CSCE 222-501 Discrete Structures for Computing
Fall 2014 – Hyunyoung Lee

Problem Set 9

Due dates: Electronic submission of hw9.tex and hw9.pdf files of this homework is due on **Monday 11/24/2014 before 23:59** on csnet.cs.tamu.edu. A signed paper copy of the pdf file is due on **Tuesday 11/25/2014** at the beginning of class.

Name: Eric E. Gonzalez

Resources. Discrete Mathematics and its Applications 7th Ed.(Rosen)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ______________________________

In this problem set, you will earn total 100 + 15 (extra credit) points. Each question is worth 10 points unless otherwise noted.

Problem 1. Chapter 9.1, Exercise 4, page 581

Solution.
a) Antisymmetric, transitive
b) Reflexive, symmetric, transitive
c) Reflexive, symmetric, transitive
d) Reflexive, symmetric

Problem 2. (15 points) Chapter 9.1, Exercise 6, page 581

Solution.
a) $(1, 1) \notin R$. Not reflexive
If $x + y = 0$, then $y + x = 0$. Symmetric
$(1, -1) \in R$ and $(-1, 1) \in R$. $1 \neq -1$. Not anti-symmetric
$(2, -2) \in R$ and $(-2, 2) \in R$. $(2, 2) \notin R$. Not transitive

b) $x = x$ for all real numbers. Reflexive
$x = \pm y$ and $y = \pm x$. Symmetric
$(1, -1) \in R$ and $(-1, 1) \in R$. $1 \neq -1$. Not anti-symmetric
If $x = \pm y$ and $y = \pm z$, $x = \pm z$. Transitive

c) $x - x = 0$ is rational. Reflexive
$x - y$ and $y - x$ are both rational. Symmetric
$(2, 3) \in R$ and $(3, 2) \in R$. $2 \neq 3$. Not anti-symmetric
$x - y$ is rational and $y - z$ is rational, therefore $x - z$ is rational. Transitive

d) $(1,1) \notin R$. Not reflexive
$(1,2) \in R$, $(2,1) \notin R$. Not symmetric
$x = 0 = y$. Anti-symmetric.
$(8,4) \in R$ and $(4,2) \in R$, however $(2,4) \notin R$. Not transitive

e) $x^2 \geq 0$ for all real numbers. Reflexive
$xy = yx$. Symmetric
$(4,2) \in R$ and $(2,4) \in R$. $2 \neq 4$. Not anti-symmetric
$(-2,0) \in R$ and $(0,4) \in R$, however $(-2,4) \notin R$. Not transitive

f) $(1,1) \notin R$. Not reflexive
$xy = yx$. Symmetric
$(1,0) \in R$ and $(0,1) \in R$. $1 \neq 0$. Not anti-symmetric
$(1,0) \in R$ and $(0,4) \in R$, however $(1,4) \notin R$. Not transitive

g) $(2,2) \notin R$. Not reflexive
$(1,2) \in R$, $(2,1) \notin R$. Not symmetric
$x = y$. Anti-symmetric.
This function is not reflexive, symmetric, or transitive for any value of x other than 1.

h) $(2,2) \notin R$ Not reflexive
$(y,x) \in R$ for any (x,y) because either $x = 1$ or $y = 1$. Symmetric
$(1,2) \in R$ and $(2,1) \in R$. $1 \neq 2$. Not anti-symmetric
$(2,1) \in R$ and $(1,2) \in R$, however $(2,2) \notin R$. Not transitive

Problem 3. Chapter 9.1, Exercise 42, page 583

Solution.
$R_1 = \emptyset$
$R_2 = \{(0,0)\}$
$R_3 = \{(0,1)\}$
$R_4 = \{(1,0)\}$
$R_5 = \{(1,1)\}$
$R_6 = \{(0,0),(0,1)\}$
$R_7 = \{(0,0),(1,0)\}$
$R_8 = \{(0,0),(1,1)\}$
$R_9 = \{(0,1),(1,0)\}$
$R_{10} = \{(0,1),(1,1)\}$
$R_{11} = \{(1,0),(1,1)\}$
$R_{12} = \{(0,0),(0,1),(1,0)\}$
$R_{13} = \{(0,0),(0,1),(1,1)\}$
$R_{14} = \{(0,0),(1,0),(1,1)\}$
$R_{15} = \{(0,1),(1,0),(1,1)\}$
$R_{16} = \{(0,0),(0,1),(1,0),(1,1)\}$

Problem 4. (20 points) Chapter 9.1, Exercise 44, page 583

Solution.
a)Reflexive: $R_8, R_{13}, R_{14}, R_{16}$
b)Irreflexive: R_1, R_3, R_9, R_4
c)Symmetric: $R_1, R_2, R_5, R_8, R_9, R_{12}, R_{15}, R_{16}$
d)Anti-Symmetric: $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_{10}, R_{11}, R_{13}, R_{14}$
e)Asymmetric: R_3, R_4
f)Transitive: $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_{10}, R_{11}, R_{13}, R_{14}, R_{16}$

