CMC - 12 Lista 3 Eric Guerra Ribeiro

1- $\begin{cases} V - Ri - V_b = 0 \\ J\dot{\omega} = \tau_m - b\omega \end{cases}$ $\quad \tau_m = k_t \cdot i \quad \omega = k_\omega V_b \quad V_b \cdot i = \tau_m \cdot \omega$

$V_b \cdot \cancel{i} = k_\omega \cdot k_t \cdot \cancel{i} \cdot V_b$

$k_\omega = \frac{1}{k_t} \Rightarrow V_b = k_t \omega$

$$V - Ri - k_t \cdot \omega = 0$$

$$J\dot{\omega} = k_t \cdot i - b\omega \Rightarrow i = \frac{J\dot{\omega} + b\omega}{k_t}$$

$$R\frac{J\dot{\omega} + b\omega}{k_t} + k_t\omega = V(t) = k_{ff}\,\omega_r + k_p(\omega_r - \omega)$$

$$\frac{RJ}{k_t}\dot{\omega} + \left(\frac{Rb}{k_t} + k_t + k_p\right)\omega - (k_{ff} + k_p)\,\omega_r = 0$$

$$\dot{\omega} + \frac{Rb + k_t^2 + k_p k_t}{RJ}\omega - \frac{k_t(k_{ff} + k_p)}{RJ}\omega_r = 0$$

$\omega(t) = \omega_h + \omega_p$

i) Solução Particular: $\omega_p = \frac{k_t(k_{ff} + k_p)}{Rb + k_t^2 + k_p k_t}\omega_r$

ii) Solução Homogênea: $\omega = e^{\lambda t} \Rightarrow e^{\lambda t}\left(\lambda + \frac{Rb + k_t^2 + k_p k_t}{RJ}\right) = 0$

$$\lambda = -\frac{Rb + k_t^2 + k_p k_t}{RJ} \Rightarrow \omega_h = c_1 e^{-\frac{Rb + k_t^2 + k_p k_t}{RJ}t}$$

$$\therefore \omega(t) = c_1 \cdot e^{-\frac{Rb + k_t^2 + k_p k_t}{RJ}t} + \frac{k_t(k_{ff} + k_p)}{Rb + k_t^2 + k_t k_p}\omega_r$$

a) $\lim_{t \to \infty} \omega = \frac{k_t(k_{ff} + k_p)}{Rb + k_t^2 + k_t k_p}\omega_r = \omega_r \Rightarrow k_t k_{ff} + \cancel{k_t k_p} = Rb + k_t^2 + \cancel{k_t k_p}$

$$\therefore k_{ff} = \frac{Rb + k_t^2}{k_t} = k_t + \frac{Rb}{k_t}$$

b) $\tau = \frac{RJ}{Rb + k_t^2 + k_p k_t} \Rightarrow Rb + k_t^2 + k_p k_t = \frac{RJ}{\tau} \Rightarrow k_p = \frac{RJ}{k_t \tau} - \frac{Rb + k_t^2}{k_t}$

$$k_p = \frac{RJ}{k_t \tau} - \frac{Rb}{k_t} - k_t = \frac{R}{k_t}\left(\frac{J}{\tau} - b\right) - k_t$$

2- $\begin{cases} V - Ri - V_b = 0 \\ J\dot{\omega} = \tau_m - b\omega \end{cases}$ $\quad \tau_m = k_t i$, $\quad V_b = k_t \cdot \omega$, $\quad V = k_v(k_p(\theta_r - \theta) - \omega)$

$$i = \frac{V - V_b}{R} \Rightarrow J\ddot{\theta} = k_t \cdot i - b\dot{\theta} \Rightarrow J\ddot{\theta} = \frac{k_v(k_p(\theta_r - \theta) - \dot{\theta}) - k_t\dot{\theta}}{R} k_t - b\dot{\theta}$$

$$J\ddot{\theta} - \frac{k_t k_v k_p \theta_r}{R} + \frac{k_t k_v k_p \theta}{R} + \frac{k_t k_v \dot{\theta}}{R} + \frac{k_t^2 \dot{\theta}}{R} + b\dot{\theta} = 0$$

$$J\ddot{\theta} + \frac{k_t k_v + k_t^2 + Rb}{R}\dot{\theta} + \frac{k_t k_v k_p}{R}\theta = \frac{k_t k_v k_p}{R}\theta_r$$

$$\ddot{\theta} + \frac{k_t k_v + k_t^2 + Rb}{JR}\dot{\theta} + \frac{k_t k_v k_p}{JR}\theta = \frac{k_t k_v k_p}{JR}\theta_r \Rightarrow \omega_n^2 = \frac{k_t k_v k_p}{JR} \text{ e } 2\xi\omega_n = \frac{k_t k_v + k_t^2 + Rb}{JR}$$

$$\Rightarrow 2JR\xi\omega_n = k_t k_v + k_t^2 + Rb \Rightarrow k_v = \frac{2JR\xi\omega_n - Rb}{k_t} - k_t$$

$$k_p = \frac{JR\omega_n^2}{k_t k_v} = \frac{JR\omega_n^2}{2JR\xi\omega_n - k_t^2 - Rb}$$

3- $\begin{cases} \dot{h} = v\psi \\ \dot{\psi} = \omega = k_\psi(k_p(h_r - h) - \psi) = -k_\psi k_p h - k_\psi \psi + k_\psi k_p h_r \end{cases}$

$$\frac{d}{dt}\begin{bmatrix} h \\ \psi \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & v \\ -k_\psi k_p & -k_\psi \end{bmatrix}}_{A} \begin{bmatrix} h \\ \psi \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ k_\psi k_p \end{bmatrix}}_{B} h_r$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} h \\ \psi \end{bmatrix} + \underbrace{0}_{D} \cdot h_r$$