*****Three problems are discarded and three new problems are added in boldface.*****

CSCE 222-501 Discrete Structures for Computing

Fall 2014 – Hyunyoung Lee

Problem Set 7

Due dates: Electronic submission of hw7.tex and hw7.pdf files of this homework is due on **11/3/2014 before 23:59** on csnet.cs.tamu.edu. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **11/4/2014** at the beginning of class.

Name: Eric E. Gonzalez

Resources. Discrete Mathematics and its Applications 7th Ed.(Rosen)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1. Section 6.1, Exercise 14, page 396

Solution.
The number of strings with length $n = 2^n$, providing 2^n numbers of choices.
The number of bit strings with 1 at the beginning and end $= n - 2$ choices.
The number of strings with 1s at the beginning and end is 2^{n-2}.

Problem 2. Section 6.1 Exercise 16, page 396

Solution.
Let S = the number of possible strings with x
Let T = the number of possible strings without x
$S = 26^4 = 456976$
$T = 25^4 = 390625$
$S - T = 456976 - 390625$
$= 66351$

Problem 3. Section 6.1, Exercise 32, page 397

Solution.
(a) 26^8
(b) $26 * 25 * 24 * 23 * 22 * 21 * 20 * 19$
(c) 25^7
(d) $25 * 24 * 23 * 22 * 21 * 20 * 19$
(e) 25^6
(f) 25^6
(g) 25^4
(h) $2(26^6) - 26^4$

Problem 4. Section 6.1, Exercise 70, page 398

Solution.
$n = 2^n$ by the Product Rule
So, $2^n = 2^{2^n}$
Thus, there are 2^{2^n} different truth tables for n variables.

Problem 5. Section 6.2, Exercise 8, page 405

Solution.
$f : S \to T$ and $|S| \geq |T|$
$|T| = n$ and $|S| \geq n + 1$
As such, n+1 elements are set to map to n elements.
So, by pigeon-hole principle, at least one pair of elements maps to the same spot, making $f(s_1) = f(s_2)$.
Therefore, the function f is not one-to-one.

Problem 6. Section 6.2, Exercise 16, page 405

Solution.
If we arrange the set into pairs that add to 16, then we get (1,15),(3,13),(5,11),(7,9).
Since there are 4 pairs of numbers that add to 16, we need to choose at least 5 to ensure that two numbers will be a matching pair and add up to 16.

Problem 7. Section 6.2, Exercise 46, page 407

Solution.
Assume each box contains at most $n_i - 1$ objects.
If n is the maximum number of objects that can be placed into a t number of boxes, then:
$n = n_1 - 1... + n_t - 1$
$= n_1... + ...n_t - t$
$< n_t - t + 1$
It is not possible to place all $n_t - t + 1$ into the t boxes. As such, for some i, the ith box must contain at least n_i objects.

Problem 8. Section 6.3, Exercise 12, page 413

Solution.
(a) $\binom{12}{3} = 220$
(b) $\binom{12}{0} + \binom{12}{1} + \binom{12}{2} + \binom{12}{3} = 299$
(c) $\binom{12}{3} + \binom{12}{4} + \binom{12}{5} + \binom{12}{6} + \binom{12}{7} + \binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12} = 4017$
(d) $\binom{12}{6} = 924$

Problem 9. Section 6.3 Exercise 18 on page 413

Solution.
(a) $2^8 = 256$
(b) $\binom{8}{3} = 56$
(c) $\binom{8}{5} + \binom{8}{4} + \binom{8}{3} + \binom{8}{2} + \binom{8}{1} + \binom{8}{0} = 219$
(d) $\binom{8}{4} = 70$

Problem 10. Section 6.3, Exercise 22, page 414

Solution.
(a) $7! = 5040$
(b) $6! = 720$
(c) $5! = 120$
(d) $5! = 120$
(e) $4! = 24$
(f) 0, as there are no permutations available. B can't be followed by both A and F simultaneously.

~~Section 6.4, Exercise 14, page 421~~

~~Section 6.4, Exercise 16, page 421~~

~~Section 6.4, Exercise 38, page 422 (read the hint carefully!)~~

Checklist:
- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework on csnet?
- ☐ Did you submit (c) a signed hardcopy of the pdf file in class?