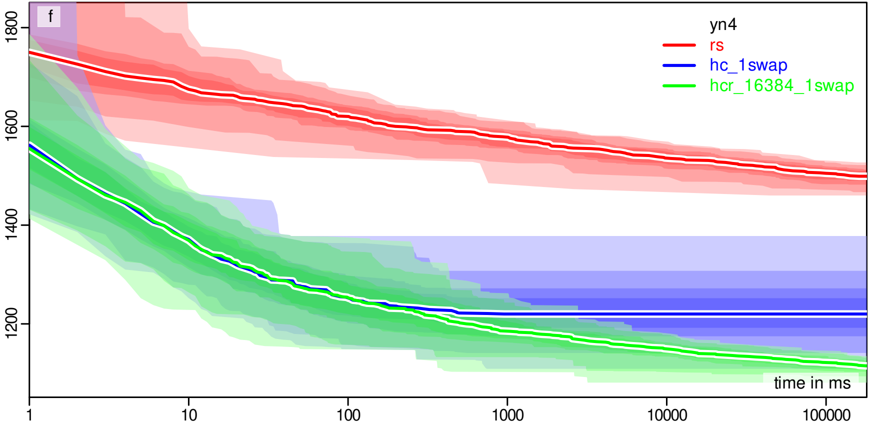

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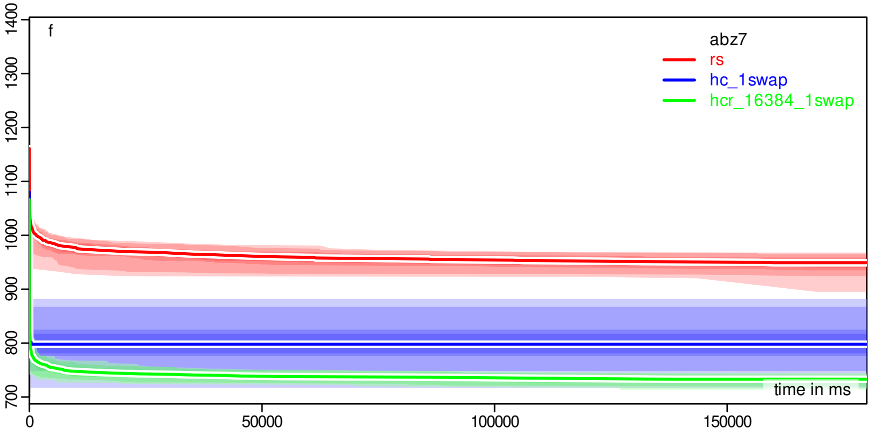

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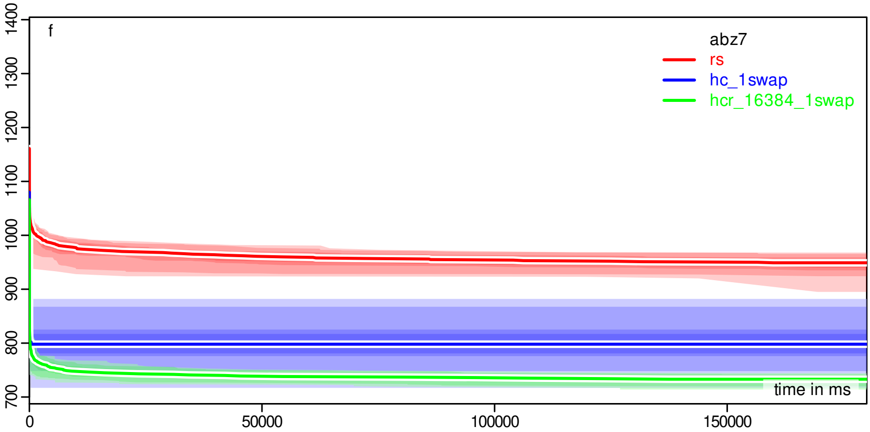

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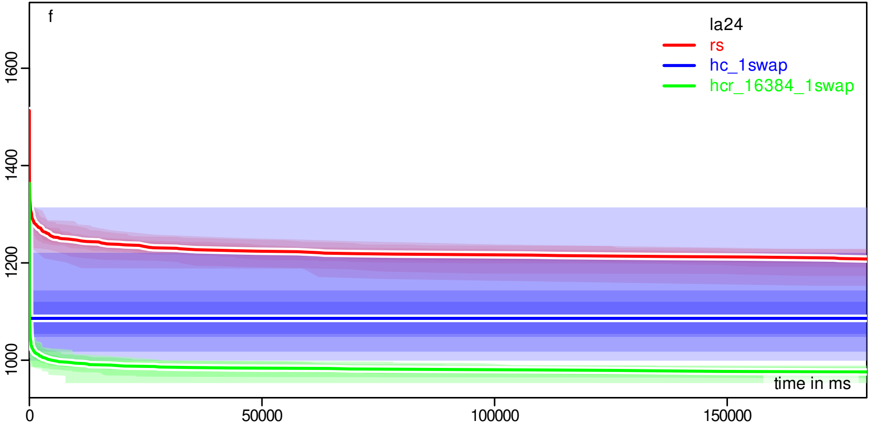

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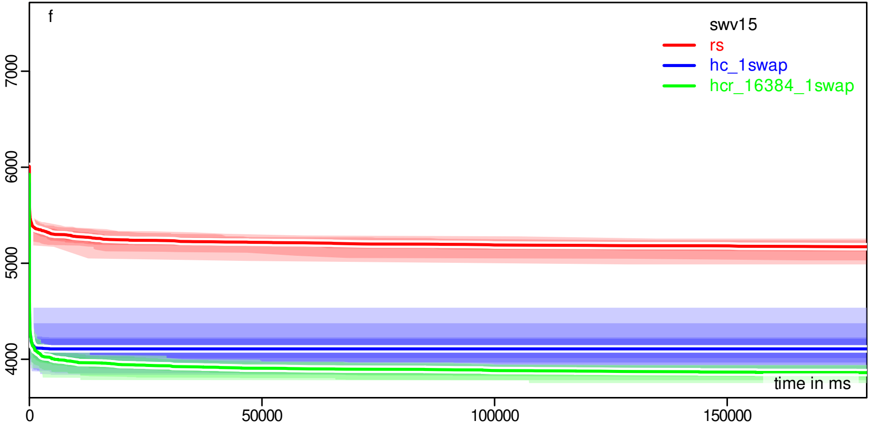

