

# CS 560: Homework 6

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## Question 1:

$$\begin{aligned}KB = \\&\neg b \vee \neg c \\&b \vee \neg a \\&c \vee \neg a\end{aligned}$$

## Question 2:

$a$

## Question 3:

$$\begin{aligned}KB = \\&\neg b \vee \neg c \\&b \vee \neg a \\&c \vee \neg a \\&a\end{aligned}$$

## Question 4:

Yes, none of the conjuncts have more than a single positive literal.

## Question 5:

$$\begin{aligned}C = \\&\neg b \vee \neg c \\&b \vee \neg a \\&c \vee \neg a \\&a\end{aligned}$$

Resolve:  $b \vee \neg a$  and  $a$

Result:  $b$

$C =$   
 $\neg b \vee \neg c$   
 $b \vee \neg a$   
 $c \vee \neg a$   
 $a$   
 $b$

Remove:  $b \vee \neg a$

Because:  $b \subset b \vee \neg a$

$C =$   
 $\neg b \vee \neg c$   
 $c \vee \neg a$   
 $a$   
 $b$

Resolve:  $c \vee \neg a$  and  $a$

Result:  $c$

$C =$   
 $\neg b \vee \neg c$   
 $c \vee \neg a$   
 $a$   
 $b$   
 $c$

Remove:  $b \vee \neg a$

Because:  $c \subset c \vee \neg a$

$C =$   
 $\neg b \vee \neg c$   
 $a$   
 $b$   
 $c$

Resolve:  $\neg b \vee \neg c$

and  $c$

Result:  $\neg b$

$C =$   
 $\neg b \vee \neg c$   
 $a$   
 $b$   
 $c$   
 $\neg b$

Since our consequence set contains both  $b$  and  $\neg b$  our knowledgebase is false because there is no interpretation of  $b$  that will model KB. Therefore  $KB'$  is false and therefore  $KB \models \neg a$ .

## Question 6

**Interpretations:**

$a$	$b$	$c$	$a \vee b \vee c$	$\neg a \vee \neg b$
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0

**Models:**

$a$	$b$	$c$	$a \vee b \vee c$	$\neg a \vee \neg b$
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1

**Question 7:**

No, coming to  $g$  through a resolution of two clauses does not imply that  $g$  is true in every model of  $KB$ . Rather, it implies that there is a model of  $KB$  in which  $g$  is true. In other words,  $KB \vdash g$ . It is possible that a different resolution rule could have been applied that resulted in the opposite, or both could be done over the course of a proof procedure.

1.  $b \vee \neg b \vee c$

$a$	$b$	$c$	$a \vee b \vee c$	$\neg a \vee \neg b$
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1

$a$	$b$	$c$	$a \vee b \vee c$	$\neg a \vee \neg b$
1	1	0	1	0
1	1	1	1	0

Since both  $b$  and  $\neg b$  are in the rule it is true for all interpretations and thus true in all models.

**2.  $a \vee \neg a \vee c$**

$a$	$b$	$c$	$a \vee \neg a \vee c$	$\neg a \vee \neg b$
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0

Since both  $a$  and  $\neg a$  are in the rule it is true for all interpretations and thus true in all models.

**3.  $c$**

$a$	$b$	$c$	$a \vee b \vee c$	$\neg a \vee \neg b$
0	0	1	1	1
0	1	1	1	1
1	0	1	1	1
1	1	1	1	0

No, there are several models of  $KB$  in which  $c$  evaluates to false.

## Question 8:

**1.**

$a(X, c) \vee b(X, d)$  with  $\neg b(e, Y)$

Resolve:  $b(e, d) \vee \neg b(e, d)$

Sub:  $\{X/e, Y/d\}$

Result:  $a(e, c)$

**2.**
 $a(X, c) \vee b(X, d) \vee f$  with  $\neg a(e, Z) \vee b(e, Y)$ 
Resolve:  $a(e, c) \vee \neg a(e, c)$ Sub:  $\{X/e, Z/c\}$ Result:  $b(e, d) \vee f \vee b(e, Y)$ Resolve:  $b(e, d) \vee b(e, d)$ Sub:  $\{X/e, Y/d\}$ Result:  $a(e, c) \vee f \neg a(e, Z)$ **Question 9:**

The first part of the question above is an example of a **unit resolution** since one of the resolvants was done with a unit literal.

**Question 10:** $\neg mortal \leftarrow mythical$  $mortal \wedge mammal \leftarrow \neg mythical$  $horned \leftarrow \neg mortal \vee mammal$  $magical \leftarrow horned$ **Question 11:** $\neg mortal \vee \neg mythical$  $mortal \vee mythical$  $mammal \vee mythical$  $horned \vee mortal$  $horned \vee \neg mammal$  $magical \vee \neg horned$ **Question 12:**

The set of clauses is not Horne or Datalog because there is more than one positive literal in multiple clauses.

