CMC - 12 Lista 2

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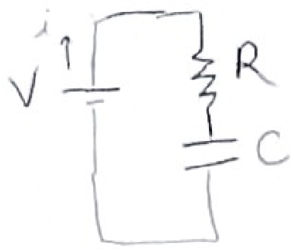
1-a)

$$V = Ri + \frac{q}{C} \Rightarrow V = R\dot{q} + \frac{q}{C}$$



$$\Rightarrow \dot{q} + \frac{1}{RC} q = \frac{V}{R}$$

i) Solução particular: $q = CV$

ii) Solução homogênea: $\dot{q} + \frac{1}{RC} q = 0 \Rightarrow q = e^{\lambda t} \Rightarrow e^{\lambda t}(\lambda + \frac{1}{RC}) = 0$

$\Rightarrow \lambda = -\frac{1}{RC} \Rightarrow q = c_1 e^{-\frac{t}{RC}}$

iii) Solução geral: $q = c_1 \; e^{-\frac{t}{RC}} + CV$

$\therefore \tau = RC = 10^4 \cdot 10^{-6} = 0.01 \, s$

b) $q(0) = 0 \Rightarrow c_1 + CV = 0 \Rightarrow c_1 = -CV$

$q(t) = CV\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow q(0.02) = 5 \cdot 10^{-6}\left(1 - e^{-\frac{0.02}{0.01}}\right)$

$q(0.02) = 5 \cdot 10^{-6}\left(1 - e^{-2}\right) \approx 4,323 \cdot 10^{-6} \, C$

2-a) $\begin{cases} V - Ri - V_b = 0 \\ J\dot{\omega} = \tau_m - b\omega \end{cases}$ e $V_b i = \tau_m \omega$, $\tau_m = k_t \cdot i$

$\omega = k_\omega \cdot V_b$

$k_t \cdot i \cdot k_\omega \cdot V_b = V_b i \Rightarrow k_\omega = \frac{1}{k_t}$

$\Rightarrow \tau_m = k_t \cdot i$ e $V_b = k_t \cdot \omega$

$V - Ri - V_b = 0$

$V - Ri - k_t \omega = 0 \Rightarrow i = \dfrac{V - k_t \cdot \omega}{R}$

$J\dot{\omega} = k_t \cdot i - b\omega \Rightarrow J\dot{\omega} = k_t \cdot \dfrac{V - k_t \cdot \omega}{R} - b\omega$

$\dot{\omega} + \dfrac{k_t^2 + bR}{JR}\omega - \dfrac{k_t \cdot V}{JR} = 0 \Rightarrow \dot{\omega} \; e^{\frac{k_t^2 + bR}{JR}t} + \dfrac{k_t^2 + bR}{JR} e^{\frac{k_t^2 + bR}{JR}t} - \dfrac{k_t \cdot V}{JR} \cdot e^{\frac{k_t^2 + bR}{JR}t} = 0$

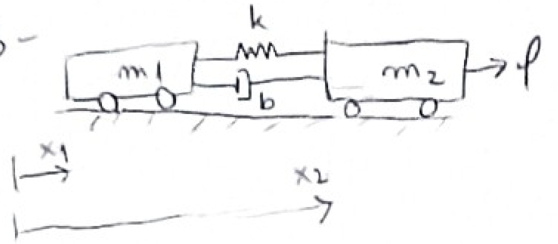
$e^{\int \frac{k_t^2 + bR}{JR} dt} = e^{\frac{k_t^2 + bR}{JR}t}$ $\quad \dfrac{d\left(\omega \, e^{\frac{k_t^2 + bR}{JR}t}\right)}{dt} = \dfrac{k_t \cdot V}{JR} \cdot e^{\frac{k_t^2 + bR}{JR}t}$

$$\omega \cdot e^{\frac{k_t^2+bR}{JR}t} = \frac{k_t V}{k_t^2+bR}\, e^{\frac{k_t^2+bR}{JR}t} + C$$

$$\omega = C\, e^{-\frac{k_t^2+bR}{JR}\cdot t} + \frac{k_t V}{k_t^2+bR} \Rightarrow \tau = \frac{JR}{k_t^2+bR}$$

b) $\omega \to \infty$: $\omega = \dfrac{k_t \cdot V}{k_t^2+bR} = 1{,}3743 \cdot 10^3$ rad/s

3-



Carrinho 1:

$$m_1 \ddot{x}_1 = -k(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2)$$

Carrinho 2:

$$m_2 \ddot{x}_2 = f - k(x_2 - x_1) - b(\dot{x}_2 - \dot{x}_1)$$

$$\begin{cases} \dot{x}_1 = \dot{x}_1 \\ \ddot{x}_1 = -\frac{b}{m_1}\dot{x}_1 + \frac{b}{m_1}\dot{x}_2 - \frac{k}{m_1}x_1 + \frac{k}{m_1}x_2 \\ \dot{x}_2 = \dot{x}_2 \\ \ddot{x}_2 = \frac{b}{m_2}\dot{x}_1 - \frac{b}{m_2}\dot{x}_2 + \frac{k}{m_2}x_1 - \frac{k}{m_2}x_2 + \frac{f}{m_2} \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{b}{m_1} & \frac{k}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & \frac{b}{m_2} & -\frac{k}{m_2} & -\frac{b}{m_2} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix}}_{B} f$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} f$$