1)

ⓐ $\boxed{H(z) = H_1(z)\left(H_2(z) - H_3(z)H_4(z)\right)}$

$$H_1(z) = Z\left\{\tfrac{1}{2}\delta(n) + \tfrac{1}{4}\delta(n-1) + \delta(n-2)\right\}$$

$$\boxed{H_1(z) = \tfrac{1}{2} + \tfrac{1}{4}z^{-1} + z^{-2},\ ROC: z \neq 0}$$

$$H_2(z) = H_3(z) = Z\{n\,u(n) + u(n)\}$$

$$= \frac{z^{-1}}{(1-z^{-1})^2} + \frac{1}{1-z^{-1}}$$

$$= \frac{z^{-1} + (1-z^{-1})}{(1-z^{-1})^2}$$

$$\boxed{H_2(z) = H_3(z) = \frac{1}{(1-z^{-1})^2},\ ROC: z > 1}$$

$$H_4(z) = Z\{\delta(n-2)\}$$

$$\boxed{H4(z) = z^{-2},\ ROC: z \neq 0}$$

$$\boxed{H_{12}(z) = H_1(z)(H_2(z)) = \frac{\frac{1}{2} + \frac{1}{4}z^{-1} + z^{-2}}{(1-z^{-1})^2}}\ ROC: z > 1$$

$$\boxed{H_{34}(z) = H_3(z)H_4(z) = \frac{z^{-2}}{(1-z^{-1})^2},\ ROC: z > 1}$$

$$\boxed{H_{134}(z) = H_1(z)H_{34}(z) = \frac{\frac{1}{2}z^{-2} + \frac{1}{4}z^{-3} + z^{-4}}{(1-z^{-1})^2},\ ROC: z > 1}$$

$$H(z) = H_{12}(z) - H_{134}(z) = \frac{\frac{1}{2} + \frac{1}{4}z^{-1} + z^{-2} - \frac{1}{2}z^{-2} - \frac{1}{4}z^{-3} - z^{-4}}{(1-z^{-1})^2}$$

ⓑ

$$\boxed{H(z) = \frac{\frac{1}{2} + \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2} - \frac{1}{4}z^{-3} - z^{-4}}{(1-z^{-1})^2}}$$

2)

a) $x(n) = ((1+n)u(n))$

$= u(n) + n \cdot u(n)$

$$\sum_{-\infty}^{\infty} u(n) z^{-n}$$

$$= \sum_{0}^{\infty} z^{-n}$$

$$= \sum_{0}^{\infty} (z^{-1})^n$$

$$= \frac{1}{1-z^{-1}} \text{ for } |z^{-1}| < 1$$

so $|z| > 1$

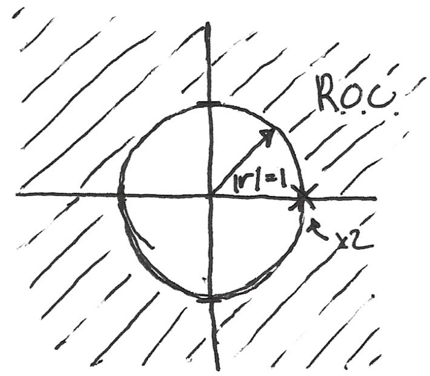
$$\sum_{-\infty}^{\infty} n\, u(n) z^{-n}$$

$$= \sum_{0}^{\infty} n z^{-n}$$

$$= \sum_{0}^{\infty} n (z^{-1})^n$$

$$= \frac{z^{-1}}{(1-z^{-1})^2} \text{ for } |z^{-1}| < 1$$

so $|z| > 1$



ⓐ $\boxed{X(z) = \frac{1}{(1-z^{-1})^2}}$

b)

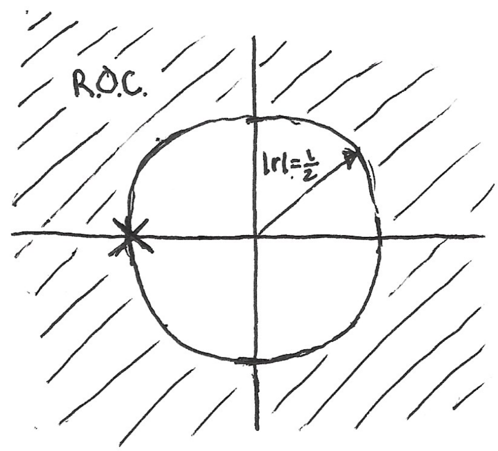
$x(n) = (-1)^n 2^{-n} u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} (-1)^n 2^{-n} u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\tfrac{1}{2}\right)^n (z^{-1})^n$$

$$= \sum_{n=0}^{\infty} \left(\tfrac{-1}{2} z^{-1}\right)^n$$

$$= \frac{1}{1-\left(\frac{-1}{2} z^{-1}\right)} \text{ for } \left|\tfrac{-1}{2} z^{-1}\right| < 1$$



