Data Science UW Methods for Data Analysis

More on Hypothesis Testing, The Central Limit Theorem,
And an introduction to Regression
Lecture 4
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'Excellent health statistics - smokers are less likely to die of age related illnesses.'

Topics

> Review

> More on hypothesis testing

> Central Limit Theorem

> Introduction to Regression

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Review

> Sampling Methods

> Law of Large Numbers

> Hypothesis Testing

– Normal testing

– One tailed vs Two tailed

– P-values

– T-test (Student's, Welch's)

– Chi-Squared

– Fisher's Exact

> Outliers

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Fisher's Exact Test

	Cat. 1	Cat. 2
Successes	A	B
Failures	C	D

> Hypothesis: (Assume we know the total # of successes)
  – Null: The proportion of successes in Category 1 is not less than that in Category 2.
  – Alternative: The proportion of successes in Category 1 is less than in Category 2.

We observe this outcome

| 0 | 5 | | 1 | 4 | | 2 | 3 | | 3 | 2 | | 4 | 1 | | 5 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 2 | | 4 | 3 | | 3 | 4 | | 2 | 5 | | 1 | 6 | | 0 | 7 |

Same outcome or worse, with the same marginals (row sums).

Hypothesis Testing Summary (so far)

> If data is normal,
>> – If you know population mean and variance,
>>> > Use standard normal 'z-test'.
>>
>> – If you just know population mean,
>>> > Use t-test (unpaired data).
>>> > Use Welch's t-test (paired data).

> For categorical comparison tests,
>> – If the sample/subgroup size is large enough,
>>> > Use Chi-squared test
>>
>> – If the sample/subgroup size is small,
>>> > Use Fisher's Exact test.

> How do we know the data is normal?

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Testing for Normality

> Kolmogorov-Smirnov test (K-S test).

– Tests if two distributions are similar.

> Consider the Normal Cumulative Distribution Function (CDF).

Normal Cumulative Probability
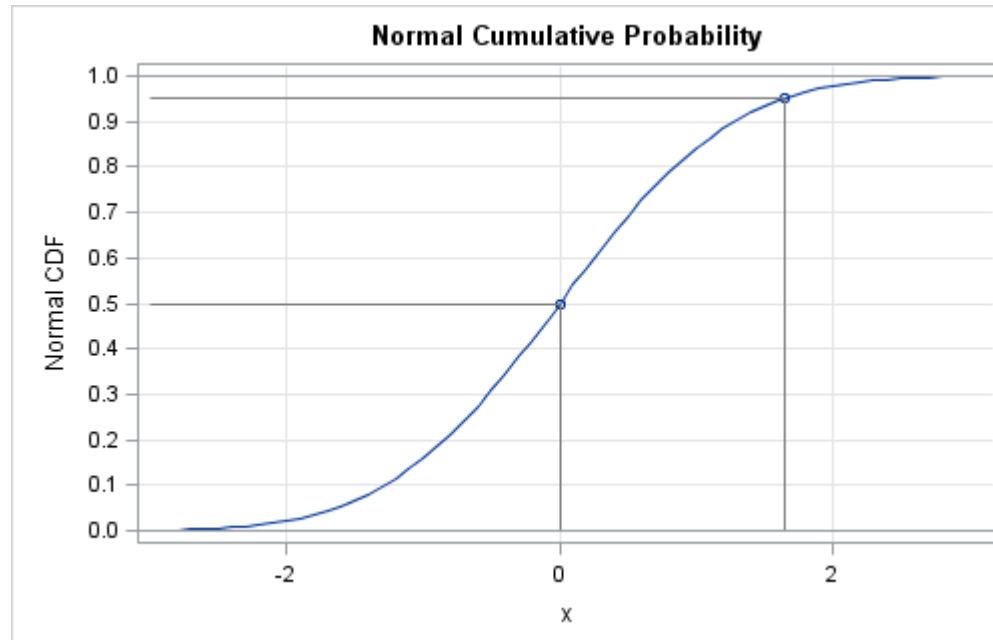
> Any similar distribution should have a similar CDF.

Testing for Normality

> The K-S statistic is just the maximum vertical distance between two CDFs.

> Note: the K-S test can test departure from any hypothetical distribution, not just normal.

> R-demo

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Testing for Normality

> Also, the Shapiro-Wilk test can tell us a test statistic for normality.

  – Tests the difference in expected and sample 'moments'.
  – Moments:
    > 1st moment = mean
    > 2nd moment = variance
    > 3rd moment = skewness
    > 4th moment = kurtosis
    > …
  – Slightly more powerful than the K-S test.

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Testing for Normality

> ALWAYS do a qq-plot to look at normality. (qqnorm())
> R-demo.

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Testing Between Multiple Groups

> What if we had multiple groups and we wanted to compare their means?

> Why can't we just do multiple two-sample t-tests for all pairs?

– Results in increased probability of accepting a false hypothesis.

