1)

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 3x(n-1)$$

- Solution to homogeneous equation

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

• Assume the form $y_h = \lambda^n$

- So,

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$\lambda^{n-2}(\lambda+1)(\lambda-4) = 0$$

$$\boxed{\lambda = \{-1, 4\}}$$

- Therefore,

$$y_h(n) = +C_1(-1)^n + C_2(4)^n$$

- For impulse response: $x(n) = \delta(n)$, $y(-1) = 0$, $y(-2) = 0$

$$y(0) = 3\cancel{y(0-1)} + 4\cancel{y(0-2)} + \delta(0) + \cancel{3\delta(0-1)} = 1$$

$$y(1) = 3y(0) + \cancel{4y(-1)} + \cancel{\delta(1)} + 3\delta(0) = 3 + 3 = 6$$

- Solving y_h for $C_{1/2}$

$$y_h(0) = C_1(-1)^0 + C_2(4)^0 = C_1 + C_2, \quad y_h(1) = C_1(-1)^1 + C_2(4)^1 = -C_1 + 4C_2$$

- Setting equal to $y(n)$

$$y(0) = 1 = C_1 + C_2 \qquad y(1) = 6 = -C_1 + 4C_2$$

$$C_1 = 1 - C_2 \longrightarrow 6 = -(1 - C_2) + 4C_2$$

$$6 = -1 + 5C_2$$

$$\boxed{C_1 = -2/5} \longleftarrow \boxed{C_2 = 7/5}$$

- Plugging back in C_1/C_2

$$\boxed{h(n) = \{(-2/5)(-1)^n + (\tfrac{7}{5})(4)^n\}\, u(n)}$$

2)

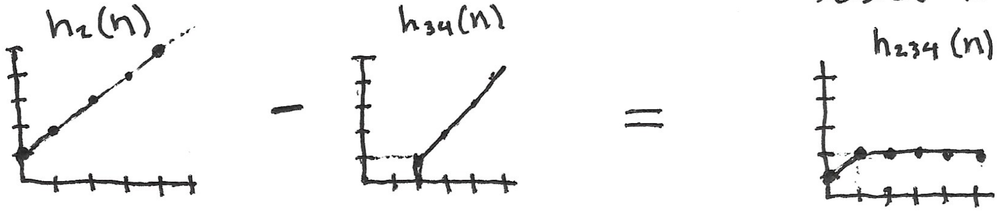
a. $\boxed{h(n) = h_1(n) * \left(h_2(n) - h_3(n) * h_4(n)\right)}$ (a)

b. • $h_{34}(n) = h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2)$

$\underline{h_{34}(n) = (n-1)u(n-2)}$ • Time shift of $h_3(n)$ by 2.

• $h_{234}(n) = h_2(n) - h_{34}(n) = (n+1)u(n) - (n-1)u(n-2)$

• Easier to solve graphically.



$\underline{h_{234}(n) = \delta(n) + u(n-1) \times 2}$

• $h(n) = h_1(n) * h_{234}(n) = \left(\tfrac{1}{2}\delta(n) + \tfrac{1}{4}\delta(n-1) + \delta(n-2)\right) * \left(\delta(n) + 2u(n-1)\right)$

$h(0) = (\tfrac{1}{2})(1) = \tfrac{1}{2}$, $h(1) = (\tfrac{1}{2})(2) + (\tfrac{1}{4})(1) = \tfrac{5}{4}$,

$h(2) = (\tfrac{1}{2})(2) + (\tfrac{1}{4})(2) + 1 = \tfrac{10}{4}$, $h(3) = (\tfrac{1}{2})(2) + (\tfrac{1}{4})(2) + (1)(2) = \tfrac{14}{4}$

(b) $\boxed{h(n) = \tfrac{1}{2}\delta(n) + \tfrac{5}{4}\delta(n-1) + \tfrac{10}{4}\delta(n-2) + \tfrac{14}{4}u(n-3)}$

c.

$x(n)$	..	0	1	0	0*	3	0	-4	0	...	
$h(n)$	...	0	0	0	1/2	5/4	10/4	14/4	14/4	...	
$x(-n)$	... 0	-4	0	3	0	0	1	0	0	...	