ⓑ $\boxed{X(z) = \frac{1}{1+\frac{1}{2} z^{-1}} \text{ for } |z| > \tfrac{1}{2}}$

3) $y(n) = \sum_{k=-\infty}^{n} x(k)$

$y(n) = y(n-1) + x(n)$

$y(n) - y(n-1) = x(n)$ · Linearity properties

$Y(z) - Y(z)z^{-1} = X(z)$ · Time shifting

$Y(z)(1-z^{-1}) = X(z)$

$$\boxed{Y(z) = \frac{X(z)}{1-z^{-1}}}$$ $z \neq 1$ ← unless zero cancels

4)

a) $X(z) = \dfrac{1}{1+\frac{3}{2}z^{-1} - \frac{1}{2}z^{-2}}$ → $\dfrac{(-\frac{3}{2}) \pm \sqrt{(\frac{3}{2})^2 - 4(1)(-\frac{1}{2})}}{2(1)}$

$= \dfrac{(\frac{-3}{2}) \pm \sqrt{\frac{9}{4}+2}}{2}$

$X(z) = \dfrac{1}{\left(1 - \frac{\frac{-3}{2}+\sqrt{17/4}}{2}z^{-1}\right)\left(1 - \frac{\frac{-3}{2}-\sqrt{17/4}}{2}z^{-1}\right)}$ $= \dfrac{(-\frac{3}{2}) \pm \sqrt{17/4}}{2}$

$A\left(1 - \frac{\frac{-3}{2}+\sqrt{17/4}}{2}z^{-1}\right) + B\left(1 - \frac{\frac{-3}{2}-\sqrt{17/4}}{2}z^{-1}\right) = 1$

let $z = \dfrac{2}{\frac{-3}{2} - \sqrt{17/4}}$ $\quad A\left(1 - \left(\frac{\frac{-3}{2}+\sqrt{17/4}}{2}\right)\left(\frac{2}{\frac{-3}{2}-\sqrt{17/4}}\right)\right) = A\left(1 - \left(\frac{-3+2\sqrt{17/4}}{-3-2\sqrt{17/4}}\right)\right)$

$\left(\dfrac{-3+2\sqrt{17/4}}{-3-2\sqrt{17/4}}\right)\left(\dfrac{-3+2\sqrt{17/4}}{-3+2\sqrt{17/4}}\right) = \dfrac{9-12\sqrt{17/4}+17}{9-4(17/4)} =$

Not worth it

$\frac{1}{2}z^{-2} - \frac{3}{2}z^{-1} + 1 \,\big)\, 1$

1

$\begin{array}{r} 1 \quad +\frac{3}{2}z^{-1} - \frac{11}{4}z^{-2} \\ 1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2} \,\big)\, 1 \qquad\qquad\qquad \\ 1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2} \qquad \\ \hline \frac{3}{2}z^{-1} - \frac{1}{2}z^{-2} \qquad \\ \frac{3}{2}z^{-1} - \frac{9}{4}z^{-2} + \frac{3}{4}z^{-3} \\ \hline +\frac{7}{4}z^{-2} \ldots \end{array}$

This function has negative poles outside of the unit circle. It blows up and is very ugly algebreically. $\boxed{X(0) = 1,\ X(1) = -\frac{3}{2},\ X(2) = \frac{7}{4}}$

The function will not converge so $X(\infty)$ is undefined. + and − ∞

5) let $y(n) = y(n-1) + y(n-2) + x(n)$

- To find the impulse response we first put the system to rest $(y(n-1) = y(n-2) = 0)$ and set $x(n)$ to the delta function $(\delta(n))$. The impulse response will be the result

n	$\delta(n)$	$y(n-2)$	$y(n-1)$	$y(n)$
0	1	0	0	1
1	0	0	1	1
2	0	1	1	2
3	0	1	2	3
4	0	2	3	5
5	0	3	5	8

$\}$ fib

$$y(n) = y(n-1) + y(n-2) + x(n) \xrightarrow{Z} Y(z) = Y(z)z^{-1} + Y(z)z^{-2} + X(z)$$

$$Y(z)\left(1 - z^{-1} - z^{-2}\right) = X(z)$$

$$H(z) = \frac{1}{1 - z^{-1} - z^{-2}}$$ ... no obvious roots ... quadratic formula?

