CSCE 222-501 Discrete Structures for Computing
Fall 2014 – Hyunyoung Lee

Problem Set 10

Due dates: Electronic submission of hw10.tex and hw10.pdf files of this homework is due on **Monday 12/8/2014 before 23:59** on csnet.cs.tamu.edu. A signed paper copy of the pdf file is due on **Tuesday 12/9/2014** at the beginning of class.

Name: Eric E. Gonzalez

Resources. Discrete Mathematics and its Applications 7th Ed.(Rosen)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

In this problem set, you will earn total 100 + 20 (extra credit) points. Each problem is worth ten points unless otherwise specified.

Problem 1. Chapter 13.1, Exercise 4, page 856

Solution.
(a) $S \to 1S$
$\to 11S$
$\to 111S$
$\to 11100A$
$\to 111000$
111000 belongs to language generated by G.
(b) Any sentence of the language generated by G ends with 0
11001 does not belong to the language generated by G.
(c) $L(G) = 1^m0^n/m \geq 0, n \geq 3$

Problem 2. (20 points) Chapter 13.1, Exercise 6, page 856

Solution.
(a) $\{abbb\}$
(b) $\{aba, aa\}$
(c) $\{abb, abab\}$
(d) $\{a2^n|n \geq 2\} \cup \{b^n|n \geq 1\}$
(e) $\{a^nb^{n+m}a^m|m \geq 0, n \geq 0\}$

Problem 3. (15 points) Chapter 13,1, Exercise 14, page 856

Solution.
(a) Let $L = \{10, 01, 101\}$ and $G = (V, T, S, P)$
Grammar G will generate language L if

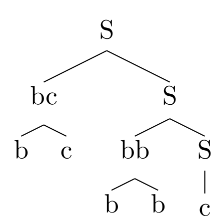
$V = \{0, 1, S\}, T = \{0, 1\}$, S is the starting symbol, and the productions are
$S \to 10, S \to 01, S \to 101$
(b) Let L be a bit string that starts with 00 and ends with one or more 1s and $G = (V, T, S, P)$
Grammar G will generate language L if
$V = \{0, 1, S, A, B\}, T = \{0, 1\}$, S is the starting symbol, and the productions are
$S \to 00AB, A \to AA, A \to 0, A \to 1, B \to BB, B \to 1$
(c) Let L be a bit string consisting of an even number of 1s followed by a final 0 and $G = (V, T, S, P)$
Grammar G will generate language L if
$V = \{0, 1, S, A, B, C\}, T = \{0, 1\}$, S is the starting symbol, and the productions are
$S \to A0, A \to \lambda, A \to BBC, A \to BCB, A \to CBB, C \to CC, B \to CB, B \to BC, B \to 1, C \to 0$
(d) Let L be a bit string that has neither two consecutive 0s nor two consecutive 1s and $G = (V, T, S, P)$
Grammar G will generate language L if
$V = \{0, 1, S, A, B\}, T = \{0, 1\}$, S is the starting symbol, and the productions are
$S \to A, A \to AA, A \to A0, A \to 01, A \to \lambda, S \to B, B \to BB, B \to B1, B \to 10, B \to \lambda$

Problem 4. (15 points) Chapter 13,1, Exercise 18, page 856

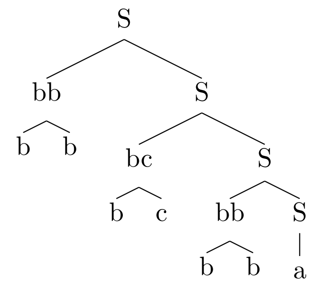
Solution.
(a) Let $L = \{01^{2n}/n \geq 0\}$ and $G = (V, T, S, P)$
Grammar G will generate language L if
$V = \{0, 1, S, A\}, T = \{0, 1\}$, S is the starting symbol, and the productions are
$S \to 0A, A \to \lambda, A \to AA, A \to 11$
(b) Let $L = \{0^n 1^{2n}/n \geq 0\}$ and $G = (V, T, S, P)$
Grammar G will generate language L if
$V = \{1, 0, S\}, T = \{0, 1\}$, S is the starting symbol, and the productions are
$S \to 0S11, S \to \lambda$
(c) Let $L = \{0^n 1^m 0^n/m \geq 0, n \geq 0\}$
Grammar G will generate language L if
$V = \{0, 1, S, A\}, T = \{0, 1\}$, S is the starting symbol, and the productions are
$S \to 0S0, S \to \lambda, S \to A, A \to AA, A \to 1$

