1) $C_K = \frac{A}{2\tau}\int_0^\tau \left(e^{j\pi\frac{1}{\tau}t} - e^{-j\pi\frac{1}{\tau}t}\right) e^{-j2\pi k\frac{1}{\tau}t}\,dt$

a) $= \frac{A}{2\tau}\int_0^\tau \left(e^{(1-2k)j\pi\frac{1}{\tau}t} - e^{(-1-2k)j\frac{1}{\tau}t}\right)dt$

$= \frac{A}{2\cancel{\tau}}\left(\left[\frac{e^{(1-2k)j\pi\frac{1}{\tau}t}}{(1-2k)\,j\pi\cancel{\frac{1}{\tau}}} - \frac{e^{(-1-2k)j\frac{\pi}{\tau}t}}{(-1-2k)\,j\pi\cancel{\frac{1}{\tau}}}\right]_0^\tau\right)$

$\frac{A}{j2\pi}\left(\left[\frac{e^{(1-2k)j\pi\cancel{\frac{1}{\tau}t}}}{(1-2k)} - \frac{e^{(-1-2k)j\pi}}{(-1-2k)}\right] - \left[\frac{1}{(1-2k)} - \frac{1}{(-1-2k)}\right]\right)$

$\frac{A}{j2\pi}\left(\frac{e^{(1-2k)j\pi}-1}{(1-2k)} + \frac{+e^{(-1-2k)j\pi}-1}{(-1-2k)}\right)$ ~~after~~ All e^x are multi of $\pi \to 0$

$\frac{A}{j2\pi}\left(\frac{-1}{(1-2k)} + \frac{-1}{(-1-2k)}\right) = \frac{A}{j2\pi}\left(\frac{1+2k-1+2k}{-1-2k+2k\;4k^2}\right)$

$= \frac{A}{j2\pi}\left(\frac{4k}{-1+4k^2}\right) = \frac{A4k}{j2\pi(-1+4k^2)} = \frac{2Ak}{j\pi(1-k^2}$

$$\boxed{C_K = \frac{2Ak}{j\pi(1-4k^2)}}$$

So $\boxed{X(F) = \sum_{k=-\infty}^{\infty}\left(\frac{2Ak}{j\pi(1-4k^2)}\right)\delta\left(F-\frac{k}{\tau}\right)}$

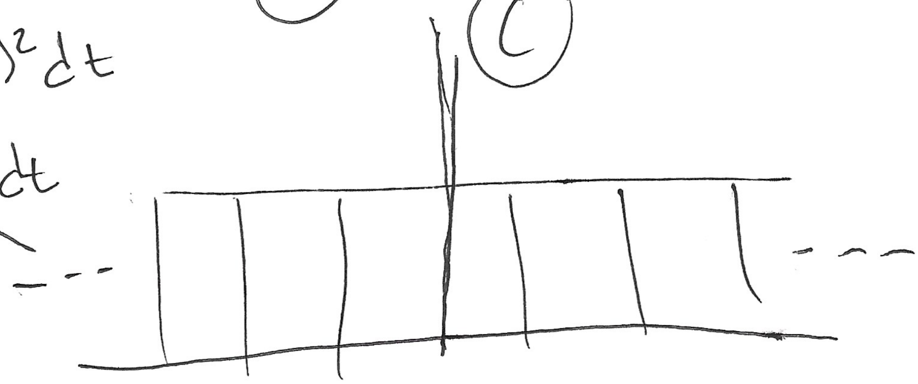
a

b) $\cancel{P_x = \frac{1}{\tau}\int_0^\tau \left(A\sin\left(\frac{\pi}{\tau}t\right)\right)^2 dt}$

$\cancel{= \frac{1}{\tau}\int_0^\tau A^2\sin^2\left(\frac{\pi}{\tau}t\right) dt}$

$\cancel{= \frac{A^2}{\tau}\Big[}$

c



b

$$\boxed{\begin{aligned} P_x &= |C_K|^2 \\ &= \sum \frac{4A^2k^2}{-\pi^2(1-4k^2)^2} \end{aligned}}$$

