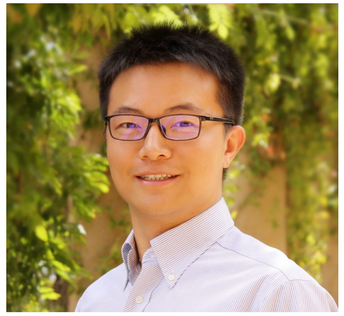
MIT HAN LAB

EfficientML.ai Lecture 05 Quantization

Part I



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Distinguished Scientist, NVIDIA

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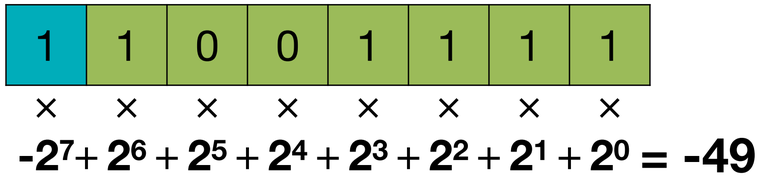
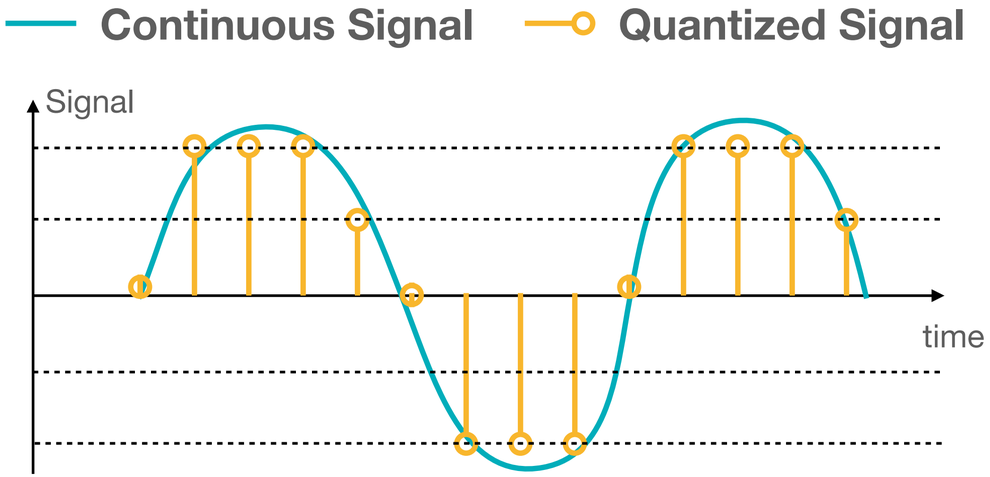
Lecture Plan

Today we will:

1. Review the numeric ***data types***, including integers and floating-point numbers (FP32, FP16, INT4, etc.)
2. Learn the basic concept of ***neural network quantization***
3. Learn three types of common neural network quantization:
    1. K-Means-based Quantization
    2. Linear Quantization
    3. Binary and Ternary Quantization

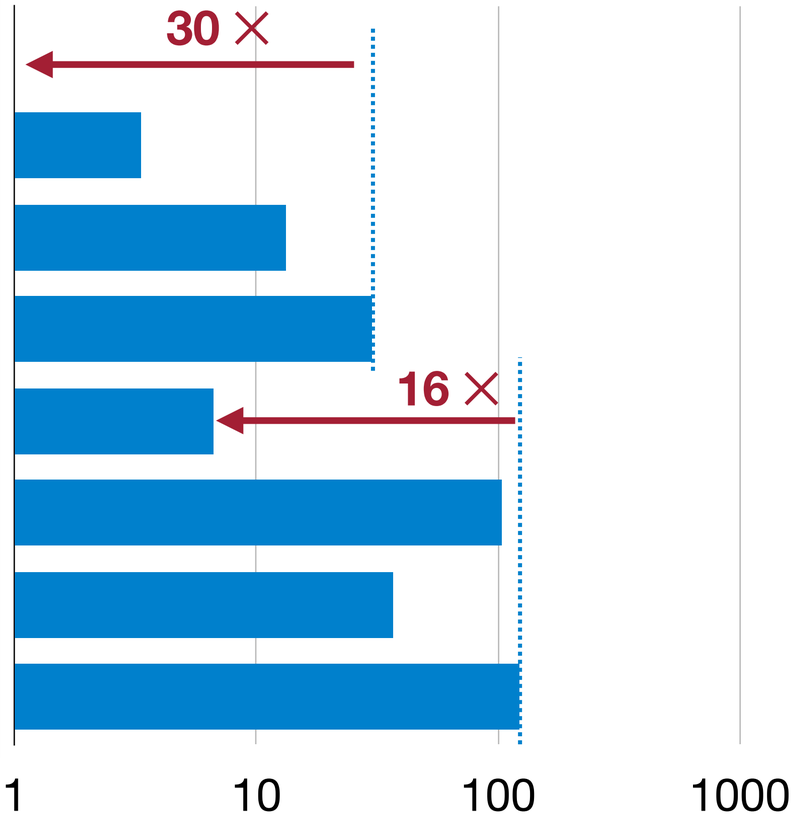




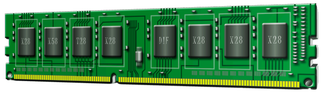
Low Bit Operations are Cheaper

Less Bit-Width → Less Energy

Operation	Energy [pJ]
8 bit int ADD	0.03
32 bit int ADD	0.1
16 bit float ADD	0.4
32 bit float ADD	0.9
8 bit int MULT	0.2
32 bit int MULT	3.1
16 bit float MULT	1.1
32 bit float MULT	3.7



Rough Energy Cost For Various Operations in 45nm 0.9V

1  = 200 × +

Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]

Low Bit Operations are Cheaper

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Operation	Energy [pJ]
8 bit int ADD	0.03
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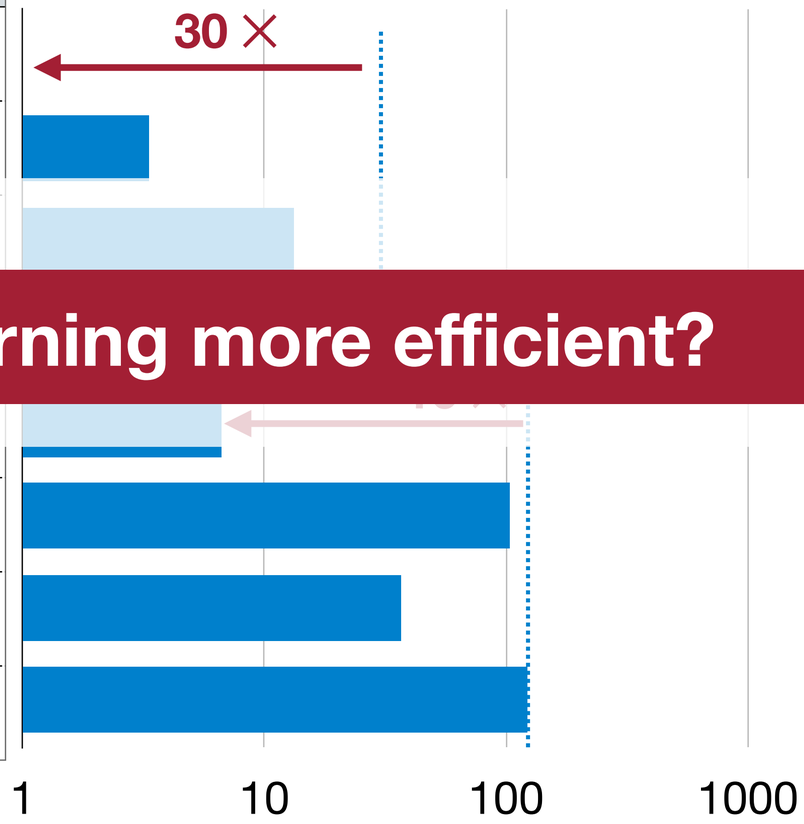
Rough Energy Cost For Various Operations in 45nm 0.9V



How should we make deep learning more efficient?

Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]

Numeric Data Types

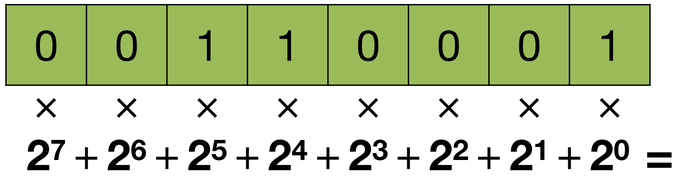
How is numeric data represented in modern computing systems?

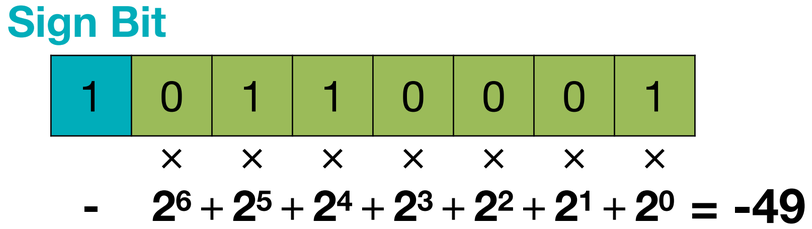
Integer

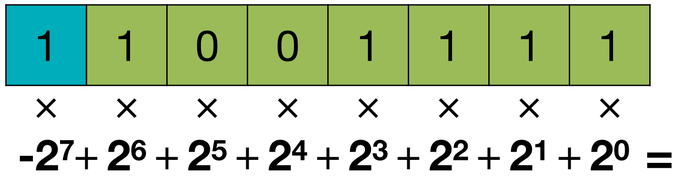
- Unsigned Integer
  - *n*-bit Range: $[0,\ 2^n - 1]$

- Signed Integer
  - Sign-Magnitude Representation
    - *n*-bit Range: $[-2^{n-1} + 1,\ 2^{n-1} - 1]$
    - Both 000…00 and 100…00 represent 0

  - Two's Complement Representation
    - *n*-bit Range: $[-2^{n-1},\ 2^{n-1} - 1]$
    - 000…00 represents 0
    - 100…00 represents -2^{n-1}







Fixed-Point Number

Integer . Fraction

“Decimal” Point

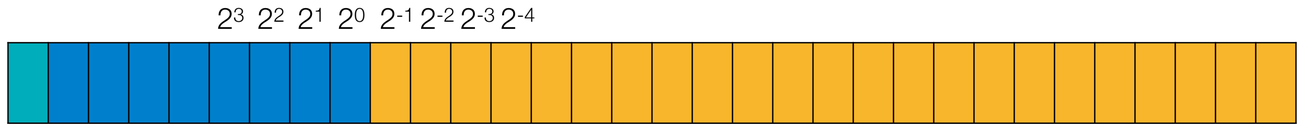
$$-2^3 + 2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} = 3.0625$$

$$\left(-2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0\right) \times 2^{-4} = 49 \times 0.0625 = 3.0625$$

(using 2’s complement representation)

Floating-Point Number

Example: 32-bit floating-point number in IEEE 754

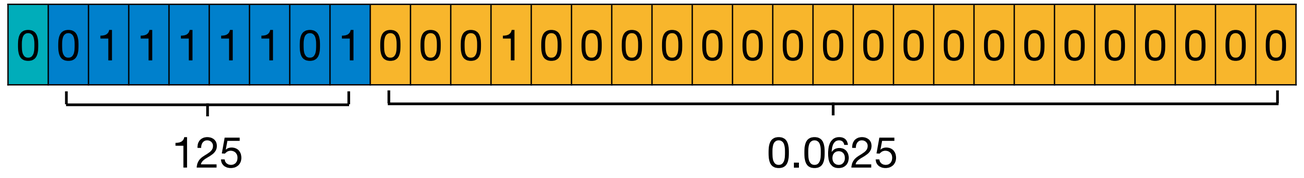


Sign **8 bit Exponent** **23 bit Fraction** (significant / mantissa)

$$(-1)^{\text{sign}} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$$ ← Exponent Bias = 127 = $2^{8-1}-1$

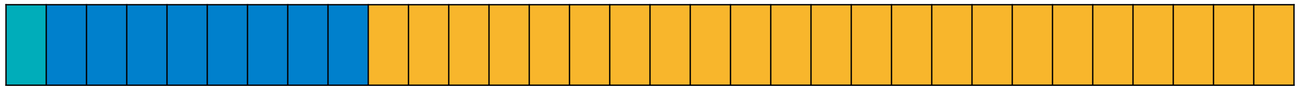
How to represent **0.265625**?

0.265625 = $1.0625 \times 2^{-2} = (1 + \underline{0.0625}) \times 2^{\underline{125}-127}$



Floating-Point Number

Example: 32-bit floating-point number in IEEE 754



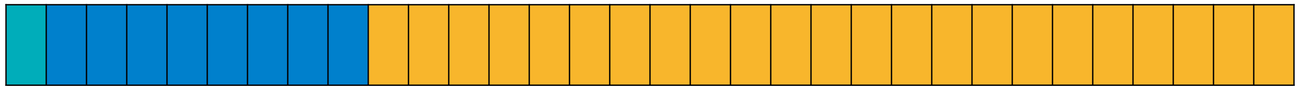
Sign **8 bit Exponent** **23 bit Fraction** (significant / mantissa)

$$(-1)^{\text{sign}} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$$ ← **Exponent Bias = 127 = $2^{8-1}-1$**

How should we represent 0?

Floating-Point Number

Example: 32-bit floating-point number in IEEE 754



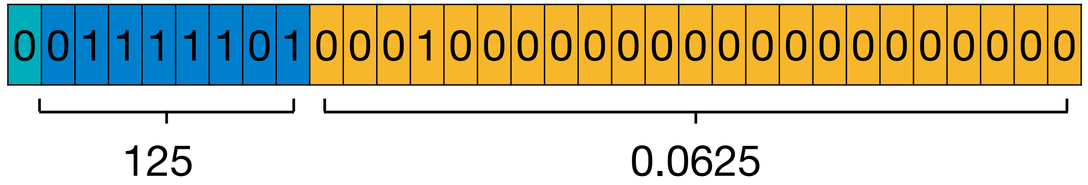
Sign **8 bit Exponent** **23 bit Fraction**

$(-1)^{sign} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$

Should have been $(-1)^{sign} \times (1 + \textbf{Fraction}) \times 2^{0-127}$

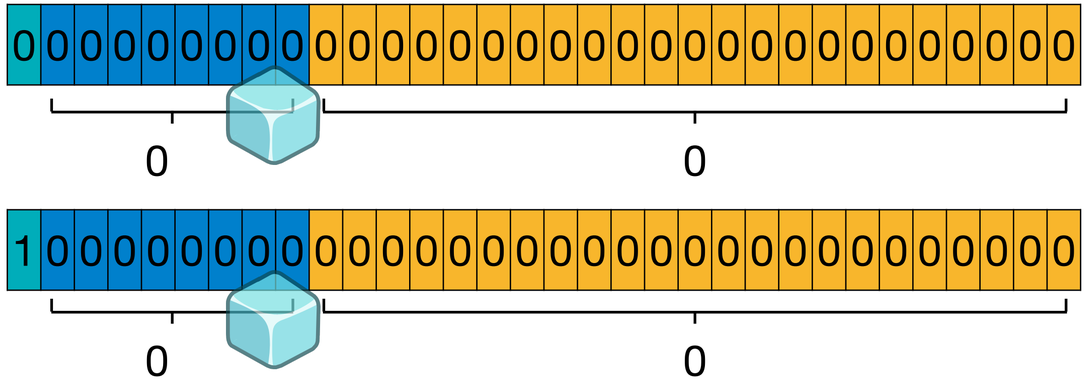
But we force to be $(-1)^{sign} \times \textbf{Fraction} \times 2^{1-127}$

Normal Numbers, Exponent≠0



$0.265625 = 1.0625 \times 2^{-2} = (1 + \underline{0.0625}) \times 2^{\underline{125}-127}$

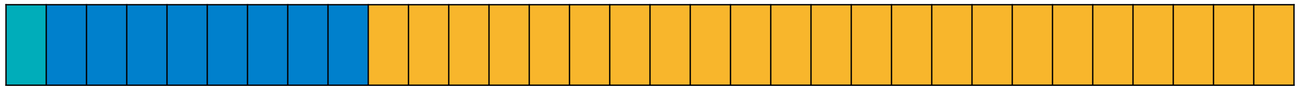
Subnormal Numbers, Exponent=0



$0 = 0 \times 2^{-126}$

Floating-Point Number

Example: 32-bit floating-point number in IEEE 754



Sign 8 bit Exponent **23 bit Fraction**

$(-1)^{sign} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$

Normal Numbers, Exponent≠0

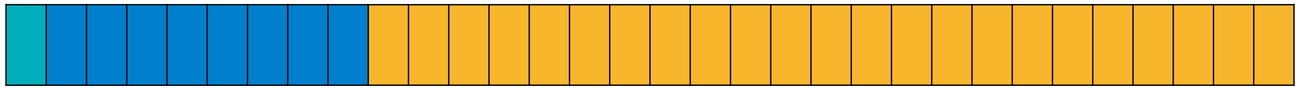
$(-1)^{sign} \times \textbf{Fraction} \times 2^{1-127}$

Subnormal Numbers, Exponent=0

What is the smallest positive subnormal value?

