CSCE 222-501 Discrete Structures for Computing
Fall 2014 – Hyunyoung Lee

Problem Set 5

Due dates: Electronic submission of hw5.tex and hw5.pdf files of this homework is due on **10/20/2014 before 23:59** on csnet.cs.tamu.edu. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **10/21/2014** at the beginning of class.

Name: Eric E. Gonzalez

Resources. (Discrete Mathematics and its Applications 7th Edition by Rosen)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____

Problem 1. Section 2.4, Exercise 6 (b), (c), (d), (g) and (h), pages 167–168

Solution.
(b) 1, 3, 6, 10, 15, 21, 28, 36, 45, 55
(c) 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025
(d) 1, 1, 1, 2, 2, 2, 2, 2, 3, 3
(g) 1, 2, 2, 4, 8, 11, 33, 37, 148, 153
(h) 1, 2, 2, 2, 2, 3, 3, 3, 3, 3

Problem 2. Section 2.4, Exercise 16 (c), (d), (e), (f) and (g), page 168

Solution.
(c) $a_n = 4 - \frac{n(n+1)}{2}$
(d) $a_n = -2^{n+2} + 3$
(e) $a_n = 2(n+1)!$
(f) $a_n = 3 * 2^n n!$
(g) $a_n = \frac{2n-1+(-1)^{n-1}}{4} + 7(-1)^n$

Problem 3. Section 5.1, Exercise 6, page 329 (use mathematical induction)

Solution.
Basis: P(1) = $(1+1)! - 1 = 2 - 1 = 1$, which makes P(1) true.

Inductive Step:
P(k) = (k + 1)! - 1
Assume P(k) is true.
P(k+1) = $(k+1)! - 1 + (k+1)(k+1)!$
$= (k+1)! - 1 + (k+1)(k+1)!$
$= (k+2)(k+1)! - 1$
$= (k+2)! - 1$
$= [(k+1)+1]! - 1$
Hence, true by mathematical induction.

Problem 4. Section 5.1, Exercise 8, page 329 (use mathematical induction)

Solution.
Basis: P(0) $= 2(-7)^0 = 2$
$(1-(-7)^1)/4 = 8/4 = 2$, which makes P(0) true

Inductive Step:
Assume P(k) is true
P(k+1) $= \frac{(1-(-7)^{k+1})}{4}$

$\frac{(1-(-7)^{k+1})}{4} + 2(-7)^{k+1} = \frac{(1-(-7)^{k+2})}{4}$
$1-(-7)^{k+1} + 8(-7)^{k+1} = 1-(-7)^{k+2}$
$(-7)^{k+1} - 8(-7)^{k+1} = (-7)^{k+2}$ multiplied both sides by -1
$\frac{(-7)^{k+1}-8(-7)^{k+1}=(-7)^{k+1}(-7)}{(-7)^{k+1}}$
$1-8=-7$
Hence, true by induction.

Problem 5. Section 5.1, Exercise 10, page 330 (use mathematical induction)

Solution.
(a) $\frac{1}{1*2} = \frac{1}{1} - \frac{1}{2} = \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$
$\frac{1}{1*2} + \frac{1}{2*3} + ... + \frac{1}{n*(n+1)} = (\frac{1}{1} - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + ... + (\frac{1}{n} - \frac{1}{n+1})$
$=\frac{1}{1} - \frac{1}{n+1}$
$=\frac{n+1-1}{n+1}$
$=\frac{n}{n+1}$

(b) Basis: P(1) $= \frac{1}{1*2} = \frac{1}{2}$
$\frac{1}{1+1} = \frac{1}{2}$ which makes P(1) true

Inductive Step:
P(k) $= \frac{1}{1*2} + \frac{1}{2*3} + ... + \frac{1}{k*(k+1)} = \frac{k}{(k+1)}$
Assume P(k) is true.
P(k+1) $= (\frac{1}{1*2} + \frac{1}{2*3} + ... + \frac{1}{k*(k+1)}) + \frac{1}{(k+1)(k+2)}$
$= \frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)}$
$= \frac{1}{(k+1)}[k + \frac{1}{(k+2)}]$
$= \frac{1}{(k+1)}[\frac{k^2+2k+1}{(k+2)}]$
$= \frac{1}{(k+1)} * \frac{(k+1)^2}{(k+2)}$
$= \frac{k+1}{k+2}$
$= \frac{k+1}{(k+1)+1}$
Hence, true by induction.

