

# Data Structures

Abstraction and Design Using Java

Third Edition

ELLIOT B. KOFFMAN AND PAUL A. T. WOLFGANG





# **DATA STRUCTURES**

Abstraction and Design  
Using Java

**THIRD EDITION**

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# Preface

Our goal in writing this book was to combine a strong emphasis on problem solving and software design with the study of data structures. To this end, we discuss applications of each data structure to motivate its study. After providing the specification (interface) and the implementation (a Java class), we then cover case studies that use the data structure to solve a significant problem. Examples include maintaining an ordered list, evaluating arithmetic expressions using a stack, finding the shortest path through a maze, and Huffman coding using a binary tree and a priority queue. In the implementation of each data structure and in the solutions of the case studies, we reinforce the message “Think, then code” by performing a thorough analysis of the problem and then carefully designing a solution (using pseudo-code and UML class diagrams) before the implementation. We also provide a performance analysis when appropriate. Readers gain an understanding of why different data structures are needed, the applications they are suited for, and the advantages and disadvantages of their possible implementations.

## Intended Audience

This book was written for anyone with a curiosity or need to know about data structures, those essential elements of good programs and reliable software. We hope that the text will be useful to readers with either professional or educational interests.

It is intended as a textbook for the second programming course in a computing curriculum involving the study of data structures, especially one that emphasizes Object-Oriented Design (OOD). The text could also be used in a more-advanced course in algorithms and data structures. Besides coverage of the basic data structures and algorithms (lists, stacks, queues, trees, recursion, sorting), there are chapters on sets and maps, balanced binary search trees, graphs, and an online appendix on event-oriented programming. Although we expect that most readers will have completed a first programming course in Java, there is an extensive review chapter (included as an appendix) for those who may have taken a first programming course in a different language, or for those who need a refresher in Java.

## Emphasis on the Java Collections Framework

The book focuses on the interfaces and classes in the Java Collections Framework. We begin the study of a new data structure by specifying an abstract data type as an interface, which we adapt from the Java API. Readers are encouraged throughout the text to use the Java Collections Framework as a resource for their programming.

Our expectation is that readers who complete this book will be familiar with the data structures available in the Java Collections Framework and will be able to use them in their future programming. However, we also expect that they will want to know how the data structures are implemented, so we provide thorough discussions of classes that implement these data structures. Each class follows the approach taken by the Java designers where appropriate. However, when their industrial-strength solutions appear to be too complicated for beginners to understand, we have provided simpler implementations but have tried to be faithful to their approach.

## Think, then Code

To help you “Think, then code” we discuss problem solving and introduce appropriate software design tools throughout the textbook. For example, Chapter 1 focuses on OOD and Class Hierarchies. It introduces the Uniform Modeling Language (also covered in Appendix B) to document an OOD. It introduces the use of interfaces to specify abstract data types and to facilitate contract programming and describes how to document classes using Javadoc-style comments. There is also coverage of exceptions and exception handling. Chapter 2 introduces the Java Collections Framework and focuses on the List interface, and it shows how to use big-O notation to analyze program efficiency. In Chapter 3, we cover different testing strategies in some detail including a discussion of test-driven design and the use of the JUnit program to facilitate testing.

## Features of the Third Edition

We had two major goals for the third edition. The first was to bring the coverage of Java up to Java 8 by introducing new features of Java where appropriate. For example, we use the Java 7 diamond operator when creating new `Collection` objects. We use the Java 8 `StringJoiner` in place of the older `StringBuilder` for joining strings.

A rather significant change was to introduce Java 8 lambda expressions and functional interfaces as a way to facilitate functional programming in Java in a new Section 6.4. Using these features significantly improved the code.

The second major goal was to provide additional emphasis on testing and debugging. To facilitate this, we moved our discussion of testing and debugging from an appendix to Chapter 3 and expanded our coverage of testing including more discussion of JUnit. We also added a new section that introduced test-driven development.

A third goal was to ease the transition to Java for Python programmers. When introducing Java data structures (for example, arrays, lists, sets, and maps), we compared them to equivalent Python data structures.

Other changes to the text included reorganizing the chapter on lists and moving the discussion of algorithm analysis to the beginning of the chapter so that big-O notation could be used to compare the efficiency of different List implementations. We also combined the chapters on stacks and queues and increased our emphasis on using Deque as an alternative to the legacy Stack class. We also added a discussion of Timsort, which is used in Java 8, to the chapter on sorting algorithms. Finally, some large case studies and an appendix were moved to online supplements.

## Case Studies

We illustrate OOD principles in the design and implementation of new data structures and in the solution of approximately 20 case studies. Case studies follow a five-step process (problem specification, analysis, design, implementation, and testing). As is done in industry, we sometimes perform these steps in an iterative fashion rather than in strict sequence. Several case studies have extensive discussions of testing and include methods that automate the testing process. Some case studies are revisited in later chapters, and solutions involving different data structures are compared. We also provide additional case studies on the Web site for the textbook ([www.wiley.com/college/koffman](http://www.wiley.com/college/koffman)), including one that illustrates a solution to the same problem using several different data structures.

## Prerequisites

Our expectation is that the reader will be familiar with the Java primitive data types including `int`, `boolean`, `char`, and `double`; control structures including `if`, `case`, `while`, `for`, and `try-catch`; the `String` class; the one-dimensional array; input/output using either `JOptionPane` dialog windows or text streams (class `Scanner` or `BufferedReader`) and console input/output. For those readers who lack some of the concepts or who need some review, we provide complete coverage of these topics in Appendix A. Although labeled an Appendix, the review chapter provides full coverage of the background topics and has all the pedagogical features (discussed below) of the other chapters. We expect most readers will have some experience with Java programming, but someone who knows another programming language should be able to undertake the book after careful study of the review chapter. We do not require prior knowledge of inheritance, wrapper classes, or `ArrayLists` as we cover them in the book (Chapters 1 and 2).

## Pedagogy

The book contains the following pedagogical features to assist inexperienced programmers in learning the material.

- **Learning objectives** at the beginning of each chapter tell readers what skills they should develop.
- **Introductions** for each chapter help set the stage for what the chapter will cover and tie the chapter contents to other material that they have learned.
- **Case Studies** emphasize problem solving and provide complete and detailed solutions to real-world problems using the data structures studied in the chapter.
- **Chapter Summaries** review the contents of the chapter.
- **Boxed Features** emphasize and call attention to material designed to help readers become better programmers.



**Pitfall** boxes help readers identify common problems and how to avoid them.



**Design Concept** boxes illuminate programming design decisions and trade-offs.



**Programming Style** boxes discuss program features that illustrate good programming style and provide tips for writing clear and effective code.



**Syntax** boxes are a quick reference for the Java structures being introduced.

- **Self-Check and Programming Exercises** at the end of each section provide immediate feedback and practice for readers as they work through the chapter.
- **Quick-Check, Review Exercises, and Programming Projects** at the end of each chapter review chapter concepts and give readers a variety of skill-building activities, including longer projects that integrate chapter concepts as they exercise the use of data structures.

## Theoretical Rigor

In Chapter 2, we discuss algorithm efficiency and big-O notation as a measure of algorithm efficiency. We have tried to strike a balance between pure “hand waving” and extreme rigor when determining the efficiency of algorithms. Rather than provide several paragraphs of

formulas, we have provided simplified derivations of algorithm efficiency using big-O notation. We feel this will give readers an appreciation of the performance of various algorithms and methods and the process one follows to determine algorithm efficiency without bogging them down in unnecessary detail.

## Overview of the book

Chapter 1 introduces Object Oriented Programming, inheritance, and class hierarchies including interfaces and abstract classes. We also introduce UML class diagrams and Javadoc-style documentation. The `Exception` class hierarchy is studied as an example of a Java class hierarchy.

Chapter 2 introduces the Java Collections Framework as the foundation for the traditional data structures. These are covered in separate chapters: lists (Chapter 2), stacks, queues and deques (Chapter 4), Trees (Chapters 6 and 9), Sets and Maps (Chapter 7), and Graphs (Chapter 10). Each new data structure is introduced as an abstract data type (ADT). We provide a specification of each ADT in the form of a Java interface. Next, we implement the data structure as a class that implements the interface. Finally, we study applications of the data structure by solving sample problems and case studies.

Chapter 3 covers different aspects of testing (e.g. top-down, bottom-up, white-box, black-box). It includes a section on developing a JUnit test harness and also a section on Test-Driven Development. It also discusses using a debugger to help find and correct errors.

Chapter 4 discusses stacks, queues, and deques. Several applications of these data structures are provided.

Chapter 5 covers recursion so that readers are prepared for the study of trees, a recursive data structure. This chapter could be studied earlier. There is an optional section on list processing applications of recursion that may be skipped if the chapter is covered earlier.

Chapter 6 discusses binary trees, including binary search trees, heaps, priority queues, and Huffman trees. It also shows how Java 8 lambda expressions and functional interfaces support functional programming.

Chapter 7 covers the Set and Map interfaces. It also discusses hashing and hash tables and shows how a hash table can be used in an implementation of these interfaces. Building an index for a file and Huffman Tree encoding and decoding are two case studies covered in this chapter.

Chapter 8 covers various sorting algorithms including mergesort, heapsort, quicksort and Timsort.

Chapter 9 covers self-balancing search trees, focusing on algorithms for manipulating them. Included are AVL and Red-Black trees, 2-3 trees, 2-3-4 trees, B-trees, and skip-lists.

Chapter 10 covers graphs. We provide several well-known algorithms for graphs, including Dijkstra's shortest path algorithm and Prim's minimal spanning tree algorithm. In most programs, the last few chapters would be covered in a second course in algorithms and data structures.

## Supplements and Companion Web Sites

The following supplementary materials are available on the Instructor's Companion Web Site for this textbook at [www.wiley.com/college/koffman](http://www.wiley.com/college/koffman). Items marked for students are accessible on the Student Companion Web Site at the same address.

- Additional homework problems with solutions
- Additional case studies, including one that illustrates a solution to the same problem using several different data structures
- Source code for all classes in the book (for students and instructors)
- PowerPoint slides
- Electronic test bank for instructors
- Solutions to end-of-section odd-numbered self-check and programming exercises (for students)
- Solutions to all exercises for instructors
- Solutions to chapter-review exercises for instructors
- Sample programming project solutions for instructors
- Additional homework and laboratory projects, including cases studies and solutions

## Acknowledgments

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# *Object-Oriented Programming and Class Hierarchies*

## Chapter Objectives

- ◆ To learn about interfaces and their role in Java
- ◆ To understand inheritance and how it facilitates code reuse
- ◆ To understand how Java determines which method to execute when there are multiple methods with the same name in a class hierarchy
- ◆ To become familiar with the Exception hierarchy and the difference between checked and unchecked exceptions
- ◆ To learn how to define and use abstract classes as base classes in a hierarchy
- ◆ To learn the role of abstract data types and how to specify them using interfaces
- ◆ To study class `Object` and its methods and to learn how to override them
- ◆ To become familiar with a class hierarchy for shapes
- ◆ To understand how to create packages and to learn more about visibility

This chapter describes important features of Java that support Object-Oriented Programming (OOP). Object-oriented languages allow you to build and exploit hierarchies of classes in order to write code that may be more easily reused in new applications. You will learn how to extend an existing Java class to define a new class that inherits all the attributes of the original, as well as having additional attributes of its own. Because there may be many versions of the same method in a class hierarchy, we show how polymorphism enables Java to determine which version to execute at any given time.

We introduce interfaces and abstract classes and describe their relationship with each other and with actual classes. We introduce the abstract class `Number`. We also discuss class `Object`, which all classes extend, and we describe several of its methods that may be used in classes you create.

As an example of a class hierarchy and OOP, we describe the Exception class hierarchy and explain that the Java Virtual Machine (JVM) creates an Exception object whenever an error occurs during program execution. Finally, you will learn how to create packages in Java and about the different kinds of visibility for instance variables (data fields) and methods.

## Inheritance and Class Hierarchies

- 1.1 ADTs, Interfaces, and the Java API
  - 1.2 Introduction to Object-Oriented Programming
  - 1.3 Method Overriding, Method Overloading, and Polymorphism
  - 1.4 Abstract Classes
  - 1.5 Class Object and Casting
  - 1.6 A Java Inheritance Example—The Exception Class Hierarchy
  - 1.7 Packages and Visibility
  - 1.8 A Shape Class Hierarchy
- Case Study:* Processing Geometric Figures

## 1.1 ADTs, Interfaces, and the Java API

In earlier programming courses, you learned how to write individual classes consisting of attributes and methods (operations). You also learned how to use existing classes (e.g., `String` and `Scanner`) to facilitate your programming. These classes are part of the Java Application Programming Interface (API).

One of our goals is to write code that can be reused in many different applications. One way to make code reusable is to encapsulate the data elements together with the methods that operate on that data. A new program can then use the methods to manipulate an object's data without being concerned about details of the data representation or the method implementations. The encapsulated data together with its methods is called an abstract data type (ADT).

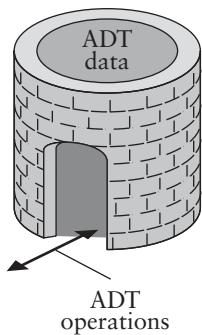
Figure 1.1 shows a diagram of an ADT. The data values stored in the ADT are hidden inside the circular wall. The bricks around this wall are used to indicate that these data values cannot be accessed except by going through the ADT's methods.

A class provides one way to implement an ADT in Java. If the data fields are private, they can be accessed only through public methods. Therefore, the methods control access to the data and determine the manner in which the data is manipulated.

Another goal of this text is to show you how to write and use ADTs in programming. As you progress through this book, you will create a large collection of ADT implementations (classes) in your own program library. You will also learn about ADTs that are available for you to use through the Java API.

Our principal focus will be on ADTs that are used for structuring data to enable you to more easily and efficiently store, organize, and process information. These ADTs are often called *data structures*. We introduce the Java Collections Framework (part of the Java API), which provides implementation of these common data structures, in Chapter 2 and study it throughout the text. Using the classes that are in the Java Collections Framework will make it much easier for you to design and implement new application programs.

**FIGURE 1.1**  
Diagram of an ADT



### Interfaces

A Java interface is a way to specify or describe an ADT to an applications programmer. An interface is like a contract that tells the applications programmer precisely what methods are available and describes the operations they perform. It also tells the applications programmer

what arguments, if any, must be passed to each method and what result the method will return. Of course, in order to make use of these methods, someone else must have written a class that *implements the interface* by providing the code for these methods.

The interface tells the coder precisely what methods must be written, but it does not provide a detailed algorithm or prescription for how to write them. The coder must “program to the interface,” which means he or she must develop the methods described in the interface without variation. If each coder does this job well, that ensures that other programmers can use the completed class exactly as it is written, without needing to know the details of how it was coded.

There may be more than one way to implement the methods; hence, several classes may implement the interface, but each must satisfy the contract. One class may be more efficient than the others at performing certain kinds of operations (e.g., retrieving information from a database), so that class will be used if retrieval operations are more likely in a particular application. The important point is that the particular implementation that is used will not affect other classes that interact with it because every implementation satisfies the contract.

Besides providing the complete definition (implementation) of all methods declared in the interface, each implementer of an interface may declare data fields and define other methods not in the interface, including constructors. An interface cannot contain constructors because it cannot be instantiated—that is, one cannot create objects, or instances, of it. However, it can be represented by instances of classes that implement it.

**EXAMPLE 1.1** An automated teller machine (ATM) enables a user to perform certain banking operations from a remote location. It must support the following operations.

1. Verify a user’s Personal Identification Number (PIN).
2. Allow the user to choose a particular account.
3. Withdraw a specified amount of money.
4. Display the result of an operation.
5. Display an account balance.

A class that implements an ATM must provide a method for each operation. We can write this requirement as the interface ATM and save it in file ATM.java, shown in Listing 1.1. The keyword `interface` on the header line indicates that an interface is being declared. If you are unfamiliar with the documentation style shown in this listing, read about Java documentation at the end of Section A.7 in Appendix A.

.....  
**LISTING 1.1**

Interface ATM.java

```
/** The interface for an ATM. */
public interface ATM {

    /** Verifies a user's PIN.
        @param pin The user's PIN
        @return Whether or not the User's PIN is verified
    */
    boolean verifyPIN(String pin);

    /** Allows the user to select an account.
        @return a String representing the account selected
    */
}
```

```

String selectAccount();

/** Withdraws a specified amount of money
    @param account The account from which the money comes
    @param amount The amount of money withdrawn
    @return Whether or not the operation is successful
*/
boolean withdraw(String account, double amount);

/** Displays the result of an operation
    @param account The account for the operation
    @param amount The amount of money
    @param success Whether or not the operation was successful
*/
void display(String account, double amount, boolean success);

/** Displays the result of a PIN verification
    @param pin The user's pin
    @param success Whether or not the PIN was valid
*/
void display(String pin, boolean success);

/** Displays an account balance
    @param account The account selected
*/
void showBalance(String account);
}

```

The interface definition shows the heading only for several methods. Because only the headings are shown, they are considered *abstract methods*. Each actual method with its body must be defined in a class that implements the interface. Therefore, a class that implements this interface must provide a void method called `verifyPIN` with an argument of type `String`. There are also two display methods with different signatures. The first is used to display the result of a withdrawal, and the second is used to display the result of a PIN verification. The keywords `public abstract` are optional (and usually omitted) in an interface because all interface methods are public abstract by default.

---



## SYNTAX Interface Definition

### FORM:

```

public interface interfaceName {
    abstract method declarations
    constant declarations
}

```

### EXAMPLE:

```

public interface Payable {
    public abstract double calcSalary();
    public abstract boolean salaried();
    public static final double DEDUCTIONS = 25.5;
}

```

### MEANING:

Interface `interfaceName` is defined. The interface body provides headings for abstract methods and constant declarations. Each abstract method must be defined in a class

that implements the interface. Constants defined in the interface (e.g., DEDUCTIONS) are accessible in classes that implement the interface or in the same way as static fields and methods in classes (see Section A.4).

#### NOTES:

The keywords `public` and `abstract` are implicit in each abstract method declaration, and the keywords `public static final` are implicit in each constant declaration. We show them in the example here, but we will omit them from now on.

Java 8 also allows for static and default methods in interfaces. They are used to add features to existing classes and interfaces while minimizing the impact on existing programs. We will discuss default and static methods when describing where they are used in the API.

## The `implements` Clause

The class headings for two classes that implement interface ATM are

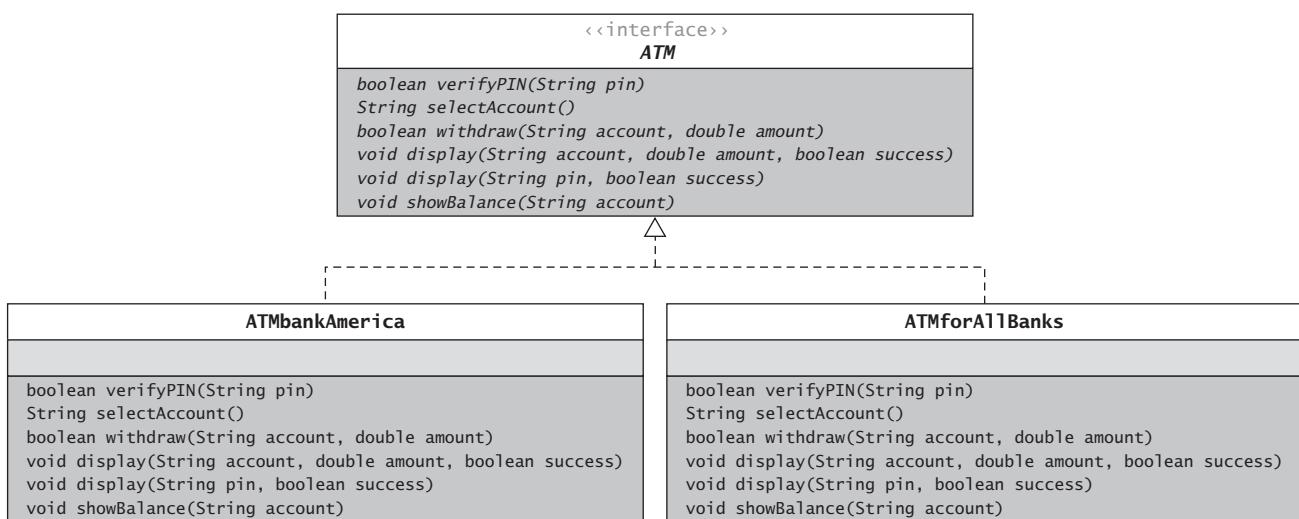
```
public class ATMBankAmerica implements ATM
public class ATMforAllBanks implements ATM
```

Each class heading ends with the clause `implements ATM`. When compiling these classes, the Java compiler will verify that they define the required methods in the way specified by the interface. If a class implements more than one interface, list them all after `implements`, with commas as separators.

Figure 1.2 is a UML (Unified Modeling Language) diagram that shows the ATM interface and these two implementing classes. Note that a dashed line from the class to the interface is used to indicate that the class implements the interface. We will use UML diagrams throughout this text to show relationships between classes and interfaces. Appendix B provides detailed coverage of UML diagrams.

**FIGURE 1.2**

UML Diagram Showing the ATM Interface and Its Implementing Classes





## PITFALL

### Not Properly Defining a Method to Be Implemented

If you neglect to define method `verifyPIN` in class `ATMforAllBanks` or if you use a different method signature, you will get the following syntax error:

```
class ATMforAllBanks should be declared abstract; it does not define method
verifyPIN(String) in interface ATM.
```

The above error indicates that the method `verifyPin` was not properly defined. Because it contains an abstract method that is not defined, Java incorrectly believes that `ATM` should be declared to be an abstract class. If you use a result type other than `boolean`, you will also get a syntax error.



## PITFALL

### Instantiating an Interface

An interface is not a class, so you cannot instantiate an interface. The statement

```
ATM anATM = new ATM(); // invalid statement
```

will cause the following syntax error:

```
interface ATM is abstract; cannot be instantiated.
```

## Declaring a Variable of an Interface Type

In the previous programming pitfall, we mentioned that you cannot instantiate an interface. However, you may want to declare a variable that has an interface type and use it to reference an actual object. This is permitted if the variable references an object of a class type that implements the interface. After the following statements execute, variable `ATM1` references an `ATMbankAmerica` object, and variable `ATM2` references an `ATMforAllBanks` object, but both `ATM1` and `ATM2` are type `ATM`.

```
ATM ATM1 = new ATMbankAmerica(); // valid statement
ATM ATM2 = new ATMforAllBanks(); // valid statement
```

## EXERCISES FOR SECTION 1.1

### SELF-CHECK

- What are the two parts of an ADT? Which part is accessible to a user and which is not? Explain the relationships between an ADT and a class, between an ADT and an interface, and between an interface and classes that implement the interface.
- Correct each of the following statements that is incorrect, assuming that class `PDGUI` and class `PDConsoleUI` implement interface `PDUUserInterface`.
  - `PDGUI p1 = new PDConsoleUI();`
  - `PDGUI p2 = new PDUUserInterface();`

```

c. PDUserInterface p3 = new PDUserInterface();
d. PDUserInterface p4 = new PDConsoleUI();
e. PDGUI p5 = new PDUserInterface();
    PDUserInterface p6 = p5;
f. PDUserInterface p7;
    p7 = new PDConsoleUI();

```

3. Explain how an interface is like a contract.
4. What are two different uses of the term *interface* in programming?

### PROGRAMMING

1. Define an interface named Resizable with just one abstract method, `resize`, that is a void method with no parameter.
2. Write a Javadoc comment for the following method of a class Person. Assume that class Person has two String data fields `familyName` and `givenName` with the obvious meanings. Provide preconditions and postconditions if needed.

```

public int compareTo(Person per) {
    if (familyName.compareTo(per.familyName) == 0)
        return givenName.compareTo(per.givenName);
    else
        return familyName.compareTo(per.familyName);
}

```

3. Write a Javadoc comment for the following method of class Person. Provide preconditions and postconditions if needed.

```

public void changeFamilyName(boolean justMarried, String newFamily) {
    if (justMarried)
        familyName = newFamily;
}

```

4. Write method `verifyPIN` for class ATMbankAmerica assuming this class has a data field `pin` (type `String`).



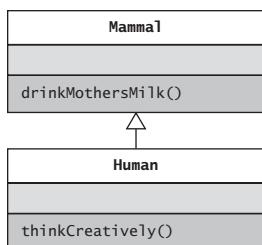
## 1.2 Introduction to Object-Oriented Programming (OOP)

In this course, you will learn to use features of Java that facilitate the practice of OOP. A major reason for the popularity of OOP is that it enables programmers to reuse previously written code saved as classes, reducing the time required to code new applications. Because previously written code has already been tested and debugged, the new applications should also be more reliable and therefore easier to test and debug.

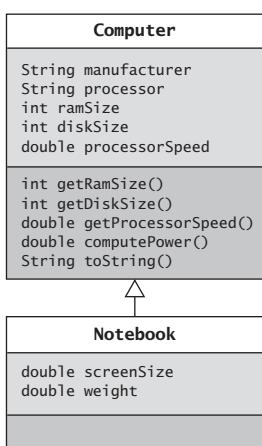
However, OOP provides additional capabilities beyond the reuse of existing classes. If an application needs a new class that is similar to an existing class but not exactly the same, the programmer can create it by extending, or inheriting from, the existing class. The new class (called the subclass) can have additional data fields and methods for increased functionality. Its objects also inherit the data fields and methods of the original class (called the superclass).

Inheritance in OOP is analogous to inheritance in humans. We all inherit genetic traits from our parents. If we are fortunate, we may even have some earlier ancestors who have left us

**FIGURE 1.3**  
Classes Mammal and Human



**FIGURE 1.4**  
Classes NoteBook and Computer



an inheritance of monetary value. As we grow up, we benefit from our ancestors' resources, knowledge, and experiences, but our experiences will not affect how our parents or ancestors developed. Although we have two parents to inherit from, Java classes can have only one parent.

Inheritance and hierarchical organization allow you to capture the idea that one thing may be a refinement or an extension of another. For example, an object that is a `Human` is a `Mammal` (the superclass of `Human`). This means that an object of type `Human` has all the data fields and methods defined by class `Mammal` (e.g., method `drinkMothersMilk()`), but it may also have more data fields and methods that are not contained in class `Mammal` (e.g., method `thinkCreatively()`). Figure 1.3 shows this simple hierarchy. The solid line in the UML class diagram shows that `Human` is a subclass of `Mammal`, and, therefore, `Human` objects can use methods `drinkMothersMilk` and `thinkCreatively`. Objects farther down the hierarchy are more complex and less general than those farther up. For this reason an object that is a `Human` is a `Mammal`, but the converse is not true because every `Mammal` object does not necessarily have the additional properties of a `Human`. Although this seems counterintuitive, the subclass `Human` is actually more powerful than the superclass `Mammal` because it may have additional attributes that are not present in the superclass.

## A Superclass and Subclass Example

To illustrate the concepts of inheritance and class hierarchies, let's consider a simple case of two classes: `Computer` and `Notebook`. A `Computer` object has a `manufacturer`, `processor`, `RAM`, and `disk`. A notebook computer is a kind of computer, so it has all the properties of a computer plus some additional features (`screen size` and `weight`). There may be other subclasses, such as tablet computer or game computer, but we will ignore them for now. We can define class `Notebook` as a subclass of class `Computer`. Figure 1.4 shows the class hierarchy.

### Class Computer

Listing 1.2 shows class `Computer.java`. It is defined like any other class. It contains a constructor, several accessors, a `toString` method, and a method `computePower`, which returns the product of its RAM size and processor speed as a simple measure of its power.

#### LISTING 1.2

Class `Computer.java`

```

/** Class that represents a computer. */
public class Computer {
    // Data Fields
    private String manufacturer;
    private String processor;
    private double ramSize;
    private int diskSize;
    private double processorSpeed;

    // Methods
    /** Initializes a Computer object with all properties specified.
     * @param man The computer manufacturer
     * @param processor The processor type
     * @param ram The RAM size
     * @param disk The disk size
     * @param procSpeed The processor speed
     */
    public Computer(String man, String processor, double ram,
                    int disk, double procSpeed) {


```

```

        manufacturer = man;
this.processor = processor;
        ramSize = ram;
        diskSize = disk;
        processorSpeed = procSpeed;
    }

    public double computePower() { return ramSize * processorSpeed; }
    public double getRamSize() { return ramSize; }
    public double getProcessorSpeed() { return processorSpeed; }
    public int getDiskSize() { return diskSize; }
    // Insert other accessor and modifier methods here.

    public String toString() {
        String result = "Manufacturer: " + manufacturer +
            "\nCPU: " + processor +
            "\nRAM: " + ramSize + " gigabytes" +
            "\nDisk: " + diskSize + " gigabytes" +
            "\nProcessor speed: " + processorSpeed + " gigahertz";
        return result;
    }
}

```

## Use of this.

In the constructor for the Computer class, the statement

```
this.processor = processor;
```

sets data field processor in the object under construction to reference the same string as parameter processor. The prefix `this.` makes data field processor visible in the constructor. This is necessary because the declaration of processor as a parameter hides the data field declaration.



### PITFALL

#### Not Using `this.` to Access a Hidden Data Field

If you write the preceding statement as

```
processor = processor; // Copy parameter processor to itself.
```

you will not get an error, but the data field `processor` in the `Computer` object under construction will not be initialized and will retain its default value (`null`). If you later attempt to use data field `processor`, you may get an error or just an unexpected result. Some IDEs will provide a warning if `this.` is omitted.

## Class Notebook

In the `Notebook` class diagram in Figure 1.4, we show just the data fields declared in class `Notebook`; however, `Notebook` objects also have the data fields that are inherited from class `Computer` (`processor`, `ramSize`, and so forth). The first line in class `Notebook` (Listing 1.3),

```
public class Notebook extends Computer {
```

indicates that class `Notebook` extends class `Computer` and inherits its data and methods. Next, we define any additional data fields

```
// Data Fields
private double screenSize;
private double weight;
```

## Initializing Data Fields in a Subclass

The constructor for class `Notebook` must begin by initializing the four data fields inherited from class `Computer`. Because those data fields are private to the superclass, Java requires that they be initialized by a superclass constructor. Therefore, a superclass constructor must be invoked as the first statement in the constructor body using a statement such as

```
super(man, proc, ram, disk, procSpeed);
```

This statement invokes the superclass constructor with the signature `Computer(String, String, double, int, double)`, passing the four arguments listed to the constructor. (A method signature consists of the method's name followed by its parameter types.) The following constructor for `Notebook` also initializes the data fields that are not inherited. Listing 1.3 shows class `Notebook`.

```
public Notebook(String man, String proc, double ram, int disk,
                double procSpeed, double screen, double wei) {
    super(man, proc, ram, disk, procSpeed);
    screenSize = screen;
    weight = wei;
}
```



### SYNTAX `super( . . . );`

#### FORM:

```
super();
super(argumentList);
```

#### EXAMPLE:

```
super(man, proc, ram, disk, procSpeed);
```

#### MEANING:

The `super()` call in a class constructor invokes the superclass's constructor that has the corresponding `argumentList`. The superclass constructor initializes the inherited data fields as specified by its `argumentList`. The `super()` call must be the first statement in a constructor.

### LISTING 1.3

Class `Notebook`

```
/** Class that represents a notebook computer. */
public class Notebook extends Computer {
    // Data Fields
    private double screenSize;
    private double weight;

    // Methods
    /** Initializes a Notebook object with all properties specified.
     * @param man The computer manufacturer
     * @param proc The processor type
     * @param ram The RAM size
```

```

@param disk The disk size
@param procSpeed The processor speed
@param screen The screen size
@param wei The weight
*/
public Notebook(String man, String proc, double ram, int disk,
                double procSpeed, double screen, double wei) {
    super(man, proc, ram, disk, procSpeed);
    screenSize = screen;
    weight = wei;
}
}

```

## The No-Parameter Constructor

If the execution of any constructor in a subclass does not invoke a superclass constructor, Java automatically invokes the no-parameter constructor for the superclass. Java does this to initialize that part of the object inherited from the superclass before the subclass starts to initialize its part of the object. Otherwise, the part of the object that is inherited would remain uninitialized.



### PITFALL

#### Not Defining the No-Parameter Constructor

If no constructors are defined for a class, the no-parameter constructor for that class will be provided by default. However, if any constructors are defined, the no-parameter constructor must also be defined explicitly if it needs to be invoked. Java does not provide it automatically because it may make no sense to create a new object of that type without providing initial data field values. (It was not defined in class `Notebook` or `Computer` because we want the client to specify some information about a `Computer` object when that object is created.) If the no-parameter constructor is defined in a subclass but is not defined in the superclass, you will get a syntax error `constructor not defined`. You can also get this error if a subclass constructor does not explicitly call a superclass constructor. There will be an implicit call to the no-parameter superclass constructor, so it must be defined.

## Protected Visibility for Superclass Data Fields

The data fields inherited from class `Computer` have private visibility. Therefore, they can be accessed only within class `Computer`. Because it is fairly common for a subclass method to reference data fields declared in its superclass, Java provides a less restrictive form of visibility called *protected visibility*. A data field (or method) with protected visibility can be accessed in the class defining it, in any subclass of that class, or in any class in the same package. Therefore, if we had used the declaration

```
protected String manufacturer;
```

in class `Computer`, the following assignment statement would be valid in class `Notebook`:

```
manufacturer = man;
```

We will use protected visibility on occasion when we are writing a class that we intend to extend. However, in general, it is better to use private visibility because subclasses may be written by different programmers, and it is always a good practice to restrict and control access to the superclass data fields. We discuss visibility further in Section 1.7.

### **Is-a versus Has-a Relationships**

One misuse of inheritance is confusing: the *has-a* relationship with the *is-a* relationship. The *is-a* relationship between classes means that one class is a subclass of the other class. For example, a game computer is a computer with specific attributes that make it suitable for gaming applications (enhanced graphics, fast processor) and is a subclass of the `Computer` class. The *is-a* relationship is achieved by extending a class.

The *has-a* relationship between classes means that one class has the second class as an attribute. For example, a game box is not really a computer (it is a kind of entertainment device), but it has a computer as a component. The *has-a* relationship is achieved by declaring a `Computer` data field in the game box class.

Another issue that sometimes arises is determining whether to define a new class in a hierarchy or whether a new object is a member of an existing class. For example, netbook computers have recently become very popular. They are smaller portable computers that can be used for general-purpose computing but are also used extensively for Web browsing. Should we define a separate class `NetBook`, or is a netbook computer a `Notebook` object with a small screen and low weight?

---

## **EXERCISES FOR SECTION 1.2**

### **SELF-CHECK**

1. Explain the effect of each valid statement in the following fragment. Indicate any invalid statements.

```
Computer c1 = new Computer();
Computer c2 = new Computer("Ace", "AMD", 8.0, 500, 3.5);
Notebook c3 = new Notebook("Ace", "AMD", 4.0, 500, 3.0);
Notebook c4 = new Notebook("Bravo", "Intel", 4.0, 750, 3.0, 15.5, 5.5);
System.out.println(c2.manufacturer + ", " + c4.processor);
System.out.println(c2.getDiskSize() + ", " + c4.getRamSize());
System.out.println(c2.toString() + "\n" + c4.toString());
```

2. Indicate where in the hierarchy you might want to add data fields for the following and the kind of data field you would add.

- Cost
- The battery identification
- Time before battery discharges
- Number of expansion slots
- Wireless Internet available

3. Can you add the following constructor to class `Notebook`? If so, what would you need to do to class `Computer`?

```
public Notebook() {}
```

### **PROGRAMMING**

1. Write accessor and modifier methods for class `Computer`.
2. Write accessor and modifier methods for class `Notebook`.



## 1.3 Method Overriding, Method Overloading, and Polymorphism

In the preceding section, we discussed inherited data fields. We found that we could not access an inherited data field in a subclass object if its visibility was private. Next, we consider inherited methods. Methods generally have public visibility, so we should be able to access a method that is inherited. However, what if there are multiple methods with the same name in a class hierarchy? How does Java determine which one to invoke? We answer this question next.

### Method Overriding

Let's use the following main method to test our class hierarchy.

```
/** Tests classes Computer and Notebook. Creates an object of each and
 * displays them.
 * @param args[] No control parameters
 */
public static void main(String[] args) {
    Computer myComputer =
        new Computer("Acme", "Intel", 4, 750, 3.5);
    Notebook yourComputer =
        new Notebook("DellGate", "AMD", 4, 500,
                    2.4, 15.0, 7.5);
    System.out.println("My computer is:\n" + myComputer.toString());
    System.out.println("\nYour computer is:\n" +
                       yourComputer.toString());
}
```

In the second call to `println`, the method call

`yourComputer.toString()`

applies method `toString` to object `yourComputer` (type `Notebook`). Because class `Notebook` doesn't define its own `toString` method, class `Notebook` inherits the `toString` method defined in class `Computer`. Executing this method displays the following output lines:

```
My computer is:
Manufacturer: Acme
CPU: Intel
RAM: 4.0 gigabytes
Disk: 750 gigabytes
Speed: 3.5 gigahertz

Your computer is:
Manufacturer: DellGate
CPU: AMD
RAM: 4.0 gigabytes
Disk: 500 gigabytes
Speed: 2.4 gigahertz
```

Unfortunately, this output doesn't show the complete state of object `yourComputer`. To show the complete state of a notebook computer, we need to define a `toString` method for class `Notebook`. If class `Notebook` has its own `toString` method, it will override the inherited method and will be invoked by the method call `yourComputer.toString()`. We define method `toString` for class `Notebook` next.

```
public String toString() {
    String result = super.toString() +
                   "\nScreen size: " + screenSize + " inches" +
                   "\nWeight: " + weight + " pounds";
    return result;
}
```

This method `Notebook.toString` returns a string representation of the state of a `Notebook` object. The first line

```
String result = super.toString()
uses method call super.toString() to invoke the toString method of the superclass (method Computer.toString) to get the string representation of the four data fields that are inherited from the superclass. The next two lines append the data fields defined in class Notebook to this string.
```



## SYNTAX `super`.

### FORM:

`super.methodName()`  
`super.methodName(argumentList)`

### EXAMPLE:

`super.toString()`

### MEANING:

Using the prefix `super.` in a call to method `methodName` calls the method with that name defined in the superclass of the current class.