Floating-Point Number

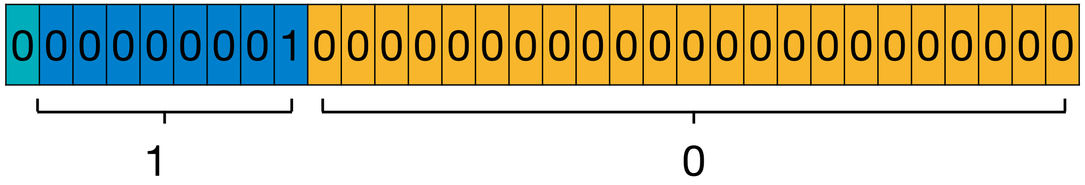
Example: 32-bit floating-point number in IEEE 754



Sign **8 bit Exponent** **23 bit Fraction**

$(-1)^{sign} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$

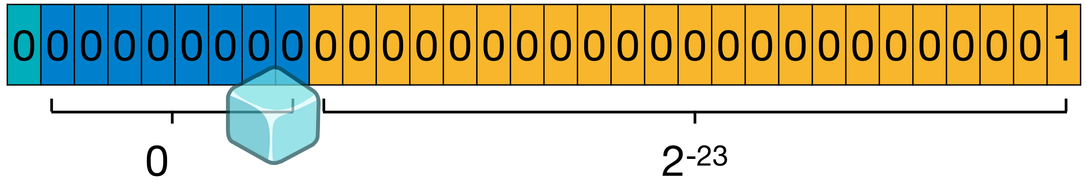
Normal Numbers, Exponent≠0



$2^{-126} = (1 + \underline{0}) \times 2^{\underline{1}-127}$

$(-1)^{sign} \times \textbf{Fraction} \times 2^{\textbf{1}-127}$

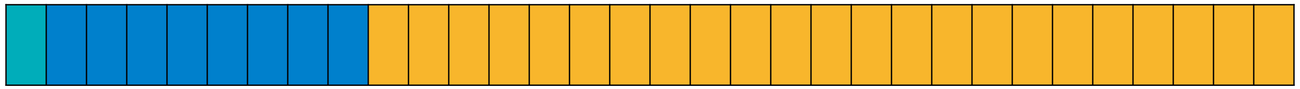
Subnormal Numbers, Exponent=0



$2^{-149} = 2^{-23} \times 2^{-126}$

Floating-Point Number

Example: 32-bit floating-point number in IEEE 754



Sign 8 bit Exponent **23 bit Fraction**

$(-1)^{sign} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$

Normal Numbers, Exponent≠0

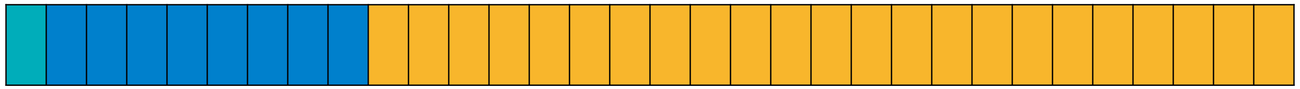
$(-1)^{sign} \times \textbf{Fraction} \times 2^{1-127}$

Subnormal Numbers, Exponent=0

What is the largest positive subnormal value?

Floating-Point Number

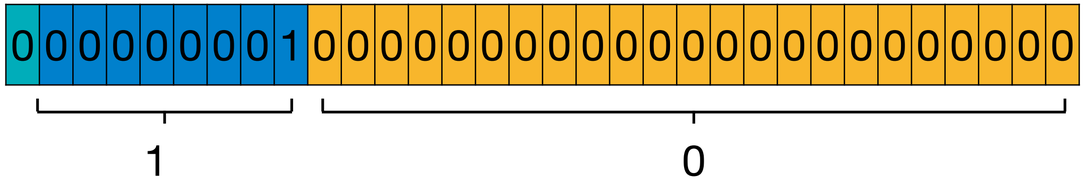
Example: 32-bit floating-point number in IEEE 754



Sign **8 bit Exponent** **23 bit Fraction**

$(-1)^{sign} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$

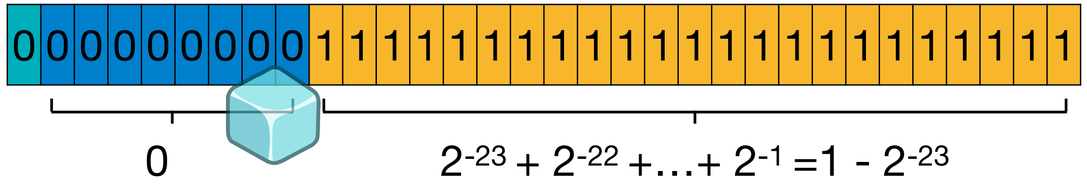
Normal Numbers, Exponent≠0



$2^{-126} = (1 + \underline{0}) \times 2^{\underline{1}-127}$

$(-1)^{sign} \times \textbf{Fraction} \times 2^{\textbf{1}-127}$

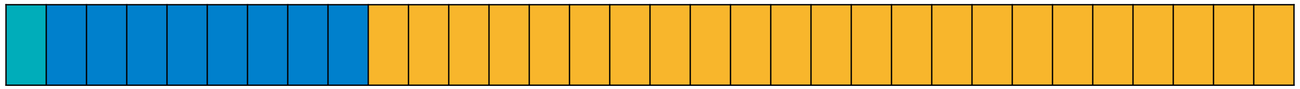
Subnormal Numbers, Exponent=0



$2^{-126} - 2^{-149} = (1 - 2^{-23}) \times 2^{-126}$

Floating-Point Number

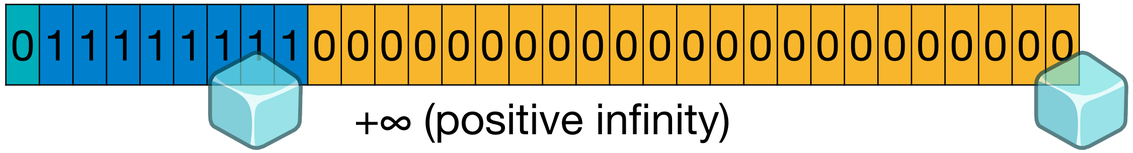
Example: 32-bit floating-point number in IEEE 754



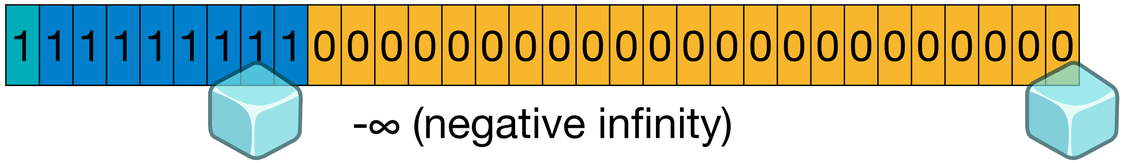
Sign **8 bit Exponent** **23 bit Fraction**

$(-1)^{sign} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$

Normal Numbers, Exponent≠0



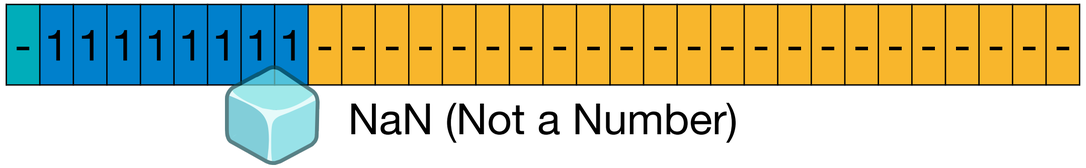
+∞ (positive infinity)



-∞ (negative infinity)

$(-1)^{sign} \times \textbf{Fraction} \times 2^{1-127}$

Subnormal Numbers, Exponent=0



NaN (Not a Number)

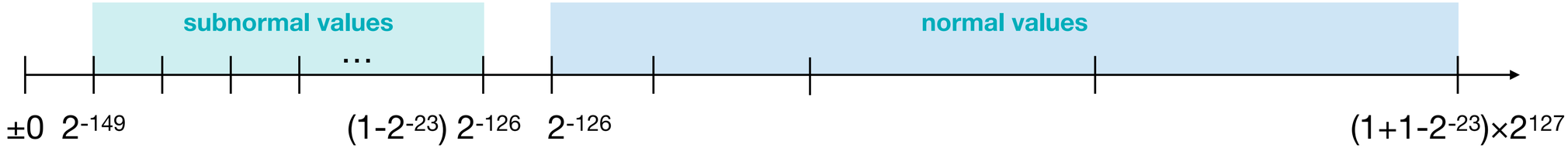
much waste. revisit in fp8.

Floating-Point Number

Example: 32-bit floating-point number in IEEE 754

Sign **8 bit Exponent** **23 bit Fraction**

Exponent	Fraction=0	Fraction≠0	Equation
$00_H = 0$	±0	subnormal	$(-1)^{sign} \times \mathbf{Fraction} \times 2^{1-127}$
$01_H \ldots FE_H = 1 \ldots 254$	normal		$(-1)^{sign} \times (1 + \mathbf{Fraction}) \times 2^{\mathbf{Exponent}-127}$
$FF_H = 255$	±INF	NaN	



Floating-Point Number

Exponent Width → Range; Fraction Width → Precision

IEEE 754 Single Precision 32-bit Float (IEEE FP32)

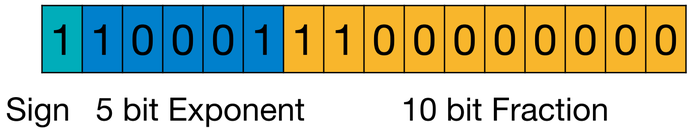
IEEE 754 Half Precision 16-bit Float (IEEE FP16)

Google Brain Float (BF16)

Exponent (bits)	Fraction (bits)	Total (bits)
8	23	32
5	10	16
8	7	16

Numeric Data Types

- **Question**: What is the following IEEE half precision (IEEE FP16) number in decimal?



Exponent Bias = 15_{10}

  - Sign: -
  - Exponent: 10001_2 - 15_{10} = 17_{10} - 15_{10} = 2_{10}
  - Fraction: 1100000000_2 = 0.75_{10}
  - Decimal Answer = - (1 + 0.75) × 2^2 = -1.75 × 2^2 = -7.0_{10}

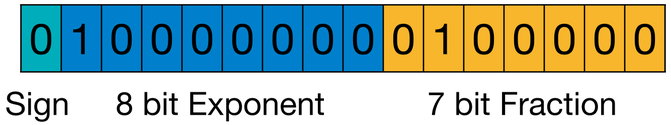
Numeric Data Types

- **Question**: What is the decimal 2.5 in Brain Float (BF16)?

$$2.5_{10} = 1.\underline{25}_{10} \times 2^{\underline{1}}$$

Exponent Bias = 127_{10}

  - Sign: +
  - Exponent Binary: $1_{10} + 127_{10} = 128_{10} = 10000000_2$
  - Fraction Binary: $0.25_{10} = 0100000_2$
  - Binary Answer



Floating-Point Number

Exponent Width → Range; Fraction Width → Precision

	Exponent (bits)	Fraction (bits)	Total (bits)
IEEE 754 Single Precision 32-bit Float (IEEE FP32) 	8	23	32
IEEE 754 Half Precision 16-bit Float (IEEE FP16) 	5	10	16
Nvidia FP8 (E4M3)  * FP8 E4M3 does not have INF, and S.1111.111$_2$ is used for NaN. * Largest FP8 E4M3 normal value is S.1111.110$_2$ =448.	4	3	8
Nvidia FP8 (E5M2) for gradient in the backward  * FP8 E5M2 have INF (S.11111.00$_2$) and NaN (S.11111.XX$_2$). * Largest FP8 E5M2 normal value is S.11110.11$_2$ =57344.	5	2	8

INT4 and FP4

Exponent Width → Range; Fraction Width → Precision

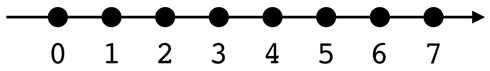
INT4

S			
0	0	0	1
0	1	1	1

=1

=7

-1,-2,-3,-4,-5,-6,-7,-8
0, 1, 2, 3, 4, 5, 6, 7



-1,-2,-3,-4,-5,-6,-7,-8
0, 1, 2, 3, 4, 5, 6, 7

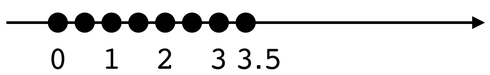
FP4 (E1M2)

S	E	M	M
0	0	0	1
0	1	1	1

=0.25×2^{1-0}=0.5

=(1+0.75)×2^{1-0}=3.5

-0,-0.5,-1,-1.5,-2,-2.5,-3,-3.5
0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5



-0,-1,-2,-3,-4,-5,-6,-7
0, 1, 2, 3, 4, 5, 6, 7
×0.5

FP4 (E2M1)

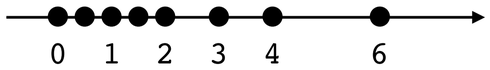
S	E	E	M
0	0	0	1
0	1	1	1

=0.5×2^{1-1}=0.5

=(1+0.5)×2^{3-1}=1

no inf, no NaN

-0,-0.5,-1,-1.5,-2,-3,-4,-6
0, 0.5, 1, 1.5, 2, 3, 4, 6



-0,-1,-2,-3,-4,-6,-8,-12
0, 1, 2, 3, 4, 6, 8, 12
×0.5

FP4 (E3M0)

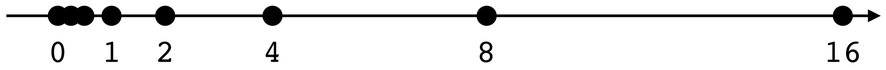
S	E	E	E
0	0	0	1
0	1	1	1

=(1+0)×2^{1-3}=0.25

=(1+0)×2^{7-3}=16

no inf, no NaN

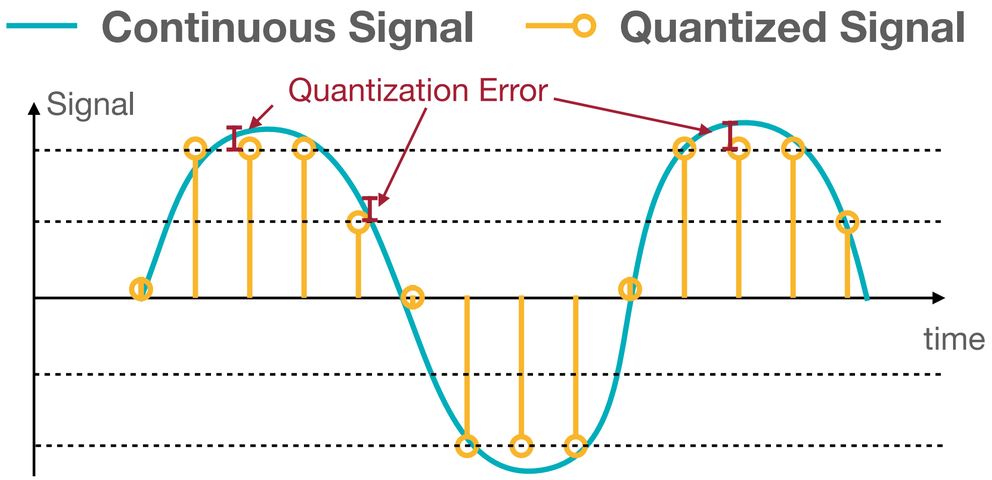
-0,-0.25,-0.5,-1,-2,-4,-8,-16
0, 0.25, 0.5, 1, 2, 4, 8, 16



-0,-1,-2,-4,-8,-16,-32,-64
0, 1, 2, 4, 8, 16, 32, 64
×0.25

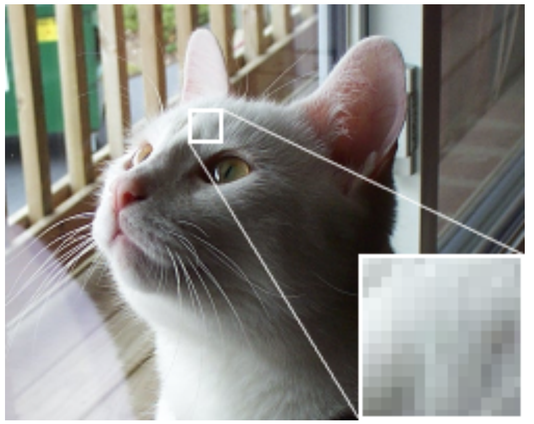
What is Quantization?

Quantization is the process of constraining an input from a continuous or otherwise large set of values to a discrete set.

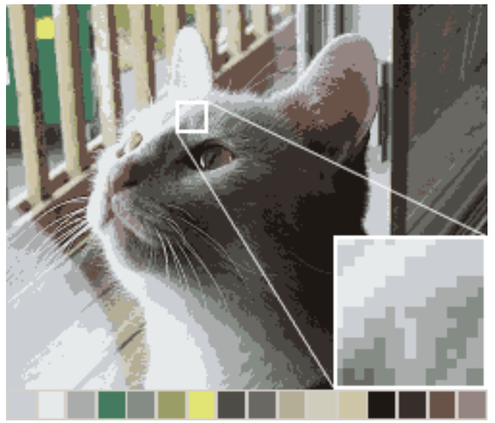


The difference between an input value and its quantized value is referred to as quantization error.