Problem 5. Chapter 9.5, Exercise 2, page 615

Solution.
a) Equivalence relation
b) Equivalence relation
c) Not an equivalence relation: not transitive because though a and b may share a common parent and b and c may share a common parent, a and c do not necessarily share the same parent.
d) Not an equivalence relation: Not transitive. Though a and b may have met, and b and c may have met, a and c may have not met.
e) Not an equivalence relation: Not transitive. Though a and b may speak the same language, and b and c may speak the same language, a and c may not necessarily speak the same language.

Problem 6. Chapter 9.5, Exercise 16, page 615

Solution.
$ab = ba$. Reflexive
$ad = bc$ is equivalent to $cb = da$. Symmetric
$ad = bc$ and $cf = de$
$af = ad/df = ad * (f/d) = b/d * (cf) = b/d(de) = de$. Transitive
As such, R is an equivalence relation.

Problem 7. Chapter 9.5, Exercise 58, page 618

Solution.
a) The relation is reflexive because B_1 can be found by rotating B_1. Since B_1 can be obtained from B_2 by a composition of rotations, $(B_2, B_1) \in R$ for any $(B_1, B_2) \in R$. Thus, the relation is symmetric. From the composition of rotations, it is made clear that R is transitive since B_1, B_2 and B_3 are in all in R.
b) $\{(B_1, B_1, B_1), (B_2, B_2, B_2), (B_3, B_3, B_3), (B_1, B_1, B_2), (B_1, B_1, B_3),$
$(B_2, B_2, B_1), (B_2, B_2, B_3), (B_3, B_3, B_1), (B_3, B_3, B_2)\}$

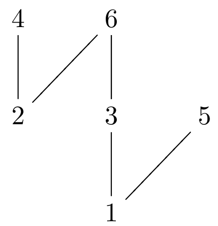
Problem 8. Chapter 9.6, Exercise 4, page 630

Solution.
a) (S, R) is not a poset: R is not antisymmetric because there may be two different people with the same height.
b) (S, R) is not a poset: R is not reflexive because it is not possible for a person to weigh more than themself.

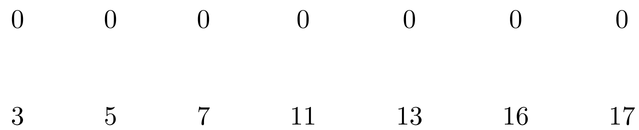
c) (S, R) is a poset. R is reflexive since $(a, a) \in R$ for all $a \in S$. For some $(a, b) \in S, a \neq b$, and $(a, b) \in R$, a is a descendant of b and, therefore, b cannot be a descendant of a. As such, $(b, a) \notin R$ and R is antisymmetric. Finally, R is transitive since $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$.
d) (S, R) is not a poset: R is not reflexive because a person has the same friends as themself.

Problem 9. Chapter 9.6, Exercise 22, page 631

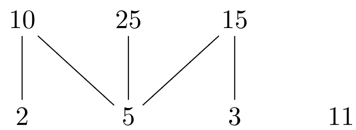
Solution.
a) The Hasse diagram of set $\{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ with divisibility condition is given by



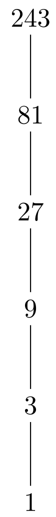
b) The Hasse diagram of set $\{3,\ 5,\ 7,\ 11,\ 13,\ 16,\ 17\}$ with divisibility condition is given by



c) The Hasse diagram of set $\{2,\ 3,\ 5,\ 10,\ 11,\ 15,\ 25\}$ with divisibility condition is given by



d) The Hasse diagram of set $\{1,\ 3,\ 9,\ 27,\ 81,\ 243\}$ with divisibility condition is given by



Problem 10. Chapter 9.6, Exercise 48, page 632

Solution.
S is reflexive since $A_1 \prec A_2$ for all integers. If $A_1 \prec A_2$ and $A_2 \prec A_1$, then $A_1 = A_2$. So, S is also antisymmetric. Finally, S is transitive because $A_1 \prec A_2$ and $A_2 \prec A_3$ implies that $A_1 \prec A_3$. Therefore, $(S, \prec)$ is a poset. Since $A_1 \in (A_1, C_1)$ and $A_2 \in (A_2, C_2)$, A_1 must be a maximal and A_2 must be a minimal since there exist no $A_1 \in S$ and $A_2 \in S$ such that $A_1 < A_2$. As such, S is a lattice.

Checklist:
- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used? (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework on csnet?
- ☐ Did you submit (c) a signed hardcopy of the pdf file in class?