## PROGRAM STYLE

### Calling Method `toString()` Is Optional

In the `println` statement shown earlier,

```
System.out.println("My computer is:\n" + myComputer.toString());
```

the explicit call to method `toString` is not required. The statement could be written as

```
System.out.println("My computer is:\n" + myComputer);
```

Java automatically applies the `toString` method to an object referenced in a `String` expression. Normally, we will not explicitly call `toString`.



## PITFALL

### Overridden Methods Must Have Compatible Return Types

If you write a method in a subclass that has the same signature as one in the superclass but a different return type, you may get the following error message: `in subclass-name cannot override method-name in superclass-name; attempting to use incompatible return type`. The subclass method return type must be the same as or a subclass of the superclass method's return type.

## Method Overloading

Let's assume we have decided to standardize and purchase our notebook computers from only one manufacturer. We could then introduce a new constructor with one less parameter for class `Notebook`.

```
public Notebook(String proc, int ram, int disk, double procSpeed,
                double screen, double wei) {
    this(DEFAULT_NB_MAN, proc, ram, disk, procSpeed, screen, wei);
}
```

The method call

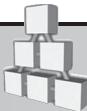
```
this(DEFAULT_NB_MAN, proc, ram, disk, procSpeed, screen, wei);
```

invokes the six-parameter constructor (see Listing 1.3), passing on the five arguments it receives and the constant string `DEFAULT_NB_MAN` (defined in class `Notebook`). The six-parameter constructor begins by calling the superclass constructor, satisfying the requirement that it be called first. We now have two constructors with different signatures in class `Notebook`. Having multiple methods with the same name but different signatures in a class is called *method overloading*.

Now we have two ways to create new `Notebook` objects. Both of the following statements are valid:

```
Notebook TTP1 = new Notebook("Intel", 4, 500, 1.8, 14, 6.5);
Notebook TTP2 = new Notebook("MicroSys", "AMD", 4, 750, 3.0, 15, 7.5);
```

The manufacturer of `TTP1` is `DEFAULT_NB_MAN`.



### SYNTAX `this( . . . );`

#### FORM:

```
this(argumentList);
```

#### EXAMPLE:

```
this(DEFAULT_NB_MAN, proc, ram, disk, procSpeed);
```

#### MEANING:

The call to `this()` invokes the constructor for the current class whose parameter list matches the argument list. The constructor initializes the new object as specified by its arguments. The invocation of another constructor (through either `this()` or `super()`) must be the first statement in a constructor.

Listing 1.4 shows the complete class `Notebook`. Figure 1.5 shows the UML diagram, revised to show that `Notebook` has a `toString` method and a constant data field. The next Pitfall discusses the reason for the `@Override` annotation preceding method `toString`.

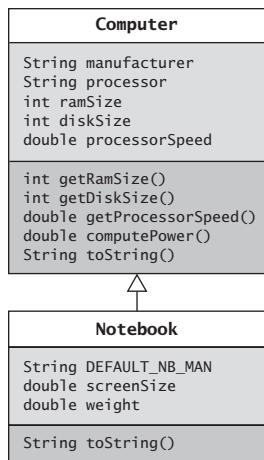
#### LISTING 1.4

Complete Class `Notebook` with Method `toString`

```
/** Class that represents a notebook computer. */
public class Notebook extends Computer {
    // Data Fields
    private static final String DEFAULT_NB_MAN = "MyBrand";
    private double screenSize;
    private double weight;
```

## **FIGURE 1.5**

## Revised UML Diagram for Computer Class Hierarchy



```
/** Initializes a Notebook object with all properties specified.
 * @param man The computer manufacturer
 * @param proc The processor type
 * @param ram The RAM size
 * @param disk The disk size
 * @param screen The screen size
 * @param wei The weight
 */
public Notebook(String man, String proc, int ram, int disk,
                double procSpeed, double screen, double wei) {
    super(man, proc, ram, disk, procSpeed);
    screenSize = screen;
    weight = wei;
}

/** Initializes a Notebook object with 6 properties specified. */
public Notebook(String proc, int ram, int disk,
                double procSpeed, double screen, double wei) {
    this(DEFAULT_NB_MAN, proc, ram, disk, procSpeed, screen, wei);
}

@Override
public String toString() {
    String result = super.toString() +
                    "\nScreen size: " + screenSize + " inches" +
                    "\nWeight: " + weight + " pounds";
    return result;
}
```



# PITFALL

## Overloading a Method When Intending to Override It

To override a method, you must use the same name and the same number and types of the parameters as the superclass method that is being overridden. If the name is the same but the number or types of the parameters are different, then the method is overloaded instead. Normally, the compiler will not detect this as an error. However, it is a sufficiently common error that a feature was added to the Java compiler so that programmers can indicate that they intend to override a method. If you precede the declaration of the method with the annotation `@Override`, the compiler will issue an error message if the method is overloaded instead of overridden.



## PROGRAM STYLE

## Precede an Overridden Method with the Annotation @Override

Whenever a method is overridden, we recommend preceding it with the annotation `@Override`. Some Java integrated development environments such as Netbeans and Eclipse will either issue a warning or add this annotation automatically.

## Polymorphism

An important advantage of OOP is that it supports a feature called *polymorphism*, which means many forms or many shapes. Polymorphism enables the JVM to determine at run time which of the classes in a hierarchy is referenced by a superclass variable or parameter. Next we will see how this simplifies the programming process.

Suppose you are not sure whether a computer referenced in a program will be a notebook or a regular computer. If you declare the reference variable

```
Computer theComputer;
```

you can use it to reference an object of either type because a type `Notebook` object can be referenced by a type `Computer` variable. In Java, a variable of a superclass type (general) can reference an object of a subclass type (specific). `Notebook` objects are `Computer` objects with more features. When the following statements are executed,

```
theComputer = new Computer("Acme", "Intel", 2, 160, 2.6);
System.out.println(theComputer.toString());
```

you would see four output lines, representing the state of the object referenced by `theComputer`.

Now suppose you have purchased a notebook computer instead. What happens when the following statements are executed?

```
theComputer = new Notebook("Bravo", "Intel", 4, 240, 2.4, 15.0, 7.5);
System.out.println(theComputer.toString());
```

Recall that `theComputer` is type `Computer`. Will the `theComputer.toString()` method call return a string with all seven data fields or just the five data fields defined for a `Computer` object? The answer is a string with all seven data fields. The reason is that the type of the object receiving the `toString` message determines which `toString` method is called. Even though variable `theComputer` is type `Computer`, it references a type `Notebook` object, and the `Notebook` object receives the `toString` message. Therefore, the method `toString` for class `Notebook` is the one called.

This is an example of polymorphism. Variable `theComputer` references a `Computer` object at one time and a `Notebook` object another time. At compile time, the Java compiler can't determine what type of object `theComputer` will reference, but at run time, the JVM knows the type of the object that receives the `toString` message and can call the appropriate `toString` method.

**EXAMPLE 1.2** If we declare the array `labComputers` as follows:

```
Computer[] labComputers = new Computer[10];
```

each subscripted variable `labComputers[i]` can reference either a `Computer` object or a `Notebook` object because `Notebook` is a subclass of `Computer`. For the method call `labComputers[i].toString()`, polymorphism ensures that the correct `toString` method is called. For each value of subscript `i`, the actual type of the object referenced by `labComputers[i]` determines which `toString` method will execute (`Computer.toString` or `Notebook.toString`).

## Methods with Class Parameters

Polymorphism also simplifies programming when we write methods that have class parameters. For example, if we want to compare the power of two computers without polymorphism, we will need to write overloaded `comparePower` methods in class `Computer`, one for each subclass parameter and one with a class `Computer` parameter. However, polymorphism enables us to write just one method with a `Computer` parameter.

---

**EXAMPLE 1.3** Method Computer.comparePowers compares the power of the Computer object it is applied to with the Computer object passed as its argument. It returns -1, 0, or +1 depending on which computer has more power. It does not matter whether this or aComputer references a Computer or a Notebook object.

```
/** Compares power of this computer and its argument computer
 * @param aComputer The computer being compared to this computer
 * @return -1 if this computer has less power,
 *         0 if the same, and
 *         +1 if this computer has more power.
 */
public int comparePower(Computer aComputer) {
    if (this.computePower() < aComputer.computePower())
        return -1;
    else if (this.computePower() == aComputer.computePower())
        return 0;
    else return 1;
}
```

---

## EXERCISES FOR SECTION 1.3

### SELF-CHECK

1. Explain the effect of each of the following statements. Which one(s) would you find in class Computer? Which one(s) would you find in class Notebook?

```
super(man, proc, ram, disk, procSpeed);
this(man, proc, ram, disk, procSpeed);
```

2. Indicate whether methods with each of the following signatures and return types (if any) would be allowed and in what classes they would be allowed. Explain your answers.

```
Computer()
Notebook()
int toString()
double getRamSize()
String getRamSize()
String getRamSize(String)
String getProcessor()
double getScreenSize()
```

3. For the loop body in the following fragment, indicate which method is invoked for each value of i. What is printed?

```
Computer comp[] = new Computer[3];
comp[0] = new Computer("Ace", "AMD", 8, 750, 3.5);
comp[1] = new Notebook("Dell", "Intel", 4, 500, 2.2, 15.5, 7.5);
comp[2] = comp[1];
for (int i = 0; i < comp.length; i++) {
    System.out.println(comp[i].getRamSize() + "\n" +
        comp[i].toString());
}
```

4. When does Java determine which `toString` method to execute for each value of i in the for statement in the preceding question: at compile time or at run time? Explain your answer.

## P R O G R A M M I N G

1. Write constructors for both classes that allow you to specify only the processor, RAM size, and disk size.
2. Complete the accessor and modifier methods for class `Computer`.
3. Complete the accessor and modifier methods for class `Notebook`.



## 1.4 Abstract Classes

In this section, we introduce another kind of class called an *abstract class*. An abstract class is denoted by the use of the word *abstract* in its heading:

```
visibility abstract class className
```

An abstract class differs from an actual class (sometimes called a concrete class) in two respects:

- An abstract class cannot be instantiated.
- An abstract class may declare abstract methods.

Just as in an interface, an abstract method is declared through a method heading in the abstract class definition. This heading indicates the result type, method name, and parameters, thereby specifying the form that any actual method declaration must take:

```
visibility abstract resultType methodName(parameterList);
```

However, the complete method definition, including the method body (implementation), does not appear in the abstract class definition.

In order to compile without error, an actual class that is a subclass of an abstract class must provide an implementation for each abstract method of its abstract superclass. The heading for each actual method must match the heading for the corresponding abstract method.

We introduce an abstract class in a class hierarchy when we need a base class for two or more actual classes that share some attributes. We may want to declare some of the attributes and define some of the methods that are common to these base classes. If, in addition, we want to require that the actual subclasses implement certain methods, we can accomplish this by making the base class an abstract class and declaring these methods abstract.

**EXAMPLE 1.4** The Food Guide Pyramid provides a recommendation of what to eat each day based on established dietary guidelines. There are six categories of foods in the pyramid: fats, oils, and sweets; meats, poultry, fish, and nuts; milk, yogurt, and cheese; vegetables; fruits; and bread, cereal, and pasta. If we wanted to model the Food Guide Pyramid, we might have each of these as actual subclasses of an abstract class called `Food`:

```
/** Abstract class that models a kind of food. */
public abstract class Food {
    // Data Field
    private double calories;

    // Abstract Methods
    /** Calculates the percent of protein in a Food object. */
```

```

public abstract double percentProtein();
/** Calculates the percent of fat in a Food object. */
public abstract double percentFat();
/** Calculates the percent of carbohydrates in a Food object. */
public abstract double percentCarbohydrates();

// Actual Methods
public double getCalories() { return calories; }
public void setCalories(double cal) {
    calories = cal;
}
}

```

The three abstract method declarations

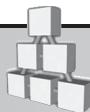
```

public abstract double percentProtein();
public abstract double percentFat();
public abstract double percentCarbohydrates();

```

impose the requirement that all actual subclasses implement these three methods. We would expect a different method definition for each kind of food. The keyword **abstract** must appear in all abstract method declarations in an abstract class. Recall that this is not required for abstract method declarations in interfaces.

---



## SYNTAX Abstract Class Definition

### FORM:

```

public abstract class className {
    data field declarations
    abstract method declarations
    actual method definitions
}

```

### EXAMPLE:

```

public abstract class Food {
    // Data Field
    private double calories;

    // Abstract Methods
    public abstract double percentProtein();
    public abstract double percentFat();
    public abstract double percentCarbohydrates();

    // Actual Methods
    public double getCalories() { return calories; }
    public void setCalories(double cal) {
        calories = cal;
    }
}

```

### INTERPRETATION:

Abstract class *className* is defined. The class body may have declarations for data fields and abstract methods as well as actual method definitions. Each abstract method declaration consists of a method heading containing the keyword **abstract**. All of the declaration kinds shown above are optional.



## PITFALL

### Omitting the Definition of an Abstract Method in a Subclass

If you write class Vegetable and forget to define method percentProtein, you will get the syntax error class Vegetable should be declared abstract, it does not define method percentProtein in class Food. Although this error message is misleading (you did not intend Vegetable to be abstract), any class with undefined methods is abstract by definition. The compiler's rationale is that the undefined method is intentional, so Vegetable must be an abstract class, with a subclass that defines percentProtein.

## Referencing Actual Objects

Because class Food is abstract, you can't create type Food objects. However, you can use a type Food variable to reference an actual object that belongs to a subclass of type Food. For example, an object of type Vegetable can be referenced by a Vegetable or Food variable because Vegetable is a subclass of Food (i.e., a Vegetable object is also a Food object).

---

**EXAMPLE 1.5** The following statement creates a Vegetable object that is referenced by variable mySnack (type Food).

```
Food mySnack = new Vegetable("carrot sticks");
```

---

## Initializing Data Fields in an Abstract Class

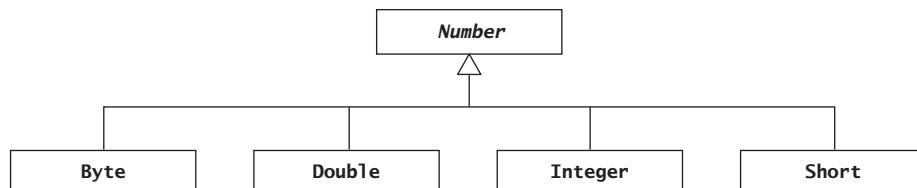
An abstract class can't be instantiated. However, an abstract class can have constructors that initialize its data fields when a new subclass object is created. The subclass constructor will use super(...) to call such a constructor.

## Abstract Class Number and the Java Wrapper Classes

The abstract class *Number* is predefined in the Java class hierarchy. It has as its subclasses all the wrapper classes for primitive numeric types (e.g., Byte, Double, Integer, and Short). A *wrapper class* is used to store a primitive-type value in an object type. Each wrapper class contains constructors to create an object that stores a particular primitive-type value. For example, Integer(35) or Integer("35") creates a type Integer object that stores the int 35. A wrapper class also has methods for converting the value stored to a different numeric type.

Figure 1.6 shows a portion of the class hierarchy with base class *Number*. Italicizing the class name *Number* in its class box indicates that *Number* is an abstract class and, therefore, cannot be instantiated. Listing 1.5 shows part of the definition for class *Number*. Two abstract methods are declared (*intValue* and *doubleValue*), and one actual method (*byteValue*) is defined.

.....  
**FIGURE 1.6**  
The Abstract Class  
*Number* and Selected  
Subclasses



In the actual implementation of `Number`, the body of `byteValue` would be provided, but we just indicate its presence in Listing 1.5.

.....

### LISTING 1.5

Part of Abstract Class `java.lang.Number`

```
public abstract class Number {
    // Abstract Methods
    /** Returns the value of the specified number as an int.
        @return The numeric value represented by this object after
        conversion to type int
    */
    public abstract int intValue();
    /** Returns the value of the specified number as a double.
        @return The numeric value represented by this object
        after conversion to type double
    */
    public abstract double doubleValue();

    ...

    // Actual Methods
    /** Returns the value of the specified number as a byte.
        @return The numeric value represented by this object
        after conversion to type byte
    */
    public byte byteValue() {
        // Implementation not shown.
        ...
    }
}
```

## Summary of Features of Actual Classes, Abstract Classes, and Interfaces

It is easy to confuse abstract classes, interfaces, and actual classes (concrete classes). Table 1.1 summarizes some important points about these constructs.

A class (abstract or actual) can extend only one other class; however, there is no restriction on the number of interfaces a class can implement. An interface cannot extend a class.

.....

**TABLE 1.1**

Comparison of Actual Classes, Abstract Classes, and Interfaces

Property	Actual Class	Abstract Class	Interface
Instances (objects) of this can be created	Yes	No	No
This can define instance variables	Yes	Yes	No
This can define methods	Yes	Yes	Yes
This can define constants	Yes	Yes	Yes
The number of these a class can extend	0 or 1	0 or 1	0
The number of these a class can implement	0	0	Any number
This can extend another class	Yes	Yes	No
This can declare abstract methods	No	Yes	Yes
Variables of this type can be declared	Yes	Yes	Yes

An abstract class may implement an interface just as an actual class does, but unlike an actual class, it doesn't have to define all of the methods declared in the interface. It can leave the implementation of some of the abstract methods to its subclasses.

Both abstract classes and interfaces declare abstract methods. However, unlike an interface, an abstract class can also have data fields and methods that are not abstract. You can think of an abstract class as combining the properties of an actual class, by providing inherited data fields and methods to its subclasses, and of an interface, by specifying requirements on its subclasses through its abstract method declarations.

## Implementing Multiple Interfaces

A class can extend only one other class, but it may extend more than one interface. For example, assume interface `StudentInt` specifies methods required for student-like classes and interface `EmployeeInt` specifies methods required for employee-like classes. The following header for class `StudentWorker`

```
public class StudentWorker implements StudentInt, EmployeeInt
```

means that class `StudentWorker` must define (provide code for) all of the abstract methods declared in both interfaces. Therefore, class `StudentWorker` supports operations required for both interfaces.

## Extending an Interface

Interfaces can also extend other interfaces. In Chapter 2 we will introduce the Java Collection Framework. This class hierarchy contains several interfaces and classes that manage the collection of objects. At the top of this hierarchy is the interface `Iterable`, which declares the method `iterator`. At the next lower level is interface `Collection`, which extends `Iterable`. This means that all classes that implement `Collection` must also implement `Iterable` and therefore must define the method `iterator`.

An interface can extend more than one other interface. In this case, the resulting interface includes the union of the methods defined in the superinterfaces. For example, we can define the interface `ComparableCollection`, which extends both `Comparable` and `Collection`, as follows:

```
public interface ComparableCollection extends Comparable, Collection { }
```

Note that this interface does not define any methods itself but does require any implementing class to implement all of the methods required by `Comparable` and by `Collection`.

---

## EXERCISES FOR SECTION 1.4

### SELF-CHECK

- What are two important differences between an abstract class and an actual class? What are the similarities?
- What do abstract methods and interfaces have in common? How do they differ?
- Explain the effect of each statement in the following fragment and trace the loop execution for each value of `i`, indicating which `doubleValue` method executes, if any. What is the final value of `x`?

```
Number[] nums = new Number[5];
nums[0] = new Integer(35);
nums[1] = new Double(3.45);
nums[4] = new Double("2.45e6");
double x = 0;
```

```

for (int i = 0; i < nums.length; i++) {
    if (nums[i] != null)
        x += nums[i].doubleValue();
}

```

4. What is the purpose of the `if` statement in the loop in Question 3? What would happen if it were omitted?

### PROGRAMMING

1. Write class `Vegetable`. Assume that a vegetable has three `double` constants: `VEG_FAT_CAL`, `VEG_PROTEIN_CAL`, and `VEG_CARBO_CAL`. Compute the fat percentage as `VEG_FAT_CAL` divided by the sum of all the constants.
2. Earlier we discussed a `Computer` class with a `Notebook` class as its only subclass. However, there are many different kinds of computers. An organization may have servers, mainframes, desktop PCs, and notebooks. There are also personal data assistants and game computers. So it may be more appropriate to declare class `Computer` as an abstract class that has an actual subclass for each category of computer. Write an abstract class `Computer` that defines all the methods shown earlier and declares an abstract method with the signature `costBenefit(double)` that returns the cost–benefit (type `double`) for each category of computer.



## 1.5 Class Object and Casting

The class `Object` is a special class in Java because it is the root of the class hierarchy, and every class has `Object` as a superclass. All classes inherit the methods defined in class `Object`; however, these methods may be overridden in the current class or in a superclass (if any). Table 1.2 shows a few of the methods of class `Object`. We discuss method `toString` next and the other `Object` methods shortly thereafter.

### The Method `toString`

You should always override the `toString` method if you want to represent an object's state (information stored). If you don't override it, the `toString` method for class `Object` will execute and return a string, but not what you are expecting.

---

**EXAMPLE 1.6** If we didn't have a `toString` method in class `Computer` or `Notebook`, the method call `aComputer.toString()` would call the `toString` method inherited from class `Object`. This method would return a string such as `Computer@ef08879`, which shows the object's class name and a special integer value that is its “hash code”—not its state. Method `hashCode` is discussed in Chapter 7.

---

**TABLE 1.2**  
The Class `Object`

Method	Behavior
<code>boolean equals(Object obj)</code>	Compares this object to its argument
<code>int hashCode()</code>	Returns an integer hash code value for this object
<code>String toString()</code>	Returns a string that textually represents the object
<code>Class&lt;?&gt; getClass()</code>	Returns a unique object that identifies the class of this object

## Operations Determined by Type of Reference Variable

You have seen that a variable can reference an object whose type is a subclass of the variable type. Because `Object` is a superclass of class `Integer`, the statement

```
Object aThing = new Integer(25);
```

will compile without error, creating the object reference shown in Figure 1.7. However, even though `aThing` references a type `Integer` object, we can't process this object like other `Integer` objects. For example, the method call `aThing.intValue()` would cause the syntax error `method intValue() not found in class java.lang.Object`. The reason for this is that the type of the reference, not the type of the object referenced, determines what operations can be performed, and class `Object` doesn't have an `intValue` method. During compilation, Java can't determine what kind of object will be referenced by a type `Object` variable, so the only operations permitted are those defined for class `Object`. The type `Integer` instance methods not defined in class `Object` (e.g., `intValue` and `doubleValue`) can't be invoked.

The method call `aThing.equals(new Integer("25"))` will compile because class `Object` has an `equals` method, and a subclass object has everything that is defined in its superclass. During execution, the `equals` method for class `Integer` is invoked, not class `Object`. (Why?)

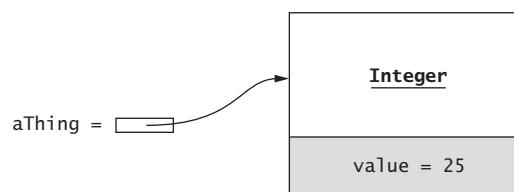
Another surprising result is that the assignment statement

```
Integer aNum = aThing; // incompatible types
```

won't compile even though `aThing` references a type `Integer` object. The syntax error: `incompatible types: found: Java.lang.Object, required: Java.lang.Integer` indicates that the expression type is incorrect (type `Object`, not type `Integer`). The reason Java won't compile this assignment is that Java is a strongly typed language, so the Java compiler always verifies that the type of the expression (`aThing` is type `Object`) being assigned is compatible with the variable type (`aNum` is type `Integer`). We show how to use casting to accomplish this in the next section.

Strong typing is also the reason that `aThing.intValue()` won't compile; the method invoked must be an instance method for class `Object` because `aThing` is type `Object`.

**FIGURE 1.7**  
Type `Integer` Object  
Referenced by `aThing`  
(Type `Object`)



### DESIGN CONCEPT

#### The Importance of Strong Typing

Suppose Java did not check the expression type and simply performed the assignment

```
Integer aNum = aThing; // incompatible types
```

Farther down the line, we might attempt to apply an `Integer` method to the object referenced by `aNum`. Because `aNum` is type `Integer`, the compiler would permit this. If `aNum` were referencing a type `Integer` object, then performing this operation would do no harm. But if `aNum` was referencing an object that was not type `Integer`, performing this operation would cause either a run-time error or an undetected logic error. It is much better to have the compiler tell us that the assignment is invalid.

## Casting in a Class Hierarchy

Java provides a mechanism, *casting*, that enables us to process the object referenced by `aThing` through a reference variable of its actual type, instead of through a type `Object` reference. The expression

```
(Integer) aThing
```

casts the type of the object referenced by `aThing` (type `Object`) to type `Integer`. The casting operation will succeed only if the object referenced by `aThing` is, in fact, type `Integer`; if not, a `ClassCastException` will be thrown.

What is the advantage of performing the cast? Casting gives us a type `Integer` reference to the object in Figure 1.7 that can be processed just like any other type `Integer` reference. The expression

```
((Integer) aThing).intValue()
```

will compile because now `intValue` is applied to a type `Integer` reference. Note that all parentheses are required so that method `intValue` will be invoked after the cast. Similarly, the assignment statement

```
Integer aNum = (Integer) aThing;
```

is valid because a type `Integer` reference is being assigned to `aNum` (type `Integer`).

Keep in mind that the casting operation does not change the object referenced by `aThing`; instead, it creates a type `Integer` reference to it. (This is called an *anonymous* or *unnamed reference*.) Using the type `Integer` reference, we can invoke any instance method of class `Integer` and process the object just like any other type `Integer` object.

The cast

```
(Integer) aThing
```

is called a downcast because we are casting from a higher type (`Object`) to a lower type (`Integer`). It is analogous to a narrowing cast when dealing with primitive types:

```
double x = . . . ;
int count = (int) x; // Narrowing cast, double is wider type than int
```

You can downcast from a more general type (a superclass type) to a more specific type (a subclass type) in a class hierarchy, provided that the more specific type is the same type as the object being cast (e.g., `(Integer) aThing`). You can also downcast from a more general type to a more specific type that is a superclass of the object being cast (e.g., `(Number) aThing`). *Upcasts* (casting from a more specific type to a more general type) are always valid; however, they are unnecessary and are rarely done.



### PITFALL

#### Performing an Invalid Cast

Assume that `aThing` (type `Object`) references a type `Integer` object as before, and you want to get its string representation. The downcast

```
(String) aThing // Invalid cast
```

is invalid and would cause a `ClassCastException` (a subclass of `RuntimeException`) because `aThing` references a type `Integer` object, and a type `Integer` object cannot be downcast to type `String` (`String` is not a superclass of `Integer`). However, the method call `aThing.toString()` is valid (and returns a string) because type `Object` has a `toString` method. (Which `toString` method would be called: `Object.toString` or `Integer.toString`?)

## Using instanceof to Guard a Casting Operation

In the preceding Pitfall, we mentioned that a `ClassCastException` occurs if we attempt an invalid casting operation. Java provides the `instanceof` operator, which you can use to guard against this kind of error.

**EXAMPLE 1.7** The following array `stuff` can store 10 objects of any data type because every object type is a subclass of `Object`.

```
Object[] stuff = new Object[10];
```

Assume that the array `stuff` has been loaded with data, and we want to find the sum of all numbers that are wrapped in objects. We can use the following loop to do so:

```
double sum = 0;
for (int i = 0; i < stuff.length; i++) {
    if (stuff[i] instanceof Number) {
        Number next = (Number) stuff[i];
        sum += next.doubleValue();
    }
}
```

The `if` condition (`stuff[i] instanceof Number`) is true if the object referenced by `stuff[i]` is a subclass of `Number`. It would be false if `stuff[i]` referenced a `String` or other nonnumeric object. The statement

```
Number next = (Number) stuff[i];
```

casts the object referenced by `stuff[i]` (type `Object`) to type `Number` and then references it through variable `next` (type `Number`). The variable `next` contains a reference to the same object as does `stuff[i]`, but the type of the reference is different (type `Number` instead of type `Object`). Then the statement

```
sum += next.doubleValue();
```

invokes the appropriate `doubleValue` method to extract the numeric value and add it to `sum`. Rather than declare variable `next`, you could write the `if` statement as

```
if (stuff[i] instanceof Number)
    sum += ((Number) stuff[i]).doubleValue();
```



### PROGRAM STYLE

#### Polymorphism Eliminates Nested if Statements

If Java didn't support polymorphism, the `if` statement in Example 1.7 would be much more complicated. You would need to write something like the following:

```
// Inefficient code that does not take advantage of polymorphism
if (stuff[i] instanceof Integer)
    sum += ((Integer) stuff[i]).doubleValue();
else if (stuff[i] instanceof Double)
    sum += ((Double) stuff[i]).doubleValue();
else if (stuff[i] instanceof Float)
    sum += ((Float) stuff[i]).doubleValue();
...
```

Each condition here uses the `instanceof` operator to determine the data type of the actual object referenced by `stuff[i]`. Once the type is known, we cast to that type and call its `doubleValue` method. Obviously, this code is very cumbersome and is more likely to be flawed than the original `if` statement. More importantly, if a new wrapper class is defined for numbers, we would need to modify the `if` statement to process objects of this new class type. So be wary of selection statements like the one shown here; their presence often indicates that you are not taking advantage of polymorphism.

### EXAMPLE 1.8

Suppose we have a class `Employee` with the following data fields:

```
public class Employee {
    // Data Fields
    private String name;
    private double hours;
    private double rate;
    private Address address;
    ...
}
```

To determine whether two `Employee` objects are equal, we could compare all four data fields. However, it makes more sense to determine whether two objects are the same employee by comparing their name and address data fields. Below, we show a method `equals` that overrides the `equals` method defined in class `Object`. By overriding this method, we ensure that the `equals` method for class `Employee` will always be called when method `equals` is applied to an `Employee` object. If we had declared the parameter type for `Employee.equals` as type `Employee` instead of `Object`, then the `Object.equals` method would be called if the argument was any data type except `Employee`.

```
/** Determines whether the current object matches its argument.
 * @param obj The object to be compared to the current object
 * @return true if the objects have the same name and address;
 *         otherwise, return false
 */
@Override
public boolean equals(Object obj) {
    if (obj == this) return true;
    if (obj == null) return false;
    if (this.getClass() == obj.getClass()) {
        Employee other = (Employee) obj;
        return name.equals(other.name) &&
               address.equals(other.address);
    } else {
        return false;
    }
}
```

If the object referenced by `obj` is not type `Employee`, we return `false`. If it is type `Employee`, we downcast that object to type `Employee`. After the downcast, the return statement calls method `String.equals` to compare the `name` field of the current object to the `name` field of object `other`, and method `Address.equals` to compare the two `address` data fields. Therefore, method `equals` must also be defined in class `Address`. The method result is `true` if both the `name` and `address` fields match, and it is `false` if one or both fields do not match. The method result is also `false` if the downcast can't be performed because the argument is an incorrect type or `null`.

## The Class Class

Every class has a `Class` object that is automatically created when the class is loaded into an application. The `Class` class provides methods that are mostly beyond the scope of this text. The important point is that each `Class` object is unique for the class, and the `getClass` method (a member of `Object`) will return a reference to this unique object. Thus, if `this.getClass() == obj.getClass()` in Example 1.8 is true, then we know that `obj` and `this` are both of class `Employee`.

---

## EXERCISES FOR SECTION 1.5

### SELF-CHECK

1. Indicate the effect of each of the following statements:

```
Object o = new String("Hello");
String s = o;
Object p = 25;
int k = p;
Number n = k;
```

2. Rewrite the invalid statements in Question 1 to remove the errors.

### PROGRAMMING

1. Write an `equals` method for class `Computer` (Listing 1.2).
2. Write an `equals` method for class `Notebook` (Listing 1.4).
3. Write an `equals` method for the following class. What other `equals` methods should be defined?

```
public class Airplane {
    // Data Fields
    Engine eng;
    Rudder rud;
    Wing[] wings = new Wing[2];
    ...
}
```

---

## 1.6 A Java Inheritance Example—The Exception Class Hierarchy

Next we show how Java uses inheritance to build a class hierarchy that is fundamental to detecting and correcting errors during program execution (run-time errors). A run-time error occurs during program execution when the Java Virtual Machine (JVM) detects an operation that it knows to be incorrect. A run-time error will cause the JVM to *throw an exception*—that is, to create an object of an exception type that identifies the kind of incorrect operation and also interrupts normal processing. Table 1.3 shows some examples of exceptions that are run-time errors. All are subclasses of class `RuntimeException`. Following are some examples of the exceptions listed in the table.

### Division by Zero

If `count` represents the number of items being processed and it is possible for `count` to be zero, then the assignment statement

```
average = sum / count;
```

**TABLE 1.3**Subclasses of `java.lang.RuntimeException`

Class	Cause/Consequence
<code>ArithmaticException</code>	An attempt to perform an integer division by zero
<code>ArrayIndexOutOfBoundsException</code>	An attempt to access an array element using an index (subscript) less than zero or greater than or equal to the array's length
<code>NumberFormatException</code>	An attempt to convert a string that is not numeric to a number
<code>NullPointerException</code>	An attempt to use a <code>null</code> reference value to access an object
<code>NoSuchElementException</code>	An attempt to get a next element after all elements were accessed
<code>InputMismatchException</code>	The token returned by a <code>Scanner next ...</code> method does not match the pattern for the expected data type

can cause a division-by-zero error. If `sum` and `count` are `int` variables, this error is indicated by the JVM throwing an `ArithmaticException`. You can easily guard against such a division with an `if` statement so that the division operation will not be performed when `count` is zero:

```
if (count == 0)
    average = 0;
else
    average = sum / count;
```

Normally, you would compute an average as a `double` value, so you could cast an `int` value in `sum` to type `double` before doing the division. In this case, an exception is not thrown if `count` is zero. Instead, `average` will have one of the special values `Double.POSITIVE_INFINITY`, `Double.NEGATIVE_INFINITY`, or `Double.NaN` depending on whether `sum` was positive, negative, or zero.

## Array Index Out of Bounds

An `ArrayIndexOutOfBoundsException` is thrown by the JVM when an index value (subscript) used to access an element in an array is less than zero or greater than or equal to the array's length. For example, suppose we define the array `scores` as follows:

```
int[] scores = new int[500];
```

The subscripted variable `scores[i]` uses `i` (type `int`) as the array index. An `ArrayIndexOutOfBoundsException` will be thrown if `i` is less than zero or greater than 499.

Array index out of bounds errors can be prevented by carefully checking the boundary values for an index that is also a loop control variable. A common error is using the array size as the upper limit rather than the array size minus 1.

**EXAMPLE 1.9** The following loop would cause an `ArrayIndexOutOfBoundsException` on the last pass, when `i` is equal to `x.length`.

```
for (int i = 0; i <= x.length; i++)
    x[i] = i * i;
```

The loop repetition test should be `i < x.length`.

## NumberFormatException and InputMismatchException

The `NumberFormatException` is thrown when a program attempts to convert a nonnumeric string (usually a data value) to a numeric value. For example, if the user types in the string "2.6e", method `parseDouble`, in the following code:

```
String speedStr = JOptionPane.showInputDialog("Enter speed");
double speed = Double.parseDouble(speedStr);
```

would throw a `NumberFormatException` because "2.6e" is not a valid numeric string (it has no exponent after the e). There is no general way to avoid this exception because it is impossible to guard against all possible data entry errors the user can make.

A similar error can occur if you are using a `Scanner` object for data entry. If `scan` is a `Scanner`, the statement

```
double speed = scan.nextDouble();
```

will throw an `InputMismatchException` if the next token scanned is "2.6e".

## Null Pointer

The `NullPointerException` is thrown when there is an attempt to access an object that does not exist; that is, the reference variable being accessed contains a special value, known as `null`. You can guard against this by testing for `null` before invoking a method.

