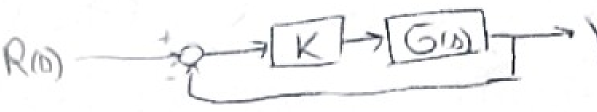
Lista 11 Eric Guerra Ribeiro

CMC-12

1- $e_{\infty,\text{rampa}} = \dfrac{1}{\lim\limits_{s\to 0} s \cdot \dfrac{K}{s(s+1)(s+2)}} = \dfrac{2}{K} \leq 1 \Rightarrow K \geq 2$



$\Rightarrow G_a(s) = KG(s)$

$G(j\omega) = \dfrac{K}{j\omega(j\omega+1)(j\omega+2)} = \dfrac{K}{j\omega(-\omega^2+3j\omega+2)} = \dfrac{K}{-\omega^3 j - 3\omega^2 + 2j\omega}$

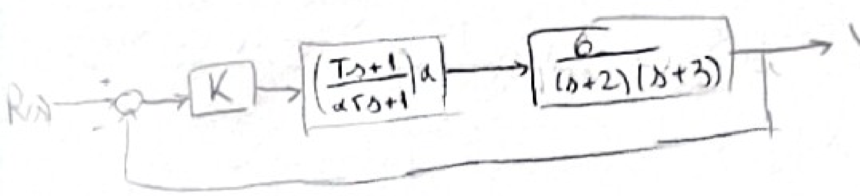
$G_a(j\omega) = \dfrac{K}{-3\omega^2 + j\omega(2-\omega^2)} \Rightarrow \angle G_a(j\omega) = -180°$ quando $2-\omega^2=0$

$\omega_{cg} = \sqrt{2}$

$|G_a(j\omega_{cg})| \cdot GM = 1 \Rightarrow GM = \left|\dfrac{-3\cdot 2}{K}\right| = \dfrac{6}{K} \geq 10^{\frac{6}{20}}$

$\Rightarrow K \leq 6 \cdot 10^{-\frac{3}{10}} \Rightarrow 2 \leq K \leq 6 \cdot 10^{-\frac{3}{10}}$

2- $e_{\infty} = \lim\limits_{s\to 0} \dfrac{1}{1 + \dfrac{K\cdot 6}{(s+2)(s+3)}} = \dfrac{1}{1+K} = \dfrac{1}{10}$



$\Rightarrow G_a(s) = \dfrac{6K\alpha}{(s+2)(s+3)} \cdot \dfrac{Ts+1}{\alpha Ts+1}$

$e_{\infty} = \dfrac{1}{20} = \lim\limits_{s\to 0} \dfrac{1}{1 + \dfrac{6K\alpha}{(s+2)(s+3)} \cdot \dfrac{Ts+1}{\alpha Ts+1}} = \dfrac{1}{1+K\alpha} \Rightarrow \alpha K = 19 \Rightarrow \alpha = \dfrac{19}{9}$

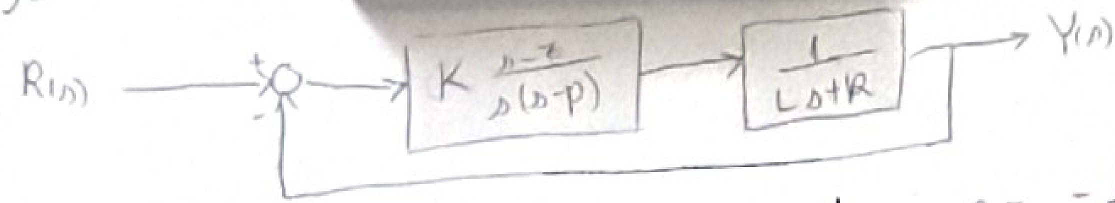
$\dfrac{1}{T} = \dfrac{\omega_{cp}}{10} \Rightarrow T = \dfrac{10}{\omega_{cp}}$

$KG(j\omega) = \dfrac{6\cdot K}{(\omega j+2)(\omega j+3)} = \dfrac{54}{6-\omega^2+j5\omega} \Rightarrow |KG(j\omega)| = \dfrac{54}{\sqrt{25\omega_{cp}^2 + (6-\omega_{cp}^2)^2}} = 1$

$2916 = 25\omega_{cp}^2 + 36 - 12\omega_{cp}^2 + \omega_{cp}^4 \Rightarrow \omega_{cp}^4 + 13\omega_{cp}^2 - 2880 = 0$

