Lista 10 CMC-12 Eric Guerra Ribeiro

2- $X(s) = ((R(s) - X(s))K_p - sX(s))K_v \cdot \dfrac{1}{s(ms+b)}$

$$s(ms+b)\frac{X(s)}{R(s)} = K_v\left(\left(1 - \frac{X(s)}{R(s)}\right)K_p - s\frac{X(s)}{R(s)}\right)$$

$$(ms^2+bs)\,G_x(s) = K_v K_p - K_v(K_p+s)\,G_x(s)$$

$$G_x(s) = \frac{K_v \cdot K_p}{ms^2+(b+K_v)s+K_vK_p} = \frac{K_v K_p/m}{s^2+\frac{b+K_v}{m}s+K_vK_p/m}$$

$$2\xi\omega_n = \frac{b+K_v}{m} \Rightarrow K_v = 2m\xi\omega_n - b$$

$$K_vK_p/m = \omega_n^2 \Rightarrow K_p = \frac{m\omega_n^2}{K_v}$$

$$PM = \operatorname{arctg}\left(\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right) \Rightarrow tg\,PM = \frac{2\xi}{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}$$

$$\Rightarrow tg^2PM = \frac{4\xi^2}{-2\xi^2+\sqrt{4\xi^4+1}} \Rightarrow \sqrt{4\xi^4+1} = \frac{4\xi^2}{tg^2PM} + 2\xi^2 = 2\xi^2\left(1+\frac{2}{tg^2PM}\right)$$

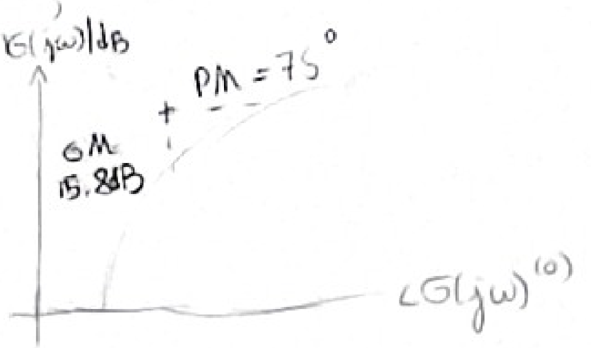
$$\Rightarrow 4\xi^4+1 = 4\xi^4\left(1+\frac{2}{tg^2PM}\right)^2 \Rightarrow 4\xi^4\left[\left(1+\frac{2}{tg^2PM}\right)^2-1\right] = 1$$

$$4\xi^4\left(\cancel{1}+\frac{4}{tg^2PM}+\frac{4}{tg^4PM}-\cancel{1}\right) = 1 \Rightarrow 16\xi^4\left(\frac{tg^2PM+1}{tg^4PM}\right) = 1$$

$$\xi^4 = \frac{1}{16}\,\frac{tg^4PM}{tg^2PM+1} = \frac{1}{16}\cdot\frac{tg^4PM}{\sec^2PM} \Rightarrow \xi = \frac{tg\,PM}{2}\cdot\sqrt{\cos PM}$$

$$\omega_b = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}} \Rightarrow \omega_n = \frac{\omega_b}{\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}}$$

3-



4- $G_a(s) = \dfrac{G(s)}{s^2}\cdot \vartheta = \dfrac{6\,(1{,}5s+1)}{s^2\,(0{,}1s+1)} = \dfrac{6\,(15s+10)}{s^2(s+10)} = \dfrac{30\,(3s+2)}{s^2(s+10)}$

$G_a(j\omega) = \dfrac{30\,(3j\omega+2)}{-\omega^2(j\omega+10)} \Rightarrow |G_a(j\omega)| = \dfrac{30}{\omega^2}\sqrt{\dfrac{4+9\omega^2}{\omega^2+100}}$

$\Rightarrow \dfrac{30}{\omega_{cp}^2}\sqrt{\dfrac{4+9\omega_{cp}^2}{\omega_{cp}^2+100}} = 1 \Rightarrow 900\left(\dfrac{4+9\omega_{cp}^2}{100+\omega_{cp}^2}\right) = \omega_{cp}^4$

$\omega_{cp}^6 + 100\,\omega_{cp}^4 - 8100\,\omega_{cp}^2 - 3600 = 0$

$\hookrightarrow \omega_{cp} = 7.29958$ rad/s

$PM_1 = PM_0 - \tau_1\omega_{cp}$
$PM_2 = PM_0 - \tau_2\omega_{cp}$ $\Rightarrow \Delta PM = (\tau_1 - \tau_2)\,\omega_{cp}$

5- $Y(s) = \left(R(s)\cdot\dfrac{K_i}{K_ps+K_i} - \dfrac{a}{s+a}\,Y(s)\right)\dfrac{K_ps+K_i}{s}\,\dfrac{1}{Ls+R}$

$G_R(s) = \left(\dfrac{K_i}{K_ps+K_i} - \dfrac{a}{s+a}\cdot G_R(s)\right)\dfrac{K_ps+K_i}{s(Ls+R)} \Rightarrow (Ls^2+Rs)G_R = K_i - \dfrac{a}{s+a}(K_ps+K_i)G_R$

$\big(s(s+a)(Ls+R) + a(K_ps+K_i)\big)G_R(s) = K_i(s+a)$

$\therefore G_R(s) = \dfrac{K_i(s+a)}{Ls^3 + (aL+R)s^2 + a(R+K_p)s + aK_i}$

$Y(s) = \left(Y(s) + N(s)\cdot\dfrac{a}{s+a}\right)(-1)\cdot\dfrac{K_ps+K_i}{s}\cdot\dfrac{1}{Ls+R} \Rightarrow s(Ls+R)G_N = -(K_ps+K_i)G_N - \dfrac{a(K_ps+K_i)}{(s+a)}$

$(Ls^2+Rs+K_ps+K_i)\,G_N(s) = -\dfrac{a(K_ps+K_i)}{(s+a)}$ $\therefore G_N(s) = -\dfrac{a(K_ps+K_i)}{(s+a)(Ls^2+(R+K_p)s+K_i)}$

$Y(s) = \left(\left(-Y(s)\cdot\dfrac{a}{s+a}\cdot\dfrac{K_ps+K_i}{s}\right) + D(s)\right)\dfrac{1}{Ls+R} \Rightarrow \left(\begin{matrix}s(s+a)(Ls+R)\\ +a(K_ps+K_i)\end{matrix}\right)G_D = s(s+a)$

$\therefore G_D(s) = \dfrac{s(s+a)}{Ls^3 + (aL+R)s^2 + a(K_p+R)s + a\cdot K_i}$