4.35 (a) No, because some values are 0.

(b) When the entire population is sampled, $\hat{t}_x = t_x$ and $\hat{t}_y = t_y$, so $\hat{B} = t_y/t_x$ and $t_x\hat{B} = t_y$.

(c) Answers will vary.

(d) Letting Z_i be the indicator variable for inclusion in the sample,

$$\begin{aligned} E[\bar{b} - B] &= E\left[\frac{1}{n}\sum_{i=1}^{N} Z_i b_i - \frac{t_y}{t_x}\right] \\ &= \frac{1}{N}\sum_{i=1}^{N} b_i - \frac{\sum_{i=1}^{N} b_i x_i}{t_x} \\ &= \frac{1}{N}\frac{(\sum_{i=1}^{N} b_i)(\sum_{j=1}^{N} x_i)}{t_x} - \frac{\sum_{i=1}^{N} b_i x_i}{t_x} \\ &= -\frac{1}{t_x}\left[\sum_{i=1}^{N} b_i x_i - N\bar{b}_U\bar{x}_u\right] \\ &= -\frac{(N-1)S_{bx}}{t_x}. \end{aligned}$$

(e) From linear models theory, if $\mathbf{Y} = \mathbf{X}\beta + \varepsilon$, with $E[\varepsilon] = 0$ and $\text{Cov}[\varepsilon] = \sigma^2\mathbf{A}$, then the weighted least squares estimator of β is

$$\hat{\beta} = (X^T\mathbf{A}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{A}^{-1}\mathbf{Y}$$

with

$$V[\hat{\beta}] = \sigma^2(X^T\mathbf{A}^{-1}\mathbf{X})^{-1}.$$

Here, $\mathbf{A} = \text{diag}(x_i^2)$, so

$$\mathbf{X}^T\mathbf{A}^{-1}\mathbf{X} = \sum_{i\in\mathcal{S}} x_i\frac{1}{x_i^2}x_i = n$$

and

$$\mathbf{X}^T\mathbf{A}^{-1}\mathbf{Y} = \sum_{i\in\mathcal{S}} x_i\frac{1}{x_i^2}y_i = \sum_{i\in\mathcal{S}} Y_i/x_i.$$

4.42

(a)

```
proc surveymeans  data = vius mean sum clm clsum;
   weight tabtrucks;
   strata stratum;
   var miles_annl;
   domain business;
run;
```

Business in which vehicle was most often used during 2002	Variable	Mean	Std Error of Mean
For-hire tra	MILES_ANNL	56452	1246.317720
Vehicle leas	MILES_ANNL	23306	790.981413
Agriculture,	MILES_ANNL	10768	415.312729
Mining	MILES_ANNL	19210	1126.305405
Utilities	MILES_ANNL	15081	830.112607
Construction	MILES_ANNL	16714	381.173223
Manufacturin	MILES_ANNL	19650	1018.600046
Wholesale tr	MILES_ANNL	23052	1184.715817
Retail trade	MILES_ANNL	17948	561.147469
Information	MILES_ANNL	14927	1396.653144
Waste manage	MILES_ANNL	14410	726.842687
Arts, entert	MILES_ANNL	9536.588165	1267.485898
Accommodatio	MILES_ANNL	20461	1300.857231
Other servic	MILES_ANNL	16818	600.239019

Domain Analysis: Business in which vehicle was most often used

Business in which vehicle was most often used during 2002	Variable	95% CL for Mean		Sum
For-hire tra	MILES_ANNL	54009.5907	58895.1519	72272793289
Vehicle leas	MILES_ANNL	21755.6067	24856.2511	20024589014
Agriculture,	MILES_ANNL	9954.2886	11582.3131	24119946651
Mining	MILES_ANNL	17002.3551	21417.4684	3411543277
Utilities	MILES_ANNL	13454.3175	16708.3561	10244675655
Construction	MILES_ANNL	15966.8537	17461.0515	75906142636
Manufacturin	MILES_ANNL	17653.3448	21646.2535	15384530602
Wholesale tr	MILES_ANNL	20729.7496	25373.8316	16963450921
Retail trade	MILES_ANNL	16848.3582	19048.0543	27470445448
Information	MILES_ANNL	12189.9160	17664.7915	5622014452
Waste manage	MILES_ANNL	12985.5076	15834.7285	10709275945
Arts, entert	MILES_ANNL	7052.3180	12020.8583	1784083855
Accommodatio	MILES_ANNL	17911.2857	23010.6416	5816313888
Other servic	MILES_ANNL	15641.1955	17994.1304	35776203775

Business in which vehicle was most often used during 2002	Variable	Std Dev	95% CL for Sum	
For-hire tra	MILES_ANNL	1608230919	6.91207E10	7.54249E10
Vehicle leas	MILES_ANNL	1213307392	1.76465E10	2.24027E10
Agriculture,	MILES_ANNL	1354386330	2.14654E10	2.67745E10
Mining	MILES_ANNL	360265917	2705422697	4117663856
Utilities	MILES_ANNL	942933274	8396528057	1.20928E10
Construction	MILES_ANNL	2821651145	7.03757E10	8.14366E10
Manufacturin	MILES_ANNL	1399406209	1.26417E10	1.81274E10
Wholesale tr	MILES_ANNL	1348917090	1.43196E10	1.96073E10
Retail trade	MILES_ANNL	1422261638	2.46828E10	3.02581E10
Information	MILES_ANNL	923917751	3811137245	7432891659
Waste manage	MILES_ANNL	901989658	8941377763	1.24772E10
Arts, entert	MILES_ANNL	353650310	1090929855	2477237856
Accommodatio	MILES_ANNL	677802928	4487821313	7144806463
Other servic	MILES_ANNL	2201296141	3.14617E10	4.00907E10