$\omega_{cp}^2 = \dfrac{-13 \pm \sqrt{169+11520}}{2} = \dfrac{\sqrt{11689}-13}{2} \Rightarrow \omega_{cp} = \sqrt{\dfrac{\sqrt{11689}-13}{2}}$

3-



$$G_a(s) = \frac{K(s-z)}{s(s-p)(Ls+R)}, \quad z = -\frac{1}{T}, \quad p = -\frac{1}{\alpha T}$$

$$\Rightarrow G_a(s) = \frac{K\left(s+\frac{1}{T}\right)}{s\left(s+\frac{1}{\alpha T}\right)(Ls+R)} = \frac{\alpha K(Ts+1)}{s(\alpha Ts+1)(Ls+R)}$$

$$G_a(s) = \underbrace{\frac{Ts+1}{\alpha Ts+1}}_{lead} \cdot \underbrace{K\alpha \cdot \frac{1}{s(Ls+R)}}_{G_a'(s)}$$

- $e_{\infty,rampa} \leq 0.05 \Rightarrow \dfrac{1}{\lim\limits_{s\to 0} s\,\alpha\dfrac{(Ts+1)}{(\alpha Ts+1)} \cdot \dfrac{K}{s(Ls+R)}} \leq \dfrac{1}{20}$

$$\Rightarrow \frac{\alpha K}{R} \leq \frac{1}{20} \Rightarrow \alpha K \geq 20R = 20, \text{ fazendo } K' = \alpha K = 20$$

$$|G_a'(j\omega_{cp})| = 1 \Rightarrow \frac{20}{|j\omega_{cp}||0.1\omega_{cp}j+1|} = 1 \Rightarrow \frac{40000}{\omega_{cp}^2(\omega_{cp}^2+100)} = 1$$

$$(\omega_{cp}^2)^2 + 100\omega_{cp}^2 - 40000 \Rightarrow \omega_{cp}^2 = \frac{-100 \pm \sqrt{10000+160000}}{2}$$

$$\omega_{cp}^2 = -50 + 50\sqrt{17} \Rightarrow \omega_{cp} = 5\sqrt{2(\sqrt{17}-1)} = 12.4962 \text{ rad/s}$$

$$PM = \underbrace{PM_{min}}_{50°} + \underbrace{\Delta PM}_{10°} = 60°$$

$$PM_0 = 180° + \angle G_a'(j\omega_{cp}) = 180° - 90° - \text{arctg}(0.1\omega_{cp}) = 90° - \text{arctg}(0.1\omega_{cp})$$

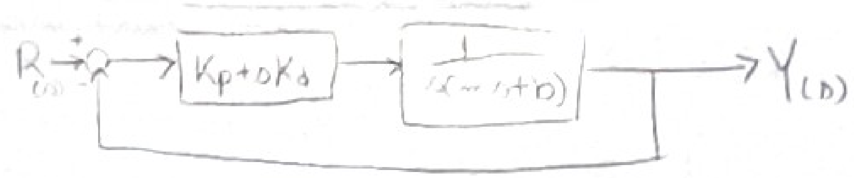
$$PM_0 = \text{arctg}\left(\frac{10}{\omega_{cp}}\right) = 38.6683°$$

$$\phi_{max} = PM - PM_0 = 21.3317°$$

$$\alpha = \frac{1-\text{sen}\,\phi_{max}}{1+\text{sen}\,\phi_{max}} = 0.4665, \quad T = \frac{1}{\omega_{cp}\sqrt{\alpha}} = 0.1172\text{ s}$$

$$\Rightarrow K = \frac{20}{\alpha} = 42.87, \quad z = -8.53, \quad p = -18.29 \quad \begin{cases} \omega_b = 26.9 \text{ rad/s} > 16 \\ e_{\infty rampa} = 0.05 \leq 0.05 \\ PM = 51.49 \geq 50° \end{cases}$$