– E.g., if we had 7 groups, there would be (7 Choose 2)=21 pairs to test. If our alpha cutoff is 5%, then we are likely to accept about 1 false hypothesis (21*0.05).

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Testing Between Multiple Groups

> Null Hypothesis:
  – All groups are just samples from the same population.

> Alternative Hypothesis:
  – At least one group has a statistically different mean.

> This type of analysis is called "ANalysis Of VAriants", or ANOVA.
  – We make data independence and normality assumptions first.
  – Our test statistic is based on:

$$statistic \sim \frac{between\ group\ variability}{within\ group\ variability}$$

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Performing Multiple Hypothesis Tests

> For non-ANOVA methods, remember that performing many hypothesis tests increases our risk of incorrectly rejecting a null-hypothesis.

> To compensate for this we decrease the p-value cutoff.

> The most common way of doing this is with the Bonferroni Correction.

$$p' = \frac{p}{(\# \, of \, Hypotheses)}$$

> This correction is argued to be too strong and other approximations for a new-p can be used instead.
  – Tukey's Range Test

> This is VERY important in genetics/bioinformatics.

> R Example
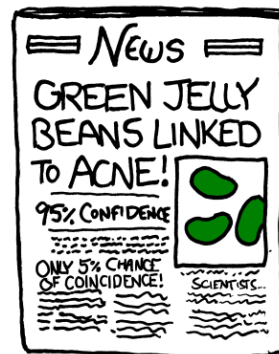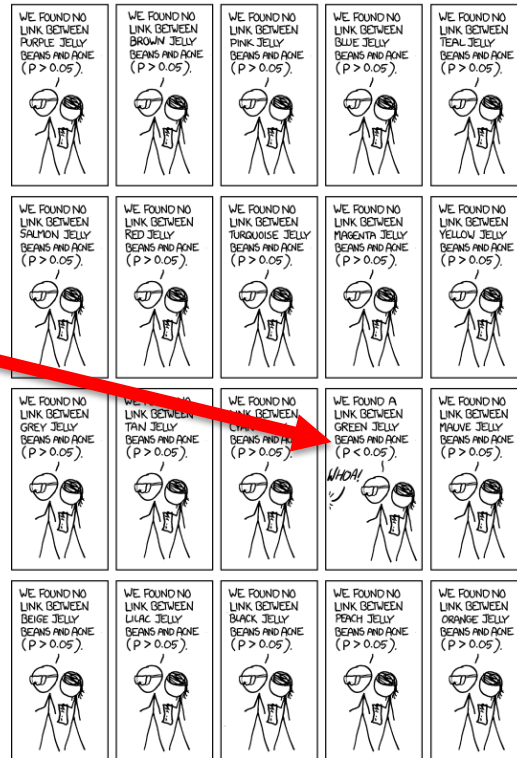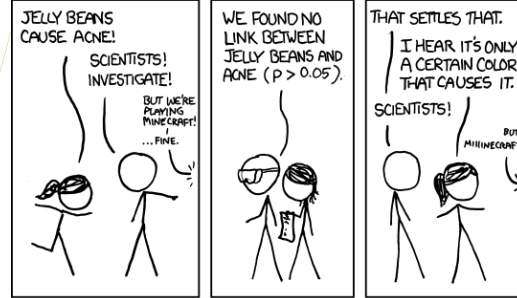
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Additional Hypothesis Tests

> Parametric test types:

- Mean comparison
- Variance comparison
- More distribution comparisons

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Central Limit Theorem

> If we sample a population over and over, the set of means of all samples are normally distributed, regardless of the population distribution.

$$\bar{X} \sim N\left(mean, \frac{variance}{\sqrt{n}}\right)$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

> Compare to Law of Large numbers ('proof' by R), shown in previous class.

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Central Limit Theorem

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

> We can use this central limit theorem to generate confidence intervals on expressing the population mean.

> We know the sample mean, sample variance, and number of samples.

> Then we know how our estimate of the population mean is distributed (from above formula).

> We can then generate 90%, 95%, … confidence intervals around our sample mean.

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Confidence Intervals

> Confidence intervals are a way to express uncertainty in *population* parameters, as estimated by the sample.

> E.g. If we create a 95% confidence interval for the population mean, say $\hat{\mu} = \bar{X} = 10 \pm 5$

– Then we say that the true population mean, μ, has a 95% chance of being between 5 and 15.

It is **<u>not</u>** correct to say:
- ~~"95% of the sample values are in this range."~~
- ~~"There is a 95% chance that the mean of another sample will be in this range."~~

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Confidence Intervals

> To create confidence intervals for population means, we use the central limit theorem and create confidence intervals based on the normal distribution.