Problem 5. Chapter 13.1, Exercise 24, page 857

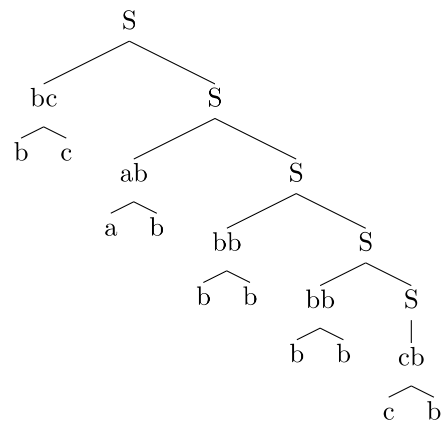
Solution.
(a)



(b)



(c)



Problem 6. (5 points) Chapter 13.2, Exercise 2 a), page 864

Solution.

	f		g	
	Input		Input	
State	0	1	0	1
s_0	s_1	s_2	1	0
s_1	s_0	s_3	1	0
s_2	s_3	s_0	0	0
s_3	s_1	s_2	1	1

Problem 7. (5 points) Chapter 13.2, Exercise 4 a), page 864

Solution.

Input	1	0	0	0	1	-
State	s_0	s_2	s_3	s_1	s_0	s_2
Output	0	0	1	1	0	-

Problem 8. Chapter 13.2, Exercise 18, page 865 (explain your FSM)

Solution.
The machine will need 5 states: s_1, s_2, s_3, s_4, s_5, aside from the starting

state s_0, to track the last five digits input. The numbers of the state indicate the numberof consecutive 1s that have been seen. The machine will output 1 if the input meets the condition, otherwise it will output 0.

Problem 9. Chapter 13.3, Exercise 8 a), b), e) and f), page 875

Solution.
(a) Let $A = \{v\}$ and $A^2 = \{vv\}$
$A \notin A^2$
Proven by counterexample that the statement is false.
(b) Let $A = \emptyset$. The hypothesis is true, as an empty set squared results in the empty set. However, the conclusion is false, as $\lambda \notin A$
Proven by counterexample that the statement is false.
(e) If A is not the empty set, the left hand side of the expression will be $A^i, i > 0$. The right hand side will contain A^0, which is the empty string λ, so the equality does not hold.
Proven by counterexample that the statement is false.
(f) Let $A = \{1, \lambda\}$ and $A^2 = \{1, 11, \lambda\}$
Hence, $\left|A^2\right| = 3$ and $|A|^2 = 4$
Proving that the equality is not true for $A = \{1, \lambda\}$, n=2
Thus, the equation is false.

Problem 10. (5 points) Chapter 13.3, Exercise 10 b), d) and f), page 875

Solution.
(b) $\{0\} * \{10\}\{1\}*$
The set allows consecutive 0s only in the beginning of the string and only a single 0 after the first 1 occurs.
01001 is not present in $\{0\} * \{10\}\{1\}*$.
(d) $\{010, 011\}\{00, 01\}$
The string 01001 is equivalent to $\{010\}\{01\}$, which cane be formed by taking 010 from the first set and 01 from the second set of the given string.
01001 is present in the given set.
(f) $\{01\} * \{01\}*$
in the set, we cannot have the continuation of two 0s, so 01001 is not present.
01001 is not present in the given set.

Problem 11. (5 points) Chapter 13.3, Exercise 16, page 876

Solution. $\{\lambda\} \cup \{1\}\{0, 1\} * \cup \{0\}\{1\} * \{0\}\{0, 1\}*$

Problem 12. (5 points) Chapter 13.3, Exercise 18, page 876

Solution. $\{\lambda\} \cup \{0\}\{1\}*$

Problem 13. (5 points) Chapter 13.3, Exercise 28, page 876

Solution. Have four states, with only q_3 final. For i = 0, 1, 2, 3, transition from si to itself on input 1 and to si+1 on input 0. Both transitions from q_3 are to itself.

Checklist:

☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework on csnet?
☐ Did you submit (c) a signed hardcopy of the pdf file in class?