4- PD → $K_p + sK_d$



$$G_f(s) = \frac{\frac{K_p+sK_d}{s(ms+b)}}{1+\frac{K_p+sK_d}{s(ms+b)}} = \frac{K_p + sK_d}{ms^2+(b+K_d)s+K_p} = \frac{\frac{K_p}{m}+s\frac{K_d}{m}}{s^2+\frac{(b+K_d)}{m}s+\frac{K_p}{m}}$$

$$s^2+\frac{(b+K_d)}{m}s+\frac{K_p}{m} \equiv (s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})$$

$$s^2+\frac{(b+K_d)}{m}s+\frac{K_p}{m} \equiv s^2+2\xi\omega_n s+\xi^2\omega_n^2+\omega_n^2(1-\xi^2) = s^2+2\xi\omega_n s+\omega_n^2$$

$$\Rightarrow \frac{K_p}{m}=\omega_n^2,\quad \frac{b+K_d}{m}=2\xi\omega_n \Rightarrow K_d = 2m\xi\omega_n - b$$

$$K_p = m\omega_n^2$$

$$G_a(s)=\frac{K_p+sK_d}{s(ms+b)} = \frac{m\omega_n^2+s(2m\xi\omega_n-b)}{s(ms+b)}$$

$$|G_a(j\omega_{cp})| = \frac{|m\omega_n^2+j\omega_{cp}(2m\xi\omega_n-b)|}{|j\omega_{cp}|\,|jm\omega_{cp}+b|} = \frac{\sqrt{(m\omega_n^2)^2+\omega_{cp}^2(2m\xi\omega_n-b)^2}}{\omega_{cp}\sqrt{m^2\omega_{cp}^2+b^2}} = 1$$

$$m^2\omega_n^4+\omega_{cp}^2(4m^2\xi^2\omega_n^2-4m\xi\omega_n b+\cancel{b^2}) = m^2\omega_{cp}^4+\cancel{b^2\omega_{cp}^2}$$

$$\omega_{cp}^4+4\xi\omega_n\left(\frac{b}{m}-\xi\omega_n\right)\omega_{cp}^2-\omega_n^4=0$$

$$\omega_{cp}^2=\frac{4\xi\omega_n(\xi\omega_n-\frac{b}{m}) \pm \sqrt{16\xi^2\omega_n^2(\frac{b}{m}-\xi\omega_n)^2+4\omega_n^4}}{2} = 2\xi\omega_n\left(\xi\omega_n-\frac{b}{m}\right)+\sqrt{4\xi^2\omega_n^2\left(\frac{b}{m}-\xi\omega_n\right)^2+\omega_n^4}$$

$$\therefore \omega_{cp}=\sqrt{2\xi\omega_n\left(\xi\omega_n-\frac{b}{m}\right)+\sqrt{4\xi^2\omega_n^2\left(\xi\omega_n-\frac{b}{m}\right)^2+\omega_n^2}}$$

$$\Delta PM = -\omega_{cp}\cdot\frac{T}{2}-\omega_{cp}\tau = -\omega_{cp}\left(\frac{T}{2}+\tau\right)$$

5- $$F_m(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \Rightarrow s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$\frac{U(z)}{E(z)} = \frac{\omega_n^2}{\frac{4}{T^2}\frac{z^2-2z+1}{z^2+2z+1}+\frac{4}{T}\xi\omega_n\frac{z-1}{z+1}+\omega_n^2} = \frac{T^2\omega_n^2(z^2+2z+1)}{4z^2-8z+4+4T\xi\omega_n z^2-4T\xi\omega_n+T^2\omega_n^2z^2+2T^2\omega_n^2z+T^2\omega_n^2}$$

$$\frac{U(z)}{E(z)}=\frac{T^2\omega_n^2(z^2+2z+1)}{(4+4T\xi\omega_n+T^2\omega_n^2)z^2+(2\omega_n^2T^2-8)z+(4-4T\xi\omega_n+T^2\omega_n^2)}$$

$$[(4+4T\xi\omega_n+T^2\omega_n^2)+(2\omega_n^2T^2-8)z^{-1}+(4-4T\xi\omega_n+T^2\omega_n^2)z^{-2}]U(z) = [T^2\omega_n^2+2T^2\omega_n^2z^{-1}+T^2\omega_n^2z^{-2}]E(z)$$

$$(4+4T\xi\omega_n+T^2\omega_n^2)u[k]+(2\omega_n^2T^2-8)u[k-1]+(4-4T\xi\omega_n+T^2\omega_n^2)u[k-2] = T^2\omega_n^2(e[k]+2e[k-1]+e[k-2])$$

$$u[k]=\frac{T^2\omega_n^2(e[k]+2e[k-1]+e[k-2])+(8-2\omega_n^2T^2)u[k-1]+(4T\xi\omega_n-4-T^2\omega_n^2)u[k-2]}{4+4T\xi\omega_n+T^2\omega_n^2}$$