$y(n) = 0.9y(n-1) + 0.1x(n)$

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low pass

$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

$\boxed{\text{So } a_1 = -0.9,\ b_0 = 0.1,\ N=1, M=0}$

$H_{lp}(\omega) = \frac{0.1}{1-0.9e^{-j\omega}}$ So: $H_{lp}(\omega - \frac{\pi}{2}) = \frac{0.1}{1-0.9e^{-j(\omega-\pi/2)}}$

$= 0.1 \frac{1}{1-(0.9)e^{-j(\omega-\pi/2)}}$

From the z transform table

(a) $\boxed{H_{lp}(\omega-\frac{\pi}{2}) = 0.1 \frac{1}{1-(0.9)e^{j\pi/2}e^{-j\omega}}}$

$\frac{1}{1-\alpha z^{-1}} \overset{DTFT^{-1}}{\Longleftrightarrow} \alpha^n u(n) \rightarrow$ $\alpha = 0.9e^{j\pi/2}$

(b) $\boxed{H_{lp}(\omega-\frac{\pi}{2}) \overset{DTFT^{-1}}{\Longrightarrow} h_{lp}(n) = (0.1)(0.9e^{j\pi/2})^n u(n)}$

$h_{lp}(n)$ has imaginary components. Real signals do not have imaginary components to modify. Information will be lost in the filter.