1. Eq. de Maxwell

10/07/24

1)
$$\oint_l \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \underbrace{\iint_S \vec{B} \cdot \vec{dS}}_{\phi_m}$$

ou

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

2)
$$\oint_l \vec{B} \cdot \vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t}$$

ou

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

3)
$$\oiint_S \vec{B} \cdot \vec{dS} = 0$$

ou

$$\vec{\nabla} \cdot \vec{B} = 0$$

4)
$$\oiint_S \vec{D} \cdot \vec{dS} = q_{en.}$$

ou

$$\vec{\nabla} \cdot \vec{E} = \rho_v / \varepsilon_0$$

PARA TODAS AS DEDUÇÕES:

$\vec{\nabla} A \rightarrow$ GRADIENTE DE A

$$\partial_x A \, \hat{a}_x + \partial_y A \, \hat{a}_y + \partial_z A \, \hat{a}_z$$

$\vec{\nabla} \times \vec{A} \longrightarrow$ ROTACIONAL DE A

$$\left(\partial_x \hat{a}_x + \partial_y \hat{a}_y + \partial_z \hat{a}_z\right) \times \left(A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z\right)$$

$\vec{\nabla} \longrightarrow$ OPERADOR VETORIAL

$$\left(\partial_x \hat{a}_x + \partial_y \hat{a}_y + \partial_z \hat{a}_z\right)$$

$\nabla \longrightarrow$ OPERADOR ESCALAR

$$\partial_x + \partial_y + \partial_z$$

$\vec{\nabla} \cdot \vec{A} \longrightarrow$ DIVERGENTE

$$\partial_x A_x + \partial_y A_y + \partial_z A_z$$

$\nabla^2 \longrightarrow$ LAPLACIANO

$$\partial_x^2 + \partial_y^2 + \partial_z^2$$

Na forma fasorial:

Não temos densidade de corrente

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{H} = \cancel{\vec{J}} + \frac{\partial\vec{D}}{\partial t}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\vec{\nabla}\times\vec{E} = -j\omega\mu\vec{H} \quad (1) \qquad \vec{\nabla}\times\vec{H} = j\omega\varepsilon\vec{E} \quad (2)$$

$$\vec{\nabla}\times\vec{H} = j\omega\varepsilon\vec{E}$$

$$(1)\rightarrow(2) \qquad \vec{\nabla}\times\left(-\frac{\vec{\nabla}\times\vec{E}}{j\omega\mu}\right) = j\omega\varepsilon\vec{E}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) = \omega^2\mu\varepsilon\vec{E}$$

Em meio LHI

$$\nabla^2\vec{E} = \nabla(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}\times\vec{\nabla}\times\vec{E}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\nabla^2\vec{E} + \nabla(\cancel{\vec{\nabla}\cdot\vec{E}})^{0}$$

Logo, $$-\nabla^2\vec{E} = \omega^2\mu\varepsilon\vec{E}$$

$$\nabla^2\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0$$

Se $k^2 = \omega^2\mu\varepsilon$ então $k = \omega\sqrt{\mu\varepsilon}$

$$\nabla^2 \vec{E} + K^2 \vec{E} = 0 \quad ,$$

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

Portanto:

$$\begin{cases} \nabla^2 E_x + K^2 E_x = 0 \\ \nabla^2 E_y + K^2 E_y = 0 \\ \nabla^2 E_z + K^2 E_z = 0 \end{cases}$$

Iremos considerar que:

$$\vec{E}(x,y,z,t) = \vec{E}(x,y) \cdot e^{j\omega t - j\beta z}$$

$$\vec{H}(x,y,z,t) = \vec{H}(x,y) \cdot e^{j\omega t - j\beta z}$$

$\beta \rightarrow$ NÚMERO DE ONDA

$$\vec{E}(x,y) = \underbrace{E_x(x,y)\hat{a}_x + E_y(x,y)\hat{a}_y}_{\text{TRANSVERSAL}} + \underbrace{E_z(x,y)\hat{a}_z}_{\text{LONGITUDINAL}}$$

$$\vec{\nabla} = \underbrace{\partial_x \hat{a}_x + \partial_y \hat{a}_y}_{\vec{\nabla}_T} + \partial_z \hat{a}_z = \vec{\nabla}_T - j\beta \hat{a}_z$$

SIMPLIFICAÇÕES

Note que derivar em relação a z é o mesmo que multiplicar por -jβ.

Assim, temos:

$$\begin{cases} \vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H} & (1) \\ \vec{\nabla} \times \vec{H} = j\omega\varepsilon\vec{E} & (2) \\ \vec{\nabla} \cdot \vec{E} = 0 & (3) \\ \vec{\nabla} \cdot \vec{H} = 0 & (4) \end{cases}$$

$$(1)\ \left(\vec{\nabla}_T - j\beta\hat{a}_z\right) \times \left(\vec{E}_T + \hat{a}_z E_z\right) = -j\omega\mu\left(\vec{H}_T + \hat{a}_z H_z\right)$$

$$(2)\ \left(\vec{\nabla}_T - j\beta\hat{a}_z\right) \times \left(\vec{H}_T + \hat{a}_z H_z\right) = j\omega\varepsilon\left(\vec{E}_T + \hat{a}_z E_z\right)$$

$$(3)\ \left(\vec{\nabla}_T - j\beta\hat{a}_z\right) \cdot \left(\vec{E}_T + \hat{a}_z E_z\right) = 0$$

$$(4)\ \left(\vec{\nabla}_T - j\beta\hat{a}_z\right) \cdot \left(\vec{H}_T + \hat{a}_z H_z\right) = 0$$

Desenvolvendo (1):

$$\underbrace{\vec{\nabla}_T \times \vec{E}_T}_{1)} + \underbrace{\vec{\nabla}_T \times \hat{a}_z}_{2)} E_z + \left(-j\beta \underbrace{\hat{a}_z \times \vec{E}_T}_{3)}\right) - j\beta\,\cancelto{0}{\hat{a}_z \times \hat{a}_z E_z}$$

$$= \overbrace{-j\omega\mu\Big(\vec{H}_T}^{5)} + \hat{a}_z H_z\Big)$$

$$\underbrace{-j\omega\mu\vec{H}_T}_{4)}$$

$$1)\ \vec{\nabla}_T \times \vec{E}_T = \left(\partial_x \hat{a}_x + \partial_y \hat{a}_y\right) \times \left(E_x \hat{a}_x + E_y \hat{a}_y\right)$$

$$\partial_x E_y \hat{a}_z - \partial_y E_x \hat{a}_z$$

$$\hat{a}_z(\partial_x E_y - \partial_y E_x) \rightarrow \text{LONGITUDINAIS}$$

2) $\vec{\nabla}_T \times \hat{a}_z E_z = (\partial_x \hat{a}_x + \partial_y \hat{a}_y) \times \hat{a}_z E_z$

$$-\partial_x E_z \hat{a}_y + \partial_y E_z \hat{a}_x = \vec{\nabla} E_z \times \hat{a}_z \nearrow \text{TRANS-VERSAIS}$$

$$\vec{\nabla} E_z \times \hat{a}_z = (\partial_x E_z \hat{a}_x + \partial_y E_z \hat{a}_y + \partial_z E_z \hat{a}_z) \times \hat{a}_z$$

$$-\partial_x E_z \hat{a}_y + \partial_y E_z \hat{a}_x$$

1) e 5) COMP. LONGITUDINAIS

2), 3) e 4) COMP. TRANSVERSAIS

LOGO, PODEMOS AGRUPAR:

1) e 5) $\vec{\nabla}_T \times \vec{E}_T = -j\omega\mu H_z \hat{a}_z$

Ser operador transversal não importa, pois de ambas formas resultará na mesma resposta

$$\vec{\nabla} \times \vec{E}_T + j\omega\mu H_z \hat{a}_z = 0$$

2), 3) e 4)

$$\vec{\nabla}_T \times \hat{a}_z E_z - j\beta \hat{a}_z \times \vec{E}_T = -j\omega\mu \vec{H}_T$$

$$\vec{\nabla} E_z \times \hat{a}_z - j\beta \hat{a}_z \times \vec{E}_T = -j\omega\mu \vec{H}_T$$

DA MESMA FORMA PARA A EQ. (2):

$$\vec{\nabla} \times \vec{H}_T - j\omega\varepsilon E_z \hat{a}_z = 0$$

e

$$\vec{\nabla} H_z \times \hat{a}_z - j\beta \hat{a}_z \times \vec{H}_T = j\omega\varepsilon \vec{E}_T$$

ALÉM DISSO, (3) VIRA:

$$\left(\vec{\nabla}_T - j\beta \hat{a}_z\right) \cdot \left(\vec{E}_T + \hat{a}_z E_z\right) = 0$$

$$\vec{\nabla}_T \cdot \vec{E}_T + \cancelto{0}{\vec{\nabla}_T \cdot \hat{a}_z} E_z - j\beta \cancelto{0}{\hat{a}_z \cdot \vec{E}_T} - j\beta E_z = 0$$

$$\vec{\nabla}_T \cdot \vec{E}_T - j\beta E_z = 0$$

e (4):

$$\vec{\nabla}_T \cdot \vec{H}_T - j\beta H_z = 0$$

Logo, A PARTIR DAS EQUAÇÕES ABAIXO:

$$\begin{cases} \vec{\nabla} E_z \times \hat{a}_z - j\beta \hat{a}_z \times \vec{E}_T = -j\omega\mu \vec{H}_T \\ \vec{\nabla} H_z \times \hat{a}_z - j\beta \hat{a}_z \times \vec{H}_T = j\omega\varepsilon \vec{E}_T \end{cases}$$

$$\hat{a}_z \times \left(\vec{\nabla} E_z \times \hat{a}_z - j\beta \hat{a}_z \times \vec{E}_T \right) = \hat{a}_z \times \left(-j\omega\mu \vec{H}_T \right)$$

$$\underbrace{\hat{a}_z \times \left(\vec{\nabla} E_z \times \hat{a}_z \right)} - j\beta\, \hat{a}_z \times \left(\hat{a}_z \times \vec{E}_T \right) = -j\omega\mu\, \hat{a}_z \times \vec{H}_T$$

$$\vec{\nabla}_T E_z + j\beta \vec{E}_T = -j\omega\mu\, \hat{a}_z \times \vec{H}_T$$

e PARA A SEGUNDA EQ.