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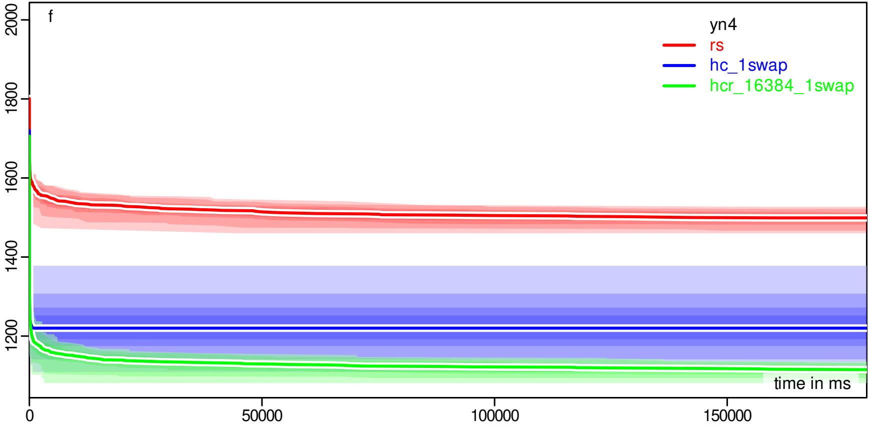

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Improved Algorithm Concept 2

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- Another problem is: With every restart we throw away all accumulated knowledge and information of the current run.
- Restarts are therefore also wasteful.

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- Notice: Whether or not a point x is a local optimum, is determined entirely by the unary search operator!
- If we had a different operator with a bigger neighborhood, then maybe $x^{\times}$ would no longer be a local optimum and we could still improve the results after reaching it...

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- On the other end of the spectrum, we could simply swap all jobs in our points x randomly. Is this a good idea?Probably not: It would turn our algorithm into random sampling!
- We should respect the causality: small changes to the solution cause small changes in the objective value – big changes will lead to unpredictable results.

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- Theoretically, we could always escape from any local optimum.

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- Theoretically, we could always escape from any local optimum, but the probability may sometimes be very very small.

Experiment and Analysis

So what do we get?

- I execute the program 101 times for each of the instances `abz7`, `la24`, `swv15`, and `yn4`

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	hcr_16384_1swap	3752	**3859**	**3861**	**42**	92s	11'756'497
	hc_nswap	**3602**	3880	3872	112	70s	8'351'112
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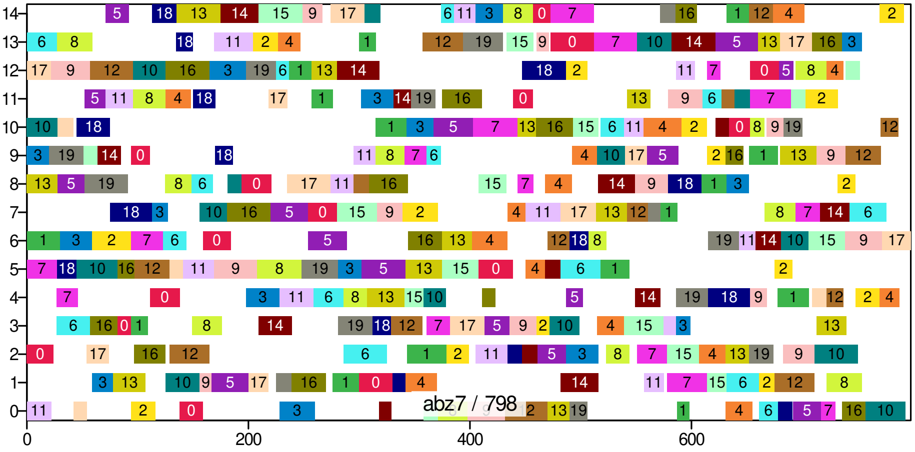
`hc_1swap`: median result of 3 min of hill climber using `1swap`



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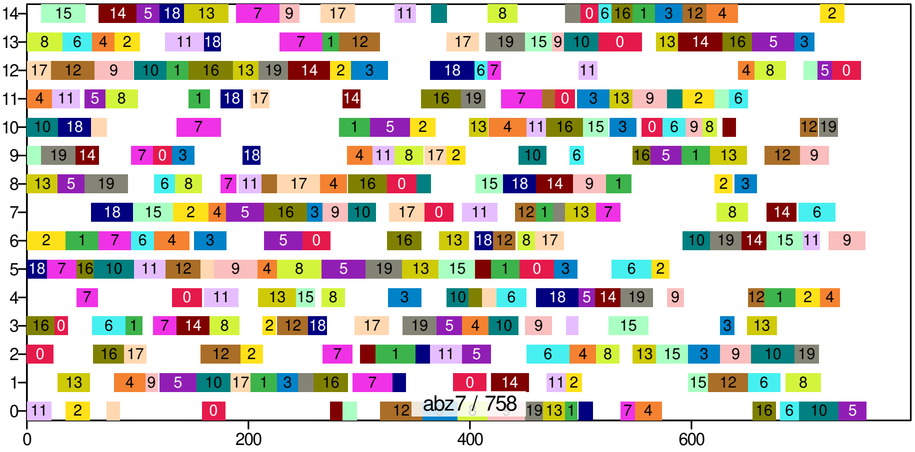

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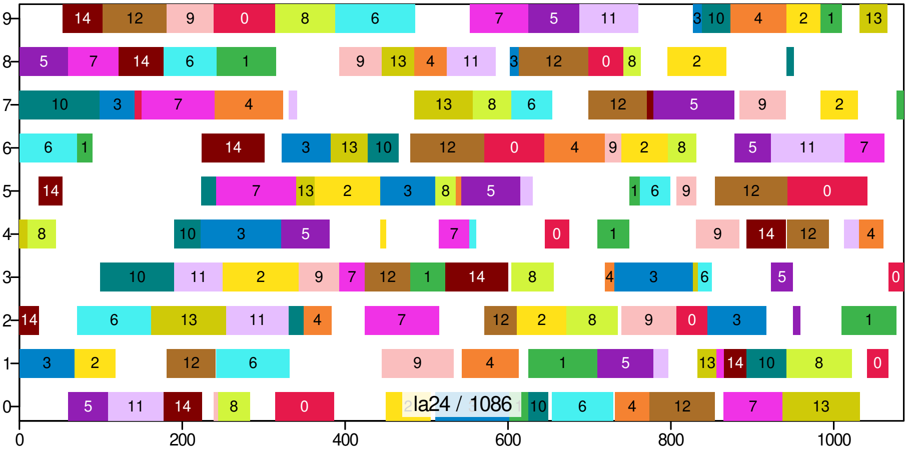

