CMC-12 [illegible] Lista 8

1- $Y_{(s)} = \left((-Y_{(s)})(K_p + K_d s) + D_{(s)} \right) \dfrac{1}{s(ms+b)}$

$\Rightarrow \dfrac{Y_{(s)}}{D_{(s)}} = \dfrac{1}{s(ms+b)} - \dfrac{K_p + K_d s}{s(ms+b)} \cdot \dfrac{Y_{(s)}}{D_{(s)}}, \quad G_D(s) = \dfrac{Y_{(s)}}{D_{(s)}}$

$\Rightarrow \left(1 + \dfrac{K_p + K_d s}{s(ms+b)}\right) G_D(s) = \dfrac{1}{s(ms+b)} \Rightarrow G_D(s) = \dfrac{1}{ms^2 + (b+K_d)s + K_p}$

$$G_D(j\omega) = \frac{1}{K_p - m\omega^2 + (b+K_d)\omega j} \Rightarrow \begin{cases} |G_D(j\omega)| = \dfrac{1}{\sqrt{(K_p - m\omega^2)^2 + (b+K_d)^2\omega^2}} \\ \angle G_D(j\omega) = -\arctan\left(\dfrac{(b+K_d)\omega}{K_p - m\omega^2}\right) \end{cases}$$

$y(t) = A_d |G_D(j\omega)| \operatorname{sen}\left(\omega t + \phi_d + \angle G_D(j\omega)\right)$

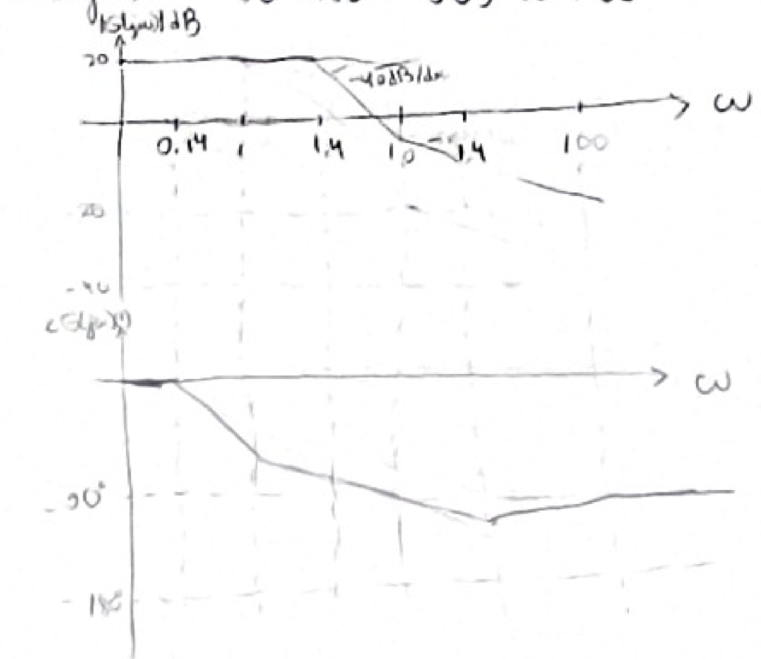
2- $G(s) = \dfrac{2(s+10)}{(s^2+2s+2)} \Rightarrow G(j\omega) = \dfrac{2(j\omega+10)}{(2j\omega + 2 - \omega^2)}$

$\begin{cases} \text{zero: } \omega = 10 \\ \text{polos: complexos conjugados } \omega_m = \sqrt{2} = 1{,}41 \end{cases}$

$G(0) = 10 \Rightarrow |G(0)|_{dB} = 20\,dB$

$|G(j\omega)| \to 0, \ \omega \to \infty$ $\qquad |G(j\omega)| = 10, \ \omega = 0$

$\angle G(j\omega) \to 90° - 180° = -90°, \ \omega \to \infty$ $\qquad \angle G(j\omega) = 0°, \ \omega = 0$



4 - $G_{R_1}(s) = \dfrac{\frac{K}{ms+b}}{1+\frac{K}{ms+b}} = \dfrac{K/m}{s+\frac{b+K}{m}}$

$e_\infty = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left(Y(s) - R(s)\right) = \lim_{s\to 0} s\left(R(s)\cdot G_{P_{fim}} - R(s)\right)$

$e_\infty = \lim_{s\to 0} \dfrac{s}{s}\left(\dfrac{K/m}{s+\frac{b+K}{m}} - 1\right) = \dfrac{\frac{K}{m}}{\frac{b+K}{m}} - 1 = -\dfrac{b}{b+K}$

