$X(t) = A\,\mathrm{Sin}(\pi F_0 t), \quad 0 \le t \le \tau, \quad F_0 = 1/\tau$

4.1 a

$$C_k = \frac{1}{T_p}\int_{T_p} X(t)\, e^{-j2\pi k F_0 t}\, dt$$

$$= \frac{A}{T_p}\int_0^{\tau} \mathrm{Sin}(\pi F_0 t)\, e^{-j2\pi k F_0 t}\, dt$$

$$= \frac{A}{T_p}\int_0^{\tau} \left(\frac{1}{2j}\right)\left(e^{j\pi F_0 t} - e^{-j\pi F_0 t}\right) e^{-j2\pi k F_0 t}\, dt$$

$$= \frac{AF_0}{2j}\int_0^{\tau} \left(e^{(j\pi F_0 t - j2\pi k F_0 t)} - e^{(-j\pi F_0 t - j2\pi k F_0 t)}\right) dt$$

$$= \frac{AF_0}{2j}\int_0^{\tau} e^{(1-2k)j\pi F_0 t} - e^{(-1-2k)j\pi F_0 t}\, dt$$

$$= \frac{AF_0}{2j}\left[\frac{1}{(1-2k)j\pi F_0} e^{(1-2k)j\pi F_0 t} - \frac{1}{(-1-2k)j\pi F_0} e^{(-1-2k)j\pi F_0 t}\right]_0^{\tau}$$

$$= \frac{AF_0}{2j(j\pi F_0)}\left[\frac{e^{(1-2k)j\pi F_0 t}}{1-2k} + \frac{e^{(-1-2k)j\pi F_0 t}}{1+2k}\right]_0^{\tau}$$

$$= \frac{A}{-2\pi}\left[(1-4k^2)^{-1}\left((1+2k)e^{(1-2k)j\pi F_0 t} + (1-2k)e^{(-1-2k)j\pi F_0 t}\right)\right]_0^{\tau}$$

$$= \frac{A}{-2\pi(1-4k^2)}\left[(1+2k)\left(e^{(1-2k)j\pi} - 1\right) + (1-2k)\left(e^{(-1-2k)j\pi F_0} - 1\right)\right]$$

$$= \frac{-A}{2\pi(1-4k^2)}\left[(1+2k)\left(e^{j\pi}e^{-2\pi kj} - 1\right) + (1-2k)\left(e^{-j\pi}e^{-2\pi kj} - 1\right)\right]$$

$$= \frac{-A}{2\pi(1-k^2 4)}\left[(-1-2k) + (-1+2k) + (1+2k)e^{(1-2k)j\pi} + (1-2k)e^{(-1-2k)j\pi}\right]$$

$$= \frac{-A}{2\pi(1-k^2 4)}\left[-2 + e^{(1-2k)j\pi} + 2k e^{(1-2k)j\pi} + e^{(-1-2k)j\pi} - 2k e^{(-1-2k)j\pi}\right]$$

$$= \frac{-A}{2\pi(1-4k^2)}\left[-2 + e^{-2kj\pi}\underbrace{\left(e^{j\pi} + e^{-j\pi}\right)}_{2\mathrm{Cos}(\pi) = 2} + 2k e^{-2\pi jk}\underbrace{\left(e^{j\pi} - e^{-j\pi}\right)}_{j2\,\mathrm{Sin}(\pi) = 0}\right]$$

$$= \frac{-A}{2\pi(1-4k^2)}\left[-2 - 2e^{-2kj\pi}\right] = \frac{-A}{2\pi(1-4k^2)}\left[-2 - 2\left(\underbrace{\mathrm{Cos}(-2\pi k)}_{\to 1} + \underbrace{j\,\mathrm{Sin}(-2\pi k)}_{\to 0}\right)\right]$$

$$= \frac{-A}{2\pi(1-4k)}\left[-2-2\right] = \boxed{\frac{2A}{\pi(1-4k^2)}}$$

$P_x = \frac{1}{T_p}\int_{T_p} |x(t)|^2 dt \qquad x(t) = A\sin(\pi F_0 t),\ 0 \le t \le \tau$