- Repeatedly sample from the population.
- Calculate the mean for each sample.
- Use the average of the sample means as the population estimate and create a C.I. based on the s.d. of the sample means.
- R demo
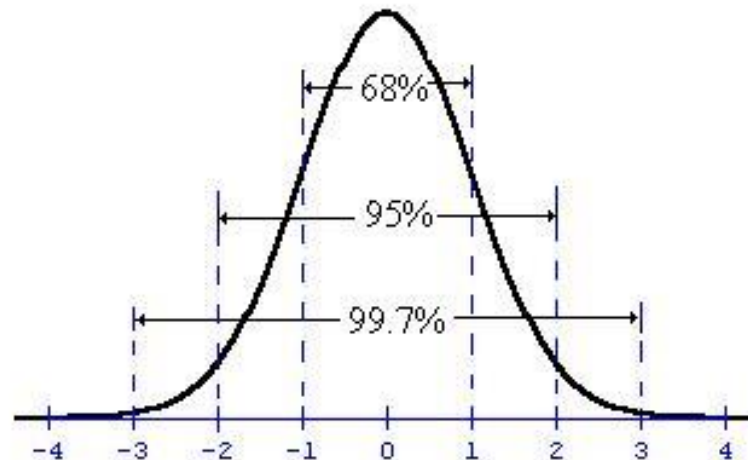
Regression Models

> The goal of regression is to produce a model that represents the best fit to some observed data.

> Typically the model is a function describing some type of curve (lines, parabolas, etc.) that is determined by a set of parameters (e.g., slope and intercept).

