Formal Type Inferencer for System-F in Lean

Yifan Song
Msc Computer Science
Delft University of Technology
Delft, Netherlands
ericyfsong@gmail.com

***Abstract*—or Math in Paper Title or Abstract.**

***Index Terms*—Lean, Type System, Type Theory, Verification**

I. Introduction

Type system is a common tool for verifying the correctness of a program.

II. STLC with Recursive Function

A. Definition of Abstract Syntax, Type and Context

The syntax, type and context of simple typed lambda caculus(STLC) with recursive function is defined the same as in the project description. The only difference is the added $@[derive\ decidable\_eq]$ notation. This is to tell lean that the comparison whether two types are equal is decidable. So we which is essential for the type checker to be a decidable algorithm.

B. Type Relations for STLC

Before implementing and formalizing the type inferencer for STLC, we need to implement the type relations for STLC so that we can type annotate an expression and judge the type inferencer afterwards.

The type relations for literal terms are straight forward, taking expression ETrue as an example:

$$\frac{}{\vdash ETrue : TBool} \tag{1}$$

In the corresponding lean code, this is represented as '$True\_typed\ \Gamma : ctx : typed\ \Gamma\ exp.ETrue\ ty.TBool$' This is basically the same for $IsZero$, $Pred$ and $Succ$:

$$\frac{}{\vdash EIsZero : TFun(TNat, TBool)} \tag{2}$$

$$\frac{}{\vdash EPred : TFun(TNat, TNat)} \tag{3}$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash EVar(x) : T} \tag{4}$$

For Var expression, we need to check its type in the given exvironment, then get the type of this expression.

$$\frac{\Gamma \vdash e1 : TFun(T1, T12) \quad \Gamma \vdash e2 : T1}{\Gamma \vdash EApp(e1, e2) : T_{12}} \tag{5}$$

Here we need to introduce the type of the two sub-expressions for for the application expression. We can and only can get the type when the first expression has a function type while the second expression has the type of the first parameter of that function.

$$\frac{\Gamma \vdash t : TBool \quad \Gamma \vdash thn : T \quad \Gamma \vdash els : T}{\Gamma \vdash EIf(t, thn, els) : T} \tag{6}$$

For if then else braches expression, we need to ensure the test expression has type bool, while both branches share same type as the whole expression.

The above cases all not include changing the context, while type relations for Lam and Rec, which both define a new function and introduce local variables, add elements to the context.

$$\frac{\Gamma, x : T1 \vdash b : T2}{\Gamma \vdash EAbs(x : T1, b) : TFun(T1, T2)} \tag{7}$$

$$\frac{\Gamma, x : T1, f : TFun(T1, T2) \vdash b : T2}{\Gamma \vdash ERec(f, x : T1, T2, b) : TFun(T1, T2)} \tag{8}$$

The type relation for Rec is basically an extension of the Lam. It need to check the declared return type is same as the type of body with parameter and the functino itself with their types are added to the context.

```
1   | RecTyped (G : ctx) (f: string)
2       (x : string) (aa bb A: ty)
3       (e : exp)
4       (p1 : (ty.TFun aa bb) = A)
5       (p2: typed (ctx.ctx_snoc
6           (ctx.ctx_snoc G x aa)
7           f (ty.TFun aa bb)) e bb)
8       : typed G (exp.ERec f x aa bb e) A
```

Here is a code snippet for the type relation for recursive functions. A type A is introduced for all rules to represent the final type of the expression for all cases as shown in line 2, because it surprisingly simplifies the proof of the completeness of the type inference algorithm to only a $simp$ term for all cases there.

C. Some STLC with Recursive Examples

This section is for the exercise 3 in the project. Here we define a function that checks if two natural numbers are equal in STLC.

```
rec f(x : Nat) : Nat -> Bool:
  return lambda y : Nat:
    if x == 0:
```

```
    if y == 0:
      return true
    else:
      return false
  else:
    if y == 0:
      return false
    else:
      return (f(x - 1))(y - 1)
```

Because the language we defined only supports lambda with one parameter, to compare two natural numbers, we need the outer function to get a nat and return a function from nat to bool.

In the inner function, we substract both numbers by one and check if they reach zero at the same time.

