CSCE 222-501 Discrete Structures for Computing
Fall 2014 – Hyunyoung Lee

Problem Set 8

Due dates: Electronic submission of hw8.tex and hw8.pdf files of this homework is due on **11/17/2014 before 23:59** on csnet.cs.tamu.edu. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **11/18/2014** at the beginning of class.

Name: Eric E. Gonzalez

Resources. Discrete Mathematics and its Applications 7th Ed.(Rosen)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ______________________________________

In this problem set, you will earn total $100 + 20$ (extra credit) points.

Problem 1. (10 points) Section 8.1, Exercise 2, page 510 [Hint: Let P_n denote the number of permutations of a set with n elements. The initial condition is $P_0 = 1$.]

Solution.
a) Let pn be the number of permutations of a set with n elements. We can make a recurrence relation choosing a location for the first element, then permuting the remaining n - 1 elements. Thus, p(n) = np(n - 1).
b) p(1) = 1
p(2) = 2
p(3) = 6
Iterating, we see p(n) = n(n - 1)(n - 2)...(2)p(1)
so p(n) = n!

Problem 2. (10 points) Section 8.1, Exercise 12, page 511 [Hint: Let S_n denote the number of ways to climb n stairs. One of the initial conditions is $S_0 = 1$.]

Solution.
a) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$
b) $a_0 = 1; a_1 = 1; a_2 = 2$
c) The number of ways one can climb the 8 flights of stairs: a_8
Sequence: $a_3 = 4; a_4 = 7; a_5 = 13; a_6 = 24; a_7 = 44; a_8 = 81$

Problem 3. (10 points) Section 8.1, Exercise 20, page 511

Solution.
a) Recurrence Relation: $a_{5n} = a_{5(n-1)} + a_{5(n-2)} forn \geq 2$
Initial Conditions: $a_0 = 1; a_5 = 1$
b) Number of different ways driver can pay toll: a_45
Sequence: $a_{10} = 2$; $a_{15} = 3$; $a_{20} = 5$; $a_{25} = 8$; $a_{30} = 13$; $a_{35} = 21$; $a_{40} = 34$; $a_{45} = 55$

Problem 4. (10 points) Section 8.1, Exercise 28, page 512. This problem has two parts as below.

Solution.
a) (4 points) Show that the Fibonacci numbers satisfy ...
For $n \geq 5$,
$fn = f_{n-1} + f_{n-2}$
$= (f_{n-2} + f_{n-3}) + (f_{n-3} + f_{n-4})$
$= f_{n-3} + f_{n-4} + 2f_{n-3} + f_{n-4}$
$= 3f_{n-3} + 2f_{n-4}$
$= 3f_{n-4} + 3f_{n-5} + 2f_{n-4}$
$= 5f_{n-4} + 3f_{n-5}$

b) (6 points) Use this recurrence relation to show that ... (prove by induction on n)
Since $f_5 = f_3 + f_4 = 5$, P(1) is true. Assume that P(n) is true. $f_{5n} = 5k$ for some integer k.
Then, $f_{5(n+1)} = f_{5n+5}$
$= 5f_{5(n+1)4} + 3f_{5(n+1)5}$
$= 5f_{5n+1} + 3f_{5n}$
$= 5f_{5n+1} + 15k$
$= 5(f_{5n+1} + 3k)$ which is divisible by 5. Therefore, P (N+1) is true.

Problem 5. (15 points) Section 8.1, Exercise 32 a), b), c) and d), page 512

Solution.
a) The number of moves needed to move n disks from peg 1 to peg 3 is a_n. With just one disk, one must move it to peg 2, then peg 3. Thus, $a_1 = 2$. With n+1 disks, one must move n disks from peg 1 to peg 3, then move the largest disk to peg 2, and then move the n disks from peg 3 back to peg 1 so that one may move the largest disk to peg 3. Then, one may finally move the n disks back to peg 3, giving a recurrence relation of $a_n = 3a_{n-1} + 2$
b) Assume $a_n = 3^n - 1$.
Base step: $a_1 = 2 = 3^1 - 1$
Inductive step: $a_{n+1} = 3a_n + 2 = 3(3^n - 1) + 2 = 3^{n+1} - 1$
Thus, the claim is proven by induction.
c) By the product rule, there are 3^n different methods of disc placement.
d) According to part (b), there are 3^n different arrangements.