$$\vec{\nabla}_T H_z + j\beta \vec{H}_T = j\omega\varepsilon\, \hat{a}_z \times \vec{E}_T$$

$$\vec{H}_T = \frac{\omega\varepsilon}{\beta} \hat{a}_z \times \vec{E}_T + \frac{\cancel{j}}{\beta} \vec{\nabla}_T H_z$$

$$\vec{\nabla}_T E_z + j\beta \vec{E}_T =$$

$$-j\omega\mu\, \hat{a}_z \times \left(\frac{\omega\varepsilon}{\beta} \hat{a}_z \times \vec{E}_T + \frac{\cancel{j}}{\beta} \vec{\nabla}_T H_z \right)$$

$$\vec{\nabla}_T E_z + j\beta \vec{E}_T = -\frac{j\omega^2\mu\varepsilon}{\beta}\,\hat{a}_z \times \big(\underbrace{\hat{a}_z \times \vec{E}_T}_{-\vec{E}_T}\big)$$

$$+ \frac{\omega\mu}{\beta}\,\hat{a}_z \times \vec{\nabla}_T H_z$$

$\hat{a}_z \times$ $\qquad\qquad$ $\hat{a}_z \times$

$$\vec{\nabla}_T E_z + j\beta \vec{E}_T = \frac{j\omega^2\mu\varepsilon}{\beta}\vec{E}_T + \frac{\omega\mu}{\beta}\,\hat{a}_z \times \vec{\nabla}_T H_z$$

$$\hat{a}_z \times \vec{\nabla}_T E_z + j\beta\,\hat{a}_z \times \vec{E}_T = \frac{j\omega^2\mu\varepsilon}{\beta}\,\hat{a}_z \times \vec{E}_T$$

$$+ \frac{\omega\mu}{\beta}\,\hat{a}_z \times \big(\hat{a}_z \times \vec{\nabla}_T H_z\big)$$

$$\hat{a}_z \times \vec{\nabla}_T E_z + \hat{a}_z \times \vec{E}_T \Big(\underbrace{j\beta - \frac{j\omega^2\mu\varepsilon}{\beta}}\Big) = -\frac{\omega\mu}{\beta}\vec{\nabla}_T H_z$$

$$\frac{j\beta^2 - j\omega^2\mu\varepsilon}{\beta}$$

$$\frac{j\beta^2 - jk^2}{\beta} \qquad\qquad K_c^2 = K^2 - \beta^2$$

$$-jK_c^2/\beta$$

$$\hat{a}_z \times \vec{\nabla}_T E_z + \left(\hat{a}_z \times \vec{E}_T\right) \cdot -j \frac{K_c^2}{\beta} = -\frac{\omega\mu}{\beta} \vec{\nabla}_T H_z$$

$$\hat{a}_z \times \vec{E}_T = -j \frac{\omega\mu}{K_c^2} \vec{\nabla}_T H_z - j \frac{\beta}{K_c^2} \hat{a}_z \times \vec{\nabla}_T E_z$$

AGORA, PARA $\vec{H}_T$:

$$\vec{H}_T = \frac{\omega\varepsilon}{\beta} \hat{a}_z \times \vec{E}_T + \frac{j}{\beta} \vec{\nabla}_T H_z$$

$$\vec{H}_T = \frac{\omega\varepsilon}{\beta} \left(-j \frac{\omega\mu}{K_c^2} \vec{\nabla}_T H_z - j \frac{\beta}{K_c^2} \hat{a}_z \times \vec{\nabla}_T E_z \right)$$

$$+ \frac{j}{\beta} \vec{\nabla}_T H_z$$

$$\vec{H}_T = \vec{\nabla}_T H_z \left(-j \frac{\omega^2\mu\varepsilon}{\beta K_c^2} + \frac{j}{\beta} \right) - j \frac{\cancel{\beta}}{K_c^2} \frac{\omega\varepsilon}{\cancel{\beta}} \hat{a}_z \times \vec{\nabla}_T E_z$$

$$\vec{H}_T = \frac{-j\overbrace{\omega^2\mu\varepsilon}^{K^2} + jK_c^2}{\beta K_c^2} \vec{\nabla}_T H_z - j \frac{\omega\varepsilon}{K_c^2} \hat{a}_z \times \vec{\nabla}_T E_z$$

$$K_c^2 = K^2 - \beta^2$$

$$\vec{H}_T = -j \frac{\beta}{K_c^2} \vec{\nabla}_T H_z - j \frac{\omega\varepsilon}{K_c^2} \hat{a}_z \times \vec{\nabla}_T E_z$$

DESSA FORMA, CHEGA-SE EM:

$$\begin{cases} \hat{a}_z \times \hat{a}_z \times \vec{E}_T = -\dfrac{j\beta}{K_c^2}\left(\dfrac{\omega\mu}{\beta}\, \hat{a}_z \times \vec{\nabla}_T H_z + \hat{a}_z \times \hat{a}_z \times \vec{\nabla}_T E_z\right) \\ \vec{H}_T = -\dfrac{j\beta}{K_c^2}\left(\dfrac{\omega\varepsilon}{\beta}\, \hat{a}_z \times \vec{\nabla}_T E_z + \vec{\nabla}_T H_z\right) \end{cases}$$

EM EVIDÊNCIA

PARA AMBAS EQUAÇÕES

$$\begin{cases} \vec{E}_T = -\dfrac{j\beta}{K_c^2}\left(\vec{\nabla}_T E_z - \dfrac{\omega\mu}{\beta}\, \hat{a}_z \times \vec{\nabla}_T H_z\right) \\ \vec{H}_T = -\dfrac{j\beta}{K_c^2}\left(\dfrac{\omega\varepsilon}{\beta}\, \hat{a}_z \times \vec{\nabla}_T E_z + \vec{\nabla}_T H_z\right) \end{cases}$$

$$\frac{\omega\mu}{\beta} = \eta_{TE} \quad e \quad \frac{\omega\varepsilon}{\beta} = \frac{1}{\eta_{TM}}$$

IMPEDÂNCIA INTRÍNSECA

DO MEIO

LEMBRE QUE:

OLHE PÁG. 5 e 6

$$\begin{cases} \vec{\nabla}_T \times \vec{E}_T + j\omega\mu H_z \hat{a}_z = 0 & (1) \\ \vec{\nabla}_T \times \vec{H}_T - j\omega\varepsilon E_z \hat{a}_z = 0 & (2) \\ \vec{\nabla}_T \cdot \vec{E}_T - j\beta E_z = 0 \\ \vec{\nabla}_T \cdot \vec{H}_T - j\beta H_z = 0 \end{cases}$$

DESSA FORMA, SUBSTITUINDO AS DUAS EQUAÇÕES PASSADAS:

$$(1) \rightarrow \vec{\nabla}_T \times \left(-\frac{j\beta}{K_c^2} \left(\cancelto{0}{\vec{\nabla}_T E_z} - \frac{\omega\mu}{\beta} \hat{a}_z \times \vec{\nabla}_T H_z \right) \right)$$

$$\vec{\nabla} \times (\nabla A) = 0 \qquad\qquad + j\omega\mu H_z \hat{a}_z = 0$$

$$\frac{j\omega\mu}{K_c^2} \underbrace{\vec{\nabla}_T \times \left(\hat{a}_z \times \vec{\nabla}_T H_z \right)}_{\hat{a}_z \nabla_T^2 H_z} + j\omega\mu H_z \hat{a}_z = 0$$

$$\hat{a}_z \nabla_T^2 H_z + j\omega\mu H_z \hat{a}_z = 0$$

$$\frac{j\omega\mu}{K_c^2} \hat{a}_z \nabla_T^2 H_z + j\omega\mu H_z \hat{a}_z = 0$$

OU AINDA

$$\frac{\cancel{j\omega\mu}}{K_c^2} \nabla_T^2 H_z + \cancel{j\omega\mu} H_z = 0$$

$$\nabla_T^2 H_z + K_c^2 H_z = 0$$

e

(2)

$$\vec{\nabla} \times \vec{H}_T - j\omega\varepsilon E_z \hat{a}_z = 0$$

$$\vec{\nabla} \times \left(-\frac{\cancel{j}\beta}{K_c^2} \left(\frac{\omega\varepsilon}{\beta} \hat{a}_z \times \vec{\nabla}_T E_z + \cancel{\vec{\nabla}_T H_z}^{\,0} \right) \right)$$

$$- j\omega\varepsilon E_z \hat{a}_z = 0$$

$$\left(-j\omega\varepsilon / K_c^2 \right) \hat{a}_z \nabla_T^2 E_z - j\omega\varepsilon E_z \hat{a}_z = 0$$

OU AINDA

$$\nabla_T^2 E_z \left(-\cancel{j\omega\varepsilon} / K_c^2 \right) - \cancel{j\omega\varepsilon} E_z = 0$$

$$\nabla_T^2 E_z + K_c^2 E_z = 0$$

ASSIM, TEMOS O PAR DE EQUAÇÕES DIFEREN-CIAIS:

EQ. DE HELMHOLTZ

- $\nabla_T^2 H_z + K_c^2 H_z = 0$
- $\nabla_T^2 E_z + K_c^2 E_z = 0$

RESOLVENDO, BASTA SUBSTITUIR E_z e H_z NAS EQUAÇÕES DA PÁG. 10. PARA ACHAR

AS COMPONENTES TRANSVERSAIS.

Como $\nabla_T^2 = \partial_x^2 + \partial_y^2 \longrightarrow$ COMP. TRANS.