> "Best fit" means that there is an optimal set of parameters according to an evaluation criteria we choose.
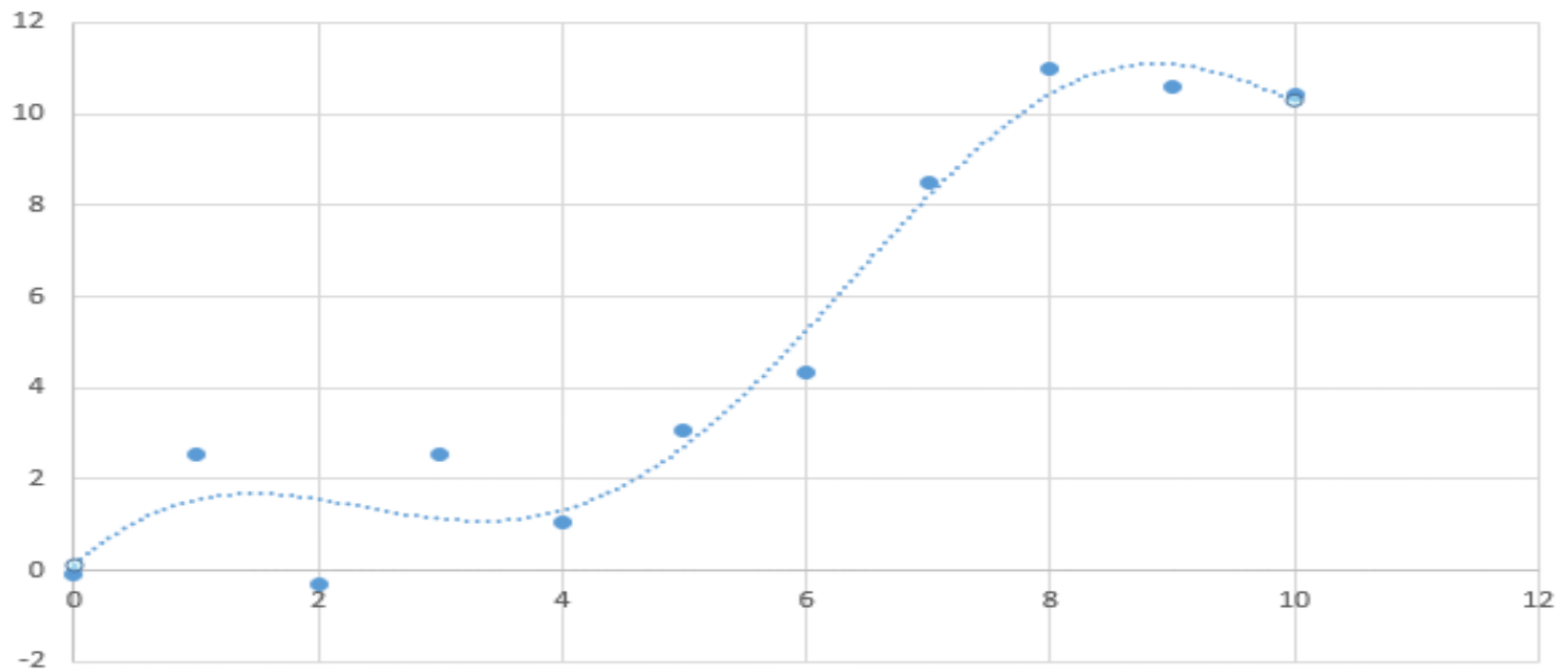
