Yeditepe University

CSE480/591 Special Topics: Optimization with Metaheuristics

Quiz 1 Solutions

Date: 17 October 2025 11:20 - 11:50 (30 mins)

Question 1 (50 points) Solve only one option: Option A or Option B

Option A for Question 1

A chemical plant must produce 250 kg of a cleaning compound by mixing two ingredients, X and Y. The profit per kilogram is 5 TL for X and 7 TL for Y. At least 80 kg of X must be used. At most 120 kg of Y can be used. Additionally, the amount of Y cannot exceed 1.5 times the amount of X. How many kilograms of X and Y should be used to maximize total profit?

(a) Define the decision variables.
Let X and Y ingredients be the decision variables of the problem.

(b) Formulate the constraints and any necessary non-negativity conditions.

$$\begin{aligned} X + Y &= 250 && \text{(total mix)} \\ X &\geq 80 && \text{(min. usage of } X\text{)} \\ Y &\leq 120 && \text{(max. usage of } Y\text{)} \\ Y &\leq 1.5\,X && \text{(process rule)} \\ X, Y &\geq 0 && \text{(non-negativity)} \end{aligned}$$

(c) Write the objective function to be maximized (in TL).

$$\max\ 5X + 7$$

(d) Identify the feasible corner points (extreme points) of the solution space and list these values of the decision variables.
The relevant intersections are:

Point	How obtained	Feasible?	Reason
$(130, 120)$	$Y = 120 \ \wedge \ X + Y = 250$	**Yes**	All constraints satisfied
$(250, 0)$	$Y = 0 \ \wedge \ X + Y = 250$	**Yes**	All constraints satisfied
$(80, 170)$	$X = 80 \ \wedge \ X + Y = 250$	**No**	$Y = 170 > 120$ and $Y > 1.5X = 120$
$(100, 150)$	$Y = 1.5X \ \wedge \ X + Y = 250$	**No**	$Y = 150 > 120$
$(0, 250)$	$X = 0 \ \wedge \ X + Y = 250$	**No**	$X < 80$

Using $Y = 250 - X$, the constraints imply:

$$X \geq 80 \ \Rightarrow\ Y \leq 170, \qquad Y \leq 120 \ \Rightarrow\ X \geq 130, \qquad Y \leq 1.5X \ \Rightarrow\ 250 - X \leq 1.5X \Rightarrow X \geq 100.$$

Thus X must satisfy $X \geq \max\{80, 100, 130\} = 130$ and $X \leq 250$ (since $Y \geq 0$). Hence the feasible segment on $Y = 250 - X$ has the *two* corner points:

$$(130, 120) \quad \text{and} \quad (250, 0).$$

.

(e) Compute the total profit at each corner point, then report the optimal solution by stating the corresponding decision variable values and the maximum profit (in TL).

$$(130, 120) : 5 \cdot 130 + 7 \cdot 120 = 650 + 840 = \mathbf{1490} \text{ TL},$$
$$(250, 0) : 5 \cdot 250 + 7 \cdot 0 = 1250 \text{ TL}.$$

Optimal mix: $X^\star = 130\,\text{kg},\ Y^\star = 120\,\text{kg}$ with $\boxed{1490 \text{ TL}}$.

Option B for Question 1

Definition of Graph Coloring Problem For a graph $G = (V, E)$, a function

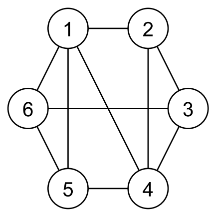
$$c : V \to \{1, 2, \ldots, k\}$$

assigns a color to each vertex $v \in V$ such that for every edge $(u, v) \in E$, the condition $c(u) \neq c(v)$ holds. The smallest value of k for which this condition is satisfied is called the *chromatic number* of the graph and is denoted by $\chi(G)$.

(a) Define the decision variables. Check the Week 1 lecture slide.

(b) Formulate the constraints. Check the Week 1 lecture slide.

(c) Write the objective function of the problem. Check the Week 1 lecture slide.



(d) Using your formulation, solve the given graph and report the value of the objective function.
Let A, B, and C be different colors.

$$c(1) = A, \quad c(2) = B, \quad c(3) = A, \quad c(4) = C, \quad c(5) = B, \quad c(6) = C,$$

so the instance is 3-colorable.

Question 2 (50 points)

We have a shoe factory which produces three product lines—*sneakers* (J_1), *boots* (J_2), and *loafers* (J_3)—on three machines. Each job (shoe type) must be processed on a job-specific ordered sequence of machines, and each operation has a job-specific processing time on its required machine. At any time, each machine can process at most one operation, and a job can be processed by at most one machine at a time. Processing is non-preemptive. The goal is to finish all work as quickly as possible (minimize the makespan).

The three machines are responsible for the following operations:

$$M_0 = \text{Cutting}, \quad M_1 = \text{Stitching}, \quad M_2 = \text{Finishing}.$$

Processing routes and times (minutes):

$$\begin{array}{ll} \textbf{Sneakers } (J_1): & M_0\,(10) \rightarrow M_2\,(5) \rightarrow M_1\,(15) \\ \textbf{Boots } (J_2): & M_1\,(12) \rightarrow M_0\,(8) \rightarrow M_2\,(14) \\ \textbf{Loafers } (J_3): & M_2\,(9) \rightarrow M_1\,(7) \rightarrow M_0\,(11) \end{array}$$

Our aim is to minimize the makespan (completion time of the last finished operation).

(a) **Create the problem instance** I formally from the scenario (list machines, jobs, ordered machine sequences, and processing times).

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++++++++++++++++++++++++++++++
instance Shoe with 3 jobs and 3 machines
3 3
0 10  2 5   1 15
1 12  0 8   2 14
2  9  1 7   0 11
++++++++++++++++++++++++++++++
```

Each line after the two header lines corresponds to one job (in job-index order). On each job line, three *(machine_id, processing_time)* pairs specify the required machine and its processing time for that job.

(b) **Construct a feasible schedule** for this instance and show it using a *Gantt chart* (machine-wise timelines with start/finish times). A hand-drawn Gantt chart is acceptable.

$$\begin{array}{l|l} M_1 & \boxed{J_2: 0-12}\ \boxed{J_3: 12-19}\ \boxed{J_1: 19-34} \\ M_0 & \boxed{J_1: 0-10}\ \boxed{\text{idle}: 10-12}\ \boxed{J_2: 12-20}\ \boxed{J_3: 20-31} \\ M_2 & \boxed{J_3: 0-9}\ \boxed{\text{idle}: 9-10}\ \boxed{J_1: 10-15}\ \boxed{\text{idle}: 15-20}\ \boxed{J_2: 20-34} \end{array}$$

(Any equivalent feasible schedule also earns full credit.)

(c) For your feasible schedule in (b), **state the makespan** (numerical value) and indicate which operation completes last.

The last completion is at time 34 (both J_1 on M_1 and J_2 on M_2 finish at $t = 34$). Since this equals the lower bound, the schedule is **optimal**: $\boxed{C_{\max} = 34}$. (Finding an optimal schedule is *not required* to earn full credit.)