- $(\partial_x^2 + \partial_y^2) H_z + K_c^2 H_z = 0$
- $(\partial_x^2 + \partial_y^2) E_z + K_c^2 E_z = 0$

OLHAR EQ. DA PÁGINA 11

$$E_x = -\frac{j\beta}{K_c^2}\left(\partial_x E_z + \underbrace{\frac{\omega\mu}{\beta}}_{\eta_{TE}} \partial_y H_z\right)$$

$$E_y = -\frac{j\beta}{K_c^2}\left(\partial_y E_z - \frac{\omega\mu}{\beta} \partial_x H_z\right)$$

$$H_x = -\frac{j\beta}{K_c^2}\left(\partial_x H_z - \underbrace{\frac{\omega\varepsilon}{\beta}}_{1/\eta_{TM}} \partial_y E_z\right)$$

$$H_y = -\frac{j\beta}{K_c^2}\left(\frac{\omega\varepsilon}{\beta} \partial_y E_z + \partial_y H_z\right)$$

MODOS DE PROPAGAÇÃO

MODO HÍBRIDO

$E_z \neq 0$ e $H_z \neq 0$

MODO TEM	MODO TE	MODO TM
$E_z = 0$	$E_z = 0$	$E_z \neq 0$
$H_z = 0$	$H_z \neq 0$	$H_z = 0$

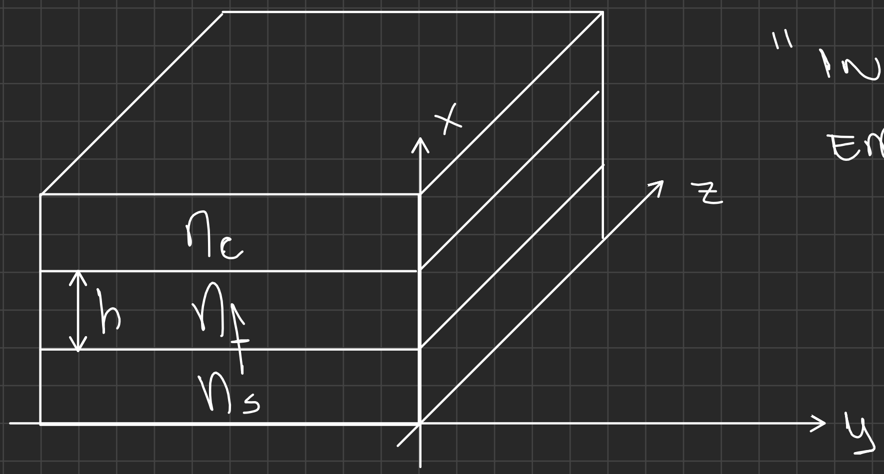
IMPEDÂNCIA INTRÍNSECA DO MEIO

$$\eta_{TE} = \frac{\omega\mu}{\beta} = \frac{\eta_0}{\sqrt{1-(\omega_c/\omega)^2}}$$

$$\eta_{TM} = \frac{1}{\omega\varepsilon/\beta} = \eta_0\sqrt{1-(\omega_c/\omega)^2}$$

ESTÁ RELACIONADO COM A RESISTÊNCIA DO MEIO À PROPAGAÇÃO DE ONDAS ELETROMAGNÉTICAS.

ANÁLISE SLAB SIMÉTRICO $(n_c = n_s)$



"INFINITO" EM y e z.

CONSIDERAÇÕES:

- $n_i \, (i = c, f, s)$ é o índice de refração do meio.
- Como o guia é simétrico, $n_c = n_s$.

PÁG. 11

$$\begin{cases} \vec{E}_T = -\dfrac{j\beta}{K_c^2}\left(\vec{\nabla}_T E_z - \dfrac{\omega\mu}{\beta}\,\hat{a}_z \times \vec{\nabla}_T H_z\right) \\[2ex] \vec{H}_T = -\dfrac{j\beta}{K_c^2}\left(\dfrac{\omega\varepsilon}{\beta}\,\hat{a}_z \times \vec{\nabla}_T E_z + \vec{\nabla}_T H_z\right) \end{cases} \rightarrow \partial_y = 0$$

POIS NÃO HÁ VARIAÇÃO EM y.

$$\vec{E}_T = -\frac{j\beta}{K_c^2}\left(\partial_x E_z\,\hat{a}_x - \eta_{TE}\,\partial_x H_z\,\hat{a}_y\right)$$

$$\vec{H}_T = -\frac{j\beta}{K_c^2}\left(\frac{1}{\eta_{TM}}\,\partial_x E_z\,\hat{a}_y + \partial_x H_z\,\hat{a}_x\right)$$

COMO E_z e H_z VARIAM ESPACIALMENTE APENAS COM x, EM OUTRAS PALAVRAS, $\partial_x E_z \neq 0$, $\partial_y E_z = 0$ e $\partial_z E_z = 0$, PODE-SE USAR A DERIVADA ORDINÁRIA AO INVÉS DA DERIVADA PARCIAL.

LOGO O PAR DE EQUAÇÕES VIRA:

$$\vec{E}_T = -\frac{j\beta}{K_c^2}\left(\frac{dE_z}{dx}\hat{a}_x - \eta_{TE}\frac{dH_z}{dx}\hat{a}_y\right)$$

$$\vec{H}_T = -\frac{j\beta}{K_c^2}\left(\frac{1}{\eta_{tm}}\frac{dE_z}{dx}\hat{a}_y + \frac{dH_z}{dx}\hat{a}_x\right)$$

$$E_x = -\frac{j\beta}{K_c^2}\frac{dE_z}{dx} \qquad E_y = \frac{j\eta_{TE}\beta}{K_c^2}\frac{dH_z}{dx}$$

$$H_x = -\frac{j\beta}{K_c^2}\frac{dH_z}{dx} \qquad H_y = -\frac{j\beta}{K_c^2}\cdot\frac{1}{\eta_{tm}}\frac{dE_z}{dx}$$

E O MESMO VALE PARA AS EQ. DE HELMHOLTZ.

$$\nabla_T^2 H_z + K_c^2 H_z = 0 \longrightarrow \frac{d^2 H_z}{dx^2} + K_c^2 H_z = 0$$

$$\nabla_T^2 E_z + K_c^2 E_z = 0 \longrightarrow \frac{d^2 E_z}{dx^2} + K_c^2 E_z = 0$$

MODO TM

$$E_x = -j\frac{\beta}{K_c^2}\frac{dE_z}{dx}$$

$$H_y = -j\frac{\beta}{K_c^2}\cdot\frac{1}{\eta_{tm}}\frac{dE_z}{dx}$$

$$\frac{d^2E_z}{dx^2} + K_c^2 E_z = 0$$

MODO TE

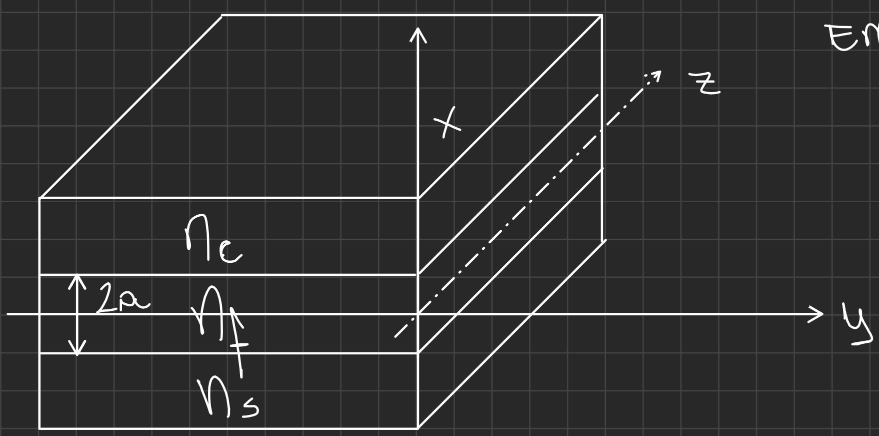
$$H_x = -j\frac{\beta}{K_c^2}\frac{dH_z}{dx}$$

$$E_y = j\frac{\eta_{TE}\,\beta}{K_c^2}\frac{dH_z}{dx}$$

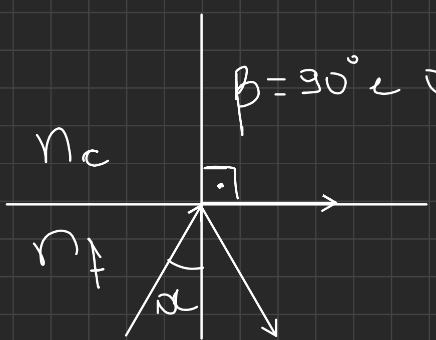
$$\frac{d^2H_z}{dx^2} + K_c^2 H_z = 0$$

ANÁLISE SLAB ASSIMÉTRICO

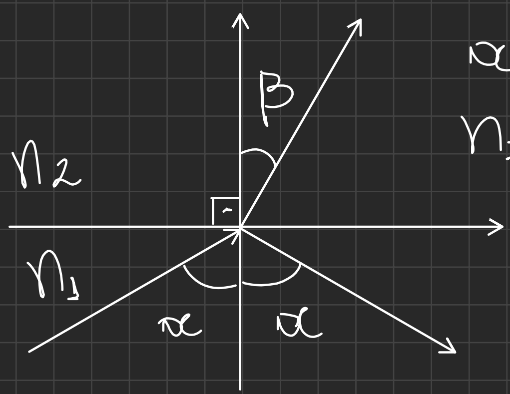
"INFINITO" EM y e z.



$0 < n_c < n_s < n_f$



$\beta = 90°$ e $0° < \alpha < 90°$



$\alpha > \beta$

$n_2 > n_1$

$$\operatorname{sen}\alpha = \frac{n_c}{n_f} \longrightarrow n_c < n_f$$

$$\operatorname{sen}\alpha\, n_1 = \operatorname{sen}\beta\, n_2$$

$$\frac{\operatorname{sen}\alpha}{\operatorname{sen}\beta} = \frac{n_2}{n_1}$$

Logo, o ÂNGULO CRÍTICO

é DADO POR:

$$\arcsen\left(\frac{n_c}{n_f}\right) < \alpha < 90°$$ → INCLINAÇÃO NECESSÁRIA PARA QUE A LUZ INCIDA e PERMANEÇA CONFINADA.