Original Image



16-Color Image

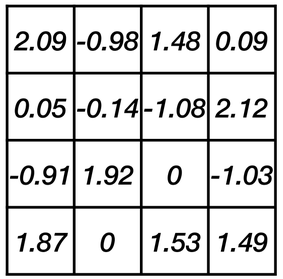


Images are in the public domain.

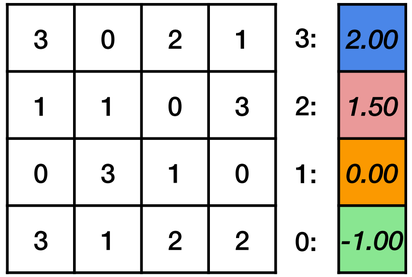
"Palettization"

Quantization [Wikipedia]

Neural Network Quantization: Agenda

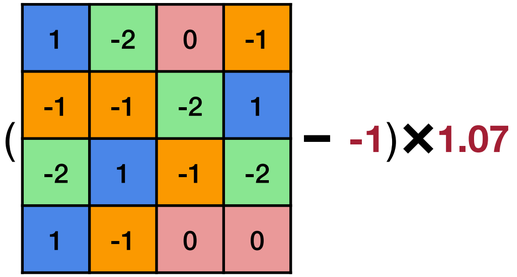


2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
-0.91	1.92	0	-1.03
1.87	0	1.53	1.49



3	0	2	1	3:	2.00
1	1	0	3	2:	1.50
0	3	1	0	1:	0.00
3	1	2	2	0:	-1.00

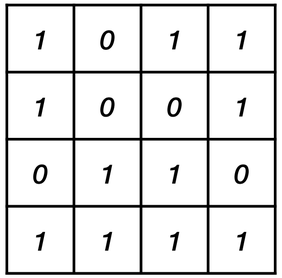
K-Means-based Quantization



1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

(− -1) × 1.07

Linear Quantization

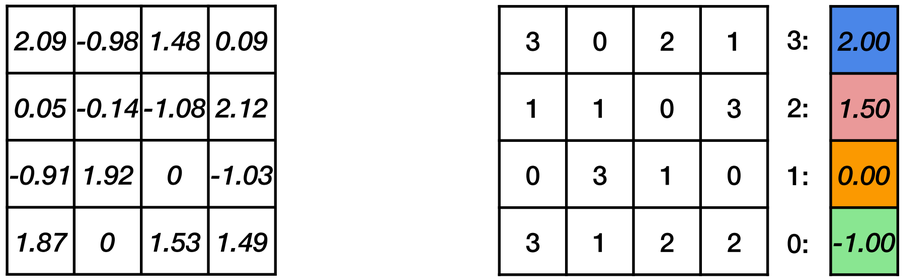


1	0	1	1
1	0	0	1
0	1	1	0
1	1	1	1

Binary/Ternary Quantization

Storage	Floating-Point Weights
Computation	Floating-Point Arithmetic

Neural Network Quantization: Agenda



K-Means-based Quantization

Linear Quantization

Binary/Ternary Quantization

Storage	Floating-Point Weights	Integer Weights; Floating-Point Codebook
Computation	Floating-Point Arithmetic	Floating-Point Arithmetic

Neural Network Quantization

Weight Quantization

weights
(32-bit float)

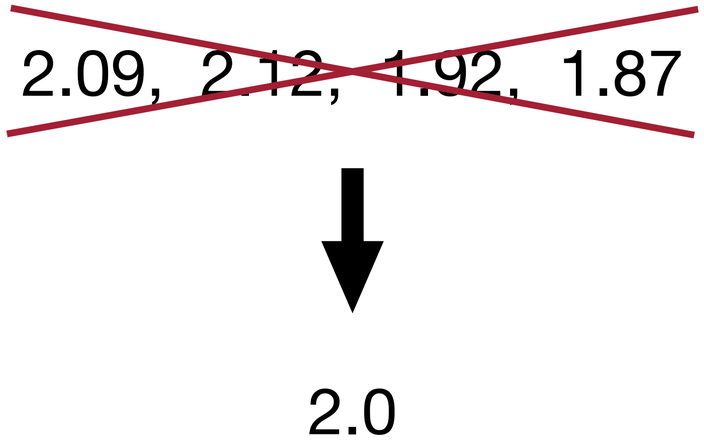
2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*

Neural Network Quantization

Weight Quantization

weights
(32-bit float)

2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*



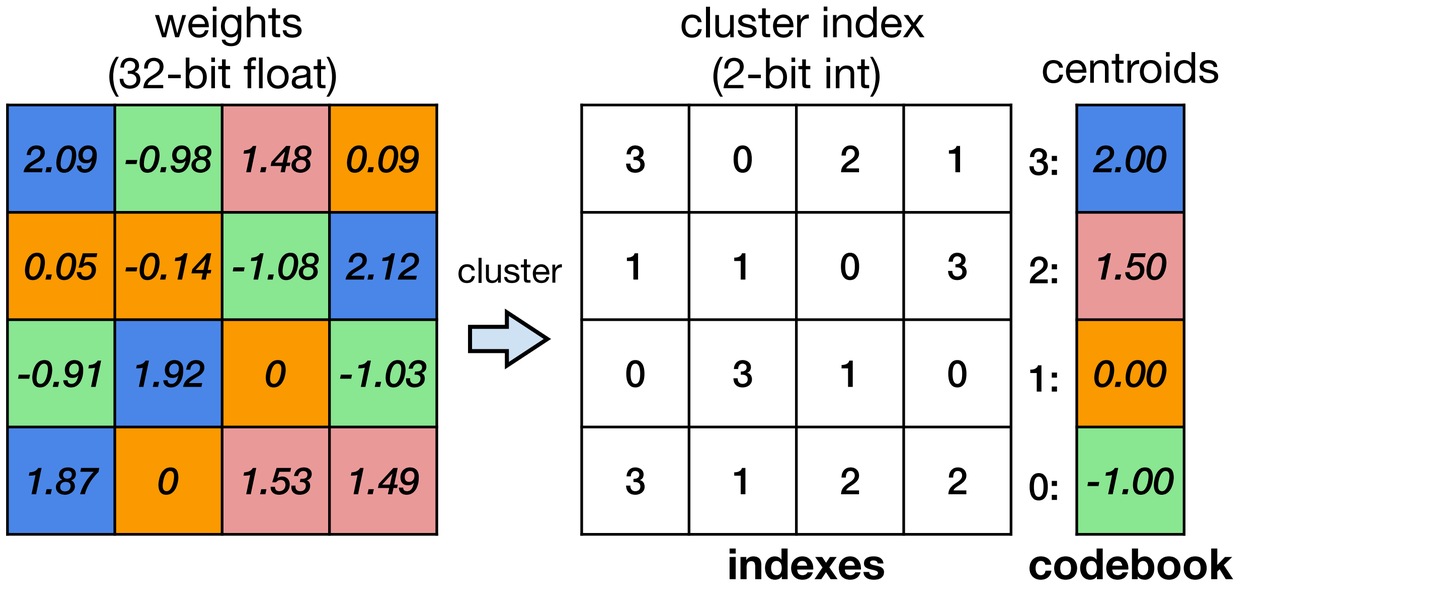
K-Means-based Weight Quantization

weights
(32-bit float)

2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*

Deep Compression [Han *et al.*, ICLR 2016]

K-Means-based Weight Quantization



reconstructed weights
(32-bit float)

2.00	-1.00	1.50	0.00
0.00	0.00	-1.00	2.00
-1.00	2.00	0.00	-1.00
2.00	0.00	1.50	1.50

storage

32 bit × 16
= 512 bit = 64 B

+

2 bit × 16
= 32 bit = 4 B

+

32 bit × 4
= 128 bit = 16 B

= 20 B

3.2 × smaller

Assume N-bit quantization, and #parameters = $M >> 2^N$.

32 bit × M
= $32M$ bit

N bit × M
= NM bit

~~32 bit × 2^N
= 2^{N+5} bit~~

$32/N$ × smaller

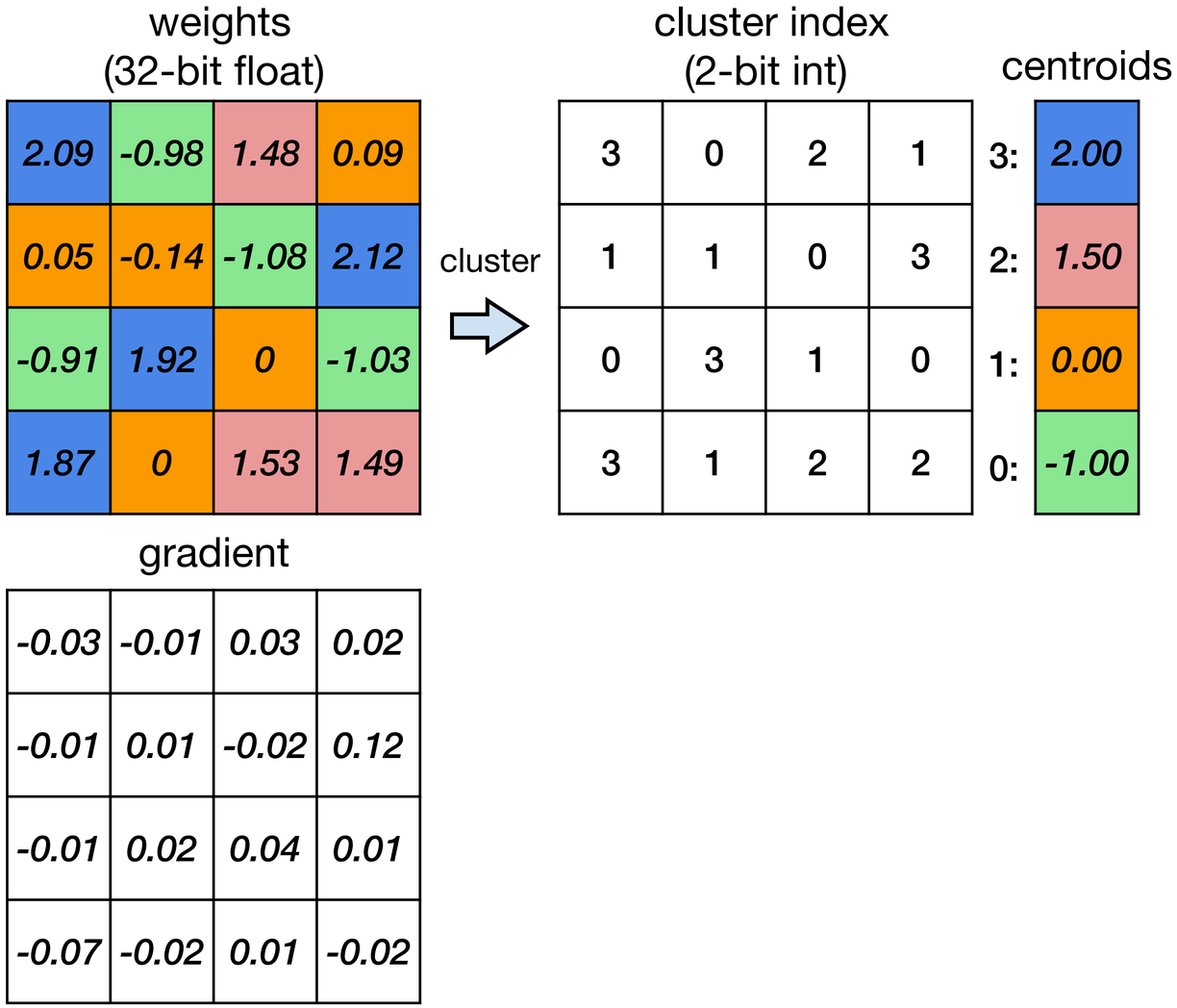
quantization error

0.09	0.02	-0.02	0.09
0.05	-0.14	-0.08	0.12
0.09	-0.08	0	-0.03
-0.13	0	0.03	-0.01

Deep Compression [Han *et al.*, ICLR 2016]

K-Means-based Weight Quantization

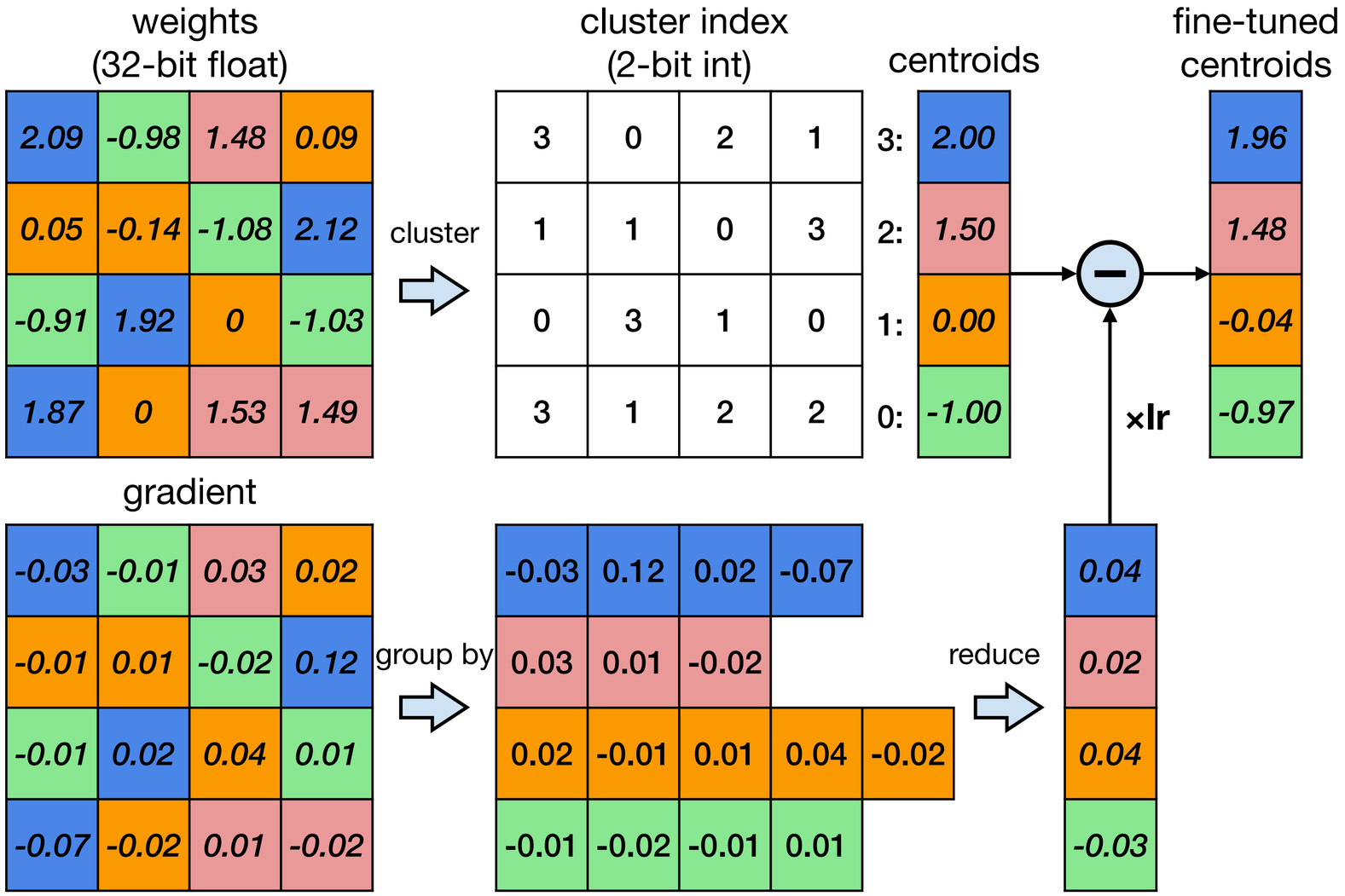
Fine-tuning Quantized Weights



Deep Compression [Han *et al.*, ICLR 2016]

