

(1) 線性生產函數

$$Q = \alpha L + \beta K \rightarrow Q = A + B$$

若兩倍功效  $Q = 2A + 2B$

$$|MRTS| = \frac{dK}{dL} = \frac{dB}{dA}$$



設  $Q = 10$

(3) Cobb-Douglas

$$Q = AL^\alpha K^\beta$$



$$|MRTS| \text{ 邊際替代率} = \frac{dK}{dL} = \frac{MPL}{MPK}$$

$$MPL = \frac{dQ}{dL} = \alpha A L^{\alpha-1} K^\beta$$

$$MPK = \frac{dQ}{dK} = \beta A L^\alpha K^{\beta-1}$$

$$|MRTS| = \frac{\alpha K}{\beta L}$$

①  $Q = \alpha L + \beta K$

$MPL =$  勞動 + 單位總產量變動

②  $Q = \min(\alpha L, \beta K)$

$$Q = AL^\alpha K^\beta$$

例  $Q = 21L + 9L^2 - L^3$

(1)  $MPL = -3L^2 + 18L + 21 \left( \frac{dQ}{dL} \right)$

$$\frac{dMPL}{dL} = 18 - 6L = 0 \quad \Rightarrow L = 18 \quad L = 3$$

(2) 令  $MPL = 0 \quad 0 = -3L^2 + 18L + 21$

$$0 = -L^2 + 6L + 7$$

$$0 = (L-1)(L+7)$$

$$L = 1 \text{ or } -7$$

$$APL = 21 + 9L - L^2 \quad L = 7$$

$$\frac{APL}{dL} = 9 - 2L = 0 \quad L = 4.5$$

(A) 孩子  $Q = 5A + 10B$

(B)  $8 = \min\{L/2, K\}$

(A) 完全替代 (線性)

$A/107 \approx 600/8 \times \frac{1}{3}$