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Regression: Linear
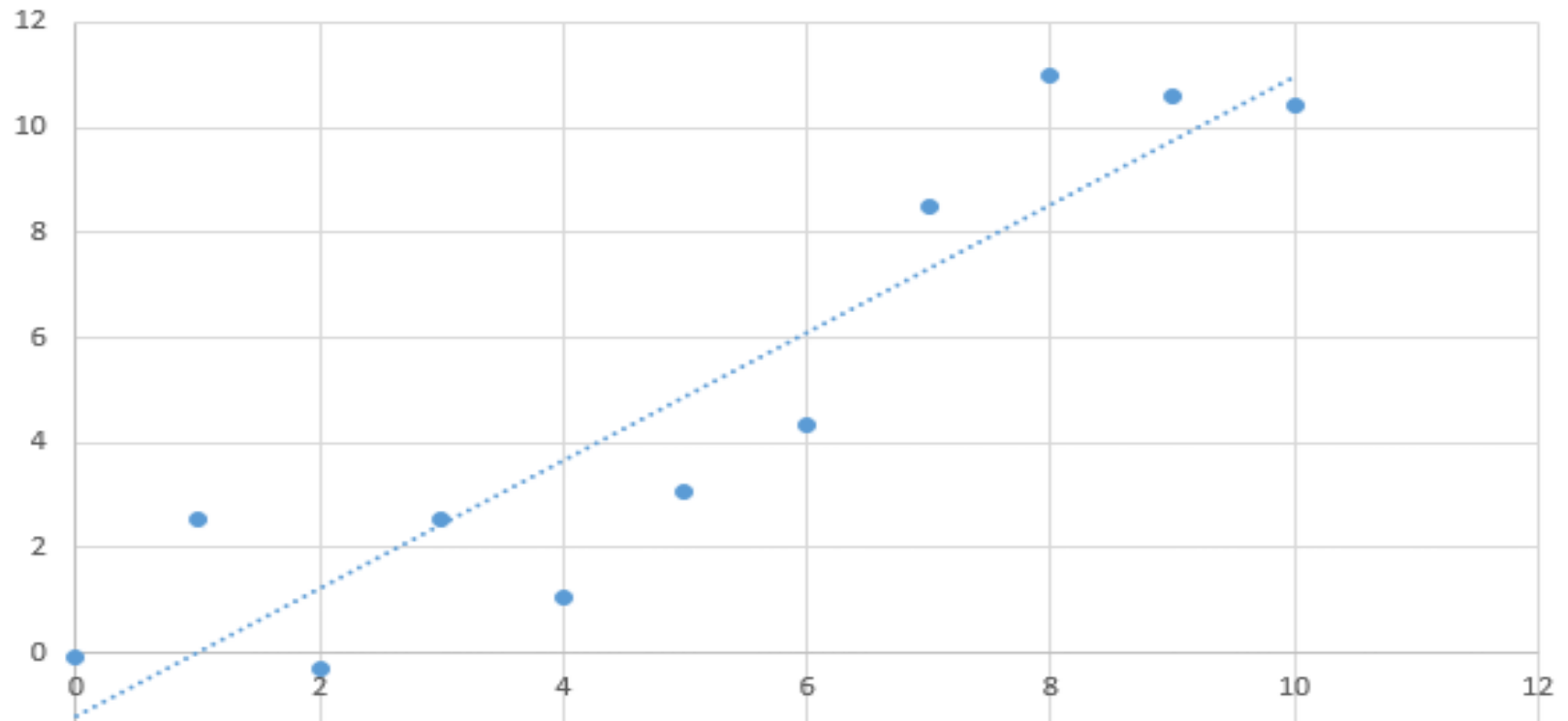
Regression: quadratic
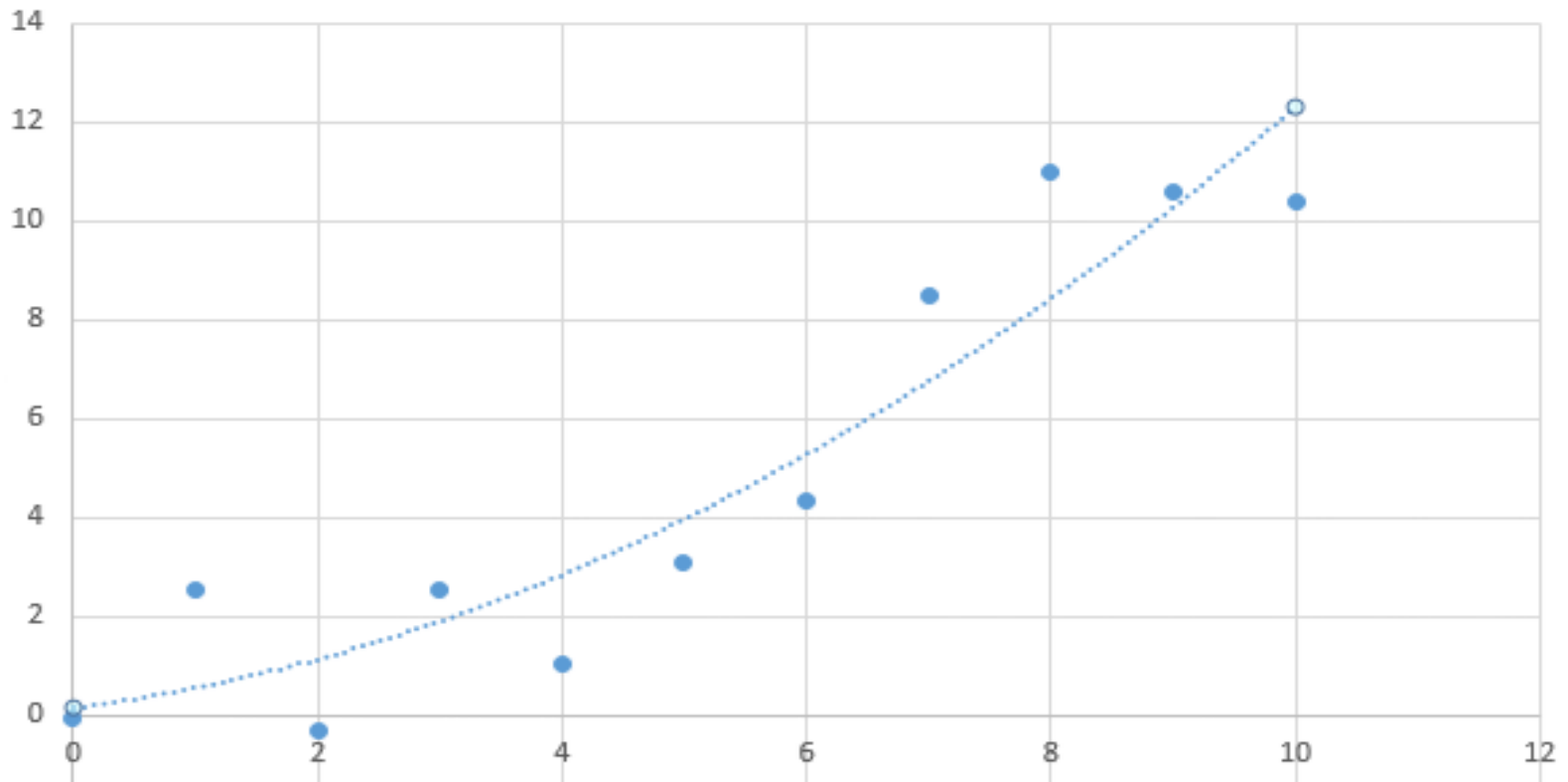
Regression: High Order

