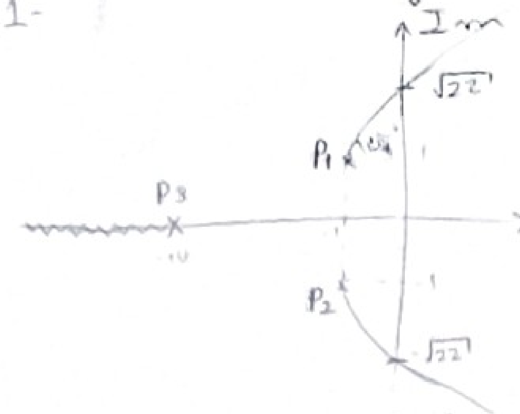
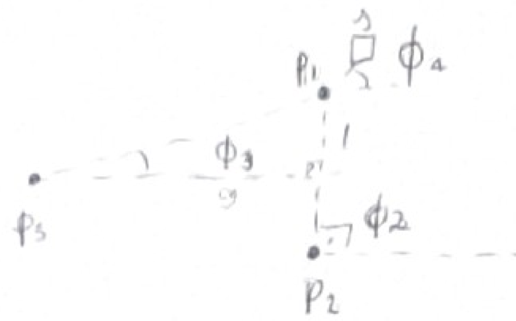
CMC - 12 Eric Guerra Ribeiro Lista 7

1-





$$G_p = \frac{KG}{1+KG} = \frac{\frac{KN}{D}}{1+\frac{KN}{D}} = \frac{KN}{D+KN}$$

$$-\phi_1 - \phi_2 - \phi_3 = -180^\circ$$

$$\phi_3 = \tan^{-1}\left(\frac{1}{9}\right), \ \phi_2 = 90^\circ$$

$$\phi_1 = 90^\circ - \tan^{-1}\left(\frac{1}{9}\right)$$

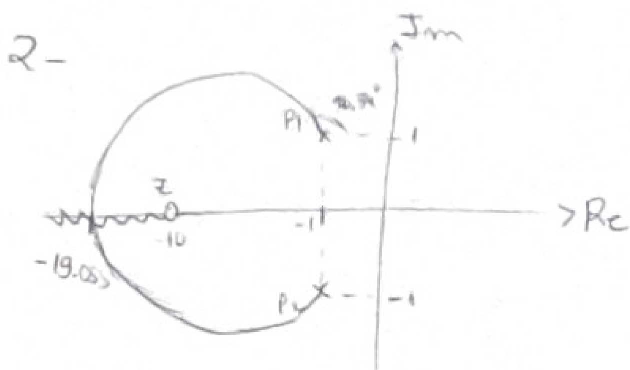
$$\therefore \phi_1 = 83.66^\circ$$

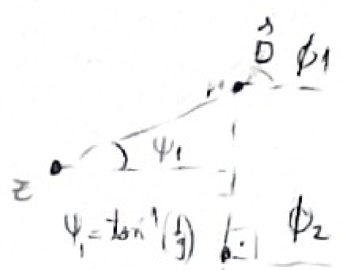
$$D_p(s) = s^3 + 12s^2 + 22s + 20 + 10K$$

$$D_p(j\omega) = -\omega^3 j - 12\omega^2 + 22\omega j + 20 + 10K = 0$$

$$\begin{cases} -\omega^3 + 22\omega = 0 \Rightarrow \omega = 0 \text{ ou } \omega = \pm\sqrt{22} \\ 20 + 10K - 12\omega^2 = 0 \Rightarrow \cancel{K = -\frac{1}{2}} \text{ ou } K = \frac{12 \cdot 22 - 20}{10} = 24.4 \text{ SI} \end{cases}$$

2-





$$\psi_1 - \phi_1 - \phi_2 = -180^\circ$$

$$\phi = 90^\circ + \psi_1 = 96.34^\circ$$

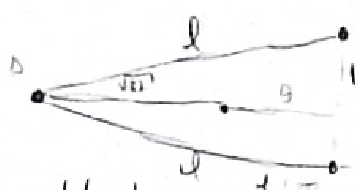
$$\frac{d}{ds}\left[\frac{1}{G(s)}\right] = 0$$

$$\frac{d}{ds}\left[10 \ \frac{s^2+2s+2}{s+10}\right] = 0$$

$$10 \ \frac{(2s+2)(s+10) - 1(s^2+2s+2)}{(s+10)^2} = 0$$

$$2s^2 + 22s + 20 - s^2 - 2s - 2 = 0$$

$$s^2 + 20s + 18 = 0 \Rightarrow s = \frac{-20 \pm \sqrt{4 \cdot 100 - 4 \cdot 18}}{2} = -10 \pm \sqrt{82} \begin{cases} -0.944 \\ -19.05 \end{cases}$$