K-Means-based Weight Quantization

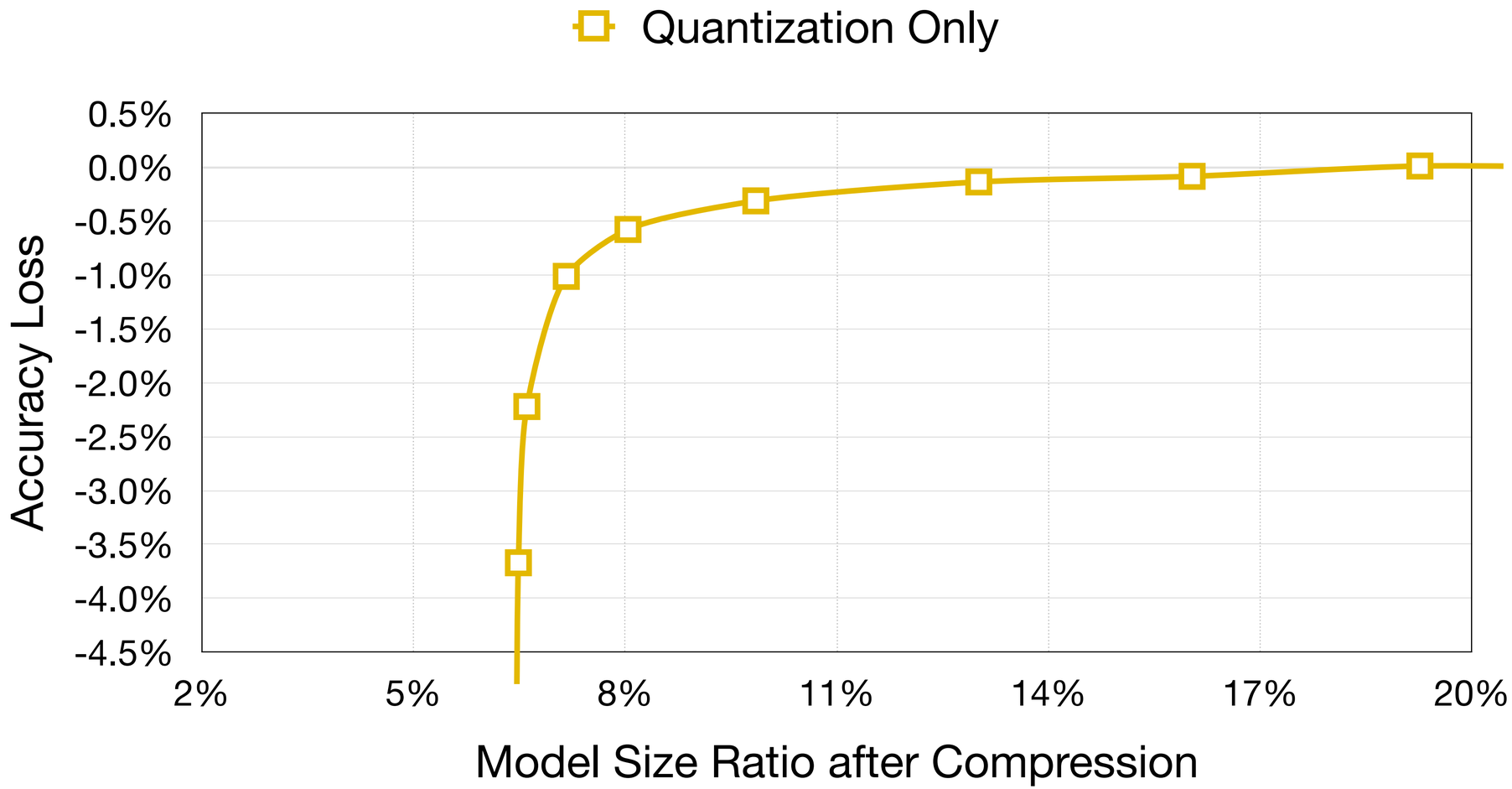
Fine-tuning Quantized Weights



Deep Compression [Han *et al.*, ICLR 2016]

K-Means-based Weight Quantization

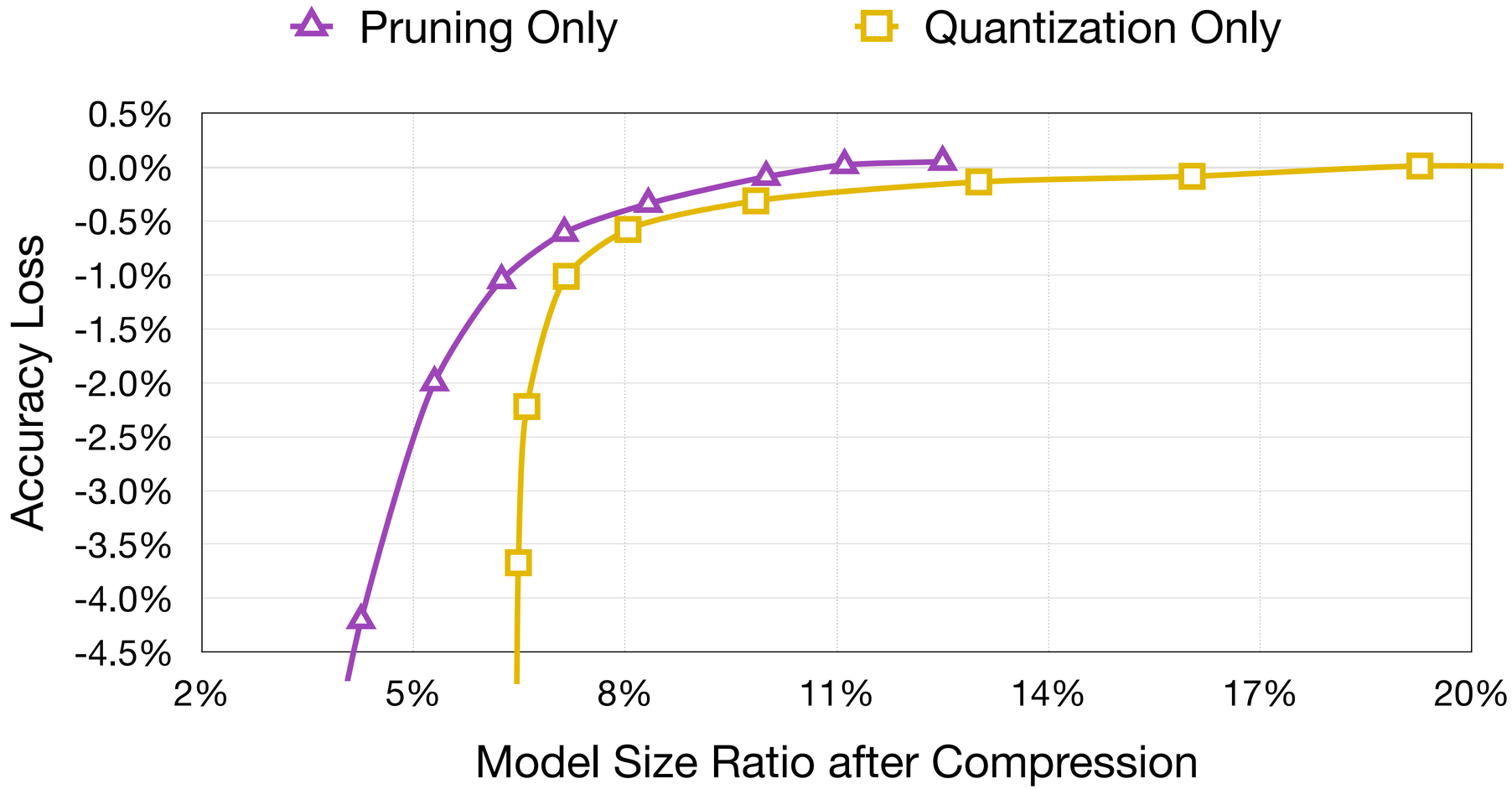
Accuracy vs. compression rate for AlexNet on ImageNet dataset



Deep Compression [Han *et al.*, ICLR 2016]

K-Means-based Weight Quantization

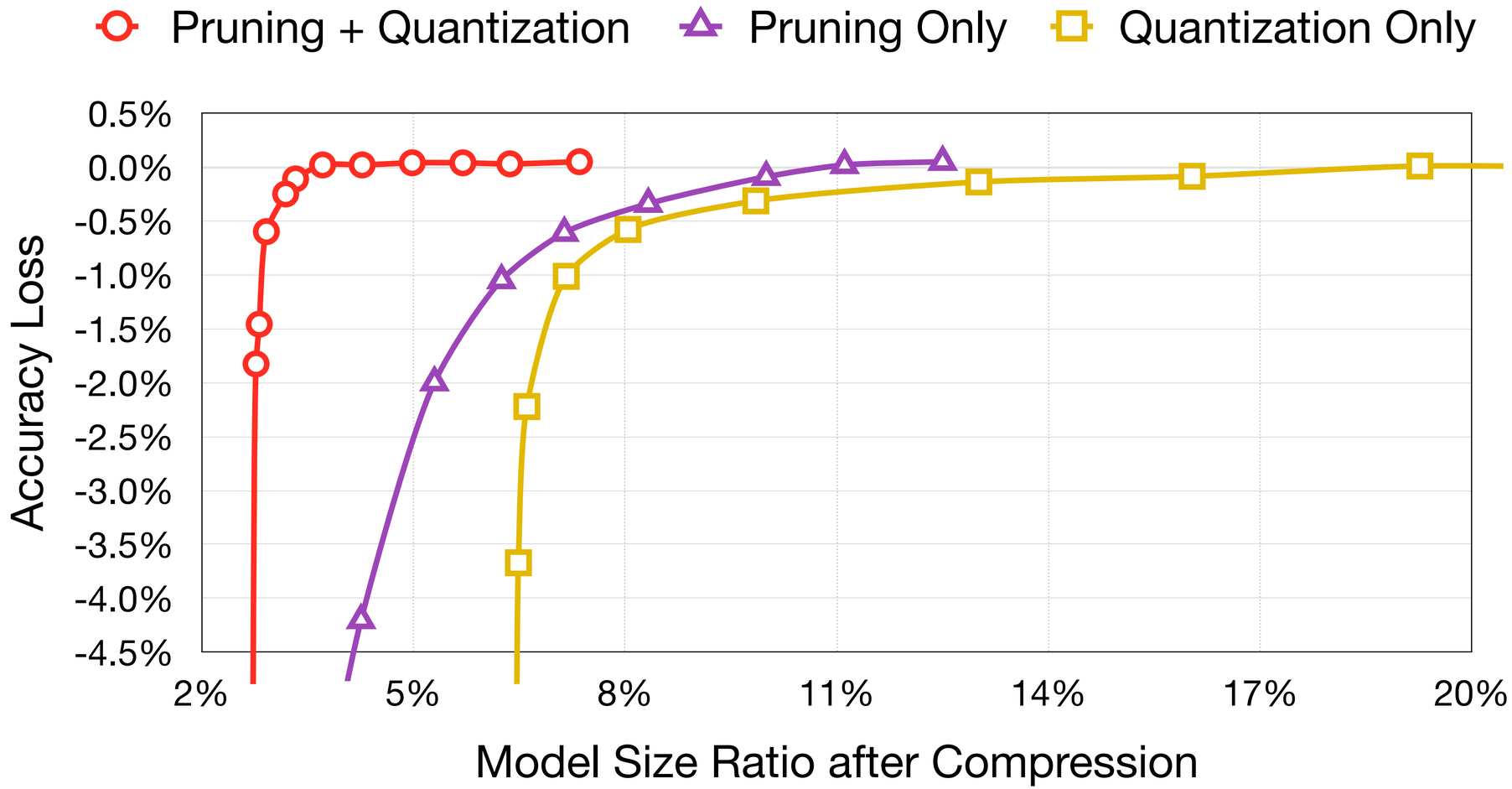
Accuracy vs. compression rate for AlexNet on ImageNet dataset



Deep Compression [Han *et al.*, ICLR 2016]

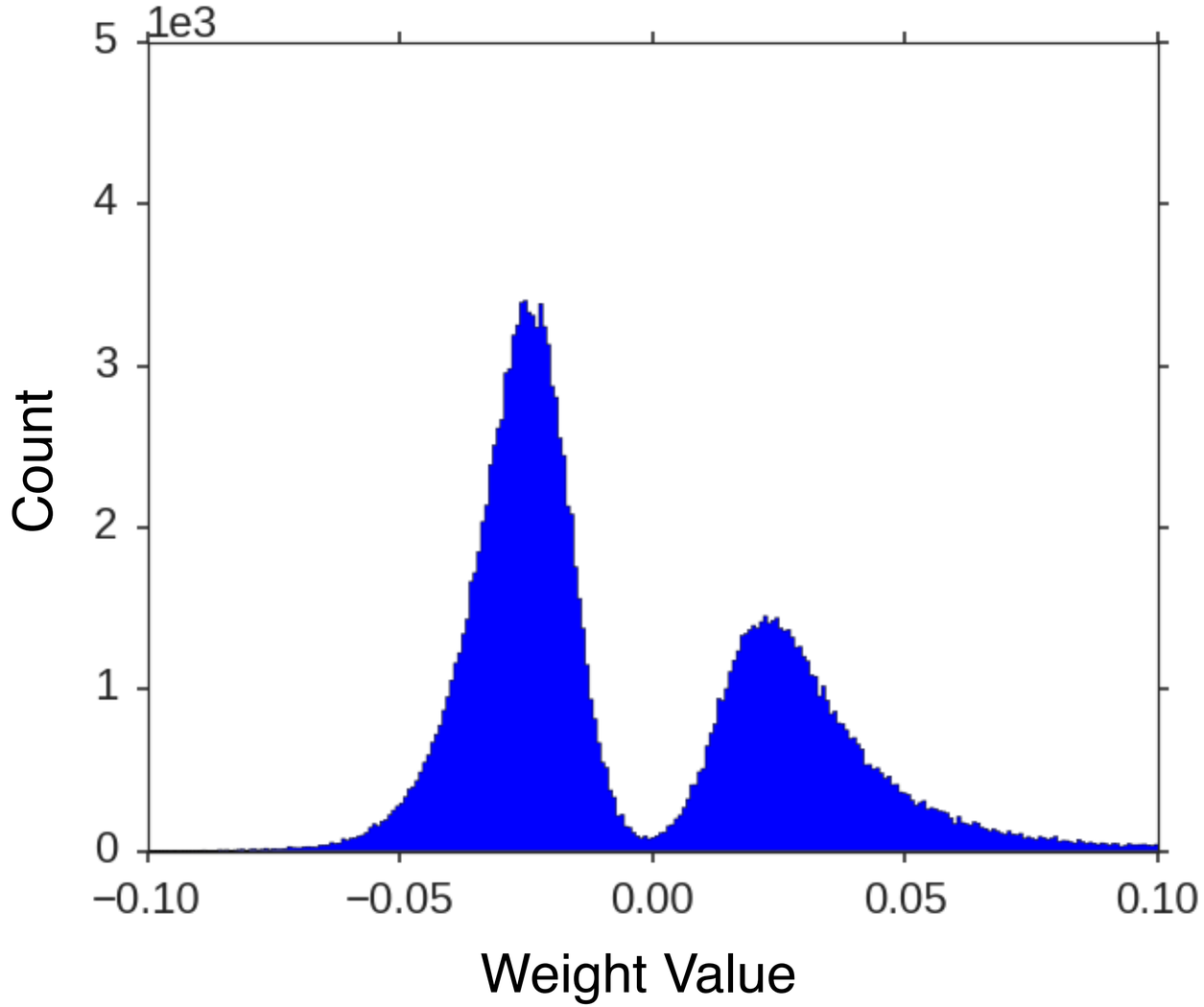
K-Means-based Weight Quantization

Accuracy vs. compression rate for AlexNet on ImageNet dataset



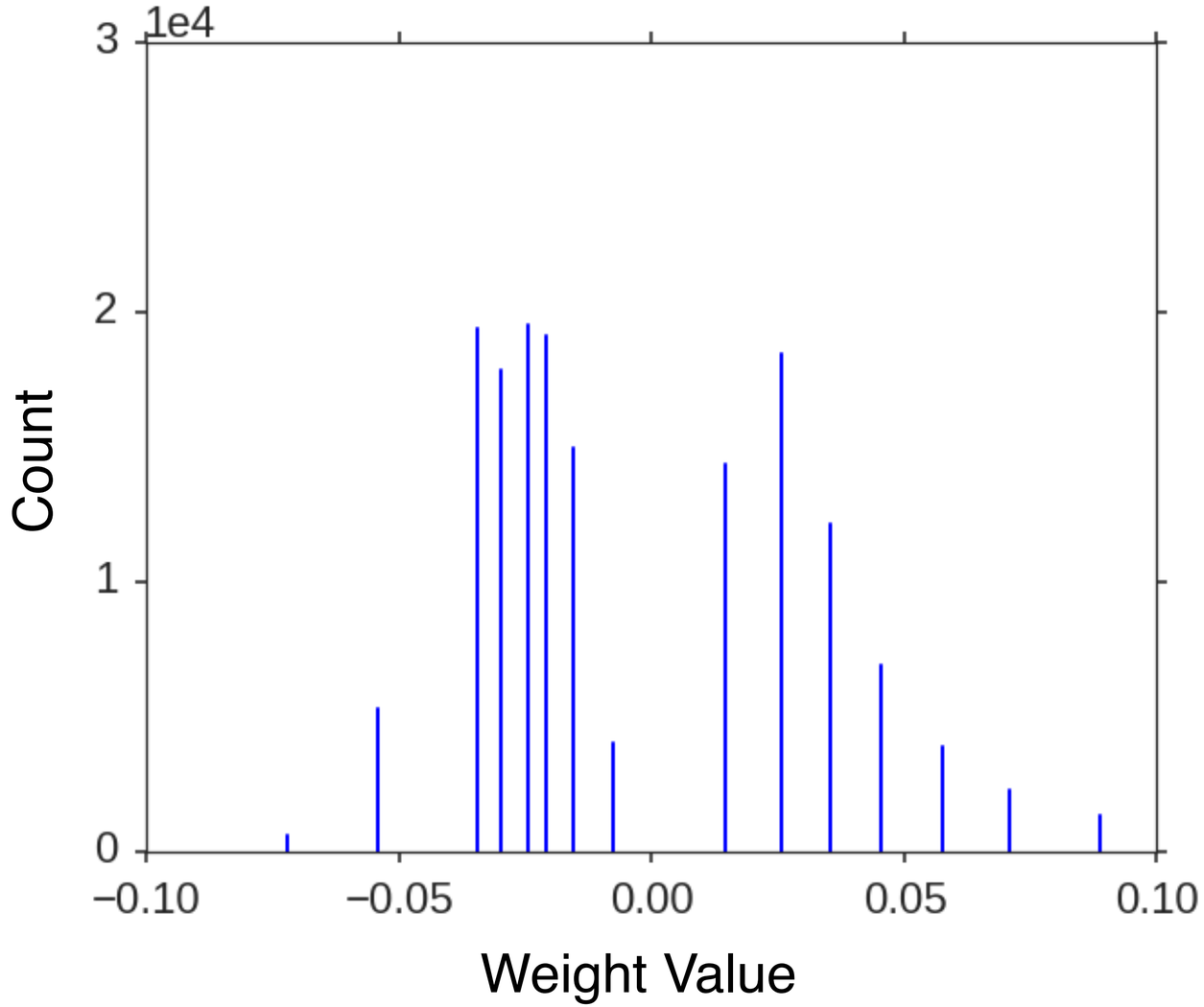
Deep Compression [Han *et al.*, ICLR 2016]