$$= F_0 \int_0^\tau |A\sin(\pi F_0 t)|^2 dt$$

$$= F_0 A^2 \int_0^\tau \sin^2(\pi F_0 t)\, dt$$

$$= F_0 A^2 \left[\frac{t}{2} - \frac{\sin(2\pi F_0 t)}{4\pi F_0}\right]_0^\tau$$ Using integral calculator

$$= F_0 A^2 \left[\left(\frac{\tau}{2} - \frac{\sin(2\pi)}{4\pi F_0}\right) - \left(\frac{0}{2} - \frac{\sin(0)}{4\pi F_0}\right)\right]$$

$$= F_0 A^2 \left[\frac{\tau}{2}\right]$$

$$\boxed{P_x = \frac{A^2}{2}}$$

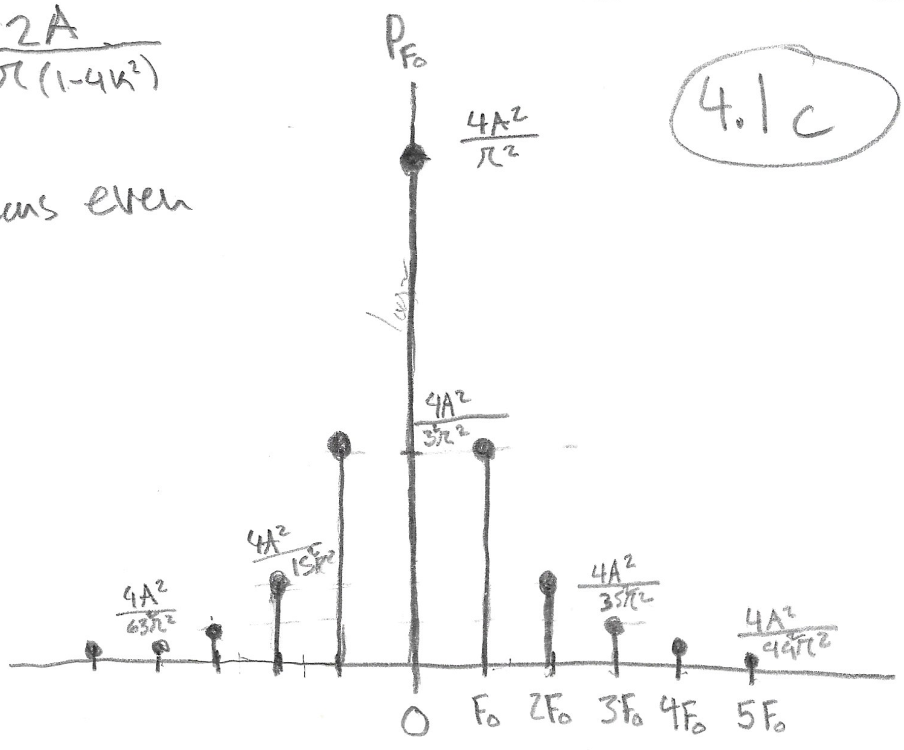
4.1 b

$P_x = |C_k|^2, \quad C_k = \frac{2A}{\pi(1-4k^2)}$

$P_x = \left|\frac{2A}{\pi(1-4k^2)}\right|^2$, k^2 means even

4.1c

K	$P_x/4A^2$
0	$1/\pi^2$
±1	$1/3^2\pi^2$
±2	$1/15^2\pi^2$
±3	$1/35^2\pi^2$
±4	$1/63^2\pi^2$
±5	$1/99^2\pi^2$



Parsevals Relation: $\frac{1}{T_P}\int_{T_P}|x(t)|^2dt = \sum_{k=-\infty}^{\infty}|C_k|^2$

