Report

Virtual joint modeling

Inverce kinematic of robot

$$\cos(q2) = \frac{x^2 + y^2 - {l_1}^2 - {l_2}^2}{2 * l_1 * l_2}$$

$$\sin(q2) = \sqrt{(1 - \cos(q2) ** 2)}$$

$$q2 = atan2(\sin(q2), \cos(q2))$$

$$q1 = atan2(x, y) - atan2(l_1 * \sin(q2), l_1 + l_2 * \cos(q2))$$

$$q3 = -(q1 + q2)$$

Forward kinematic of robot

$$T = T_{base_i} T_z(d_i) T_z(\theta_{i,1}) R_z(q_{i,1}) T_x(l_{i,1}) T_{3D}(\theta_{i,2-7}) \\ R_z(q_{i,2}) T_x(l_{i,2}) T_{3D}(\theta_{i,8-13}) R_z(q_{i,3}) T_{tool_i}$$

Where $\theta_{i,j}$ is j^{th} virtual joint of i^{th} leg

$$T_{3D}(\theta_{i,j-(j+5)}) = T_x(\theta_{i,j}) T_y(\theta_{i,j+1}) T_z(\theta_{i,j+2}) \\ R_x(\theta_{i,j+3}) R_y(\theta_{i,j+4}) R_z(\theta_{i,j+5})$$

For each leg $K_{c,i}^0 = \left(J_{\theta,i} K_{\theta,i}^{-1} J_{\theta,i}^T\right)^{-1}$ where J_θ is Jacobian with respect to virtual joint variables and K_θ is aggregated stiffness matrix

```
Ka,0,0,0,0,0,0,0,0,0,0,0,0
 0,                0,0,0,0,0,0
 0,                0,0,0,0,0,0
 0,      K22       0,0,0,0,0,0
 0,                0,0,0,0,0,0
 0,                0,0,0,0,0,0
 0,                0,0,0,0,0,0
 0,0,0,0,0,0,0,
 0,0,0,0,0,0,0,
 0,0,0,0,0,0,0,      K22,
 0,0,0,0,0,0,0,
 0,0,0,0,0,0,0,
 0,0,0,0,0,0,0,
```

where

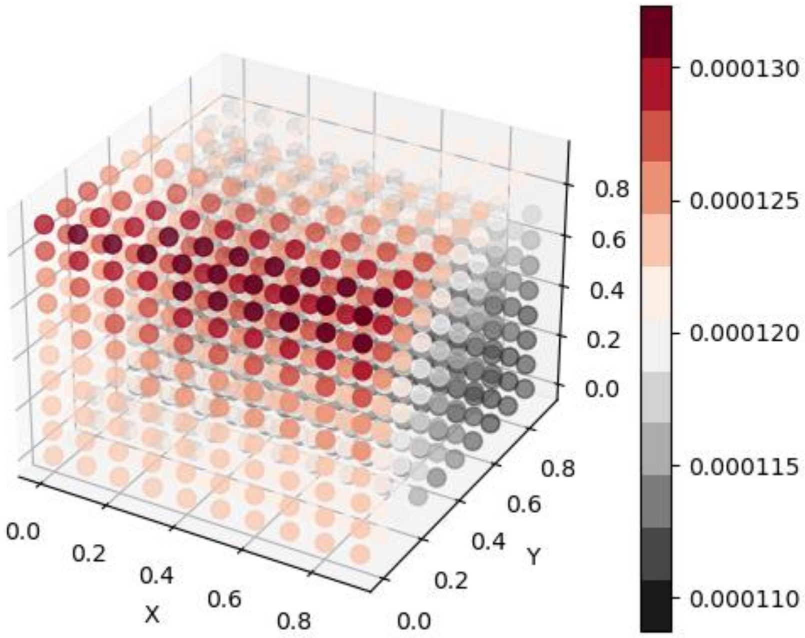
$$K_{22} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI_\rho}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\ 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix}$$

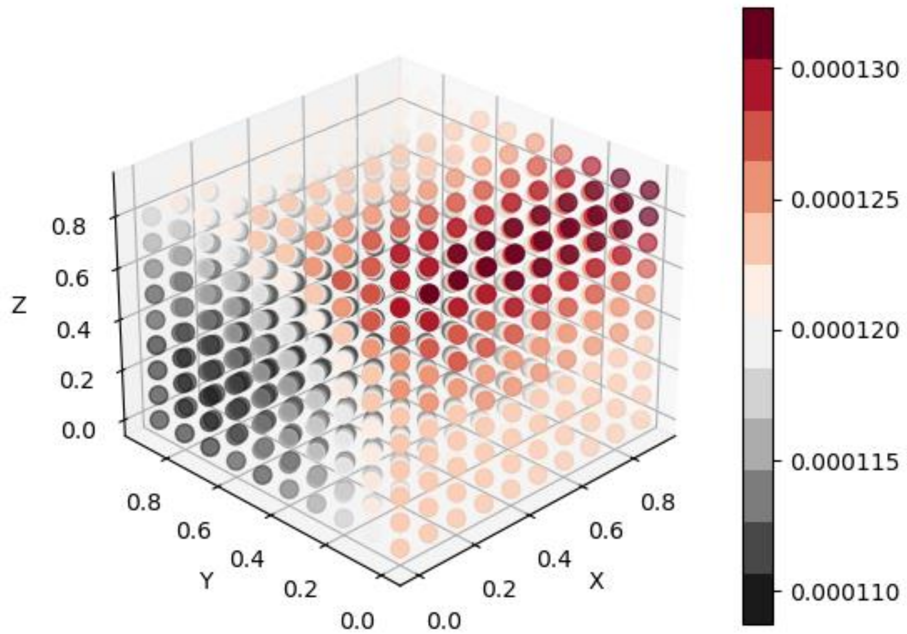
Cartesian stiffness matrix of leg $K_{c,i} = K_{c,i}^0 - K_{c,i}^0 J_{q,i} K_{Cq,i}$

where $K_{Cq,i} = \left(J_{q,i}^T (K_{C,i}^0)^{-1} J_{q,i}\right)^{-1} J_{q,i}^T (K_{C,i}^0)^{-1}$

J_q is Jacobian with respect to the passive joint variables

Cartesian stiffness matrix of whole robot equal sum of $K_{c,i.}$ and $W = K_c \Delta t$





Result

Link to github https://github.com/EriKarasik/ARHW2