$\cancel{t = \sum_{k=-\infty}^{\infty}}$

2) $x_a(t) = \begin{cases} Ae^{-at}, & t \geq 0 \\ 0, & t \leq 0 \end{cases}$

C.T.F.T. $X(F) = \int_{-\infty}^{\infty} x_a(t) e^{-2\pi j F t} dt$

$$\boxed{\measuredangle \frac{Aa - Aj2\pi F}{a^2 - (2\pi F)^2} \qquad \measuredangle X(t) = \arctan\left(\frac{-2\pi F}{a}\right)}$$

$$X(F) = \int_0^{\infty} A e^{-at} e^{-2\pi j F t} dt$$

(a)

$$= A \int_0^{\infty} e^{(-a - j2\pi F)t} dt$$

$$= A \left[\frac{e^{(-a-j2\pi F)t}}{(-a - j2\pi F)} \right]_0^{\infty}$$

• Since $a > 0$

$$= \frac{A}{-a - j2\pi F} \left[e^{-\infty} - e^{0} \right]$$

$$\boxed{X(F) = \frac{A}{a + j2\pi F}}$$

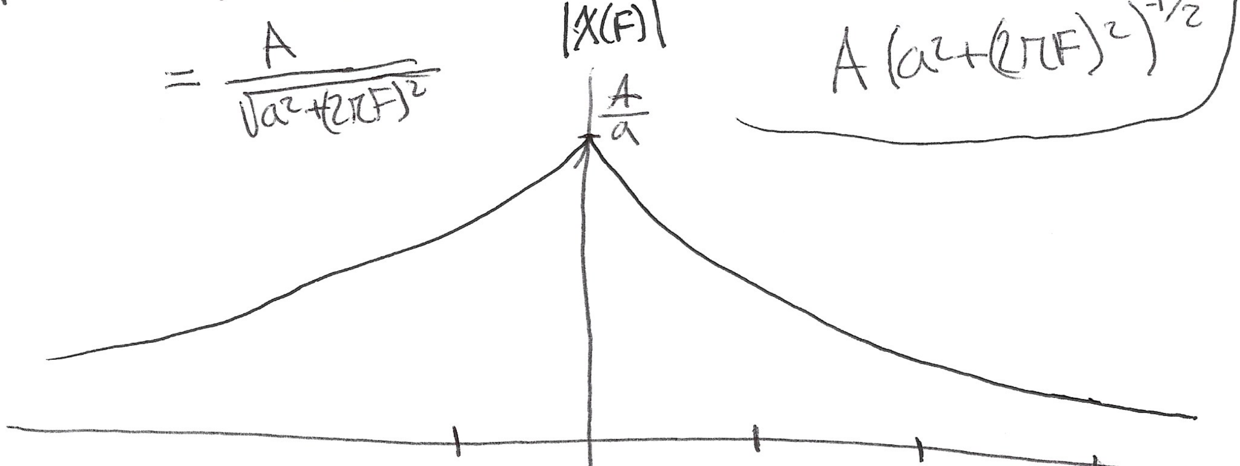
$$\cancel{|X(F)| = \sqrt{\left(\frac{A}{a + j2\pi F}\right)^2}}$$