**Question 13:** $daughter(Daughter, Mother) \vee male(Daughter) \vee \neg mother(Mother, Daughter)$  $mother(mary, nancy)$  $\neg male(nancy)$ **Question 14:**

The above is neither Horne nor Datalog because the first statement has more than one positive literal.

## Question 15:

Initialize  $C$  with  $KB$ :

$C =$

$daughter(Daughter, Mother) \vee male(Daughter) \vee \neg mother(Mother, Daughter)$   
 $mother(mary, nancy)$   
 $\neg male(nancy)$

Resolve:  $mother(mary, nancy) \vee \neg mother(Mother, Daughter)$

Sub:  $\{Mother/mary, Daughter/nancy\}$

Result:  $daughter(nancy, mary) \vee male(nancy)$

$C =$

$daughter(Daughter, Mother) \vee male(Daughter) \vee \neg mother(Mother, Daughter)$   
 $mother(mary, nancy)$   
 $\neg male(nancy)$   
 $daughter(nancy, mary) \vee male(nancy)$

Prune:

$daughter(nancy, mary) \vee male(nancy) \subset daughter(Daughter, Mother) \vee male(Daughter) \vee$

$C =$

$mother(mary, nancy)$   
 $\neg male(nancy)$   
 $daughter(nancy, mary) \vee male(nancy)$

Resolve:  $\neg male(nancy) \vee male(nancy)$

Sub:  $\{\}$

Result:  $daughter(nancy, mary)$

$C =$

$mother(mary, nancy)$   
 $\neg male(nancy)$   
 $daughter(nancy, mary) \vee male(nancy)$   
 $daughter(nancy, mary)$

Prune:  $daughter(nancy, mary) \subset daughter(nancy, mary) \vee male(nancy)$

$C =$

$mother(mary, nancy)$   
 $\neg male(nancy)$   
 $daughter(nancy, mary)$

$daughter(nancy, mary)$  is in the consequence set.

## Question 16:

$yes \leftarrow daughter(nancy, mary)$

$yes \vee \neg daughter(nancy, mary)$

Use:  $daughter(Daughter, Mother) \vee male(Daughter) \vee \neg mother(Mother, Daughter)$

Subs: {  $Daughter/nancy, Mother/mary$  }

$yes \vee male(nancy) \vee \neg mother(mary, nancy)$

Use:  $male(nancy)$

Subs: { }

$yes \vee \neg mother(mary, nancy)$

Use:  $mother(mary, nancy)$

Subs: { }

$yes$

## Question 17:

Rewrite  $KB$ :

$poor(X) \vee \neg student(X)$

$student(john) \vee student(tim)$

$?poor(X)$

$yes(X) \vee \neg poor(X)$

Use:  $poor(X) \vee \neg student(X)$

Subs: { }

$yes(X) \vee \neg student(X)$

Use:  $student(john) \vee student(tim)$

Subs: {  $X/tim$  }

$yes(tim) \vee student(john)$

Use:  $\neg student(X)$

Subs: {  $X1/john$  }

$yes(tim)$

## Question 18:

No, it is not possible to prove that 'tweety' can fly without the CKA. This is because we need to able reason about  $abnormal(tweety)$ . The failure to resolve the top down proof follows:

$?fly(tweety)$

$yes \leftarrow fly(tweety)$

Use:  $fly(X) \leftarrow bird(X) \wedge \neg abnormal(X)$

Subs:  $\{X/tweety\}$

$yes \leftarrow bird(tweety) \wedge \neg abnormal(tweety)$

Use:  $bird(tweety)$

Subs:  $\{\}$

$yes \leftarrow \neg abnormal(tweety)$

This is where the proof without CKA gets stuck. If we were able to assume that any knowledge that can not be derived from the knowledgebase is false we would be able to conclude that  $abnormal(tweety)$  would evaluate to false. This would allow us to complete the proof.