Problem 6. Section 5.1, Exercise 14, page 330 (use mathematical induction)

Solution.
Basis: P(1) $= 1 * 2^1 = (1-1) * 2^{1+1} + 2 = 2$

Inductive Step:
P(k) = $(k-1)*2^{k+1}+2$
P(k+1) = $(k-1)*2^{k+1}+2+(k+1)*2^{k+1}$
$= 2^{k+1}[(k-1)+(k+1)]+2$
$= 2^{k+1}[2k]+2$
$= 2^{k+2}*k+2$
$= k*2^{k+2}+2$
Hence, true by induction.

Problem 7. Section 5.1, Exercise 24, page 330 (use mathematical induction)

Solution.
Basis: P(1): $\frac{2-1}{2} \geq \frac{1}{2}$ which holds

Inductive Step:
P(k): $\frac{1}{2k} \leq \frac{1*3*5*...*2k-1}{2*4*6*...*2k}$
P(k+1): $\frac{1}{2(k+1)} \leq \frac{1}{2k} * \frac{2(k+1)-1}{2(k+1)}$
$\frac{1}{2k+2} \leq \frac{2k+1}{2k(2k+2)}$

$\frac{1}{2(k+1)} = \frac{1}{2k} * \frac{2k}{(2k+2)}$
$\leq [\frac{1*3*5*...*2k-1}{2*4*6*...*2k}] * \frac{2k}{2k+2}$ ... (by assumption)
$\leq [\frac{1*3*5*...*2k-1}{2*4*6*...*2k}] * \frac{2k+1}{2k+2}$
$\leq [\frac{1*3*5*...*(2k-1)*(2k+1)}{2*4*6*...*2k*(2k+2)}]$
$\leq [\frac{1*3*5*...*(2(k+1)-1)}{2*4*6*...*2(k+1)}]$
Hence, true by induction.

Problem 8. Section 5.1, Exercise 32, page 330 (use mathematical induction)

Solution.
Basis: P(1) = $1^3+2*1=3$ which holds true, since it is divisible by 3

Inductive Step:
P(k) =k^3+2k
P(k+1) = $(k+1)^3+2(k+1)$
$= k^3+1^3+3k^2+3k+2k+2$
$= k^3+2k+3k^2+3k+3$
$= (k^3+2k)+3(k^2+k+1)$ which is divisible by 3
Hence, true by induction.

Problem 9. Section 5.2, Exercise 12, page 342

Solution.
Basis: $1 = 2^0$ represents numbers in binary form.
By the inductive hypothesis, assume that every positive integer up to k can be represented as a sum of powers of 2. If k + 1 is odd, then k is even. If k + 1 is

even, then $(k+1)/2$ is a positive integer, and can be written as a sum of distinct powers of two. After increasing every exponent by 1, it doubles the value and provides the sum for k+1.

Problem 10. Section 5.2, Exercise 30, page 344

Solution.
The flaw is that the inductive step requires that both $a^k = 1$
and $a^{k-1} = 1$. The base case has to be sufficient
to support the inductive hypothesis in the first step. Here it isnt,
as there are multiple values for which the statement is not supported. The statement breaks at k=1.

Before attempting the problems from Chapter 5, make sure that you have carefully read Chapter 5.

Checklist:
- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework on csnet?
- ☐ Did you submit (c) a signed hardcopy of the pdf file in class?