Eric Guerra Ribeiro
CMC - 12 Lista 1

1 - C

$\ddot{y} + 2\dot{y} + 3y = 0$ (1): linear de 2ª ordem, homogênea

$5\dddot{y} + 2x\dot{y} + y = \operatorname{sen} x$ (2): linear de 3ª ordem, não-homogênea

$\ddot{y}(1+y) = 1 \Rightarrow \ddot{y} + y\ddot{y} = 1$: não linear, de ordem 2, não homogênea

a) F (2 é linear) b) F (1 é linear) c) V d) F (3 não é linear)

e) F (3 não é linear)

2 - $\ddot{x} + 1.4\dot{x} + x = 1$, $x(0) = 0$ e $\dot{x}(0) = 0$

i) Solução Particular:

$x = 1$

ii) Solução homogênea: $\ddot{x} + 1.4\dot{x} + x = 0$

$$x = e^{\lambda t} \Rightarrow e^{\lambda t}(\lambda^2 + 1.4\lambda + 1) = 0 \Rightarrow \lambda^2 + 1.4\lambda + 1 = 0$$

$$\lambda = \frac{-1.4 \pm \sqrt{1.96 - 4}}{2} = \frac{-1.4 \pm \sqrt{-2.04}}{2} = \frac{-1.4 \pm \sqrt{2.04}\, i}{2}$$

$$\lambda = -0.7 \pm \sqrt{0.51}\, i \Rightarrow x = c_1 \cdot e^{(-0.7 + \sqrt{0.51}\, i)t} + c_2\, e^{(-0.7 - \sqrt{0.51}\, i)t}$$

$x \in \mathbb{R} \Rightarrow Im(x) \equiv 0 \;\forall t$:

$$e^{-0.7t}\left(Re(c_1)\operatorname{sen}(\sqrt{0.51}\, t) - Re(c_2)\operatorname{sen}(\sqrt{0.51}\, t)\right) + e^{-0.7t}\left(Im(c_1)\cos(\sqrt{0.51}\, t) + Im(c_2)\cos(\sqrt{0.51}\, t)\right) = 0$$

$$\Rightarrow Re(c_1) = Re(c_2) = \frac{\alpha}{2} \text{ e } Im(c_1) = -Im(c_2) = \frac{-\beta}{2}$$

$$x = \left(\frac{\alpha}{2} - \frac{\beta}{2} i\right) e^{(-0.7 + \sqrt{0.51}\, i)t} + \left(\frac{\alpha}{2} + \frac{\beta}{2} i\right) e^{(-0.7 - \sqrt{0.51}\, i)t}$$

$$x(t) = e^{-0.7t}\Big(\frac{\alpha}{2}\cos(\sqrt{0.51}\, t) \cancel{- \frac{\beta}{2} i \cos(\sqrt{0.51}\, t)} \cancel{+ \frac{\alpha}{2} i \operatorname{sen}(\sqrt{0.51}\, t)} + \frac{\beta}{2}\operatorname{sen}(\sqrt{0.51}\, t) + \frac{\alpha}{2}\cos(\sqrt{0.51}\, t) \cancel{+ \frac{\beta}{2} i \cos(\sqrt{0.51}\, t)} \cancel{- \frac{\alpha}{2} i \operatorname{sen}(\sqrt{0.51}\, t)} + \frac{\beta}{2}\operatorname{sen}(\sqrt{0.51}\, t)\Big)$$

$$x(t) = e^{-0.7t}\left(\alpha\cos(\sqrt{0.51}\, t) + \beta\operatorname{sen}(\sqrt{0.51}\, t)\right)$$

iii) Solução Geral:

$$x(t) = e^{-0.7t}\left(\alpha\cos(\sqrt{0.51}\, t) + \beta\operatorname{sen}(\sqrt{0.51}\, t)\right) + 1$$

iv) PVI:

$x(0) = 0 \Rightarrow \alpha + 1 = 0 \Rightarrow \alpha = -1$

$\dot{x}(t) = \frac{d}{dt}\left(e^{-0.7t}\left(-\cos(\sqrt{0.51}\, t) + \beta \operatorname{sen}(\sqrt{0.51}\, t)\right) + 1\right)$

$\dot{x}(t) = -0.7\, e^{-0.7t}\left(-\cos(\sqrt{0.51}\, t) + \beta \operatorname{sen}(\sqrt{0.51}\, t)\right) + \sqrt{0.51}\, e^{-0.7t}\left(\operatorname{sen}(\sqrt{0.51}\, t) + \beta \cos(\sqrt{0.51}\, t)\right)$

$\dot{x}(0) = 0 \Rightarrow 0 = 0.7 + \sqrt{0.51}\,\beta \Rightarrow \beta = \frac{-0.7}{\sqrt{0.51}} = \frac{-0.7\sqrt{0.51}}{0.51} = -\frac{7}{51}\sqrt{51}$

$\therefore\ x(t) = 1 - e^{-0.7t}\left(\cos\left(\frac{\sqrt{51}\, t}{10}\right) + \frac{7\sqrt{51}}{51} \operatorname{sen}\left(\frac{\sqrt{51}\, t}{10}\right)\right)$

3- $m\ddot{x} + b\dot{x} + kx = f$, $\quad x_1 = x$, $\quad x_2 = \dot{x} = \dot{x}_1$

$$\Rightarrow \begin{cases} x_2 = \dot{x}_1 \Rightarrow \dot{x}_1 = x_2 \\ m\dot{x}_2 + b x_2 + k x_1 = f \Rightarrow \dot{x}_2 = -\frac{b}{m} x_2 - \frac{k}{m} x_1 + \frac{f}{m} \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f$$

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B} f$$

4- $ml\ddot{\theta}(t) + b\dot{\theta}(t) + mg \operatorname{sen}\theta(t) = 0 \Rightarrow \ddot{\theta} + \frac{b}{ml}\dot{\theta} + \frac{g}{l}\operatorname{sen}\theta = 0$

$\theta_1 = \theta \Rightarrow \dot{\theta}_1 = \theta_2$

$\theta_2 = \dot{\theta} = \dot{\theta}_1 \quad \dot{\theta}_2 + \frac{b}{ml}\theta_2 + \frac{g}{l}\operatorname{sen}\theta_1 = 0$

$$\begin{cases} \dot{\theta}_1 = \theta_2 \\ \dot{\theta}_2 = -\frac{g}{l}\operatorname{sen}\theta_1 - \frac{b}{ml}\theta_2 \end{cases} \Rightarrow f\left(t, \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}\right) = \begin{bmatrix} \theta_2 \\ -\frac{g}{l}\operatorname{sen}\theta_1 - \frac{b}{ml}\theta_2 \end{bmatrix}$$