PARA O MODO TE, TEMOS:

$$\begin{cases} H_x = -j\dfrac{\beta}{k_i^2}\dfrac{dH_z}{dx} \\ E_y = j\dfrac{\eta_{TE}\,\beta}{k_i^2}\dfrac{dH_z}{dx} \\ \dfrac{d^2H_z}{dx^2} + k_i^2 H_z = 0 \end{cases} \qquad \eta_{TE} = \frac{\omega\mu}{\beta}$$

$, \; i = c, f, s$

$$K_c^2 = K^2 - \beta^2 \quad , \quad K^2 = \omega^2\mu\varepsilon$$

$$K_i^2 = \underbrace{K_0^2 n_i^2}_{\text{ASSUME O PAPEL DO } K^2} - \beta^2$$

ASSUME O PAPEL DO K^2

$$e \quad \beta = K_0 n_{eff}$$

Logo, $K_i^2 = K_0^2 n_i^2 - K_0^2 n_{eff}^2$

$$K_i^2 = K_0^2 (n_i^2 - n_{eff}^2)$$

$$K_i = K_0 \sqrt{n_i^2 - n_{eff}^2}$$

Como ESTÁ SENDO ANALISADO O MODO TE, $E_z = 0$, Logo TODAS AS EQ. DA PÁG. 18 PARA MODO TM SÃO NULA, EM OUTRAS PALAVRAS $E_x = 0$, $H_y = 0$ e $E_z = 0$.

$$\begin{cases} \dfrac{d^2 H_z}{dx^2} + K_c^2 H_z = 0, & x \geq a \\ \dfrac{d^2 H_z}{dx^2} + K_f^2 H_z = 0, & |x| \leq a \\ \dfrac{d^2 H_z}{dx^2} + K_s^2 H_z = 0, & x \leq -a \end{cases}$$

- $\dfrac{d^2 H_z}{dx^2} + K_c^2 H_z = 0$

$$r^2 + K_c^2 = 0$$

$$r = \pm\sqrt{-K_c^2} \qquad K_c = j\alpha_c$$

$$K_c^2 = -\alpha_c^2$$

$$r = \pm \alpha_c$$

Logo,

$$H_z(x) = A e^{-\alpha_c x} + B e^{\alpha_c x}, \quad x \geq a$$

DECRESCE EXPONENCIALMENTE

- $$\frac{d^2 H_z}{dx^2} + K_f^2 H_z = 0$$

$$r^2 + K_f^2 = 0$$

$$r = \pm j K_f$$

$$H_z(x) = C \operatorname{sen}(K_f x) + D \cos(K_f x), \quad |x| \leq a$$

- $$\frac{d^2 H_z}{dx^2} + K_s^2 H_z = 0$$

$$r^2 + K_s^2 = 0$$

$$r = \pm \sqrt{-K_s^2} \qquad K_s = j\alpha_s$$

$$r = \pm \sqrt{\alpha_s^2} \qquad K_s^2 = -\alpha_s^2$$

$$r = \pm \alpha_s$$

$$H_z(x) = E e^{\alpha_s x} + F e^{-\alpha_s x}, \quad x \leq -a$$

DECRESCE EXPONENCIALMENTE

$$H_z(x) = \begin{cases} Ae^{-\alpha_c x} + Be^{\alpha_c x}, & x \geq a \\ C\,\text{sen}(K_f x) + D\cos(K_f x), & |x| \leq a \\ Ee^{\alpha_s x} + Fe^{-\alpha_s x}, & x \leq -a \end{cases}$$

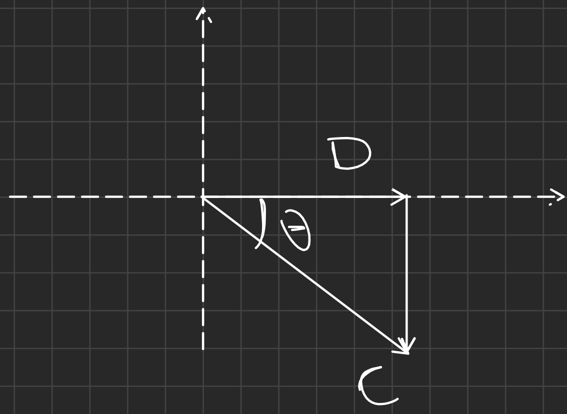
COMO QUEREMOS QUE A LUZ FIQUE O MÁXIMO CONFINADO, TUDO QUE "VAZE" PARA C e S DEVE SER REDUZIDO O MÁXIMO POSSÍVEL. Logo, B e F DEVEM SER IGUAIS A ZERO POIS:

$$\lim_{x \to \pm\infty} H_z(x) = 0$$

$$H_z(x) = \begin{cases} Ae^{-\alpha_c x}, & x \geq a \\ C\,\text{sen}(K_f x) + D\cos(K_f x), & |x| \leq a \\ Ee^{\alpha_s x}, & x \leq -a \end{cases}$$

$$C\,\underbrace{\text{sen}(K_f x)}_{\cos(K_f - 90^\circ)} + D\cos(K_f x)$$

$$C\cos K_f x - 90^\circ + D\cos(K_f x)$$



$$C \,\mathrm{sen}(K_f x) + D \cos(K_f x)$$

$$= \underbrace{\sqrt{D^2 + C^2}}_{H_0} \cdot \cos\Big(K_f x - \underbrace{tg^{-1}\Big(\frac{C}{D}\Big)}_{\phi}\Big)$$

Logo,

$$H_z(x) = \begin{cases} A e^{-\alpha_c x}, & x \geq a \\ H_0 \,\mathrm{sen}(K_f x + \phi), & |x| \leq a \\ E e^{\alpha_s x}, & x \leq -a \end{cases}$$

Como $H_z(x)$ DEVE SER UMA FUNÇÃO CONTÍNUA

$$\lim_{x \to a^+} H_z(x) = \lim_{x \to a^-} H_z(x)$$

$$A e^{-\alpha_c a} = H_0 \,\mathrm{sen}(K_f a + \phi)$$

$$A = H_0 \,\mathrm{sen}(K_f a + \phi)\, e^{\alpha_c a}$$

$$\lim_{x \to -a^+} H_z(x) = \lim_{x \to -a^-} H_z(x)$$

$$E\, e^{-\alpha_s a} = H_0 \operatorname{sen}(-K_f a + \phi)$$

$$E = H_0 \cdot \operatorname{sen}(-(-(\phi - K_f a)))\, e^{\alpha_s a}$$

$$E = -H_0 \operatorname{sen}(K_f a - \phi)\, e^{\alpha_s a}$$

Logo,

$$H_z(x) = \begin{cases} H_0 \operatorname{sen}(K_f a + \phi)\, e^{\alpha_s (a - x)}, & x \geq a \\ H_0 \operatorname{sen}(K_f x + \phi), & |x| \leq a \\ -H_0 \operatorname{sen}(K_f a - \phi)\, e^{\alpha_s (a + x)}, & x \leq -a \end{cases}$$

Porém, LEMBRE DA PÁG. 19

$$H_x = -j \frac{\beta}{K_i^2} \frac{dH_z}{dx} \qquad E_y = j \frac{\eta_{TE} \beta}{K_i^2} \frac{dH_z}{dx}$$

$$K_c^2 = -\alpha_c^2$$

$$H_x(x) = -j\beta \begin{cases} -\dfrac{H_0 \alpha_c}{K_c^2} \operatorname{sen}(K_f a + \phi)\, e^{\alpha_c (a-x)} & , \; x \geq a \\ \dfrac{H_0 K_f}{K_f^2} \cos(K_f x + \phi) & , \; |x| \leq a \\ -\dfrac{H_0 \alpha_s}{K_s^2} \operatorname{sen}(K_f a - \phi)\, e^{\alpha_s (a+x)} & , \; x \leq -a \end{cases}$$

$$K_s^2 = -\alpha_s^2$$

$$H_x(x) = -j\beta \begin{cases} \dfrac{H_0}{\alpha_c} \operatorname{sen}(K_f a + \phi)\, e^{\alpha_c (a-x)} & , \; x \geq a \\ \dfrac{H_0}{K_f} \cos(K_f x + \phi) & , \; |x| \leq a \\ \dfrac{H_0}{\alpha_s} \operatorname{sen}(K_f a - \phi)\, e^{\alpha_s (a+x)} & , \; x \leq -a \end{cases}$$

E PARA $E_y(x)$:

$$E_y(x) = j\,\eta_{TE}\,\beta \cdot \begin{cases} \dfrac{H_0}{\alpha_c} \operatorname{sen}(K_f a + \phi)\, e^{\alpha_c (a-x)} & , \; x \geq a \\ \dfrac{H_0}{K_f} \cos(K_f x + \phi) & , \; |x| \leq a \\ \dfrac{H_0}{\alpha_s} \operatorname{sen}(K_f a - \phi)\, e^{\alpha_s (a+x)} & , \; x \leq -a \end{cases}$$

Se notar para $E_y(x=a^+) = E_y(x=a^-)$, temos:

$$\frac{1}{K_f} \cdot \cos(K_f a + \phi) = \frac{1}{\alpha_c} \operatorname{sen}(K_f a + \phi)$$

$$\frac{\alpha_c}{K_f} = tg\, K_f a + \phi \longrightarrow tg^{-1}\left(\frac{\alpha_c}{K_f}\right) = K_f a + \phi$$

O mesmo para $E_y(x=-a^+) = E_y(x=-a^-)$:

$$\frac{1}{K_f} \cos(K_f a - \phi) = \frac{1}{\alpha_s} \operatorname{sen}(K_f a - \phi)$$

$$\frac{\alpha_s}{K_f} = tg(K_f a - \phi) \longrightarrow tg^{-1}\left(\frac{\alpha_s}{K_f}\right) = K_f a - \phi$$

$$\operatorname{arctg} A \pm \operatorname{arctg} B = \operatorname{arctg}\left(\frac{A \pm B}{1 \mp AB}\right)$$

Somando as eq. em azul:

$$2K_f a = tg^{-1}\left(\frac{\alpha_s}{K_f}\right) + tg^{-1}\left(\frac{\alpha_c}{K_f}\right)$$

$$2K_f a = tg^{-1}\left(\frac{\frac{\alpha_s}{K_f} + \frac{\alpha_c}{K_f}}{1 - \frac{\alpha_s \alpha_c}{K_f^2}}\right)$$

$$2K_f a = tg^{-1}\left(\frac{(\alpha_s+\alpha_c)K_f}{K_f^2-\alpha_s\alpha_c}\right)$$

$$tg(2K_f a) = \frac{(\alpha_s+\alpha_c)K_f}{K_f^2-\alpha_s\alpha_c}$$

$u = K_f a \qquad w = a\alpha_c \qquad v = a\alpha_s$

$$tg(2u) = \frac{\left(\frac{v}{a}+\frac{w}{a}\right)\frac{u}{a}}{\frac{u^2}{a^2}-\frac{v}{a}\cdot\frac{w}{a}}$$

$$= \frac{\frac{1}{a^2}(v+w)u}{\frac{1}{a^2}(u^2-vw)}$$

$$tg(2u) = \frac{(v+w)u}{(u^2-vw)}$$

LEMBRANDO

$$K_i = K_0\sqrt{n_i^2-n_{eff}^2}$$

$$u = aK_f = aK_0\sqrt{n_f^2-n_{eff}^2}$$

$$v = a\alpha_s = -ajK_s \qquad \alpha_s = -jK_s$$

$$= -ajK_0\underbrace{\sqrt{n_s^2-n_{eff}^2}}_{<0}$$

$$= -ajK_0 j\sqrt{n_{eff}^2-n_s^2} = aK_0\sqrt{n_{eff}^2-n_s^2}$$

$$W = a\,\alpha_c = a\,K_0\sqrt{n_{eff}^2 - n_c^2}$$

$$\begin{cases} tg^{-1}\left(\dfrac{\alpha_c}{K_f}\right) = K_f a + \phi \\ tg^{-1}\left(\dfrac{\alpha_s}{K_f}\right) = K_f a - \phi \end{cases}$$

SOMANDO AMBAS:

$$2K_f a = tg^{-1}\left(\frac{\alpha_s}{K_f}\right) + tg^{-1}\left(\frac{\alpha_c}{K_f}\right)$$

$$2u = tg^{-1}\left(\frac{v}{u}\right) + tg^{-1}\left(\frac{w}{u}\right) + m\pi$$

SUBTRAÍNDO AMBAS:

$$2\phi = tg^{-1}\left(\frac{\alpha_c}{K_f}\right) - tg^{-1}\left(\frac{\alpha_s}{K_s}\right)$$

$$2\phi = tg^{-1}\left(\frac{w}{u}\right) - tg^{-1}\left(\frac{v}{u}\right) + m\pi$$

RESOLVENDO AS EQ. PARA u:

$$u = \sqrt{1 - \underbrace{\left(\frac{\beta^2 - K_0^2 n_s^2}{K_0^2 n_f^2 - K_0^2 n_s^2}\right)}_{b}}\;\underbrace{\left[K_0\, a\sqrt{n_f^2 - n_s^2}\right]}_{V}$$

$b \rightarrow$ CONSTANTE DE PROPAGAÇÃO NORMALIZADA.