Regression Models

> Which one of the preceding examples is correct?

> In a sense, all of them are. They all give decent approximations to the data.

> It's hard to tell, just from looking at these plots whether any of them in fact will continue to perform well as more data is received.

> We don't know if these models will generalize

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Linear Regression

> <u>Response (Dependent) variable</u>: the variable of primary interest in a study- the one you are trying to predict or explain.

> <u>Explanatory (Independent) variable</u>: the variable that attempts to explain the observed outcomes of the response variable.

> There are two types of parameters in linear models:

– The intercept (y-intercept).

– The slope, rise over run, or change in Y divided by change in X

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Linear Regression

> When x = 0, then y = b.

> When x = -(b/m) then y = 0.
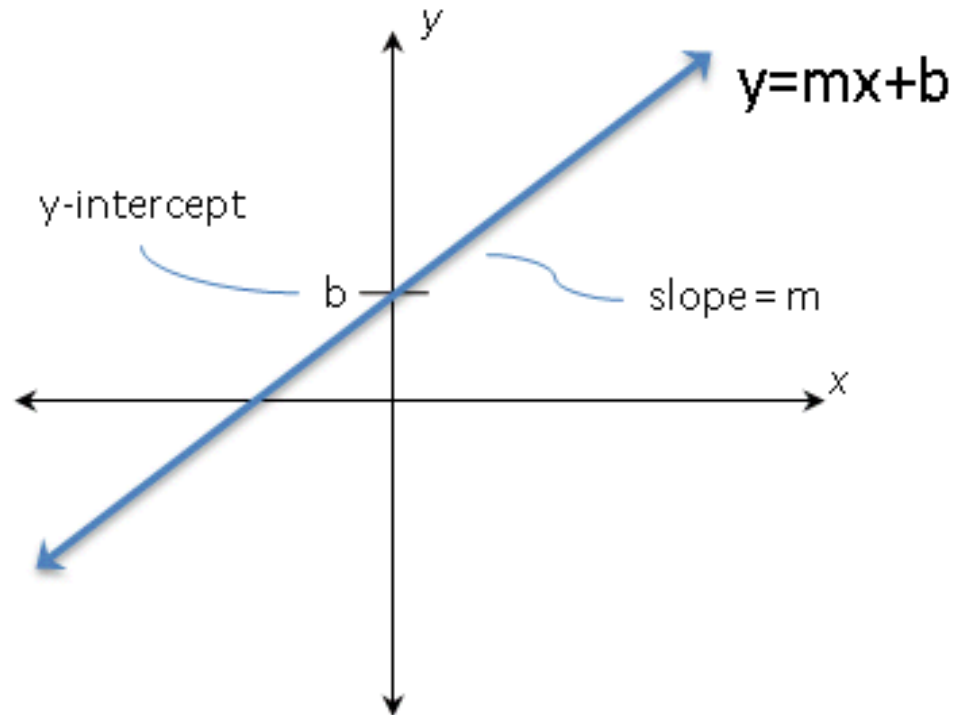
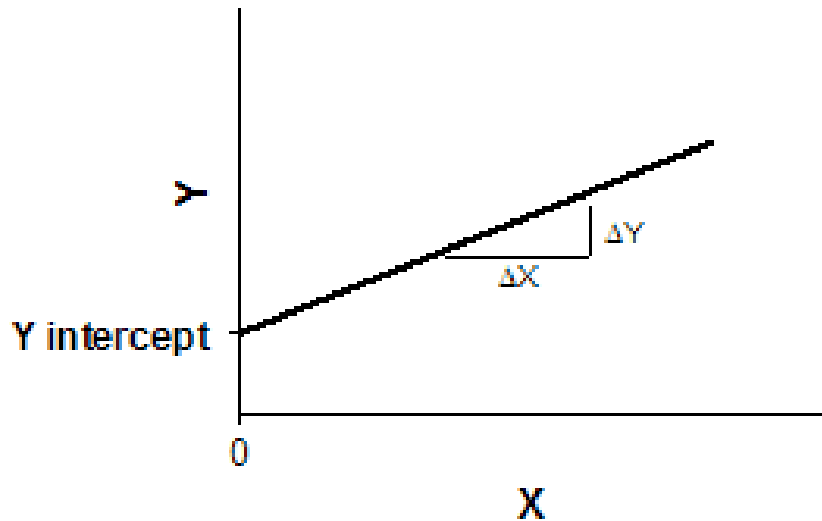# Linear Regression

> Interpret slope: $m = \dfrac{rise}{run} = \dfrac{\Delta y}{\Delta x}$

– If x changes by Δx, then y must change by Δy in order for the slope to stay the same (and it must).

Given two points, (x1,y1), (x2,y2)
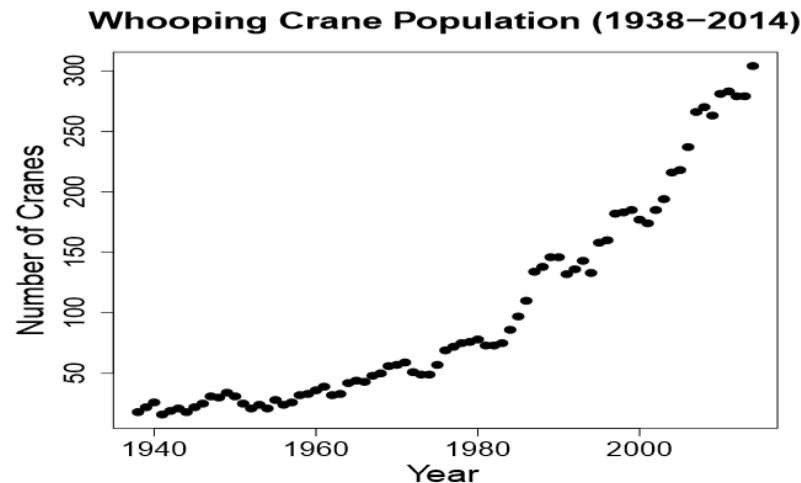
$$m = \frac{(y2 - y1)}{(x2 - x1)} = \frac{(y1 - y2)}{(x1 - x2)}$$

R example!

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Linear Regression

> Consider the relationship between Whooping Crane population and year below.
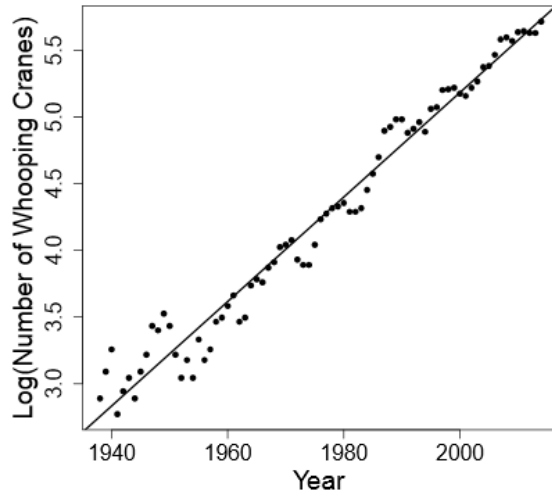

Whooping Crane Population (1938–2014)

