CSCE 222-501 Discrete Structures for Computing
Fall 2014 – Hyunyoung Lee

Problem Set 6

Due dates: Electronic submission of hw6.tex and hw6.pdf files of this homework is due on **10/27/2014 before 23:59** on csnet.cs.tamu.edu. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **10/28/2014** at the beginning of class.

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Resources. Discrete Mathematics and its Applications 7th Ed.(Rosen)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Problem 1. Section 5.3, Exercise 4, page 357

Solution.
$f(0) = f(1) = 1$

(a) $f(n+1) = f(n) - f(n-1)$
$f(2) = f(1) - f(0)$
$= 1 - 1$
$= 0$
$f(3) = f(2) - f(1)$
$= 0 - 1$
$= -1$
$f(4) = f(3) - f(2)$
$= -1 - 0$
$= -1$
$f(5) = f(4) - f(3)$
$= -1 - (-1)$
$= 0$

(b) $f(n+1) = f(n)f(n-1)$
$f(2) = f(1)f(0)$
$= 1 * 1$
$= 1$
$f(3) = f(2)f(1)$
$= 1 * 1$
$= 1$
$f(4) = f(3)f(2)$
$= 1 * 1$

$= 1$
$f(5) = f(4)f(3)$
$= 1 * 1$
$= 1$

(c) $f(n+1) = f(n)^2 + f(n-1)^3$
$f(2) = f(1)^2 + f(0)^3$
$= 1^2 + 1^3$
$= 1 + 1$
$= 2$
$f(3) = f(2)^2 + f(1)^3$
$= 2^2 + 1^3$
$= 4 + 1$
$= 5$
$f(4) = f(3)^2 + f(2)^3$
$= 5^2 + 2^3$
$= 25 + 8$
$= 33$
$f(5) = f(4)^2 + f(3)^3$
$= 33^2 + 5^3$
$= 1089 + 125$
$= 1214$

(d)
$f(n+1) = f(n)/f(n-1)$
$f(2) = f(1)/f(0)$
$= 1/1 = 1$
$f(3) = f(2)/f(1)$
$= 1/1 = 1$
$f(4) = f(3)/f(2)$
$= 1/1 = 1$
$f(5) = f(4)/f(3)$
$= 1/1 = 1$

Problem 2. Section 5.3, Exercise 6, page 357

Solution.
(a) The function is well-defined
$f(n) = -f(n-1)$ for $n \geq 1$.
Basis step: $f(0) = 1 = (-1)^0$
Induction step: $f(n) = -f(n-1)$
$= -(-1)^{n-1}$
$= (-1)^n$
Therefore, claim proven by induction.
(b) The function is well-defined
2^m if $n = 3k$,
0 if $n = 3k + 1$,

and 2^{m+1} if $n = 3k + 2$
for some integer k.
Basis step:
$f(0) = 1 = 2^0$
$f(1) = 0$
$f(2) = 2 = 2^{0+1}$
Induction step: $f(n) = 2f(n - 3)$ for $n \geq 3$
Therefore, $f(n) = 2^k$, or, $= 0$, or, $= 2^{k+1}$
The claim is proven by strong induction.
c) Function is invalid because no base case can be reached.
d) Function is not well-defined because $f(1)$ and $2f(1-1)$ equal different values.
e) The function is defined by $f(n) = 2^{\lfloor n/2 \rfloor + 1}$
Basis Step:
$f(0) = 2 = 2^1 = 2^{\lfloor 0/2 \rfloor + 1}$
$f(1) = f(0) = 2 = 2^1 = 2^{\lfloor 1/2 \rfloor + 1}$
Induction Step:
$f(n) = 2f(n - 2)$
$= 2 * 2^{\lfloor (n-2)/2 \rfloor + 1}$
$= 2 * 2^{\lfloor n/2 \rfloor} = 2^{\lfloor n/2 \rfloor + 1}$
Thus, the claim is proven by induction.

Problem 3. Section 5.3, Exercise 8, page 358

Solution.
a) $a_1 = 2$ and $a_n = 4 + a_{n-1}$ for $n \geq 2$.
b) $a_1 = 0$, $a_2 = 2$, and $a_n = a_{n-2}$ for $n \geq 3$.
c) $a_1 = 2$ and $a_n = a_{n-1} + 2n$ for $n \geq 2$.
d) $a_1 = 1$ and $a_n = a_{n-1} + 2n - 1$ for $n \geq 2$.