$V \rightarrow$ FREQUÊNCIA NORMALIZADA. ESTA DEPENDE DOS ÍNDICES DE REFRAÇÃO, ESPESSURA E COMPRIMENTO DE ONDA.

$$u = \sqrt{1-b}\,.\,V \longrightarrow v = \sqrt{b}\,V$$

$$u^2 + v^2 = V^2$$

E AINDA

$$\left(\frac{w}{u}\right)^2 = \frac{\beta^2 - K_0^2 n_c^2}{K_0^2 n_f^2 - \beta^2} = \frac{\beta^2 - K_0^2 n_c^2 + K_0^2 n_s^2 - K_0^2 n_s^2}{K_0^2 n_f^2 - \beta^2 + K_0^2 n_s^2 - K_0^2 n_s^2}$$

$$= \frac{b + \delta}{1 - b}$$

$$\delta = \frac{K_0^2 n_s^2 - K_0^2 n_c^2}{K_0^2 n_f^2 - K_0^2 n_s^2} = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2}$$

PARÂMETRO DE ASSIMETRIA

$$\frac{w}{u} = \sqrt{\frac{b+\delta}{1-b}} = \sqrt{\frac{b}{1-b} + \frac{\delta}{1-b}}$$

$$\frac{W}{U} = \left(\frac{V^2}{U^2} + \delta \cdot \frac{V^2}{U^2} \right)^{1/2}$$

$$V = \sqrt{b}\, V \qquad U = \sqrt{1-b}\, V$$

$$\frac{V}{U} = \frac{\sqrt{b}}{\sqrt{1-b}} \qquad \frac{1}{\sqrt{1-b}} = \frac{V}{U}$$

$$\frac{W^2}{U^2} = \frac{V^2}{U^2} + \delta \cdot \frac{V^2}{U^2}$$

$$W^2 - V^2 = \delta V^2$$

VALE RESSALTAR QUE SE $\delta = 0$, SIGNIFICA QUE

$$n_s = n_c$$

OU SEJA, TEMOS UM GUIA SIMÉTRICO.

POR FIM:

$$2U = tg^{-1}\left(\frac{V}{U}\right) + tg^{-1}\left(\frac{W}{U}\right) + m\pi$$

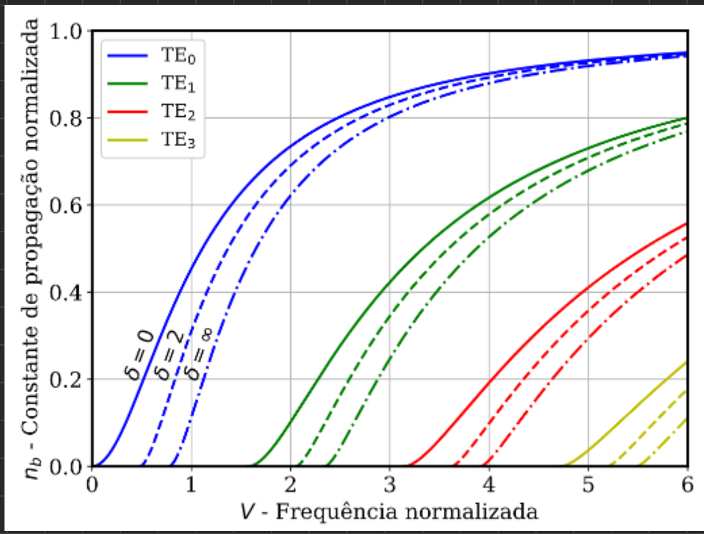
VIRA

$$2\sqrt{1-b}\, V = tg^{-1}\left(\sqrt{\frac{b}{1-b}}\right) + tg^{-1}\left(\sqrt{\frac{b+\delta}{1-b}}\right) + m\pi$$

DESSA FORMA, TIRAMOS A DEPENDÊNCIA DE β, ONDE

$$\beta = K_0 n_{eff} \longrightarrow \text{TERMOS DESCONHECIDOS}$$

b



V

PARA ACHARMOS A FREQUÊNCIA NORMALIZADA DE CORTE, $b=0$, ENTÃO:

$$2V_c^m = tg^{-1}(\sqrt{\delta}) + m\pi$$

$$K_o = W\sqrt{\mu_o \varepsilon_o}$$

$$K_o = \frac{2\pi f}{C_o}$$

$$V_c^m = \frac{tg^{-1}(\sqrt{\delta}) + m\pi}{2}$$

$$K_o a\sqrt{n_f^2 - n_s^2} = V$$

$$V = \frac{2\pi f}{C_o} a\sqrt{n_f^2 - n_s^2}$$

$$\frac{2\pi f_c^m}{C_o} \cdot a\sqrt{n_f^2 - n_s^2} = \frac{tg^{-1}(\sqrt{\delta}) + m\pi}{2}$$

$$f_c^m = \frac{tg^{-1}(\sqrt{\delta}) + m\pi}{2 \cdot 2\pi \frac{a}{C_o} \cdot \sqrt{n_f^2 - n_s^2}}$$

PARA CONHECERMOS TODOS OS MODOS TE GUIADOS,

$$V_c^m \leq V$$

$$\frac{tg^{-1}(\sqrt{\delta}) + m\pi}{2} \leq V$$

$$m \leq \frac{2V - tg^{-1}\sqrt{\delta}}{\pi}$$

Logo, A QUANTIDADE DE MODOS M é O MAIOR INTEIRO MENOR QUE m.

Se $m = 2{,}45 \rightarrow M = 2$, $m = 2{,}98 \rightarrow M = 2$.

PARA ANALISARMOS O MODO TM, O GRUPO DE EQUAÇÕES AGORA é:

OLHE PÁG. 18

$$\left\{\begin{aligned} E_x &= -\frac{j\beta}{K_c^2} \frac{dE_z}{dx} \\ H_y &= -\frac{j\beta}{K_c^2} \cdot \frac{1}{\eta_{tm}} \frac{dE_z}{dx} \\ \frac{d^2 E_z}{dx^2} &+ K_c^2 E_z = 0 \end{aligned}\right.$$

COMO OS PASSOS JÁ FORAM REALIZADOS NO MODO TE, ADIANTAREMOS AS RESPOSTAS:

$$E_z(x) = E_0 \begin{cases} \sin(K_f a + \phi)\, e^{-\alpha_c (x-a)} & , x \geq a \\ \sin(K_f x + \phi) & , |x| \leq a \\ -\sin(K_f a - \phi)\, e^{\alpha_s (x+a)} & , x \leq -a \end{cases}$$

$$E_x(x) = -j\beta E_0 \begin{cases} \alpha_c^{-1} \sin(K_f a + \phi)\, e^{-\alpha_c (x-a)} & , x \geq a \\ K_f^{-1} \cos(K_f x + \phi) & , |x| \leq a \\ \alpha_s^{-1} \sin(K_f a - \phi)\, e^{\alpha_s (x+a)} & , x \leq -a \end{cases}$$

$$H_y(x) = -j\omega E_0 \begin{cases} \frac{n_c^2}{\alpha_c} \sin(K_f a + \phi)\, e^{-\alpha_c (x-a)} & , x \geq a \\ \frac{n_f^2}{K_f} \cos(K_f x + \phi) & , |x| \leq a \\ \frac{n_s^2}{\alpha_s} \sin(K_f a - \phi)\, e^{\alpha_s (x+a)} & , x \leq -a \end{cases}$$

OLHE QUE OS TERMOS EM COMUM PARA E_x SÃO OS MESMOS PARA O MODO TE, LOGO AS CONDIÇÕES DO MODO TE JÁ SÃO

SUFICIENTES PARA E_x. RESTA APENAS $H_y(x)$.

$$\lim_{x\to a^+} H_y(x) = \lim_{x\to a^-} H_y(x):$$

$$\frac{n_c^2}{\alpha_c}\sin(K_f a+\phi) = \frac{n_f^2}{K_f}\cos(K_f a+\phi)$$

$$\frac{\alpha_c}{K_f}\cdot\underbrace{\left(\frac{n_f}{n_c}\right)^2}_{p_c} = tg(K_f a+\phi)$$

$$tg^{-1}\left(\frac{\alpha_c}{K_f}p_c\right) = K_f a+\phi$$

$$\lim_{x\to -a^+} H_y(x) = \lim_{x\to -a^-} H_y(x):$$

$$\frac{n_s^2}{\alpha_s}\underbrace{\sin(\,K_f a-\phi)}_{\sin(K_f a-\phi)} = \frac{n_f^2}{K_f}\underbrace{\cos(-K_f a+\phi)}_{\cos(K_f a-\phi)}$$

$$tg(K_f a-\phi) = \frac{\alpha_s}{K_f}\underbrace{\left(\frac{n_f}{n_s}\right)^2}_{p_s}$$

$$K_f a-\phi = tg^{-1}\left(\frac{\alpha_s}{K_f}p_s\right)$$

SOMANDO AS DUAS EQUAÇÕES:

$$2K_f a = tg^{-1}\left(\frac{\alpha_s}{K_f} p_s\right) + tg^{-1}\left(\frac{\alpha_c}{K_f} p_c\right)$$

$$\operatorname{arctg} A \pm \operatorname{arctg} B = \operatorname{arctg}\left(\frac{A \pm B}{1 \mp AB}\right)$$

$$2K_f a = tg^{-1}\left(\frac{\frac{\alpha_s}{K_f} p_s + \frac{\alpha_c}{K_f} p_c}{1 - \frac{\alpha_s}{K_f} p_s \frac{\alpha_c}{K_f} p_c}\right)$$

$$2K_f a = tg^{-1}\left(\frac{\alpha_s p_s + \alpha_c p_c}{K_f} \cdot \frac{K_f^2}{K_f^2 - \alpha_s \alpha_c p_s p_c}\right)$$

$$2K_f a = tg^{-1}\left(\frac{K_f(\alpha_s p_s + \alpha_c p_c)}{K_f^2 - \alpha_s \alpha_c p_s p_c}\right)$$

$$tg(2K_f a) = \frac{K_f(\alpha_s p_s + \alpha_c p_c)}{K_f^2 - \alpha_s \alpha_c p_s p_c}$$