## Question 19:

$yes \leftarrow fly(tweety)$

Use:  $fly(X) \leftarrow bird(X) \wedge \neg abnormal(X)$

Subs:  $\{X/tweety\}$

$yes \leftarrow bird(tweety) \wedge \neg abnormal(tweety)$

Use:  $bird(tweety)$

Subs:  $\{\}$

$yes \leftarrow \neg abnormal(tweety)$

**Recursion to prove  $\sim abnormal(tweety)$  with NaF**

$yes \leftarrow abnormal(tweety)$

Use:  $abnormal(X) \leftarrow toy(X)$

Subs:  $\{X/tweety\}$

$yes \leftarrow toy(tweety)$

**FAIL**

$yes \leftarrow abnormal(tweety)$

Use:  $abnormal(X) \leftarrow dead(X)$

Subs:  $\{X/tweety\}$

$yes \leftarrow dead(tweety)$

**FAIL**

$yes \leftarrow abnormal(tweety)$

**FAIL**



Add  $\sim abnormal(tweety)$  to  $C$

$yes \leftarrow \neg abnormal(tweety)$

Use:  $\sim abnormal(tweety)$

Subs:  $\{ \}$

$yes$

## Question 20:

$abnormal(X) \leftarrow toy(X) \vee dead(X)$

$toy(X) \leftarrow X = gun$

$dead(X) \leftarrow X = elvis$

## Question 21:

$abnormal(X) \longleftrightarrow toy(X) \vee dead(X)$

$toy(X) \longleftrightarrow X = gun$

$dead(X) \longleftrightarrow X = elvis$

## Question 22:

$toy(X) \vee dead(X) \leftarrow abnormal(X)$

$X = gun \leftarrow toy(X)$

$X = elvis \leftarrow dead(X)$

### CNF

$toy(X) \vee dead(X) \vee \neg abnormal(X)$

$X = gun \vee \neg toy(X)$

$X = elvis \vee \neg dead(X)$

## Question 23:

$KB =$

$bird(tweety)$

$fly(X) \vee \neg bird(X) \vee abnormal(X)$

$abnormal(X) \vee \neg toy(X)$

$abnormal(X) \vee \neg dead(X)$

$toy(gun)$

$dead(elvis)$

$toy(X) \vee dead(X) \vee \neg abnormal(X)$

$X = gun \vee \neg toy(X)$

$X = elvis \vee \neg dead(X)$

$?fly(tweety)$

$yes \vee \neg fly(tweety)$

Resolve:  $fly(X) \vee \neg bird(X) \vee abnormal(X)$

Sub:  $\{X/tweety\}$

Result:  $\neg bird(tweety) \vee abnormal(tweety)$

$yes \vee \neg bird(tweety) \vee abnormal(tweety)$

Resolve:  $bird(tweety)$

Sub:  $\{\}$

Result:  $abnormal(tweety)$

$yes \vee abnormal(tweety)$

Resolve:  $toy(X) \vee dead(X) \vee \neg abnormal(X)$

Sub:  $\{X/tweety\}$

Result:  $toy(tweety) \vee dead(tweety)$

$yes \vee toy(tweety) \vee dead(tweety)$

Resolve:  $X = gun \vee \neg toy(X)$

Sub:  $\{X/tweety\}$

Result:  $tweety = gun \vee dead(tweety)$

$yes \vee tweety = gun \vee dead(tweety)$

Resolve:  $X = elvis \vee \neg dead(X)$

Sub:  $\{X/tweety\}$

Result:  $tweety = gun \vee tweety = elvis$

Resolve:

Sub:  $\{\}$

Result:

**Not sure how this is supposed to work**

## Question 24:

### Helper Functions

In [158]:

```

1  from hw4standard import *
2
3  def PrettyCNF(expr):
4      #print(expr, "\n\n")
5      s = ""
6      for literal in expr:
7          if s != "": s += " v "
8          #print(type(literal))
9          if type(literal) is list:
10             if literal[0] == "not":
11                 s += "!"
12                 atom = literal[1]
13             else:
14                 atom = literal
15             s += prettyexpr(atom)
16     return s
17
18 def PrettyConsequence(consequence):
19     '''prints an entire consequent set'''
20     for i in consequence:
21         print(PrettyCNF(i))
22
23 def PrettyResolution(unit, clause, result):
24     print("Resolving", PrettyCNF(clause), "with", PrettyCNF(unit))
25     print("Result:", PrettyCNF(result))
26
27 def sameclause(a,b):
28     subs = {}
29     ret = unify(a,b,subs)
30     if ret == False or subs == {}:
31         return ret
32     # same clause if bottom just has variables and each is different
33     seen = []
34     for bottom in ret.values:
35         if not isVar(bottom):
36             return False
37         if bottom in seen:
38             return False
39         seen.append(bottom)
40     return True
41
42
43 def negate(literal):
44     if type(literal) is list and literal[0] == "not":
45         return literal[1]
46     return ['not', literal]
47
48
49 def resolve(unit, clause):
50     '''Takes in a unit clause and a disjunct clause and returns a reso
51
52     # negate the unit clause for resolution
53     neg_unit = negate(unit)
54
55     # look for matching disjunct in clause
56     for disjunct in clause:
57