Problem 6. (10 points) Section 8.2, Exercise 4 c), d), e) and f), page 524

Solution.
c) Characteristic equation: $r^2 - 6r + 8 = 0$
Solution: $\alpha_n = 3(2^n) + 1(4^n)$
d) Characteristic equation: $r^2 - 2r + 1 = 0$
Solution: $\alpha_n = 4 - 3n$
e) Characteristic equation: $r^2 - 0r - 1 = 0$
Solution: $\alpha_n = 2(1^n) + 3(-1)^n$
f) Characteristic equation: $r^2 + 4r - 5 = 0$
Solution: $\alpha_n = 3(-3^n) - 2n(-3)^n$

Problem 7. (10 points) Section 8.2, Exercise 8, page 524–525

Solution.
a) L_n is the number of lobsters caught in year n for the model of $L_n = (1/2)L_{n-1} + (1/2)L_{n-2}$
b) Characteristic equation: $r^2 - (1/2)r - (1/2) = 0 = (1/2)(2r+1)(r-1)$
Roots: $r_1 = -(1/2)$ and $r_2 = 1$
Solution: $r_1 = -(1/2)$ and $r_2 = 1$, so $a_n = k_1(-1/2)^n + k_2$.
Thus, $(-1/2)k_1 + k_2 = 100000$, and $(1/4)k_1 + k_2 = 300000$.
Solving for system of equations gives $k_1 = 800000/3$, $k_2 = 700000/3$.
Therefore, $a_n = (800000/3)(-1/2)^n + (700000/3)$.

Problem 8. (10 points) Section 8.4, Exercise 6 a), b), c) and d), page 549

Solution.
a) Generating function of $(-1)_n$ is $1/(x-1)$.
b) Generating function of $(2^n)_{n\geq 1}, a_0 = 1$ is $2x/(1-2x)$.
c) Generating function of $(n-1)_n$ is $-1 + x^2/(1-x)^2$.
d) Generating function of $(1/(n+1)!)_n$ is $(e^x - 1)/x$.

Problem 9. (15 points) Section 8.4, Exercise 8 a), b) and c), page 549

Solution.
a) $x^6 + 3x^4 + 3x^2 + 1$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, ...) = (1, 0, 3, 0, 3, 0, 1, 0, ...)$
$a_n = 0$ for all $n \geq 7$
b) $(3x-1)^3 = 27x^3 - 27x^2 + 9x - 1$
$(a_0, a_1, a_2, a_3, a_4, ...) = (-1, 9, -27, 27, 0, ...)$
$a_n = 0$ for all $n \geq 4$
c) Sequence for generating function of $1/(1-2x^2)$:
$a_{2n} = 2^n$
$a_{2n+1} = 0$ for all $n \geq 0$

Problem 10. (Extra credits: 5 points) Section 8.4, Exercise 6 e) and f), page 549

Solution.
e) $a_n = \binom{n}{2}$
Generating function: $x^2/(1-x)^3$
f) $a_n = \binom{10}{n+1}$
Generating function: $(-1 + (1+x)^{10})/x$

Problem 11. (Extra credits: 15 points) Section 8.4, Exercise 8 d), e) and f), page 549

Solution.
d) Sequence for generating function of $x^2/(1-x)^3$: $a_n = \binom{n}{2}$
e) Sequence for generating function of $4x + 9x^2 + 27x^3 + 81x^4 + ...$: $a_0 = 0, a_1 = 4$, and $a_n = 3^n$ for all $n \geq 2$
f) Sequence for generating function of $(1+x^3)/(1+x)^3$: $a_0 = 1$ and $a_n = (-1)^n * 3n$ for all $n \geq 1$

Checklist:

☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework on csnet?
☐ Did you submit (c) a signed hardcopy of the pdf file in class?