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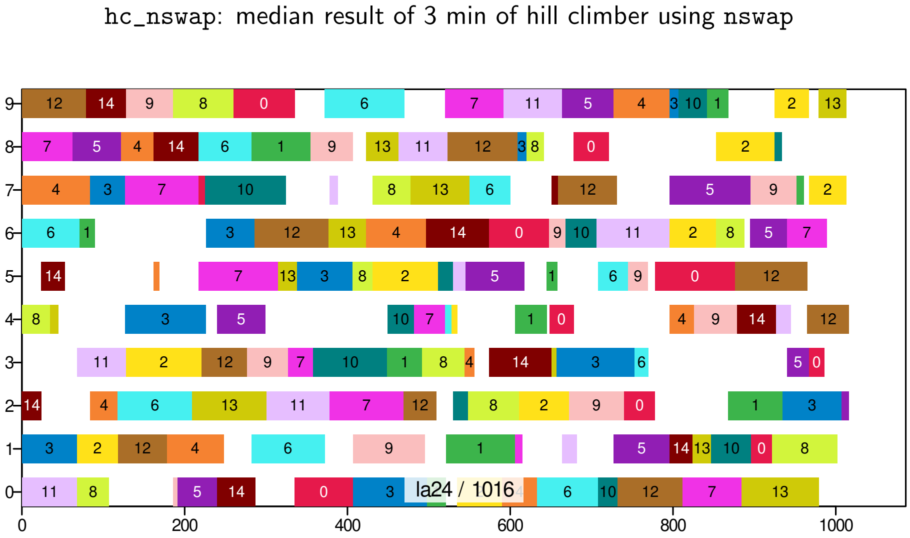

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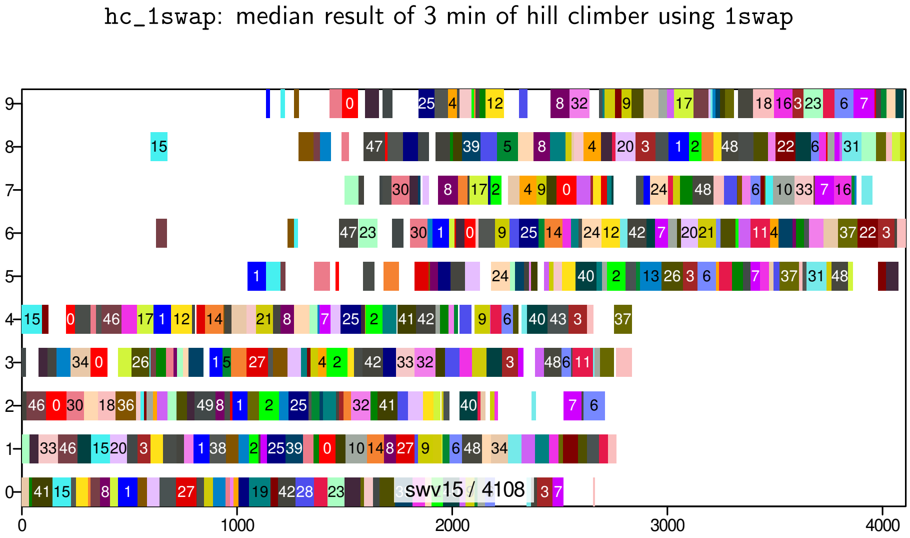

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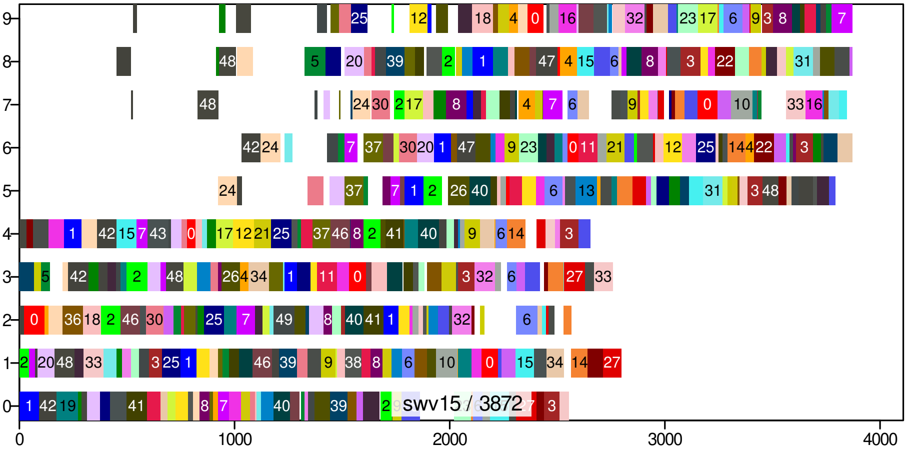

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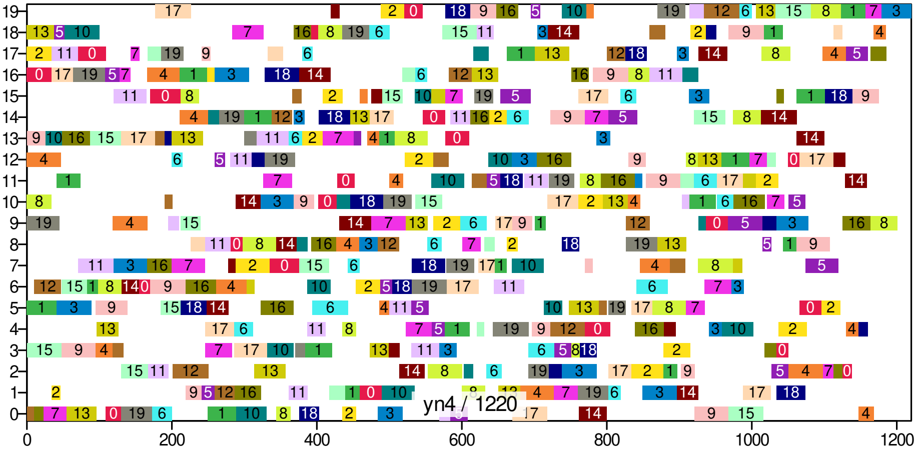

