Reasoning Axioms and the Game-Theoretical Substrate of Intelligence:
Objectifying Reason as the Final Frontier of Science

Eric Robert Lawson

November 13, 2025

Abstract

This paper introduces *reasoning axioms*, primitive structures classifiable as mathematical or game-theoretical, which form the foundational building blocks of bounded reasoning systems—whether physical, cognitive, or symbolic. By objectifying and operationalizing reasoning itself, and integrating reward-structured exploration and meta-reasoning, a universal substrate emerges in which game-theoretical reasoning can be formalized with the same rigor as mathematics. Within this framework, reward functions guide the emergence of higher-order reasoning patterns, enabling systems to discover, optimize, and evaluate reasoning processes across domains—from strategic play to scientific discovery. We argue that this formal substrate constitutes a frontier of science: the systematic axiomatization, analysis, and exploration of reasoning itself.

1 Introduction

Logic and mathematics provide axiomatic foundations for static knowledge, while game theory models dynamic interaction. However, reasoning as an operational process lacks a formal medium encompassing both static and dynamic reasoning. By objectifying reasoning as composable entities, we identify *reasoning axioms* that are domain-neutral yet classifiable as mathematical or game-theoretical. These form a universal reasoning substrate, enabling game-theoretical reasoning as a first-class axiomatic process.

These reasoning axioms provide a foundation, but to understand their significance and operational potential, we first examine reasoning as an object and the distinctions between its mathematical and game-theoretical domains.

2 Motivation and Background

2.1 Reasoning as an Operational Object

Building on the *OrganismCore* framework, reasoning can be represented as a composable, operational object—instantiated, transformed, and optimized. This allows meta-reasoning and cross-domain interaction.

2.2 Mathematical vs. Game-Theoretical Reasoning Domains

- **Mathematical reasoning:** Static, truth-preserving domains (e.g., proof systems, algebra, topology).

- **Game-theoretical reasoning:** Dynamic, strategic domains (e.g., decision systems, adaptive interaction).

Both are bounded systems under the same substrate, differing only in temporal or interactive constraints.

Having established the operational and domain-specific perspectives of reasoning, we can now formalize the primitive inferential units—reasoning axioms—that underpin all bounded reasoning processes.

3   Reasoning Axioms and Domain Classification

A **Reasoning Axiom** is a primitive inferential transformation satisfying:

1. Closure under compositional reasoning;

2. Representability as a computable object;

3. Preservation of meta-invariance across equivalent representations.

Formally:
$$A_r \in \begin{cases} \mathcal{M} & \text{if the axiom is static and truth-preserving,} \\ \mathcal{G} & \text{if the axiom is dynamic or adversarial.} \end{cases}$$

While the reasoning substrate itself is universal, each axiom is valid only within the specific reasoning primitive space and reward-structured context in which it was derived. For example, a chess axiom describing an optimal rook endgame strategy applies within the chess reasoning space but does not directly inform GPS navigation or other domains. Applying axioms across domains requires careful alignment of the underlying primitives and reward structures.

4   Operationalization of Game-Theoretical Proofs

With reasoning axioms defined, we can now explore how these primitives enable formal proofs and optimization in dynamic, game-theoretical contexts.

Defined in an operational substrate, game-theoretical reasoning can be *proven* like mathematical theorems by:

1. Defining the environment (agents, goals, rules);

2. Expressing interactions as composable reasoning processes;

3. Deriving equilibrium or optimal reasoning paths as invariant proofs.

This extends to real-world systems such as medical diagnosis, strategy optimization, or adaptive control.

4.1 Reward-Structured and Self-Referential Reasoning

Reasoning in goal-directed domains depends on reward functions encoding incentivized behavior. Each reasoning axiom not only governs inference but embeds a reward structure reflecting alignment with objectives. Crucially, self-referentiality allows reasoning to be introspective: axioms can be derived over reasoning processes themselves, yielding a fully operationalized meta-layer.

This perspective enables *gamification* of reasoning: the system discovers, tests, and refines its own axioms. We define a **meta-Reasoning DNA Unit (meta-RDU)** as a construct optimizing reasoning structure across spaces, creating a universal optimization layer.

