# Formal Type Inferencer for System-F in Lean

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Abstract—or Math in Paper Title or Abstract. Index Terms-Lean, Type System, Type Theory, Verification

### I. INTRODUCTION

Type system is a common tool for verifying the correctness of a program.

## II. STLC WITH RECURSIVE FUNCTION

# A. Definition of Abstract Syntax, Type and Context

The syntax, type and context of simple typed lambda caculus(STLC) with recursive function is defined the same as in the project description. The only difference is the added @[derive decidable\_eq] notation. This is to tell lean that the comparison whether two types are equal is decidable. So we which is essential for the type checker to be a decidable algorithm.

#### B. Type Relations for STLC

Before implementing and formalizing the type inferencer for STLC, we need to implement the type relations for STLC so that we can type annotate an expression and judge the type inferencer afterwards.

The type relations for literal terms are straight forward, taking expression ETrue as an example:

$$\overline{\vdash ETrue : TBool} \tag{1}$$

In the corresponding lean code, this is represented as ' $True\_typed \ \Gamma : ctx : typed \ \Gamma \ exp.ETrue \ ty.TBool$ ' This is basically the same for IsZero, Pred and Succ:

$$\overline{\vdash EIsZero: TFun(TNat, TBool)} \tag{2}$$

$$\vdash EPred : TFun(TNat, TNat)$$
 (3)

$$\frac{x:T\in\Gamma}{\Gamma\vdash EVar(x):T}\tag{4}$$

For Var expression, we need to check its type in the given exvironment, then get the type of this expression.

$$\frac{\Gamma \vdash e1 : TFun(T1, T12) \quad \Gamma \vdash e2 : T1}{\Gamma \vdash EApp(e1, e2) : T_{12}} \tag{5}$$

Here we need to introduce the type of the two sub-expressions ||rec|| f(x : Nat) : Nat ->for for the application expression. We can and only can get 2 return lambda y: Nat: the type when the first expression has a function type while 3

the second expression has the type of the first parameter of that function.

$$\frac{\Gamma \vdash t : TBool \quad \Gamma \vdash thn : T \quad \Gamma \vdash els : T}{\Gamma \vdash EIf(t, thn, els) : T} \tag{6}$$

For if then else braches expression, we need to ensure the test expression has type bool, while both branches share same type as the whole expression.

The above cases all not include changing the context, while type relations for Lam and Rec, which both define a new function and introduce local variables, add elements to the context.

$$\frac{\Gamma, x: T1 \vdash b: T2}{\Gamma \vdash EAbs(x: T1, b): TFun(T1, T2)}$$
(7)  
$$\frac{\Gamma, x: T1, f: TFun(T1, T2) \vdash b: T2}{\Gamma \vdash ERec(f, x: T1, T2, b): TFun(T1, T2)}$$
(8)

The type relation for Rec is basically an extension of the Lam. It need to check the declared return type is same as the type of body with parameter and the functino itself with their types are added to the context.

```
RecTyped (G: ctx) (f: string)
   (x : string) (aa bb A: ty)
   (e : exp)
   (p1 : (ty.TFun aa bb) = A)
   (p2: typed (ctx.ctx_snoc
       (ctx.ctx_snoc G x aa)
       f (ty.TFun aa bb)) e bb)
    : typed G (exp. ERec f x aa bb e) A
```

Here is a code snippet for the type relation for recursive functions. A type A is introduced for all rules to represent the final type of the expression for all cases as shown in line 2, because it surprisingly simplifies the proof of the completeness of the type inference algorithm to only a simp term for all cases there.

# C. Some STLC with Recursive Examples

This section is for the exercise 3 in the project. Here we define a function that checks if two natural numbers are equal in STLC.

```
if x == 0:
```

```
if y == 0:
    return true
else:
    return false
else:
    if y == 0:
    return false

return false
else:
    return false
else:
    return (f(x - 1))(y - 1)
```

Because the language we defined only supports lambda with one parameter, to compare two natural numbers, we need the outer function to get a nat and return a function from nat to bool.

In the inner function, we substract both numbers by one and check if they reach zero at the same time.

The proof of:  $\vdash f: TFun(TNat, TFun(TNat, TBool))$ , is just a lot of  $apply\ typed\_sth$  and refl. These refls comes because of those type As introduced in typed.

# D. Maintaining the Integrity of the Specifications

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$$a + b = \gamma \tag{9}$$

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TABLE I TABLE TYPE STYLES

Table	Table Column Head		
Head	Table column subhead	Subhead	Subhead
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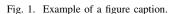


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