$K_f a = u \qquad \alpha_c a = w \qquad \alpha_s a = v$

$$tg(2u) = \frac{\frac{u}{a}\left(\frac{v p_s}{a} + \frac{w}{a} p_c\right)}{\left(\frac{u}{a}\right)^2 - \frac{v}{a} \cdot \frac{w}{a} p_s p_c}$$

$$\operatorname{tg}(2u) = \frac{u(v p_s + w p_c)}{u^2 - v w p_s p_c}$$

SUBTRAINDO AS DUAS EQUAÇÕES:

$$2\phi = \operatorname{tg}^{-1}\left(\frac{\alpha_c}{K_f} p_c\right) - \operatorname{tg}^{-1}\left(\frac{\alpha_s}{K_f} p_s\right)$$

Como AS DUAS EQUAÇÕES EM VERDE:

$$2V\sqrt{1-b} = m\pi + \operatorname{tg}^{-1}\left(p_c\sqrt{\frac{b+\delta}{1-b}}\right) + \operatorname{tg}^{-1}\left(p_s\sqrt{\frac{b}{1-b}}\right)$$

PARA A FREQUÊNCIA DE CORTE, $b=0$:

$$2V_c^m = m\pi + \operatorname{tg}^{-1}\left(p_c\sqrt{\delta}\right)$$

$$K_0 a\sqrt{n_f^2 - n_s^2} = V$$

$$\frac{2\pi f}{c_0} a\sqrt{n_f^2 - n_s^2}$$

$$f_c^m = \frac{m\pi + \operatorname{tg}^{-1}\left(p_c\sqrt{\delta}\right)}{4\pi \frac{a}{c_0}\sqrt{n_f^2 - n_s^2}}$$

Se o GUIA FOR SIMÉTRICO, $\delta = 0$ e

$$V_c^m = \frac{m\pi}{2} \quad e \quad f_c^m = \frac{m\pi}{4\pi \frac{a}{c_0}\sqrt{n_f^2 - n_s^2}}$$

PARA CONHECERMOS TODOS OS MODOS TM GUIADOS,

$$V_c^m \leq V$$

$$\frac{tg^{-1}(p_c\sqrt{\delta}) + m\pi}{2} \leq V$$

$$m \leq \frac{2V - tg^{-1}(p_c\sqrt{\delta})}{\pi}$$

Logo, A QUANTIDADE DE MODOS M é O MAIOR INTEIRO MENOR QUE m.

Se $m = 2{,}45 \rightarrow M = 2$, $m = 2{,}98 \rightarrow M = 2$.

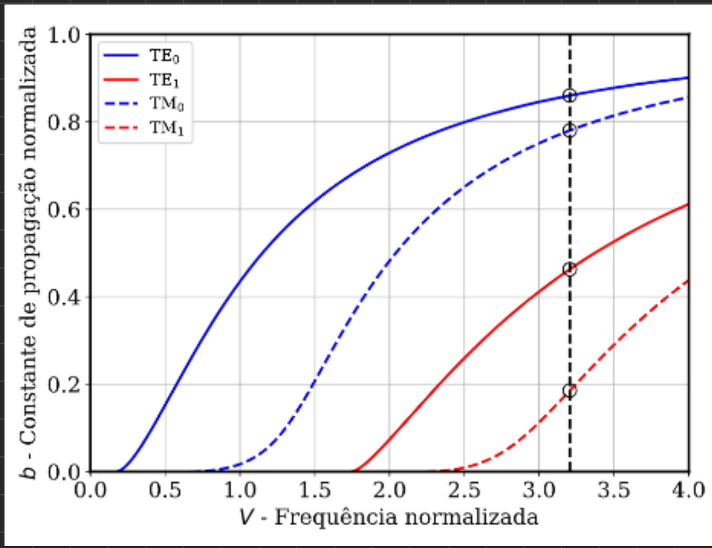
$$p_c = \left(\frac{n_f}{n_c}\right)^2 \quad e \quad n_f > n_c$$

$$p_c > 1$$

Logo $M_{TM} \leq M_{TE}$

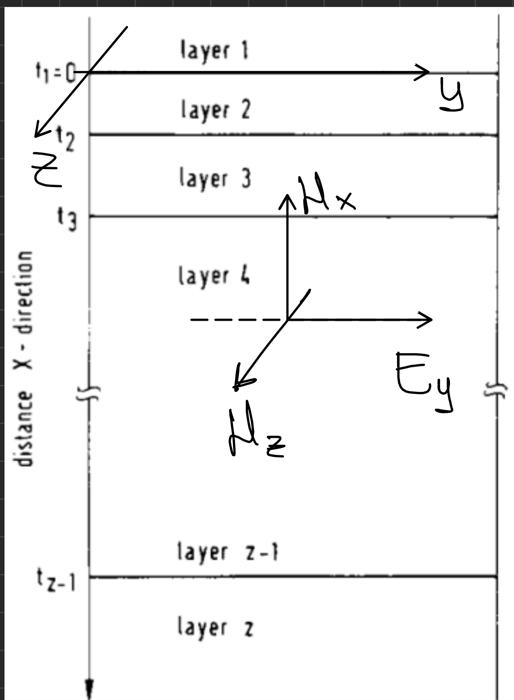
e

$$f_{c_{TM}}^m \geq f_{c_{TE}}^m$$



TRANSMISSION MATRIX METHOD – TMM

CONSIDERANDO UM GUIA DIELÉTRICO COMPOSTO DE Z CAMADAS, ONDE A CAMADA j POSSUI ÍNDICE n_j.



A COMPONENTE y É DADA POR:

$$E_y(x,z,t) = E_y(x)\, e^{wt - z\beta}$$

$$\frac{d E_y^j(x)}{dx^2} + (\underbrace{k_0^2 n_j^2 - \beta^2}_{k_j^2}) E_y^j(x) = 0$$

$$k_j = \sqrt{k_0^2 n_j^2 - \beta^2}$$

$$k_j = \left(-1\left(\beta^2 - k_0^2 n_j^2\right)\right)^{1/2}$$

$$r^2 + k_j^2 = 0 \qquad k_j = j\alpha_j$$

$$r = \pm j k_j \qquad \alpha_j = \sqrt{\beta^2 - k_0^2 n_j^2}$$

$$r = \pm \alpha_j$$

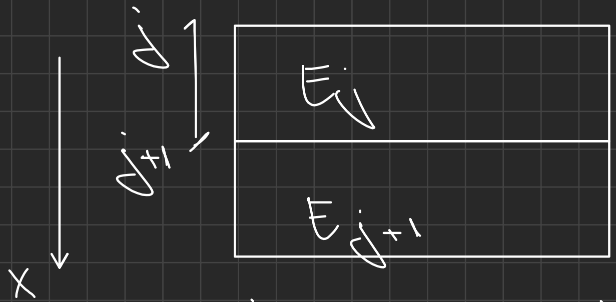
Logo, $E_y^j(x) = A_j e^{\alpha_j x} + B_j e^{-\alpha_j x}$

PARA CADA CAMADA j:

$$E_y^j(x) = A_j e^{\alpha_j (x - t_j)} + B_j e^{-\alpha_j (x - t_j)}$$

NO MODO TE, AO APLICAR AS CONDIÇÕES DE CONTORNO:

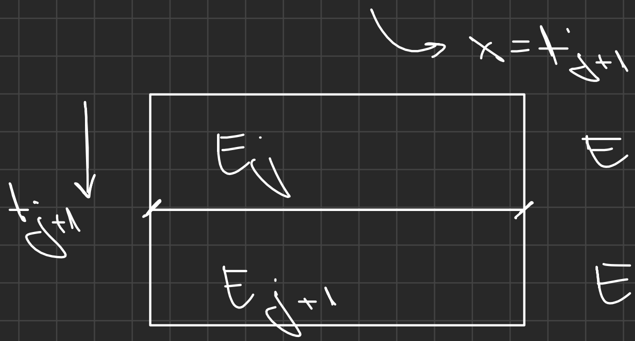
(1) $E_j(t_{j+1}) = E_{j+1}(t_{j+1})$ CONTÍNUO



DIFERENCIÁVEL

(2) $\left.\frac{dE_y^j}{dx}\right|_{t_{j+1}} = \left.\frac{dE_y^{j+1}}{dx}\right|_{t_{j+1}}$

DESSA MANEIRA, OS COEFICIENTE DA j-ÉSIMA CAMADA DEVEM SER IGUAIS AOS COEFICIENTES DA CAMADA SEGUINTE, NO PONTO QUE AS SEPARAM, OU SEJA,



$$E_j = A_j e^{\alpha_j d_1} + B_j e^{-\alpha_j d_1}$$

$$E_{j+1} = A_{j+1} e^{\alpha_{j+1} \cdot 0} + B_{j+1} e^{-\alpha_{j+1} \cdot 0}$$

COND. (1) $\quad A_j e^{\tau_j} + B_j e^{-\tau_j} = A_{j+1} + B_{j+1}$

O MESMO DEVE OCORRER PARA A DERIVADA

$$E_j(x) = A_j e^{\alpha_j (x - t_j)} + B_j e^{-\alpha_j (x - t_j)}$$

$$E_{j+1}(x) = A_{j+1} e^{\alpha_{j+1}(x - t_j)} + B_{j+1} e^{-\alpha_{j+1}(x - t_{j+1})}$$

$$E_j'(x) = A_j \alpha_j e^{\alpha_j (x - t_j)} - B_j \alpha_j e^{-\alpha_j (x - t_j)}$$

$$E_{j+1}'(x) = A_{j+1} \cdot \alpha_{j+1} e^{\alpha_{j+1}(x - t_{j+1})} - B_{j+1} \alpha_{j+1} e^{-\alpha_{j+1}(x - t_{j+1})}$$

NO PONTO QUE SEPARA AMBOS, ($x = t_{j+1}$) DEVEM SER IGUAIS PARA MANTER-SE DIFERENCIÁVEL.

$$A_j \alpha_j e^{\alpha_j d} - B_j \alpha_j e^{-\alpha_j d}$$

$$= A_{j+1} \cdot \alpha_{j+1} - B_{j+1} \alpha_{j+1}$$

COND. 2 $A_j\left(\frac{\alpha_j}{\alpha_{j+1}}\right) e^{\sigma_j} - B_j\left(\frac{\alpha_j}{\alpha_{j+1}}\right) e^{-\sigma_j} = A_{j+1} - B_{j+1}$

VALE DESTACAR A PRESENÇA DO TERMO "d" NO EXPONENCIAL. COMO ANALISA-SE O LIMIAR DAS CAMADAS j E $j+1$, "d" É A ESPESSURA DA CAMADA j.