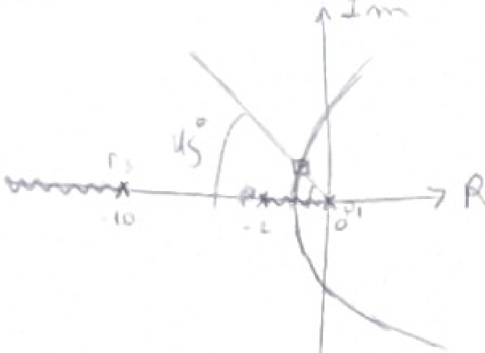


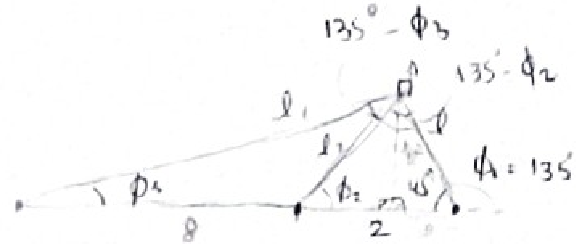
$$l^2 = 1 + (9+\sqrt{82})^2$$
$$l^2 = 82 + 18\sqrt{82} + 82$$
$$l^2 = 2(82 + 9\sqrt{82})$$
$$l^2 = 2\sqrt{82}(9+\sqrt{82})$$

$$K = \left|\frac{1}{G(s)}\right| = \frac{10\overbrace{|s+1-j|}^{l}\overbrace{|s+1+j|}^{l}}{\underbrace{|s+10|}_{\sqrt{82}}} = 20(\sqrt{82}+9)$$

3 - Considerando polos dominantes.

$$\xi = \frac{1}{\sqrt{\left(\frac{\pi}{\ln M_p}\right)^2 + 1}} = 0.707 \Rightarrow \beta = \arccos \xi = 45^\circ$$





$$-\phi_1 - \phi_2 - \phi_3 = -180^\circ$$

$$\phi_3 = 45^\circ - \phi_2$$

$$tg\,\phi_2 = \frac{\frac{l}{\sqrt{2}}}{2 - \frac{l}{\sqrt{2}}} = \frac{l}{2\sqrt{2} - l} \qquad tg\,\phi_3 = \frac{1 - tg\,\phi_2}{1 + tg\,\phi_2}$$

$$tg\,\phi_3 = \frac{\frac{l}{\sqrt{2}}}{10 - \frac{l}{\sqrt{2}}} \Rightarrow \frac{\frac{l}{\sqrt{2}}}{10 - \frac{l}{\sqrt{2}}} = \frac{1 - \frac{l}{2\sqrt{2} - l}}{1 + \frac{l}{2\sqrt{2} - l}} = \frac{\sqrt{2} - l}{\sqrt{2}} = 1 - \frac{l}{\sqrt{2}}$$

$$\frac{l}{\sqrt{2}} = 10 - \frac{10l}{\sqrt{2}} - \frac{l}{\sqrt{2}} + \frac{l^2}{2} \Rightarrow \frac{l^2}{2} - \frac{12l}{\sqrt{2}} + 10 = 0$$

$$l^2 - 12\sqrt{2}\,l + 20 = 0 \Rightarrow l = \frac{12\sqrt{2} \pm \sqrt{288 - 80}}{2}$$

$$l = 2(3\sqrt{2} \pm \sqrt{13}) \quad \begin{cases} 15.7 \rightarrow \text{Não convém, pois } l < 2 \\ 1.274 \end{cases}$$

$$\therefore l = 2(3\sqrt{2} - \sqrt{13})$$

$$l_1^2 = 100 + l^2 - 2 \cdot 10 \cdot l \cdot \frac{1}{\sqrt{2}} = 100 + 12\sqrt{2}\,l - 20 - 10\sqrt{2}\,l = 80 + 2\sqrt{2}\,l$$

$$l_1 = \sqrt{80 + 2\sqrt{2}\,l}$$

$$l_2^2 = 4 + l^2 - 2 \cdot 2l \frac{1}{\sqrt{2}} = 4 + 12\sqrt{2}\,l - 20 - 2\sqrt{2}\,l = 10\sqrt{2}\,l - 16$$

$$l_2 = \sqrt{10\sqrt{2}\,l - 16} \qquad K = \left|\frac{1}{G}\right| = \frac{\overset{l}{|s|}\,\overset{l_2}{|s+2|}\,\overset{l_1}{|s+10|}}{5} = 3.311$$

$$G_f = \frac{5K}{s^3 + 12s^2 + 20s + 5K}$$, para $K = 3.311$ polos: $-10.1980 + 0j$; $-0.9010 \pm 0.9010j$