> Possible regression solutions:
  – Transform the response variable, to linearize the relationship.
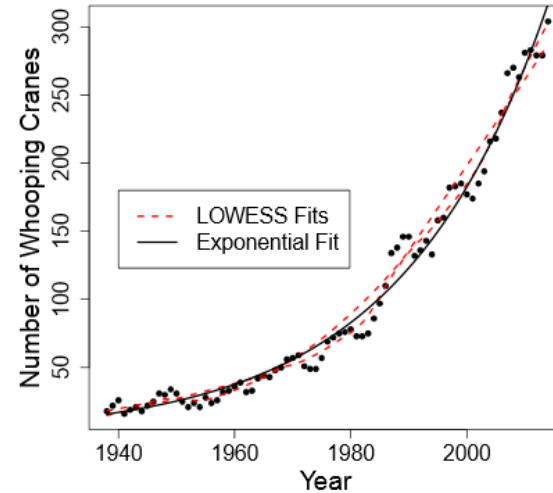 – Fit a nonlinear regression model to the data.

Linear Regression

> How would we decide on a 'best' model?

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Linear Regression

> We use the method of least squares to find the best fit line: $y_i = mx_i + b + \varepsilon_i$

$$\min_{m,b} \sum_{i=1}^{n} (\varepsilon_i)^2 = \min_{a,b} \sum_{i=1}^{n} (y_i - (mx_i + b))^2$$

> Explicit solutions exist (using calculus).

> Computers are really good at finding minimums of equations. We let them do this for us.

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Linear Regression

> The method of least squares finds the best fit line.

– The mean of the errors from the best fit line is zero.

– This means there is no 'bias' in our prediction.

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Linear Regression

> Linear regression is the most common.
  – Fit a line (2D), plane (3D), or a hyperplane to the observed data.

> We need to define an error metric for a line through points.
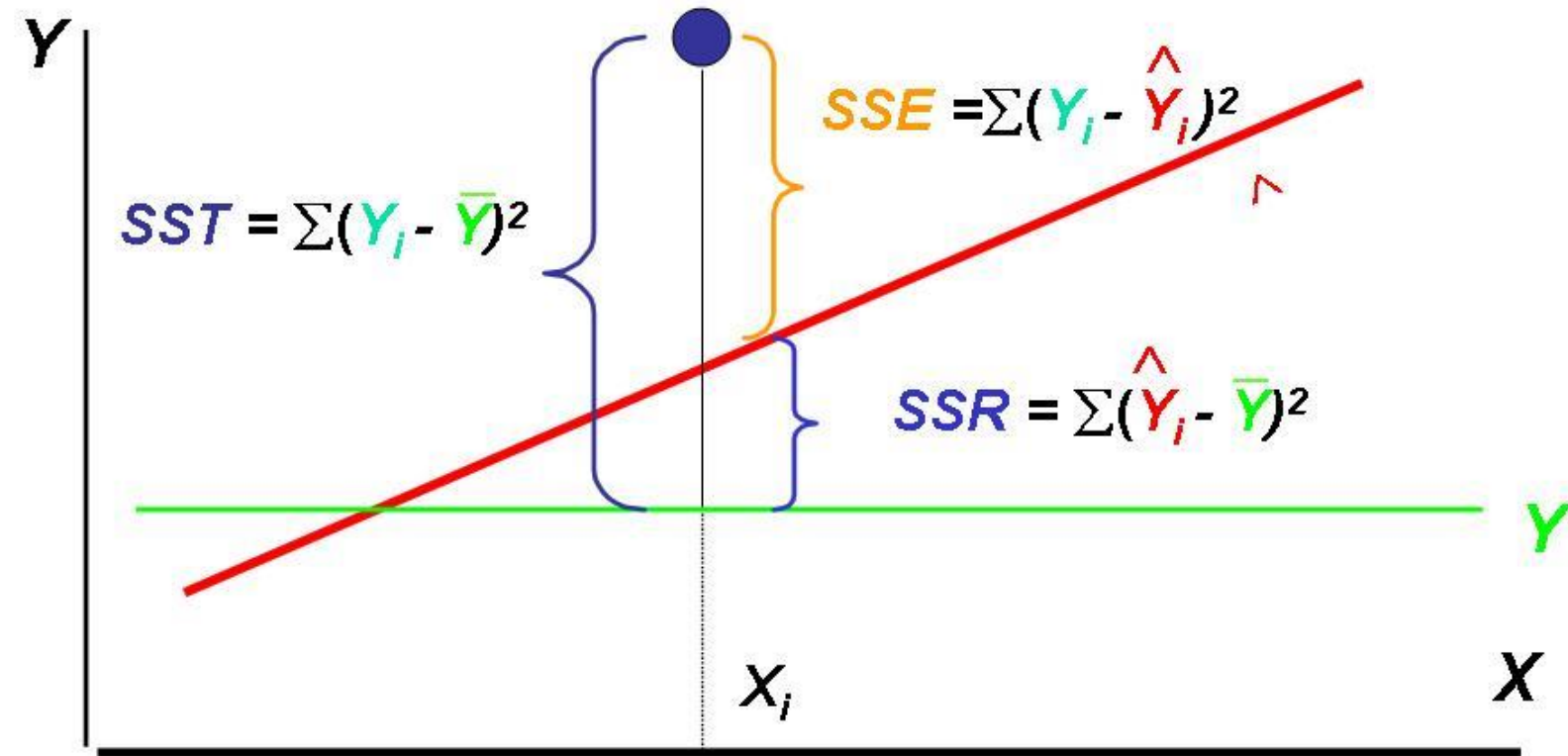