## The Exception Class Hierarchy

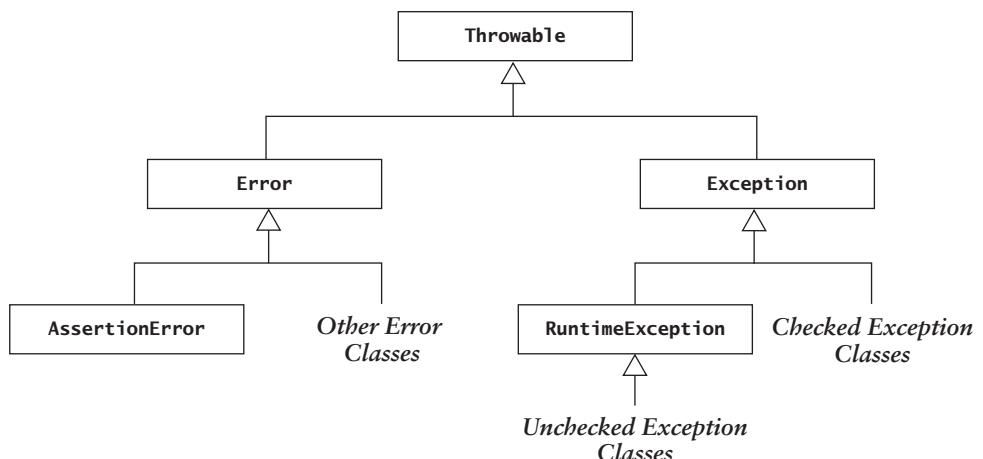
The exceptions in Table 1.3 are all subclasses of `Runtime`. All Exception classes are defined within a class hierarchy that has the class `Throwable` as its superclass (see the UML diagram in Figure 1.8). The UML diagram shows that classes `Error` and `Exception` are subclasses of `Throwable`. Each of these classes has subclasses that are shown in the figure. We will focus on class `Exception` and its subclasses in this chapter. Because `RuntimeException` is a subclass of `Exception`, it is also a subclass of `Throwable` (the subclass relationship is transitive).

## The Class `Throwable`

The class `Throwable` is the superclass of all exceptions. The methods that you will use from class `Throwable` are summarized in Table 1.4. Because all exception classes are subclasses of class `Throwable`, they can call any of its methods including `getMessage`, `printStackTrace`, and `toString`. If `ex` is an `Exception` object, the call

```
ex.printStackTrace();
```

**FIGURE 1.8**  
Summary of  
Exception Class  
Hierarchy



**TABLE I.4**Summary of Commonly Used Methods from the `java.lang.Throwable` Class

Method	Behavior
<code>String getMessage()</code>	Returns the detail message
<code>void printStackTrace()</code>	Prints the stack trace to <code>System.err</code>
<code>String toString()</code>	Returns the name of the exception followed by the detail message

displays a stack trace, discussed in Appendix A (Section A.11). The statement

```
System.out.println(ex.getMessage());
```

displays a detail message (or error message) describing the exception. The statement

```
System.out.println(ex.toString());
```

displays the name of the exception followed by the detail message.

## Checked and Unchecked Exceptions

There are two categories of exceptions: *checked* and *unchecked*. A checked exception is an error that is normally not due to programmer error and is beyond the control of the programmer. All exceptions caused by input/output errors are considered checked exceptions. For example, if the programmer attempts to access a data file that is not available because of a user or system error, a `FileNotFoundException` is thrown. The class `IOException` and its subclasses (see Table 1.5) are checked exceptions. Even though checked exceptions are beyond the control of the programmer, the programmer must be aware of them and must handle them in some way (discussed later). All checked exceptions are subclasses of `Exception`, but they are not subclasses of `RuntimeException`. Figure 1.9 is a more complete diagram of the Exception hierarchy.

The *unchecked* exceptions represent error conditions that may occur as a result of programmer error or of serious external conditions that are considered unrecoverable. For example, exceptions such as `NullPointerException` or `ArrayIndexOutOfBoundsException` are unchecked exceptions that are generally due to programmer error. These exceptions are all subclasses of `RuntimeException`. While you can sometimes prevent these exceptions via defensive programming, it is impractical to try to prevent them all or to provide exception handling for all of them. Therefore, you can handle these exceptions, but Java does not require you to do so.

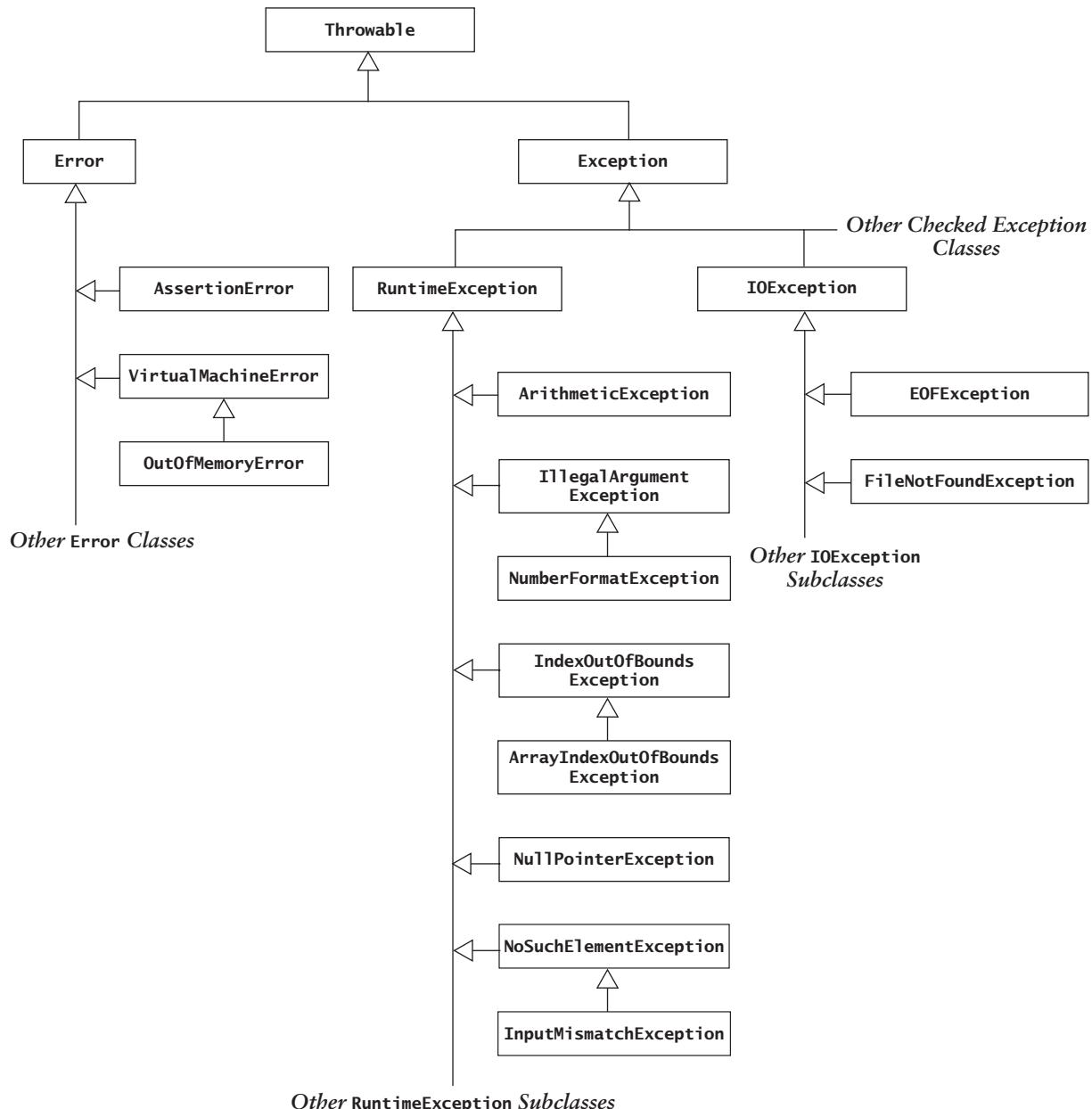
The class `Error` and its subclasses represent errors that are due to serious external conditions. An example of such an error is `OutOfMemoryError`, which is thrown when there is no memory available. You can't foresee or guard against these kinds of errors. You can attempt to handle these exceptions, but you are strongly discouraged from trying to do so because you probably will be unsuccessful. For example, if an `OutOfMemoryError` is thrown, there is no memory available to process the exception-handling code, so the exception would be thrown again.

**TABLE I.5**Class `java.io.IOException` and Some Subclasses

Exception Class	Cause
<code>IOException</code>	Some sort of input/output error
<code>EOFException</code>	Attempt to read beyond the end of data with a <code>DataInputStream</code>
<code>FileNotFoundException</code>	Inability to find a file

**FIGURE 1.9**

Exception Hierarchy Showing Selected Checked and Unchecked Exceptions



How do we know which exceptions are checked and which are unchecked? Exception classes that are subclasses of `RuntimeException` and `Error` are unchecked. All other exception classes are checked exceptions.

We discuss Java exceptions further in Appendix A. Section A.11 describes how to use the try-catch statement to handle different kinds of exceptions; Section A.12 shows how to write statements that throw exceptions when your code detects an error during run time.

## Handling Exceptions to Recover from Errors

Exceptions enable Java programmers to write code that can report errors and sometimes recover from them. The key to this process is the **try-catch** sequence. We will cover the essentials of catching and throwing exceptions in this section. A complete discussion is provided in Sections A.11 and A.12.

### The try-catch Sequence

The **try-catch** sequence is used to catch and handle exceptions. It resembles an **if-then-else** statement. It consists of one **try** block followed by one or more **catch** clauses. The statements in the **try** block are executed first. If they execute without error, the **catch** clauses are skipped. If a statement in the **try** block throws an exception, the rest of the statements in the **try** block are skipped and execution continues with the statements in the **catch** clause for that particular type of exception. If there is no **catch** clause for that exception type, the exception is rethrown to the calling method. If the **main** method is reached and no appropriate **catch** clause is located, the program terminates with an unhandled exception error. A **try** block with two **catch** clauses follows.

```
try {
    // Execute the following statements until an exception is thrown
    ...
    // Skip the catch blocks if no exceptions were thrown
} catch (ExceptionTypeA ex) {
    // Execute this catch block if an exception of type ExceptionTypeA
    // was thrown in the try block
    ...
} catch (ExceptionTypeB ex) {
    // Execute this catch block if an exception of type ExceptionTypeB
    // was thrown in the try block
    ...
}
```

A **catch** clause header resembles a method header. The expression in parentheses in the **catch** clause header is like a method parameter declaration (the parameter is *ex*). The statements in curly braces, the **catch** block, execute if the exception that was thrown is the specified exception type or is a subclass of that exception type.



### PITFALL

#### Unreachable catch block

In the above, `ExceptionTypeA` cannot be a superclass of `ExceptionTypeB`. If it is, `ExceptionTypeB` is considered unreachable because its exceptions would be caught by the first `catch` clause.

## Using try-catch to Recover from an Error

One common source of exceptions is user input. For example, the `Scanner nextInt` method is supposed to read a type `int` value. If an `int` is not the next item read, Java throws an `InputMismatchException`. Rather than have this problem terminate the program, you can read the data value in a **try** block and catch an `InputMismatchException` in the **catch** clause. If one is thrown, you can give the user another chance to enter an integer as shown in method `getintValue`, as follows.

```

/** Reads an integer using a scanner.
 * @return the first integer read.
 */
public static int getIntValue(Scanner scan) {
    int nextInt = 0;           // next int value
    boolean validInt = false; // flag for valid input
    while (!validInt) {
        try {
            System.out.println("Enter number of kids:");
            nextInt = scan.nextInt();
            validInt = true;
        } catch (InputMismatchException ex) {
            scan.nextLine(); // clear buffer
            System.out.println("Bad data -- enter an integer:");
        }
    }
    return nextInt;
}

```

The **while** loop repeats while **validInt** is **false** (its initial value). The **try** block attempts to read a type **int** value using Scanner **scan**. If the user enters an integer, **validInt** is set to **true** and the **try-catch** statement and **while** loop are exited. The integer data value will be returned as the method result.

If the user enters a data item that is not an integer, however, Java throws an **InputMismatchException**. This is caught by the **catch** clause

```
catch (InputMismatchException ex)
```

The first statement in the **catch** block clears the Scanner buffer, and the user is prompted to enter an integer. Because **validInt** is still **false**, the **while** loop repeats until the user successfully enters an integer.

## Throwing an Exception When Recovery Is Not Obvious

In the last example, method **getIntValue** was able to recover from a bad data item by giving the user another chance to enter data. In some cases, you may be able to write code that detects certain kinds of errors, but there may not be an obvious way to recover from them. In these cases, the best approach is just to throw an exception reporting the error to the method that called it. The caller can then catch the exception and handle it.

Method **processPositiveInteger** requires a positive integer as its argument. If the argument is not positive, there is no reason to continue executing the method because the result may be meaningless, or the method execution may cause a different exception to be thrown, which could confuse the method caller. There is also no obvious way to correct this error because the method has no way of knowing what **n** should be, so a **try-catch** sequence would not fix the problem.

```

public static void processPositiveInteger(int n) {
    if (n < 0)
        throw new IllegalArgumentException(
            "Invalid negative argument");
    else {
        // Process n as required
        //...
        System.out.println("Finished processing " + n);
    }
}

```

If the argument **n** is not positive, the statement

```
throw new IllegalArgumentException("Invalid negative argument");
```

executes and throws an `IllegalArgumentException` object. The string in the last line is stored in the exception object's `message` data field, and the method is exited, returning control to the caller. The caller is then responsible for handling this exception. If possible, the caller may be able to recover from this error and would attempt to do so.

The `main` method, which follows, calls both `getIntValue` and `processPositiveInteger` in the `try` block. If an `IllegalArgumentException` is thrown, the message `invalid negative argument` is displayed, and the program terminates with an error indication. If no exception is thrown, the program exits normally.

```
public static void main(String[] args) {
    Scanner scan = new Scanner (System.in);
    try {
        int num = getIntValue(scan);
        processPositiveInteger(num);
    } catch (IllegalArgumentException ex) {
        System.err.println(ex.getMessage());
        System.exit(1); // error indication
    }
    System.exit(0); //normal exit
}
```

## EXERCISES FOR SECTION 1.6

### SELF-CHECK

1. Explain the key difference between checked and unchecked exceptions. Give an example of each kind of exception. What criterion does Java use to decide whether an exception is checked or unchecked?
2. What is the difference between the kind of unchecked exceptions in class `Error` and the kind in class `Exception`?
3. List four subclasses of `RuntimeException`.
4. List two subclasses of `IOException`.
5. What happens in the `main` method preceding the exercises if an exception of a different type occurs in method `processPositiveInteger`?
6. Trace the execution of method `getIntValue` if the following data items are entered by a careless user. What would be displayed?  
ace  
7.5  
-5
7. Trace the execution of method `main` preceding the exercises if the data items in Question 6 were entered. What would be displayed?

## 1.7 Packages and Visibility

### Packages

You have already seen packages. The Java API is organized into packages such as `java.lang`, `java.util`, `java.io`, and `javax.swing`. The package to which a class belongs is declared by the first statement in the file in which the class is defined using the keyword `package`, followed

by the package name. For example, we could begin each class in the computer hierarchy (class `Notebook` and class `Computer`) with the line:

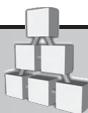
```
package computers;
```

All classes in the same package are stored in the same directory or folder. The directory must have the same name as the package. All the classes in the folder must declare themselves to be in the package.

Classes that are not part of a package may access only public members (data fields or methods) of classes in the package. If the application class is not in the package, it must reference the classes by their complete names. The complete name of a class is `packageName.className`. However, if the package is imported by the application class, then the prefix `packageName.` is not required. For example, we can reference the constant `GREEN` in class `java.awt.Color` as `Color.GREEN` if we import package `java.awt`. Otherwise, we would need to use the complete name `java.awt.Color.GREEN`.

## The No-Package-Declared Environment

So far we have not specified packages, yet objects of one class could communicate with objects of another class. How does this work? Just as there is a default visibility, there is a default package. Files that do not specify a package are considered part of the default package. Therefore, if you don't declare packages, all your classes belong to the same package (the default package).



### SYNTAX Package Declaration

#### FORM:

```
package packageName;
```

#### EXAMPLE:

```
package computers;
```

#### INTERPRETATION:

This declaration appears as the first line of the file in which a class is defined. The class is now considered part of the package. This file must be contained in a folder with the same name as the package.



### PROGRAM STYLE

#### When to Package Classes

The default package facility is intended for use during the early stages of implementing classes or for small prototype programs. If you are developing an application that has several classes that are part of a hierarchy of classes, you should declare them all to be in the same package. The package declaration will keep you from accidentally referring to classes by their short names in other classes that are outside the package. It will also restrict the visibility of protected members of a class to only its subclasses outside the package (and to other classes inside the package) as intended.

## Package Visibility

So far, we have discussed three layers of visibility for classes and class members (data fields and methods): private, protected, and public. There is a fourth layer, called *package visibility*, that sits between private and protected. Classes, data fields, and methods with package visibility are accessible to all other methods of the same package but are not accessible to methods outside of the package. By contrast, classes, data fields, and methods that are declared protected are visible within subclasses that are declared outside the package, in addition to being visible to all members of the package.

We have used the visibility modifiers `private`, `public`, and `protected` to specify the visibility of a class member. If we do not use one of these visibility modifiers, then the class member has package visibility and it is visible in all classes of the same package, but not outside the package. Note that there is no visibility modifier `package`; package visibility is the default if no visibility modifier is specified.

## Visibility Supports Encapsulation

The rules for visibility control how encapsulation occurs in a Java program. Table 1.6 summarizes the rules in order of decreasing protection. Note that private visibility is for members of a class that should not be accessible to anyone but the class, not even classes that extend it. Except for inner classes, it does not make sense for a class to be private. It would mean that no other class can use it.

Also, note that package visibility (the default if a visibility modifier is not given) allows the developer of a library to shield classes and class members from classes outside the package. Typically, such classes perform tasks required by the public classes within the package.

Use of protected visibility allows the package developer to give control to other programmers who want to extend classes in the package. Protected data fields are typically essential to an object. Similarly, protected methods are those that are essential to an extending class.

Table 1.6 shows that public classes and members are universally visible. Within a package, the public classes are those that are essential to communicating with objects outside the package.

**TABLE 1.6**

Summary of Kinds of Visibility

Visibility	Applied to Classes	Applied to Class Members
<code>private</code>	Applicable to inner classes. Accessible only to members of the class in which it is declared	Visible only within this class
Default or package	Visible to classes in this package	Visible to classes in this package
<code>protected</code>	Applicable to inner classes. Visible to classes in this package and to classes outside the package that extend the class in which it is declared	Visible to classes in this package and to classes outside the package that extend this class
<code>public</code>	Visible to all classes	Visible to all classes. The class defining the member must also be public



## PITFALL

### Protected Visibility Can Be Equivalent to Public Visibility

The intention of protected visibility is to enable a subclass to access a member (data field or method) of a superclass directly. However, protected members can also be accessed within any class that is in the same package. This is not a problem if the class with the protected members is declared to be in a package; however, if it is not, then it is in the default package. Protected members of a class in the default package are visible in all other classes you have defined that are not part of an actual package. This is generally not a desirable situation. You can avoid this dilemma by using protected visibility only with members of classes that are in explicitly declared packages. In all other classes, use either public or private visibility because protected visibility is virtually equivalent to public visibility.

## EXERCISES FOR SECTION 1.7

### SELF-CHECK

1. Consider the following declarations:

```
package pack1;
public class Class1 {
    private int v1;
    protected int v2;
    int v3;
    public int v4;
}

package pack1;
public class Class2 {...}

package pack2;
public class Class3 extends pack1.Class1 {...}

package pack2;
public class Class4 {...}
```

- a. What visibility must variables declared in pack1.Class1 have in order to be visible in pack1.Class2?
- b. What visibility must variables declared in pack1.Class1 have in order to be visible in pack2.Class3?
- c. What visibility must variables declared in pack1.Class1 have in order to be visible in pack2.Class4?

## 1.8 A Shape Class Hierarchy

In this section, we provide a case study that illustrates some of the principles in this chapter. For each case study, we will begin with a statement of the problem (Problem). Then we analyze the problem to determine exactly what is expected and to develop an initial strategy for solution (Analysis). Next, we design a solution to the problem, developing and refining an algorithm (Design). We write one or more Java classes that contain methods for the algorithm steps (Implementation). Finally, we provide a strategy for testing the completed classes and discuss special cases that should be investigated (Testing). We often provide a separate class that does the testing.

## CASE STUDY Processing Geometric Figures

**Problem** We would like to process some standard geometric shapes. Each `Figure` object will be one of three standard shapes (rectangle, circle, and right triangle). We would like to be able to do standard computations, such as finding the area and perimeter, for any of these shapes.

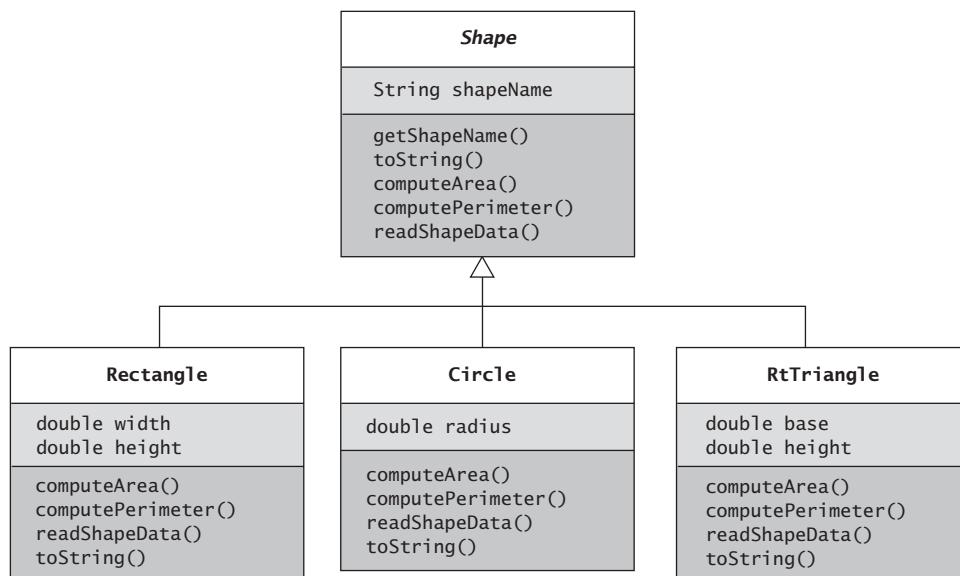
**Analysis** For each of the geometric shapes we can process, we need a class that represents the shape and knows how to perform the standard computations on it (i.e., find its area and perimeter). These classes will be `Rectangle`, `Circle`, and `RtTriangle`. To ensure that these shape classes all define the required computational methods (finding area and perimeter), we will make them abstract methods in the base class for the shape hierarchy. If a shape class does not have the required methods, we will get a syntax error when we attempt to compile it.

Figure 1.10 shows the class hierarchy. We used abstract class `Shape` as the base class of the hierarchy. We didn't consider using an actual class because there are no actual objects of the base class type. The single data field `shapeName` stores the kind of shape object as a `String`.

**Design** We will discuss the design of the `Rectangle` class here. The design of the other classes is similar and is left as an exercise. Table 1.7 shows class `Rectangle`. Class `Rectangle` has data fields `width` and `height`. It has methods to compute area and perimeter, a method to read in the attributes of a rectangular object (`readShapeData`), and a `toString` method.

**FIGURE 1.10**

Abstract Class `Shape` and Its Three Actual Subclasses



**TABLE 1.7**  
Class Rectangle

Data Field	Attribute
double width	Width of a rectangle
double height	Height of a rectangle
Method	Behavior
double computeArea()	Computes the rectangle area (width × height)
double computePerimeter()	Computes the rectangle perimeter ( $2 \times \text{width} + 2 \times \text{height}$ )
void readShapeData()	Reads the width and height
String toString()	Returns a string representing the state

**Implementation** Listing 1.6 shows abstract class Shape.

**LISTING 1.6**

Abstract Class Shape (Shape.java)

```
/** Abstract class for a geometric shape. */
public abstract class Shape {

    /** The name of the shape */
    private String shapeName = "";
    /** Initializes the shapeName.
     * @param shapeName the kind of shape
     */
    public Shape(String shapeName) {
        this.shapeName = shapeName;
    }

    /** Get the kind of shape.
     * @return the shapeName
     */
    public String getShapeName() { return shapeName; }

    @Override
    public String toString() { return "Shape is a " + shapeName; }

    // abstract methods
    public abstract double computeArea();
    public abstract double computePerimeter();
    public abstract void readShapeData();
}
```

Listing 1.7 shows class Rectangle.

**LISTING 1.7**

Class Rectangle (Rectangle.java)

```
import java.util.Scanner;

/** Represents a rectangle.
 * Extends Shape.
 */
public class Rectangle extends Shape {

    // Data Fields
    /** The width of the rectangle. */
    private double width = 0;
    /** The height of the rectangle. */
    private double height = 0;

    // Constructors
    public Rectangle() {
        super("Rectangle");
    }

    /** Constructs a rectangle of the specified size.
     * @param width the width
     * @param height the height
     */
    public Rectangle(double width, double height) {
        super("Rectangle");
        this.width = width;
        this.height = height;
    }

    // Methods
    /** Get the width.
     * @return The width
     */
    public double getWidth() {
        return width;
    }

    /** Get the height.
     * @return The height
     */
    public double getHeight() {
        return height;
    }

    /** Compute the area.
     * @return The area of the rectangle
     */
    @Override
    public double computeArea() {
        return height * width;
    }

    /** Compute the perimeter.
     * @return The perimeter of the rectangle
     */
}
```

```

@Override
public double computePerimeter() {
    return 2 * (height + width);
}

/** Read the attributes of the rectangle. */
@Override
public void readShapeData() {
    Scanner in = new Scanner(System.in);
    System.out.println("Enter the width of the Rectangle");
    width = in.nextDouble();
    System.out.println("Enter the height of the Rectangle");
    height = in.nextDouble();
}

/** Create a string representation of the rectangle.
 * @return A string representation of the rectangle
 */
@Override
public String toString() {
    return super.toString() + ": width is " + width + ", height is " +
        height;
}
}

```

**Testing** To test the shape hierarchy, we will write a program that will prompt for the kind of figure, read the parameters for that figure, and display the results. The code for ComputeAreaAndPerimeter is shown in Listing 1.8. The `main` method is very straightforward, and so is `displayResult`. The `main` method first calls `getShape`, which displays a list of available shapes and prompts the user for the choice. The reply is expected to be a single character. The nested `if` statement determines which shape instance to return. For example, if the user's choice is C (for Circle), the statement

```

return new Circle();

```

returns a reference to a new `Circle` object.

After the new shape instance is returned to `myShape` in `main`, the statement

```

myShape.readShapeData();

```

uses polymorphism to invoke the correct member function `readShapeData` to read the shape object's parameter(s). The methods `computeArea` and `computePerimeter` are then called to obtain the values of the area and perimeter. Finally, `displayResult` is called to display the result.

A sample of the output from `ComputeAreaAndPerimeter` follows.

```

Enter C for circle
Enter R for Rectangle
Enter T for Right Triangle
R
Enter the width of the Rectangle
120
Enter the height of the Rectangle
200
Shape is a Rectangle: width is 120.0, height is 200.0
The area is 24000.00
The perimeter is 640.00

```

**LISTING 1.8**

ComputeAreaAndPerimeter.java

```
import java.util.Scanner;

/*
    Computes the area and perimeter of selected figures.
    @author Koffman and Wolfgang
*/
public class ComputeAreaAndPerimeter {

    /**
     * The main program performs the following steps.
     * 1. It asks the user for the type of figure.
     * 2. It asks the user for the characteristics of that figure.
     * 3. It computes the perimeter.
     * 4. It computes the area.
     * 5. It displays the result.
     * @param args The command line arguments -- not used
    */
    public static void main(String args[]) {
        Shape myShape;
        double perimeter;
        double area;
        myShape = getShape();      // Ask for figure type
        myShape.readShapeData();   // Read the shape data
        perimeter = myShape.computePerimeter(); // Compute perimeter
        area = myShape.computeArea();           // Compute the area
        displayResult(myShape, area, perimeter); // Display the result
        System.exit(0);                      // Exit the program
    }

    /**
     * Ask the user for the type of figure.
     * @return An instance of the selected shape
    */
    public static Shape getShape() {
        Scanner in = new Scanner(System.in);
        System.out.println("Enter C for circle");
        System.out.println("Enter R for Rectangle");
        System.out.println("Enter T for Right Triangle");
        String figType = in.next();
        if (figType.equalsIgnoreCase("c")) {
            return new Circle();
        }
        else if (figType.equalsIgnoreCase("r")) {
            return new Rectangle();
        }
        else if (figType.equalsIgnoreCase("t")) {
            return new RtTriangle();
        }
        else {
            return null;
        }
    }
}
```

```

/** Display the result of the computation.
 * @param area The area of the figure
 * @param perim The perimeter of the figure
 */
private static void displayResult(Shape myShape, double area, double perim) {
    System.out.println(myShape);
    System.out.printf("The area is %.2f%nThe perimeter is %.2f%n",
                      area, perim);
}
}

```



## PROGRAM STYLE

### Using Factory Methods to Return Objects

The method `getShape` is an example of a *factory method* because it creates a new object and returns a reference to it. The author of the `main` method does not need to know what kinds of shapes are available. Knowledge of the available shapes is confined to the `getShape` method. This function must present a list of available shapes to the user and decode the user's response to return an instance of the desired shape. If you add a new geometric shape class to the class hierarchy, you only need to modify the `if` statement in the factory method so that it can create and return an object of that type.

## EXERCISES FOR SECTION 1.8

### SELF-CHECK

1. Explain why `Shape` cannot be an actual class.
2. Explain why `Shape` cannot be an interface.

### PROGRAMMING

1. Write class `Circle`.
2. Write class `RtTriangle`.



# Chapter Review

- ◆ Inheritance and class hierarchies enable you to capture the idea that one thing may be a refinement or an extension of another. For example, a plant is a living thing. Such *is-a* relationships create the right balance between too much and too little structure. Think of inheritance as a means of creating a refinement of an abstraction. The entities farther down the hierarchy are more complex and less general than those higher up. The entities farther down the hierarchy may inherit data members (attributes) and methods from those farther up, but not vice versa. A class that inherits from another class **extends** that class.
- ◆ Encapsulation and inheritance impose structure on object abstractions. Polymorphism provides a degree of flexibility in defining methods. It loosens the structure a bit in order to make methods more accessible and useful. *Polymorphism* means “many forms.” It captures the idea that methods may take on a variety of forms to suit different purposes.
- ◆ All exceptions in the `Exception` class hierarchy are derived from a common superclass called `Throwable`. This class provides methods for collecting and reporting the state of the program when an exception is thrown. The commonly used methods are `getMessage` and `toString`, which return a detail message describing what caused the exception to be thrown, and `printStackTrace`, which prints the exception and then shows the line where the exception occurred and the sequence of method calls leading to the exception.
- ◆ There are two categories of exceptions: checked and unchecked. Checked exceptions are generally due to an error condition external to the program. Unchecked exceptions are generally due to a programmer error or a dire event.
- ◆ The keyword **interface** defines an interface. A Java interface can be used to specify an abstract data type (ADT), and a Java class can be used to implement an ADT. A class that implements an interface must define the methods that the interface declares.
- ◆ The keyword **abstract** defines an abstract class or method. An abstract class is like an interface in that it leaves method implementations up to subclasses, but it can also have data fields and actual methods. You use an abstract class as the superclass for a group of classes in a hierarchy.
- ◆ Visibility is influenced by the package in which a class is declared. You assign classes to a package by including the statement `package packageName;` at the top of the file. You can refer to classes within a package by their direct names when the package is imported through an import declaration.

## Java Constructs Introduced in This Chapter

<code>abstract</code>	<code>private</code>	<code>super(...)</code>
<code>extends</code>	<code>protected</code>	<code>this.</code>
<code>instanceof</code>	<code>public</code>	<code>this(...)</code>
<code>interface</code>	<code>super.</code>	
<code>package</code>		

## Java API Classes Introduced in This Chapter

<code>java.lang.Byte</code>	<code>java.lang.Number</code>
<code>java.lang.Float</code>	<code>java.lang.Object</code>
<code>java.lang.Integer</code>	<code>java.lang.Short</code>

## User-Defined Interfaces and Classes in This Chapter

ComputeAreaAndPerimeter	Food	Student
Computer	Notebook	StudentInt
Employee	Rectangle	StudentWorker
EmployeeInt	Shape	

## Quick-Check Exercises

- What does *polymorphism* mean, and how is it used in Java? What is method overriding? Method overloading?
- What is a method signature? Describe how it is used in method overloading.
- Describe the use of the keywords **super** and **this**.
- Indicate whether each error or exception in the following list is checked or unchecked: `IOException`, `EOFException`, `VirtualMachineError`, `IndexOutOfBoundsException`, `OutOfMemoryError`.
- When would you use an abstract class, and what should it contain?
- An \_\_\_\_\_ specifies the requirements of an ADT as a contract between the \_\_\_\_\_ and \_\_\_\_\_; a \_\_\_\_\_ implements the ADT.
- An interface can be implemented by multiple classes. (True/False)
- Describe the difference between *is-a* and *has-a* relationships.
- Which can have more data fields and methods: the superclass or the subclass?
- You can reference an object of a \_\_\_\_\_ type through a variable of a \_\_\_\_\_ type.
- You cast an object referenced by a \_\_\_\_\_ type to an object of a \_\_\_\_\_ type in order to apply methods of the \_\_\_\_\_ type to the object. This is called a \_\_\_\_\_.
- The four kinds of visibility in order of decreasing visibility are \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.

## Review Questions

- Which method is invoked in a particular class when a method definition is overridden in several classes that are part of an inheritance hierarchy? Answer the question for the case in which the class has a definition for the method and also for the case where it doesn't.
- Explain how assignments can be made within a class hierarchy and the role of casting in a class hierarchy. What is strong typing? Why is it an important language feature?
- If Java encounters a method call of the following form:  
`superclassVar.methodName()`  
 where `superclassVar` is a variable of a superclass that references an object whose type is a subclass, what is necessary for this statement to compile? During run time, will method `methodName` from the class that is the type of `superclassVar` always be invoked, or is it possible that a different method `methodName` will be invoked? Explain your answer.
- Assume the situation in Question 3, but method `methodName` is not defined in the class that is the type of `superclassVar`, although it is defined in the subclass type. Rewrite the method call so that it will compile.
- Explain the process of initializing an object that is a subclass type in the subclass constructor. What part of the object must be initialized first? How is this done?
- What is default or package visibility?
- Indicate what kind of exception each of the following errors would cause. Indicate whether each error is a checked or an unchecked exception.
  - Attempting to create a `Scanner` for a file that does not exist
  - Attempting to call a method on a variable that has not been initialized
  - Using `-1` as an array index

8. Discuss when abstract classes are used. How do they differ from actual classes and from interfaces?
9. What is the advantage of specifying an ADT as an interface instead of just going ahead and implementing it as a class?
10. Define an interface to specify an ADT **Money** that has methods for arithmetic operations (addition, subtraction, multiplication, and division) on real numbers having exactly two digits to the right of the decimal point, as well as methods for representing a **Money** object as a string and as a real number. Also, include methods **equals** and **compareTo** for this ADT.
11. Answer Review Question 10 for an ADT **Complex** that has methods for arithmetic operations on a complex number (a number with a real and an imaginary part). Assume that the same operations (+, -, \*, /) are supported. Also, provide methods **toString**, **equals**, and **compareTo** for the ADT **Complex**.
12. Like a rectangle, a parallelogram has opposite sides that are parallel, but it has a corner angle, theta, that is less than 90 degrees. Discuss how you would add parallelograms to the class hierarchy for geometric shapes (see Figure 1.10). Write a definition for class **Parallelogram**.

## Programming Projects

1. A veterinary office wants to store information regarding the kinds of animals it treats. Data includes diet, whether the animal is nocturnal, whether its bite is poisonous (as for some snakes), whether it flies, and so on. Use a superclass **Pet** with abstract methods and create appropriate subclasses to support about 10 animals of your choice.
2. A student is a person, and so is an employee. Create a class **Person** that has the data attributes common to both students and employees (name, social security number, age, gender, address, and telephone number) and appropriate method definitions. A student has a grade-point average (GPA), major, and year of graduation. An employee has a department, job title, and year of hire. In addition, there are hourly employees (hourly rate, hours worked, and union dues) and salaried employees (annual salary). Define a class hierarchy and write an application class that you can use to first store the data for an array of people and then display that information in a meaningful way.
3. Create a pricing system for a company that makes individualized computers, such as you might see on a Web site. There are two kinds of computers: notebooks and desktop computers. The customer can select the processor speed, the amount of memory, and the size of the disk drive. The customer can also choose a CD drive (CD ROM, CD-RW), a DVD drive, or both. For notebooks, there is a choice of screen size. Other options are a modem, a network card, or a wireless network. You should have an abstract class **Computer** and subclasses **DeskTop** and **Notebook**. Each subclass should have methods for calculating the price of a computer, given the base price plus the cost of the different options. You should have methods for calculating memory price, hard drive price, and so on. There should be a method to calculate shipping cost.
4. Write a banking program that simulates the operation of your local bank. You should declare the following collection of classes.

### Class Account

Data fields: **customer** (type **Customer**), **balance**, **accountNumber**, **transactions** array (type **Transaction**[]). Allocate an initial **Transaction** array of a reasonable size (e.g., 20) and provide a **realloc** method that doubles the size of the **Transaction** array when it becomes full.

