CMC-12 Lista 4 [illegible] Gouvea Ribeiro

1-

$$F(s) = \frac{3s+5}{s^3+4s^2+5s+2} = \frac{3s+5}{(s+2)(s+1)^2}$$

-1	1	4	5	2
-1	1	3	2	0
-2	1	2	0	
	1	0		

$$F(s) = \frac{C_1}{s+2} + \frac{C_2}{s+1} + \frac{C_3}{(s+1)^2} = \frac{C_1(s+1)^2 + C_2(s+2)(s+1) + C_3(s+2)}{(s+2)(s+1)^2}$$

$$F(s) = \frac{(C_1+C_2)s^2 + (2C_1+3C_2+C_3)s + (C_1+2C_2+2C_3)}{(s+2)(s+1)^2} = \frac{3s+5}{(s+2)(s+1)^2}$$

$$\begin{cases} C_1+C_2 = 0 \Rightarrow C_1 = -C_2 \\ 2C_1+3C_2+C_3 = 3 \\ C_1+2C_2+2C_3 = 5 \end{cases} \Rightarrow \begin{cases} C_2 + C_3 = 3 \\ C_2 + 2C_3 = 5 \end{cases} \Rightarrow C_3 = 2,\ C_2 = 1,\ C_1 = -1$$

$$\therefore F(s) = -\frac{1}{s+2} + \frac{1}{s+1} + \frac{2}{(s+1)^2} \Rightarrow f(t) = -e^{-2t} + e^{-t} + 2te^{-t}$$

$$f(t) = (2t+1)e^{-t} - e^{-2t}$$

2- $\ddot{x} + 6\dot{x} + 18x = 18$, $x(0) = 1$, $\dot{x}(0) = 3$

$$s^2X - s\,x(0) - \dot{x}(0) + 6sX - 6x(0) + 18X = \frac{18}{s}$$

$$(s^2+6s+18)X = s + 9 + \frac{18}{s} = \frac{s^2+9s+18}{s}$$

$$\Rightarrow X = \frac{s^2+9s+18}{s(s^2+6s+18)} = \frac{C_1}{s} + \frac{C_2 s + C_3}{s^2+6s+18} = \frac{(C_1+C_2)s^2 + (6C_1+C_3)s + 18C_1}{s(s^2+6s+18)}$$

$$\begin{cases} C_1+C_2 = 1 \Rightarrow C_2 = 0 \\ 6C_1+C_3 = 9 \Rightarrow C_3 = 3 \\ 18C_1 = 18 \Rightarrow C_1 = 1 \end{cases} \Rightarrow X = \frac{1}{s} + \frac{3}{(s+3)^2+3^2}$$

$$\Rightarrow x(t) = 1 + e^{-3t}\,\mathrm{sen}(3t)$$

3- $$\begin{cases} V = Ri + L\dot{i} + V_b \\ J\ddot{\theta} = \tau_m - b\dot{\theta} \end{cases}$$

$\tau_m = K_t\, i$ $\quad$ $\tau_m \omega = V_b \cdot i \Rightarrow V_b = K_t\,\omega$

$\omega = K_\omega \cdot V_b$

$$\begin{cases} V = Ri + L\dot{i} + K_t\dot{\theta} \\ J\ddot{\theta} + b\dot{\theta} = K_t \cdot i \Rightarrow i = \dfrac{J\ddot{\theta} + b\dot{\theta}}{K_t} \Rightarrow \dot{i} = \dfrac{J\dddot{\theta} + b\ddot{\theta}}{K_t} \end{cases}$$

$$\frac{L}{K_t}J\dddot{\theta} + \frac{bL+RJ}{K_t}\ddot{\theta} + \frac{bR+K_t^2}{K_t}\dot{\theta} = V$$

$$s^3 \frac{LJ}{K_t} \Theta(s) + s^2 \frac{bL+RJ}{K_t} \Theta(s) + s \frac{bR+K_t^2}{K_t} \Theta(s) = V(s)$$

$$\frac{\Theta(s)}{V(s)} = G(s) = \frac{K_t}{s\left(LJ s^2 + (bL+RJ)s + (bR+K_t^2)\right)}$$

4- $m\ddot{x} + (b+K_v)\dot{x} + K_p K_v x = K_p K_v\, u(t)$

$$s^2 m X + s(b+K_v)X + K_p K_v X = K_p K_v U$$

$$\Rightarrow (m s^2 + (b+K_v)s + K_p K_v)X = K_p K_v U$$

$$\frac{X(s)}{U(s)} = G(s) = \frac{K_p K_v}{m s^2 + (b+K_v)s + K_p K_v}$$