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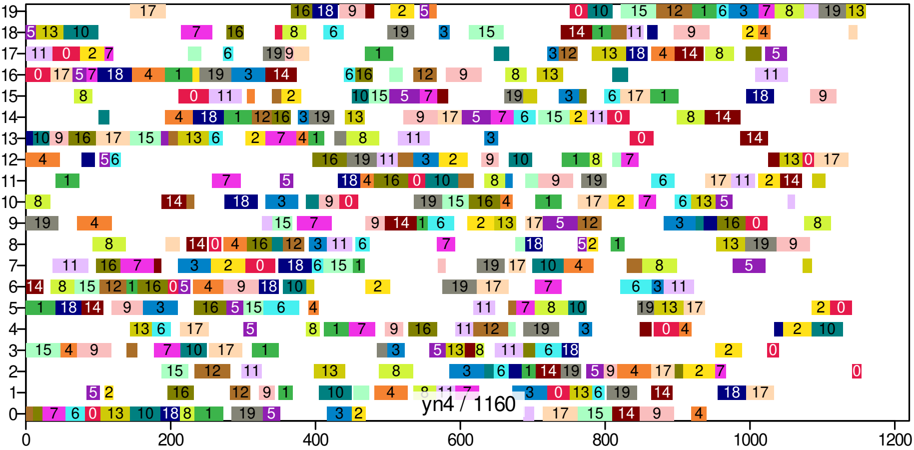

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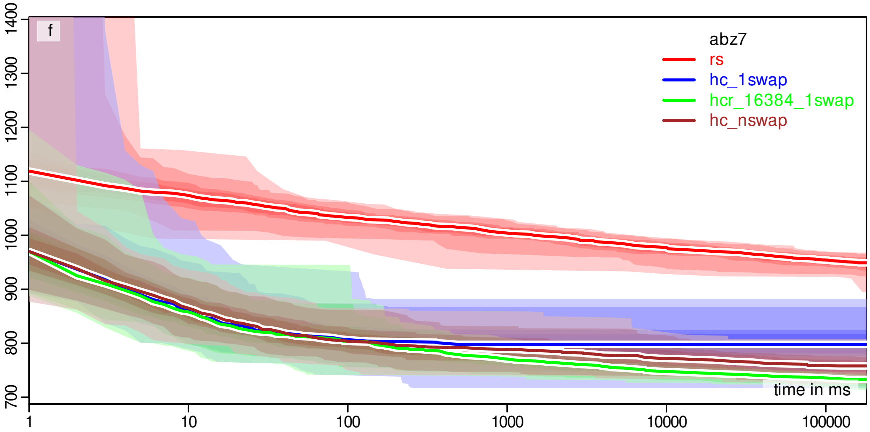

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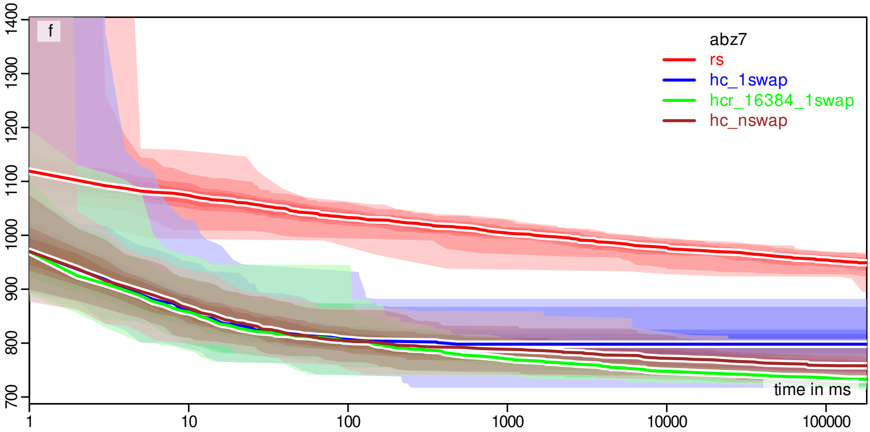

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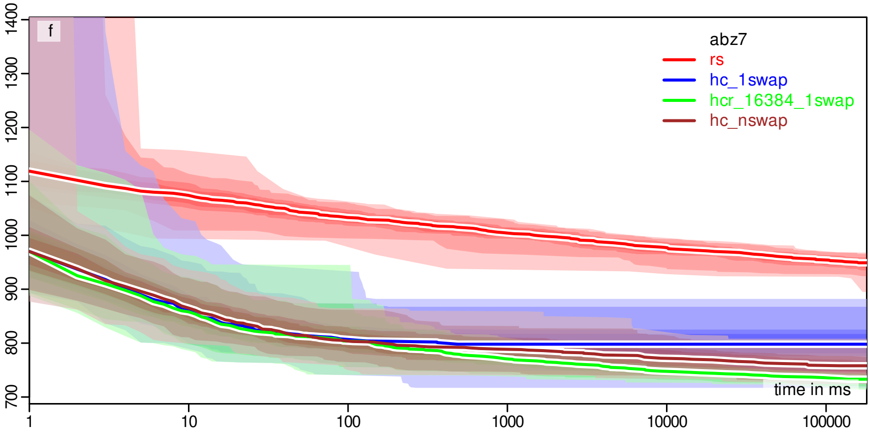
- `hc_nswap` first behaves like `hc_1swap`, because most of the `nswap` moves are the same as `1swap` moves.

Progress over Time



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- The rare larger moves allow it to escape from local optima that would trap `hc_1swap`.

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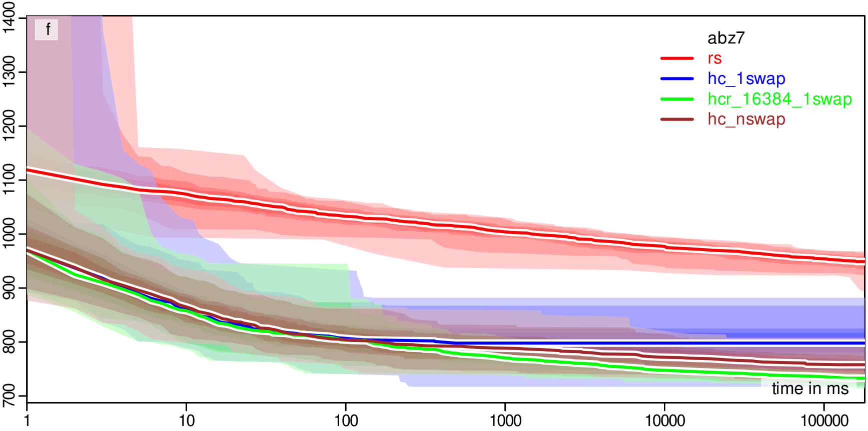