$y(-2) = 1/2$, $y(-1) = 5/4$, $y(0) = 10/4$, $y(1) = (\tfrac{1}{2})(3) + (\tfrac{14}{4})$,

$y(2) = (\tfrac{5}{4})(3) + (\tfrac{14}{4})$, $y(3) = (-4)(\tfrac{1}{2}) + (3)(\tfrac{10}{4}) + (\tfrac{14}{4})$,

$y(4) = (-4)(5/4) + (\tfrac{14}{4})(4)$, $y(5) = (-4)(\tfrac{10}{4}) + (\tfrac{14}{4})(4)$, $y(6) = 0$

$y(n)$	1/2	5/4	10/4	20/4	29/4	36/4	36/4	16/4	0
n	-2	-1	0	1	2	3	4	5	6

(c)

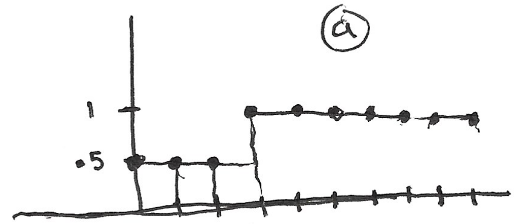
$$\boxed{y(n) = \tfrac{1}{2}\delta(n+2) + \tfrac{5}{4}\delta(n+1) + \tfrac{10}{4}\delta(n) + \tfrac{20}{4}\delta(n-1) + \tfrac{29}{4}\delta(n-2) + \tfrac{36}{4}\left(\delta(n-3) + \delta(n-4)\right) + \tfrac{16}{4}\delta(n-5)}$$

3) * Calculations for parts a, b, c are done in Python. Scripts are attached w/ the submission. (q3.py)

a. $y(n) = (1/2)x(n) + (1/2)x(n-3) + y(n-1)$

$h(n) = (1/2)\delta(n) + (1/2)\delta(n-3) + h(n-1)$

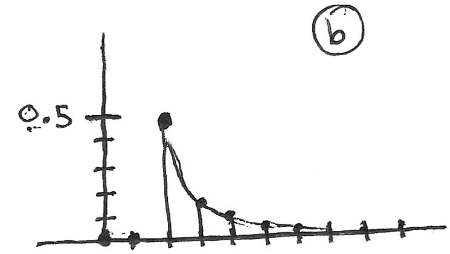
$\boxed{[.5, .5, .5, 1, 1, 1, 1, 1, 1, 1]}$



b. $y(n) = (1/2)x(n-2) + (1/3)y(n-1) + (1/8)y(n-2)$

$h(n) = (1/2)\delta(n-2) + (1/3)h(n-1) + (1/8)h(n-2)$

$\boxed{[0.0, 0.0, 0.5, 0.1\overline{6}, 0.1180\overline{5}, 0.060185, 0.03482, 0.01913, 0.01073, 0.00597]}$

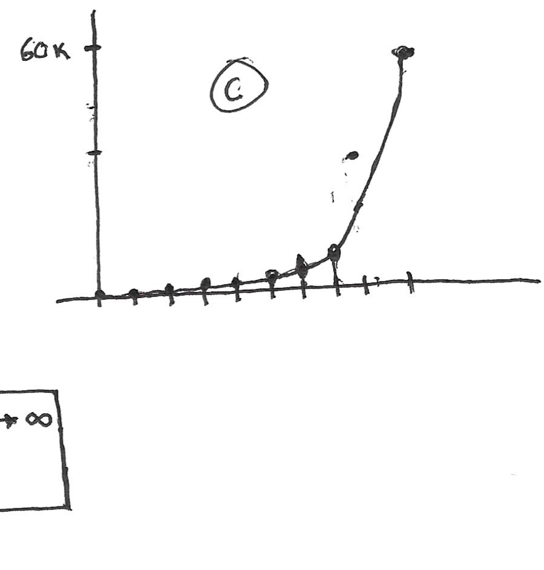


c. let $z(n) = x(n) + 2z(n-1)$

$y(n) = z(n) + 3y(n-1)$

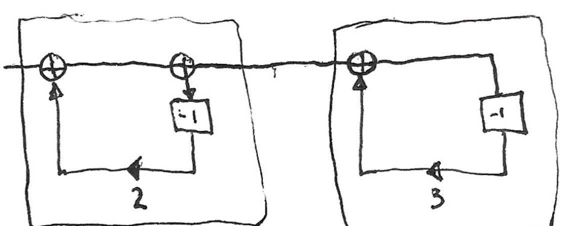
$h(n) = \delta(n) + z(n-1) + h(n-1)$

$\boxed{[1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025]}$



d. $\boxed{\text{All of these impulse responses extend to } n \to \infty. \text{ Therefore, all of these systems are IIR.}}$

e.



$h_1(n) = 2^n u(n)$ $h_2(n) = 3^n u(n)$

$h(n) = h_1(n) * h_2(n)$

$\boxed{h(n) = \sum_{k=-\infty}^{\infty} 2^k u(k) \cdot 3^{n-k} u(n-k)}$