SOMANDO E SUBTRAÍNDO AMBAS EQUAÇÕES AZUIS E ISOLANDO A_{j+1} E B_{j+1}, RESPECTIVAMENTE:

$$(+)\cdot \quad 2A_{j+1} = A_j\left(\frac{\alpha_j}{\alpha_{j+1}}\right) e^{\sigma_j} + A_j e^{\sigma_j}$$

$$- B_j\left(\frac{\alpha_j}{\alpha_{j+1}}\right) e^{-\sigma_j} + B_j e^{-\sigma_j}$$

$$A_{j+1} = A_j/2 \left[\left(\frac{\alpha_j}{\alpha_{j+1}} + 1\right) \cdot e^{\sigma_j}\right]$$

$$+ B_j/2 \left[\left(1 - \frac{\alpha_j}{\alpha_{j+1}}\right) e^{-\sigma_j}\right]$$

$$\ominus .\quad 2B_{j+1} = -A_j\left(\frac{x_j}{x_{j+1}}\right)e^{\sigma_j} + A_j e^{\sigma_j}$$

$$+ B_j\left(\frac{x_j}{x_{j+1}}\right)e^{-\sigma_j} + B_j e^{-\sigma_j}$$

$$B_{j+1} = A_j/2\left[\left(1 - \frac{x_j}{x_{j+1}}\right)e^{\sigma_j}\right]$$

$$+ B_j/2\left[\left(1 + \frac{x_j}{x_{j+1}}\right)e^{-\sigma_j}\right]$$

Como matriz:

$$\begin{bmatrix} A_{j+1} \\ B_{j+1} \end{bmatrix} = 1/2 \begin{bmatrix} \left(\frac{x_j}{x_{j+1}} + 1\right)e^{\sigma_j} & \left(1 - \frac{x_j}{x_{j+1}}\right)e^{-\sigma_j} \\ \left(1 - \frac{x_j}{x_{j+1}}\right)e^{\sigma_j} & \left(\frac{x_j}{x_{j+1}} + 1\right)e^{-\sigma_j} \end{bmatrix} \begin{bmatrix} A_j \\ B_j \end{bmatrix}$$

$$\downarrow$$

MATRIZ DE TRANSFORMAÇÃO

$$\begin{bmatrix} A \\ B \end{bmatrix}_{j+1} = \begin{bmatrix} T \end{bmatrix}_j \begin{bmatrix} A \\ B \end{bmatrix}_j$$

OU AINDA:

$$\begin{bmatrix} A \\ B \end{bmatrix}_j = \begin{bmatrix} T \end{bmatrix}_{j-1} \begin{bmatrix} A \\ B \end{bmatrix}_{j-1}$$

$$\begin{bmatrix} A \\ B \end{bmatrix}_j = \begin{bmatrix} T \end{bmatrix}_{j-1} \begin{bmatrix} T \end{bmatrix}_{j-2} \begin{bmatrix} A \\ B \end{bmatrix}_{j-2}$$

$$\begin{bmatrix} A \\ B \end{bmatrix}_j = \begin{bmatrix} T \end{bmatrix}_{j-1} \begin{bmatrix} T \end{bmatrix}_{j-2} \begin{bmatrix} T \end{bmatrix}_{j-3} \cdots \begin{bmatrix} T \end{bmatrix}_1 \begin{bmatrix} A \\ B \end{bmatrix}_1$$

PARA A CAMADA Z (ÚLTIMA CAMADA):

$$\begin{bmatrix} A \\ B \end{bmatrix}_Z = \underbrace{\begin{bmatrix} T \end{bmatrix}_{Z-1} \begin{bmatrix} T \end{bmatrix}_{Z-2} \begin{bmatrix} T \end{bmatrix}_{Z-3} \cdots \begin{bmatrix} T \end{bmatrix}_1}_{\begin{bmatrix} T \end{bmatrix}_{WG}} \begin{bmatrix} A \\ B \end{bmatrix}_1$$

PARA DETERMINAR $\begin{bmatrix} T \end{bmatrix}_Z$ E $\begin{bmatrix} T \end{bmatrix}_1$, LEMBRE:

PRIMEIRA E ÚLTIMA CAMADA

$$E_y^j(x) = A_j e^{\alpha_j(x-t_j)} + B_j e^{-\alpha_j(x-t_j)}$$

CAMADA 1: $E_y^1(x) = A_1 e^{\alpha_1 x} + B_1 e^{-\alpha_1 x}$

$$\lim_{x \to -\infty} E_y^1(x) = 0, \text{ Logo } A_1 \neq 0 \text{ e } B_1 = 0$$

$$A_1 = a \text{ e } B_1 = 0$$

CAMADA 2: $E_y^z(x) = A_z e^{\alpha_z(x-t_z)} + B_z e^{-\alpha_z(x-t_z)}$

$$\lim_{x \to \infty} E_y^z(x) = 0, \text{ Assim } A_z = 0 \text{ e } B_z \neq 0$$

$$A_z = 0 \text{ e } B_z = b$$

Em OUTRAS PALAVRAS:

$$\begin{bmatrix} A \\ B \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} a$$

e

$$\begin{bmatrix} A \\ B \end{bmatrix}_z = \begin{bmatrix} 0 \\ 1 \end{bmatrix} b$$

$$\begin{bmatrix} A \\ B \end{bmatrix}_z = \underbrace{\begin{bmatrix} T \end{bmatrix}_{z-1} \begin{bmatrix} T \end{bmatrix}_{z-2} \begin{bmatrix} T \end{bmatrix}_{z-3} \cdots \begin{bmatrix} T \end{bmatrix}_1}_{\begin{bmatrix} T \end{bmatrix}_{WG}} \begin{bmatrix} A \\ B \end{bmatrix}_1$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} b = [T]_{WG} \begin{bmatrix} 1 \\ 0 \end{bmatrix} a$$

Como $Q.L.C \{[T]_{z-1}\} = Q.L.C \{[T]_{z-2}\} =$

$Q.L.C \{[T]_{z-3}\} = \dots Q.L.C \{[T]_{1}\} = \{2,2\}$ ENTÃO

$[T]_{z-1}[T]_{z-2}[T]_{z-3} \cdots [T]_{1}$ RESULTA EM UMA

MATRIZ $[T]_{WG}$ CUJA $Q.L.C \{[T]_{WG}\} = \{2,2\}$

(*) Q.L.C → QUANTIDADE DE LINHAS e COLUNAS

Assim,

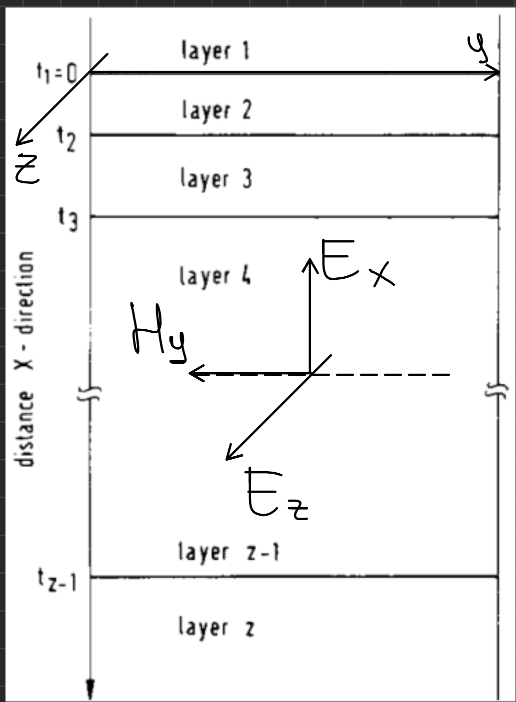
$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} b = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} a$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} b = \begin{bmatrix} t_{11} \\ t_{21} \end{bmatrix} a$$

$$t_{11}\, a = 0 \quad e \quad t_{21}\, a = b$$

$$t_{11}(\beta) = 0.$$

MÉTODO TMM PARA MODO TM.