```

```

58     subs = {}
59     # find one and get subs
60     if unify(neg_unit, disjunct, subs):
61         sub_clause = substitute(clause, subs)
62         sub_neg_unit = substitute(neg_unit, subs)
63
64         # remove instance that was resolved
65         sub_clause.remove(sub_neg_unit)
66
67         # return new clause
68         return sub_clause
69 # TEST
70 #print(resolve(['canary', 'tweety'], [['normal', 'X'], ['not', ['canary', 'X']
71
72 def can_resolve(unit, clause):
73     '''Checks to see if a resolution is possible'''
74     neg_unit = negate(unit)
75     for disjunct in clause:
76         if unify(neg_unit, disjunct, {}):
77             return True
78     return False
79 # TEST
80 #can_resolve(['canary', 'tweety'], [['normal', 'X'], ['not', ['canary', 'X']
81
82
83 def already_in_c_set(resolution, c_set):
84     ''' See if a clause is already in a set'''
85     for disjunct in c_set:
86         if sameclause(resolution, disjunct):
87             return True
88     return False
89 #TEST
90 #print(alread_in_c_set(['canary', 'tweety'], kb))
91 #print(alread_in_c_set(['canary', 'SHOULD FAIL'], kb))
92
93 def merge_sets(s1, s2):
94     '''Merges to sets without overlap'''
95     new_set = s1.copy()
96
97     for expression in s2:
98         if not already_in_c_set(expression, new_set):
99             new_set.append(expression)
100     return new_set
101 # TEST
102 #print(merge_sets(kb[:3], kb[1:5]))

```

## Prove Function

```

In [159]: 1 def prove(kb, query):
2
3     # declare consequence set
4     consequence = kb.copy()
5
6     # negate query
7     neg_query = negate(query)
8
9     # add query negation to consequence set
10    consequence.append([neg_query])
11
12    #PrettyConsequence(consequence)
13
14    # loop until the consequence set is empty
15    while 1:
16
17        temp_set = [] # new resolutions before adding to consequence
18
19        # look for units and try to resolve
20        for unit in consequence:
21            if len(unit) == 1:
22
23                # unit clause found, give fresh variables
24                unit_clause = freshvariables(unit[0])
25
26                # look at every expression in the current consequence set
27                for expression in consequence:
28
29                    # if it can be resolved do so and add the result to
30                    if can_resolve(unit_clause, expression):
31                        new_clause = resolve(unit_clause, expression)
32                        PrettyResolution([unit_clause], expression, new_clause)
33                        print("\n")
34                        # success if empty disjunct
35                        if new_clause == []:
36                            return True
37
38                        # otherwise check to see if resolution is in temp set
39                        # add to temp set if it is not.
40                        else:
41                            if not already_in_c_set(new_clause, temp_set):
42                                temp_set.append(new_clause)
43                                PrettyConsequence(merge_sets(consequence, temp_set))
44                                print("\n")
45
46        # if no further possible resolutions, fail
47        if temp_set == []:
48            return False
49        # otherwise, merge temp and consequence and run again
50        else:
51            consequence = merge_sets(consequence, temp_set)
52

```

## Demo 1

```
In [160]: 1 kb = [[['ostrich', 'sam']],
2          [['canary', 'tweety']],
3          [['bird', 'X'], ['not', ['ostrich', 'X']]],
4          [['bird', 'X'], ['not', ['canary', 'X']]],
5          [['fly', 'X'], ['not', ['bird', 'X']], ['not', ['normal', 'X']]],
6          [['not', ['normal', 'X']], ['not', ['ostrich', 'X']]],
7          [['normal', 'X'], ['not', ['canary', 'X']]]]
8
9
10 print(prove(kb, ['fly', 'tweety']))
```

Resolving bird(X) v !ostrich(X) with ostrich(sam)  
Result: bird(sam)

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
```

Resolving !normal(X) v !ostrich(X) with ostrich(sam)  
Result: !normal(sam)

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
```

Resolving bird(X) v !canary(X) with canary(tweety)  
Result: bird(tweety)

