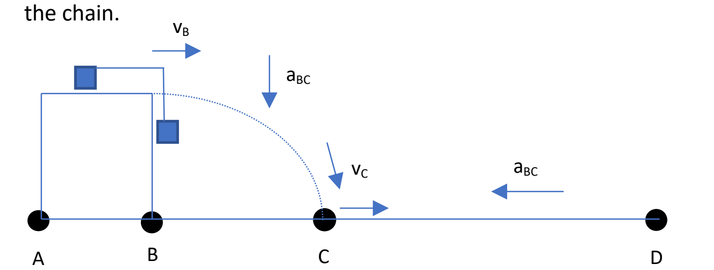
Uber Problem – "Jerky" Jerry's Jabberwocky Jumper (Calculus version)

Problem Description

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.



The problem is divided into three parts, described with points A, B, C, D:

A: Point at which jumper starts

B: Point at which jumper leaves the platform

C: Point at which jumper hits the ground

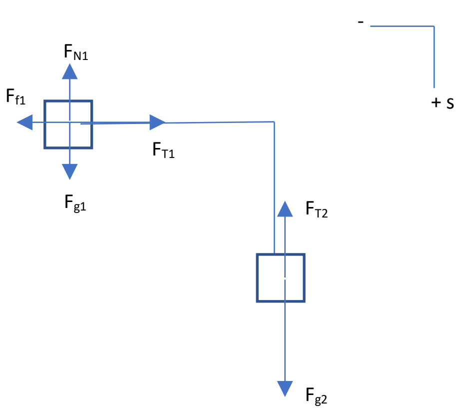
D: Point at which jumper slides to a stop

Givens:

Total mass of Jerry and Jumper	$m_1 = 76$	kg
Total mass of Barrel and rocks	$m_2 = 178$	kg
Total mass of Chain	$m_C = 58$	kg
Length of Chain (and Δx_{AB})	$L_C = 8$	m
Height of Platform (y_B)	$h = 17$	m
Coefficient of friction between jumper and platform	$\mu_P = 0.11$	
Total horizontal distance from platform to final location	$\Delta x_{BD} = 106$	m

Method and Strategy

Part AB:



Step 1: Find acceleration in terms of position

Using the system method, the sum of all the forces in the s direction is:

$$\Sigma F_s = F_{g2} - F_{f1} = m_s a_s$$

However, F_{f1} is currently unknown because it is equal to μF_N and F_N is not given. Therefore, to solve for that, the jumper system is isolated to get the sum of the forces in the y direction of the jumper only:

$$\Sigma F_y = F_N - F_g = m_1 a_y$$
$$F_N - (76)(9.8) = 0$$
$$\underline{F_N = 744.80\ N}$$

With that, the system method can be used to find acceleration in terms of position x where x is the length of the chain off of the platform:

$$\Sigma F_s = F_{g2} - \mu F_N = m_s a_s$$
$$(9.8)\left(178 + \left(\frac{x}{8}\right)(58)\right) - (0.11)(744.8) = 312 a_s$$
$$1744.4 + 71.05x - 81.928 = 312 a_s$$
$$\underline{a_s = 0.22772x + 5.3284}$$

Step 2: Find velocity in terms of position

Now, using the well known identity $a[x]dx = vdv$, velocity in terms of position can be derived:

$$a[x]dx = vdv$$
$$(0.2272x + 5.3284)dx = vdv$$
$$\int_0^x (0.22772x + 5.3284)\,dx = \int_0^v (v)\,dv$$
$$0.11386x^2 + 5.3284x = \frac{1}{2}v^2$$
$$\underline{v = \sqrt{0.22772x^2 + 10.657x}}$$

Step 3: Find v_B

With v[x], the velocity at point B can be determined if 8 m (the length of chain) is plugged in for x, given that the jumper starts at rest:

$$v_B = \sqrt{0.22772(8)^2 + 10.657(8)}$$
$$v_B = \sqrt{99.7968}$$
$$\underline{v_B = 9.9915\ m/s}$$

Part BC:

Step 1: Find t_{BC}

Part BC is purely algebraic, projectile motion. Using the v_B value derived from part and the given information, the following kinematic equation in the y-DIR can be applied to solve for t_{BC}

$$y_C = \frac{1}{2}a_{BCy}{t_{BC}}^2 + v_{By}t_{BC} + y_B$$
$$0 = -4.9{t_{BC}}^2 + 17$$
$$\underline{t_{BC} = 1.8626\ s}$$

Step 2: Find Δx_{CD}

With t_{BC}, Δx_{BC} can be derived using the same kinematic equation used in step 1, except applied in the x direction:

$$x_C = \frac{1}{2}a_{BCx}{t_{BC}}^2 + v_{Bx}t_{BC} + x_B$$
$$\Delta x_{BC} = 9.9915(1.8626)$$
$$\underline{\Delta x_{BC} = 18.610\ m}$$

Since Δx_{BD} is given as 106 m, Δx_{CD} can be derived with simple subtraction:

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$
$$\Delta x_{CD} = 106 - 18.61$$
$$\underline{\Delta x_{CD} = 87.390\ m}$$

Step 3: Find v_C, not componentized

The components of v_C can be easily determined from a different kinematic equation:

y-DIR:

$$v_{Cy} = v_{0y} + a_{BCy}t_{BC}$$
$$v_{Cy} = (-9.8)(1.8626)$$
$$\underline{v_{Cy} = -18.254\ m/s}$$

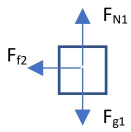
x-DIR:

$$v_{Cx} = v_{0x} + a_{BCx}t_{BC}$$
$$\underline{v_{Cx} = 9.9915\ m/s}$$

Now that the components of v_C are known, v_C can be solved for using the Pythagorean theorem:

$$v_C = \sqrt{{v_{Cx}}^2 + {v_{Cy}}^2}$$
$$v_C = \sqrt{(9.9915)^2 + (-18.254)^2}$$
$$\underline{v_C = 20.810\ m/s}$$

Part CD:



Step 1: Find a_{CD}

According to the problem, only 75% of the velocity is transitioned into the horizontal direction at C. So v_C, instead of being 20.810 m/s as derived in part BC, would actually be 0.75(20.810), which is approximately equal to 15.608 m/s. Now, with the correct v_C for part CD, the following kinematic equation can be used to find a_{CD}:

$${v_D}^2 = {v_C}^2 + 2a_{CD}\Delta x_{CD}$$
$$0 = (15.608)^2 + 2a_{CD}(87.390)$$
$$\underline{a_{CD} = -1.3937\ m/s^2}$$

Final Step: Find μ_G

For the last part, all that is necessary to find μ_G is to write out the sum of the forces in the x-direction:

$$\Sigma F_x: -F_{f2} = m_1 a_{CD}$$
$$-\mu F_{N1} = m_1 a_{CD}$$
$$-\mu(744.8) = (76)(-1.3937)$$
$$\boxed{\mu = 0.14222}$$