CS201: Discrete Mathematics (Fall 2023)
Written Assignment #1
(100 points maximum but 105 points in total)
Deadline: 11:59pm on Oct 12 (please submit via Blackboard)
PLAGIARISM WILL BE PUNISHED SEVERELY

Q.1 (**5p**) Consider the following propositions:

p : You get an A on the final.

q : You do all the assignments.

r : You get an A in this course.

Translate the following statements to formulas using p, q, r and logical connectives.

(a) (**1p**) You get an A either in this course or on the final.

(b) (**1p**) To get an A in this course, it is necessary for you to do all the assignments.

(c) (**1p**) You do all the assignments, but you don't get an A on the final; nevertheless, you get an A in this course.

(d) (**1p**) If you don't get an A in this course, then you don't get an A on the final or don't do all the assignments.

(e) (**1p**) You get an A in this course if and only if you do all the assignments and get an A on the final.

Q.2 (**10p**) Construct a truth table for each of the following compound propositions:

(a) (**1p**) $p \oplus \neg p$

(b) (**2p**) $(p \to q) \land (\neg p \leftrightarrow q)$

(c) (**2p**) $(p \oplus q) \to (p \lor \neg q)$

(d) (**5p**) $(p \to \neg q) \leftrightarrow (r \to (p \lor \neg q))$

Q.3 (**15p**) Use logical equivalences to prove the following statements. Please write out the names of laws used at each step (see the lecture slides for examples).

(a) (**4p**) $\neg(p \to q) \to p$ is a tautology.

(b) (**3p**) $(p \land \neg q) \to r$ and $p \to (q \lor r)$ are equivalent.

(c) (**8p**) $(p \to q) \to ((r \to p) \to (r \to q))$ is a tautology.

Q.4 (**10p**) Determine whether or not the following pairs of statements are logically equivalent, and explain your answer. (Truth tables are not necessary if your explanation is clear.)

(a) (**2p**) $p \oplus q$ and $\neg p \lor \neg q$

(b) (**2p**) $\neg q \land (p \leftrightarrow q)$ and $\neg p$

(c) (**3p**) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

(d) (**3p**) $(p \to q) \to r$ and $p \to (q \to r)$

Q.5 (**5p**) Determine for which values of p, q, r the statement $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true and for which values it is false, and explain your reasoning. Do not use a truth table.

Q.6 (**5p**) Prove that if $p \to q$, $\neg p \to \neg r$, $s \vee r$, then $q \vee s$. Please write out the names of inference rules used at each step (see the lecture slides for examples).

Q.7 (**5p**) Prove that if $p \wedge q$, $q \to \neg(p \wedge r)$, $s \to r$, then $\neg s$. Please write out the names of inference rules used at each step (see the lecture slides for examples).

Q.8 (**5p**) Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++". Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at SUSTech.

(a) (**1p**) There is a student who either can speak Russian or knows C++.

(b) (**1p**) There is a student who can speak Russian but who doesn't know C++.

(c) (**1p**) Every student can speak Russian and knows C++.

(d) (**1p**) No student can speak Russian or knows C++.

(e) (**1p**) If a student can speak Russian then he/she does not know C++.

Q.9 (**8p**) Let $L(x, y)$ be the statement "x loves y", where the domain for both x and y consists of all people in the world. Translate the following statements to quantified formulas. (Hint: you can use $=, \neq$ to connect variables.)

(a) (**1p**) Everybody loves somebody.

(b) (**2p**) There is someone who loves only himself or herself but no other person.

(c) (**5p**) There are exactly two people whom Lynn loves.

Q.10 (**5p**) Express the negations of each of the following statements such that all negation symbols *immediately precede* predicates.

(a) (**1p**) $\exists z \forall y \forall x T(x, y, z)$

(b) (**2p**) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

(c) (**2p**) $\forall x \exists y (P(x, y) \to Q(x, y))$

Q.11 (**10p**) Consider this argument: "All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners." Answer the following questions.

(a) (**3p**) Define the predicates and translate each sentence to a quantified formula.

(b) (**7p**) Show the formal proof steps and explain which rule of inference is used for each step.

Q.12 (**5p**) Prove that $\sqrt[3]{2}$ is irrational.

Q.13 (**5p**) Prove that there is an irrational number between every two distinct rational numbers. (Hint: you can use the in-class learned fact that $\sqrt{2}$ is irrational.)

Q.14 (**12p**) Prove that all integral solutions to the equation

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{e} + \frac{1}{2}$$

such that $m, n \geq 3$ and $e > 0$ are in this table:

m	n	e
3	3	6
3	4	12
3	5	30
4	3	12
5	3	30

(Hint: Use proof by cases.)