CMC - 12 Lista 6 Enzo Guerra Ribeiro

1 -
$$Y = K\frac{5}{(s+2)(s+3)(s+4)}(R-Y) \Rightarrow Y\left(1+\frac{5K}{(s+2)(s+3)(s+4)}\right) = \frac{5K}{(s+2)(s+3)(s+4)}R$$

$$\frac{Y}{R} = \frac{\left(\frac{5K}{(s+2)(s+3)(s+4)}\right)}{\left(\frac{s^3+9s^2+26s+24+5K}{(s+2)(s+3)(s+4)}\right)} = \frac{5K}{s^3+9s^2+26s+24+5K} = G_{f(s)}$$

$$\frac{26\cdot 9-24-5K}{9} > 0 \Rightarrow 260-24-26-5K > 0 \Rightarrow 5K < 210$$
$$K < 42$$

s^3: 1 26

s^2: 9 24+5K

s^1: $\frac{26\cdot 9 - 24 - 5K}{9}$

s^0: 24+5K

$$24+5K > 0 \Rightarrow K > -\frac{24}{5} = -4.8$$

$a = -4.8$ e $b = 42$ $K \in]-4.8, 42[$

2 -
$$Y(s) = \left(Y(s)\frac{Kp\cdot s + Ki}{s} + D(s)\right)\cdot\frac{1}{ms+b}$$

$$(ms+b)Y(s) = \frac{Kp\cdot s+Ki}{s}Y(s) + D(s)$$

$$\left(ms+b-\frac{Kp\cdot s+Ki}{s}\right)Y(s) = D(s)$$

$$\frac{Y(s)}{D(s)} = G_{f,D}(s) = \frac{s}{ms^2+(b-Kp)s-Ki}$$

$$e_\infty = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\cdot G_{f,D}\cdot D(s) = \lim_{s\to 0}\frac{s^2}{ms^2+(b-Kp)s-Ki}\cdot\frac{1}{s^2} = -\frac{1}{Ki}$$

3 -
$$\begin{cases}\dot y = v\\ \dot v = \frac{Kf}{m}\frac{u^2}{(y_{max}-y)^2} - \frac{b}{m}v - g\end{cases} \Rightarrow \frac{d}{dt}\begin{bmatrix}y\\v\end{bmatrix} = \begin{bmatrix}v\\ \frac{Kf}{m}\cdot\frac{u^2}{(y_{max}-y)^2} - \frac{b}{m}v - g\end{bmatrix} = 0$$

$v_0 = 0$

$$\Rightarrow \frac{Kf}{m}\frac{u_0^2}{(y_{max}-y_0)^2} - \frac{b}{m}\cancel{v_0}^{0} - g = 0 \Rightarrow u_0^2 = \frac{mg}{Kf}(y_{max}-y_0)^2 \Rightarrow u_0 = \sqrt{\frac{mg}{Kf}}\,(y_{max}-y_0)$$

$$\frac{d}{dt}\begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial y}(v)\Big|_{y_0,v_0,u_0} & \frac{\partial}{\partial v}(v)\Big|_{y_0,v_0,u_0} \\ \frac{\partial}{\partial y}\left(\frac{KI}{m}\frac{u^2}{(y_{max}-y)^2} - \frac{bv}{m} - g\right)\Big|_{y_0,v_0,u_0} & \frac{\partial}{\partial v}\left(\frac{KI}{m}\frac{u^2}{(y_{max}-y)^2} - \frac{bv}{m} - g\right)\Big|_{y_0,v_0,u_0} \end{bmatrix} \begin{bmatrix} y-y_0 \\ v-v_0 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{\partial}{\partial u}(v)\Big|_{y_0,v_0,u_0} \\ \frac{\partial}{\partial u}\left(\frac{KI}{m}\frac{u^2}{(y_{max}-y)^2} - \frac{bv}{m} - g\right)\Big|_{y_0,v_0,u_0} \end{bmatrix} [u-u_0]$$

$$\frac{d}{dt}\begin{bmatrix} y \\ v \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \delta y \\ \delta v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{2KI\,u_0^2}{m(y_{max}-y_0)^3} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} \delta y \\ \delta v \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2KI}{m}\frac{u_0}{(y_{max}-y_0)^2} \end{bmatrix}\delta u$$

$$\frac{d}{dt}\begin{bmatrix} \delta y \\ \delta v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{2g}{y_{max}-y_0} & -\frac{b}{m} \end{bmatrix}}_{A}\begin{bmatrix} \delta y \\ \delta v \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \sqrt{\frac{Kg}{m}}\cdot\frac{2}{y_{max}-y_0} \end{bmatrix}}_{B}\delta u$$