Before Quantization: Continuous Weight



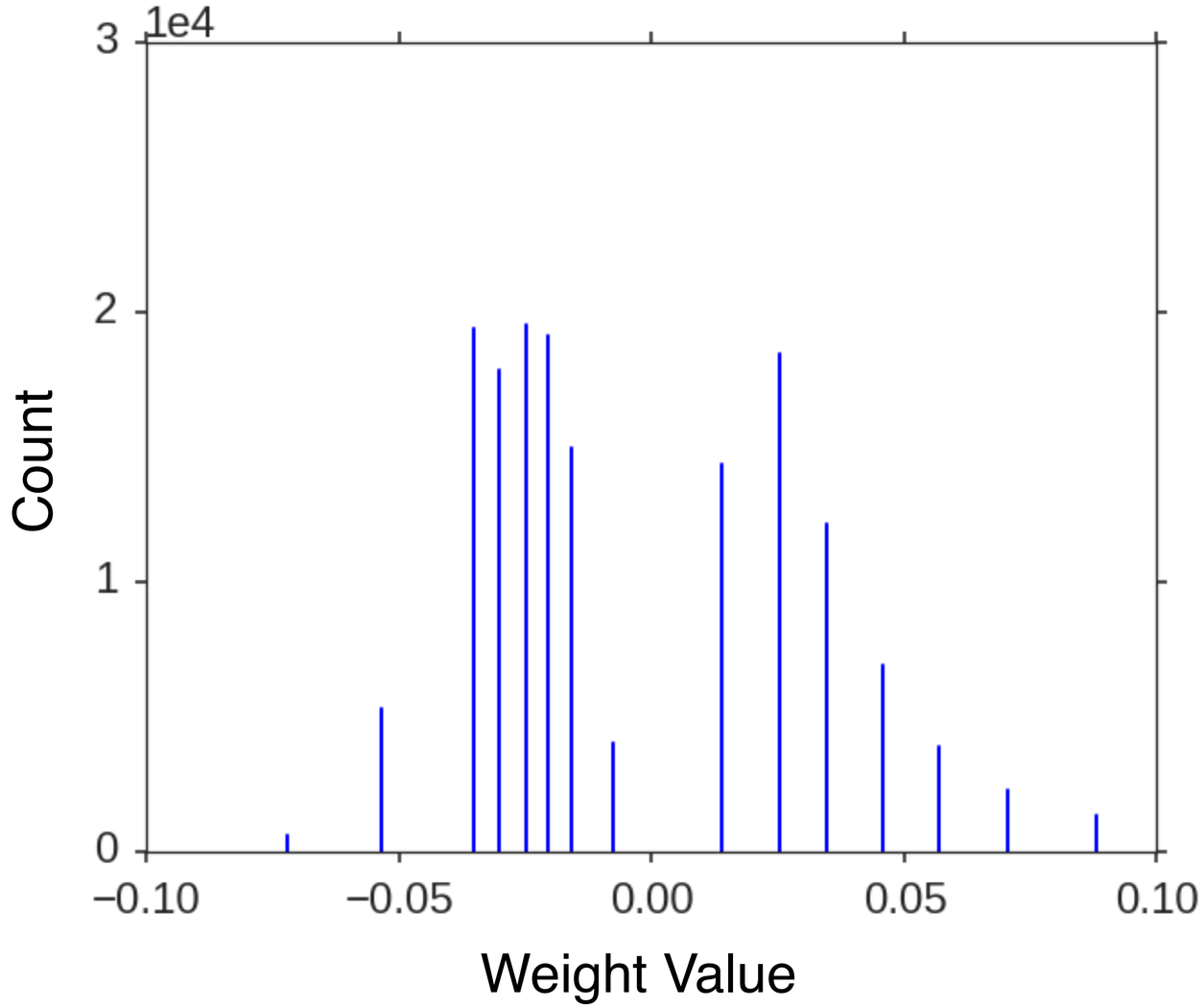
Deep Compression [Han *et al.*, ICLR 2016]

After Quantization: Discrete Weight



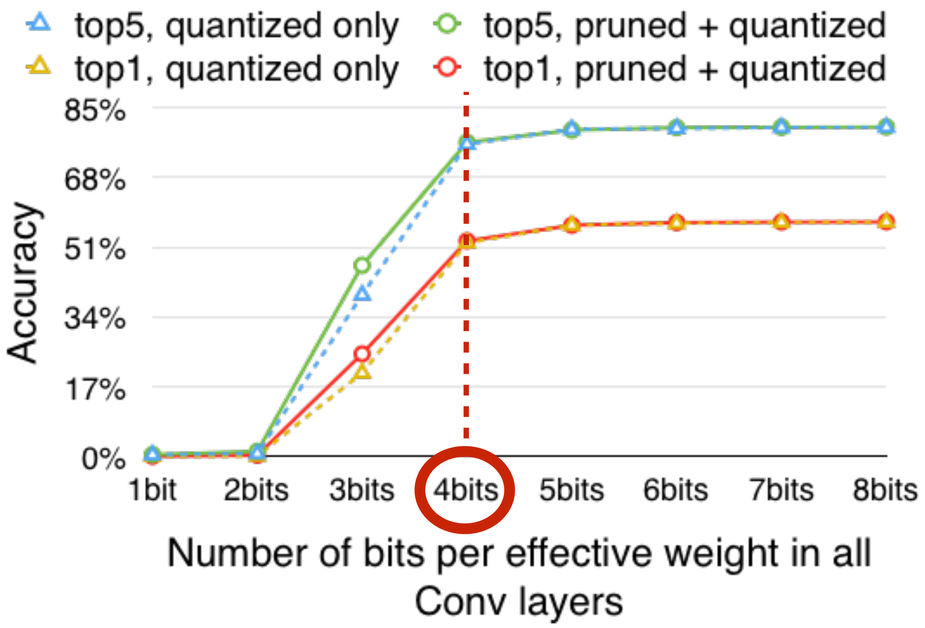
Deep Compression [Han *et al.*, ICLR 2016]

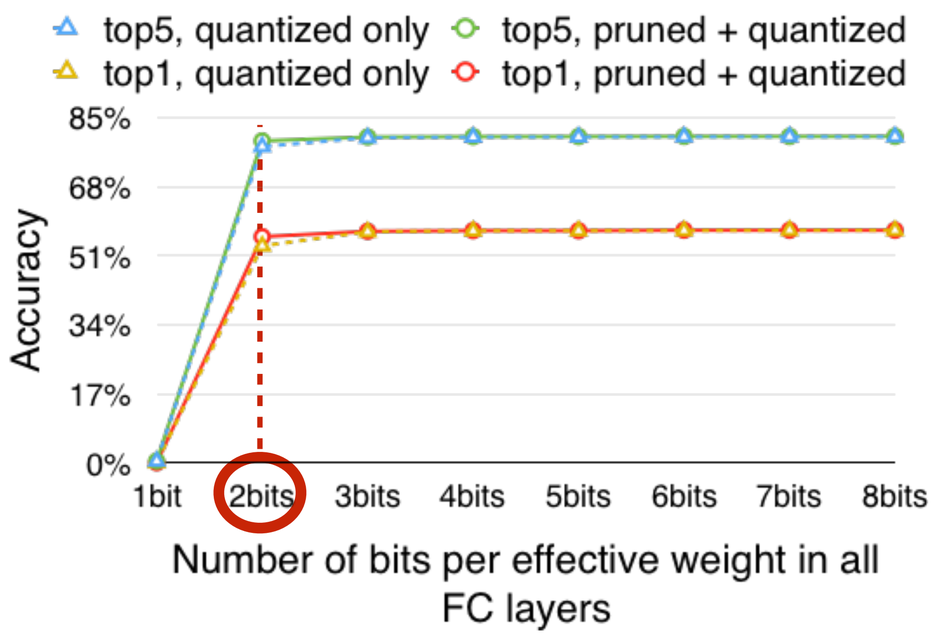
After Quantization: Discrete Weight after Retraining



Deep Compression [Han *et al.*, ICLR 2016]

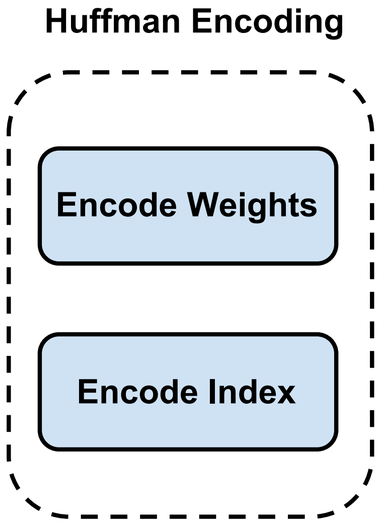
How Many Bits do We Need?

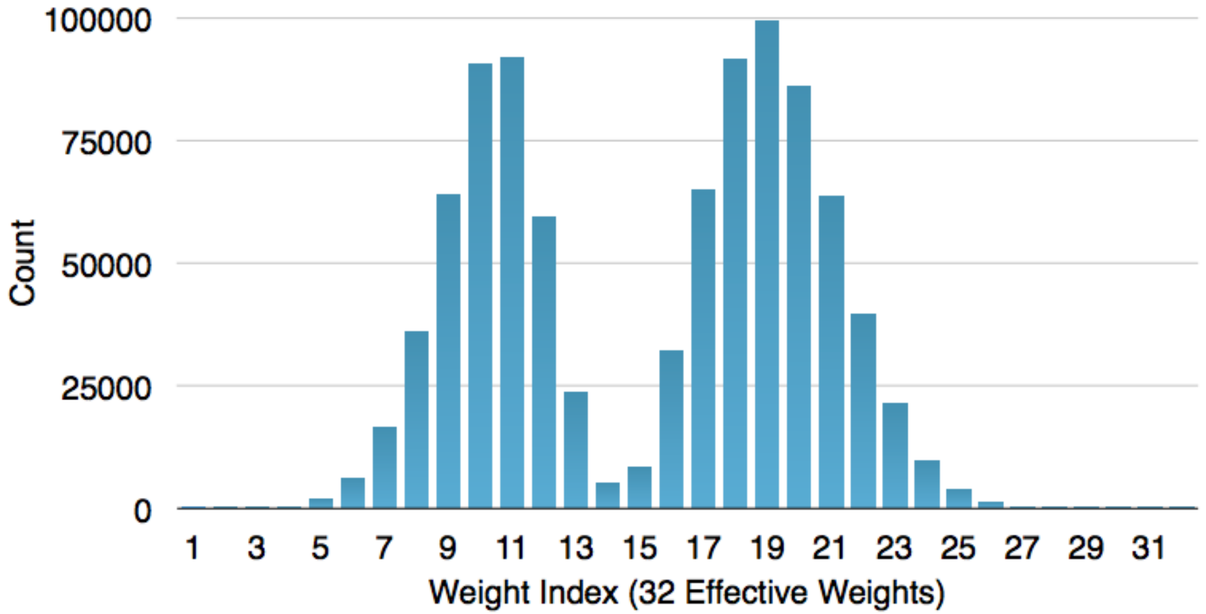




Deep Compression [Han *et al.*, ICLR 2016]

Huffman Coding

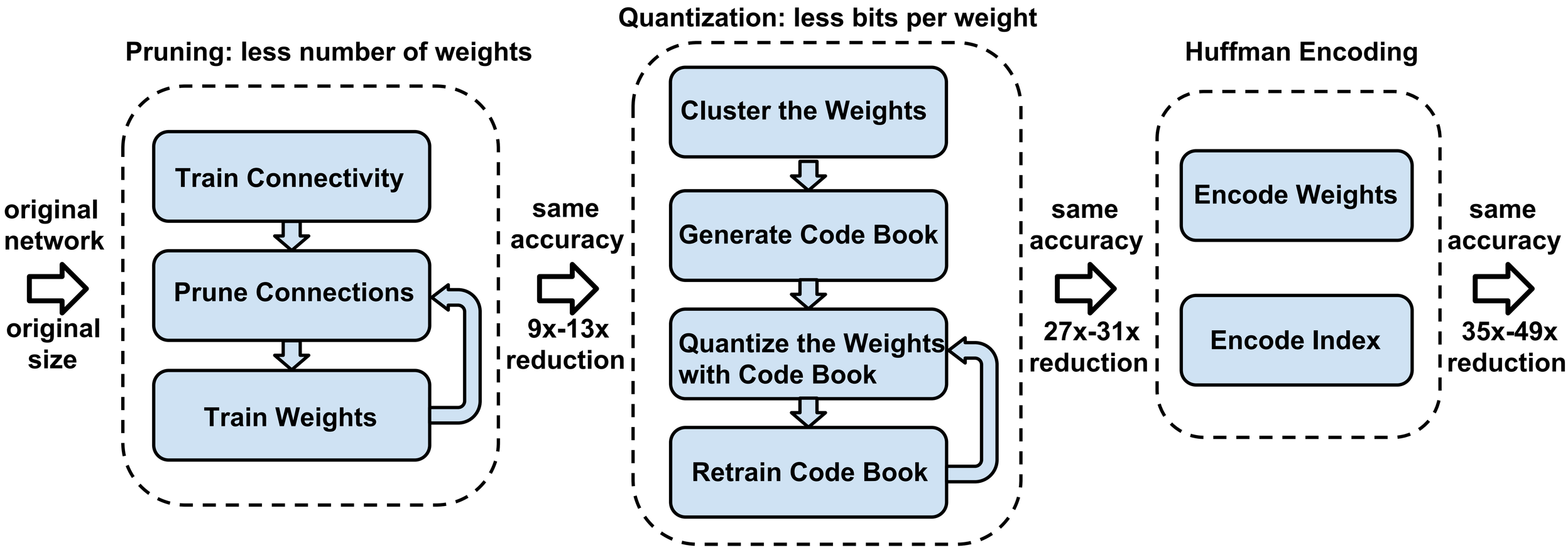




- In-frequent weights: use more bits to represent
- Frequent weights: use less bits to represent

Deep Compression [Han *et al.*, ICLR 2016]

Summary of Deep Compression



Deep Compression [Han *et al.*, ICLR 2016]

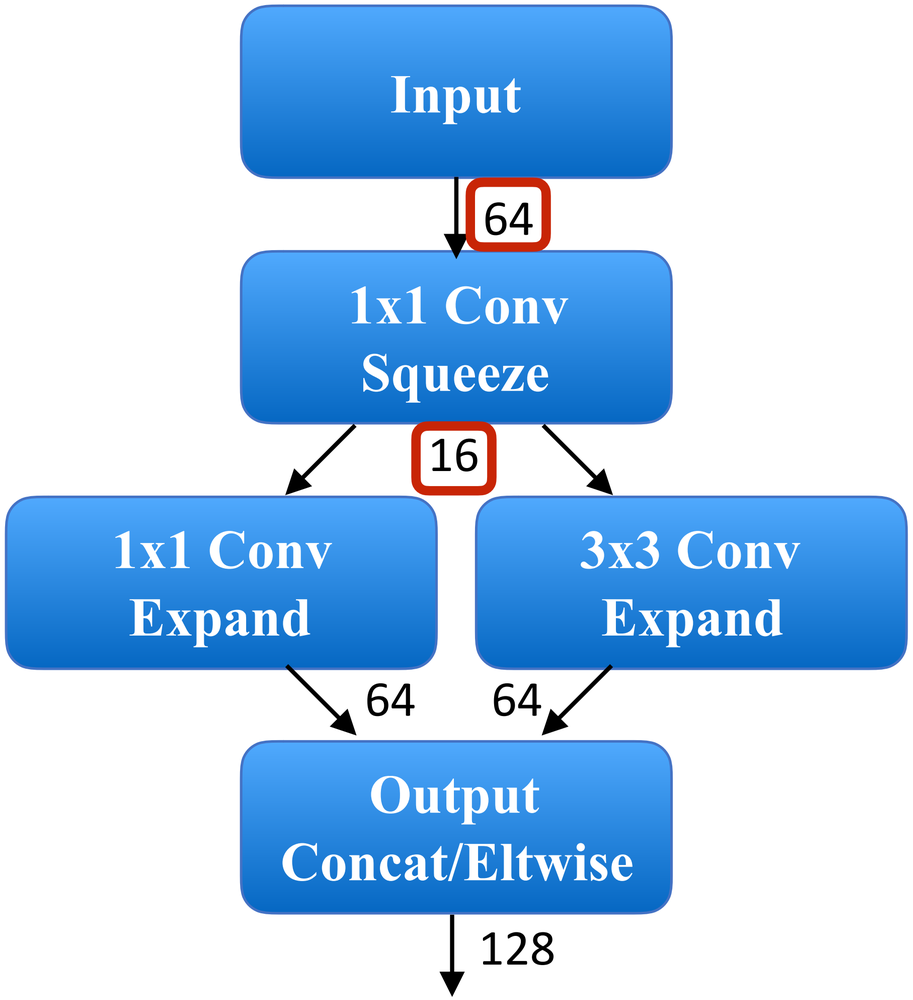
Deep Compression Results

Network	Original Size	Compressed Size	Compression Ratio	Original Accuracy	Compressed Accuracy
LeNet-300	1070KB	27KB	**40x**	98.36%	98.42%
LeNet-5	1720KB	44KB	**39x**	99.20%	99.26%
AlexNet	240MB	6.9MB	**35x**	80.27%	80.30%
VGGNet	550MB	11.3MB	**49x**	88.68%	89.09%
GoogleNet	28MB	2.8MB	**10x**	88.90%	88.92%
ResNet-18	44.6MB	4.0MB	**11x**	89.24%	89.28%

Can we make compact models to begin with?

