$$H_z(\omega) = (1 - re^{j\theta}e^{-j\omega})(1 - re^{-j\theta}e^{-j\omega})$$

S.38 a

$$= (1 - re^{-j(\omega-\theta)})(1 - re^{-j(\omega+\theta)})$$

$$|H_z(\omega)|^2 = H_z(\omega) H_z^*(\omega)$$

$$= \left|\left[(1-re^{-j(\omega-\theta)})(1-re^{j(\omega-\theta)})\right]\left[(1-re^{-j(\omega+\theta)})(1-re^{j(\omega+\theta)})\right]\right|^2$$

$$= \left|\left[1 - r(e^{j(\omega-\theta)} + e^{-j(\omega+\theta)}) + r^2 e^0\right]\left[1 - r(e^{j(\omega+\theta)} + e^{-j(\omega+\theta)}) + r^2 e^0\right]\right|^2$$

$$= \left|(1 - 2r\cos(\omega-\theta) + r^2)(1 - 2r\cos(\omega+\theta) + r^2)\right|^2$$

$|H_z(\omega)|_{dB} = 10 \log_{10} |H_z(\omega)|$ • we will be able to divide by 2.

$$= \frac{1}{2}\left(10 \log_{10} |H_z(\omega)|^2\right)$$

$$= \frac{1}{2}\left(10 \log_{10} \left|(1-2r\cos(\omega-\theta)+r^2)(1-2r\cos(\omega+\theta)+r^2)\right|^2\right)$$

• all real

$$\boxed{|H_z(\omega)|_{dB} = 10 \log_{10}\left[1-2r\cos(\omega-\theta)+r^2\right] + 10 \log_{10}\left[1-2r\cos(\omega+\theta)+r^2\right]}$$

S.38 b

$$\measuredangle H_z(\omega) = \arctan\left(\frac{\mathrm{Im}(H_z(\omega))}{\mathrm{Re}(H_z(\omega))}\right)$$

$$= \arctan\left(\frac{\mathrm{Im}}{\mathrm{Re}}\left\{1 - r\cos(\omega-\theta) - rj\sin(\omega-\theta)\right\}\right)$$

$$+ \arctan\left(\frac{\mathrm{Im}}{\mathrm{Re}}\left\{1 - r\cos(\omega+\theta) - rj\sin(\omega+\theta)\right\}\right)$$

$$\boxed{\measuredangle H_z(\omega) = \arctan\left(\frac{r\sin(\omega-\theta)}{1-r\cos(\omega-\theta)}\right) + \arctan\left(\frac{r\sin(\omega+\theta)}{1-r\cos(\omega+\theta)}\right)}$$

S.38 c

$$\tau_g^z(\omega) = \frac{d}{d\omega} \measuredangle H_z(\omega)$$

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

Using Wolfram Alpha $\boxed{\frac{d}{d\omega} H_z(\omega) = \frac{r^2 - r\cos(\omega-\theta)}{1+r^2+2r\cos(\omega-\theta)} + \frac{r^2 - r\cos(\omega+\theta)}{1+r^2+2r\cos(\omega+\theta)}}$

5.38 d

$H_p(\omega) = 1/H_z(\omega)$

$H_p(\omega) = (H_z(\omega))^{-1}$

$|H_p(\omega)| = |H_z(\omega)|^{-1}$

$10 \log_{10} |H_p(\omega)| = 10 \log_{10} |H_z(\omega)|^{-1}$

$10 \log_{10} |H_p(\omega)| = -(10 \log_{10} |H_z(\omega)|)$

$\boxed{|H_p(\omega)|_{dB} = -|H_z(\omega)|_{dB}}$

Ran out of time