Methods: **getBalance**, **getCustomer**, **toString**, **setCustomer**

### Class SavingsAccount extends Account

Methods: **deposit**, **withdraw**, **addInterest**

### Class CheckingAccount extends Account

Methods: **deposit**, **withdraw**, **addInterest**

**Class Customer**

Data fields: name, address, age, telephoneNumber, customerNumber

Methods: Accessors and modifiers for data fields plus the additional abstract methods `getSavingsInterest`, `getCheckInterest`, and `getCheckCharge`.

Classes Senior, Adult, Student, all these classes extend `Customer`

Each has constant data fields `SAVINGS_INTEREST`, `CHECK_INTEREST`, `CHECK_CHARGE`, `good!` and `OVERDRAFT_PENALTY` that define these values for customers of that type, and each class implements the corresponding accessors.

**Class Bank**

Data field: `accounts` array (type `Account[]`). Allocate an array of a reasonable size (e.g., 100) and provide a reallocate method.

Methods: `addAccount`, `makeDeposit`, `makeWithdrawal`, `getAccount`

**Class Transaction**

Data fields: `customerNumber`, `transactionType`, `amount`, `date`, and `fees` (a string describing unusual fees)

Methods: `processTran`

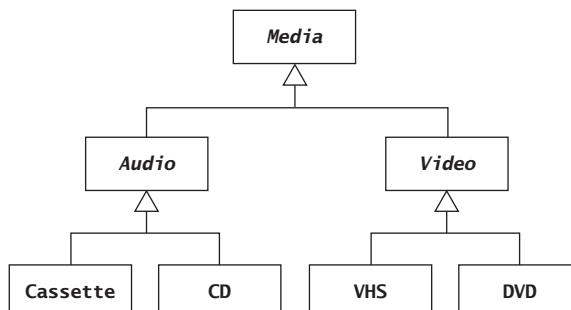
You need to write all these classes and an application class that interacts with the user. In the application, you should first open several accounts and then enter several transactions.

5. You have a sizable collection of music and videos and want to develop a database for storing and processing information about this collection. You need to develop a class hierarchy for your media collection that will be helpful in designing the database. Try the class hierarchy shown in Figure 1.11, where `Audio` and `Video` are media categories. Then CDs and cassette tapes would be subclasses of `Audio`, and DVDs and VHS tapes would be subclasses of `Video`.

If you go to the video store to get a movie, you can rent or purchase only movies that are recorded on VHS tapes or DVDs. For this reason, class `Video` (and also classes `Media` and `Audio`) should be abstract classes because there are no actual objects of these types. However, they are useful classes to help define the hierarchy.

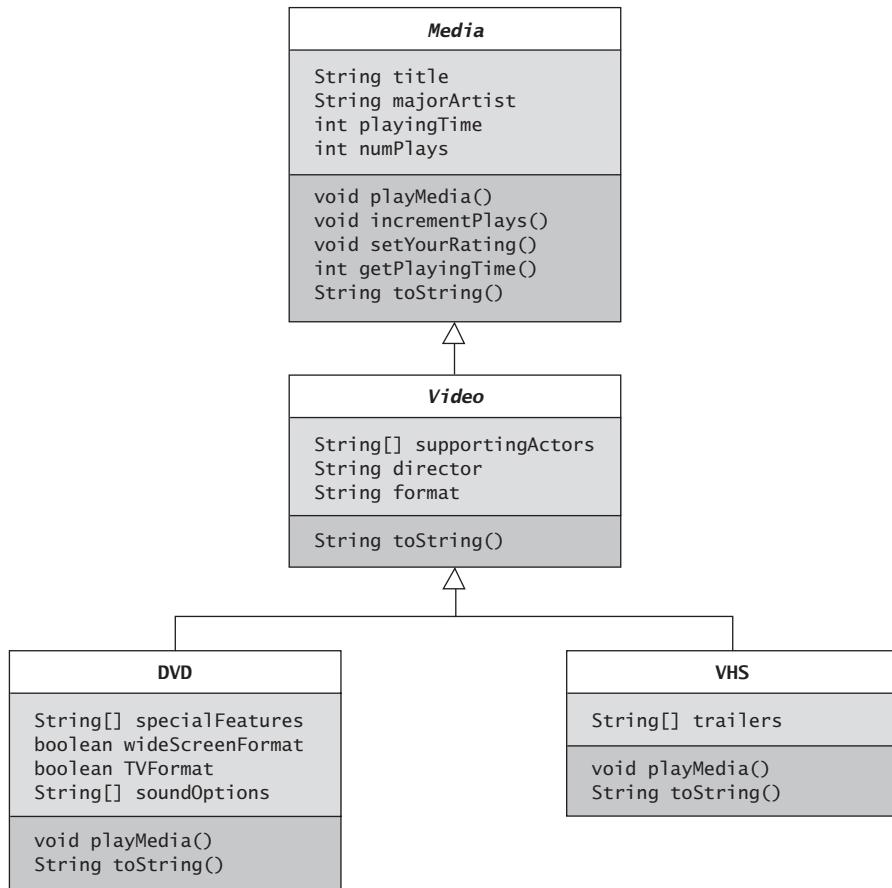
Class `Media` should have data fields and methods common to all classes in the hierarchy. Every media object has a title, major artist, distributor, playing time, price, and so on. Class `Video` should have additional data fields for information describing movies recorded on DVDs and videotapes. This would include information about the supporting actors, the producer, the director, and the movie's rating. Class `DVD` would have specific information about DVD movies only, such as the format of the picture and special features on the disk. Figure 1.12 shows a possible class diagram for `Media`, `Video`, and subclasses of `Video`.

**FIGURE 1.11**  
Media Class Hierarchy



**FIGURE 1.12**

Class `Video` and Its  
Supers and Subclasses



Provide methods to load the media collection from a file and write it back out to a file. Also, provide a method to retrieve the information for a particular item identified by its title and a method to retrieve all your items for a particular artist.

6. Add shape classes `Square` and `EquilateralTriangle` to the figures hierarchy in Section 1.7. Modify class `ComputeAreaAndPerim` (Listing 1.8) to accept the new figures.
7. Complete the `Food` class hierarchy in Section 1.4. Read and store a list of your favorite foods. Show the total calories for these foods and the overall percentages of fat, protein, and carbohydrates for this list. To find the overall percentage, if an item has 200 calories and 10 percent is fat calories, then that item contributes 20 fat calories. You need to find the totals for fat calories, protein calories, and carbohydrate calories and then calculate the percentages.
8. A hospital has different kinds of patients who require different procedures for billing and approval of procedures. Some patients have insurance and some do not. Of the insured patients, some are on Medicare, some are in HMOs, and some have other health insurance plans. Develop a collection of classes to model these different kinds of patients.
9. A company has two different kinds of employees: professional and nonprofessional. Generally, professional employees have a monthly salary, whereas nonprofessional employees are paid an hourly rate. Similarly, professional employees have a certain number of days of vacation, whereas nonprofessional employees receive vacation hours based on the number of hours they have worked. The amount contributed for health insurance is also different for each kind of employee. Use an abstract class `Employee` to store information common to all employees and to declare methods for calculating weekly salary and computing health care contributions and vacation days earned that week. Define subclasses `Professional` and `Nonprofessional`. Test your class hierarchy.

10. Implement class `AMTBandkAmerica` in Section 1.1.
11. For the shape class hierarchy discussed in Section 1.8, consider adding classes `DrawableRectangle`, `DrawableCircle`, and so on that would have additional data fields and methods that would enable a shape to be drawn on a monitor. Provide an interface `DrawableInt` that specifies the methods required for drawing a shape. Class `DrawableRectangle`, for example, should extend `Rectangle` and implement this interface. Draw the new class hierarchy and write the new interface and classes. Using the Java Abstract Window Toolkit (AWT) to draw objects is described in Appendix Section C.7.

## Answers to Quick-Check Exercises

1. *Polymorphism* means “many forms.” Method overriding means that the same method appears in a subclass and a superclass. Method overloading means that the same method appears with different signatures in the same class.
2. A signature is the form of a method determined by its name and arguments. For example, `doIt(int, double)` is the signature for a method `doIt` that has one type `int` parameter and one type `double` parameter. If several methods in a class have the same name (method overloading), Java applies the one with the same signature as the method call.
3. The keyword `this` followed by a dot and a name means use the named member (data field or method) of the object to which the current method is applied rather than the member with the same name declared locally in the method. The keyword `super`. means use the method (or data field) with this name defined in the superclass of the object, not the one belonging to the object. Using `super(...)` as a method call in a constructor tells Java to call a constructor for the superclass of the object being created. Similarly, using `this(...)` as a method call in a constructor tells Java to call another constructor for the same class but with a different parameter list. The `super(...)` or `this(...)` call must be the first statement in a subclass constructor.
4. `VirtualMachineError`, `OutOfMemoryError`, and `IndexOutOfBoundsException` are unchecked; the rest are checked.
5. An abstract class is used as a parent class for a collection of related subclasses. An abstract class cannot be instantiated. The abstract methods (identified by modifier `abstract`) defined in the abstract class act as placeholders for the actual methods. Also, you should define data fields that are common to all the subclasses in the abstract class. An abstract class can have actual methods as well as abstract methods.
6. An interface specifies the requirements of an ADT as a contract between the developer and the user; a class implements the ADT.
7. True.
8. An *is-a* relationship between classes means that one class is a subclass of a parent class. A *has-a* relationship means that one class has data members of the other class type.
9. Subclass.
10. You can reference an object of a subclass type through a variable of a superclass type.
11. You cast an object referenced by a superclass type to an object of a subclass type in order to apply methods of the subclass type to the object. This is called a *downcast*.
12. The four kinds of visibility in order of decreasing visibility are *public*, *protected*, *package*, and *private*.



# *Lists and the Collections Framework*

## Chapter Objectives

- ◆ To understand the meaning of big-O notation and how it is used as a measure of an algorithm's efficiency
- ◆ To become familiar with the List interface and the Java Collections Framework
- ◆ To understand how to write an array-based implementation of the List interface
- ◆ To study the differences between single-, double-, and circular-linked list data structures
- ◆ To learn how to implement a single-linked list
- ◆ To learn how to implement the List interface using a double-linked list
- ◆ To understand the Iterator interface
- ◆ To learn how to implement the Iterator for a linked list
- ◆ To become familiar with the Java Collections Framework

**S**o far we have one data structure that you can use in your programming—the array. Giving a programmer an array and asking her to develop software systems is like giving a carpenter a hammer and asking him to build a house. In both cases, more tools are needed. The Java designers attempted to supply those tools by providing a rich set of data structures written as Java classes. The classes are all part of a hierarchy called the Java Collections Framework. We will discuss classes from this hierarchy in the rest of the book, starting in this chapter with the classes that are considered lists.

A list is an expandable collection of elements in which each element has a position or index. Some lists enable their elements to be accessed in arbitrary order (called *random access*) using a position value to select an element. Alternatively, you can start at the beginning and process the elements in sequence. We will also discuss iterators and their role in facilitating sequential access to lists.

In this chapter, we will discuss the `ArrayList` and linked lists (class `LinkedList`) and their similarities and differences. We will show that these classes are subclasses of the abstract class `AbstractList` and that they implement the `List` interface.

First, we will discuss algorithm efficiency and how to characterize the efficiency of an algorithm. You will learn about big-O notation, which you can use to compare the relative efficiency of different algorithms.

## Lists and the Collections Framework

- 2.1 Algorithm Efficiency and Big-O
- 2.2 The List Interface and ArrayList Class
- 2.3 Applications of ArrayList
- 2.4 Implementation of an ArrayList Class
- 2.5 Single-Linked Lists
- 2.6 Double-Linked Lists and Circular Lists
- 2.7 The LinkedList Class and the Iterator, ListIterator, and Iterable Interfaces
- 2.8 Application of the LinkedList Class
  - Case Study:* Maintaining an Ordered List
- 2.9 Implementation of a Double-Linked List Class
- 2.10 The Collections Framework Design

## 2.1 Algorithm Efficiency and Big-O

Whenever we write a new class, we will discuss the efficiency of its methods so that you know how they compare to similar methods in other classes. You can't easily measure the amount of time it takes to run a program with modern computers. When you issue the command

```
java MyProgram
```

(or click the Run button of your integrated development environment [IDE]), the operating system first loads the Java Virtual Machine (JVM). The JVM then loads the .class file for MyProgram, it then loads other .class files that MyProgram references, and finally your program executes. (If the .class files have not yet been created, the Java IDE will compile the source file before executing the program.) Most of the time it takes to run your program is occupied with the first two steps. If you run your program a second time immediately after the first, it may seem to take less time. This is because the operating system may have kept the files in a local memory area called a cache. However, if you have a large enough or complicated enough problem, then the actual running time of your program will dominate the time required to load the JVM and .class files.

Because it is very difficult to get a precise measure of the performance of an algorithm or program, we normally try to approximate the effect of a change in the number of data items,  $n$ , that an algorithm processes. In this way, we can see how an algorithm's execution time increases with respect to  $n$ , so we can compare two algorithms by examining their growth rates.

For many problems, there are algorithms that are relatively obvious but inefficient. Although every day computers are getting faster, with larger memories, there are algorithms whose growth rate is so large that no computer, no matter how fast or with how much memory, can solve the problem above a certain size. Furthermore, if a problem that has been too large to be solved can now be solved with the latest, biggest, and fastest supercomputer, adding a few more inputs may make the problem impractical, if not impossible, again. Therefore, it is important to have some idea of the relative efficiency of different algorithms. Next, we see how we might obtain such an idea by examining three methods in the following examples.

---

**EXAMPLE 2.1** Consider the following method, which searches an array for a value:

```
public static int search(int[] x, int target) {
    for (int i = 0; i < x.length; i++) {
        if (x[i] == target)
            return i;
    }
    // target not found
    return -1;
}
```

If the target is not present in the array, then the `for` loop body will be executed `x.length` times. If the target is present, it could be anywhere. If we consider the average over all cases where the target is present, then the loop body will execute `x.length/2` times. Therefore, the total execution time is directly proportional to `x.length`. If we doubled the size of the array, we would expect the time to double (not counting the overhead discussed earlier).

---

**EXAMPLE 2.2** Now let us consider another problem. We want to find out whether two arrays have no common elements. We can use our `search` method to search one array for values that are in the other.

```
/** Determine whether two arrays have no common elements.
 * @param x One array
 * @param y The other array
 * @return true if there are no common elements
 */
public static boolean areDifferent(int[] x, int[] y) {
    for (int i = 0; i < x.length; i++) {
        if (search(y, x[i]) != -1)
            return false;
    }
    return true;
}
```

The loop body will execute at most `x.length` times. During each iteration, it will call method `search` to search for current element, `x[i]`, in array `y`. The loop body in `search` will execute at most `y.length` times. Therefore, the total execution time would be proportional to the product of `x.length` and `y.length`.

---

**EXAMPLE 2.3** Let us consider the problem of determining whether each item in an array is unique. We could write the following method:

```
/** Determine whether the contents of an array are all unique.
 * @param x The array
 * @return true if all elements of x are unique
 */
public static boolean areUnique(int[] x) {
    for (int i = 0; i < x.length; i++) {
        for (int j = 0; j < x.length; j++) {
            if (i != j && x[i] == x[j])
                return false;
        }
    }
    return true;
}
```

If all values are unique, the outer loop will execute  $x.length$  times. For each iteration of the outer loop, the inner loop will also execute  $x.length$  times. Thus, the total number of times the body of the inner loop will execute is  $(x.length)^2$ .

---

**EXAMPLE 2.4** The method we showed in Example 2.3 is very inefficient because we do approximately twice as many tests as necessary. We can rewrite the inner loop as follows:

```
/** Determine whether the contents of an array are all unique.
 * @param x The array
 * @return true if all elements of x are unique
 */
public static boolean areUnique(int[] x) {
    for (int i = 0; i < x.length; i++) {
        for (int j = i + 1; j < x.length; j++) {
            if (x[i] == x[j])
                return false;
        }
    }
    return true;
}
```

We can start the inner loop index at  $i + 1$  because we have already determined that elements preceding this one are unique. The first time, the inner loop will execute  $x.length - 1$  times. The second time, it will execute  $x.length - 2$  times, and so on. The last time, it will execute just once. The total number of times it will execute is

$$x.length - 1 + x.length - 2 + \dots + 2 + 1$$

The series  $1 + 2 + 3 + \dots + (n - 1)$  is a well-known series that has a value of

$$\frac{n \times (n - 1)}{2} \text{ or } \frac{n^2 - n}{2}$$

Therefore, this sum is

$$\frac{x.length^2 - x.length}{2}$$


---

## Big-O Notation

Today, the type of analysis just illustrated is more important to the development of efficient software than measuring the milliseconds in which a program runs on a particular computer. Understanding how the execution time (and memory requirements) of an algorithm grows as a function of increasing input size gives programmers a tool for comparing various algorithms and how they will perform. Computer scientists have developed a useful terminology and notation for investigating and describing the relationship between input size and execution time. For example, if the time is approximately doubled when the number of inputs,  $n$ , is doubled, then the algorithm grows at a linear rate. Thus, we say that the growth rate has an order of  $n$ . If, however, the time is approximately quadrupled when the number of inputs is doubled, then the algorithm grows at a quadratic rate. In this case, we say that the growth rate has an order of  $n^2$ .

In the previous section, we looked at four methods: one whose execution time was related to  $x.length$ , another whose execution time was related to  $x.length \times y.length$ , one whose execution time was related to  $(x.length)^2$ , and one whose execution time was related to  $(x.length)^2 \times y.length$ . Computer scientists use the notation  $O(n)$  to represent the first case,  $O(n \times m)$  to represent the second, and  $O(n^2)$  to represent the third and fourth, where  $n$  is  $x.length$  and  $m$  is  $y.length$ . The symbol  $O$  (which you will see in a variety of typefaces and styles in computer science literature) can be thought of as an abbreviation for “order of magnitude.” This notation is called *big-O notation*.

Often, a simple way to determine the big-O of an algorithm or program is to look at the loops and to see whether the loops are nested. Assuming that the loop body consists only of simple statements, a single loop is  $O(n)$ , a pair of nested loops is  $O(n^2)$ , a nested loop pair inside another is  $O(n^3)$ , and so on. However, you also must examine the number of times the loop executes.

Consider the following:

```
for (i = 1; i < x.length; i *= 2) {
    // Do something with x[i]
}
```

The loop body will execute  $k - 1$  times, with  $i$  having the following values: 1, 2, 4, 8, 16, 32, . . . ,  $2^k$  until  $2^k$  is greater than  $x.length$ . Since  $2^{k-1} = x.length < 2^k$  and  $\log_2 2^k$  is  $k$ , we know that  $k - 1 = \log_2(x.length) < k$ . Thus, we say that this loop is  $O(\log n)$ . The logarithm function grows slowly. The log to the base 2 of 1,000,000 is approximately 20. Typically, in analyzing the running time of algorithms, we use logarithms to the base 2.

## Formal Definition of Big-O

Consider a program that is structured as follows:

```
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        Simple Statement
    }
}
for (int k = 0; i < n; k++) {
    Simple Statement 1
    Simple Statement 2
    Simple Statement 3
    Simple Statement 4
    Simple Statement 5
}
Simple Statement 6
Simple Statement 7
...
Simple Statement 30
```

Let us assume that each *Simple Statement* takes one unit of time and that the *for* statements are free. The nested loop executes a *Simple Statement*  $n^2$  times. Then five *Simple Statements* are executed  $n$  times in the loop with control variable  $k$ . Finally, 25 *Simple Statements* are executed after this loop. We would then conclude that the expression

$$T(n) = n^2 + 5n + 25$$

shows the relationship between processing time and  $n$  (the number of data items processed in the loop), where  $T(n)$  represents the processing time as a function of  $n$ . It should be clear that the  $n^2$  term dominates as  $n$  becomes large.

In terms of  $T(n)$ , formally, the big-O notation

$$T(n) = O(f(n))$$

means that there exist two constants,  $n_0$  and  $c$ , greater than zero, and a function,  $f(n)$ , such that for all  $n > n_0$ ,  $cf(n) \geq T(n)$ . In other words, as  $n$  gets sufficiently large (larger than  $n_0$ ), there is some constant  $c$  for which the processing time will always be less than or equal to  $cf(n)$ , so  $cf(n)$  is an upper bound on the performance. The performance will never be worse than  $cf(n)$  and may be better.

If we can determine how the value of  $f(n)$  increases with  $n$ , we know how the processing time will increase with  $n$ . The growth rate of  $f(n)$  will be determined by the growth rate of the fastest-growing term (the one with the largest exponent), which in this case is the  $n^2$  term. This means that the algorithm in this example is an  $O(n^2)$  algorithm rather than an  $O(n^2 + 5n + 25)$  algorithm. In general, it is safe to ignore all constants and drop the lower-order terms when determining the order of magnitude for an algorithm.

**EXAMPLE 2.5** Given  $T(n) = n^2 + 5n + 25$ , we want to show that this is indeed  $O(n^2)$ . Thus, we want to show that there are constants  $n_0$  and  $c$  such that for all  $n > n_0$ ,  $cn^2 > n^2 + 5n + 25$ .

One way to do this is to find a point where

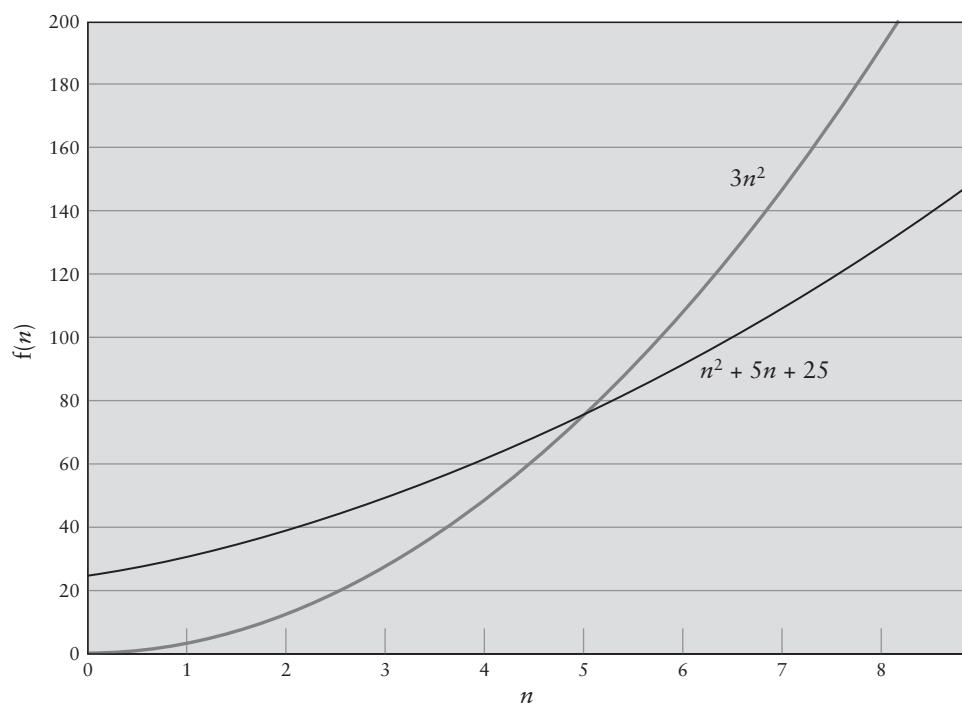
$$cn^2 = n^2 + 5n + 25$$

If we let  $n$  be  $n_0$  and solve for  $c$ , we get

$$c = 1 + 5/n_0 + 25/n_0^2$$

We can evaluate the expression on the right easily when  $n_0$  is  $5(1 + 5/5 + 25/25)$ . This gives us a  $c$  of 3. So  $3n^2 > n^2 + 5n + 25$  for all  $n$  greater than 5, as shown in Figure 2.1.

**FIGURE 2.1**  
 $3n^2$  versus  $n^2 + 5n + 25$



**EXAMPLE 2.6** Consider the following program loop:

```
for (int i = 0; i < n - 1; i++) {
    for (int j = i + 1; j < n; j++) {
        3 simple statements
    }
}
```

The first time through the outer loop, the inner loop is executed  $n - 1$  times; the next time,  $n - 2$ ; and the last time, once. The outer loop is executed  $n$  times. So we get the following expression for  $T(n)$ :

$$3(n - 1) + 3(n - 2) + \dots + 3(2) + 3(1)$$

We can factor out the 3 to get

$$3((n - 1) + (n - 2) + \dots + 2 + 1)$$

The sum  $1 + 2 + \dots + (n - 2) + (n - 1)$  (in parentheses above) is equal to

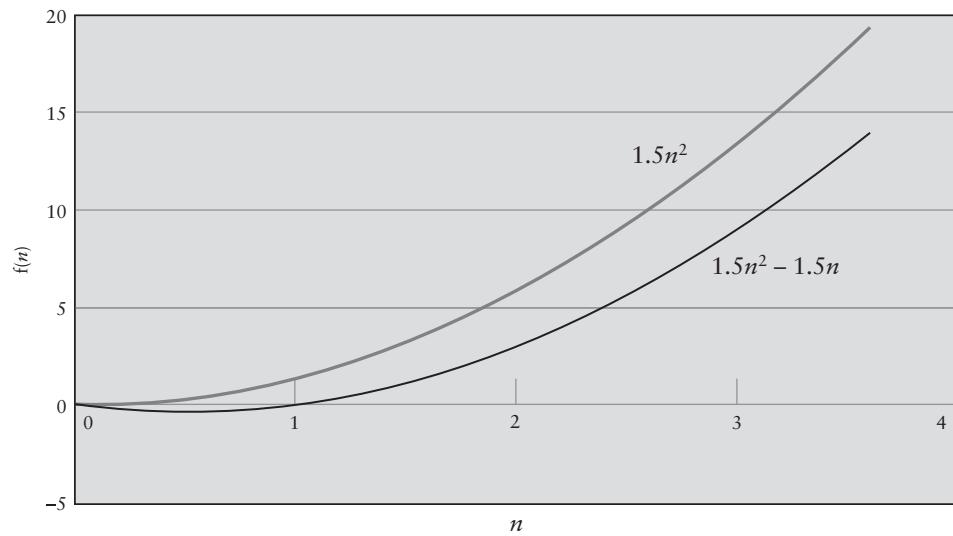
$$\frac{n^2 - n}{2}$$

Thus, our final  $T(n)$  is

$$T(n) = 1.5n^2 - 1.5n$$

This polynomial is zero when  $n$  is 1. For values greater than 1,  $1.5n^2$  is always greater than  $1.5n^2 - 1.5n$ . Therefore, we can use 1 for  $n_0$  and 1.5 for  $c$  to conclude that our  $T(n)$  is  $O(n^2)$  (see Figure 2.2).

.....  
**FIGURE 2.2**  
 $1.5n^2$  versus  $1.5n^2 - 1.5n$



If  $T(n)$  is the form of a polynomial of degree  $d$  (the highest exponent), then it is  $\mathbf{O}(n^d)$ . A mathematically rigorous proof of this is beyond the scope of this text. An intuitive proof is demonstrated in the previous two examples. If the remaining terms have positive coefficients, find a value of  $n$  where the first term is equal to the remaining terms. As  $n$  gets bigger than this value, the  $n^d$  term will always be bigger than the remaining terms.

We use the expression  $O(1)$  to represent a constant growth rate. This is a value that doesn't change with the number of inputs. The simple steps all represent  $O(1)$ . Any finite number of  $O(1)$  steps is still considered  $O(1)$ .

## Summary of Notation

In this section, we have used the symbols  $T(n)$ ,  $f(n)$ , and  $O(f(n))$ . Their meaning is summarized in Table 2.1.

**TABLE 2.1**

Symbols Used in Quantifying Software Performance

Symbol	Meaning
$T(n)$	The time that a method or program takes as a function of the number of inputs, $n$ . We may not be able to measure or determine this exactly
$f(n)$	Any function of $n$ . Generally, $f(n)$ will represent a simpler function than $T(n)$ , for example, $n^2$ rather than $1.5n^2 - 1.5n$
$O(f(n))$	Order of magnitude. $O(f(n))$ is the set of functions that grow no faster than $f(n)$ . We say that $T(n) = O(f(n))$ to indicate that the growth of $T(n)$ is bounded by the growth of $f(n)$

## Comparing Performance

Throughout this text, as we discuss various algorithms, we will discuss how their execution time or storage requirements grow as a function of the problem size using this big-O notation. Several common growth rates will be encountered and are summarized in Table 2.2.

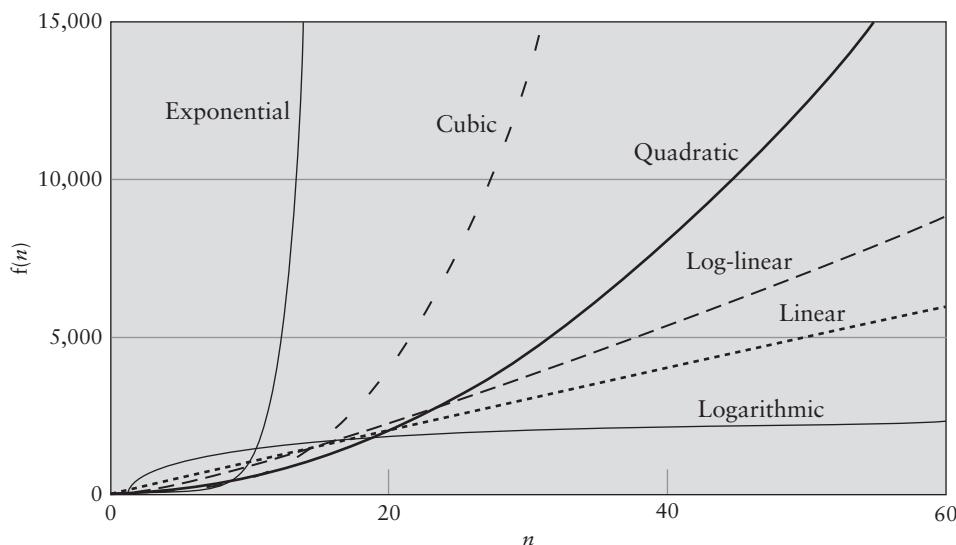
**TABLE 2.2**

Common Growth Rates

Big-O	Name
$O(1)$	Constant
$O(\log n)$	Logarithmic
$O(n)$	Linear
$O(n \log n)$	Log-linear
$O(n^2)$	Quadratic
$O(n^3)$	Cubic
$O(2^n)$	Exponential
$O(n!)$	Factorial

Figure 2.3 shows the growth rate of a logarithmic, a linear, a log-linear, a quadratic, a cubic, and an exponential function by plotting  $f(n)$  for a function of each type. Note that for small values of  $n$ , the exponential function is smaller than all of the others. As shown, it is not until  $n$  reaches 20 that the linear function is smaller than the quadratic. This illustrates two points. For small values of  $n$ , the less efficient algorithm may be actually more efficient. If you know that you are going to process only a limited amount of data, the  $O(n^2)$  algorithm may be much more appropriate than the  $O(n \log n)$  algorithm that has a large constant factor. However, algorithms with exponential growth rates can start out small but very quickly grow to be quite large.

**FIGURE 2.3**  
Different Growth Rates



The raw numbers in Figure 2.3 can be deceiving. Part of the reason is that big-O notation ignores all constants. An algorithm with a logarithmic growth rate  $O(\log n)$  may be more complicated to program, so it may actually take more time per data item than an algorithm with a linear growth rate  $O(n)$ . For example, at  $n = 25$ , Figure 2.3 shows that the processing time is approximately 1800 units for an algorithm with a logarithmic growth rate and 2500 units for an algorithm with a linear growth rate. Comparisons of this sort are pretty meaningless. The logarithmic algorithm may actually take more time to execute than the linear algorithm for this relatively small data set. Again, what is important is the growth rate of these two kinds of algorithms, which tells you how the performance of each kind of algorithm changes with  $n$ .

---

**EXAMPLE 2.7** Let's look at how growth rates change as we double the value of  $n$  (say, from  $n = 50$  to  $n = 100$ ). The results are shown in Table 2.3. The third column gives the ratio of processing times for the two different data sizes. For example, it shows that it will take 2.35 times as long to process 100 numbers as it would to process 50 numbers with an  $O(n \log n)$  algorithm.

**TABLE 2.3**  
Effects of Different Growth Rates

$O(f(n))$	$f(50)$	$f(100)$	$f(100)/f(50)$
$O(1)$	1	1	1
$O(\log n)$	5.64	6.64	1.18
$O(n)$	50	100	2
$O(n \log n)$	282	664	2.35
$O(n^2)$	2500	10,000	4
$O(n^3)$	12,500	100,000	8
$O(2^n)$	$1.126 \times 10^{15}$	$1.27 \times 10^{30}$	$1.126 \times 10^{15}$
$O(n!)$	$3.0 \times 10^{64}$	$9.3 \times 10^{57}$	$3.1 \times 10^{93}$

## Algorithms with Exponential and Factorial Growth Rates

Algorithms with exponential and factorial (even faster) growth rates have an effective practical upper limit on the size of problem they can be used for, even with faster and faster computers. For example, if we have an  $O(2^n)$  algorithm that takes an hour for 100 inputs, adding the 101st input will take a second hour, adding 5 more inputs will take 32 hours (more than a day!), and adding 14 inputs will take 16,384 hours, which is almost 2 years! This relation is the basis for cryptographic algorithms—algorithms that *encrypt* text using a special key to make it unreadable by anyone who intercepts it and does not know the key. Encryption is used to provide security for sensitive data sent over the Internet. Some cryptographic algorithms can be broken in  $O(2^n)$  time, where  $n$  is the number of bits in the key. A key length of 40 bits is considered breakable by a modern computer, but a key length of 100 (60 bits longer) is not because the key with a length of 100 bits will take approximately a billion-billion ( $10^{18}$ ) times as long as the 40-bit key to crack.

---

## EXERCISES FOR SECTION 2.1

### SELF-CHECK

- Determine how many times the output statement is executed in each of the following fragments. Indicate whether the algorithm is  $O(n)$  or  $O(n^2)$ .
  - ```
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        System.out.println(i + " " + j);
```
  - ```
for (int i = 0; i < n; i++)
    for (int j = 0; j < 2; j++)
        System.out.println(i + " " + j);
```
  - ```
for (int i = 0; i < n; i++)
    for (int j = n - 1; j >= i; j--)
        System.out.println(i + " " + j);
```
  - ```
for (int i = 1; i < n; i++)
    for (int j = 0; j < i; j++)
        if (j % i == 0)
            System.out.println(i + " " + j);
```
- For the following  $T(n)$ , find values of  $n_0$  and  $c$  such that  $cn^3$  is larger than  $T(n)$  for all  $n$  larger than  $n_0$ .  

$$T(n) = n^3 - 5n^2 + 20n - 10$$
- How does the performance grow as  $n$  goes from 2000 to 4000 for the following? Answer the same question as  $n$  goes from 4000 to 8000. Provide tables similar to Table 2.3.
  - $O(\log n)$
  - $O(n)$
  - $O(n \log n)$
  - $O(n^2)$
  - $O(n^3)$
- According to the plots in Figure 2.3, what are the processing times at  $n = 20$  and at  $n = 40$  for each of the growth rates shown?

### PROGRAMMING

- Write a program that compares the values of  $y1$  and  $y2$  in the following expressions for values of  $n$  up to 100 in increments of 10. Does the result surprise you?

```
y1 = 100 * n + 10
y2 = 5 * n * n + 2
```

## 2.2 The List Interface and ArrayList Class

An *array* is an indexed data structure, which means you can select its elements in arbitrary order as determined by the subscript value. You can also access the elements in sequence using a loop that increments the subscript. However, you can't do the following with an array object:

- Increase or decrease its length, which is fixed.
- Add an element at a specified position without shifting the other elements to make room.
- Remove an element at a specified position without shifting the other elements to fill in the resulting gap.

The classes that implement the Java `List` interface (part of Java API `java.util`) all provide methods to do these operations and more. Table 2.4 shows some of the methods in the Java `List` interface.

These methods perform the following operations:

- Return a reference to an element at a specified location (method `get`).
- Find a specified target value (method `get`).
- Add an element at the end of the list (method `add`).
- Insert an element anywhere in the list (method `add`).
- Remove an element (method `remove`).
- Replace an element in the list with another (method `set`).
- Return the size of the list (method `size`).
- Sequentially access all the list elements without having to manipulate a subscript.

The symbol `E` in Table 2.4 is a *type parameter*. Type parameters are analogous to method parameters. In the declaration of an interface or class, the type parameter represents the data type of all objects stored in a collection.

Although all of the classes we study in this chapter support the operations in Table 2.4, they do not do them with the same degree of efficiency. The kinds of operations you intend to perform in a particular application should influence your decision as to which `List` class to use.

One feature that the array data structure provides that these classes don't is the ability to store primitive-type values. The `List` classes all store references to `Objects`, so all primitive-type values must be wrapped in objects. Autoboxing facilitates this.