From question 4.1a we have $\frac{1}{T_P}\int_{T_P}|x(t)|^2dt = \frac{A^2}{2}$

4.1 d

So we need to solve

$$\sum_{k=-\infty}^{\infty}\left|\frac{2A}{\pi(1-4k^2)}\right|^2$$

$$= |C_0|^2 + 2\sum_{k=1}^{\infty}\left|\frac{2A}{\pi(1-4k^2)}\right|^2$$

$$= |C_0|^2 + \frac{8A^2}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(1-4k^2)^2}$$

$$= |C_0|^2 + \frac{8A^2}{\pi^2}\sum_{1}^{\infty}\frac{1}{1-8k^2+16k^4}$$

↳ Calculator for $\sum_1^{100} \rightarrow .11685\ldots$

$$= \frac{4A^2}{\pi^2} + A^2 \cdot .094715249$$

$$= \frac{(4+.934802)A^2}{\pi^2}$$

$$= .499999833\,A^2 \approx \frac{A^2}{2}$$

$$\boxed{\text{So } \frac{1}{T_P}\int_{T_P}|x(t)|^2dt = \sum_{k=-\infty}^{\infty}|C_k|^2}$$

OOPS! Just saw that $\sum_{k-\infty}^{\infty}\frac{1}{(1-4k^2)^2} = \pi^2/8$, that would have made life easier.

$$X(F) = A\int_0^\infty e^{-at} e^{-j2\pi F t} dt$$

$$= A\int_0^\infty e^{(-a-j2\pi F)t} dt$$

$$= \frac{A}{-a-j2\pi F}\left[e^{(-a-j2\pi F)t}\right]_0^\infty$$

$$= \frac{A}{-a-j2\pi F}\left[0 - 1\right]$$

4.2a

$$\boxed{X(F) = \frac{A}{a+j2\pi F}}$$

- Multiply top/bottom by cplx*

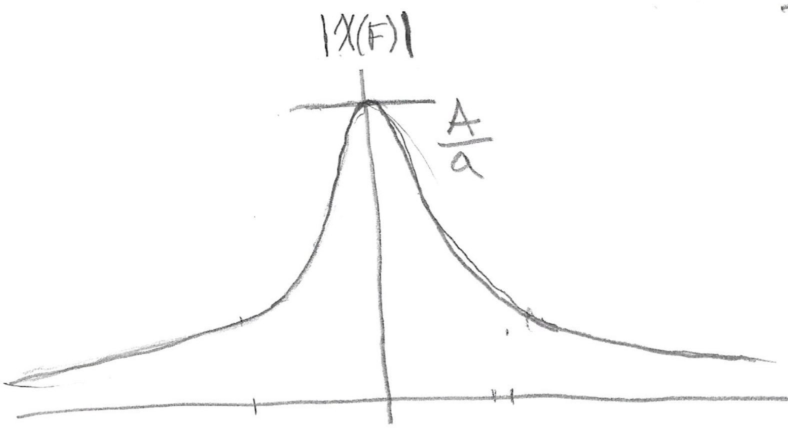
$$X(F) = \frac{A(a-j2\pi F)}{a^2+2\pi F}$$