Least Squares Linear Regression
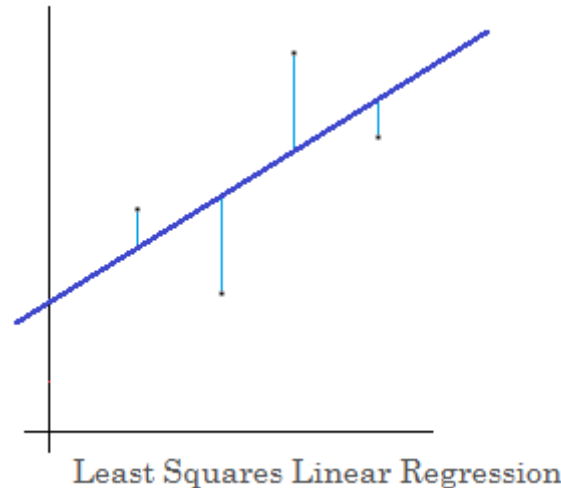
> We use the sum of the squared error between the predicted and actual.

> R demo.

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Linear Regression

> With modeling, we are interested in the SSE, SSR, SST.

$$SSE = \sum (Y_i - \hat{Y}_i)^2$$

$$SST = \sum (Y_i - \bar{Y})^2$$

$$SSR = \sum (\hat{Y}_i - \bar{Y})^2$$
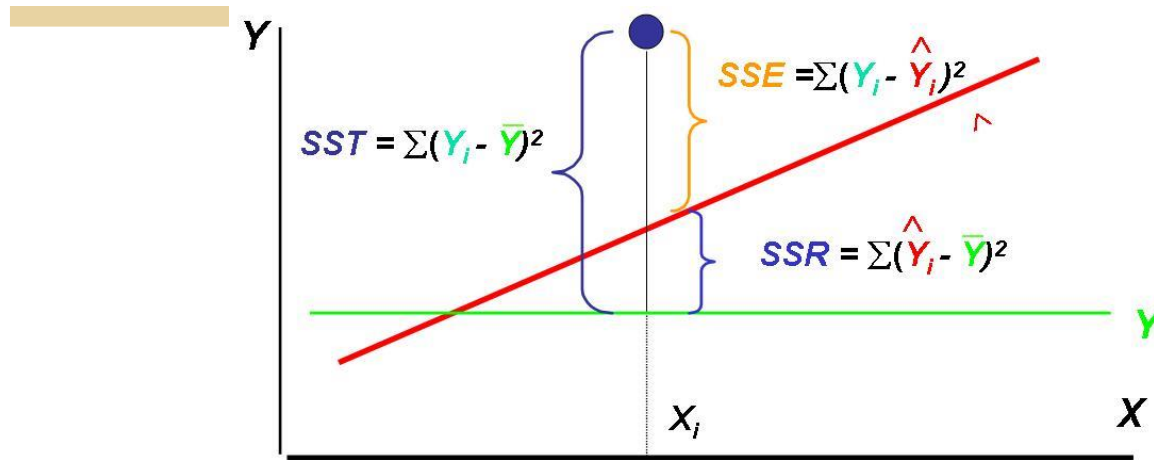
Linear Regression

> R-squared is called the coefficient of determination.

> It indicates how well the data fits a specified model.

> For linear models, we define this as:

$$R^2 = 1 - \frac{SSE}{SST} = \frac{SSR}{SST}$$

Linear Regression

> We can also measure accuracy of the line using Root Mean Squared Error (RMSE).

– Using this as an estimate of the error means we are losing one more degree of freedom than the standard deviation, so we write the RMSE as

$$RMSE = \frac{SSE}{n-2}$$

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Assignment

> Complete Homework 4:
  – Fit a linear model to the Chicago Diabetes Hospital data as follows:
    > Transform the data to sum across all zip codes.
    > Then plot and fit a line to the following:
      – Num. Hospitalizations vs. Crude Admittance Rate
      – Δ Num. Hospitalizations vs. Δ Crude Admittance Rate
    > Summarize your finding of each model.
      – Be sure to interpret the slope for each model (think of the units).
  – You should submit:
    > Just one R-script.
    > Read Statistical Thinking for Programmers Chapters 6 and 7.
  – Email me a summary of your final project idea.
    > Include data sources