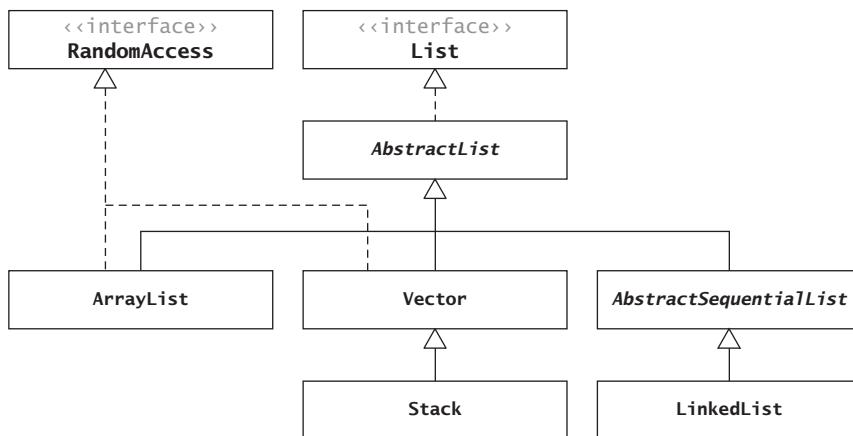
Figure 2.4 shows an overview of the `List` interface and the four actual classes that implement it. We will study the `ArrayList` and `LinkedList` classes in this chapter; we will study the `Stack`

**TABLE 2.4**

Methods of Interface `java.util.List<E>`

Method	Behavior
<code>public E get(int index)</code>	Returns the data in the element at position <code>index</code>
<code>public E set(int index, E anEntry)</code>	Stores a reference to <code>anEntry</code> in the element at position <code>index</code> . Returns the data formerly at position <code>index</code>
<code>public int size()</code>	Gets the current size of the <code>List</code>
<code>public boolean add(E anEntry)</code>	Adds a reference to <code>anEntry</code> at the end of the <code>List</code> . Always returns <code>true</code>
<code>public void add(int index, E anEntry)</code>	Adds a reference to <code>anEntry</code> , inserting it before the item at position <code>index</code>
<code>int indexOf(E target)</code>	Searches for <code>target</code> and returns the position of the first occurrence, or <code>-1</code> if it is not in the <code>List</code>
<code>E remove(int index)</code>	Removes the entry formerly at position <code>index</code> and returns it

**FIGURE 2.4**  
The `java.util.List`  
Interface and Its  
Implementers



class Chapter 4. Class `Vector` has been *deprecated*, which means it is included for historical reasons. Deprecated classes have been replaced by better classes and should not be used in new applications. We will briefly discuss the `RandomAccess` interface and the two abstract classes `AbstractList` and `AbstractSequentialList` in Section 2.10.

## The `ArrayList` Class

The simplest class that implements the `List` interface is the `ArrayList` class. An `ArrayList` object is an improvement over an array object in that it supports all of the operations just listed. `ArrayList` objects are used most often when a programmer wants to be able to grow a list by adding new elements to the end but still needs the capability to access the elements stored in the list in arbitrary order. These are the features we would need for an e-mail address book application: New entries should be added at the end, and we would also need to find e-mail addresses for entries already in the address book. The size of an `ArrayList` automatically increases as new elements are added to it, and the size decreases as elements are removed. An `ArrayList` object has an instance method `size` that returns its current size.

Each `ArrayList` object has a *capacity*, which is the number of elements it can store. If you add a new element to an `ArrayList` whose current size is equal to its capacity, the capacity is automatically increased.



### FOR PYTHON PROGRAMMERS

The Python `list` class is similar to the Java `ArrayList` class. Both can store a collection of objects and both automatically expand when extra space is needed. Both have methods to add elements, insert elements, and get the list length. However, you cannot use array index notation (e.g., `scores[3]`) with an `ArrayList` but you can with a Python `list`.

---

### EXAMPLE 2.8 The Statements

```
List<String> yourList;
yourList = new ArrayList<String>();
```

```
List<String> myList = new ArrayList<String>();
```

declare `List` variables `myList` and `yourList` whose elements will reference `String` objects. The actual lists referenced by `myList` and `yourList` are `ArrayList<String>` objects. Variable `yourList` is declared as type `List` in the first statement and then it is created as an `ArrayList` in the

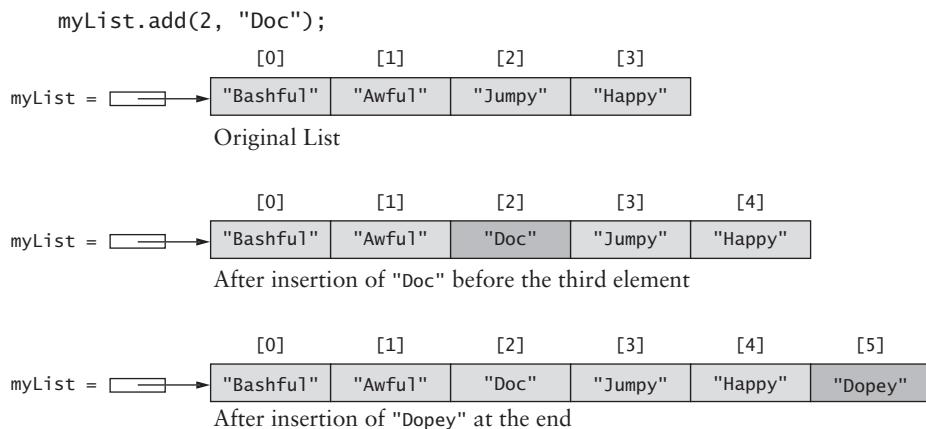
second statement. The third statement both declares the type of `myList` (`List`) and creates it as an `ArrayList`. Initially, `myList` is empty; however, it has an initial capacity of 10 elements (the default capacity).

The statements

```
myList.add("Bashful");
myList.add("Awful");
myList.add("Jumpy");
myList.add("Happy");
```

add references to four strings as shown in the top diagram of Figure 2.5. The value of `myList.size()` is now 4.

The middle diagram of Figure 2.5 shows `ArrayList` object `myList` after the insertion of the reference to "Doc" at the element with subscript 2:




---

The new size is 5. The strings formerly referenced by the elements with subscripts 2 and 3 are now referenced by the elements with subscripts 3 and 4. This is the same as what happens when someone cuts into a line of people waiting to buy tickets; everyone following the person who cuts in moves back one position in the line.

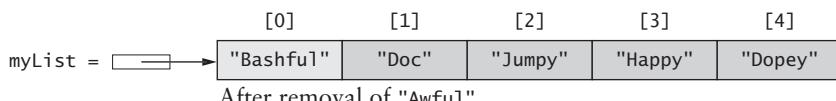
The bottom diagram in Figure 2.5 shows the effect of the statement

```
myList.add("Dopey");
```

which adds a reference to "Dopey" at the end of the `ArrayList`. The size of `myList` is now 6.

Similarly, if you remove an element from an `ArrayList` object, the size automatically decreases, and the elements following the one removed shift over to fill the vacated space. This is the same as when someone leaves a ticket line; the people in back all move forward. Here is object `myList` after using the statement

```
myList.remove(1);
```



to remove the element with subscript 1. Note that the strings formerly referenced by subscripts 2 through 5 are now referenced by subscripts 1 through 4 (in the darker color), and the size has decreased by 1.

Even though an `ArrayList` is an indexed collection, you can't access its elements using a subscript. Instead, you use the `get` method to access its elements. For example, the statement

```
String dwarf = myList.get(2)
```

stores a reference to the string object "Jumpy" in variable `dwarf`, without changing `myList`.

You use the `set` method to store a value in an `ArrayList`. The method call

```
myList.set(2, "Sneezy")
```

stores a reference to string "Sneezy" at index 2, replacing the reference to string "Jumpy". However, variable `dwarf` would still reference the string "Jumpy".

[0]	[1]	[2]	[3]	[4]
myList =  → [ ]	"Bashful"	"Doc"	"Sneezy"	"Happy"

After replacing "Jumpy" with "Sneezy"

You can also search an `ArrayList`. The method call

```
myList.indexOf("Jumpy")
```

would return `-1` after the reference to "Jumpy" was replaced, indicating an unsuccessful search. The method call

```
myList.indexOf("Sneezy")
```

would return `2`.



## PITFALL

### Using Subscripts with an `ArrayList`

If you use a subscript with an `ArrayList` (e.g., `myList[i]`), you will get the syntax error `array type required for [] but ArrayList found`. This means that subscripts can be used only with arrays, not with indexed collections.

## Generic Collections

The statement

```
List<String> myList = new ArrayList<String>();
```

uses a language feature introduced in Java 5.0 called *generic collections* (or *generics*). Generics allow you to define a collection that contains references to objects of a specific type. The declaration for `myList` specifies that it is a `List` of `String` where `String` is a *type parameter*. Furthermore, `myList` references an `ArrayList<String>` object. Therefore, only references to objects of type `String` can be stored in `myList`, and all items retrieved would be of type `String`.



## SYNTAX Creating a Generic Collection

### FORM:

```
CollectionClassName<E> variable = new CollectionClassName<>();
CollectionClassName<E> variable = new CollectionClassName<E>();
```

### EXAMPLE:

```
List<Person> people = new ArrayList<>();
List<String> myList = new ArrayList<String>();
ArrayList<Integer> numList = new ArrayList<>();
```

### MEANING:

An initially empty `CollectionClassName<E>` object is created that can be used to store references to objects of type `E` (the type parameter). The actual object type stored in

an object of type *CollectionClassName*<E> is specified when the object is created. If the *CollectionClassName* on the left is an interface, the *CollectionClassName* on the right must be a class that implements it. Otherwise, it must be the same class or a subclass of the one on the left.

The examples above show different ways to create an *ArrayList*. In this text, we normally specify the interface name on the left of the = operator and the implementing class name on the right as shown in the first two examples. Since the type parameter E must be the same on both sides of the assignment operator, Java 7 introduced the diamond operator <> which eliminates the need to specify the type parameter twice. We will follow this convention. In some cases, we will declare the variable type in one statement and create it in a later statement.

In earlier versions of Java, generic collections were not supported. In these versions, you use the statement

```
List yourList = new ArrayList();
```

to create an initially empty *ArrayList*. Each element of *yourList* is a type *Object* reference. The data types of the actual objects referenced by elements of *yourList* are not specified, and in fact, different elements can reference objects of different types.

Use of the adjective “generic” is a bit confusing. A nongeneric collection in Java is very general in that it can store objects of different data types. A generic collection, however, can store objects of one specified data type only. Therefore, generics enable the compiler to do more strict type checking to detect errors at compile time instead of at run time. They also eliminate the need to downcast from type *Object* to a specific type. For these reasons, we will always use generic collections.



## PROGRAM STYLE

The examples above show different ways to create an *ArrayList*. In this text, we normally specify the interface name on the left of the = operator and the implementing class name on the right as shown in the first two examples in the Syntax box above. Since the type parameter E must be the same on both sides of the assignment operator, Java 7 introduced the diamond operator <> that eliminates the need to specify the type parameter twice. We will follow this convention. In some cases, we will declare the variable type in one statement and create it in a later statement.



## PITFALL

### Adding Incompatible Type Objects to a Generic ArrayList

The advantage of generics is that the compiler can ensure that all operations involving objects referenced by a generic *ArrayList* are “safe” and will not cause exceptions during run time. Any type of incompatibility will be detected during compilation. If *myList* is type *ArrayList*<*String*>, the statement

```
myList.add(35);
```

will not compile because 35 (type *int*) is not compatible with type *String*.

## EXERCISES FOR SECTION 2.2

### SELF-CHECK

1. Describe the effect of each of the following operations on object myList as shown at the bottom of Figure 2.5. What is the value of myList.size() after each operation?

```
myList.add("Pokey");
myList.add("Campy");
int i = myList.indexOf("Happy");
myList.set(i, "Bouncy");
myList.remove(myList.size() - 2);
String temp = myList.get(1);
myList.set(1, temp.toUpperCase());
```

### PROGRAMMING

1. Write the following static method:

```
/** Replaces each occurrence of oldItem in aList with newItem. */
public static void replace(ArrayList<String> aList, String oldItem,
                           String newItem)
```

2. Write the following static method:

```
/** Deletes the first occurrence of target in aList. */
public static void delete(ArrayList<String> aList, String target)
```



## 2.3 Applications of ArrayList

We illustrate two applications of ArrayList objects next.

**EXAMPLE 2.9** The following statements create an ArrayList<Integer> object and load it with the values stored in a type int[] array. The statement

```
some.add(numsNext);
```

retrieves a value from array nums (type int[]), automatically wraps it in an Integer object, and stores a reference to that object in some (type ArrayList<Integer>).

The println statement shows how the list grows as each number is inserted.

```
List<Integer> some = new ArrayList<>();
int[] nums = {5, 7, 2, 15};
for (int numsNext : nums) {
    some.add(numsNext);
    System.out.println(some);
}
```

Loop exit occurs after the last Integer object is stored in some. The output displayed by this fragment follows:

```
[5]
[5, 7]
[5, 7, 2]
[5, 7, 2, 15]
```

The following fragment computes and displays the sum (29) of the `Integer` object values in `ArrayList` `some`. Note that we can use the enhanced for loop with an array and an `ArrayList`. We will talk more about this in Section 2.7.

```
int sum = 0;
for (Integer someText : some) {
    sum += someNext;
}
System.out.println("sum is " + sum);
```

Although it may seem wasteful to carry out these operations when you already have an array of `ints`, the purpose of this example is to illustrate the steps needed to process a collection of `Integer` objects referenced by an `ArrayList<Integer>`.

## A Phone Directory Application

If we want to write a program to store a list of names and phone numbers, we can use class `DirectoryEntry` to represent each item in our phone directory.

```
public class DirectoryEntry {
    String name;
    String number;
}
```

We can declare an `ArrayList<DirectoryEntry>` object to store a phone directory (`theDirectory`) with our friends' names and phone numbers:

```
private List<DirectoryEntry> theDirectory =
    new ArrayList<>();
```

We can use the statement

```
theDirectory.add(new DirectoryEntry("Jane Smith", "555-549-1234"));
```

to add an entry to `theDirectory`. If we want to retrieve the entry for a particular name (`String aName`), we can use the statements

```
int index = theDirectory.indexOf(new DirectoryEntry(aName, ""));
```

to locate the entry for the person referenced by `aName`. Method `indexOf` searches `theDirectory` by applying the `equals` method for class `DirectoryEntry` to each element of `theDirectory`. We are assuming that method `DirectoryEntry.equals` compares the `name` field of each element to the `name` field of the argument of `indexOf` (an anonymous object with the desired name). The statement

```
if (index != -1)
    dE = theDirectory.get(index);
else
    dE = null;
```

uses `ArrayList.get` to retrieve the desired entry (name and phone number) if found and stores a reference to it in `dE` (type `DirectoryEntry`). Otherwise, `null` is stored in `dE`.

## EXERCISES FOR SECTION 2.3

### SELF-CHECK

- What does the following code fragment do?

```
List<Double> myList = new ArrayList<>();
myList.add(3.456);
myList.add(5.0);
double result = myList.get(0) + myList.get(1);
System.out.println("Result is " + result);
```

## P R O G R A M M I N G

1. Write a method `addOrChangeEntry` for a class that has a data field `theDirectory` (type `ArrayList<DirectoryEntry>`) where class `DirectoryEntry` is described just before the exercises. Assume class `DirectoryEntry` has an accessor method `getNumber` and a setter method `setNumber`.

```
/** Add an entry to theDirectory or change an existing entry.
 * @param aName The name of the person being added or changed
 * @param newNumber The new number to be assigned
 * @return The old number, or if a new entry, null
 */
public String addOrChangeEntry(String aName, String newNumber) {
```

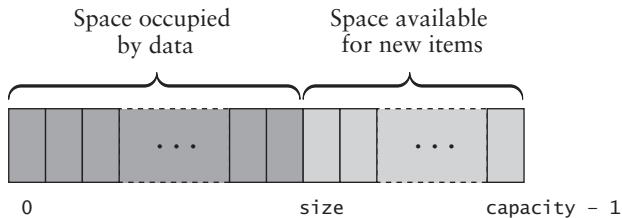
2. Write a `removeEntry` method for the class in programming exercise 1. Use `ArrayList` methods `indexOf` and `remove`.

```
/** Remove an entry.
 * @param aName The name of the person being removed
 * @return The entry removed, or null if there is no entry for aName
 */
public Entry removeEntry(String aName) {
```

## 2.4 Implementation of an `ArrayList` Class

We will implement our own version of the `ArrayList` class called `KWArrayList`. Just as Java does for an `ArrayList`, we use a Java array internally to contain the data of a `KWArrayList`, as shown in Figure 2.6. The physical size of the array is indicated by the data field `capacity`. The number of data items is indicated by the data field `size`. The elements between `size` and `capacity` are available for the storage of new items.

**FIGURE 2.6**  
Internal Structure of  
`KWArrayList`



We are assuming the following data fields in the discussion of our `KWArrayList` class. This is not exactly how it is done in Java, but it will give you a feel for how to write the class methods. The constructor shown in the following code allocates storage for the underlying array and initializes its capacity to 10. We will not provide a complete implementation because we expect you to use the standard `ArrayList` class provided by the Java API (package `java.util`).

We show the definition of a generic class `KWArrayList<E>` where `E` is the parameter type. The actual parameter type is specified when a generic `KWArrayList` object is declared. The data type of the references stored in the underlying array `theData` (type `E[]`) is also determined when the `KWArrayList` object is declared. If no parameter type is specified, the implicit parameter type is `Object`, and the underlying data array is type `Object[]`.

```
import java.util.*;
/** This class implements some of the methods of the Java ArrayList class. It
 * does not implement the List interface.
*/
```

```

public class KWArrayList<E> {
    // Data Fields
    /** The default initial capacity */
    private static final int INITIAL_CAPACITY = 10;

    /** The underlying data array */
    private E[] theData;

    /** The current size */
    private int size = 0;

    /** The current capacity */
    private int capacity = 0;
    ...
}

```

## The Constructor for Class KWArrayList<E>

The constructor declaration follows. Because the constructor is for a generic class, the type parameter <E> is implied but it must not appear in the constructor heading.

```

public KWArrayList() {
    capacity = INITIAL_CAPACITY;
    theData = (E[]) new Object[capacity];
}

```

The statement

```

theData = (E[]) new Object[capacity];

```

allocates storage for an array with type `Object` references and then casts this array object to type `E[]` so that it is type compatible with variable `theData`. Because the actual type corresponding to `E` is not known, the compiler issues the warning message: `KWArrayList.java` uses unchecked or unsafe operations. Don't be concerned about this warning—everything is fine.



### PROGRAM STYLE

Java provides an annotation that enables you to compile the constructor without an error message. If you place the statement

```
@SuppressWarnings("unchecked")
```

before the constructor, the compiler warning will not appear.



### PITFALL

#### Declaring a Generic Array

Rather than use the approach shown in the above constructor, you might try to create a generic array directly using the statement

```
theData = new E[capacity]; // Invalid generic array type.
```

However, this statement will not compile because Java does not allow you to create an array with an unspecified type. Remember, `E` is a type parameter that is not specified until a generic `ArrayList` object is created. Therefore, the constructor must create an array of type `Object[]` since `Object` is the superclass of all types and then downcast this array object to type `E[]`.

## The add(E anEntry) Method

We implement two add methods with different signatures. The first appends an item to the end of a KWArrayList; the second inserts an item at a specified position. If size is less than capacity, then to append a new item:

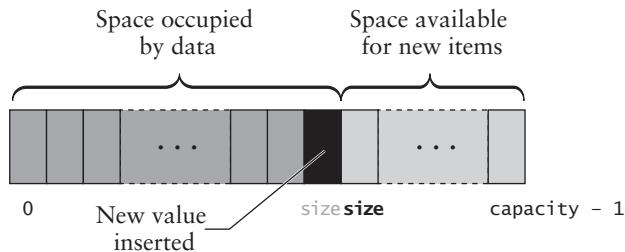
- Insert the new item at the position indicated by the value of size.
- Increment the value of size.
- Return true to indicate successful insertion.

This sequence of operations is illustrated in Figure 2.7. The add method is specified in the Collection interface, which is discussed in Section 2.10. The Collection interface is a super-interface to the List interface. The add method must return a boolean to indicate whether or not the insertion is successful. For an ArrayList, this is always true. The old value of size is in gray; its new value is in black.

If the size is already equal to the capacity, we must first allocate a new array to hold the data and then copy the data to this new array. The method reallocate (explained shortly) does this. The code for the add method follows.

```
public boolean add(E anEntry) {
    if (size == capacity) {
        reallocate();
    }
    theData[size] = anEntry;
    size++;
    return true;
}
```

**FIGURE 2.7**  
Adding an Element to  
the End of a  
KWArrayList



### PROGRAM STYLE

#### Using the Postfix (or Prefix) Operator with a Subscript

Some programmers prefer to combine the two statements before return in the add method and write them as

```
theData[size++] = theValue;
```

This is perfectly valid. Java uses the current value of size as the array subscript and then increments it. The only difficulty is the fact that two operations are written in one statement and are carried out in a predetermined order (first access array and then increment subscript). If you write the prefix operator (`++size`) by mistake, the subscript will increase before array access.

## The add(int index, E anEntry) Method

To insert an item into the middle of the array (anywhere but the end), the values that are at the insertion point and beyond must be shifted over to make room. In Figure 2.8, the arrow with label 1 shows the first element moved, the arrow with label 2 shows the next element moved, and so on. This data move is done using the following loop:

```
for (int i = size; i > index; i--) {
    theData[i] = theData[i - 1];
}
```

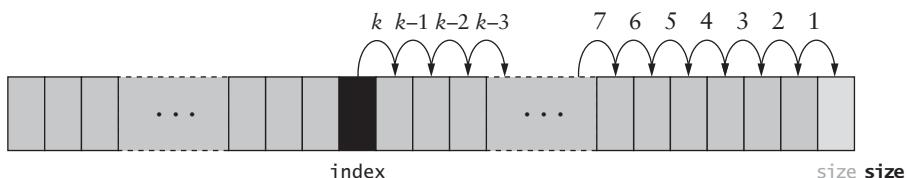
Note that the array subscript starts at `size` and moves toward the beginning of the array (down to `index + 1`). If we had started the subscript at `index + 1` instead, we would duplicate the value at `index` in each element of the array following it. Before we execute this loop, we need to be sure that `size` is not equal to `capacity`. If it is, we must call `reallocate`.

After increasing the capacity (if necessary) and moving the other elements, we can then add the new item. The complete code, including a test for an out-of-bounds value of `index`, follows:

```
public void add(int index, E anEntry) {
    if (index < 0 || index > size) {
        throw new ArrayIndexOutOfBoundsException(index);
    }
    if (size == capacity) {
        reallocate();
    }
    // Shift data in elements from index to size - 1
    for (int i = size; i > index; i--) {
        theData[i] = theData[i - 1];
    }
    // Insert the new item.
    theData[index] = anEntry;
    size++;
}
```

**FIGURE 2.8**

Making Room to Insert an Item into an Array



## The set and get Methods

Methods `set` and `get` throw an exception if the array index is out of bounds; otherwise, method `get` returns the item at the specified index. Method `set` inserts the new item (parameter `newValue`) at the specified index and returns the value (`oldValue`) that was previously stored at that index.

```
public E get(int index) {
    if (index < 0 || index >= size) {
        throw new ArrayIndexOutOfBoundsException(index);
    }
    return theData[index];
}

public E set(int index, E newValue) {
    if (index < 0 || index >= size) {
        throw new ArrayIndexOutOfBoundsException(index);
    }
}
```

```

        E oldValue = theData[index];
        theData[index] = newValue;
        return oldValue;
    }
}

```

## The remove Method

To remove an item, the items that follow it must be moved forward to close up the space. In Figure 2.9, the arrow with label 1 shows the first element moved, the arrow with label 2 shows the next element moved, and so on. This data move is done using the `for` loop in method `remove` shown next. The item removed is returned as the method result.

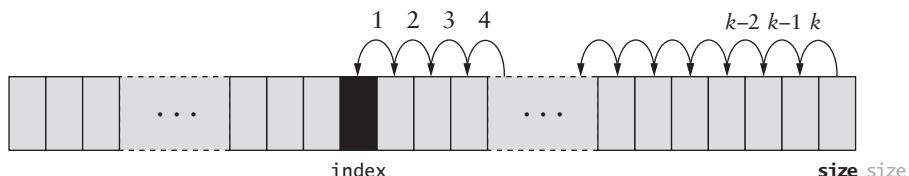
```

public E remove(int index) {
    if (index < 0 || index >= size) {
        throw new ArrayIndexOutOfBoundsException(index);
    }
    E returnValue = theData[index];
    for (int i = index + 1; i < size; i++) {
        theData[i - 1] = theData[i];
    }
    size--;
    return returnValue;
}

```

**FIGURE 2.9**

Removing an Item from an Array



## The reallocate Method

The `reallocate` method creates a new array that is twice the size of the current array and then copies the contents of the current array into the new one. The `Arrays.copyOf` method makes a copy of the given array truncating if the new array is shorter or padding with `null`s if the new array is larger, so that the copy has the specified length. The reference variable `theData` is then set to reference this new array. The code is as follows:

```

private void reallocate() {
    capacity = 2 * capacity;
    theData = Arrays.copyOf(theData, capacity);
}

```

The reason for doubling is to spread out the cost of copying. We discuss this next.

## Performance of the KWArrayList Algorithms

The `set` and `get` methods are each a few lines of code and contain no loops. Thus, we say that these methods execute in constant time, or  $O(1)$ .

If we insert into (or remove from) the middle of a `KWArrayList`, then at most  $n$  items have to be shifted. Therefore, the cost of inserting or removing an element is  $O(n)$ . What if we have to allocate a larger array before we can insert? Recall that when we reallocate the array, we double its size. Doubling an array of size  $n$  allows us to add  $n$  more items before we need to do another array copy. Therefore, we can add  $n$  new items after we have copied over  $n$  existing items. This averages out to 1 copy per add. Because each new array is twice the size of the previous one, it will take only about 20 reallocate operations to create an array that can store over a million references ( $2^{20}$  is greater than 1,000,000). Therefore, reallocation is effectively an  $O(1)$  operation, so the insertion is still  $O(n)$ .

## EXERCISES FOR SECTION 2.4

### SELF-CHECK

1. Trace the execution of the following:

```
int[] anArray = {0, 1, 2, 3, 4, 5, 6, 7};
for (int i = 3; i < anArray.length - 1; i++)
    anArray[i + 1] = anArray[i];
```

and the following:

```
int[] anArray = {0, 1, 2, 3, 4, 5, 6, 7};
for (int i = anArray.length - 1; i > 3; i--)
    anArray[i] = anArray[i - 1];
```

What are the contents of `anArray` after the execution of each loop?

2. Write statements to remove the middle object from a `KWArrayList` and place it at the end.

### PROGRAMMING

1. Implement the `indexOf` method of the `KWArrayList<E>` class.
2. Provide a constructor for class `KWArrayList<E>` that accepts an `int` argument that represents the initial array capacity. Use this instead of `INITIAL_CAPACITY`.



## 2.5 Single-Linked Lists

The `ArrayList` has the limitation that the `add` and `remove` methods operate in linear ( $O(n)$ ) time because they require a loop to shift elements in the underlying array (see Figures 2.8 and 2.9). In this section, we introduce a data structure, the linked list, that overcomes this limitation by providing the ability to add or remove items anywhere in the list in constant ( $O(1)$ ) time. A linked list is useful when you need to insert and remove elements at arbitrary locations (not just at the end) and when you do frequent insertions and removals.

One example would be maintaining an alphabetized list of students in a course at the beginning of a semester while students are adding and dropping courses. If you were using an `ArrayList`, you would have to shift all names that follow the new person's name down one position before you could insert a new student's name. Figure 2.10 shows this process. The names in gray were all shifted down when Barbara added the course. Similarly, if a student drops the course, the names of all students after the one who dropped (in gray in Figure 2.11) would be shifted up one position to close up the space.

Another example would be maintaining a list of students who are waiting to register for a course. Instead of having the students waiting in an actual line, you can give each student a

**FIGURE 2.10**  
Removing a Student  
from a Class List

Before adding Browniten, Barbara	After adding Browniten, Barbara
<b>Abidoye, Olandunni</b>	<b>Abidoye, Olandunni</b>
<b>Boado, Annabelle</b>	<b>Boado, Annabelle</b>
Butler, James	<b>Browniten, Barbara</b>
Chee, Yong-Han	Butler, James
Debaggis, Tarra	Chee, Yong-Han
:	Debaggis, Tarra
	:

**FIGURE 2.11**  
Removing a Student  
from a Class List

Before dropping Boado, Annabelle	After dropping Boado, Annabelle
<b>Abidoye, Olandunni</b>	<b>Abidoye, Olandunni</b>
<b>Boado, Annabelle</b>	Browniten, Barbara
Browniten, Barbara	Butler, James
Butler, James	Chee, Yong-Han
Chee, Yong-Han	Debaggis, Tarra
Debaggis, Tarra	:
:	

number, which is the student's position in the line. If someone drops out of the line, everyone with a higher number gets a new number that is 1 lower than before. If someone cuts into the line because they "need the course to graduate," everyone after this person gets a new number, which is one higher than before. The person maintaining the list is responsible for giving everyone their new number after a change. Figure 2.12 illustrates what happens when Alice is inserted and given the number 1: Everyone whose number is  $\geq 1$  gets a new number. This process is analogous to maintaining the names in an `ArrayList`; each person's number is that person's position in the list, and some names in the list are shifted after every change.

**FIGURE 2.12**  
Inserting into a  
Numbered List of  
Students Waiting to  
Register

Before inserting Alice at position 1	After inserting Alice at position 1
<b>0. Warner, Emily</b>	<b>0. Warner, Emily</b>
1. Dang, Phong	<b>1. Franklin, Alice</b>
2. Feldman, Anna	2. Dang, Phong
3. Barnes, Aaron	3. Feldman, Anna
4. Torres, Kristopher	4. Barnes, Aaron
:	5. Torres, Kristopher

A better way to do this would be to give each person the name of the next person in line, instead of his or her own position in the line (which can change frequently). To start the registration process, the person who is registering students calls the person who is at the head of the line. After he or she finishes registration, the person at the head of the line calls the next person, and so on. Now what if person A lets person B cut into the line after her? Because B will now register after A, person A must call B. Also, person B must call the person who originally followed A. Figure 2.13 illustrates what needs to be done to insert Alice in the list after Emily. Only the two highlighted entries need to be changed (Emily must call Alice instead of Phong, and Alice must call Phong). Although Alice is shown at the bottom of Figure 2.13 (third column), she is really the second student in the list. The first four students in the list are Emily Warner, Alice Franklin, Phong Dang, and Anna Feldman.

What happens if someone drops out of our line? In this case, the name of the person who follows the one who drops out must be given to the person who comes before the one who drops out. This is illustrated in Figure 2.14. If Aaron drops out, only one entry needs to be changed (Anna must call Kristopher instead of Aaron).

**FIGURE 2.13**  
Inserting into a List  
Where Each Student  
Knows Who Is Next

Before inserting Alice		After inserting Alice	
Person in line	Person to call	Person in line	Person to call
Warner, Emily	Dang, Phong	<b>Warner, Emily</b>	<b>Franklin, Alice</b>
Dang, Phong	Feldman, Anna	Dang, Phong	Feldman, Anna
Feldman, Anna	Barnes, Aaron	Feldman, Anna	Barnes, Aaron
Barnes, Aaron	Torres, Kristopher	Barnes, Aaron	Torres, Kristopher
Torres, Kristopher	...	Torres, Kristopher	...
:	...		...
		<b>Franklin, Alice</b>	<b>Dang, Phong</b>

**FIGURE 2.14**

Removing a Student from a List Where Each Student Knows Who Is Next

Before dropping Aaron		After dropping Aaron	
Person in line	Person to call	Person in line	Person to call
Warner, Emily	Franklin, Alice	Warner, Emily	Franklin, Alice
Dang, Phong	Feldman, Anna	Dang, Phong	Feldman, Anna
Feldman, Anna	Barnes, Aaron	<b>Feldman, Anna</b>	<b>Torres, Kristopher</b>
Barnes, Aaron	Torres, Kristopher	Barnes, Aaron	Torres, Kristopher
Torres, Kristopher	...	Torres, Kristopher	...
Franklin, Alice	Dang, Phong	Franklin, Alice	Dang, Phong

Using a linked list is analogous to the process just discussed and illustrated in Figures 2.13 and 2.14 for storing our list of student names. After we find the position of a node to be inserted or removed, the actual insertion or removal is done in constant time and no shifts are required. Each element in a linked list, called a node, stores information and a link to the next node in the list. For example, for our list of students in Figure 2.14, the information "Warner, Emily" would be stored in the first node, and the link to the next node would reference a node whose information part was "Franklin, Alice". Here are the first three nodes of this list:

"Warner, Emily" ==> "Franklin, Alice" ==> "Dang, Phong"

We discuss how to represent and manipulate a linked list next.

## A List Node

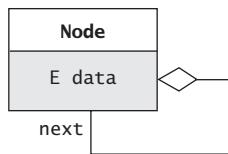
A *node* is a data structure that contains a data item and one or more links. A *link* is a reference to a node. A UML (Unified Modeling Language) diagram of this relationship is shown in Figure 2.15. This shows that a Node contains a data field named data of type E and a reference (as indicated by the open diamond) to a Node. The name of the reference is next, as shown on the line from the Node to itself.

Figure 2.16 shows four nodes linked together to form the list "Tom" ==> "Dick" ==> "Harry" ==> "Sam". In this figure, we show that data references a String object. In subsequent figures, we will show the string value inside the Node. We will explain the purpose of the box in the left margin when we define class `KWSingleLinkedList`.

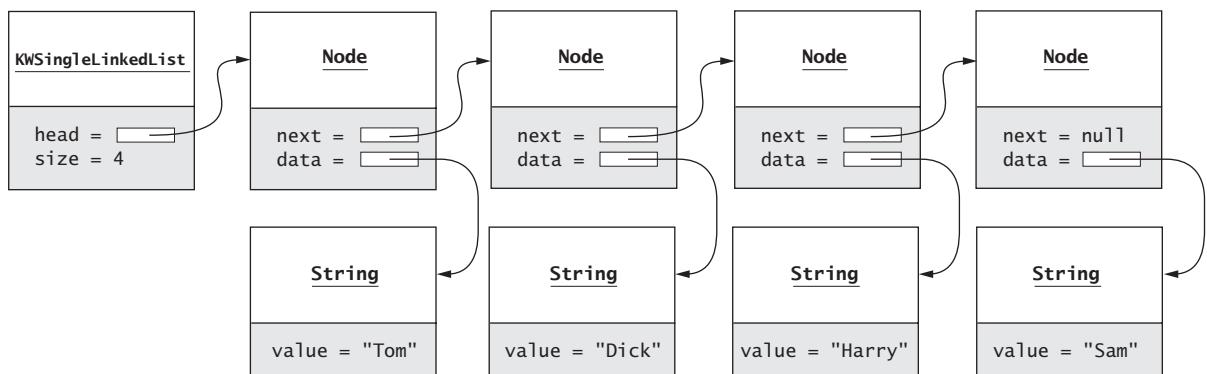
Next, we define a class `Node<E>` (see Listing 2.1) as an inner class that can be placed inside a generic list class. When each node is created, the type parameter E specifies the type of data stored in the node.

**FIGURE 2.15**

Node and Link

**FIGURE 2.16**

Nodes in a Linked List



**LISTING 2.1**

An Inner Class Node

```
.....  

LISTING 2.1  

An Inner Class Node  

/** A Node is the building block for a single-linked list. */  

private static class Node<E> {  

    // Data Fields  

    /** The reference to the data. */  

    private E data;  

    /** The reference to the next node. */  

    private Node<E> next;  

    // Constructors  

    /** Creates a new node with a null next field.  

     * @param dataItem The data stored  

     */  

    private Node(E dataItem) {  

        data = dataItem;  

        next = null;  

    }  

    /** Creates a new node that references another node.  

     * @param dataItem The data stored  

     * @param nodeRef The node referenced by new node  

     */  

    private Node(E dataItem, Node<E> nodeRef) {  

        data = dataItem;  

        next = nodeRef;  

    }  

}
```

The keyword `static` in the class header indicates that the `Node<E>` class will not reference its outer class. (It can't because it has no methods other than constructors.) In the Java API documentation, static inner classes are also called *nested classes*.

Generally, we want to keep the details of the `Node` class private. Thus, the qualifier `private` is applied to the class as well as to the data fields and the constructor. However, the data fields and methods of an inner class are visible anywhere within the enclosing class (also called the *parent class*).

The first constructor stores the data passed to it in the instance variable `data` of a new node. It also sets the `next` field to `null`. The second constructor sets the `next` field to reference the same node as its second argument. We didn't define a default constructor because none is needed.

## Connecting Nodes

We can construct the list shown in Figure 2.16 using the following sequence of statements:

```
Node<String> tom = new Node<>("Tom");  

Node<String> dick = new Node<>("Dick");  

Node<String> harry = new Node<>("Harry");  

Node<String> sam = new Node<>("Sam");  

tom.next = dick;  

dick.next = harry;  

harry.next = sam;
```

The assignment statement

```
tom.next = dick;
```

stores a reference (link) to the node with data "Dick" in the variable `next` of node `tom`.