Deep Compression [Han *et al.*, ICLR 2016]

SqueezeNet



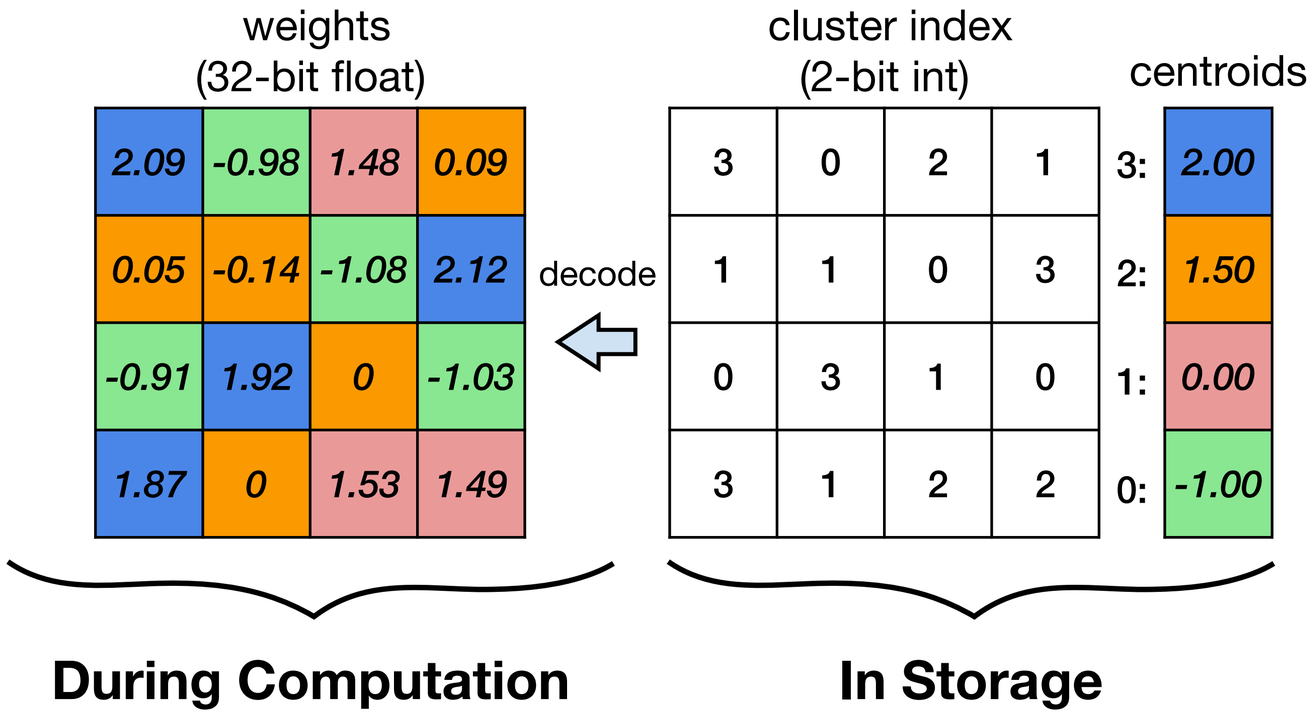
SqueezeNet: AlexNet-level accuracy with 50x fewer parameters and <0.5MB model size [Iandola *et al.*, arXiv 2016]

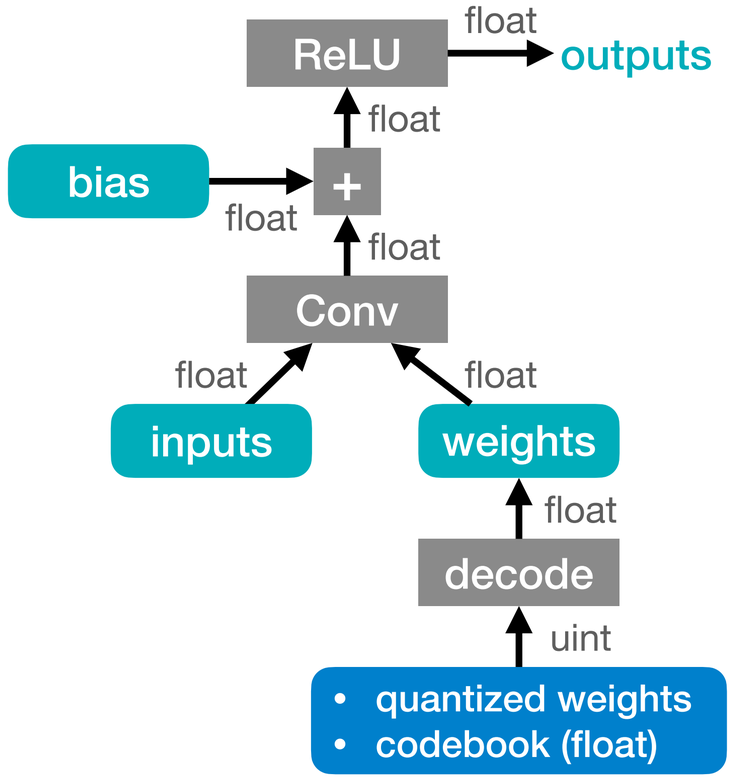
Deep Compression on SqueezeNet

Network	Approach	Size	Ratio	Top-1 Accuracy	Top-5 Accuracy
AlexNet	-	240MB	1x	57.2%	80.3%
AlexNet	SVD	48MB	5x	56.0%	79.4%
AlexNet	Deep Compression	6.9MB	35x	57.2%	80.3%
SqueezeNet	-	4.8MB	50x	57.5%	80.3%
SqueezeNet	Deep Compression	0.47MB	510x	57.5%	80.3%

SqueezeNet: AlexNet-level accuracy with 50x fewer parameters and <0.5MB model size [Iandola *et al.*, arXiv 2016]

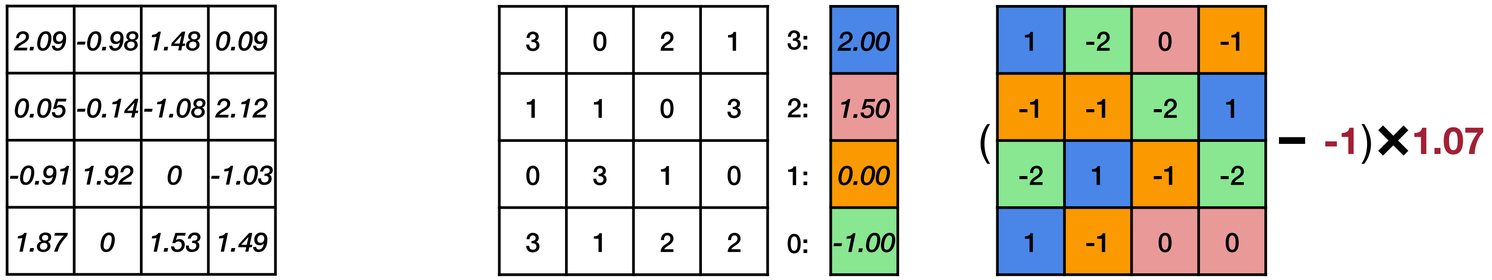
K-Means-based Weight Quantization





- The weights are decompressed using a lookup table (*i.e.*, codebook) during runtime inference.
- K-Means-based Weight Quantization only saves storage cost of a neural network model.
  - All the computation and memory access are still floating-point.

Neural Network Quantization



		K-Means-based Quantization	Linear Quantization	Binary/Ternary Quantization
Storage	Floating-Point Weights	Integer Weights; Floating-Point Codebook	Integer Weights	
Computation	Floating-Point Arithmetic	Floating-Point Arithmetic	Integer Arithmetic	

Linear Quantization

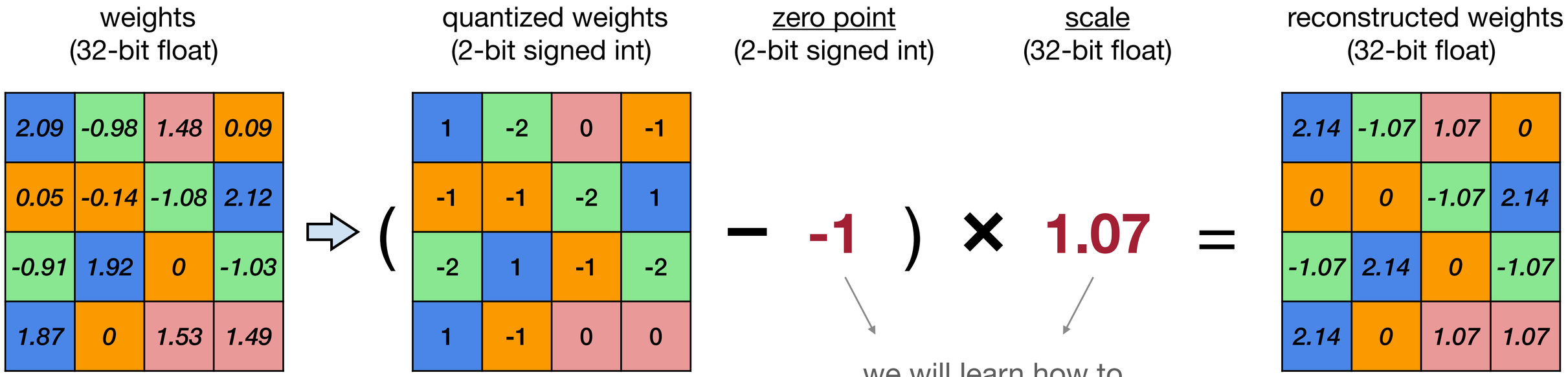
What is Linear Quantization?

weights
(32-bit float)

2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*

What is Linear Quantization?

An affine mapping of integers to real numbers



we will learn how to determine these parameters

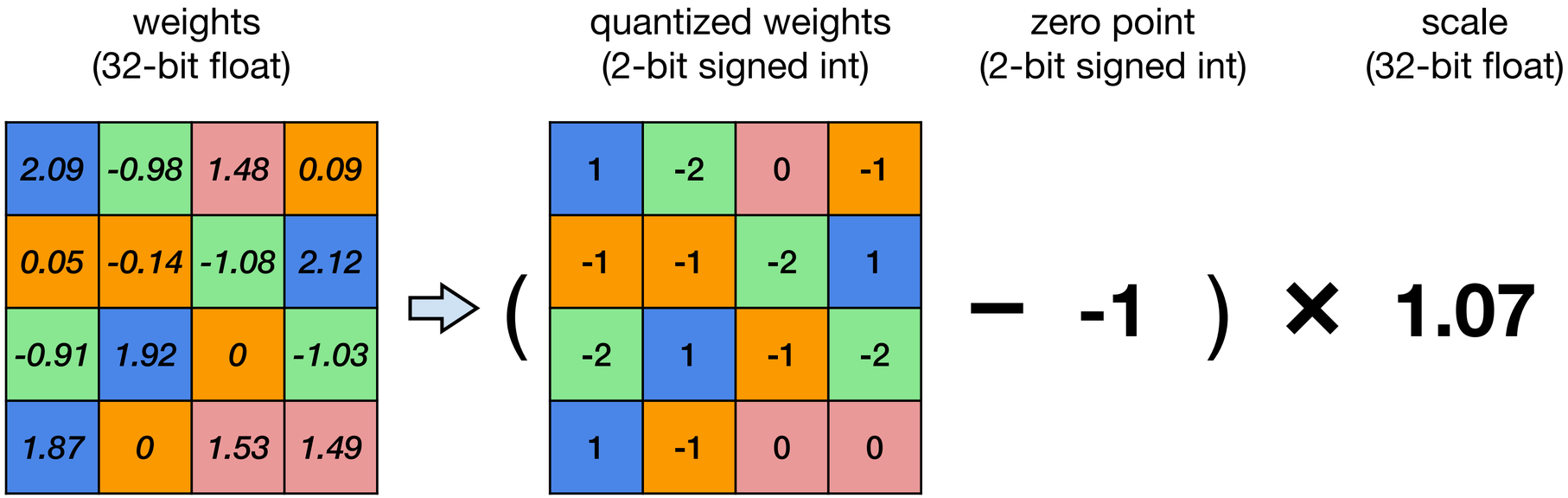
Binary	Decimal
01	1
00	0
11	-1
10	-2

quantization error

-0.05	0.09	0.41	0.09
0.05	-0.14	-0.01	-0.02
0.16	-0.22	0	0.04
-0.27	0	0.46	0.42

Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$



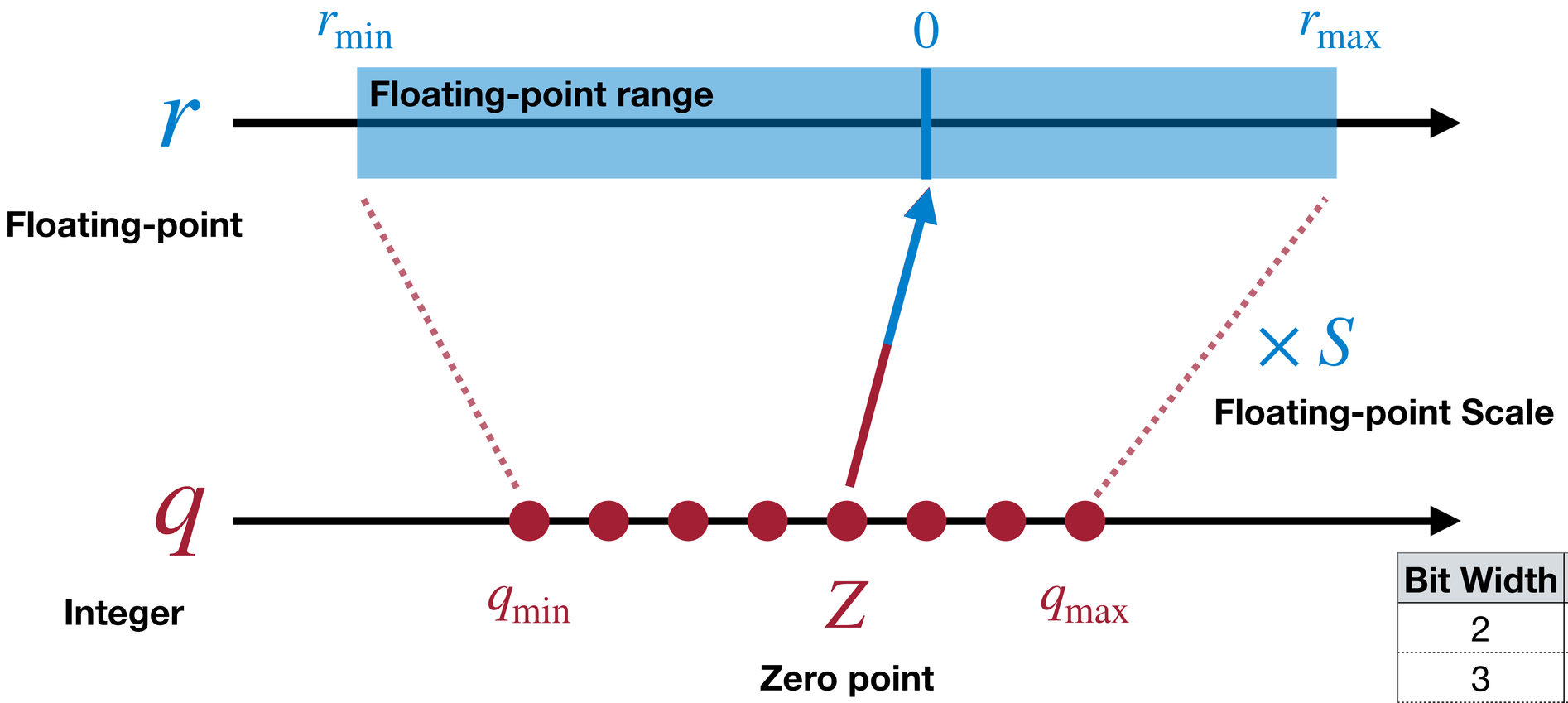
- quantization parameter
- allow real number *r=0* be exactly representable by a quantized integer Z

