CMC-12 Lista 5 Erui Guerra Pinheiro

1- $\sigma = \xi \omega_n$, $\omega_d = \omega_n \sqrt{1-\xi^2}$

$t_r|_0^{100\%} = t_r$: $\dfrac{\pi - \arccos \xi}{\omega_n \sqrt{1-\xi^2}}$ $M_p = e^{\frac{-\xi \pi}{\sqrt{1-\xi^2}}}$

$$\Rightarrow \frac{-\xi \pi}{\sqrt{1-\xi^2}} = \ln M_p \Rightarrow -\xi\pi = (\ln M_p)\sqrt{1-\xi^2}$$

$$\Rightarrow \xi^2 \pi^2 = (\ln^2 M_p)(1-\xi^2) \Rightarrow \xi^2(\pi^2 + \ln^2 M_p) = \ln^2 M_p$$

$$\xi^2 = \frac{1}{\left(\frac{\pi}{\ln M_p}\right)^2 + 1} \Rightarrow \xi = \sqrt{\frac{1}{1 + \left(\frac{\pi}{\ln M_p}\right)^2}}$$

$$\omega_n = \frac{\pi - \arccos \xi}{t_r \sqrt{1-\xi^2}} \Rightarrow \sigma = \xi \frac{\pi - \arccos \xi}{t_r\sqrt{1-\xi^2}} = \frac{\xi \omega_d}{\sqrt{1-\xi^2}}$$

$$\omega_d = \frac{\pi - \arccos \xi}{t_r}$$

2- MATLAB

3- $I(s) = \dfrac{K_p s + K_i}{s} \cdot \dfrac{1}{Ls + R}\left(R(s) - I(s)\right)$

$$I(s)\left(1 + \frac{K_p s + K_i}{s(Ls+R)}\right) = \frac{K_p s + K_i}{s(Ls+R)} R(s)$$

$$\frac{I(s)}{R(s)} = \frac{\frac{K_p \cdot s + K_i}{\cancel{s(Ls+R)}}}{\frac{Ls^2 + (K_p + R)s + K_i}{\cancel{s(Ls+R)}}} = \frac{\frac{K_p}{L} \cdot s + \frac{K_i}{L}}{s^2 + \frac{K_p + R}{L}s + \frac{K_i}{L}}$$

$$s^2 + \frac{K_p+R}{L}s + \frac{K_i}{L} = (s - p_1)(s - p_2) = (s - p_1)(s - \overline{p_1}) = s^2 - 2Re(p_1)s + |p_1|^2$$

$$\begin{cases} \frac{K_p + R}{L} = -2(-\xi\omega_n) = 2\xi\omega_n \Rightarrow K_p = 2\xi\omega_n \cdot L - R \\ \frac{K_i}{L} = (\xi\omega_n)^2 + (\omega_n\sqrt{1-\xi^2})^2 \quad \omega_n^2(\xi^2 + 1 - \xi^2) = \omega_n^2 \Rightarrow K_i = \omega_n^2 \cdot L \end{cases}$$

4-

$$X(s) = (K_p + K_d s) \frac{1}{ms^2+bs} \left(R(s) \cdot F(s) - X(s) \right)$$

$$\Rightarrow X(s) \left(1 + \frac{K_p + K_d s}{ms^2+bs} \right) = \frac{(K_p + K_d s) F(s)}{ms^2+bs} R(s)$$

$$\frac{X(s)}{R(s)} = \frac{\frac{(K_p + K_d s) F(s)}{ms^2+bs}}{\frac{ms^2+(b+K_d)s+K_p}{ms^2+bs}} = \frac{(K_p+K_d s) F(s)}{ms^2+(b+K_d)s+K_p}$$

$$\frac{X(s)}{R(s)} = \frac{(K_p + K_d s) \frac{F(s)}{m}}{s^2 + \frac{(b+K_d)}{m} s + \frac{K_p}{m}} = \frac{\omega_m^2}{s^2 + 2\xi\omega_m s + \omega_m^2}$$

$$\begin{cases} (K_p + K_d \cdot s) \cdot \frac{F(s)}{m} = \omega_m^2 \\ \frac{b+K_d}{m} = 2\xi\omega_m \Rightarrow K_d = 2m\xi\omega_m - b \\ \frac{K_p}{m} = \omega_m^2 \Rightarrow K_p = m\omega_m^2 \end{cases}$$

$$F(s) = \frac{\omega_m^2}{\frac{K_p}{m} + \frac{K_d}{m} s} = \frac{\omega_m^2}{\omega_m^2 + (2\xi\omega_m - \frac{b}{m}) s} = \frac{1}{1 + \frac{2\xi\omega_m - \frac{b}{m}}{\omega_m^2} s}$$

$$C(s) = K_p + K_d s = (2m\xi\omega_m - b) s + m\omega_m^2$$