Problem 4. Section 5.3, Exercise 12, page 358

Solution.
Basis Step: $P(1) = f_1^2 = f_1 f_1 = 1 * 1 = 1$
Induction Step:
$P(k + 1) = f_1^2 + f_2^2 + ... + f_k^2 + f_{k+1}^2 = f_k f_k + 1 + f_{k+1}^2$
$= f_k f_{k+1} + f_{k+1} f_{k+1}$
$= f_{k+1}[f_k + f_{k+1}]$
$= f_{k+1} f_{k+2}$
Therefore, true by induction.

Problem 5. Section 5.3, Exercise 14, page 358

Solution.
Basis Step: $P(1) = f_{1+1} f_{1-1} - f_1^2 = f_2 f_0 - f_1 f_1 = 1 * 0 - 1 * 1 = (-1)^1$
Induction Step:
$P(k + 1) = f_{(k+1)+1} f_{(k+1)-1} - f_{k+1}^2$
$= f_{k+2} f_k - f_{k+1}^2$
$= f_{k+2} f_k - f_{k+1} f_{k+1}$

$= (f_{k+1} + f_k)f_k - f_{k+1}fk + 1$
$= f_{k+1}f_k + f_k^2 - f_{k+1}[f_k + f_{k-1}]$
$= f_{k+1}f_k + f_k^2 - f_{k+1}f_k - f_{k+1}f_{k-1}$
$f_k^2 - f_{k+1}f_{k-1}$
$= -(-1)^k$
$= -1(-1)^k$
$= (-1)^{k+1}$
Therefore, proven by induction.

Problem 6. Section 5.3, Exercise 16, page 358

Solution.
Basis Step: $f_0 - f_1 + f_2 = 0 - 1 + 1 = 1 - 1 = f_1 - 1$
Induction Step:
$\Sigma_{k=0}^{2n+2}(-1)^k f_k$
$=f_{2n-1} - 1 - f_{2n+1} + f_{2n+2}$
$=f_{2n-1} + f_{2n} - 1$
$=f_{2n+1} - 1$
$=f_{2(n+1)-1} - 1$
Therefore, claim proven by induction.

Problem 7. Section 5.3, Exercise 20, page 358

Solution.
max:
$max(a_i, a_{i+1}) = a_i if a_i \geq a_{i+1} for i = 0, 1, 2, ..., n-1$
$max(a_1, a_2, ..., a_n) = max(max(a_1, a_2, ..., a_{n-1}), a_n)$

min:
$min(a_i, a_{i+1}) = a_i if a_i \leq a_{i+1} for i = 0, 1, 2, ..., n-1$
$min(a_1, a_2, ..., a_n) = min(min(a_1, a_2, ..., a_{n-1}), a_n)$

Problem 8. Section 5.3, Exercise 26, page 358

Solution.
(a)
1st Application: $(2, 3), (3, 2)$
2nd Application: $(4, 6), (5, 5), (6, 4)$
3rd Application: $(6, 9), (7, 8), (8, 7), (9, 6)$
4th Application: $(8, 12), (9, 11), (10, 10), (11, 9), (12, 8)$
5th Application: $(10, 15), (11, 14), (12, 13), (13, 12), (14, 11), (15, 11)$

(b)
Basis Step:
$5|a + b$ with $(0, 0)$
$5|0 + 0$, so basis holds
Inductive Hypothesis: Assume 5—a+b
Recursive Step:

True by strong induction. Cause I ran out of time.

(c)
Basis Step:
$5|a + b$ when a=0 and b=0
$5|0$, so basis holds
Inductive Hypothesis: Assume $5|a + b$ for $(a, b) \in S$
Recursive Step:
Show that (a+2, b+3) and (a+3, b+2) are divisible by 5
5—a+b is true by I.H.
5—a+2+b+3 = 5—a+b+5
5—5
Thus, we can prove the claim by structural induction.

Problem 9. Section 5.3, Exercise 36, page 359

Solution.
Let w_1 = ab
Let w_2 = cd
$w_1 w_2$ = abcd
$(w_1 w_2)^R$ = dcba

Recursive Step:
w_1^R = ba
w_2^R = dc
$w_2^R w_1^R$ = dcba
$= (w_1 w_2)^R$

Problem 10. Section 5.3, Exercise 44, page 359

Solution.
Let T be a binary tree with a single root node.
$l(T) = 1, i(T) = 0$
$l(T) = l(T_1) + l(T_2)$
$= (i(T_1) + 1) + (i(T_2) + 1)$
As such, for any binary tree T, $l(T) = i(T) + 1$
The claim is proven by structural induction.

Checklist:
□ Did you add your name?
□ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
□ Did you sign that you followed the Aggie honor code?
□ Did you solve all problems?
□ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework on csnet?
□ Did you submit (c) a signed hardcopy of the pdf file in class?