$$\cancel{|X(F)| = \frac{A}{\sqrt{a^2 + 4j\pi F a +}}}$$

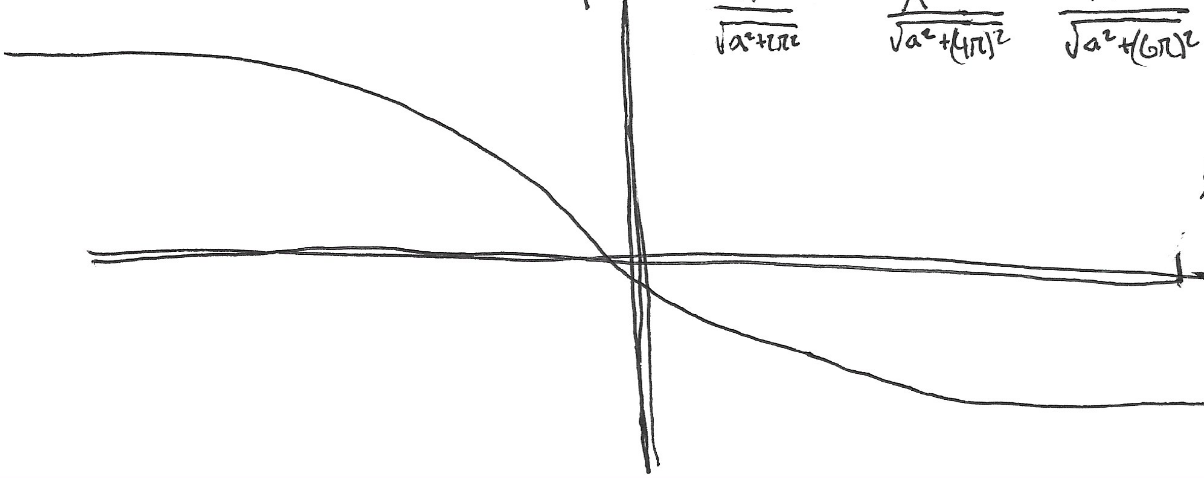
$$A \left(a^2 + (2\pi F)^2\right)^{-1/2}$$

$$\|X(F)\| = \frac{|A|}{|a + 2\pi F j|}$$

$$= \frac{A}{\sqrt{a^2 + (2\pi F)^2}}$$



$\measuredangle X(F)$ $\quad \frac{A}{\sqrt{a^2 + 2\pi^2}} \quad \frac{A}{\sqrt{a^2 + (4\pi)^2}} \quad \frac{A}{\sqrt{a^2 + (6\pi)^2}}$



2) $x(t) = A e^{-a|t|} = \begin{cases} A e^{-at}, & t \geq 0 \\ A e^{at}, & t \leq 0 \end{cases}$

C.T.F.T $X(F) = \int_{-\infty}^{\infty} x(t) e^{-2\pi j F t} dt$

b

$$A \left(\int_{-\infty}^{0} e^{at} e^{-2\pi j F t} dt + \int_{0}^{\infty} e^{-at} e^{-2\pi j F t} dt \right)$$

$$A \left(\int_{-\infty}^{0} e^{(a-2\pi j F)t} dt + \int_{0}^{\infty} e^{(-a-2\pi j F)t} dt \right)$$

$$A \left(\left[\frac{e^{(a-2\pi j F)t}}{a-2\pi j F} \right]_{-\infty}^{0} + \left[\frac{e^{(-a-2\pi j F)t}}{-a-2\pi j F} \right]_{0}^{\infty} \right)$$

$$A \left(\left[\frac{1-0}{a-2\pi j F} \right] + \left[\frac{0-1}{-a-2\pi j F} \right] \right)$$

$$A \left(\frac{(-a-2\pi j F) + (-a+2\pi j F)}{-a^2 + (2\pi j F)^2} \right)$$

$$A \left(\frac{-2a}{-a^2 - (2\pi F)^2} \right)$$

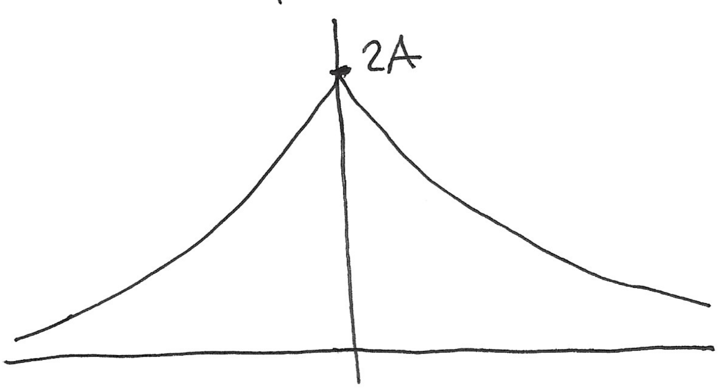
$$\boxed{X(F) = \frac{2Aa}{a^2 + (2\pi F)^2}}$$

$$|X(F)| = \frac{|2Aa|}{|a^2 + (2\pi F)^2|}$$

- No imagineary component

$|X(F)| = X(F)$

$|X(F)|$



$\measuredangle X(F)$