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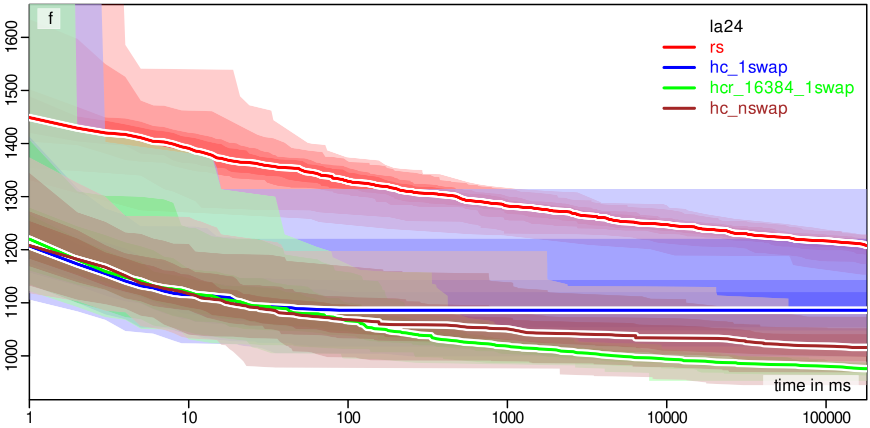

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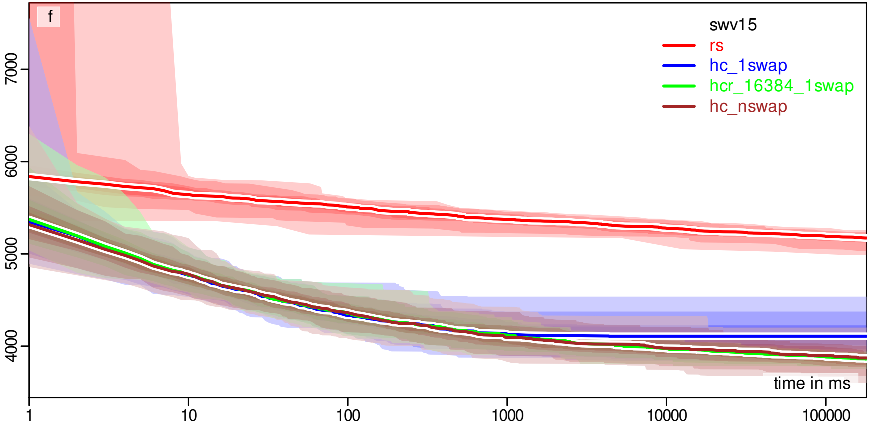

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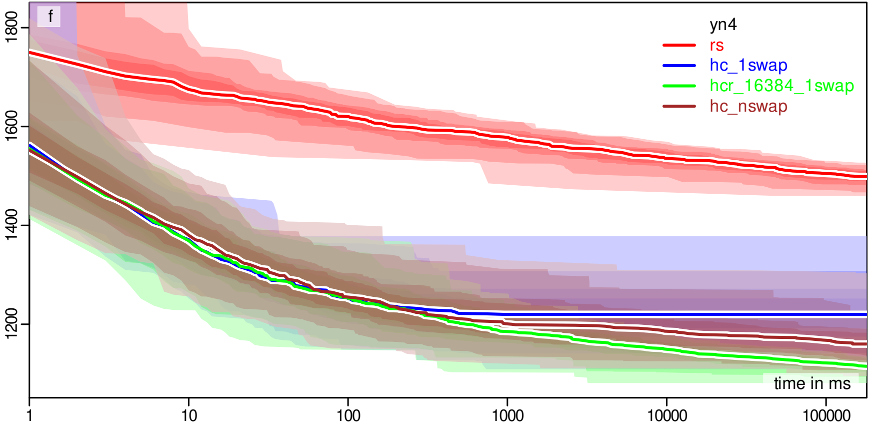

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- Now we know two ways to improve the performance of our hill climber:
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  2. we can use a unary operator with larger neighborhood that still mostly makes small steps.
- It is only natural to try to combine these two improvements.

Configuring the Algorithm

		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	hc_1swap	717	800	798	28	0s	16'978
	hc_nswap	724	758	758	17	35s	7'781'762
la24	hc_1swap	999	1095	1086	56	0s	6'612
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Configuring the Algorithm

- The `hc_nswap` improves longer than `hc_1swap`
- We can expect that the number L of unsuccessful steps before a restart should be higher now.

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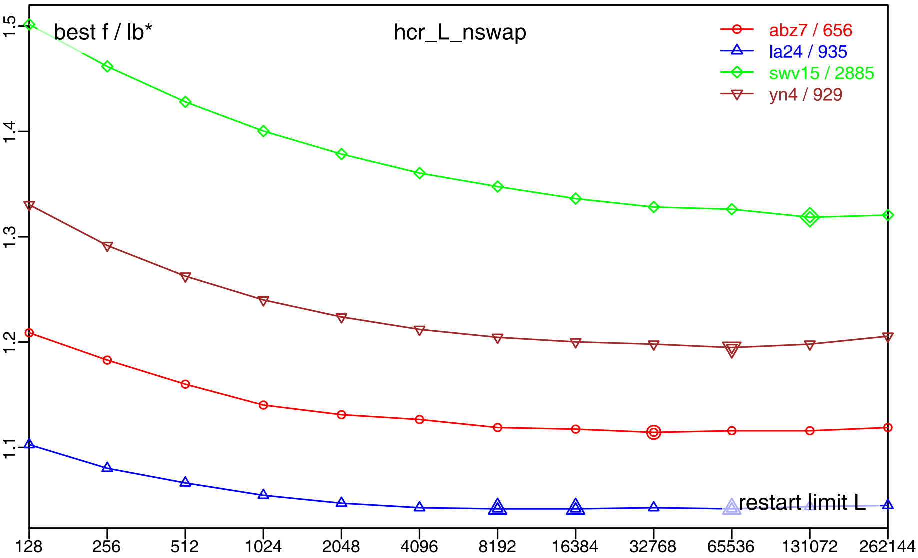

Configuring the Algorithm

- The hc_nswap improves longer than hc_1swap
- We can expect that the number L of unsuccessful steps before a restart should be higher now.
- Let's choose $L = 65'536$, i.e., hcr_65536_nswap.