## A Single-Linked List Class

Java does not have a class that implements single-linked lists. Instead, it has a more general double-linked list class, which we will be discussed in the next section. However, we will create a `KWSingleLinkedList` class to show you how these operations could be implemented.

```
/** Class to represent a linked list with a link from each node to the next
 *  node. SingleLinkedList does not implement the List interface.
 */
public class KWSingleLinkedList<E> {
    /** Reference to list head. */
    private Node<E> head = null;
    /** The number of items in the list */
    private int size = 0;
    ...
}
```

KWSingleLinkedList
head = null size = 0

A new `KWSingleLinkedList` object has two data fields, `head` and `size`, with initial values `null` and 0, respectively, as shown in the diagram in the margin. The data field `head` will reference the first list node called the *list head*. Method `addFirst` below inserts one element at a time to the front of the list, thereby changing the node pointed to by `head`. In the call to the constructor for `Node`, the argument `head` references the current first list node. A new node is created, which is referenced by `head` and is linked to the previous list head. Variable `data` of the new list head references `item`.

```
/** Add an item to the front of the list.
 * @param item The item to be added
 */
public void addFirst(E item) {
    head = new Node<>(item, head);
    size++;
}
```

The following fragment creates a linked list `names` and builds the list shown in Figure 2.16 using method `addFirst`:

```
KWSingleLinkedList<String> names = new KWSingleLinkedList<>();
names.addFirst("Sam");
names.addFirst("Harry");
names.addFirst("Dick");
names.addFirst("Tom");
```

## Inserting a Node in a List

If we have a reference `harry` to node "Harry", we can insert a new node, "Bob", into the list after "Harry" as follows:

```
Node<String> bob = new Node<>("Bob");
bob.next = harry.next; // Step 1
harry.next = bob; // Step 2
```

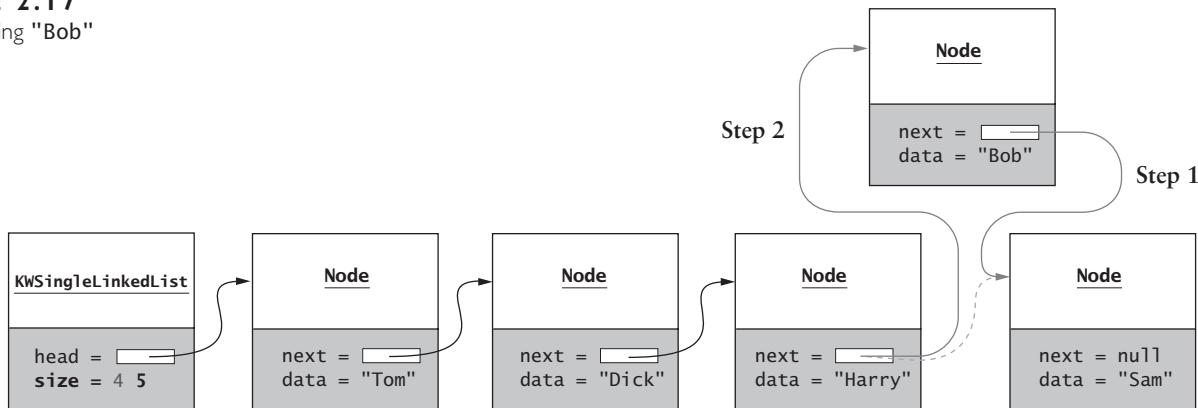
The linked list now is as shown in Figure 2.17. We show the number of the step that created each link alongside it.

We can generalize this by writing the method `addAfter` as follows:

```
/** Add a node after a given node
 * @param node The node preceding the new item
 * @param item The item to insert
 */
private void addAfter(Node<E> node, E item) {
    node.next = new Node<>(item, node.next);
    size++;
}
```

**FIGURE 2.17**

After Inserting "Bob"



We declare this method `private` since it should not be called from outside the class. This is because we want to keep the internal structure of the class hidden. Such private methods are known as helper methods because they will help implement the public methods of the class. Later we will see how `addAfter` is used to implement the public `add` methods.

## Removing a Node

If we have a reference, `tom`, to the node that contains "Tom", we can remove the node that follows "Tom":

```
tom.next = tom.next.next;
```

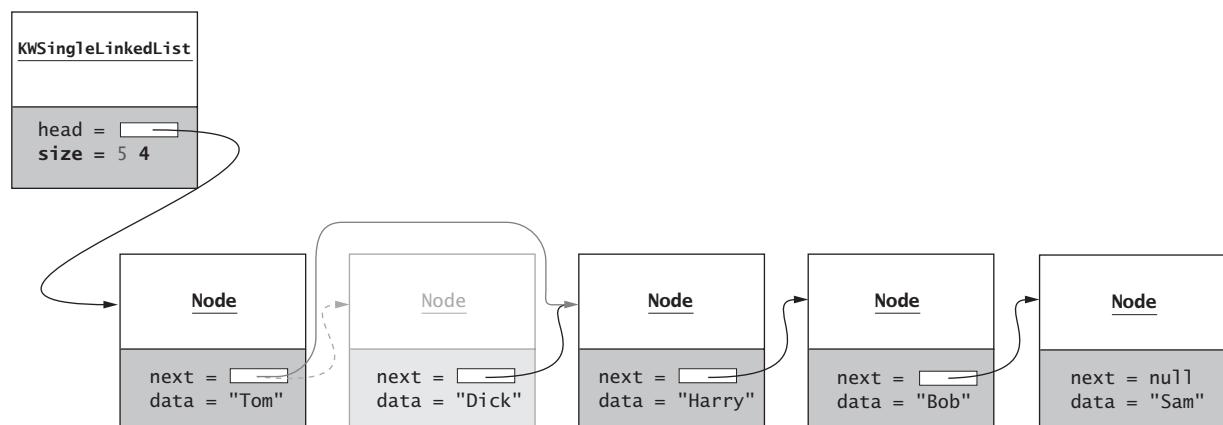
The list is now as shown in Figure 2.18. Note that we did not start with a reference to "Dick" but instead began with a reference to "Tom". To delete a node, we need a reference to the node that precedes it, not the node being deleted. (Recall from our registration list example that the person in front of the one dropping out of line must be told to call the person who follows the one who is dropping out.)

Again, we can generalize this by writing the `removeAfter` method:

```
/** Remove the node after a given node
 * @param node The node before the one to be removed
 * @return The data from the removed node, or null
 *         if there is no node to remove
 */
```

**FIGURE 2.18**

After Removing "Dick"



```

private E removeAfter(Node<E> node) {
    Node<E> temp = node.next;
    if (temp != null) {
        node.next = temp.next;
        size--;
        return temp.data;
    } else {
        return null;
    }
}

```

The `removeAfter` method works on all nodes except for the first one. For that we need a special method, `removeFirst`:

```

/** Remove the first node from the list
 * @return The removed node's data or null if the list is empty
 */
private E removeFirst() {
    Node<E> temp = head;
    if (head != null) {
        head = head.next;
    }
    // Return data at old head or null if list is empty
    if (temp != null) {
        size--;
        return temp.data;
    } else {
        return null;
    }
}

```

## Completing the SingleLinkedList Class

We conclude our illustration of the single-linked list data structure by showing how it can be used to implement a limited subset of the methods required by the `List` interface (see Table 2.5). Specifically, we will write the `get`, `set`, and `add`, methods. Methods `size`, `indexOf`, and `remove` are left as exercises. Recall from the `ArrayList` class that each of these methods takes an index parameter, but we showed above that the methods to add and remove a node need a reference to a node. We need an additional helper method to get a node at a given index.

**TABLE 2.5**

Methods of Interface `java.util.List<E>`

Method	Behavior
<code>public E get(int index)</code>	Returns the data in the element at position <code>index</code>
<code>public E set(int index, E anEntry)</code>	Stores a reference to <code>anEntry</code> in the element at position <code>index</code> . Returns the data formerly at position <code>index</code>
<code>public int size()</code>	Gets the current size of the <code>List</code>
<code>public boolean add(E anEntry)</code>	Adds a reference to <code>anEntry</code> at the end of the <code>List</code> . Always returns <code>true</code>
<code>public void add(int index, E anEntry)</code>	Adds a reference to <code>anEntry</code> , inserting it before the item at position <code>index</code>
<code>int indexOf(E target)</code>	Searches for <code>target</code> and returns the position of the first occurrence, or <code>-1</code> if it is not in the <code>List</code>
<code>E remove(int index)</code>	Removes the entry formerly at position <code>index</code> and returns it

```

    /**
     * Find the node at a specified position
     * @param index The position of the node sought
     * @return The node at index or null if it does not exist
     */
    private Node<E> getNode(int index) {
        Node<E> node = head;
        for (int i = 0; i < index && node != null; i++) {
            node = node.next;
        }
        return node;
    }

```

## The get and set Methods

Using the getNode method, the get and set methods are straightforward:

```

    /**
     * Get the data at index
     * @param index The position of the data to return
     * @return The data at index
     * @throws IndexOutOfBoundsException if index is out of range
     */
    public E get(int index) {
        if (index < 0 || index >= size) {
            throw new IndexOutOfBoundsException(Integer.toString(index));
        }
        Node<E> node = getNode(index);
        return node.data;
    }

    /**
     * Store a reference to anEntry in the element at position index.
     * @param index The position of the item to change
     * @param newValue The new data
     * @return The data previously at index
     * @throws IndexOutOfBoundsException if index is out of range
     */
    public E set(int index, E newValue) {
        if (index < 0 || index >= size) {
            throw new IndexOutOfBoundsException(Integer.toString(index));
        }
        Node<E> node = getNode(index);
        E result = node.data;
        node.data = newValue;
        return result;
    }

```

## The add Methods

After verifying that the index is in range, the index is checked for the special case of adding at the first element. If the index is zero, then the addFirst method is used to insert the new item; otherwise the addAfter method is used. Note that getNode (called before addAfter) returns a reference to the predecessor of the node to be inserted.

```

    /**
     * Insert the specified item at index
     * @param index The position where item is to be inserted
     * @param item The item to be inserted
     * @throws IndexOutOfBoundsException if index is out of range
     */
    public void add(int index, E item) {
        if (index < 0 || index > size) {
            throw new IndexOutOfBoundsException(Integer.toString(index));
        }

```

```

        if (index == 0) {
            addFirst(item);
        } else {
            Node<E> node = getNode(index-1);
            addAfter(node, item);
        }
    }
}

```

The `List` interface also specifies an `add` method without an index that adds (appends) an item to the end of a list. It can be easily implemented by calling the `add(int index, E item)` method using `size` as the index.

```

/** Append item to the end of the list
 * @param item The item to be appended
 * @return true (as specified by the Collection interface)
 */
public boolean add(E item) {
    add(size, item);
    return true;
}

```

## EXERCISES FOR SECTION 2.5

### SELF-CHECK

- What is the big-O for the single-linked list `get` operation?
- What is the big-O for the `set` operation?
- What is the big-O for each `add` method?
- Draw a single-linked list of `Integer` objects containing the integers 5, 10, 7, and 30 and referenced by `head`. Complete the following fragment, which adds all `Integer` objects in a list. Your fragment should walk down the list, adding all integer values to `sum`.

```

int sum = 0;
Node<Integer> nodeRef = _____;
while (nodeRef != null) {
    int next = _____;
    sum += next;
    nodeRef = _____;
}

```

- For the single-linked list in Figure 2.16, data field `head` (type `Node<String>`) references the first node. Explain the effect of each statement in the following fragments.

```

a. head = new Node<>("Shakira", head.next);
b. Node<String> nodeRef = head.next;
    nodeRef.next = nodeRef.next.next;
c. Node<String> nodeRef = head;
    while (nodeRef.next != null)
        nodeRef = nodeRef.next;
    nodeRef.next = new Node<>("Tamika");
d. Node<String> nodeRef = head;
    while (nodeRef != null && !nodeRef.data.equals("Harry"))
        nodeRef = nodeRef.next;
    if (nodeRef != null) {
        nodeRef.data = "Sally";
        nodeRef.next = new Node<>("Harry", nodeRef.next.next);
    }
}

```

## P R O G R A M M I N G

1. Using the single-linked list shown in Figure 2.16, and assuming that `head` references the first `Node` and `tail` references the last `Node`, write statements to do each of the following:
  - a. Insert "Bill" before "Tom".
  - b. Insert "Sue" before "Sam".
  - c. Remove "Bill".
  - d. Remove "Sam".
2. Write method `size`.
3. Write method `indexOf`.
4. Write method `remove`. Use helper methods `getNode`, `removeFirst` and `removeAfter`. Method `remove` should throw an exception if `index` is out-of-bounds.
5. Write the `remove` method whose method heading follows.

```
/** Remove the first occurrence of element item.
    @param item The item to be removed
    @return true if item is found and removed; otherwise, return false.
*/
public boolean remove(E item)
```

6. Write the following method `add` for class `SingleLinkedList<E>` without using any helper methods.

```
/** Insert a new item before the one at position index,
    starting at 0 for the list head. The new item is inserted between the one
    at position index-1 and the one formerly at position index.
    @param index The index where the new item is to be inserted
    @param item The item to be inserted
    @throws IndexOutOfBoundsException if the index is out of range
*/
public void add(int index, E item)
```

## 2.6 Double-Linked Lists and Circular Lists

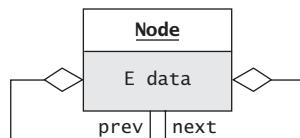
Our single-linked list data structure has some limitations:

- Insertion at the front of the list is  $O(1)$ . Insertion at other positions is  $O(n)$ , where  $n$  is the size of the list.
- We can insert a node only after a node for which we have a reference. For example, to insert "Bob" in Figure 2.17, we needed a reference to the node containing "Harry". If we wanted to insert "Bob" before "Sam" but did not have a reference to "Harry", we would have to start at the beginning of the list and search until we found a node whose next node was "Sam".
- We can remove a node only if we have a reference to its predecessor node. For example, to remove "Dick" in Figure 2.18, we needed a reference to the node containing "Tom". If we wanted to remove "Dick" without having this reference, we would have to start at the beginning of the list and search until we found a node whose next node was "Dick".
- We can move in only one direction, starting at the list head, whereas with an `ArrayList` we can move forward (or backward) by incrementing (or decrementing) the index.

We can overcome these limitations by adding a reference to the previous node in the `Node` class, as shown in the UML class diagram in Figure 2.19. The open diamond indicates that both `prev` and `next` are references whose values can be changed. Our *double-linked list* is shown in Figure 2.20.

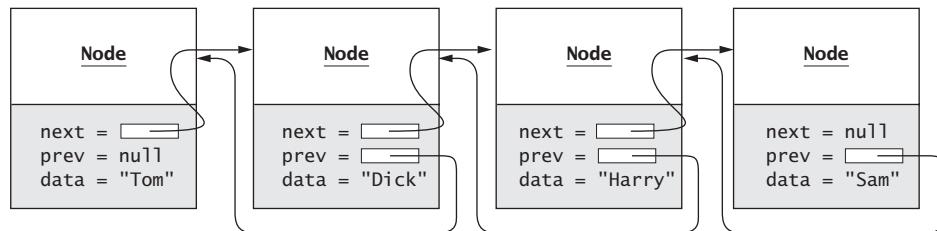
**FIGURE 2.19**

Double-Linked List Node UML Diagram



**FIGURE 2.20**

A Double-Linked List



## PITFALL

### Falling Off the End of a List

If `nodeRef` is at the last list element and you execute the statement

```
nodeRef = nodeRef.next;
```

`nodeRef` will be set to `null`, and you will fall off the end of the list. This is not an error. However, if you execute this statement again, you will get a `NullPointerException`, because `nodeRef.next` is undefined when `nodeRef` is `null`.

## The Node Class

The `Node` class for a double-linked list has references to the data and to the next and previous nodes. The declaration of this class follows.

```

/*
 * A Node is the building block for a double-linked list.
 */
private static class Node<E> {
    /**
     * The data value.
     */
    private E data;
    /**
     * The link to the next node.
     */
    private Node<E> next = null;
    /**
     * The link to the previous node.
     */
    private Node<E> prev = null;

    /**
     * Construct a node with the given data value.
     * @param dataItem The data value
     */
    private Node(E dataItem) {
        data = dataItem;
    }
}

```

## Inserting into a Double-Linked List

If `sam` is a reference to the node containing "Sam", we can insert a new node containing "Sharon" into the list before "Sam" using the following statements. Before the insertion, we can refer to the predecessor of `sam` as `sam.prev`. After the insertion, this node will be referenced by `sharon.prev`.

```
Node<String> sharon = new Node<>("Sharon");
// Link new node to its neighbors.
sharon.next = sam; // Step 1
sharon.prev = sam.prev; // Step 2
// Link old predecessor of sam to new predecessor.
sam.prev.next = sharon; // Step 3
// Link to new predecessor.
sam.prev = sharon; // Step 4
```

The three nodes affected by the insertion are shown in Figures 2.21 and 2.22. The old links are shown in black, and the new links are shown in gray. Next to each link we show the number of the step that creates it. Figure 2.21 shows the links after Steps 1 and 2, and Figure 2.22 shows the links after Steps 3 and 4.

## Removing from a Double-Linked List

If we have a reference, `harry`, to the node that contains "Harry", we can remove that node without having a named reference to its predecessor:

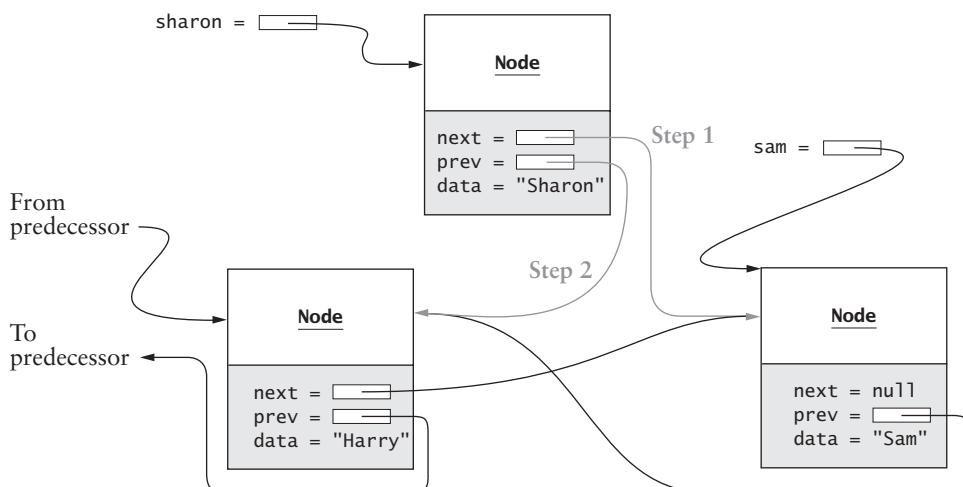
```
harry.prev.next = harry.next; // Step 1
harry.next.prev = harry.prev; // Step 2
```

The list is now as shown in Figure 2.23.

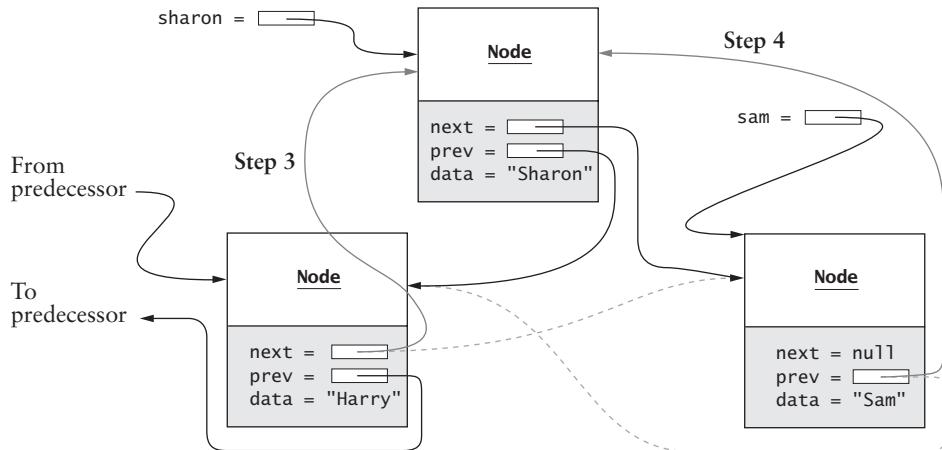
## A Double-Linked List Class

So far we have shown just the internal Nodes for a linked list. A double-linked list object would consist of a separate object with data fields `head` (a reference to the first list `Node`), `tail` (a reference to the last list `Node`), and `size` (the number of `Nodes`) (see Figure 2.24). Because both ends of the list are directly accessible, now insertion at either end is  $O(1)$ ; insertion elsewhere is still  $O(n)$ .

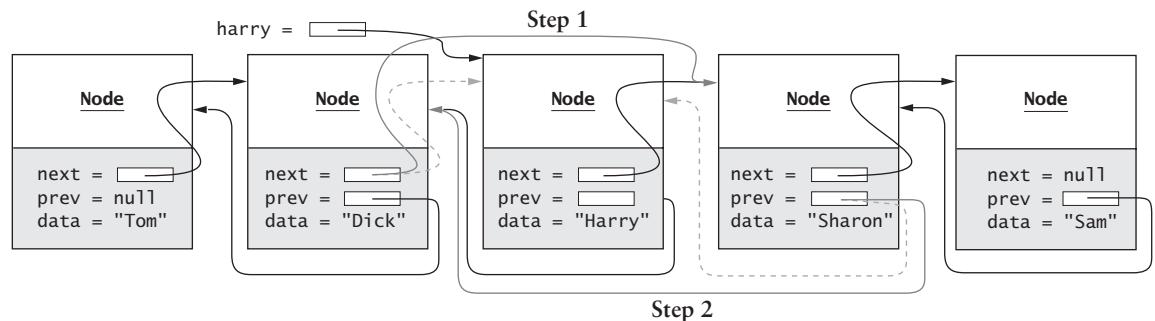
**FIGURE 2.21**  
Steps 1 and 2 in  
Inserting "Sharon"



**FIGURE 2.22**  
After Inserting "Sharon"  
before "Sam"



**FIGURE 2.23**  
Removing "Harry" from a Double-Linked List



**FIGURE 2.24**  
A Double-Linked List  
Object



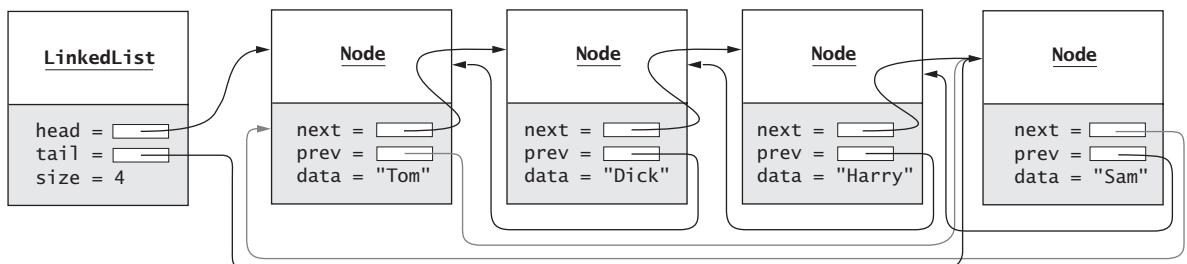
## Circular Lists

We can create a *circular* list from a double-linked list by linking the last node to the first node (and the first node to the last one). If `head` references the first list node and `tail` references the last list node, the statements

```
head.prev = tail;
tail.next = head;
```

would accomplish this (see Figure 2.25).

**FIGURE 2.25**  
A Circular Linked List



You could also create a circular list from a single-linked list by executing just the statement

```
tail.next = head;
```

This statement connects the last list element to the first list element. If you keep a reference to only the last list element, `tail`, you can access the last element and the first element (`tail.next`) in O(1) time.

One advantage of a circular list is that you can continue to traverse in the forward (or reverse) direction even after you have passed the last (or first) list node. This enables you to visit all the list elements from any starting point. In a list that is not circular, you would have to start at the beginning or at the end if you wanted to visit all the list elements. A second advantage of a circular list is that you can never fall off the end of the list. There is a disadvantage: you must be careful not to set up an infinite loop.

## EXERCISES FOR SECTION 2.6

### SELF-CHECK

1. Answer the following questions about lists.
  - a. Each node in a single-linked list has a reference to \_\_\_\_\_ and \_\_\_\_\_.
  - b. In a double-linked list, each node has a reference to \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.
  - c. To remove an item from a single-linked list, you need a reference to \_\_\_\_\_.
  - d. To remove an item from a double-linked list, you need a reference to \_\_\_\_\_.
2. For the double-linked list in Figure 2.20, explain the effect of each statement in the following fragments.
  - a. 

```
Node<String> nodeRef = tail.prev;
nodeRef.prev.next = tail;
tail.prev = nodeRef.prev;
```
  - b. 

```
Node<String> nodeRef = head;
head = new Node<>("Tamika");
head.next = nodeRef;
nodeRef.prev = head;
```
  - c. 

```
Node<String> nodeRef = new Node<>("Shakira");
nodeRef.prev = head;
nodeRef.next = head.next;
head.next.prev = nodeRef;
head.next = nodeRef;
```

### PROGRAMMING

1. For the double-linked list shown in Figure 2.20, assume `head` references the first list node and `tail` references the last list node. Write statements to do each of the following:
  - a. Insert "Bill" before "Tom".
  - b. Insert "Sue" before "Sam".
  - c. Remove "Bill".
  - d. Remove "Sam".

## 2.7 The LinkedList Class and the Iterator, ListIterator, and Iterable Interfaces

### The LinkedList Class

The `LinkedList` class, part of the Java API package `java.util`, is a double-linked list that implements the `List` interface. A selected subset of the methods from this Java API is shown in Table 2.6. Because the `LinkedList` class, like the `ArrayList` class, implements the `List` interface, it contains many of the methods found in the `ArrayList` class as well as some additional methods.

**TABLE 2.6**

Selected Methods of the `java.util.LinkedList<E>` Class

Method	Behavior
<code>public void add(int index, E obj)</code>	Inserts object <code>obj</code> into the list at position <code>index</code>
<code>public void addFirst(E obj)</code>	Inserts object <code>obj</code> as the first element of the list
<code>public void addLast(E obj)</code>	Adds object <code>obj</code> to the end of the list
<code>public E get(int index)</code>	Returns the item at position <code>index</code>
<code>public E getFirst()</code>	Gets the first element in the list. Throws <code>NoSuchElementException</code> if the list is empty
<code>public E getLast()</code>	Gets the last element in the list. Throws <code>NoSuchElementException</code> if the list is empty
<code>public boolean remove(E obj)</code>	Removes the first occurrence of object <code>obj</code> from the list. Returns <code>true</code> if the list contained object <code>obj</code> ; otherwise, returns <code>false</code>
<code>public int size()</code>	Returns the number of objects contained in the list

### The Iterator

Let's say we want to process each element in a `LinkedList`. We can use the following loop to access the list elements in sequence, starting with the one at index 0.

```
// Access each list element.
for (int index = 0; index < aList.size(); index++) {
    E nextElement = aList.get(index);
    // Do something with the element at position index (nextElement)
    . .
}
```

The loop is executed `aList.size()` times; thus it is  $O(n)$ . During each iteration, we call the method `get` to retrieve the element at position `index`.

If we assume that the method `get` begins at the first list node (`head`), each call to method `get` must advance a local reference (`nodeRef`) to the node at position `index` using a loop such as:

```
// Advance nodeRef to the element at position index.
Node<E> nodeRef = head;
for (int j = 0; j < index; j++) {
    nodeRef = nodeRef.next;
}
```

This loop (in method `get`) executes `index` times, so it is also  $O(n)$ . Therefore, the performance of the nested loops used to process each element in a `LinkedList` is  $O(n^2)$  and is very inefficient. We would like to have an alternative way to access the elements in a linked list sequentially.

We can use the concept of an iterator to accomplish this. Think of an *iterator* as a moving place marker that keeps track of the current position in a particular linked list. The `Iterator` object for a list starts at the first element in the list. The programmer can use the `Iterator` object's `next` method to retrieve the next element. Each time it does a retrieval, the `Iterator` object advances to the next list element, where it waits until it is needed again. We can also ask the `Iterator` object to determine whether the list has more elements left to process (method `hasNext`). `Iterator` objects throw a `NoSuchElementException` if they are asked to retrieve the next element after all elements have been processed.

**EXAMPLE 2.10** Assume `iter` is declared as an `Iterator` object for `LinkedList myList`. We can replace the fragment shown at the beginning of this section with the following.

```
// Access each list element.
while (iter.hasNext()) {
    E nextElement = iter.next();
    // Do something with the next element (nextElement).
    . .
}
```

This fragment is  $O(n)$  instead of  $O(n^2)$ . All that remains is to determine how to declare `iter` as an `Iterator` for `LinkedList` object `myList`. We show how to do this in the next section and discuss `Iterator` a bit more formally.

## The Iterator Interface

The interface `Iterator<E>` is defined as part of API package `java.util`. Table 2.7 summarizes the methods declared by this interface.

The `List` interface declares the method `iterator`, which returns an `Iterator` object that will iterate over the elements of that list. (The requirement for the `iterator` method is actually in the `Collection` interface, which is the superinterface for the `List` interface. We discuss the `Collection` interface in Section 2.9.)

In the following loop, we process all items in `List<Integer> aList` through an `Iterator`.

```
// Obtain an Iterator to the list aList.
Iterator<Integer> itr = aList.iterator();
while (itr.hasNext()) {
    int value = itr.next();
    // Do something with value.
    . .
}
```

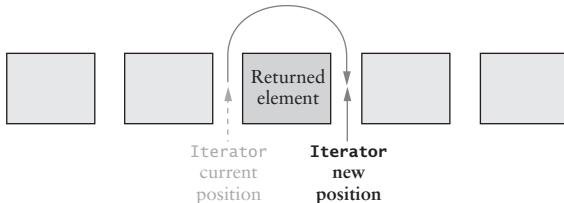
TABLE 2.7

The `java.util.Iterator<E>` Interface

Method	Behavior
<code>boolean hasNext()</code>	Returns <code>true</code> if the <code>next</code> method returns a value
<code>E next()</code>	Returns the next element. If there are no more elements, throws the <code>NoSuchElementException</code>
<code>void remove()</code>	Removes the last element returned by the <code>next</code> method

**FIGURE 2.26**

Advancing an **Iterator** via the `next` Method



An **Iterator** does not refer to or point to a particular object at any given time. Rather, you should think of an **Iterator** as pointing between objects within a list. The method `hasNext` tells us whether or not calling the `next` method will succeed. If `hasNext` returns `true`, then a call to `next` will return the next object in the list and advance the **Iterator** (see Figure 2.26). If `hasNext` returns `false`, a call to `next` will cause the `NoSuchElementException` to be thrown.

You can use the `Iterator remove` method to remove elements from a list as you access them. You can remove only the element that was most recently accessed by `next`. Each call to `remove` must be preceded by a call to `next` to retrieve the next element.

**EXAMPLE 2.11** We wish to remove all elements from `aList` (type `LinkedList<Integer>`) that are divisible by a particular value. The following method will accomplish this:

```
/** Remove the items divisible by div. */
@pre LinkedList aList contains Integer objects.
@post Elements divisible by div have been removed.
*/
public static void removeDivisibleBy(LinkedList<Integer> aList, int div) {
    Iterator<Integer> iter = aList.iterator();
    while (iter.hasNext()) {
        int nextInt = iter.next();
        if (nextInt % div == 0)
            iter.remove();
    }
}
```

The method call `iter.next` retrieves the next `Integer` in the list. Its value is unboxed, and if it is divisible by `div`, the statement

```
iter.remove();
```

removes the element just retrieved from the list.



## PITFALL

### Improper use of Remove

If a call to `remove` is not preceded by a call to `next`, `remove` will throw an `IllegalStateException`. If you want to remove two consecutive elements in a list, a separate call to `next` must occur before each call to `remove`.



## PROGRAM STYLE

### Removal Using `Iterator.remove` versus `List.remove`

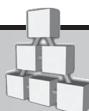
You could also use method `LinkedList.remove` to remove elements from a list. However, it is more efficient to remove multiple elements from a list using `Iterator.remove` than it would be to use `LinkedList.remove`. The `LinkedList.remove` method removes only one element at a time, so you would need to start at the beginning of the list each time and advance down the list to each element that you wanted to remove ( $O(n^2)$  process). With the `Iterator.remove` method, you can remove elements as they are accessed by the `Iterator` object without having to go back to the beginning of the list ( $O(n)$  process).

## The Enhanced for Loop

The enhanced `for` loop (also called the `for each` loop) makes it easier to sequence through arrays. It also enables sequential access to `List` objects without the need to create and manipulate an iterator. The following loop uses the enhanced `for` loop to count the number of times that `target` occurs in `myList` (type `List<String>`).

```
count = 0;
for (String nextStr : myList) {
    if (target.equals(nextStr)) {
        count++;
    }
}
```

The enhanced `for` loop creates an `Iterator` object and implicitly calls its `hasNext` and `next` methods. Other `Iterator` methods, such as `remove`, are not available.



## SYNTAX The Enhanced for loop (for each) with a Collection Class

### FORM:

```
for (formalParameter : expression) { . . . }
```

### EXAMPLE:

```
for (String nextStr : myList) { . . . }
for (int nextInt : aList) { . . . }
```

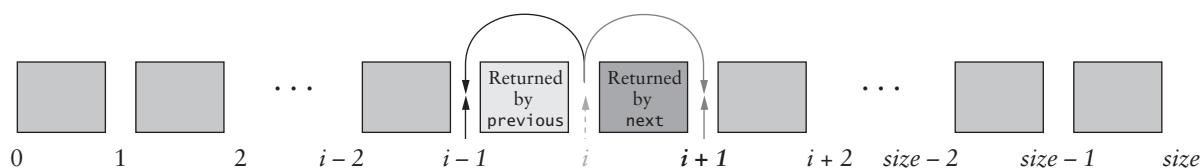
### MEANING:

During each repetition of the loop, the variable specified by `formalParameter` accesses the next element of `expression`, starting with the first element and ending with the last. The `expression` must be an array or a collection that implements the `Iterable` interface. The `Collection` interface extends the `Iterable` interface so that all classes that implement it are implementors of the `Iterable` interface (see next section).

## The `ListIterator` Interface

The `Iterator` has some limitations. It can traverse the `List` only in the forward direction. It also provides only a `remove` method, not an `add` method. Also, to start an `Iterator` somewhere other than at first `List` element, you must write your own loop to advance the `Iterator` to the desired starting position.

**FIGURE 2.27**  
The ListIterator



The Java API also contains the `ListIterator<E>` interface, which is an extension of the `Iterator<E>` interface that overcomes these limitations. Like the `Iterator`, the `ListIterator` should be thought of as being positioned between elements of the linked list. The positions are assigned an index from 0 to `size`, where the position just before the first element has index 0 and the position just after the last element has index `size`. The `next` method moves the iterator forward and returns the element that was jumped over. The `previous` method moves the iterator backward and also returns the element that was jumped over. This is illustrated in Figure 2.27, where `i` is the current position of the iterator. The methods defined by the `ListIterator` interface are shown in Table 2.8.

To obtain a `ListIterator`, you call the `listIterator` method of the `LinkedList` class. This method has two forms, as shown in Table 2.9.

**TABLE 2.8**  
The `java.util.ListIterator<E>` Interface

Method	Behavior
<code>void add(E obj)</code>	Inserts object <code>obj</code> into the list just before the item that would be returned by the next call to method <code>next</code> and after the item that would have been returned by method <code>previous</code> . If the method <code>previous</code> is called after <code>add</code> , the newly inserted object will be returned
<code>boolean hasNext()</code>	Returns <code>true</code> if <code>next</code> will not throw an exception
<code>boolean hasPrevious()</code>	Returns <code>true</code> if <code>previous</code> will not throw an exception
<code>E next()</code>	Returns the next object and moves the iterator forward. If the iterator is at the end, the <code>NoSuchElementException</code> is thrown
<code>int nextIndex()</code>	Returns the index of the item that will be returned by the next call to <code>next</code> . If the iterator is at the end, the list size is returned
<code>E previous()</code>	Returns the previous object and moves the iterator backward. If the iterator is at the beginning of the list, the <code>NoSuchElementException</code> is thrown
<code>int previousIndex()</code>	Returns the index of the item that will be returned by the next call to <code>previous</code> . If the iterator is at the beginning of the list, <code>-1</code> is returned
<code>void remove()</code>	Removes the last item returned from a call to <code>next</code> or <code>previous</code> . If a call to <code>remove</code> is not preceded by a call to <code>next</code> or <code>previous</code> , the <code>IllegalStateException</code> is thrown
<code>void set(E obj)</code>	Replaces the last item returned from a call to <code>next</code> or <code>previous</code> with <code>obj</code> . If a call to <code>set</code> is not preceded by a call to <code>next</code> or <code>previous</code> , the <code>IllegalStateException</code> is thrown

**TABLE 2.9**Methods in `java.util.LinkedList<E>` that Return `ListIterators`

Method	Behavior
<code>public ListIterator&lt;E&gt; listIterator()</code>	Returns a <code>ListIterator</code> that begins just before the first list element
<code>public ListIterator&lt;E&gt; listIterator(int index)</code>	Returns a <code>ListIterator</code> that begins just before the position <code>index</code>

**EXAMPLE 2.12** If `myList` is type `LinkedList<String>`, the statement

```
ListIterator<String> myIter = myList.listIterator(3);
```

would create a `ListIterator` object `myIter` positioned between the elements at positions 2 and 3 of the linked list. The method call

```
myIter.next()
```

would return a reference to the `String` object at position 3 and move the iterator forward; the method call

```
myIter.nextIndex()
```

would return 4. The method call

```
myIter.previous()
```

would return a reference to the `String` object at position 3 and move the iterator back to its original position. The method call

```
myIter.previousIndex()
```

would return 2. The method call

```
myIter.hasNext()
```

would return `true` if the list has at least four elements; the method call

```
myIter.hasPrevious()
```

would return `true`.