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
bird(tweety)
```

Resolving  $\text{normal}(X) \vee \neg \text{canary}(X)$  with  $\text{canary}(\text{tweety})$   
Result:  $\text{normal}(\text{tweety})$

$\text{ostrich}(\text{sam})$   
 $\text{canary}(\text{tweety})$   
 $\text{bird}(X) \vee \neg \text{ostrich}(X)$   
 $\text{bird}(X) \vee \neg \text{canary}(X)$   
 $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$   
 $\neg \text{normal}(X) \vee \neg \text{ostrich}(X)$   
 $\text{normal}(X) \vee \neg \text{canary}(X)$   
 $\neg \text{fly}(\text{tweety})$   
 $\text{bird}(\text{sam})$   
 $\neg \text{normal}(\text{sam})$   
 $\text{bird}(\text{tweety})$   
 $\text{normal}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\neg \text{fly}(\text{tweety})$   
Result:  $\neg \text{bird}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$

$\text{ostrich}(\text{sam})$   
 $\text{canary}(\text{tweety})$   
 $\text{bird}(X) \vee \neg \text{ostrich}(X)$   
 $\text{bird}(X) \vee \neg \text{canary}(X)$   
 $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$   
 $\neg \text{normal}(X) \vee \neg \text{ostrich}(X)$   
 $\text{normal}(X) \vee \neg \text{canary}(X)$   
 $\neg \text{fly}(\text{tweety})$   
 $\text{bird}(\text{sam})$   
 $\neg \text{normal}(\text{sam})$   
 $\text{bird}(\text{tweety})$   
 $\text{normal}(\text{tweety})$   
 $\neg \text{bird}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$

Resolving  $\text{bird}(X) \vee \neg \text{ostrich}(X)$  with  $\text{ostrich}(\text{sam})$   
Result:  $\text{bird}(\text{sam})$

Resolving  $\neg \text{normal}(X) \vee \neg \text{ostrich}(X)$  with  $\text{ostrich}(\text{sam})$   
Result:  $\neg \text{normal}(\text{sam})$

Resolving  $\text{bird}(X) \vee \neg \text{canary}(X)$  with  $\text{canary}(\text{tweety})$   
Result:  $\text{bird}(\text{tweety})$

Resolving  $\text{normal}(X) \vee \neg \text{canary}(X)$  with  $\text{canary}(\text{tweety})$   
Result:  $\text{normal}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\neg \text{fly}(\text{tweety})$   
Result:  $\neg \text{bird}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \text{!bird}(X) \vee \text{!normal}(X)$  with  $\text{bird}(\text{sam})$   
 Result:  $\text{fly}(\text{sam}) \vee \text{!normal}(\text{sam})$

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(tweety) v !normal(tweety)
fly(sam) v !normal(sam)
```

Resolving  $\text{normal}(X) \vee \text{!canary}(X)$  with  $\text{!normal}(\text{sam})$   
 Result:  $\text{!canary}(\text{sam})$

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(tweety) v !normal(tweety)
fly(sam) v !normal(sam)
!canary(sam)
```

Resolving  $\text{fly}(X) \vee \text{!bird}(X) \vee \text{!normal}(X)$  with  $\text{bird}(\text{tweety})$   
 Result:  $\text{fly}(\text{tweety}) \vee \text{!normal}(\text{tweety})$

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
```



```
bird(tweety)
normal(tweety)
!bird(tweety) v !normal(tweety)
fly(sam) v !normal(sam)
!canary(sam)
fly(tweety) v !normal(tweety)
```

Resolving !bird(tweety) v !normal(tweety) with bird(tweety)  
 Result: !normal(tweety)

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(tweety) v !normal(tweety)
fly(sam) v !normal(sam)
!canary(sam)
fly(tweety) v !normal(tweety)
!normal(tweety)
```

Resolving fly(X) v !bird(X) v !normal(X) with normal(tweety)  
 Result: fly(tweety) v !bird(tweety)

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(tweety) v !normal(tweety)
fly(sam) v !normal(sam)
!canary(sam)
fly(tweety) v !normal(tweety)
!normal(tweety)
fly(tweety) v !bird(tweety)
```

Resolving !normal(X) v !ostrich(X) with normal(tweety)  
 Result: !ostrich(tweety)

```

ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(tweety) v !normal(tweety)
fly(sam) v !normal(sam)
!canary(sam)
fly(tweety) v !normal(tweety)
!normal(tweety)
fly(tweety) v !bird(tweety)
!ostrich(tweety)

```

Resolving !bird(tweety) v !normal(tweety) with normal(tweety)  
 Result: !bird(tweety)

```

ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(tweety)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(tweety) v !normal(tweety)
fly(sam) v !normal(sam)
!canary(sam)
fly(tweety) v !normal(tweety)
!normal(tweety)
fly(tweety) v !bird(tweety)
!ostrich(tweety)
!bird(tweety)

```

Resolving bird(X) v !ostrich(X) with ostrich(sam)  
 Result: bird(sam)

Resolving !normal(X) v !ostrich(X) with ostrich(sam)  
 Result: !normal(sam)

Resolving  $\text{bird}(X) \vee \neg \text{canary}(X)$  with  $\text{canary}(\text{tweety})$   
Result:  $\text{bird}(\text{tweety})$

Resolving  $\text{normal}(X) \vee \neg \text{canary}(X)$  with  $\text{canary}(\text{tweety})$   
Result:  $\text{normal}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\neg \text{fly}(\text{tweety})$   
Result:  $\neg \text{bird}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$