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abz7	hc_1swap	717	800	798	28	0s	16'978
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	hcr_65536_nswap	**712**	**731**	**732**	6	96s	21'189'358
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	hc_nswap	945	1018	1016	29	25s	9'072'935
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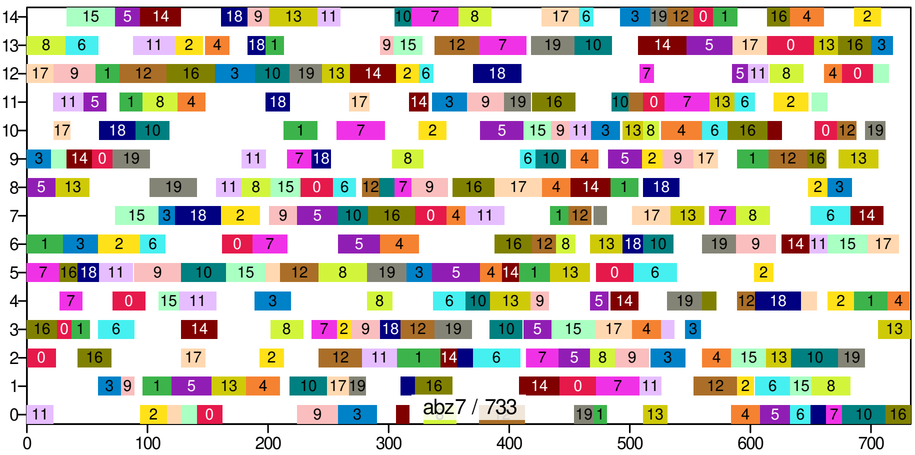
hcr_16384_1swap: median result of 3 min of hcr_16384_1swap



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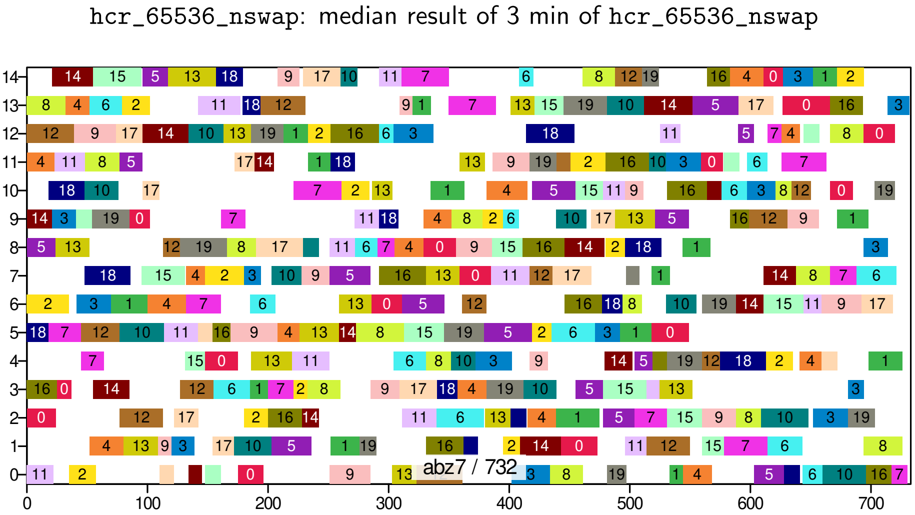

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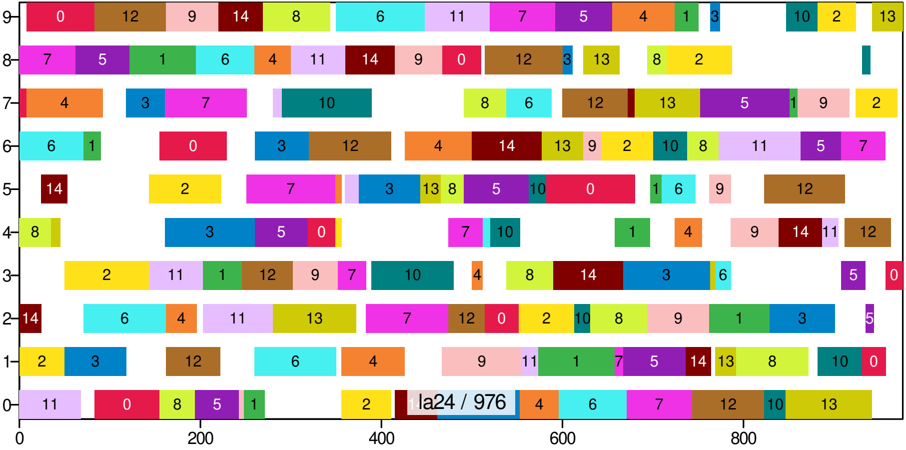

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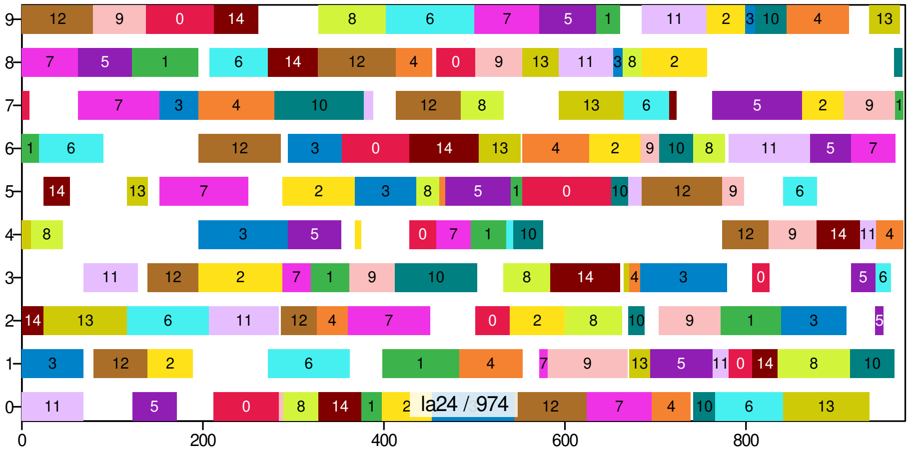