The proof of: $\vdash f : TFun(TNat, TFun(TNat, TBool))$, is just a lot of *apply typed_sth* and $refl$. These $refl$s comes because of those type As introduced in *typed*.

D. Maintaining the Integrity of the Specifications

The IEEEtran class file is used to format your paper and style the text. All margins, column widths, line spaces, and text fonts are prescribed; please do not alter them. You may note peculiarities. For example, the head margin measures proportionately more than is customary. This measurement and others are deliberate, using specifications that anticipate your paper as one part of the entire proceedings, and not as an independent document. Please do not revise any of the current designations.

III. Prepare Your Paper Before Styling

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections III-A–III-E below for more information on proofreading, spelling and grammar.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
- Do not mix complete spellings and abbreviations of units: "Wb/m^2" or "webers per square meter", not "webers/m^2". Spell out units when they appear in text: ". . . a few henries", not ". . . a few H".
- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

C. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{9}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(9)", not "Eq. (9)" or "equation (9)", except at the beginning of a sentence: "Equation (9) is . . ."

D. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

BibTeX does not work by magic. It doesn't get the bibliographic data from thin air but from .bib files. If you use BibTeX to produce a bibliography you must send the .bib files.

LaTeX can't read your mind. If you assign the same label to a subsubsection and a table, you might find that Table I has been cross referenced as Table IV-B3.

LaTeX does not have precognitive abilities. If you put a `\label` command before the command that updates the counter it's supposed to be using, the label will pick up the last counter to be cross referenced instead. In particular, a `\label` command should not go before the caption of a figure or a table.

Do not use `\nonumber` inside the `{array}` environment. It will not stop equation numbers inside `{array}` (there won't be any anyway) and it might stop a wanted equation number in the surrounding equation.

E. Some Common Mistakes

- The word “data” is plural, not singular.
- The subscript for the permeability of vacuum μ_0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter “o”.
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an “inset”, not an “insert”. The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
- Do not use the word “essentially” to mean “approximately” or “effectively”.
- In your paper title, if the words “that uses” can accurately replace the word “using”, capitalize the “u”; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones “affect” and “effect”, “complement” and “compliment”, “discreet” and “discrete”, “principal” and “principle”.
- Do not confuse “imply” and “infer”.
- The prefix “non” is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the “et” in the Latin abbreviation “et al.”.
- The abbreviation “i.e.” means “that is”, and the abbreviation “e.g.” means “for example”.

An excellent style manual for science writers is [7].

F. Authors and Affiliations

The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

G. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is “Heading 5”. Use “figure caption” for your Figure captions, and “table head” for your table title. Run-in heads, such as “Abstract”, will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

H. Figures and Tables

a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation “Fig. 1”, even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
copy	More table copy[a]		

[a]Sample of a Table footnote.

Fig. 1. Example of a figure caption.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity “Magnetization”, or “Magnetization, M”, not just “M”. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write “Magnetization (A/m)” or “Magnetization $\{A[m(1)]\}$”, not just “A/m”. Do not label axes with a ratio of quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

ACKNOWLEDGMENT

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks ...”. Instead, try “R. B. G. thanks...”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

REFERENCES

Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use "Ref. [3]" or "reference [3]" except at the beginning of a sentence: "Reference [3] was the first . . ."

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors' names; do not use "et al.". Papers that have not been published, even if they have been submitted for publication, should be cited as "unpublished" [4]. Papers that have been accepted for publication should be cited as "in press" [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

REFERENCES

[1] G. Eason, B. Noble, and I. N. Sneddon, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," Phil. Trans. Roy. Soc. London, vol. A247, pp. 529–551, April 1955.

[2] J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.

[3] I. S. Jacobs and C. P. Bean, "Fine particles, thin films and exchange anisotropy," in Magnetism, vol. III, G. T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271–350.

[4] K. Elissa, "Title of paper if known," unpublished.

[5] R. Nicole, "Title of paper with only first word capitalized," J. Name Stand. Abbrev., in press.

[6] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interface," IEEE Transl. J. Magn. Japan, vol. 2, pp. 740–741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].

[7] M. Young, The Technical Writer's Handbook. Mill Valley, CA: University Science, 1989.