- quantization parameter

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$

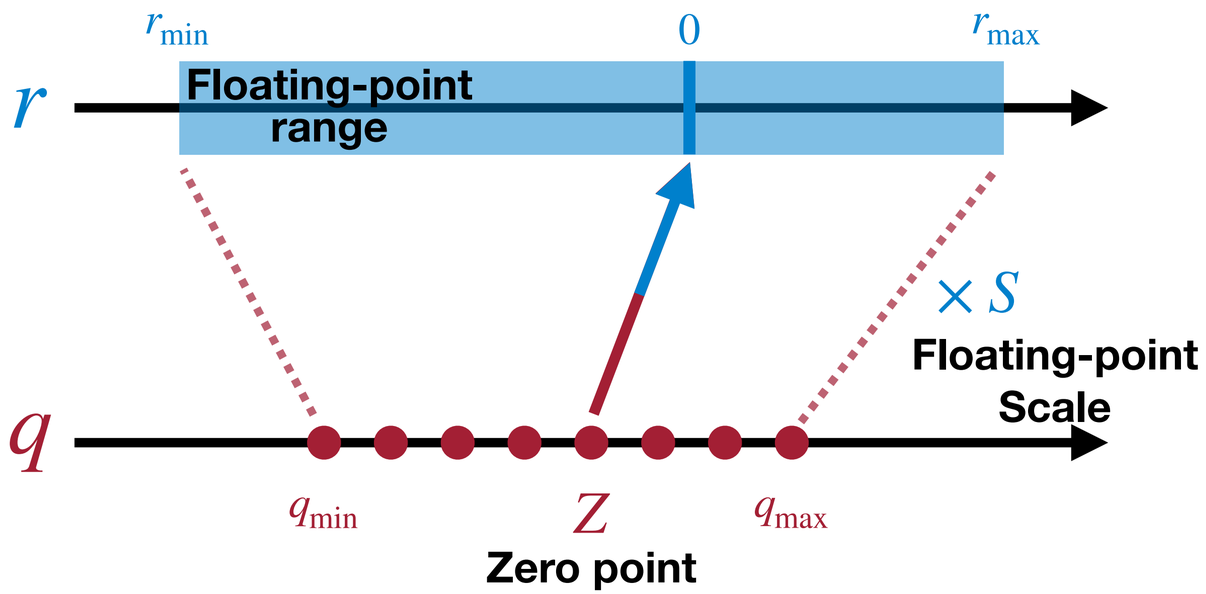


Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Scale of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



$$r_{max} = S\left(q_{max} - Z\right)$$

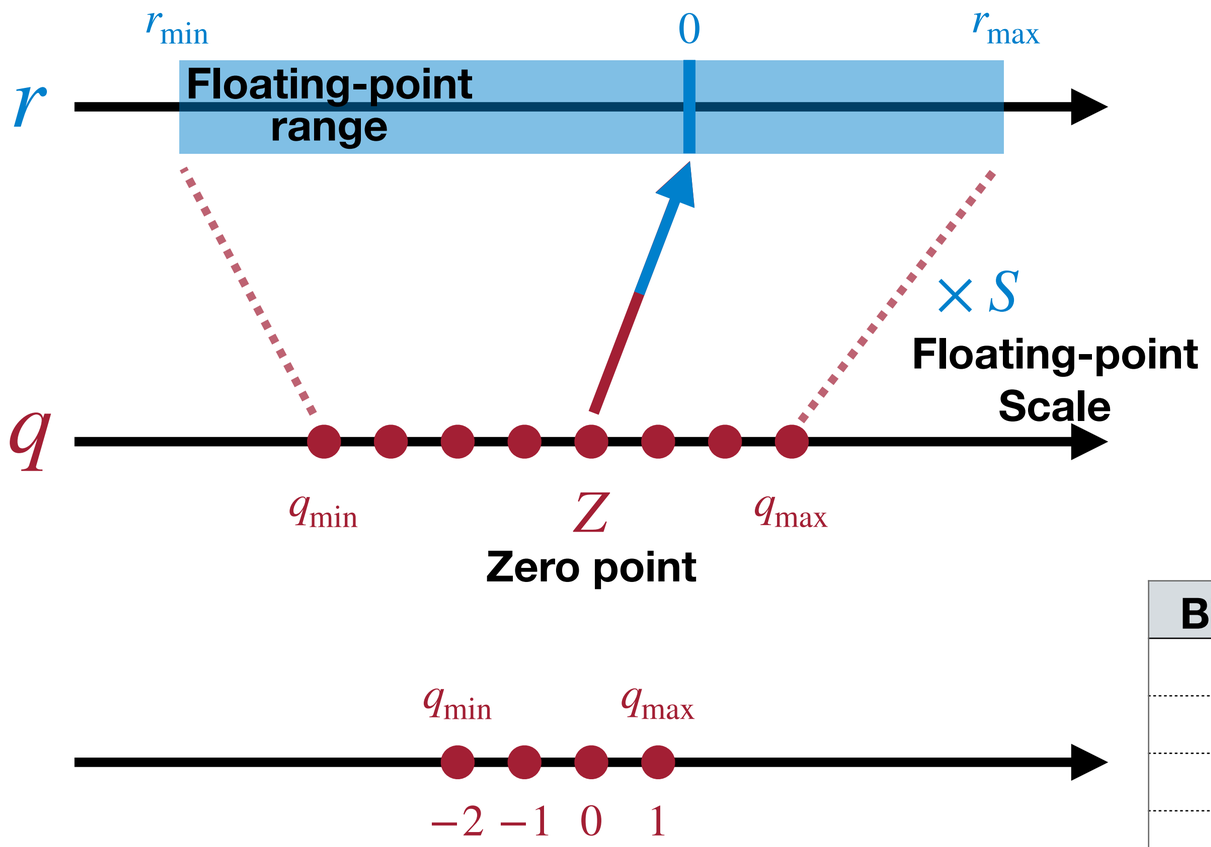
$$r_{min} = S\left(q_{min} - Z\right)$$

$$r_{max} - r_{min} = S\left(q_{max} - q_{min}\right)$$

$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

Scale of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*

Binary	Decimal
01	1
00	0
11	-1
10	-2

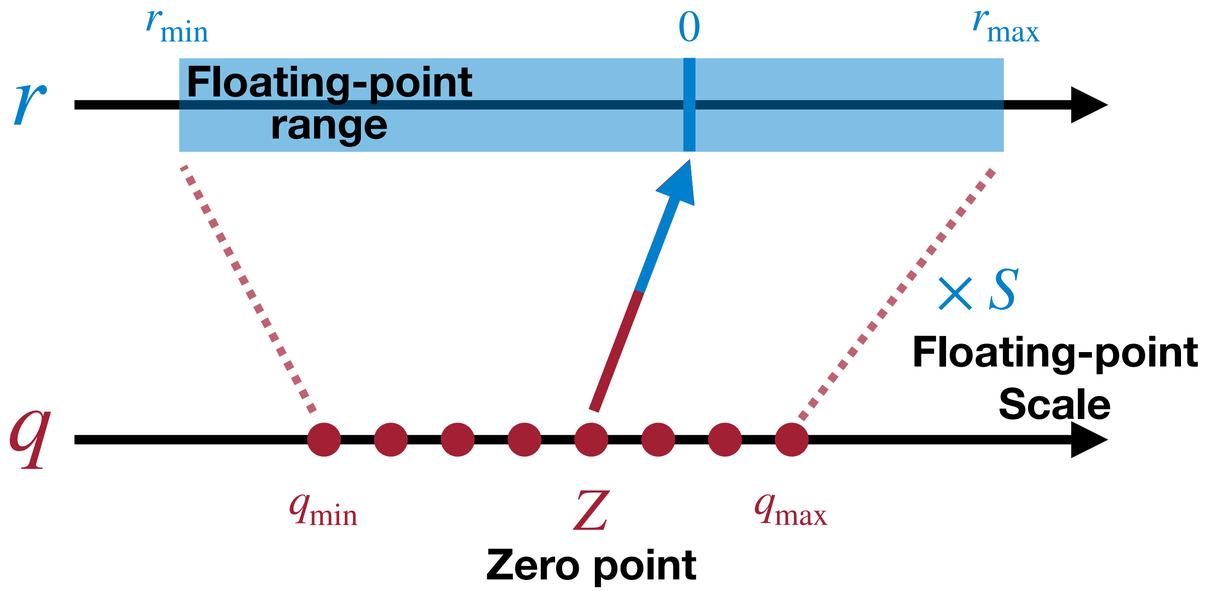
$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

$$= \frac{2.12 - (-1.08)}{1 - (-2)}$$

$$= 1.07$$

Zero Point of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



$$r_{min} = S\left(q_{min} - Z\right)$$

$$\downarrow$$

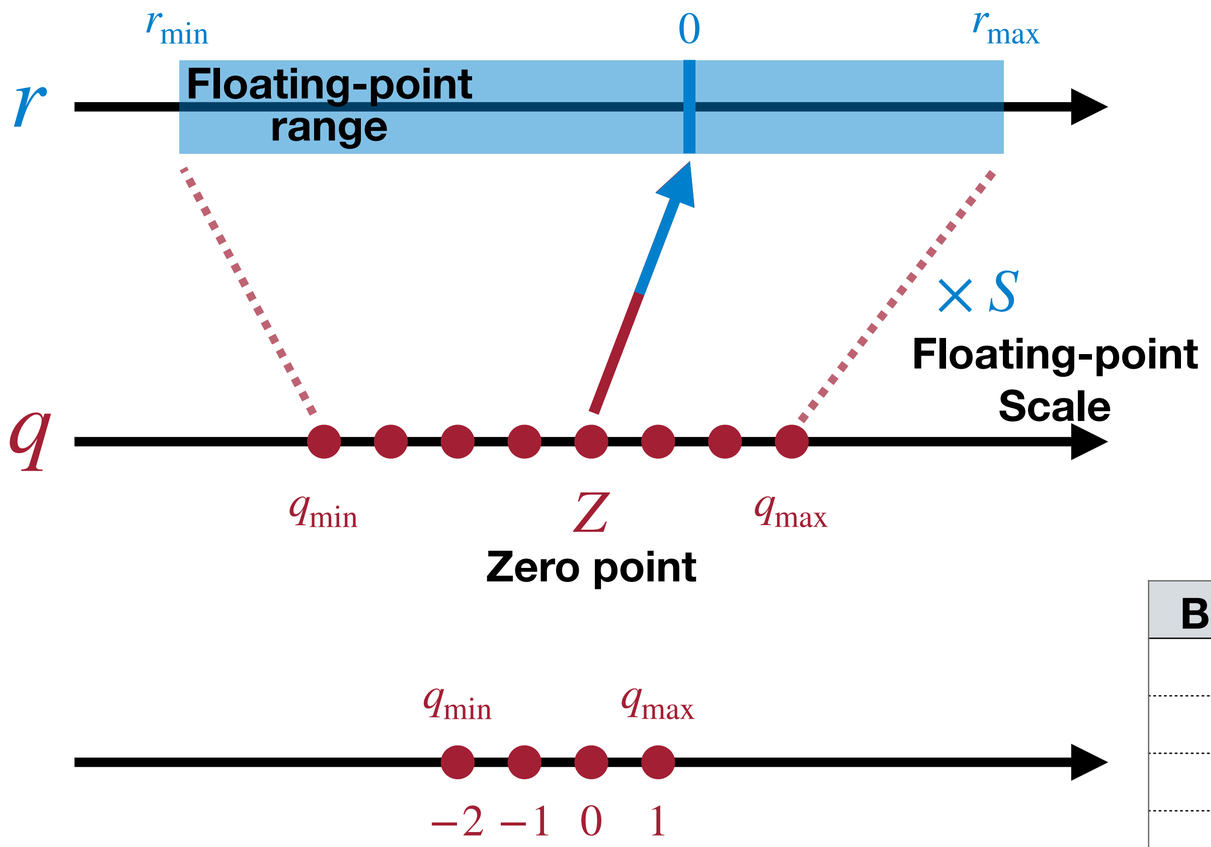
$$Z = q_{min} - \frac{r_{min}}{S}$$

$$\downarrow$$

$$Z = \text{round}\left(q_{min} - \frac{r_{min}}{S}\right)$$

Zero Point of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*

Binary	Decimal
01	1
00	0
11	-1
10	-2

$$Z = q_{min} - \frac{r_{min}}{S}$$

$$= \text{round}(-2 - \frac{-1.08}{1.07})$$

$$= -1$$

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}\left(\mathbf{q}_{\mathbf{W}} - Z_{\mathbf{W}}\right) \cdot S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\right)$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}} - Z_{\mathbf{W}}\right)\left(\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\right) + Z_{\mathbf{Y}}$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}} + Z_{\mathbf{W}}Z_{\mathbf{X}}\right) + Z_{\mathbf{Y}}$$

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$\mathbf{q_Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q_W q_X} - Z_\mathbf{W} \mathbf{q_X} - Z_\mathbf{X} \mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X} \right) + Z_\mathbf{Y}$$

Precompute

***N*-bit Integer Multiplication**
32-bit Integer Addition/Subtraction

***N*-bit Integer Addition**

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$\mathbf{q_Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q_W q_X} - Z_\mathbf{W} \mathbf{q_X} - Z_\mathbf{X} \mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X} \right) + Z_\mathbf{Y}$$

- Empirically, the scale $\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}}$ is always in the interval (0, 1).

Fixed-point Multiplication

$$\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} = 2^{-n} M_0, \quad \text{where} \quad M_0 \in [0.5, 1)$$

Bit Shift

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

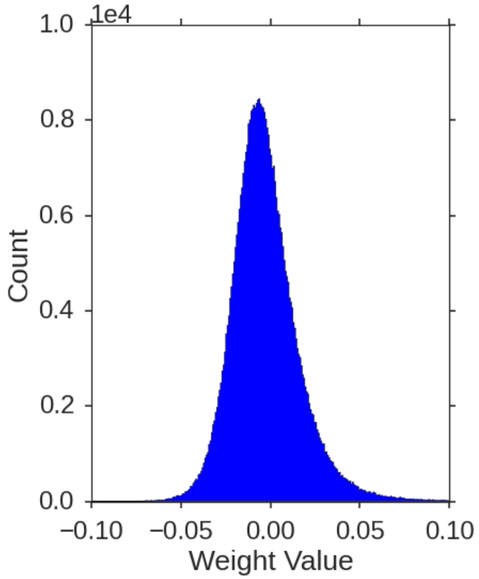
$$\mathbf{q_Y} = \boxed{\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}}} \left(\boxed{\mathbf{q_W q_X} - Z_\mathbf{W}\mathbf{q_X} - Z_\mathbf{X}\mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X}} \right) + \boxed{Z_\mathbf{Y}}$$

Precompute

Rescale to *N*-bit Integer

***N*-bit Integer Multiplication**
32-bit Integer Addition/Subtraction

***N*-bit Integer Addition**

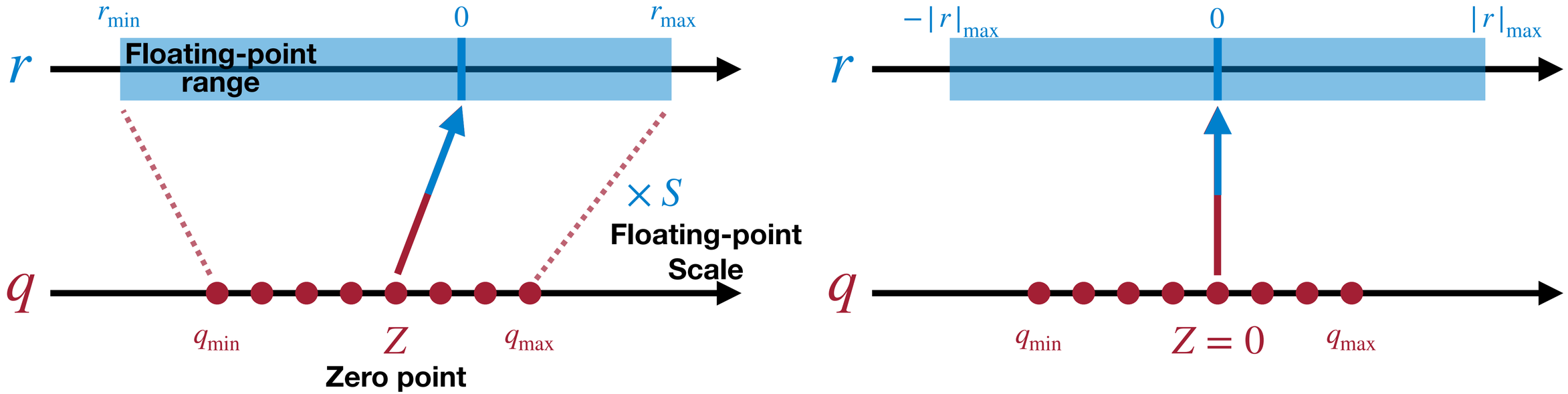


$$Z_\mathbf{W} = 0?$$

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Symmetric Linear Quantization