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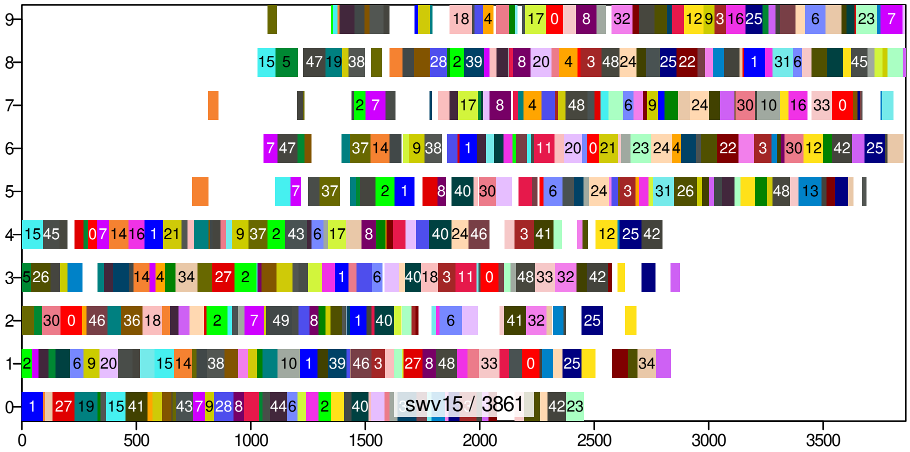

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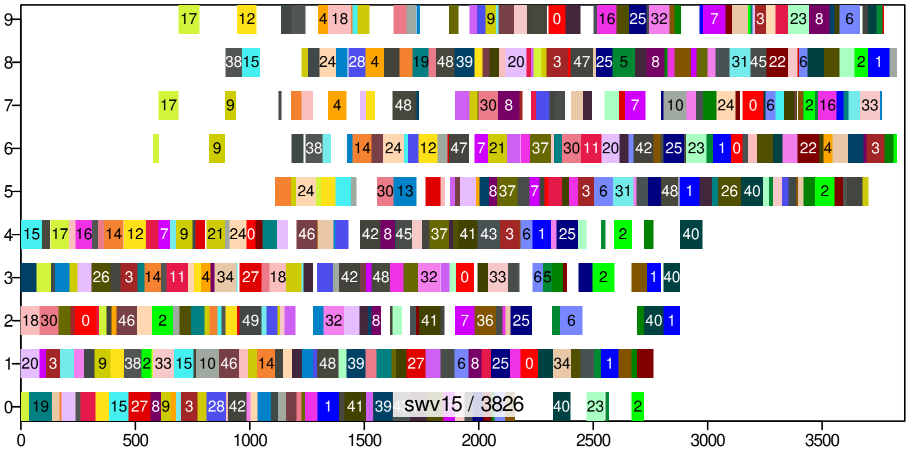

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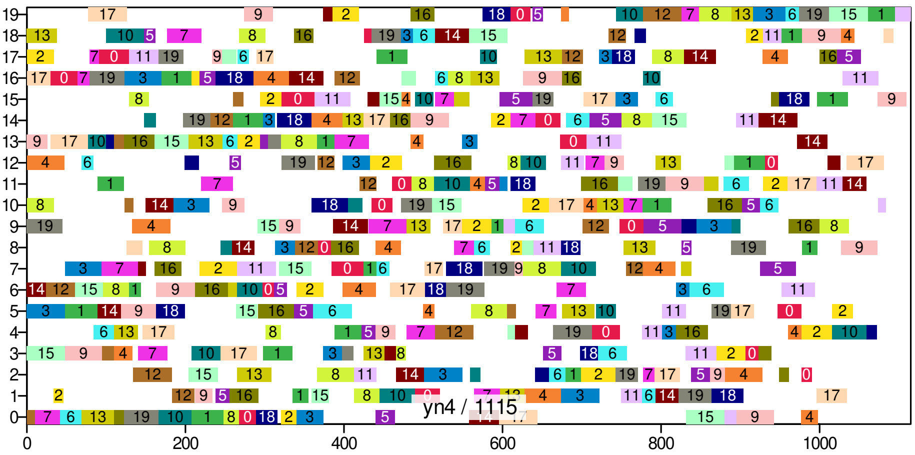

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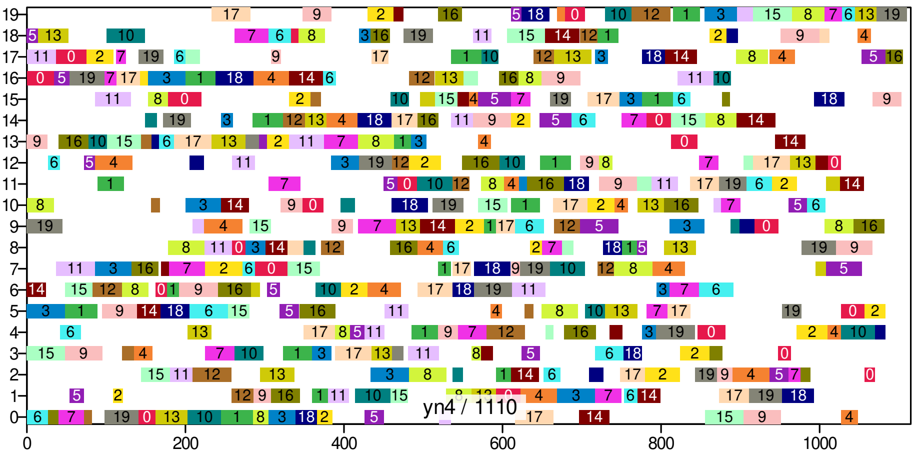

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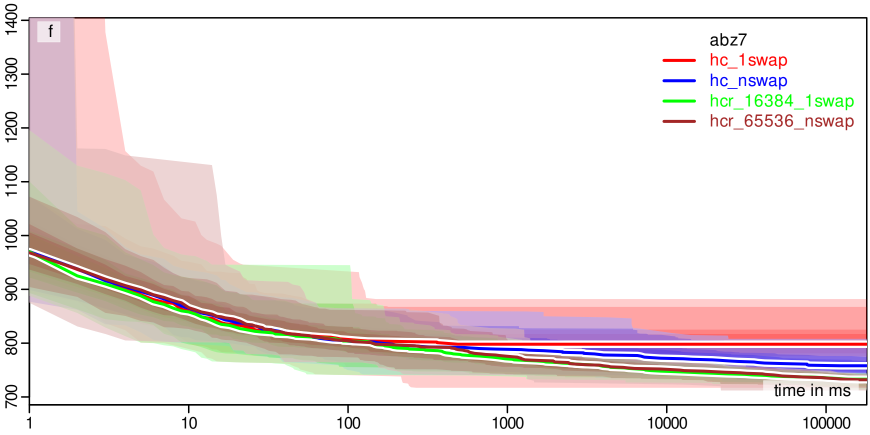