$\Rightarrow \dfrac{b}{b+K} \leq 0.1 \Rightarrow K \geq 9b$

$Y(s) = -\left(Y(s) + N(s)\right)\dfrac{K}{ms+b} \Rightarrow G_{N(s)} = -\left(G_{N(s)} + 1\right)\dfrac{K}{ms+b}$

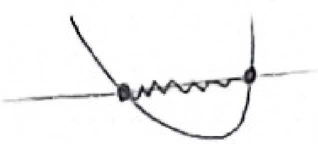
$G_{N(s)} = -\dfrac{\frac{K}{ms+b}}{1+\frac{K}{ms+b}} = -\dfrac{K}{ms+b+K} = -\dfrac{K/m}{s+\frac{b+K}{m}}$

$G_N(j\omega) = -\dfrac{K/m}{j\omega+\frac{b+K}{m}} \Rightarrow |G_N(j\omega)| = \dfrac{K/m}{\sqrt{\omega^2+\left(\frac{b+K}{m}\right)^2}} = \dfrac{K}{\sqrt{m^2\omega^2+(b+K)^2}}$

$20\log_{10}\left(\dfrac{K}{\sqrt{m^2\omega^2+(b+K)^2}}\right) \leq -20 \Rightarrow \dfrac{K}{\sqrt{m^2\omega^2+(b+K)^2}} \leq 10^{-1}$

$\dfrac{K^2}{m^2\omega^2+(b+K)^2} \leq 10^{-2} \Rightarrow K^2 \leq 10^{-2}\cdot m^2\omega^2 + 10^{-2}(b+K)^2$

$100K^2 \leq m^2\omega^2 + b^2 + 2bK + K^2 \Rightarrow 99K^2 - 2bK - m^2\omega^2 - b^2 \leq 0$



$K = \dfrac{2b \pm \sqrt{4b^2 + 4\cdot 99\cdot(m^2\omega^2+b^2)}}{2\cdot 99}$

$K = \dfrac{b \pm \sqrt{99m^2\omega^2+100b^2}}{99} \Rightarrow K_{max} = \dfrac{b+\sqrt{99m^2\omega^2+100b^2}}{99}$

$K_{min} = \max\left(9b, \dfrac{b-\sqrt{9m^2\omega^2+100b^2}}{99}\right)$

Se $K_{min} > K_{max} \Rightarrow$ não há solução

5-

$$H(s) = \left((R(s) - H(s)) K_p - \frac{s H(s)}{v} \right) \frac{K_\psi}{s} \cdot v \cdot \frac{1}{s}$$

$$G_H = \left((1 - G_H) K_p - \frac{s}{v} G_H \right) \frac{K_\psi v}{s^2}$$

$$s^2 G_H = (1 - G_H) K_p K_\psi \cdot v - s K_\psi G_H$$

$$(s^2 + K_\psi \cdot s + K_p K_\psi \cdot v) G_H = K_p K_\psi v$$

$$G_H(s) = \frac{K_p K_\psi \cdot v}{s^2 + K_\psi \cdot s + K_p K_\psi v} \Rightarrow \begin{cases} \omega_n^2 = K_p K_\psi v \\ 2\xi \omega_n = K_\psi \end{cases} \Rightarrow \begin{array}{l} K_p = \frac{\omega_n}{2\xi v} \\ K_\psi = 2\xi\omega_n \end{array}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \Rightarrow M_r^2 = \frac{1}{4\xi^2(1-\xi^2)}$$

$$\Rightarrow \xi^2(1-\xi^2) = \frac{1}{4M_r^2} \Rightarrow \xi^4 - \xi^2 + \frac{1}{4M_r^2} = 0$$

$$\xi^2 = \frac{1 \pm \sqrt{1 - \frac{4}{4M_r^2}}}{2} = \frac{1 \pm \sqrt{1 - \frac{1}{M_r^2}}}{2} = \frac{1 \pm \sqrt{1 - 10^{-\frac{M_{r_{dB}}}{10}}}}{2}$$

$$M_{r_{dB}} = 20 \log M_r \Rightarrow M_r = 10^{\frac{M_{r_{dB}}}{20}}$$

$$\omega_b = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}} \Rightarrow \omega_n = \frac{\omega_b}{\sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}}$$

$$\xi^2 < \frac{1}{2} \Rightarrow \xi = \sqrt{\frac{1 - \sqrt{1 - 10^{-\frac{M_{r_{dB}}}{10}}}}{2}}$$