6.1 Using table of continuous-time Fourier transform Properties. Will attach Table Used.

a) $y(t) = \frac{dx(t)}{dt} \xleftrightarrow{FT} Y(F) = j2\pi X(F)$

- We see that the maximum frequency component of $Y(F)$ will be the same as $X(F)$

$$\boxed{F_s = 2B}$$ a

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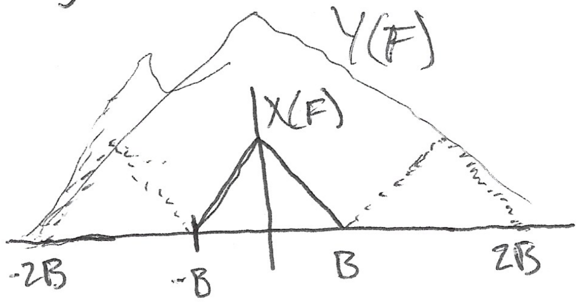
b) $y(t) = x^2(t) \xrightarrow{FT} Y(F) = X(F) * X(F)$

$$Y(F) = \int_{-\infty}^{\infty} X(\tau)X(F-\tau)\,d\tau$$



$$\boxed{F_s = 4B}$$ b

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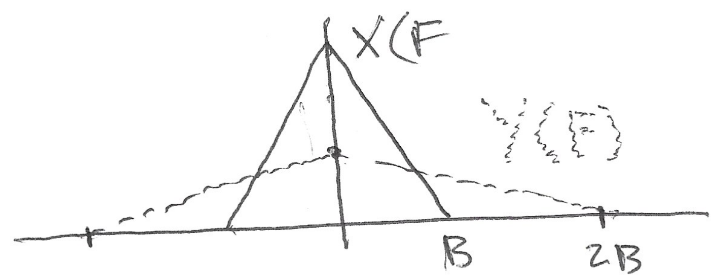
c) $y(t) = x(2t) \xrightarrow{FT} Y(F) = \frac{1}{|2|} X\left(\frac{F}{2}\right)$



$$\boxed{F_s = 4B}$$

operties of Fourier Representations

Fourier Transform	Fourier Series
$x(t) \xleftrightarrow{FT} X(j\omega)$ $y(t) \xleftrightarrow{FT} Y(j\omega)$	$x(t) \xleftrightarrow{FS;\,\omega_o} X[k]$ $y(t) \xleftrightarrow{FS;\,\omega_o} Y[k]$ Period $= T$
$ax(t) + by(t) \xleftrightarrow{FT} aX(j\omega) + bY(j\omega)$	$ax(t) + by(t) \xleftrightarrow{FS;\,\omega_o} aX[k] + bY[k]$
$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o}X(j\omega)$	$x(t - t_o) \xleftrightarrow{FS;\,\omega_o} e^{-jk\omega_o t_o}X[k]$
$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$	$e^{jk_o\omega_o t}x(t) \xleftrightarrow{FS;\,\omega_o} X[k - k_o]$
$x(at) \xleftrightarrow{FT} \frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$	$x(at) \xleftrightarrow{FS;\,a\omega_o} X[k]$
$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$	$\frac{d}{dt}x(t) \xleftrightarrow{FS;\,\omega_o} jk\omega_o X[k]$
$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(j\omega)$	—
$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{FT} \frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$	—
$\int_{-\infty}^{\infty} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FT} X(j\omega)Y(j\omega)$	$\int_{0}^{T} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FS;\,\omega_o} TX[k]Y[k]$
$x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\nu)Y(j(\omega - \nu))\,d\nu$	$x(t)y(t) \xleftrightarrow{FS;\,\omega_o} \sum_{l=-\infty}^{\infty} X[l]Y[k - l]$
$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$	$\frac{1}{T}\int_{0}^{T} \lvert x(t)\rvert^2\,dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$
$X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$
$x(t)$ real $\xleftrightarrow{FT} X^*(j\omega) = X(-j\omega)$ $x(t)$ imaginary $\xleftrightarrow{FT} X^*(j\omega) = -X(-j\omega)$ $x(t)$ real and even $\xleftrightarrow{FT} \mathrm{Im}\{X(j\omega)\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FT} \mathrm{Re}\{X(j\omega)\} = 0$	$x(t)$ real $\xleftrightarrow{FS;\,\omega_o} X^*[k] = X[-k]$ $x(t)$ imaginary $\xleftrightarrow{FS;\,\omega_o} X^*[k] = -X[-k]$ $x(t)$ real and even $\xleftrightarrow{FS;\,\omega_o} \mathrm{Im}\{X[k]\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FS;\,\omega_o} \mathrm{Re}\{X[k]\} = 0$

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6.16

$d(n) = x(n) - a\,x(n-1)$

$\sigma_d^2 = E[d^2(n)]$ -pp 433

$\sigma_d^2 = E[(x(n) - a x(n-1))^2]$

$= \underbrace{E[x^2(n)]} - 2a\underbrace{E[x(n)x(n-1)]} + \underbrace{E[a^2 x^2(n-1)]}$

$= \sigma_x^2 - 2a \cdot \gamma_x(1) + a^2\sigma_x^2$ -pp 433

$= \sigma_x^2 [1 + a^2 - 2a \underbrace{\gamma_x(1)\sigma_x^{-2}}]$

$\frac{\gamma_x(1)}{\sigma_x^2} = \frac{\gamma_x(1)}{\gamma_x(0)}$ -P.P. 434

$\boxed{\sigma_2(d) = \sigma_x^2[1 + a^2 - 2a\rho(1)]}$ (a)

- To find the minimum value we can set the derivative to 0 and solve.

$\frac{d}{da}\sigma_d^2 = 2\sigma_x^2 a - 2\rho_x(1)\sigma_x^2 = 0$

$2\sigma_x^2(a - \rho_x(1)) = 0$

- We plug result back in. $a = \rho_x(1)$

$\sigma_x^2(d) = \sigma_x^2[1 + \rho_x^2(1) - 2\rho_x^2(1)]$

$\boxed{\sigma_d^2 = \sigma_x^2[1 - \rho_x^2(1)]}$ (b)

- To find where $\sigma_d^2 < \sigma_x^2$ we say

$1 + a^2 - 2a\rho_x(1) < 1$

$a^2 - 2a\rho_x(1) < 0$

$a - 2\rho_x(1) < 0$

$\boxed{a < 2\rho_x(1)}$ (c)

for $\sigma_d^2 = \sigma_x^2[1 - \rho_x^2(1)]$

$1 - \rho_x^2(1) < 1$

$\rho_x^2(1) > 0$

So $\boxed{\rho_x(1) \neq 0}$ (c)