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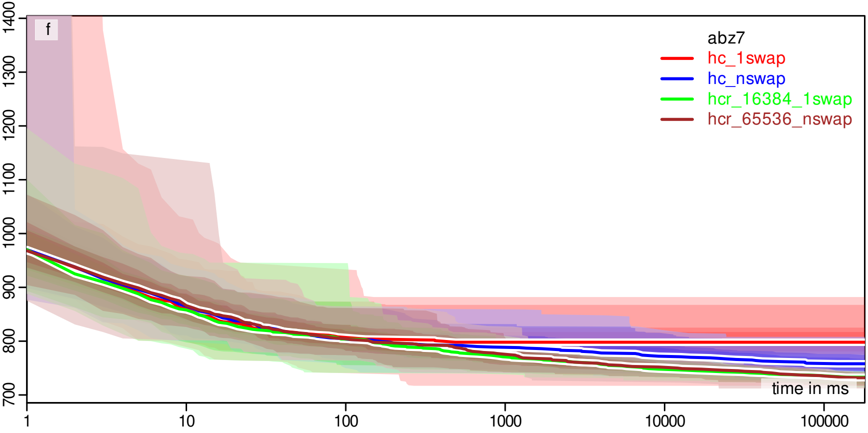

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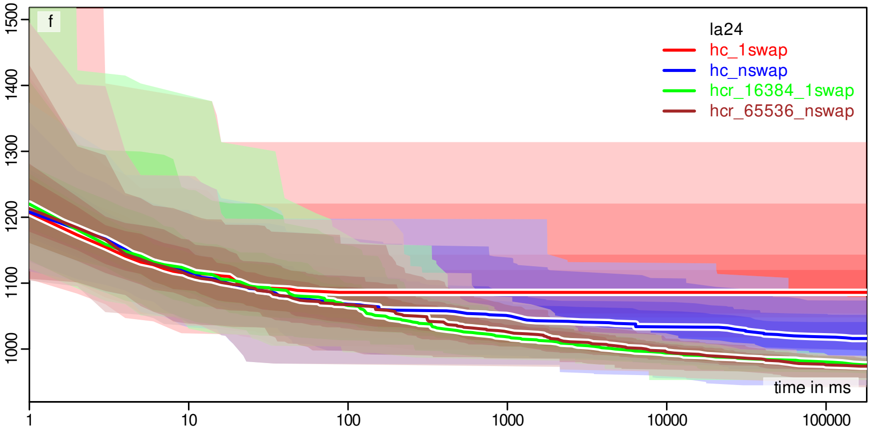

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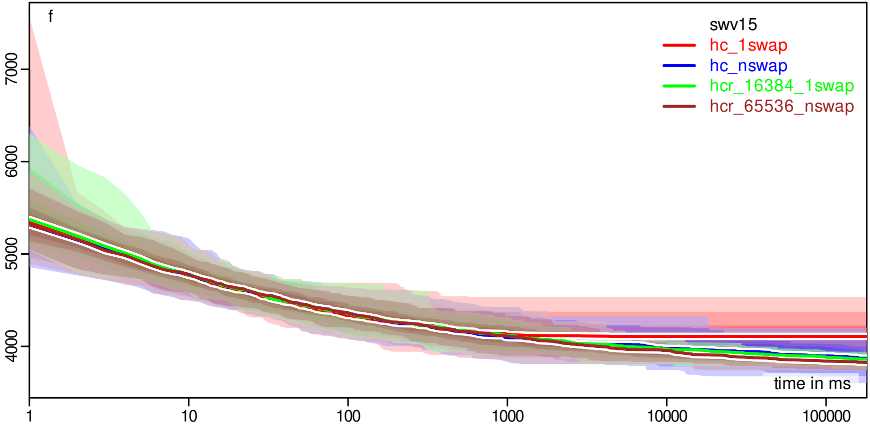

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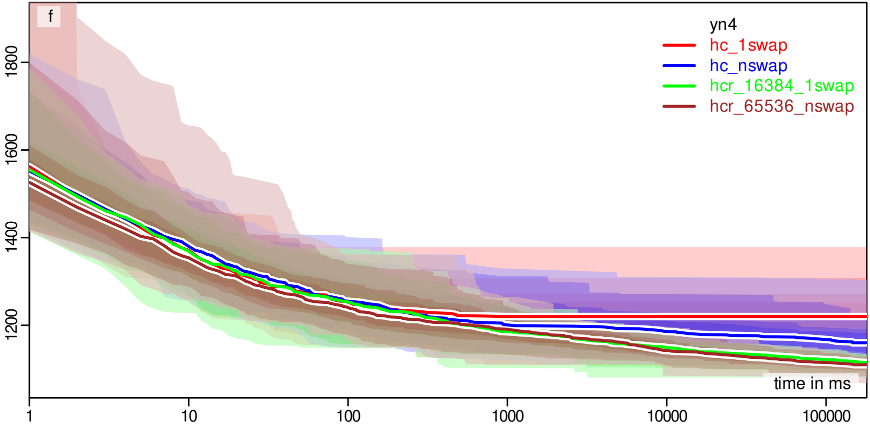

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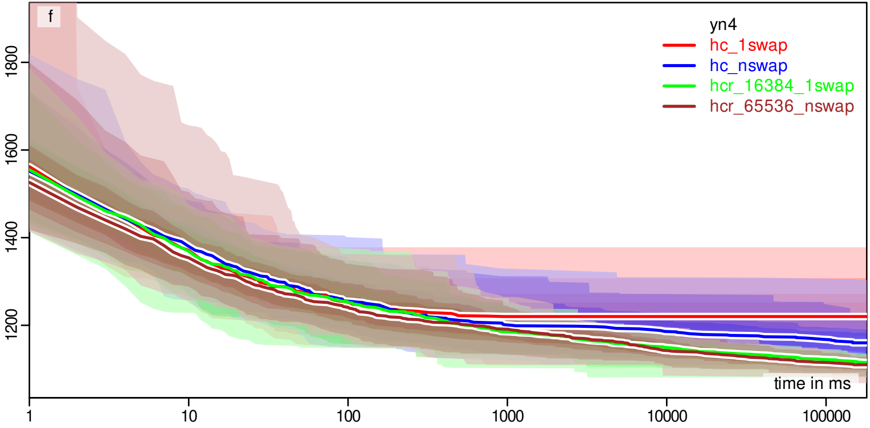

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