$$\frac{d^2 H_y^i(x)}{dx^2} + (k_0^2 n_i^2 - \beta^2) H_y^i(x) = 0$$

$$\alpha_i = \sqrt{\beta^2 - k_0^2 n_i^2}$$

$$H_y^i(x) = A_i e^{\alpha_i x} + B_i e^{-\alpha_i x}$$

As condições de contorno são:

$$① \quad H_y^{i}(t_{i+1}) = H_y^{i+1}(t_{i+1})$$

$$② \quad E_z^{i}(t_{i+1}) = E_z^{i+1}(t_{i+1})$$

$$n_i^2 = \varepsilon_i \varepsilon_0$$

$$E_z = -j\frac{\beta}{K_i^2} \cdot \frac{1}{\eta_{TM}} \frac{dH_y}{dx}$$

$$\frac{1}{\eta_{TM}} = \frac{\omega\varepsilon}{\beta}$$

$$-\frac{\cancel{j\beta}}{K_i^2} \cdot \frac{1}{\eta_{TM\,i}} \frac{d}{dx}\left(A_i e^{\alpha_i(x-t_i)} + B_i e^{-\alpha_i(x-t_i)}\right)\Big|_{t_{i+1}}$$

$$= -\frac{\cancel{j\beta}}{K_{i+1}^2} \cdot \frac{1}{\eta_{TM\,i+1}} \cdot \frac{d}{dx}\left(A_{i+1} e^{\alpha_{i+1}(x-t_{i+1})} + B_{i+1} e^{-\alpha_{i+1}(x-t_{i+1})}\right)\Big|_{t_{i+1}}$$

$$\frac{\omega \underbrace{\varepsilon_i}_{\frac{n_i^2}{\varepsilon_0}}}{\cancel{\beta}} \cdot \frac{1}{{}_2K_i^2} \cdot \left(A_i \alpha_i e^{\alpha_i(x-t_i)} - \alpha_i B_i e^{-\alpha_i(x-t_i)}\right)\Big|_{t_{i+1}}$$

$$= \frac{\omega \underbrace{\varepsilon_{i+1}}_{\frac{n_{i+1}}{\varepsilon_0}}}{\cancel{\beta}} , \frac{1}{K_{i+1}^2}\left(A_{i+1} \alpha_{i+1} \cancelto{1}{e^{\alpha_{i+1}(x-t_{i+1})}} - \alpha_{i+1} B_{i+1} \cancelto{1}{e^{-\alpha_{i+1}(x-t_{i+1})}}\right)\Big|_{t_{i+1}}$$

$$\frac{n_i^2}{\underbrace{K_i^2}_{-\alpha_i^2}}\left(A_i \alpha_i e^{\alpha_i d} - \alpha_i B_i e^{-\alpha_i d}\right) = \frac{n_{i+1}^2}{\underbrace{K_{i+1}^2}_{-\alpha_{i+1}^2}}\left(A_{i+1} \alpha_{i+1} - B_{i+1} \alpha_{i+1}\right)$$

$$\frac{\alpha_i}{\alpha_{i+1}} \cdot \left(\frac{n_i}{n_{i+1}}\right)^2 \left(A_i e^{\sigma_i} - B_i e^{-\sigma_i}\right) = A_{i+1} - B_{i+1}$$

EQ. 1 :

$$A_i e^{\sigma_i} + B_i e^{-\sigma_i} = A_{i+1} + B_{i+1}$$

EQ. 2 :

$$\frac{\alpha_i}{\alpha_{i+1}} \cdot \left(\frac{n_i}{n_{i+1}}\right)^2 \left(A_i e^{\sigma_i} - B_i e^{-\sigma_i}\right) = A_{i+1} - B_{i+1}$$

SOMANDO 1 e 2 :

$$A_{i+1} = \frac{A_i}{2}\left(\left(1 + \frac{\alpha_i}{\alpha_{i+1}} \cdot \left(\frac{n_i}{n_{i+1}}\right)^2\right) e^{\sigma_i}\right)$$

$$+ \frac{B_i}{2}\left(\left(1 - \frac{\alpha_i}{\alpha_{i+1}} \cdot \left(\frac{n_i}{n_{i+1}}\right)^2\right) e^{-\sigma_i}\right)$$

SUBTRAINDO 1 e 2 :

$$B_{i+1} = \frac{A_i}{2}\left(\left(1 - \frac{\alpha_i}{\alpha_{i+1}} \cdot \left(\frac{n_i}{n_{i+1}}\right)^2\right) e^{\sigma_i}\right)$$

$$+ \frac{B_i}{2}\left(\left(1 + \frac{\alpha_i}{\alpha_{i+1}} \cdot \left(\frac{n_i}{n_{i+1}}\right)^2\right) e^{-\sigma_i}\right)$$

$$P_i = \left(\frac{n_i}{n_{i+1}}\right)^2$$

$$A_{i+1} = \frac{A_i}{2}\left(\left(1 + \frac{x_i}{x_{i+1}} \cdot P_i\right)e^{\gamma_i}\right) + \frac{B_i}{2}\left(\left(1 - \frac{x_i}{x_{i+1}} \cdot P_i\right)e^{-\gamma_i}\right)$$

$$B_{i+1} = \frac{A_i}{2}\left(\left(1 - \frac{x_i}{x_{i+1}} \cdot P_i\right) e^{\gamma_i}\right) + \frac{B_i}{2}\left(\left(1 + \frac{x_i}{x_{i+1}} \cdot P_i\right)e^{-\gamma_i}\right)$$

$$\begin{bmatrix} A_{i+1} \\ B_{i+1} \end{bmatrix} = \frac{1}{2} \underbrace{\begin{bmatrix} \left(1 + \frac{x_i}{x_{i+1}} \cdot P_i\right)e^{\gamma_i} & \left(1 - \frac{x_i}{x_{i+1}} \cdot P_i\right)e^{-\gamma_i} \\ \left(1 - \frac{x_i}{x_{i+1}} \cdot P_i\right) e^{\gamma_i} & \left(1 + \frac{x_i}{x_{i+1}} \cdot P_i\right)e^{-\gamma_i} \end{bmatrix}}_{[T]_i} \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix}_i = [T]_{i-1} \begin{bmatrix} A \\ B \end{bmatrix}_{i-1}$$

Para $i = z$:

$$\begin{bmatrix} A \\ B \end{bmatrix}_z = \underbrace{[T]_{z-1} [T]_{z-2} [T]_{z-3} \cdots [T]_1}_{[T]_{WG}} \begin{bmatrix} A \\ B \end{bmatrix}_1$$

DESSA MANEIRA:

$$\begin{bmatrix} A \\ B \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} a$$

e

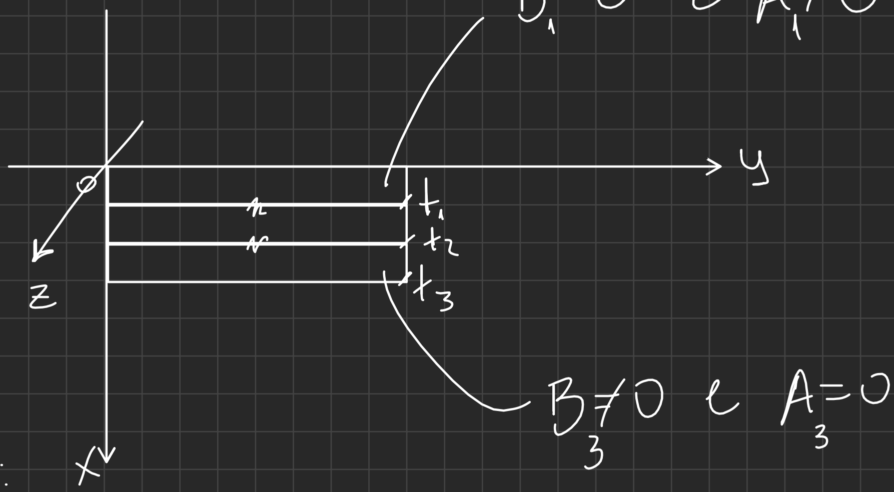
$$\begin{bmatrix} A \\ B \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} b$$

E ASSIM:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} b = \underbrace{[T]_{WG}}_{\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} a \quad , a \text{ e } b \neq 0$$

$$\begin{cases} t_{11} a = 0 \\ t_{21} a = b \end{cases} \longrightarrow t_{11}(\beta) = 0$$

Ex3:



Modo TM:

$$\begin{bmatrix} A_{j+1} \\ B_{j+1} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \left(\frac{\alpha_j P_j}{\alpha_{j+1}}+1\right)e^{\gamma_j} & \left(1-\frac{P_j\alpha_j}{\alpha_{j+1}}\right)e^{-\gamma_j} \\ \left(1-\frac{\alpha_j P_j}{\alpha_{j+1}}\right)e^{\gamma_j} & \left(\frac{\alpha_j P_j}{\alpha_{j+1}}+1\right)e^{-\gamma_j} \end{bmatrix}\begin{bmatrix} A_j \\ B_j \end{bmatrix}$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \left(\frac{\alpha_1}{\alpha_2}\cdot P_1+1\right)e^{\gamma_1} & \left(-\frac{\alpha_1}{\alpha_2}\cdot P_1+1\right)e^{-\gamma_1} \\ \left(-\frac{\alpha_1}{\alpha_2}\cdot P_1+1\right)e^{\gamma_1} & \left(\frac{\alpha_1}{\alpha_2}\cdot P_1+1\right)e^{-\gamma_1} \end{bmatrix}\begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(\frac{\alpha_1}{\alpha_2}\cdot P_1+1\right)e^{\gamma_1}\cdot a \\ \frac{1}{2}\left(-\frac{\alpha_1}{\alpha_2}\cdot P_1+1\right)e^{\gamma_1}\cdot a \end{bmatrix}$$

$$\begin{bmatrix} A_3 \overset{=0}{} \\ B_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \left(\frac{\alpha_2}{\alpha_3}\cdot P_2+1\right)e^{\gamma_2} & \left(-\frac{\alpha_2}{\alpha_3}\cdot P_2+1\right)e^{-\gamma_2} \\ \left(-\frac{\alpha_2}{\alpha_3}\cdot P_2+1\right)e^{\gamma_2} & \left(\frac{\alpha_2}{\alpha_3}P_2+1\right)e^{-\gamma_2} \end{bmatrix}\begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$$

$$0 = \frac{1}{2}\left\{ \left(\frac{x_2}{x_3}\cdot P_2 + 1\right) e^{\sigma_2} \cdot \frac{1}{2}\left(\frac{x_1}{x_2}\cdot P_1 + 1\right) e^{\sigma_1}\cdot a \right.$$
$$\left. + \left(-\frac{x_2}{x_3}\cdot P_2+1\right)e^{-\sigma_2}\cdot\frac{1}{2}\left(-\frac{x_1}{x_2}\cdot P_1+1\right)e^{\sigma_1}\cdot a\right\} \times 4/a$$

$$\left(\frac{x_2}{x_3}\cdot P_2 + 1\right) e^{\sigma_2} \left(\frac{x_1}{x_2}\cdot P_1 + 1\right) \cancel{e^{\sigma_1}}\ 1$$
$$+ \left(-\frac{x_2}{x_3}\cdot P_2+1\right)e^{-\sigma_2}\left(-\frac{x_1}{x_2}\cdot P_1+1\right)\cancel{e^{\sigma_1}}\ 1 = 0$$

$$\left(\frac{x_2}{x_3}\cdot P_2 + 1\right)\left(\frac{x_1}{x_2}\cdot P_1 + 1\right) e^{\sigma_2} = -\left(-\frac{x_1}{x_2}\cdot P_1+1\right)\left(-\frac{x_2}{x_3}\cdot P_2+1\right)e^{-\sigma_2}$$