**EXAMPLE 2.13** The Fragment

```
ListIterator<String> myIter = myList.listIterator();
while (myIter.hasNext()) {
    if (target.equals(myIter.next())) {
        myIter.set(newItem);
        break; // Exit loop
    }
}
```

searches for `target` in list `myList` (type `List<String>`) and, if `target` is present, replaces its first occurrence with `newItem`.

**Comparison of Iterator and ListIterator**

Because the interface `ListIterator<E>` is a subinterface of `Iterator<E>`, classes that implement `ListIterator` must provide all of the capabilities of both. The `Iterator` interface requires fewer methods and can be used to iterate over more general data structures—that is, structures for which an index is not meaningful and ones for which traversing in only the forward direction is required. It is for this reason that the `Iterator` is required by the

Collection interface (more general), whereas the `ListIterator` is required only by the `List` interface (more specialized). We will discuss the `Collection` interface in Section 2.10.

## Conversion between a `ListIterator` and an Index

The `ListIterator` has the methods `nextIndex` and `previousIndex`, which return the index values associated with the items that would be returned by a call to the `next` or `previous` methods. The `LinkedList` class has the method `listIterator(int index)`, which returns a `ListIterator` whose next call to `next` will return the item at position `index`. Thus, you can convert between an index and a `ListIterator`. However, remember that the `listIterator(int index)` method creates the desired `ListIterator` by creating a new `ListIterator` that starts at the beginning and then walks along the list until the desired position is found. There is a special case where `index` is equal to `size()`, but all others are an  $O(n)$  operation.

## The `Iterable` Interface

Next we show the `Iterable` interface. This interface requires only that a class that implements it provides an `iterator` method. As mentioned above, the `Collection` interface extends the `Iterable` interface, so all classes that implement the `List` interface (a subinterface of `Collection`) must provide an `iterator` method.

```
public interface Iterable<E> {
    /** Returns an iterator over the elements in this collection. */
    Iterator<E> iterator();
}
```

---

## EXERCISES FOR SECTION 2.7

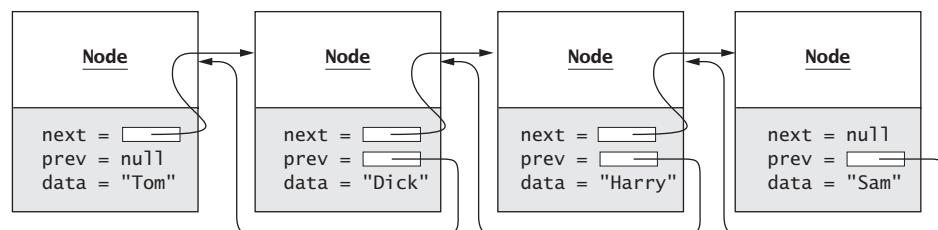
### SELF-CHECK

1. The method `indexOf`, part of the `List` interface, returns the index of the first occurrence of an object in a `List`. What does the following code fragment do?

Delete Harry

```
int indexOfSam = myList.indexOf("Sam");
ListIterator<String> iteratorToSam = myList.listIterator(indexOfSam);
iteratorToSam.previous();
iteratorToSam.remove();
```

where the internal nodes of `myList` (type `LinkedList<String>`) are shown in the figure below:



2. In Question 1, what if we change the statement

```
iteratorToSam.previous();
to
iteratorToSam.next();
```

3. In Question 1, what if we omit the statement

```
iteratorToSam.previous();
```

### PROGRAMMING

1. Write the method `indexOf` as specified in the `List` interface by adapting the code shown in Example 2.13 to return the index of the first occurrence of an object.
2. Write the method `lastIndexOf` specified in the `List` interface by adapting the code shown in Example 2.13 to return the index of the last occurrence of an object.
3. Write a method `indexOfMin` that returns the index of the minimum item in a `List`, assuming that each item in the list implements the `Comparable` interface.



## 2.8 Application of the `LinkedList` Class

In this section, we introduce a case study that uses the Java `LinkedList` class to solve a common problem: maintaining an ordered list. We will develop an `OrderedList` class.

### CASE STUDY Maintaining an Ordered List

**Problem** As discussed in Section 2.5, we can use a linked list to maintain a list of students who are registered for a course. We want to maintain this list so that it will be in alphabetical order even after students have added and dropped the course.

**Analysis** Instead of solving this problem just for a list of students, we will develop a generic `OrderedList` class that can be used to store any group of objects that can be compared. Java classes whose object types can be compared implement the `Comparable` interface, which is defined as follows:

```
/** Instances of classes that realize this interface can be compared.
 */
public interface Comparable<E> {
    /** Method to compare this object to the argument object.
     * @param obj The argument object
     * @return Returns a negative integer if this object < obj;
     *         zero if this object equals obj;
     *         a positive integer if this object > obj
     */
    int compareTo(E obj);
}
```

Therefore, a class that implements the Comparable interface must provide a `compareTo` method that returns an `int` value that indicates the relative ordering of two instances of that class. The result is negative if this object `<` argument; zero if this object equals argument; and positive if this object `>` argument.

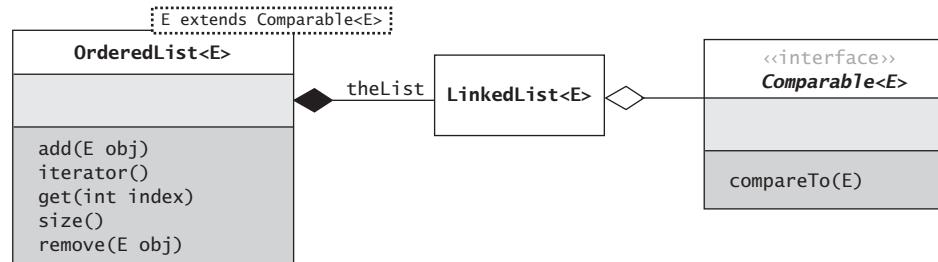
We can either extend the Java `LinkedList` class to create a new class `OrderedList`, or create an `OrderedList` class that uses a `LinkedList` to store the items. If we implement our `OrderedList` class as an extension of `LinkedList`, a client will be able to use methods in the `List` interface that can insert new elements or modify existing elements in such a way that the items are no longer in order. Therefore, we will use the `LinkedList` class as a component of the `OrderedList` class and we will implement only those methods that preserve the order of the items.

### Design

The class diagram in Figure 2.28 shows the relationships among the `OrderedList` class, the `LinkedList` class, and the `Comparable` interface. The filled diamond indicates that the `LinkedList` is a component of the `OrderedList`, and the open diamond indicates that the `LinkedList` will contain `Comparable` objects. We explain the meaning of the text `E extends Comparable<E>` shortly.

Because we want to be able to make insertions and deletions in the ordered linked list, we must implement `add` and `remove` methods. We also provide a `get` method, to access the element at a particular position, and an `iterator` method, to provide the user with the ability to access all of the elements in sequence efficiently. Table 2.10 describes the class. Although

**FIGURE 2.28**  
OrderedList  
Class Diagram



**TABLE 2.10**  
Class `OrderedList<E extends Comparable<E>>`

Data Field	Attribute
<code>private LinkedList&lt;E&gt; theList</code>	A linked list to contain the data
Method	Behavior
<code>public void add(E obj)</code>	Inserts obj into the list preserving the list's order
<code>public Iterator iterator()</code>	Returns an Iterator to the list
<code>public E get(int index)</code>	Returns the object at the specified position
<code>public int size()</code>	Returns the size of the list
<code>public boolean remove(E obj)</code>	Removes first occurrence of obj from the list. Returns true if the list contained object obj; otherwise, returns false

not shown in Figure 2.28, class `OrderedList<E>` implements `Iterable<E>` because it has an `iterator` method. Following is the start of its definition.

```
import java.util.*;  
  
/** A class to represent an ordered list. The data is stored in  
 * a linked list data field.  
 */  
  
public class OrderedDict<E extends Comparable<E>>  
    implements Iterable<E> {  
    /** A list to contain the data. */  
    private List<E> theList = new LinkedList<E>();
```

Because we want our ordered list to contain only objects that implement the `Comparable` interface, we need to tell the compiler that only classes that meet this criterion should be bound to the type parameter `E`. We do this by declaring our ordered list as `OrderedList<E extends Comparable<E>>`.



## SYNTAX Specifying Requirements on Generic Types

### FORM:

```
class ClassName<TypeParameter> extends ClassOrInterfaceName<TypeParameter>>
```

### EXAMPLE:

```
class OrderedDict<E extends Comparable<E>>
```

### MEANING:

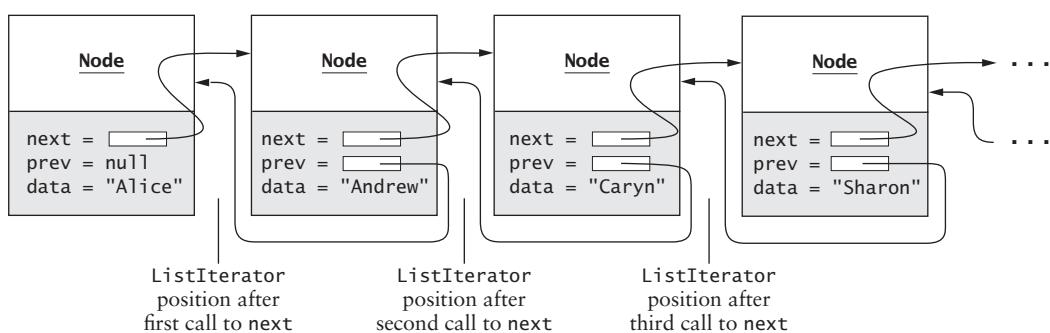
When we declare actual objects of type `ClassName<TypeParameter>`, class `TypeParameter` must extend class `ClassOrInterfaceName` or implement interface `ClassOrInterfaceName`.

## Implementation

Let's say we have an ordered list that contains the data: "Alice", "Andrew", "Caryn", "Sharon", and we want to insert "Bill" (see Figure 2.29). If we start at the beginning of the list and access "Alice", we know that "Bill" must follow "Alice", but we can't insert "Bill" yet. If we access "Andrew", we know that "Bill" must follow "Andrew", but we can't

**FIGURE 2.29**

Inserting "Bill" before "Caryn" in an Ordered List



insert "Bill" yet. However, when we access "Caryn", we know we must insert "Bill" before "Caryn". Therefore, to insert an element in an ordered list, we need to access the first element whose data is larger than the data in the element to be inserted. Once we have accessed the successor of our new node, we can insert a new node just before it. (Note that in order to access "Caryn" using the method `next`, we have advanced the iterator just past "Caryn".)

### Algorithm for Insertion

The algorithm for insertion is as follows:

1. Find the first item in the list that is greater than the item to be inserted.
2. Insert the new item before this one.

We can refine this algorithm as follows:

- 1.1 Create a `ListIterator` that starts at the beginning of the list.
- 1.2 while the `ListIterator` is not at the end and the item to be inserted is greater than or equal to the next item.
- 1.3 Advance the `ListIterator`.
2. Insert the new item before the current `ListIterator` position.

### The add Method

A straightforward coding of the insertion algorithm would be the following:

```
// WARNING - THIS DOES NOT WORK.
ListIterator<E> iter = theList.listIterator();
while (iter.hasNext())
    && obj.compareTo(iter.next()) >= 0) {
    // iter was advanced - check new position.
}
iter.add(obj);
```

Unfortunately, this does not work. When the `while` loop terminates, either we are at the end of the list or the `ListIterator` has just skipped over the first item that is greater than the item to be inserted (see Figure 2.30). In the first case, the `add` method will insert the item at the end of the list, just as we want, but in the second case, it will insert the item just after the position where it belongs. Therefore, we must separate the two cases and code the `add` method as follows:

```
/** Insert obj into the list preserving the list's order.
 * @pre The items in the list are ordered.
 * @post obj has been inserted into the list
 *       such that the items are still in order.
 * @param obj The item to be inserted
 */
public void add(E obj) {
    ListIterator<E> iter = theList.listIterator();
    // Find the insertion position and insert.
    while (iter.hasNext()) {
        if (obj.compareTo(iter.next()) < 0) {
            // Iterator has stepped over the first element
            // that is greater than the element to be inserted.
            // Move the iterator back one.
            iter.previous();
            iter.add(obj);
            return;
        }
    }
}
```

```

        // Insert the element.
        iter.add(obj);
        // Exit the loop and return.
        return;
    }
}
// assert: All items were examined and no item is larger than
// the element to be inserted.
// Add the new item to the end of the list.
iter.add(obj);
}

```

### Using Delegation to Implement the Other Methods

The other methods in Table 2.10 are implemented via delegation to the `LinkedList` class. They merely call the corresponding method in the `LinkedList`. For example, the `get` and `iterator` methods are coded as follows:

```

/** Returns the element at the specified position.
 * @param index The specified position
 * @return The element at position index
 */
public E get(int index) {
    return theList.get(index);
}

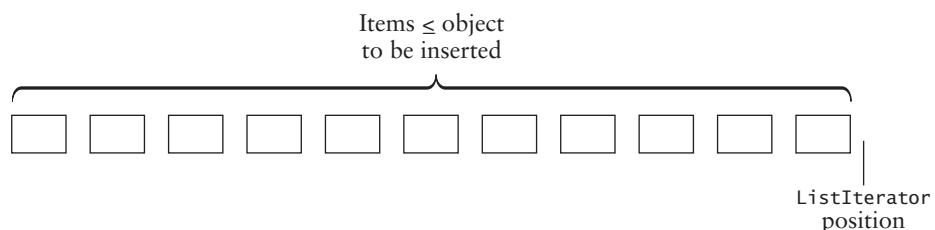
/** Returns an iterator to this OrderedList.
 * @return The iterator, positioning it before the first element
 */
public Iterator<E> iterator() {
    return theList.iterator();
}

```

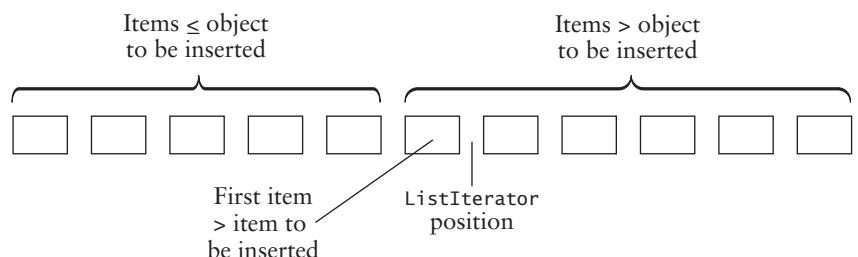
**FIGURE 2.30**

Attempted Insertion  
into an Ordered List

#### Case 1: Inserting at the end of a list



#### Case 2: Inserting in the middle of a list





## PITFALL

### Omitting <E> After ListIterator<E> or Iterable<E>

If you omit <E> in the declaration for `ListIterator<E> iter` in method `add` of `OrderedList`:

```
ListIterator<E> iter = theList.listIterator();
```

you will get an `incompatible types` syntax error when the `if` statement

```
if (obj.compareTo(iter.next()) < 0) {
```

is compiled. The reason is that the object returned by `iter.next()` will be type `Object` instead of type `E`, so the argument of `compareTo` will not be type `E` as required.

Similarly, if you omit the <E> after `Iterable` in the header for class `OrderedList`:

```
public class OrderedList<E extends Comparable<E>> implements Iterable<E> {
```

you will get an `incompatible types` syntax error if an `Iterator` method or an enhanced `for` loop is compiled. The reason is that the data type of the object returned by an `Iterator` method would be `Object`, not type `E` as required.

## Testing Class `OrderedList`

Next, we illustrate the design of a class that tests our implementation of the `OrderedList`. The next chapter provides a thorough discussion of testing.

## CASE STUDY Maintaining an Ordered List (continued)

### Testing

You can test the `OrderedList` class by storing a collection of randomly generated positive integers in an `OrderedList`. You can then insert a negative integer and an integer larger than any integer in the list. This tests the two special cases of inserting at the beginning and at the end of the list. You can then create an iterator and use it to traverse the list, displaying an error message if the current integer is smaller than the previous integer, which is an indication that the list is not ordered. You can also display the list during the traversal so that you can inspect it to verify that it is in order. Finally, you can remove the first element, the last element, and an element in the middle and repeat the traversal to show that removal does not affect the ordering. Listing 2.2 shows a class with methods that performs these tests.

Method `traverseAndShow` traverses an ordered list passed as an argument using an enhanced `for` statement to access the list elements. Each `Integer` is stored in `thisItem`. The `if` statement displays an error message if the previous value is greater than the current value (`prevItem > thisItem` is `true`). Method `main` calls `traverseAndShow` after all elements are inserted and after the three elements are removed. In method `main`, the loop

```

        for (int i = 0; i < START_SIZE; i++) {
            int anInteger = random.nextInt(MAX_INT);
            testList.add(anInteger);
        }
    }

```

fills the ordered list with randomly generated values between 0 and MAX\_INT-1. Variable random is an instance of class Random (in API java.util), which contains methods for generating pseudorandom numbers. Method Random.nextInt generates random integers between 0 and its argument. Chapter 3 provides a thorough discussion of testing.

.....

**LISTING 2.2**

```

Class TestOrderedList
import java.util.*;
public class TestOrderedList {
    /** Traverses ordered list and displays each element.
     * Displays an error message if an element is out of order.
     * @param testList An ordered list of integers
    */
    public static void traverseAndShow(OrderedList<Integer> testList) {
        int prevItem = testList.get(0);

        // Traverse ordered list and display any value that
        // is out of order.
        for (Integer thisItem : testList) {
            System.out.println(thisItem);
            if (prevItem > thisItem)
                System.out.println("**** FAILED, value is " + thisItem);
            prevItem = thisItem;
        }
    }

    public static void main(String[] args) {
        OrderedList<Integer> testList = new OrderedList<>();
        final int MAX_INT = 500;
        final int START_SIZE = 100;

        // Create a random number generator.
        Random random = new Random();
        // Fill list with START_SIZE random values.
        for (int i = 0; i < START_SIZE; i++) {
            int anInteger = random.nextInt(MAX_INT);
            testList.add(anInteger);
        }

        // Add to beginning and end of list.
        testList.add(-1);
        testList.add(MAX_INT + 1);
        traverseAndShow(testList); // Traverse and display.

        // Remove first, last, and middle elements.
        Integer first = testList.get(0);
        Integer last = testList.get(testList.size() - 1);
        Integer middle = testList.get(testList.size() / 2);
        testList.remove(first);
        testList.remove(last);
        testList.remove(middle);
        traverseAndShow(testList); // Traverse and display.
    }
}

```

## EXERCISES FOR SECTION 2.8

### SELF-CHECK

1. Why don't we implement the `OrderedList` by extending `LinkedList`? What would happen if someone called the `add` method? How about the `set` method?
2. What other methods in the `List` interface could we include in the `OrderedList` class? See the Java API documentation for a complete list of methods.
3. Why don't we provide a `ListIterator` method for the `OrderedList` class?

### PROGRAMMING

1. Write the code for the other methods of the `OrderedList` class that are listed in Table 2.10.
2. Rewrite the `OrderedList.add` method to start at the end of the list and iterate using the `ListIterator`'s `previous` method.



## 2.9 Implementation of a Double-Linked List Class

In this section, we will describe the class `KWLinkedList` that implements some of the methods of the `List` interface using a double-linked list. We will not provide a complete implementation because we expect you to use the standard `LinkedList` class provided by the Java API (in package `java.util`). The data fields for the `KWLinkedList` class are shown in Table 2.11. They are declared as shown here.

```
import java.util.*;  
  
/** Class KWLinkedList implements a double-linked list and  
 * a ListIterator. */  
public class KWLinkedList<E> {  
    // Data Fields  
    /** A reference to the head of the list. */  
    private Node<E> head = null;  
    /** A reference to the end of the list. */  
    private Node<E> tail = null;  
    /** The size of the list. */  
    private int size = 0;  
    ...
```

**TABLE 2.11**

Data Fields for Class `KWLinkedList<E>`

Data Field	Attribute
<code>private Node&lt;E&gt; head</code>	A reference to the first item in the list
<code>private Node&lt;E&gt; tail</code>	A reference to the last item in the list
<code>private int size</code>	A count of the number of items in the list

## Implementing the KWLinkedList Methods

We need to implement the methods shown earlier in Table 2.6 for the `LinkedList` class. The algorithm for the `add(int index, E obj)` method is

1. Obtain a reference, `nodeRef`, to the node at position `index`.
2. Insert a new `Node` containing `obj` before the `Node` referenced by `nodeRef`.

Similarly, the algorithm for the `get (int index)` method is

1. Obtain a reference, `nodeRef`, to the node at position `index`.
2. Return the contents of the `Node` referenced by `nodeRef`.

We also have the `listIterator (int index)` method with the following algorithm:

1. Obtain a reference, `nodeRef`, to the node at position `index`.
2. Return a `ListIterator` that is positioned just before the `Node` referenced by `nodeRef`.

These three methods all have the same first step. Therefore, we want to use a common method to perform this step.

If we look at the requirements for the `ListIterator`, we see that it has an `add` method that inserts a new item before the current position of the iterator. Thus, we can refine the algorithm for the `KWLinkedList.add (int index, E obj)` method to

1. Obtain an iterator that is positioned just before the `Node` at position `index`.
2. Insert a new `Node` containing `obj` before the `Node` currently referenced by this iterator.

Thus, the `KWLinkedList<E>` method `add` can be coded as

```
/** Add an item at position index.
 * @param index The position at which the object is to be
 * inserted
 * @param obj The object to be inserted
 * @throws IndexOutOfBoundsException if the index is out
 * of range (i < 0 || i > size())
 */
public void add(int index, E obj) {
    listIterator(index).add(obj);
}
```

Note that it was not necessary to declare a local `ListIterator` object in the `KWLinkedList` method `add`. The method call `listIterator(index)` returns an anonymous `ListIterator` object, to which we apply the `ListIterator.add` method.

Similarly, we can code the `get` method as

```
/** Get the element at position index.
 * @param index Position of item to be retrieved
 * @return The item at index
 */
public E get(int index) {
    return listIterator(index).next();
}
```

Other methods in Table 2.6 (`addFirst`, `addLast`, `getFirst`, `getLast`) can be implemented by delegation to methods `add` and `get` above.

## A Class that Implements the ListIterator Interface

We can implement most of the `KWLinkedList` methods by delegation to the class `KWListIter`, which will implement the `ListIterator` interface (see Table 2.8). Because it is an inner class

of `KWLinkedList`, its methods will be able to reference the data fields and members of the parent class (and also the other inner class, `Node`). The data fields for class `KWListIter` are shown in Table 2.12.

```
/** Inner class to implement the ListIterator interface. */
private class KWListIter implements ListIterator<E> {
    /** A reference to the next item. */
    private Node<E> nextItem;
    /** A reference to the last item returned. */
    private Node<E> lastItemReturned;
    /** The index of the current item. */
    private int index = 0;
```

Figure 2.31 shows an example of a `KWLinkedList` object and a `KWListIter` object. The `next` method would return "Harry", and the `previous` method would return "Dick". The `nextIndex` method would return 2, and the `previousIndex` method would return 1.

## The Constructor

The `KWListIter` constructor takes as a parameter the index of the `Node` at which the iteration is to begin. A test is made for the special case where the index is equal to the size; in that case the iteration starts at the tail. Otherwise, a loop starting at the head walks along the list until the node at `index` is reached.

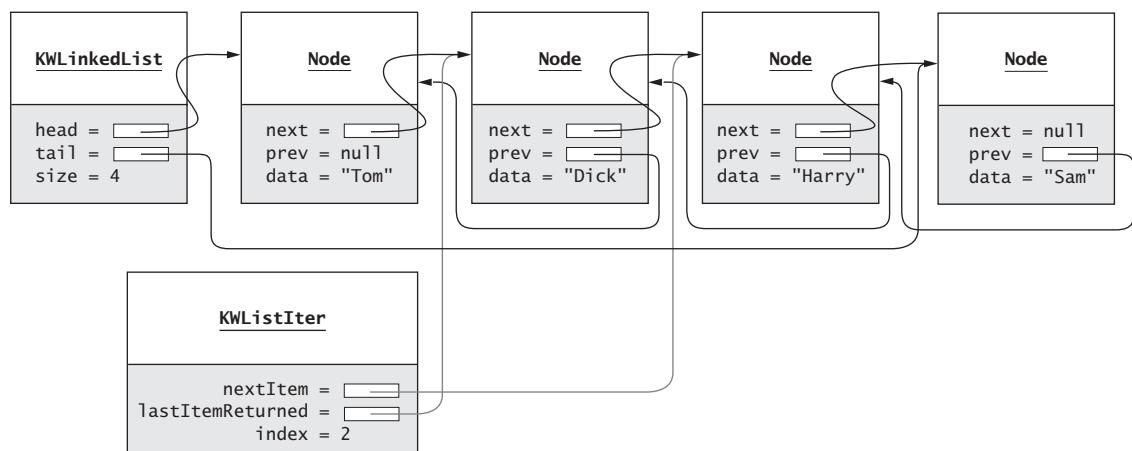
**TABLE 2.12**

Data Fields of Class `KWListIter`

<code>private Node&lt;E&gt; nextItem</code>	A reference to the next item
<code>private Node&lt;E&gt; lastItemReturned</code>	A reference to the node that was last returned by <code>next</code> or <code>previous</code>
<code>private int index</code>	The iterator is positioned just before the item at <code>index</code>

**FIGURE 2.31**

Double-Linked List with `KWListIter`



```

/** Construct a KWListIter that will reference the ith item.
 * @param i The index of the item to be referenced
 */
public KWListIter(int i) {
    // Validate i parameter.
    if (i < 0 || i > size) {
        throw new IndexOutOfBoundsException("Invalid index " + i);
    }
    lastItemReturned = null; // No item returned yet.
    // Special case of last item.
    if (i == size) {
        index = size;
        nextItem = null;
    } else { // Start at the beginning
        nextItem = head;
        for (index = 0; index < i; index++) {
            nextItem = nextItem.next;
        }
    }
}

```

## The hasNext and next Methods

The data field `nextItem` will always reference the Node that will be returned by the `next` method. Therefore, the `hasNext` method merely tests to see whether `nextItem` is `null`.

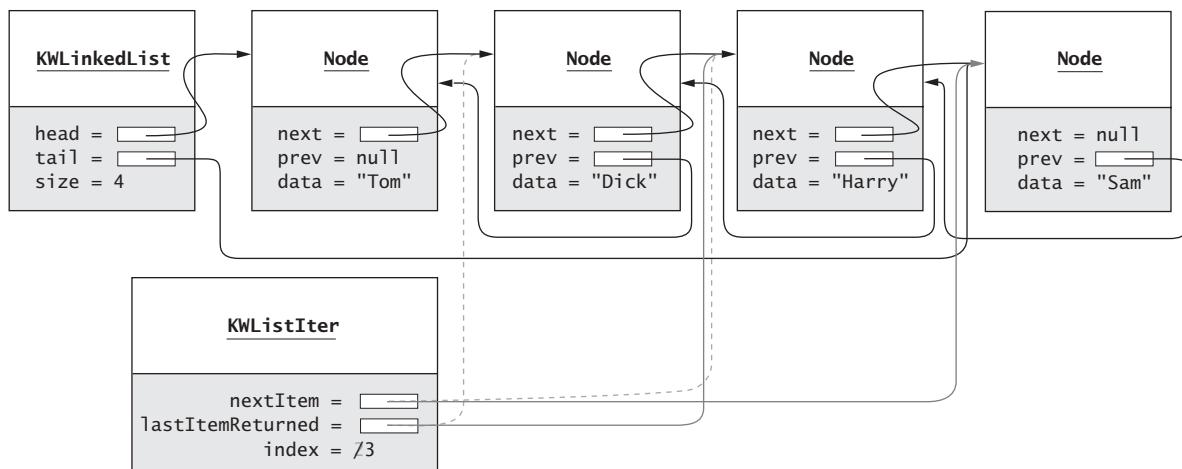
```

/** Indicate whether movement forward is defined.
 * @return true if call to next will not throw an exception
 */
public boolean hasNext() {
    return nextItem != null;
}

```

The `next` method begins by calling `hasNext`. If the result is `false`, the `NoSuchElementException` is thrown. Otherwise, `lastItemReturned` is set to `nextItem`; then `nextItem` is advanced to the next node, and `index` is incremented. The data field of the node referenced by `lastItemReturned` is returned. As shown in Figure 2.32, the previous iterator position is indicated by the dashed arrows and the new position by the gray arrows.

**FIGURE 2.32**  
Advancing a `KWListIter`



```

/** Move the iterator forward and return the next item.
 * @return The next item in the list
 * @throws NoSuchElementException if there is no such object
 */
public E next() {
    if (!hasNext()) {
        throw new NoSuchElementException();
    }
    lastItemReturned = nextItem;
    nextItem = nextItem.next;
    index++;
    return lastItemReturned.data;
}

```

## The hasPrevious and previous Methods

The `hasPrevious` method is a little trickier. When the iterator is at the end of the list, `nextItem` is `null`. In this case, we can determine that there is a previous item by checking the size—a non-empty list will have a previous item when the iterator is at the end. If the iterator is not at the end, then `nextItem` is not `null`, and we can check for a previous item by examining `nextItem.prev`.

```

/** Indicate whether movement backward is defined.
 * @return true if call to previous will not throw an exception
 */
public boolean hasPrevious() {
    return (nextItem == null && size != 0)
        || nextItem.prev != null;
}

```

The `previous` method begins by calling `hasPrevious`. If the result is `false`, the `NoSuchElementException` is thrown. Otherwise, if `nextItem` is `null`, the iterator is past the last element, so `nextItem` is set to `tail` because the previous element must be the last list element. If `nextItem` is not `null`, `nextItem` is set to `nextItem.prev`. Either way, `lastItemReturned` is set to `nextItem`, and `index` is decremented. The data field of the node referenced by `lastItemReturned` is returned.

```

/** Move the iterator backward and return the previous item.
 * @return The previous item in the list
 * @throws NoSuchElementException if there is no such object
 */
public E previous() {
    if (!hasPrevious()) {
        throw new NoSuchElementException();
    }
    if (nextItem == null) { // Iterator is past the last element
        nextItem = tail;
    } else {
        nextItem = nextItem.prev;
    }
    lastItemReturned = nextItem;
    index--;
    return lastItemReturned.data;
}

```

## The add Method

The `add` method inserts a new node before the node referenced by `nextItem`. There are four cases: add to an empty list, add to the head of the list, add to the tail of the list, and add to the middle of the list. We next discuss each case separately; you can combine them to write the method.

An empty list is indicated by head equal to null. In this case, a new Node is created, and both head and tail are set to reference it. This is illustrated in Figure 2.33.

```
/** Add a new item between the item that will be returned
   by next and the item that will be returned by previous.
   If previous is called after add, the element added is
   returned.
   @param obj The item to be inserted
 */
public void add(E obj) {
    if (head == null) { // Add to an empty list.
        head = new Node<E>(obj);
        tail = head;
    }
    ...
}
```

The `KWListIter` object in Figure 2.33 shows a value of null for `lastItemReturned` and 1 for `index`. These data fields are set at the end of the method. In all cases, data field `nextItem` is not changed by the insertion. It must reference the successor of the item that was inserted, or null if there is no successor.

If `nextItem` equals `head`, then the insertion is at the head. The new Node is created and is linked to the beginning of the list.

```
} else if (nextItem == head) { // Insert at head.
    // Create a new node.
    Node<E> newNode = new Node<E>(obj);
    // Link it to the nextItem.
    newNode.next = nextItem; // Step 1
    // Link nextItem to the new node.
    nextItem.prev = newNode; // Step 2
    // The new node is now the head.
    head = newNode;         // Step 3
```

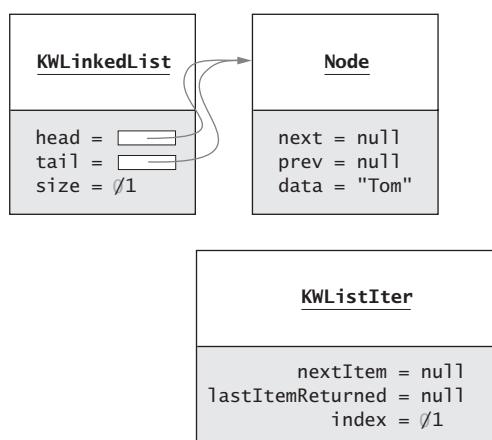
This is illustrated in Figure 2.34.

If `nextItem` is null, then the insertion is at the tail. The new node is created and linked to the tail.

```
} else if (nextItem == null) { // Insert at tail.
    // Create a new node.
    Node<E> newNode = new Node<E>(obj);
    // Link the tail to the new node.
    tail.next = newNode; // Step 1
    // Link the new node to the tail.
    newNode.prev = tail; // Step 2
    // The new node is the new tail.
    tail = newNode;      // Step 3
```

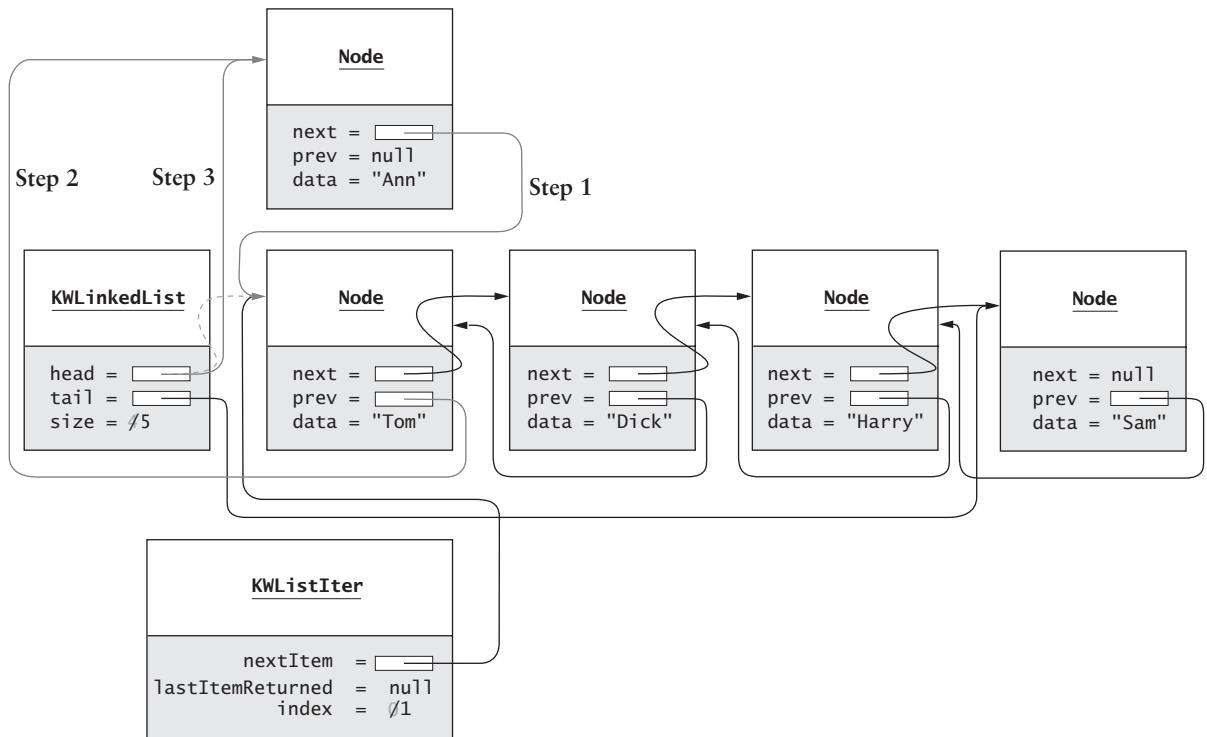
**FIGURE 2.33**

Adding to an Empty List



**FIGURE 2.34**

Adding to the Head of the List



This is illustrated in Figure 2.35.

If none of the previous cases is true, then the addition is into the middle of the list. The new node is created and inserted before the node referenced by **nextItem**.

```

} else { // Insert into the middle.
    // Create a new node.
    Node<E> newNode = new Node<E>(obj);
    // Link it to nextItem.prev.
    newNode.prev = nextItem.prev; // Step 1
    nextItem.prev.next = newNode; // Step 2
    // Link it to the nextItem.
    newNode.next = nextItem; // Step 3
    nextItem.prev = newNode; // Step 4
}

```

This is illustrated in Figure 2.36.

After the new node is inserted, both **size** and **index** are incremented and **lastItemReturned** is set to null.