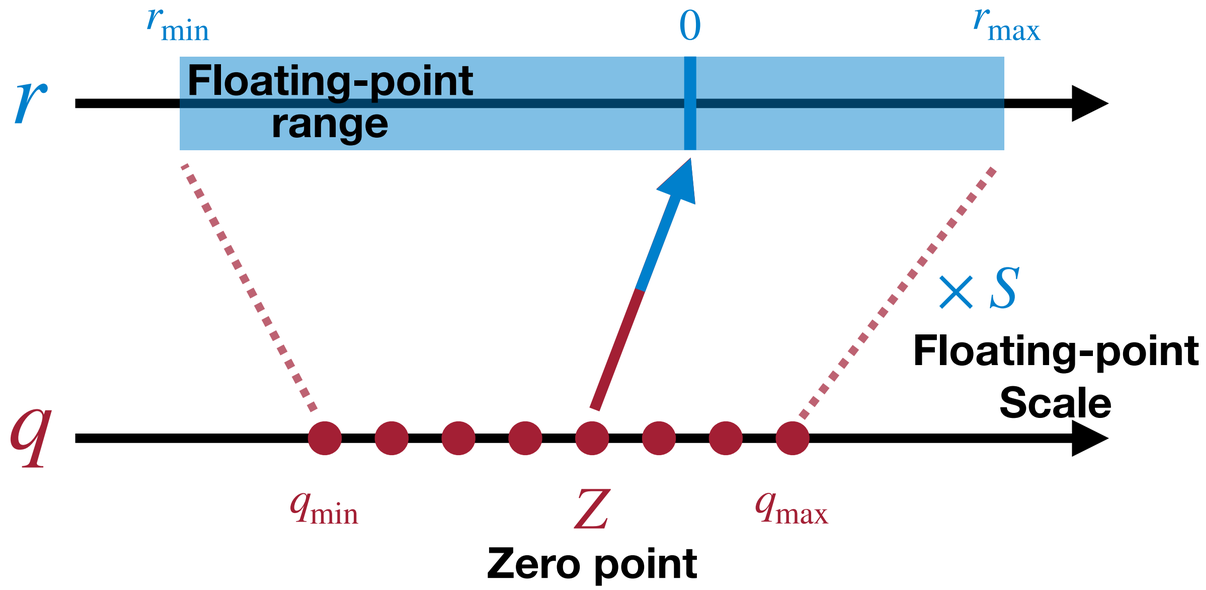
Zero point $Z = 0$ and Symmetric floating-point range

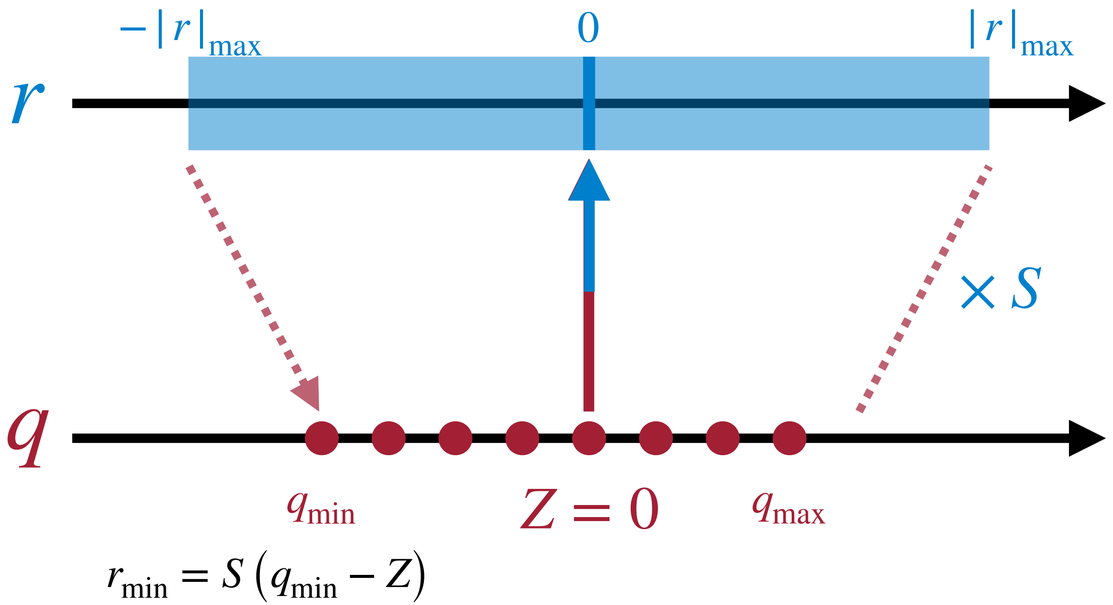


Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

Symmetric Linear Quantization

Full range mode





$$r_{min} = S\left(q_{min} - Z\right)$$

Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

$$S = \frac{r_{min}}{q_{min} - Z} = \frac{-|r|_{max}}{q_{min}} = \frac{|r|_{max}}{2^{N-1}}$$

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication, when Zw=0.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$\mathbf{q_Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q_W q_X} - Z_\mathbf{W} \mathbf{q_X} - Z_\mathbf{X} \mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X} \right) + Z_\mathbf{Y}$$

Precompute

Rescale to *N*-bit Integer

N-bit Integer Multiplication
32-bit Integer Addition/Subtraction

N-bit Integer Addition

$$Z_\mathbf{W} = 0$$

$$\mathbf{q_Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q_W q_X} - Z_\mathbf{X} \mathbf{q_W} \right) + Z_\mathbf{Y}$$

Linear Quantized Fully-Connected Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- So far, we ignore bias. Now we consider the following fully-connected layer with bias.

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}\left(\mathbf{q}_{\mathbf{W}} - Z_{\mathbf{W}}\right) \cdot S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\right) + S_{\mathbf{b}}\left(\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{b}}\right)$$

$$\downarrow \quad Z_{\mathbf{W}} = 0$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = \underline{S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}\right)} + \underline{S_{\mathbf{b}}\left(\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{b}}\right)}$$

Linear Quantized Fully-Connected Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

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$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}\left(\mathbf{q}_{\mathbf{W}} - Z_{\mathbf{W}}\right) \cdot S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\right) + S_{\mathbf{b}}\left(\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{b}}\right)$$

$$\downarrow \quad Z_{\mathbf{W}} = 0$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}\right) + S_{\mathbf{b}}\left(\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{b}}\right)$$

$$\downarrow \quad Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}}S_{\mathbf{X}}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}} + \mathbf{q}_{\mathbf{b}}\right)$$

Linear Quantized Fully-Connected Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- So far, we ignore bias. Now we consider the following fully-connected layer with bias.

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0 \downarrow Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}} S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}} + \mathbf{q}_{\mathbf{b}}\right)$$

$$\downarrow$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} + \mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}\right) + Z_{\mathbf{Y}}$$

Precompute

$$\downarrow \quad \mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} + \mathbf{q}_{bias}\right) + Z_{\mathbf{Y}}$$

We will discuss how to compute activation zero point in the next lecture.

Linear Quantized Fully-Connected Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- So far, we ignore bias. Now we consider the following fully-connected layer with bias.

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}} \mathbf{q}_{\mathbf{W}}$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}} \left(\mathbf{q}_{\mathbf{W}} \mathbf{q}_{\mathbf{X}} + \mathbf{q}_{bias} \right) + Z_{\mathbf{Y}}$$

Rescale to *N*-bit Int | **_N_-bit Int Mult. 32-bit Int Add.** | **_N_-bit Int Add**

Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

Linear Quantized Convolution Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following convolution layer.

$$\mathbf{Y} = \text{Conv}(\mathbf{W}, \mathbf{X}) + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - \text{Conv}(\mathbf{q}_{\mathbf{W}}, Z_{\mathbf{X}})$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}} \left(\text{Conv}(\mathbf{q}_{\mathbf{W}}, \mathbf{q}_{\mathbf{X}}) + \mathbf{q}_{bias} \right) + Z_{\mathbf{Y}}$$

Rescale to *N*-bit Int

***N*-bit Int Mult. 32-bit Int Add.**

***N*-bit Int Add**

Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

Linear Quantized Convolution Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following convolution layer.

$$\mathbf{Y} = \text{Conv}(\mathbf{W}, \mathbf{X}) + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

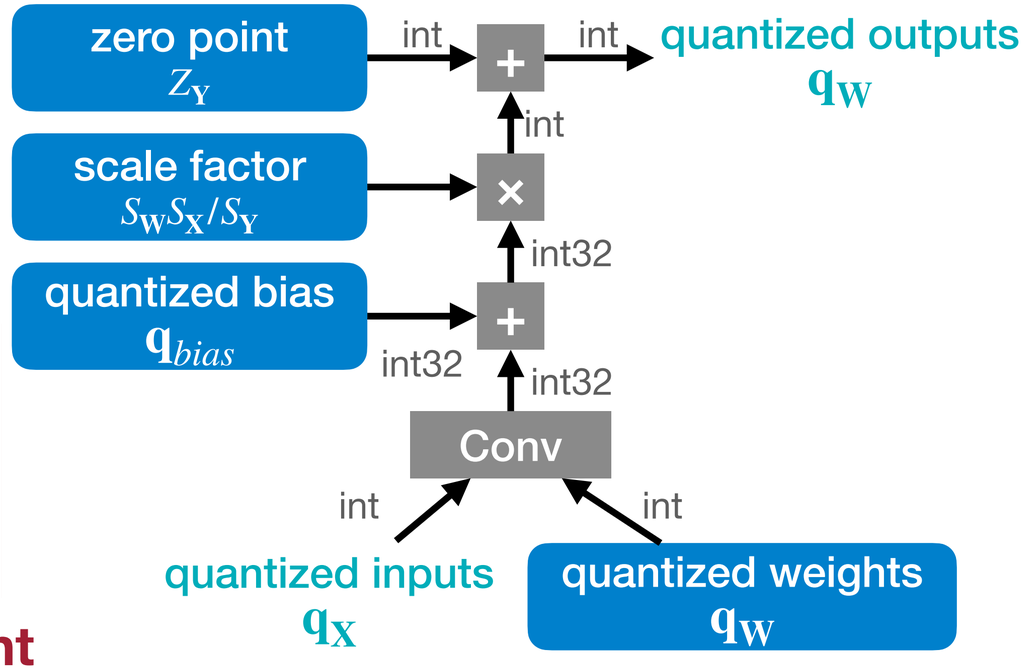
$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - \text{Conv}(\mathbf{q}_{\mathbf{W}}, Z_{\mathbf{X}})$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}} \left(\text{Conv}(\mathbf{q}_{\mathbf{W}}, \mathbf{q}_{\mathbf{X}}) + \mathbf{q}_{bias} \right) + Z_{\mathbf{Y}}$$

Rescale to *N*-bit Int

***N*-bit Int Mult. 32-bit Int Add.**

***N*-bit Int Add**

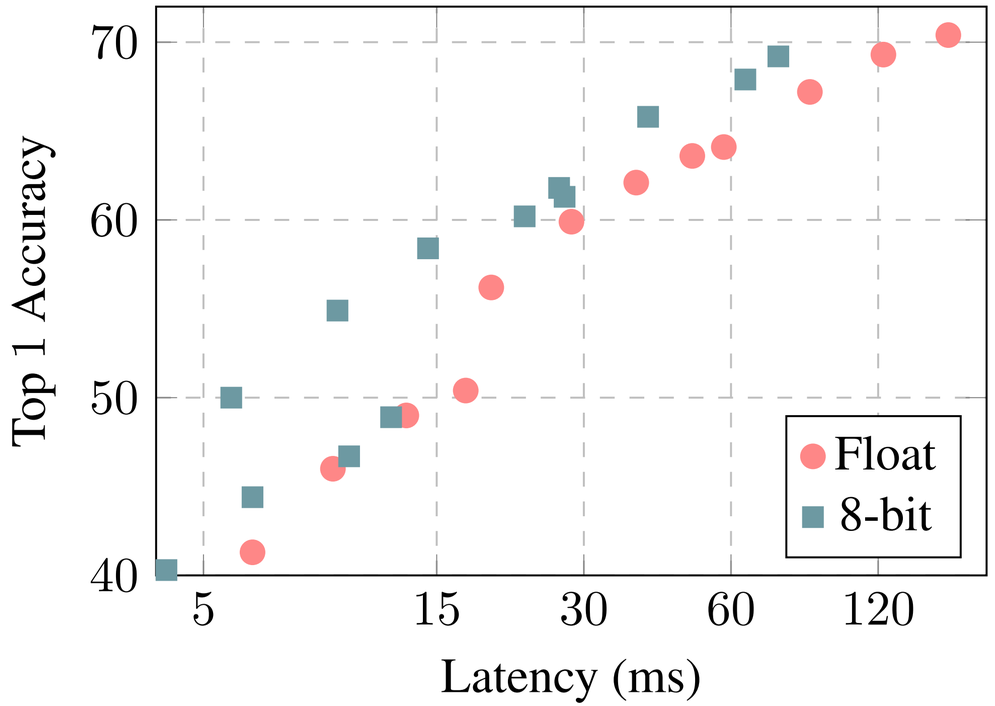


Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

INT8 Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$

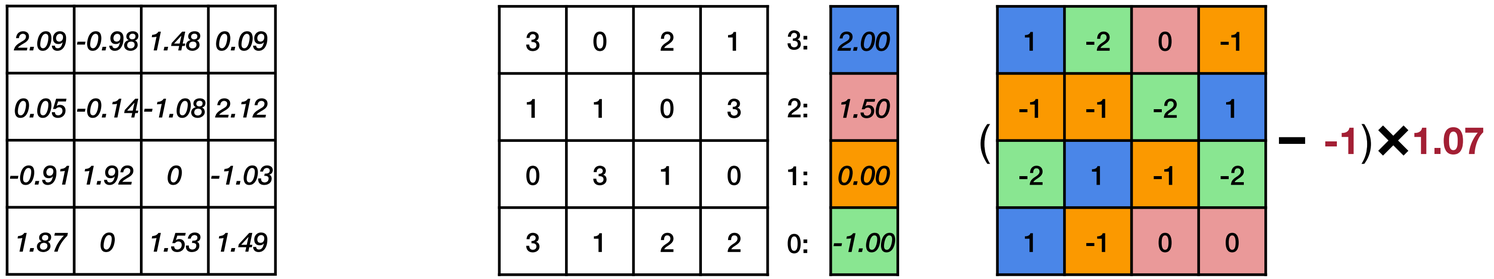
Neural Network	ResNet-50	Inception-V3
Floating-point Accuracy	76.4%	78.4%
8-bit Integer-quantized Acurracy	74.9%	75.4%



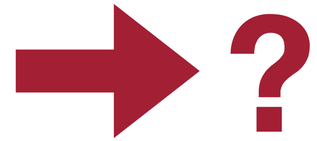
Latency-vs-accuracy tradeoff of float vs. integer-only MobileNets on ImageNet using Snapdragon 835 big cores.

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Neural Network Quantization



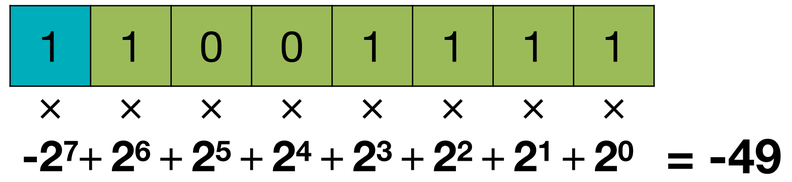
		K-Means-based Quantization	**Linear Quantization**
Storage	Floating-Point Weights	Integer Weights; Floating-Point Codebook	Integer Weights
Computation	Floating-Point Arithmetic	Floating-Point Arithmetic	Integer Arithmetic



Summary of Today's Lecture

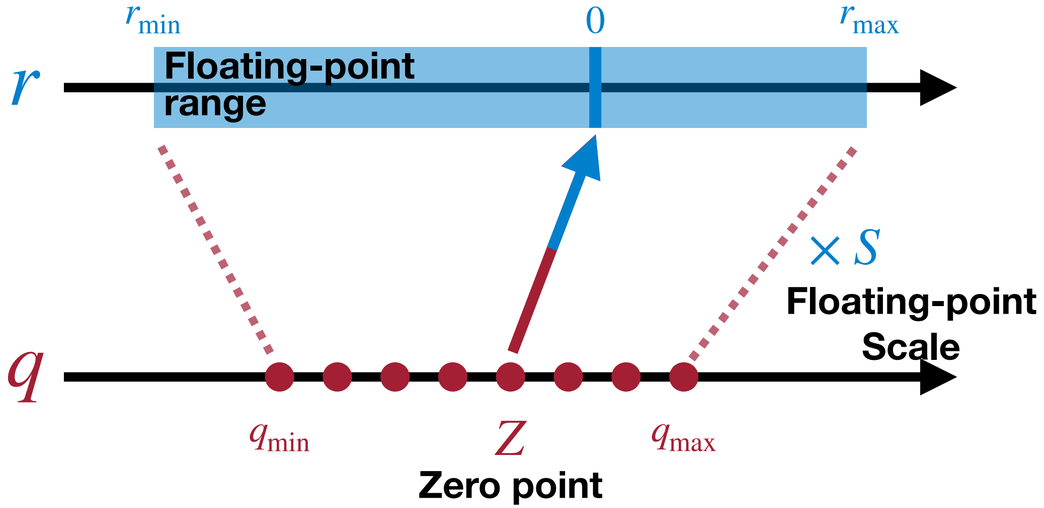
Today, we reviewed and learned

- the numeric data types used in the modern computing systems, including integers and floating-point numbers.



- the basic concept of **neural network quantization**: *converting the weights and activations of neural networks into a limited discrete set of numbers.*

- two types of common neural network quantization:
  - K-Means-based Quantization
  - Linear Quantization



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