4- $G_p(s) = \frac{600}{(s+10)(s+20)(s^2+2s+4)} = \frac{10}{s+10}\cdot\frac{20}{s+20}\cdot\frac{3}{s^2+2s+4}$

p.d. $-1 \pm \sqrt{3}\,j$

$G_p(s) \approx \frac{3}{s^2+2s+4}$ $\quad e_\infty = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s(R - G_pR) = \lim_{s\to 0} s\,R(1-G_p)$

$e_\infty = \lim_{s\to 0} s\,\frac{1}{s}\left(1 - \frac{3}{s^2+2s+4}\right) = \frac{1}{4}$

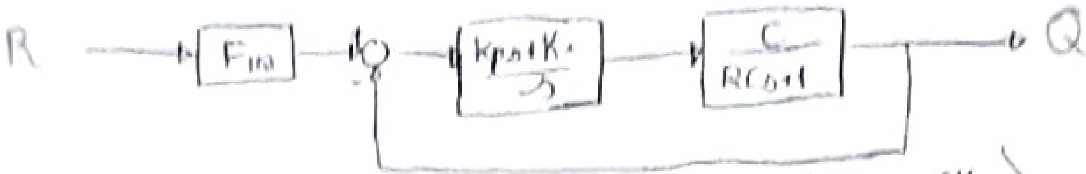
$t_r\big|_{10\%}^{90\%} = \frac{2.16\xi + 0.6}{\omega_n}$

$t_s\big|_{2\%} = \frac{-\ln\left(\frac{1}{50}\right)}{\xi\omega_n} = \frac{\ln 50}{\xi\omega_n}$

$$\begin{cases} 2\xi\omega_n = 2 \Rightarrow \xi = \frac{1}{2} \\ \omega_n^2 = 4 \Rightarrow \omega_n = 2 \end{cases}$$

5. $V = Ri + \frac{q}{C} = R\dot{q} + \frac{q}{C} \Rightarrow V(s) = Rs\,Q(s) + \frac{Q(s)}{C}$

$$\frac{Q(s)}{V(s)} = \frac{C}{RCs+1}$$



$$G_p = F \cdot \frac{\frac{K_p s + K_i}{s} \cdot \frac{C}{RCs+1}}{1 + \frac{K_p s + K_i}{s} \cdot \frac{C}{RCs+1}} = F \cdot \frac{\left(\frac{CK_p s + CK_i}{s(RCs+1)}\right)}{\left(\frac{RCs^2 + (CK_p+1)s + CK_i}{s(RCs+1)}\right)} = F\,\frac{\frac{K_p s + K_i}{R}}{s^2 + \frac{K_p+1}{R}s + \frac{K_i}{R}}$$

$$= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \Rightarrow \begin{cases} \frac{F\cdot(K_p s + K_i)}{R} = \frac{K_i}{R} \Rightarrow F = \frac{1}{\frac{K_p}{K_i}s + 1} \\ \frac{K_i}{R} = \omega_n^2 \Rightarrow K_i = R\omega_n^2 \\ 2\xi\omega_n = \frac{K_p + \frac{1}{C}}{R} \Rightarrow K_p = R\left(2\xi\omega_n - \frac{1}{C}\right) \end{cases}$$

$$F = \frac{1}{\frac{2\xi\omega_n - \frac{1}{C}}{\omega_n^2}s + 1}$$

$$M_p = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right) \Rightarrow \ln^2 M_p = \frac{\pi^2\xi^2}{1-\xi^2}$$

$$\xi = \sqrt{\frac{1}{1 + \left(\frac{\pi}{\ln M_p}\right)^2}}$$

$$t_r\Big|_{0\%}^{100\%} = \frac{\pi - \arccos\xi}{\omega_n\sqrt{1-\xi^2}} \Rightarrow \omega_n = \frac{\pi - \arccos\xi}{t_r\Big|_{0\%}^{100\%}\sqrt{1-\xi^2}}$$

$$K_p = R\left(2\xi\omega_n - \frac{1}{C}\right) \qquad K_i = R\omega_n^2$$