MAT 450 Finite Sampling
Final Exam

Name: _____

Show all steps to get full credits.

1. (35 points) A firm with 36 factories decides to check the condition of some equipment of which $M_0 = 25012$ pieces are in use. A random sample of 5 factories is taken, 10% of the samples are radomly checked in each selected factory. The number of pieces checked (m_i) and the numbers found with signs of deterioration (a_i) are as follows.

Factory	m_i	a_i
1	65	8
2	82	21
3	52	4
4	91	12
5	62	1

a) Estimate the percentage of defective pieces in use. (5 points)

b) Construct the 95% C.I. for the percentage of defective pieces in use. (5 points)

c) Construct the 95% C.I. for the percentage of defective pieces in use based on Jacknife. (5 points)

d) Now assume that the numbers found with signs of deterioration a_i's are SRS. Construct the 95% C.I. for the percentage of defective pieces in use. (5 points)

e) Compare the C.I.s in b), c) and d), and explain your findings. (5 points)

f) Estimate the median number of defective pieces in use. (5 points)

g) Construct 95% C.I. for the median number of defective pieces in use. (5 points)

2. (30 points) The following table gives population values for a small population of clusters. Now, you wish to select two psus with probabilities of inclusion proportional to M_i.

psu	M_i	y_{ij}	t_i
1	5	3,5,4,6,2	20
2	4	7,4,7,7	25
3	8	7,2,9,4,5,3,2,6	38
4	5	2,5,3,6,8	24

a) Suppose we have selected psus $\{2, 3\}$ by Lahiri's method, and subsample observations from in each psu rather than all the observations.. Now estimate the proportion of observations with values less or equal to 5, and construct 95% C.I. for the proportion. (5 points)

psu	M_i	y_{ij}
2	4	7,4
3	8	7,2,9,4

b) What is the probability of selecting psus $\{2, 3\}$ for the without replacement? (5 points)

c) What is the probability that psu 3 is selected for the without replacement? (5 points)

d) Construct 95% C.I. for the population mean using SYS estimation method based on psus $\{2, 3\}$ if applicable (5 points).

e) Now, construct 95% C.I. for the population mean using with-replacement variance (5 points).

f) Compare your confidence intervals in d), e), and explain your findings. What about the design effect of with and without- replacement sampling? (5 points)