Resolving  $\text{fly}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$  with  $\neg \text{fly}(\text{tweety})$   
Result:  $\neg \text{normal}(\text{tweety})$

Resolving  $\text{fly}(\text{tweety}) \vee \neg \text{bird}(\text{tweety})$  with  $\neg \text{fly}(\text{tweety})$   
Result:  $\neg \text{bird}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\text{bird}(\text{sam})$   
Result:  $\text{fly}(\text{sam}) \vee \neg \text{normal}(\text{sam})$

Resolving  $\text{normal}(X) \vee \neg \text{canary}(X)$  with  $\neg \text{normal}(\text{sam})$   
Result:  $\neg \text{canary}(\text{sam})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\text{bird}(\text{tweety})$   
Result:  $\text{fly}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$

Resolving  $\neg \text{bird}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$  with  $\text{bird}(\text{tweety})$   
Result:  $\neg \text{normal}(\text{tweety})$

Resolving  $\text{fly}(\text{tweety}) \vee \neg \text{bird}(\text{tweety})$  with  $\text{bird}(\text{tweety})$   
Result:  $\text{fly}(\text{tweety})$

$\text{ostrich}(\text{sam})$   
 $\text{canary}(\text{tweety})$   
 $\text{bird}(X) \vee \neg \text{ostrich}(X)$   
 $\text{bird}(X) \vee \neg \text{canary}(X)$   
 $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$   
 $\neg \text{normal}(X) \vee \neg \text{ostrich}(X)$   
 $\text{normal}(X) \vee \neg \text{canary}(X)$   
 $\neg \text{fly}(\text{tweety})$   
 $\text{bird}(\text{sam})$   
 $\neg \text{normal}(\text{sam})$   
 $\text{bird}(\text{tweety})$   
 $\text{normal}(\text{tweety})$   
 $\neg \text{bird}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$   
 $\text{fly}(\text{sam}) \vee \neg \text{normal}(\text{sam})$   
 $\neg \text{canary}(\text{sam})$   
 $\text{fly}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$   
 $\neg \text{normal}(\text{tweety})$

```
fly(tweety) v !bird(tweety)
!ostrich(tweety)
!bird(tweety)
fly(tweety)
```

Resolving !bird(tweety) with bird(tweety)  
Result:

True

## Demo 2

```
In [162]: 1 print(prove(kb,['fly','sam']))
```

```
Resolving bird(X) v !ostrich(X) with ostrich(sam)
Result: bird(sam)
```

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
```

```
Resolving !normal(X) v !ostrich(X) with ostrich(sam)
Result: !normal(sam)
```

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
```

```
Resolving bird(X) v !canary(X) with canary(tweety)
Result: bird(tweety)
```

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
```

```
Resolving normal(X) v !canary(X) with canary(tweety)
Result: normal(tweety)
```

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
```

```

bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)

```

Resolving fly(X) v !bird(X) v !normal(X) with !fly(sam)  
 Result: !bird(sam) v !normal(sam)

```

ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(sam) v !normal(sam)

```

Resolving bird(X) v !ostrich(X) with ostrich(sam)  
 Result: bird(sam)

Resolving !normal(X) v !ostrich(X) with ostrich(sam)  
 Result: !normal(sam)

Resolving bird(X) v !canary(X) with canary(tweety)  
 Result: bird(tweety)

Resolving normal(X) v !canary(X) with canary(tweety)  
 Result: normal(tweety)

Resolving fly(X) v !bird(X) v !normal(X) with !fly(sam)  
 Result: !bird(sam) v !normal(sam)

Resolving fly(X) v !bird(X) v !normal(X) with bird(sam)  
 Result: fly(sam) v !normal(sam)

```

ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)

```

```

bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(sam) v !normal(sam)
fly(sam) v !normal(sam)

```

Resolving !bird(sam) v !normal(sam) with bird(sam)  
 Result: !normal(sam)

Resolving normal(X) v !canary(X) with !normal(sam)  
 Result: !canary(sam)

```

ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(sam) v !normal(sam)
fly(sam) v !normal(sam)
!canary(sam)

```

Resolving fly(X) v !bird(X) v !normal(X) with bird(tweety)  
 Result: fly(tweety) v !normal(tweety)

```

ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(sam) v !normal(sam)
fly(sam) v !normal(sam)
!canary(sam)

```

```
fly(tweety) v !normal(tweety)
```

```
Resolving fly(X) v !bird(X) v !normal(X) with normal(tweety)
Result: fly(tweety) v !bird(tweety)
```

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(sam) v !normal(sam)
fly(sam) v !normal(sam)
!canary(sam)
fly(tweety) v !normal(tweety)
fly(tweety) v !bird(tweety)
```

```
Resolving !normal(X) v !ostrich(X) with normal(tweety)
Result: !ostrich(tweety)
```