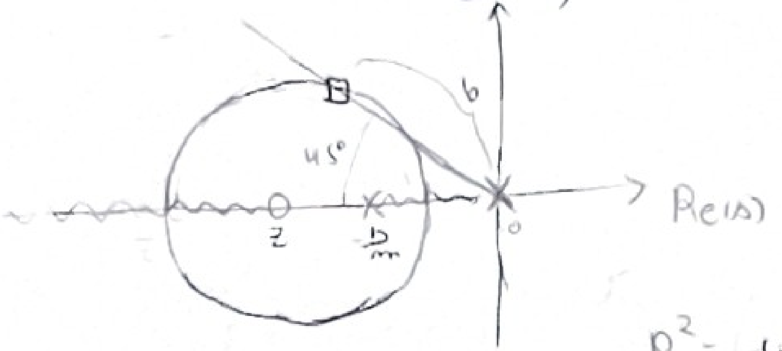
Como $5 \cdot 0.9010 < 10.19$ é válida a hipótese de polos dominantes

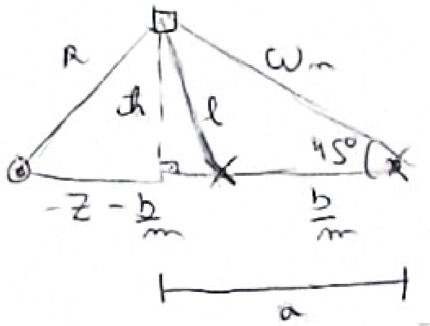
4- $PI \Rightarrow \dfrac{K_p \cdot s + K_i}{s} = \dfrac{K(s-z)}{s} \Rightarrow \begin{matrix} K_p = K \\ K_i = -z \cdot K \end{matrix}$

$$G'(s) = \frac{s-z}{s} \cdot \frac{1}{ms+b} = \frac{s-z}{s(ms+b)} = \frac{1}{m} \cdot \frac{s-z}{s\left(s+\frac{b}{m}\right)}$$

Supondo $z < 0$ e $|z| > \left|\frac{b}{m}\right|$

$\beta = \arccos \xi = 45^\circ$





$$R^2 = \omega_n^2 + z^2 - 2(-z)\omega_n \frac{\sqrt{2}}{2}$$

$$R^2 = \omega_n^2 + z^2 + \omega_n z \sqrt{2}$$

$$\text{e} \quad R^2 = \left(z + \frac{b}{m}\right) z = z^2 + \frac{b}{m} z$$

$$\Rightarrow \frac{b}{m} z = \omega_n^2 + \omega_n z \sqrt{2}$$

$$K = \frac{1}{|G'(s)|} = \frac{m \overbrace{|s|}^{\omega_n} \overbrace{\left|s + \frac{b}{m}\right|}^{l}}{\underbrace{|s-z|}_{R}}$$

$$z = \frac{\omega_n^2}{\frac{b}{m} - \omega_n\sqrt{2}} = \frac{m\omega_n^2}{b - m\omega_n\sqrt{2}}$$

$$z = -\frac{m\omega_n^2}{m\omega_n\sqrt{2} - b}$$

$$R = \sqrt{\left(z + \frac{b}{m}\right) z},$$

$$l^2 = \frac{b^2}{m^2} + \omega_n^2 - \cancel{2}\omega_n \frac{b}{m} \frac{\sqrt{2}}{\cancel{2}}$$

$$l = \sqrt{\frac{b^2}{m^2} + \omega_n^2 - \frac{\omega_n b \sqrt{2}}{m}}$$

Obs $\dfrac{m\omega_n^2}{m\omega_n\sqrt{2} - b} = 4.2678 > \dfrac{50}{1000} = \dfrac{1}{20}$

5 -

$$G(s) = (s+a)\ \frac{10}{s(s+1)(s+2)}$$

$G'(s)$

$$G_p(s) = \frac{K(s+a)\cdot G'(s)}{1+K(s+a)G'(s)} \Rightarrow D_p(s) = 1 + K(s+a)G'(s) = 0$$

$$\Rightarrow \frac{1}{KG'(s)} + s + a = 0 \Rightarrow \frac{1}{KG'(s)} + s = -a$$

$$\Rightarrow \frac{1}{\frac{1}{KG'(s)} + s} = -\frac{1}{a}$$

$G''(s)$

$$\Rightarrow \text{rlocus}\left(\frac{1}{\frac{1}{KG'(s)} + s}\right) \Rightarrow \text{rlocus}\left(\frac{1}{s\left(\frac{(s+1)(s+2)}{10K} + 1\right)}\right)$$