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a=1, b=-1, c=-1 \rightarrow \frac{1 \pm \sqrt{5}}{2}$$

so $\frac{1}{1-z^{-1}-z^{-2}} = \frac{A}{1 - \frac{1+\sqrt{5}}{2}z^{-1}} + \frac{B}{1 - \frac{1-\sqrt{5}}{2}z^{-1}}$

$$1 = A\left(1 - \frac{1-\sqrt{5}}{2}z^{-1}\right) + B\left(1 - \frac{1+\sqrt{5}}{2}z^{-1}\right)$$

let $z = \frac{2}{1+\sqrt{5}} \rightarrow 1 = A\left(1 - \frac{2(1-\sqrt{5})}{2(1+\sqrt{5})}\right) = A\left(1 - \frac{6}{-4}\right) \rightarrow A = \frac{2}{5-2\sqrt{5}}$

let $z = \frac{2}{1-\sqrt{5}} \rightarrow 1 = B\left(1 - \frac{2(1+\sqrt{5})}{2(1-\sqrt{5})}\right) = B\left(1 - \left(\frac{6+2\sqrt{5}}{-4}\right)\right) \frac{10+2\sqrt{5}}{4}$ $\quad B = \frac{5+2\sqrt{5}}{2}$

$$H(z) = \frac{5-2\sqrt{5}}{1 - \frac{1+\sqrt{5}}{2}z^{-1}} + \frac{5+2\sqrt{5}}{1 - \frac{1-\sqrt{5}}{2}z^{-1}} \xrightarrow{Z^{-1}} \boxed{h(n) = (5-2\sqrt{5})\left(\frac{1+\sqrt{5}}{2}\right)^n u(n) + (5+2\sqrt{5})\left(\frac{1-\sqrt{5}}{2}\right)^n u(n)}$$

6)

$$H(z) = \frac{1-2z^{-1}+2z^{-2}-z^{-3}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})(1-\frac{1}{5}z^{-1})} = \frac{\cancel{(1-z^{-1})}(1-z^{-1}+z^{-2})}{\cancel{(1-z^{-1})}(1-\frac{1}{2}z^{-1})(1-\frac{1}{5}z^{-1})}$$

$$\frac{-(-1) \pm \sqrt{(-1)^2-4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2}$$

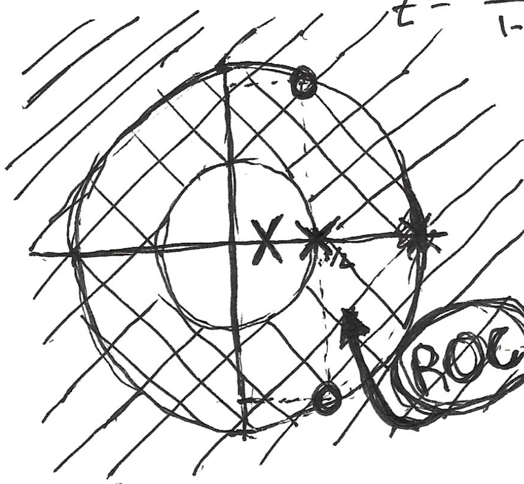
$$\boxed{H(z) = \frac{\left(1-\left(\frac{1+j\sqrt{3}}{2}\right)z^{-1}\right)\left(1-\left(\frac{1-j\sqrt{3}}{2}\right)z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{5}z^{-1}\right)}}$$

Zeros: $z = \frac{2}{1+j\sqrt{3}} \frac{(1-j\sqrt{3})}{(1-j\sqrt{3})} = \frac{2-2j\sqrt{3}}{1+3} = \frac{1-j\sqrt{3}}{2}$

$z = \frac{2}{1-j\sqrt{3}} \frac{(1+j\sqrt{3})}{(1+j\sqrt{3})} = \frac{2+j2\sqrt{3}}{1+3} = \frac{1}{2} + j\frac{\sqrt{3}}{2}$

$\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}}$

$\pm\sqrt{\frac{4}{4}}$



Zeros

Stable:

Positive real valued poles
less than 1