```
ostrich(sam)
canary(tweety)
bird(X) v !ostrich(X)
bird(X) v !canary(X)
fly(X) v !bird(X) v !normal(X)
!normal(X) v !ostrich(X)
normal(X) v !canary(X)
!fly(sam)
bird(sam)
!normal(sam)
bird(tweety)
normal(tweety)
!bird(sam) v !normal(sam)
fly(sam) v !normal(sam)
!canary(sam)
fly(tweety) v !normal(tweety)
fly(tweety) v !bird(tweety)
!ostrich(tweety)
```

```
Resolving bird(X) v !ostrich(X) with ostrich(sam)
Result: bird(sam)
```

```
Resolving !normal(X) v !ostrich(X) with ostrich(sam)
Result: !normal(sam)
```



Resolving  $\text{bird}(X) \vee \neg \text{canary}(X)$  with  $\text{canary}(\text{tweety})$   
 Result:  $\text{bird}(\text{tweety})$

Resolving  $\text{normal}(X) \vee \neg \text{canary}(X)$  with  $\text{canary}(\text{tweety})$   
 Result:  $\text{normal}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\neg \text{fly}(\text{sam})$   
 Result:  $\neg \text{bird}(\text{sam}) \vee \neg \text{normal}(\text{sam})$

Resolving  $\text{fly}(\text{sam}) \vee \neg \text{normal}(\text{sam})$  with  $\neg \text{fly}(\text{sam})$   
 Result:  $\neg \text{normal}(\text{sam})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\text{bird}(\text{sam})$   
 Result:  $\text{fly}(\text{sam}) \vee \neg \text{normal}(\text{sam})$

Resolving  $\neg \text{bird}(\text{sam}) \vee \neg \text{normal}(\text{sam})$  with  $\text{bird}(\text{sam})$   
 Result:  $\neg \text{normal}(\text{sam})$

Resolving  $\text{normal}(X) \vee \neg \text{canary}(X)$  with  $\neg \text{normal}(\text{sam})$   
 Result:  $\neg \text{canary}(\text{sam})$

Resolving  $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$  with  $\text{bird}(\text{tweety})$   
 Result:  $\text{fly}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$

Resolving  $\text{fly}(\text{tweety}) \vee \neg \text{bird}(\text{tweety})$  with  $\text{bird}(\text{tweety})$   
 Result:  $\text{fly}(\text{tweety})$

$\text{ostrich}(\text{sam})$   
 $\text{canary}(\text{tweety})$   
 $\text{bird}(X) \vee \neg \text{ostrich}(X)$   
 $\text{bird}(X) \vee \neg \text{canary}(X)$   
 $\text{fly}(X) \vee \neg \text{bird}(X) \vee \neg \text{normal}(X)$   
 $\neg \text{normal}(X) \vee \neg \text{ostrich}(X)$   
 $\text{normal}(X) \vee \neg \text{canary}(X)$   
 $\neg \text{fly}(\text{sam})$   
 $\text{bird}(\text{sam})$   
 $\neg \text{normal}(\text{sam})$   
 $\text{bird}(\text{tweety})$   
 $\text{normal}(\text{tweety})$   
 $\neg \text{bird}(\text{sam}) \vee \neg \text{normal}(\text{sam})$   
 $\text{fly}(\text{sam}) \vee \neg \text{normal}(\text{sam})$   
 $\neg \text{canary}(\text{sam})$   
 $\text{fly}(\text{tweety}) \vee \neg \text{normal}(\text{tweety})$   
 $\text{fly}(\text{tweety}) \vee \neg \text{bird}(\text{tweety})$   
 $\neg \text{ostrich}(\text{tweety})$   
 $\text{fly}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \text{!bird}(X) \vee \text{!normal}(X)$  with  $\text{normal}(\text{tweety})$   
Result:  $\text{fly}(\text{tweety}) \vee \text{!bird}(\text{tweety})$

Resolving  $\text{!normal}(X) \vee \text{!ostrich}(X)$  with  $\text{normal}(\text{tweety})$   
Result:  $\text{!ostrich}(\text{tweety})$

Resolving  $\text{fly}(\text{tweety}) \vee \text{!normal}(\text{tweety})$  with  $\text{normal}(\text{tweety})$   
Result:  $\text{fly}(\text{tweety})$

Resolving  $\text{bird}(X) \vee \text{!ostrich}(X)$  with  $\text{ostrich}(\text{sam})$   
Result:  $\text{bird}(\text{sam})$

Resolving  $\text{!normal}(X) \vee \text{!ostrich}(X)$  with  $\text{ostrich}(\text{sam})$   
Result:  $\text{!normal}(\text{sam})$

Resolving  $\text{bird}(X) \vee \text{!canary}(X)$  with  $\text{canary}(\text{tweety})$   
Result:  $\text{bird}(\text{tweety})$

Resolving  $\text{normal}(X) \vee \text{!canary}(X)$  with  $\text{canary}(\text{tweety})$   
Result:  $\text{normal}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \text{!bird}(X) \vee \text{!normal}(X)$  with  $\text{!fly}(\text{sam})$   
Result:  $\text{!bird}(\text{sam}) \vee \text{!normal}(\text{sam})$

Resolving  $\text{fly}(\text{sam}) \vee \text{!normal}(\text{sam})$  with  $\text{!fly}(\text{sam})$   
Result:  $\text{!normal}(\text{sam})$

Resolving  $\text{fly}(X) \vee \text{!bird}(X) \vee \text{!normal}(X)$  with  $\text{bird}(\text{sam})$   
Result:  $\text{fly}(\text{sam}) \vee \text{!normal}(\text{sam})$

Resolving  $\text{!bird}(\text{sam}) \vee \text{!normal}(\text{sam})$  with  $\text{bird}(\text{sam})$   
Result:  $\text{!normal}(\text{sam})$

Resolving  $\text{normal}(X) \vee \text{!canary}(X)$  with  $\text{!normal}(\text{sam})$   
Result:  $\text{!canary}(\text{sam})$

Resolving  $\text{fly}(X) \vee \text{!bird}(X) \vee \text{!normal}(X)$  with  $\text{bird}(\text{tweety})$   
Result:  $\text{fly}(\text{tweety}) \vee \text{!normal}(\text{tweety})$

Resolving  $\text{fly}(\text{tweety}) \vee \text{!bird}(\text{tweety})$  with  $\text{bird}(\text{tweety})$   
Result:  $\text{fly}(\text{tweety})$

Resolving  $\text{fly}(X) \vee \text{!bird}(X) \vee \text{!normal}(X)$  with  $\text{normal}(\text{tweety})$   
 Result:  $\text{fly}(\text{tweety}) \vee \text{!bird}(\text{tweety})$

Resolving  $\text{!normal}(X) \vee \text{!ostrich}(X)$  with  $\text{normal}(\text{tweety})$   
 Result:  $\text{!ostrich}(\text{tweety})$

Resolving  $\text{fly}(\text{tweety}) \vee \text{!normal}(\text{tweety})$  with  $\text{normal}(\text{tweety})$   
 Result:  $\text{fly}(\text{tweety})$

False

### Demo 3