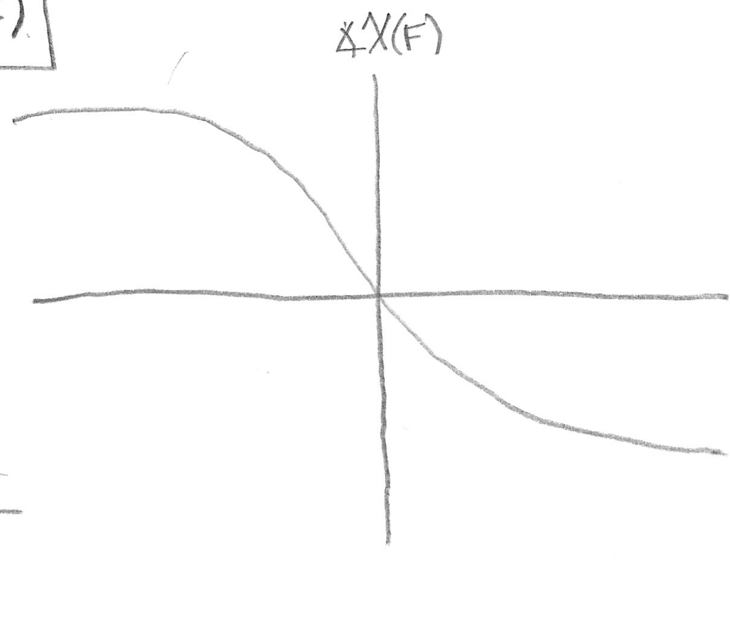
So $X_R(F) = (a^2+2\pi F)^{-1}(Aa)$

and $X_I(F) = (a^2+2\pi F)^{-1}(-2\pi F A)$

So $|X(F)| = \dfrac{\sqrt{A^2a^2 + 4\pi^2F^2A^2}}{a^2+2\pi F}$

and $\measuredangle X(F) = \arctan(a/-2\pi F)$





$$\boxed{4.2\text{ b}}$$

$$X(F) = \int_{-\infty}^{\infty} A e^{-a|t|} e^{-j2\pi F t}\, dt, \quad a > 0$$

$$= A\left(\int_0^{\infty} e^{(-a-j2\pi F)t}\, dt + \int_{-\infty}^{0} e^{(a-j2\pi F)t}\, dt\right)$$

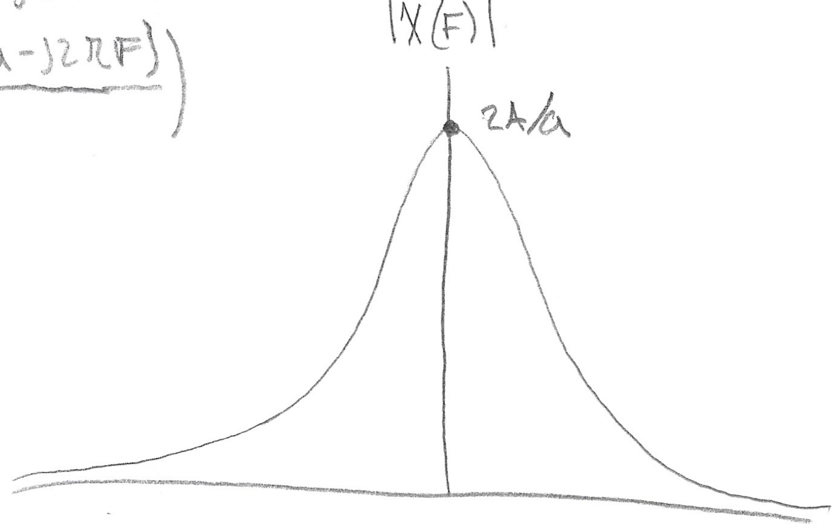
$$= A\left((-a-j2\pi F)^{-1}\left[e^{(-a-j2\pi F)t}\right]_0^{\infty} + (a-j2\pi F)^{-1}\left[e^{(a-j2\pi F)t}\right]_{-\infty}^{0}\right)$$

$$= A\left(\frac{0-1}{-a-j2\pi F} + \frac{1-0}{a-j2\pi F}\right)$$

$$= A\left(\frac{(-a+j2\pi F)+(-a-j2\pi F)}{-a^2-4\pi^2F^2}\right)$$

$$= A\left(\frac{-2a}{-a^2-4\pi^2F^2}\right)$$

$$\boxed{X(F) = \frac{2Aa}{a^2+4\pi^2F^2}}$$



$$X_{R}(F) = \frac{2Aa}{a^2+4\pi^2F^2}$$

$$X_{I}(F) = 0$$

$$\boxed{\begin{aligned} |X(F)| &= \frac{2Aa}{a^2+4\pi^2F^2} \\ \measuredangle X(F) &= 0 \end{aligned}}$$