Crucially, designing a reward function is itself an act of shaping the reasoning space: it encodes which inferential trajectories are valued and guides the system's exploration. In this sense, crafting a reward function is analogous to teaching or incentivizing an agent, defining its understanding of how to navigate the reasoning space effectively.

4.2 Emergence of Game-Theoretical Reasoning Axioms via Reward Functions

The structured incentives encoded by reward functions not only guide reasoning behavior but also give rise to higher-level patterns, forming emergent game-theoretical reasoning axioms.

In self-referential spaces, reward functions both define success and structure the reasoning space. Patterns of optimal reasoning emerge consistently as **game-theoretical reasoning axioms**, capturing transformations that lead to high-reward outcomes.

These axioms can be recursively integrated, refining navigation within the reasoning space. Their validity is contextual, relative to the primitives and reward structure from which they arise. Cross-domain applications require alignment of contextual primitives.

Formally, reward functions do more than incentivize: they generate and optimize the structure of reasoning itself, creating emergent patterns that guide meta-reasoning and the discovery of optimal strategies. The following section demonstrates how these emergent structures manifest in real-world and abstract systems, bridging theory and application.

5 Applications and Implications

5.1 Unified Proof and Optimization Framework

Expressing mathematical and game-theoretical reasoning within a single substrate establishes a universal proof environment. Optimization, equilibrium, and correctness become provable properties of reasoning itself.

5.2 Examples: Chess, Medicine, and Reality as an Adversary

Chess reasoning axioms map to move evaluations and meta-strategic inference. In medicine, the "adversary" is biological entropy or system failure. Each domain reduces to a bounded set of inferential primitives. While chess, medicine, and physical navigation all inhabit the same universal reasoning substrate, the axioms derived within one domain do not automatically transfer to another; domain-specific primitives and reward structures determine the applicability of any reasoning axiom.

5.3 Objectifying Game-Theoretical Reasoning Axioms

Toy universes, such as endgame chess puzzles, are literal manifestations of reasoning axioms. Objectifying these axioms allows construction of reasoning objects capturing why one move is superior, analogous to internalized transformations in human cognition.

Mapping reasoning spaces generated by trained models, optimized via reward functions, produces trajectories that can be compared against all alternatives. *Explainability* emerges through self-referential reasoning, akin to navigation via GPS: the path is explainable by its position in the reasoning space.

This objectification bridges human reasoning and computational meta-reasoning, where discovery of optimal paths becomes formally analyzable.

By examining these applications, we see how formalized reasoning axioms and reward-informed navigation coalesce into a broader framework, setting the stage for a conceptual redefinition of intelligence.

6 Discussion

This framework reframes intelligence as a property of reasoning spaces, not agents. Composable, self-referential reasoning enables exploration of how systems evolve toward reasoning optimality.

7 Conclusion

Integrating reasoning axioms with reward-structured exploration and meta-reasoning provides a lens for understanding the capabilities and limitations of intelligent systems. Reasoning axioms are context-dependent: their validity is defined relative to the primitives and reward structures that generated them. For example, an axiom capturing optimal chess play does not directly inform GPS navigation, illustrating that universality of the substrate does not imply universality of individual axioms.

These axioms form a formal substrate for capturing structured inference across mathematical and game-theoretical domains. While many reasoning processes can be formalized within this framework, some fall outside or even contradict a given axiomatic set, providing a natural criterion for assessing *reasonability*.

Intelligence can thus be measured by a system's ability to navigate reasoning spaces efficiently and coherently relative to domain-specific axioms. By operationalizing reasoning, meta-reasoning emerges: the discovery, refinement, and optimization of reasoning patterns themselves. Axioms serve both as building blocks and evaluative tools, revealing rational and anomalous trajectories.

Moreover, intelligence depends on a system's responsiveness to structured incentives—reward functions—that guide desirable navigation through reasoning spaces. Treating reasoning as a manipulable object opens a new scientific frontier, enabling the mapping, analysis, and understanding of both formalizable structures and the uncharted spaces beyond strict axiomatization, bridging structured inference with the broader landscape of intelligent behavior.