```
In [161]: 1 print(prove([['boy', ['goo', 'X', 'Y']], ['boy', ['foo', 'X', 'Y']]], ['boy
```

Resolving  $\text{boy}(\text{goo}(X,Y)) \vee \text{boy}(\text{foo}(X,Y))$  with  $\text{!boy}(\_10)$   
 Result:  $\text{boy}(\text{foo}(X,Y))$

$\text{boy}(\text{goo}(X,Y)) \vee \text{boy}(\text{foo}(X,Y))$   
 $\text{!boy}(X)$   
 $\text{boy}(\text{foo}(X,Y))$

Resolving  $\text{boy}(\text{goo}(X,Y)) \vee \text{boy}(\text{foo}(X,Y))$  with  $\text{!boy}(\_11)$   
 Result:  $\text{boy}(\text{foo}(X,Y))$

Resolving  $\text{boy}(\text{foo}(X,Y))$  with  $\text{!boy}(\_11)$   
 Result:

True

### Question 25:

If any of the following is true the entire disjunct is true:

- a is true
- b is true
- c is false
- d is true

### Question 26:

```
disjunction([p(true)|_]).
disjunction([n(false)|_]).
disjunction([_|Rest]) :-
    disjunction(Rest).
```

```
?- disjunction([p(A),p(B),n(C)]).
A = true ;
B = true ;
C = false ;
false.
```

```
?- disjunction([p(A),p(true),n(C)]).
A = true ;
true ;
C = false ;
false.
```

```
?- disjunction([p(A),p(false),n(C)]).
A = true ;
C = false ;
false.
```

## Question 27:

```
conjunction([X]) :- disjunction(X).
conjunction([X|Rest]) :-
    conjunction(Rest),
    disjunction(X).
```

## Question 28:

```
?- conjunction([[p(A),p(B),n(C)],[p(A),n(B)],[n(A)]]).
A = B, B = C, C = false ;
false.
```

```
?- conjunction([[p(A),p(B),n(C),p(D)],[p(A),n(B)],[n(A)],[n(D),n
(A)]]).
A = B, B = C, C = D, D = false ;
A = B, B = C, C = false ;
A = B, B = false,
D = true ;
false.
```

