

5.13a

- Let $X(n)$ be a signal produced by sampling an oscillator with fundamental frequency f_0 .
- Let P_x be the power of $X(n)$.
- Let $X_i(n)$ be an ideal sinusoid of frequency f_0 with Power P_{xi} .
- The spurious harmonic power will be given by.

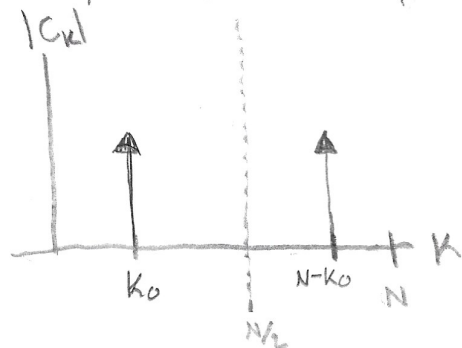
$$P_x - P_{xi}$$

- Since P_{xi} is the power of X_i , and X_i is a pure sinusoid of frequency f_0

$$P_{xi} = \frac{1}{N} \sum_{n=0}^{N-1} |X(n)|^2$$

$$P_{xi} = \sum_{n=0}^{N-1} |C_k|^2 \quad \left\{ \text{eq 4.2.11} \right.$$

- From the question we know the graph of the frequency components of a pure sinusoid has matching lines at k_0 and $N-k_0$



- From this graph we can see

$$P_{xi} = \sum_{n=0}^{N-1} |C_k|^2 = |C_{k_0}|^2 + |C_{N-k_0}|^2 = 2|C_{k_0}|^2$$

- Using the above result we can calculate the spurious harmonic power by subtracting it from the sampled sinusoid power, P_x .

$$\text{Spurious Harmonic Power} = P_x - P_{xi} = \underline{P_x - 2|C_{k_0}|^2}$$

$$\text{Since THD} = \frac{\text{spurious Harmonic Power}}{\text{total power}}$$

it follows that:

$$\text{THD} = \frac{P_x - P_{xi}}{P_x} = \frac{P_x - 2|C_{k_0}|^2}{P_x} = 1 - \frac{2|C_{k_0}|^2}{P_x}$$