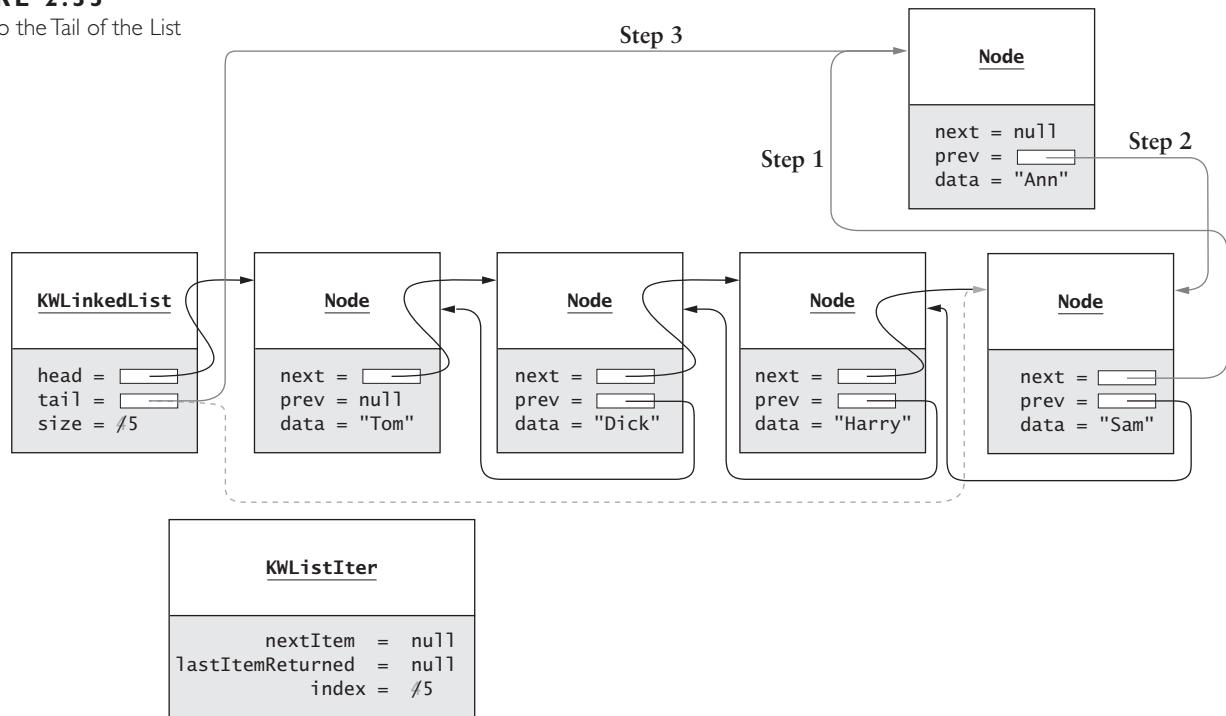
```

// Increase size and index and set lastItemReturned.
size++;
index++;
lastItemReturned = null;
} // End of method add.

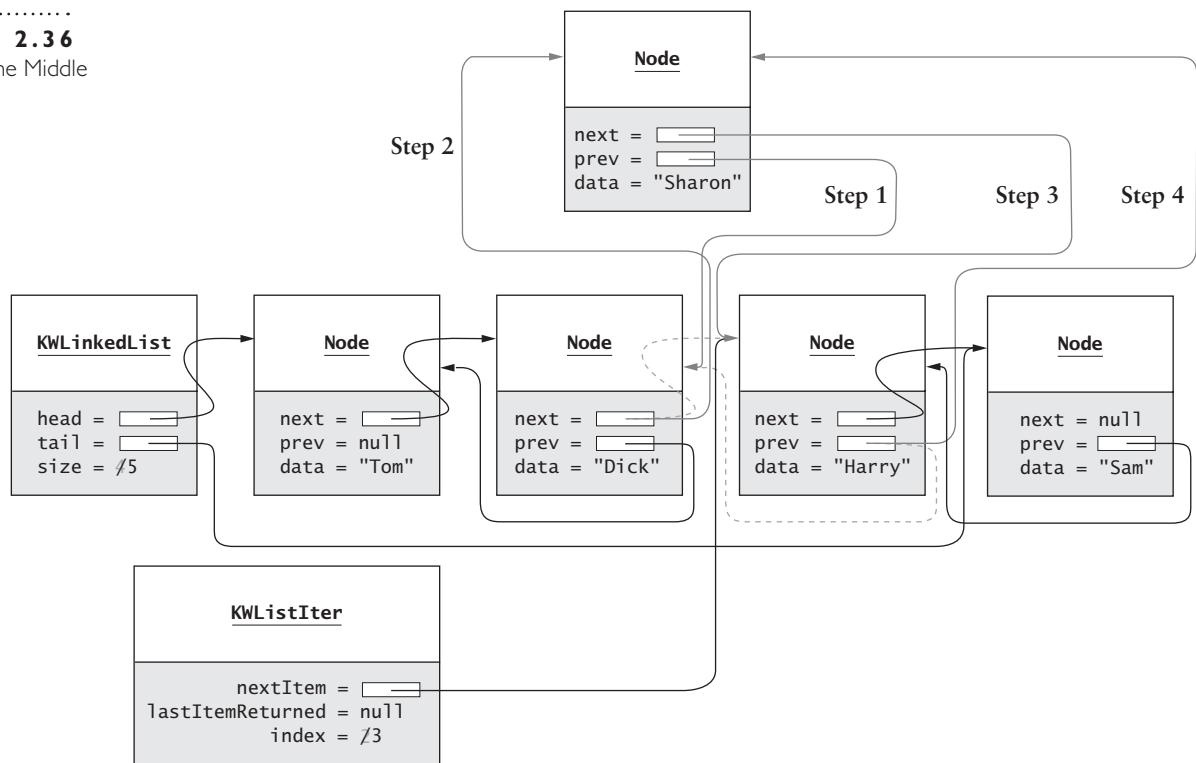
```

**FIGURE 2.35**

Adding to the Tail of the List

**FIGURE 2.36**

Adding to the Middle of the List



## Inner Classes: Static and Nonstatic

There are two inner classes in class `KWLinkedList<E>`: class `Node<E>` and class `KWListIter`. We declare `Node<E>` to be static because there is no need for its methods to access the data fields of its parent class (`KWLinkedList<E>`). We can't declare `KWListIter` to be static because its methods access and modify the data fields of the `KWLinkedList` object that creates the `KWListIter` object. An inner class that is not static contains an implicit reference to its parent object, just as it contains an implicit reference to itself. Because `KWListIter` is not static and can reference data fields of its parent class `KWLinkedList<E>`, the type parameter `<E>` is considered to be previously defined; therefore, it cannot appear as part of the class name.



### PITFALL

#### Defining `Kwlistiter` as a Generic Inner Class

If you define class `KWListIter` as

```
private class KWListIter<E>...
```

you will get an `incompatible types` syntax error when you attempt to reference data field `head` or `tail` (type `Node<E>`) inside class `KWListIter`.

## EXERCISES FOR SECTION 2.9

### SELF-CHECK

1. Why didn't we write the `hasPrevious` method as follows?

```
public boolean hasPrevious() {
    return nextItem.prev != null
        || (nextItem == null && size != 0);
}
```

2. Why must we call `next` or `previous` before we call `remove`?

3. What happens if we call `remove` after we call `add`? What does the Java API documentation say? What does our implementation do?

### PROGRAMMING

1. Implement the `KWListIter.remove` method.
2. Implement the `KWListIter.set` method.
3. Implement the `KWLinkedList.ListIterator` and `iterator` methods.
4. Implement the `KWLinkedList.addFirst`, `addLast`, `getFirst`, and `getLast` methods.

## 2.10 The Collections Framework Design

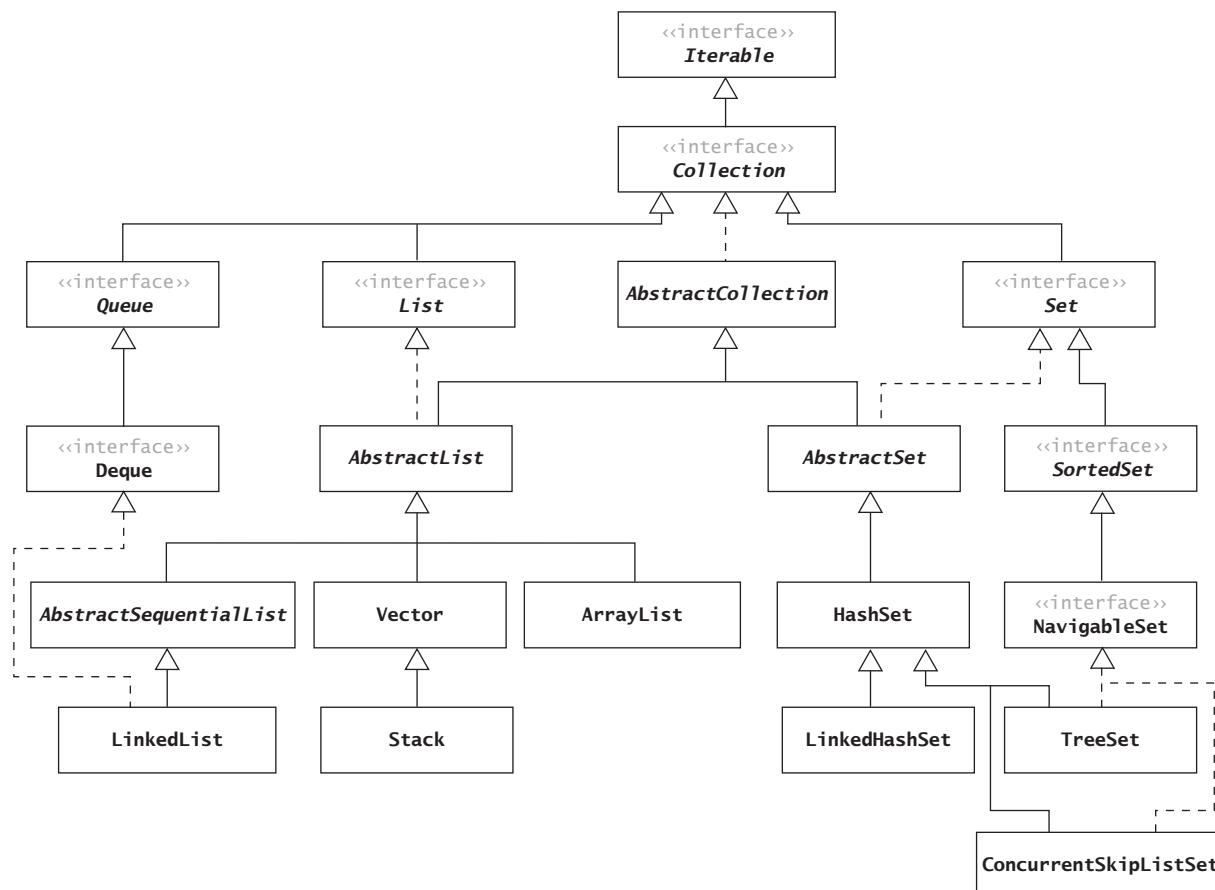
### The Collection Interface

The `Collection` interface specifies a subset of the methods specified in the `List` interface. Specifically, the `add(int, E)`, `get(int)`, `remove(int)`, `set(int, E)`, and related methods (all of which have an `int` parameter that represents a position) are not in the `Collection` interface, but the `add(E)` and `remove(Object)` methods, which do not specify a position, are included. The `iterator` method is also included in the `Collection` interface. Thus, you can use an `Iterator` to access all of the items in a `Collection`, but the order in which they are retrieved is not necessarily related to the order in which they were inserted.

The `Collection` interface is part of the Collections Framework as shown in Figure 2.37. This interface has three subinterfaces: the `List` interface, the `Queue` interface (Chapter 4), and the `Set` interface (Chapter 7). The Java API does not provide any direct implementation of the `Collection` interface. The interface is used to reference collections of data in the most general way.

**FIGURE 2.37**

The Collections Framework



## Common Features of Collections

Because it is the superinterface of `List`, `Queue`, and `Set`, the `Collection` interface specifies a set of common methods. If you look at the documentation for the Java API `java.util.Collection`, you will see that this is a fairly large set of methods and other requirements. A few features can be considered fundamental:

- Collections grow as needed.
- Collections hold references to objects.
- Collections have at least two constructors: one to create an empty collection and one to make a copy of another collection.

Table 2.13 shows selected methods defined in the `Collection` interface. We have already seen and described these methods in the discussions of the `ArrayList` and `LinkedList`. The `Iterator` provides a common way to access all of the elements in a `Collection`. For collections implementing the `List` interface, the order of the elements is determined by the index of the elements. In the more general `Collection`, the order is not specified.

In the `ArrayList` and `LinkedList`, the `add(E)` method always inserts the object at the end and always returns `true`. In the more general `Collection`, the position where the object is inserted is not specified. The `Set` interface extends the `Collection` by requiring that the `add` method not insert an object that is already present; instead, in that case it returns `false`. The `Set` interface is discussed in Chapter 7.

**TABLE 2.13**

Selected Methods of the `java.util.Collection<E>` Interface

Method	Behavior
<code>boolean add(E obj)</code>	Ensures that the collection contains the object <code>obj</code> . Returns <code>true</code> if the collection was modified
<code>boolean contains(E obj)</code>	Returns <code>true</code> if the collection contains the object <code>obj</code>
<code>Iterator&lt;E&gt; iterator()</code>	Returns an <code>Iterator</code> to the collection
<code>int size()</code>	Returns the size of the collection

## The `AbstractCollection`, `AbstractList`, and `AbstractSequentialList` Classes

If you look at the Java API documentation, you will see that the `Collection` and `List` interfaces specify a large number of methods. To help implement these interfaces, the Java API includes the `AbstractCollection` and `AbstractList` classes. You can think of these classes as a kit (or as a cake mix) that can be used to build implementations of their corresponding interface. Most of the methods are provided, but you need to add a few to make it complete.

To implement the `Collection` interface completely, you need only extend the `AbstractCollection` class, provide an implementation of the `add`, `size`, and `iterator` methods, and supply an inner class to implement the `Iterator` interface. To implement the `List` interface, you can extend the `AbstractList` class and provide an implementation of the `add(int, E)`, `get(int)`, `remove(int)`, `set(int, E)`, and `size()` methods. Since we provided these methods in our `KWArrayList`, we can make it a complete implementation of the `List` interface by changing the class declaration to

```
public class KWArrayList<E> extends AbstractList<E> implements List<E>
```

Note that the `AbstractList` class implements the `iterator` and `listIterator` methods using the index associated with the elements.

Another way to implement the `List` interface is to extend the `AbstractSequentialList` class, implement the `listIterator` and `size` methods, and provide an inner class that implements the `ListIterator` interface. This was the approach we took in our `KWLinkedList`. Thus, by changing the class declaration to

```
public class KWLinkedList<E> extends AbstractSequentialList<E>
    implements List<E>
```

it becomes a complete implementation of the `List` interface. Our `KWLinkedList` class included the `add`, `get`, `remove`, and `set` methods. These are provided by the `AbstractSequentialList`, so we could remove them from our `KWLinkedList` class and still have a complete `List` implementation.

## The List and RandomAccess Interfaces (Advanced)

The `ArrayList` and the `LinkedList` implement the `List` interface that we described in Section 2.2. Both the `ArrayList` and `LinkedList` represent a collection of objects that can be referenced using an index. This may not be the best design because accessing elements of a `LinkedList` using an index requires an  $O(n)$  traversal of the list until the item selected by the index is located. Unfortunately, the Java designers cannot easily change the design of the API since a lot of programs have been written and the users of Java do not want to go back and change their code. Also, there are other implementations of the `List` interface in which the indexed operations `get` and `set` are approximately  $O(n)$  instead of  $O(1)$ .

The `RandomAccess` interface is applied only to those implementations in which indexed operations are efficient (e.g., `ArrayList`). An algorithm can then test to see if a parameter of type `List` is also of type `RandomAccess` and, if not, copy its contents into an `ArrayList` temporarily so that the indexed operations can proceed more efficiently. After the indexed operations are completed, the contents of the `ArrayList` are copied back to the original.

---

## EXERCISES FOR SECTION 2.10

### SELF-CHECK

1. Look at the `AbstractCollection` definition in the Java API documentation. What methods are abstract? Could we use the `KWArrayList` and extend the `AbstractCollection`, but not the `AbstractList`, to develop an implementation of the `Collection` interface? How about using the `KWLinkedList` and the `AbstractCollection`, but not the `AbstractSequentialList`?

### PROGRAMMING

1. Using either the `KWArrayList` or the `KWLinkedList` as the base, develop an implementation of the `Collection` interface by extending the `AbstractCollection`. Test it by ensuring that the following statements compile:

```
Collection<String> testCollection = new KWArrayList<>();
Collection<String> testCollection = new KWLinkedList<>();
```



# Chapter Review

- ◆ We use big-O notation to describe the performance of an algorithm. Big-O notation specifies how the performance increases with the number of data items being processed by an algorithm. The best performance is  $O(1)$ , which means the performance is constant regardless of the number of data items processed.
- ◆ The `List` is a generalization of the array. As in the array, elements of a `List` are accessed by means of an index. Unlike the array, the `List` can grow or shrink. Items may be inserted or removed from any position.
- ◆ The Java API provides the `ArrayList<E>` class, which uses an array as the underlying structure to implement the `List`. We provided an example of how this might be implemented by allocating an array that is larger than the number of items in the list. As items are inserted into the list, the items with higher indices are moved up to make room for the inserted item, and as items are removed, the items with higher indices are moved down to fill in the emptied space. When the array capacity is reached, a new array is allocated that is twice the size and the old array is copied to the new one. By doubling the capacity, the cost of the copy is spread over each insertion so that the copies can be considered to have a constant time contribution to the cost of each insertion.
- ◆ A linked list data structure consists of a set of nodes, each of which contains its data and a reference to the next node in the list. In a double-linked list, each node contains a reference to both the next and the previous node in the list. Insertion into and removal from a linked list is a constant-time operation.
- ◆ To access an item at a position indicated by an index in a linked list requires walking along the list from the beginning until the item at the specified index is reached. Thus, traversing a linked list using an index would be an  $O(n^2)$  operation because we need to repeat the walk each time the index changes. The `Iterator` provides a general way to traverse a list so that traversing a linked list using an iterator is an  $O(n)$  operation.
- ◆ An iterator provides us with the ability to access the items in a `List` sequentially. The `Iterator` interface defines the methods available to an iterator. The `List` interface defines the `iterator` method, which returns an `Iterator` to the list. The `Iterator.hasNext` method tells whether there is a next item, and the `Iterator.next` method returns the next item and advances the iterator. The `Iterator` also provides the `remove` method, which lets us remove the last item returned by the `next` method.
- ◆ The `ListIterator` interface extends the `Iterator` interface. The `ListIterator` provides us with the ability to traverse the list either forward or backward. In addition to the `hasNext` and `next` methods, the `ListIterator` has the `hasPrevious` and `previous` methods. Also, in addition to the `remove` method, it has an `add` method that inserts a new item into the list just before the current iterator position.
- ◆ The `Iterable` interface is implemented by the `Collection` interface. It imposes a requirement that its implementers (all classes that implement the `Collection` interface) provide an `iterator` method that returns an `Iterator` to an instance of that collection class. The enhanced for loop makes it easier to iterate through these collections without explicitly manipulating an iterator and also to iterate through an array object without manipulating an array index.

- ◆ The Java API provides the `LinkedList` class, which uses a double-linked list to implement the `List` interface. We show an example of how this might be implemented. Because the class that realizes the `ListIterator` interface provides the `add` and `remove` operations, the corresponding methods in the linked list class can be implemented by constructing an iterator (using the `listIterator(int)` method) that references the desired position and then calling on the iterator to perform the insertion or removal.
- ◆ The `Collection` interface is the root of the Collections Framework. The `Collection` is more general than the `List` because the items in a `Collection` are not indexed. The `add` method inserts an item into a `Collection` but does not specify where it is inserted. The `Iterator` is used to traverse the items in a `Collection`, but it does not specify the order of the items.
- ◆ The `Collection` interface and the `List` interface define a large number of methods that make these abstractions useful for many applications. In our discussion of both the `ArrayList` and `LinkedList`, we showed how to implement only a few key methods. The Collections Framework includes the `AbstractCollection`, `AbstractList`, and `AbstractSequentialList` classes. These classes implement their corresponding interface except for a few key methods; these are the same methods for which we showed implementations.

## Java API Interfaces and Classes Introduced in this Chapter

<code>java.util.AbstractCollection</code>	<code>java.util.Iterator</code>
<code>java.util.AbstractList</code>	<code>java.util.LinkedList</code>
<code>java.util.AbstractSequentialList</code>	<code>java.util.List</code>
<code>java.util.ArrayList</code>	<code>java.util.ListIterator</code>
<code>java.util.Collection</code>	<code>java.util.RandomAccess</code>
<code>java.util.Iterable</code>	

## User-Defined Interfaces and Classes in this Chapter

<code>KWArrayList</code>	<code>Node</code>
<code>KWLinkedList</code>	<code>OrderedList</code>
<code>KWListIter</code>	<code>SingleLinkedList</code>

## Quick-Check Exercises

1. Elements of a `List` are accessed by means of \_\_\_\_\_.
2. A `List` can \_\_\_\_\_ or \_\_\_\_\_ as items are added or removed.
3. When we allocate a new array for an `ArrayList` because the current capacity is exceeded, we make the new array at least \_\_\_\_\_. This allows us to \_\_\_\_\_.
4. Determine the order of magnitude (big-O) for an algorithm whose running time is given by the equation  $T(n) = 3n^4 - 2n^2 + 100n + 37$ .
5. In a single-linked list, if we want to remove a list element, which list element do we need to access? If `nodeRef` references this element, what statement removes the desired element?
6. Suppose a single-linked list contains three `Nodes` with data "him", "her", and "it" and `head` references the first element. What is the effect of the following fragment?  

```
Node<String> nodeRef = head.next;
nodeRef.data = "she";
```
7. Answer Question 5 for the following fragment.  

```
Node<String> nodeRef = head.next;
head.next = nodeRef.next;
```

8. Answer Question 5 for the following fragment.  
`head = new Node<String>("his", head);`
9. An Iterator allows us to access items of a List \_\_\_\_\_.
10. A ListIterator allows us to access the elements \_\_\_\_\_.
11. The Java LinkedList class uses a \_\_\_\_\_ to implement the List interface.
12. The Collection is a \_\_\_\_\_ of the List.

## Review Questions

1. What is the difference between the size and the capacity of an ArrayList? Why might we have a constructor that lets us set the initial capacity?
2. What is the difference between the `remove(Object obj)` and `remove(int index)` methods?
3. When we insert an item into an ArrayList, why do we start shifting at the last element?
4. The Vector and ArrayList both provide the same methods, since they both implement the List interface. The Vector has some additional methods with the same functionality but different names. For example, the Vector `addElement` and `add` methods have the same functionality. There are some methods that are unique to Vector. Look at the Java API documentation and make a list of the methods that are in Vector that have equivalent methods in ArrayList and ones that are unique. Can the unique methods be implemented using the methods available in ArrayList?
5. If a loop processes  $n$  items and  $n$  changes from 1024 to 2048, how does that affect the running time of a loop that is  $O(n^2)$ ? How about a loop that is  $O(\log n)$ ? How about a loop that is  $O(n \log n)$ ?
6. What is the advantage of a double-linked list over a single-linked list? What is the disadvantage?
7. Why is it more efficient to use an iterator to traverse a linked list?
8. What is the difference between the Iterator and ListIterator interfaces?
9. How would you make a copy of a ListIterator? Consider the following:

```
ListIterator copyOfIter =
    myList.ListIterator(otherIterator.previousIndex());
```

Is this an efficient approach? How would you modify the KWLinkedList class to provide an efficient method to copy a ListIterator?

10. What is a Collection? Are there any classes in the Java API that completely implement the Collection interface?

## Programming Projects

1. Develop a program to maintain a list of homework assignments. When an assignment is assigned, add it to the list, and when it is completed, remove it. You should keep track of the due date. Your program should provide the following services:
  - Add a new assignment.
  - Remove an assignment.
  - Provide a list of the assignments in the order they were assigned.
  - Find the assignment(s) with the earliest due date.
2. We can represent a polynomial as a list of terms, where the terms are in decreasing order by exponent. You should define a class Term that contains data fields `coef` and `exponent`. For example,  $-5x^4$  has a `coef` value of -5 and an `exponent` value of 4. To add two polynomials, you traverse both lists and examine the two terms at the current iterator position. If the exponent of one is smaller than the exponent of the other, then insert the larger one into the result and advance that list's iterator. If the exponents are equal, then create a new term with that exponent and the sum of the two coefficients, and advance both iterators. For example:

$$3x^4 + 2x^2 + 3x + 7 \text{ added to } 2x^3 + -5x + 5 \text{ is } 3x^4 + 2x^3 + 2x^2 + -2x + 12$$

Write a `Polynomial` class with an inner class `Term`. The `Polynomial` class should have a data field `terms` that is of type `LinkedList <Term>`. Provide an `addpoly` method and a `readpoly` method. Method `readpoly` reads a string representing a polynomial such as " $2x^3 + -4x^2$ " and returns a polynomial list with two terms. You also need a `toString` method for class `Term` and `Polynomial` that would display this stored polynomial  $2x^3 + -4x^2$ .

3. Provide a multiple method for your polynomial class. To multiply, you iterate through polynomial A and then multiply all terms of polynomial B by the current term of polynomial A. You then add each term you get by multiplying two terms to the polynomial result. Hint: to multiply two terms, multiply their coefficients and add their exponents. For example,  $2x^3 \times 4x^2$  is  $8x^5$ .
4. Write a program to manage a list of students waiting to register for a course as described in Section 2.5. Operations should include adding a new student at the end of the list, adding a new student at the beginning of the list, removing the student from the beginning of the list, and removing a student by name.
5. A circular-linked list has no need of a head or tail. Instead, you need only a reference to a current node, which is the `nextNode` returned by the `Iterator`. Implement such a `CircularList` class. For a nonempty list, the `Iterator.hasNext` method will always return `true`.
6. The Josephus problem is named after the historian Flavius Josephus, who lived between the years 37 and 100 CE. Josephus was also a reluctant leader of the Jewish revolt against the Roman Empire. When it appeared that Josephus and his band were to be captured, they resolved to kill themselves. Josephus persuaded the group by saying, "Let us commit our mutual deaths to determination by lot. He to whom the first lot falls, let him be killed by him that hath the second lot, and thus fortune shall make its progress through us all; nor shall any of us perish by his own right hand, for it would be unfair if, when the rest are gone, somebody should repent and save himself" (Flavius Josephus, *The Wars of the Jews*, Book III, Chapter 8, Verse 7, tr. William Whiston, 1737). Yet that is exactly what happened; Josephus was left for last, and he and the person he was to kill surrendered to the Romans. Although Josephus does not describe how the lots were assigned, the following approach is generally believed to be the way it was done. People form a circle and count around the circle some predetermined number. When this number is reached, that person receives a lot and leaves the circle. The count starts over with the next person. Using the circular-linked list developed in Exercise 4, simulate this problem. Your program should take two parameters:  $n$ , the number of people who start, and  $m$ , the number of counts. For example, try  $n = 20$  and  $m = 12$ . Where does Josephus need to be in the original list so that he is the last one chosen?
7. To mimic the procedure used by Josephus and his band strictly, the person eliminated remains in the circle until the next one is chosen. Modify your program to take this into account. You may need to modify the circular-linked list class to make a copy of an iterator. Does this change affect the outcome?
8. A two-dimensional shape can be defined by its boundary-polygon, which is simply a list of all coordinates ordered by a traversal of its outline. See the following figure for an example.

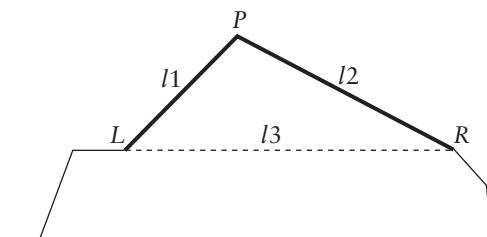


The left picture shows the original shape; the middle picture, the outline of the shape. The rightmost picture shows an abstracted boundary, using only the “most important” vertices. We can assign an importance measure to a vertex P by considering its neighbors L and R. We compute the distances LP, PR, and LR. Call these distances  $l_1$ ,  $l_2$ , and  $l_3$ . Define the importance as  $l_1 + l_2 - l_3$ .

Use the following algorithm to find the  $n$  most important points.

1. while the number of points is greater than  $n$ .
2. Compute the importance of each point.
3. Remove the least significant one.

Write a program to read a set of coordinates that form an outline and reduce the list to the  $n$  most significant ones, where  $n$  is an input value. Draw the initial and resulting shapes. Note: This problem and the algorithm for its solution are based on the paper: L. J. Latecki and R. Lakämper, “Convexity Rule for Shape Decomposition Based on Discrete Contour Evolution,” *Computer Vision and Image Understanding (CVIU)* 73(1999): 441–454.



9. As an additional feature, add a slider to your application in Project 8, showing each step of the simplification. Because a slider can go back and forth, you have to store the results of each single simplification step. Consult the Java API documentation on how to use a slider.

## Answers to Quick-Check Exercises

1. an index.
2. grow, shrink.
3. twice the size, spread out the cost of the reallocation so that it is effectively a constant-time operation.
4.  $O(n^4)$
5. The predecessor of this node. `nodeRef.next = nodeRef.next.next;`
6. Replaces "her" with "she".
7. Deletes the second list element ("she").
8. Insert a new first element containing "his".
9. sequentially.
10. both forward and backward.
11. double-linked list.
12. superinterface.



# *Testing and Debugging*

## Chapter Objectives

- ◆ To understand different testing strategies and when and how they are performed
- ◆ To introduce testing using the JUnit test framework
- ◆ To show how to write classes using test-driven development
- ◆ To illustrate how to use a debugger within a Java Integrated Development Environment (IDE)

This chapter introduces and illustrates some testing and debugging techniques. We begin by discussing testing in some detail. You will learn how to generate a proper test plan and the differences between unit and integration testing as they apply to an object-oriented design (OOD). Next we describe the JUnit Test Framework, which has become a commonly used tool for creating and running unit tests. We also describe test-driven development, a design approach in which the tests and programs are developed in parallel. Finally, we illustrate the debugger, which allows you to suspend program execution at a specified point and examine the value of variables to assist in isolating errors.

### Testing and Debugging

- 3.1** Types of Testing
- 3.2** Specifying the Tests
- 3.3** Stubs and Drivers
- 3.4** Testing Using the JUnit Framework
- 3.5** Test-Driven Development
  - Case Study:* Finding a target in an array.
- 3.6** Testing Interactive Methods
- 3.7** Debugging a Program

## 3.1 Types of Testing

---

Testing is the process of exercising a program (or part of a program) under controlled conditions and verifying that the results are as expected. The purpose of testing is to detect program defects after all syntax errors have been removed and the program compiles successfully. The more thorough the testing, the greater the likelihood that the defects will be found. However, no amount of testing can guarantee the absence of defects in sufficiently complex programs. The number of test cases required to test all possible inputs and states that each method may execute can quickly become prohibitively large. That is often why commercial software products have different versions or patches that the user must install. Version  $n$  usually corrects the defects that were still present in version  $n - 1$ .

Testing is generally done at the following levels and in the sequence shown below:

- Unit testing refers to testing the smallest testable piece of the software. In OOD, the unit will be either a method or a class. The complexity of a method determines whether it should be tested as a separate unit or whether it can be tested as part of its class.
- Integration testing involves testing the interactions among units. If the unit is the method, then integration testing includes testing interactions among methods within a class. However, generally it involves testing interactions among several classes.
- System testing is the testing of the whole program in the context in which it will be used. A program is generally part of a collection of other programs and hardware, called a *system*. Sometimes a program will work correctly until some other software is loaded onto the system and then it will fail for no apparent reason.
- Acceptance testing is system testing designed to show that the program meets its functional requirements. It generally involves use of the system in the real environment or as close to the real environment as possible.

There are two types of testing:

- *Black-box testing* tests the item (method, class, or program) based on its interfaces and functional requirements. This is also called closed-box testing or functional testing. For testing a method, the input parameters are varied over their allowed range and the results compared against independently calculated results. In addition, values outside the allowed range are tested to ensure that the method responds as specified (e.g., throws an exception or computes a nominal value). Also, the inputs to a method are not only the parameters of the method, but also the values of any global data that the method accesses.
- *White-box testing* tests the software element (method, class, or program) with the knowledge of its internal structure. Other terms used for this type of testing are glass-box testing, open-box testing, and coverage testing. The goal is to exercise as many paths through the element as possible or practical. There are various degrees of coverage. The simplest is statement coverage, which ensures that each statement is executed at least once. Branch coverage ensures that every choice at each branch (**if** statements, **switch** statements, and loops) is tested. For example, if there are only **if** statements, and they are not nested, then each **if** statement is tried with its condition true and with its condition false. This could possibly be done with two test cases: one with all of the **if** conditions true and the other with all of them false. Path coverage tests each path through a method. If there are  $n$  **if** statements, path coverage could require  $2^n$  test cases if the **if** statements are not nested (each condition has two possible values, so there could be  $2^n$  possible paths).

---

**EXAMPLE 3.1** Method `testMethod` has a nested `if` statement and displays one of four messages, path 1 through path 4, depending on which path is followed. The values passed to its arguments determine the path. The ellipses represent the other statements in each path.

```
public void testMethod(char a, char b) {
    if (a < 'M') {
        if (b < 'X') {
            System.out.println("path 1");
            ...
        } else {
            System.out.println("path 2");
            ...
        }
    } else {
        if (b < 'C') {
            System.out.println("path 3");
            ...
        } else {
            System.out.println("path 4");
            ...
        }
    }
}
```

---

To test this method, we need to pass values for its arguments that cause it to follow the different paths. Table 3.1 shows some possible values and the corresponding path.

The values chosen for `a` and `b` in Table 3.1 are the smallest and largest uppercase letters. For a more thorough test, you should see what happens when `a` and `b` are passed values that are between A and Z. For example, what happens if the value changes from L to M? We pick those values because the condition (`a < 'M'`) has different values for each of them.

Also, what happens when `a` and `b` are not uppercase letters? For example, if `a` and `b` are both digit characters (e.g., '2'), the path 1 message should be displayed because the digit characters precede the uppercase letters (see Appendix A, Table A.2). If `a` and `b` are both lowercase letters, the path 4 message should be displayed (Why?). If `a` is a digit and `b` is a lowercase letter, the path 2 message should be displayed (Why?). As you can see, the number of test cases required to test even a simple method such as `testMethod` thoroughly can become quite large.

.....  
**TABLE 3.1**  
Testing All Paths of `testMethod`

<b>a</b>	<b>b</b>	<b>Message</b>
'A'	'A'	path 1
'A'	'Z'	path 2
'Z'	'A'	path 3
'Z'	'Z'	path 4

## Preparations for Testing

Although testing is usually done after each unit of the software is coded, a test plan should be developed early in the design stage. Some aspects of a test plan include deciding how the software will be tested, when the tests will occur, who will do the testing, and what test data will be used. If the test plan is developed early in the design stage, testing can take place concurrently with the design and coding. Again, the earlier an error is detected, the easier and less expensive it is to correct it.

Another advantage of deciding on the test plan early is that this will encourage programmers to prepare for testing as they write their code. A good programmer will practice defensive programming and include code that detects unexpected or invalid data values. For example, if the parameter *n* for a method is required to be greater than zero, you can place the *if* statement

```
if (n <= 0)
    throw new IllegalArgumentException("n <= 0: " + n);
```

at the beginning of the method. This *if* statement will throw an exception and provide a diagnostic message in the event that the parameter passed to the method is invalid. Method exit will occur and the exception can be handled by the method caller.

## Testing Tips for Program Systems

Most of the time, you will be testing program systems that contain collections of classes, each with several methods. Next, we provide a list of testing tips to follow in writing these methods.

1. Carefully document each method parameter and class attribute using comments as you write the code. Also, describe the method operation using comments, following the Javadoc conventions discussed in Section A.7.
2. Leave a trace of execution by displaying the method name as you enter it.
3. Display the values of all input parameters upon entry to a method. Also, display the values of any class attributes that are accessed by this method. Check that these values make sense.
4. Display the values of all method outputs after returning from a method. Also, display any class attributes that are modified by this method. Verify that these values are correct by hand computation.

You should plan for testing as you write each module rather than after the fact. Include the output statements required for Steps 2 and 3 in the original Java code for the method. When you are satisfied that the method works as desired, you can “remove” the testing statements. One efficient way to remove them is to enclose them in an *if* (*TESTING*) block as follows:

```
if (TESTING) {
    // Code that you wish to "remove"
    .
    .
}
```

You would then define *TESTING* at the beginning of the class as *true* to enable testing,

```
private static final boolean TESTING = true;
```

or as *false* to disable testing,

```
private static final boolean TESTING = false;
```

If you need, you can define different *boolean* flags for different kinds of tests.

---

## EXERCISES FOR SECTION 3.1

### SELF-CHECK

1. Explain why a method that does not match its declaration in the interface would not be discovered during white-box testing.
2. During which phase of testing would each of the following tests be performed?
  - a. Testing whether a method worked properly at all its boundary conditions.
  - b. Testing whether class A can use class B as a component.
  - c. Testing whether a phone directory application and a word-processing application can run simultaneously on a personal computer.
  - d. Testing whether a method `search` can search an array that was returned by method `buildArray` that stores input data in the array.
  - e. Testing whether a class with an array data field can use a static method `search` defined in a class `ArraySearch`.



## 3.2 Specifying the Tests

---

In this section, we discuss how to specify the tests needed to test a program system and its components. The test data may be specified during the analysis and design phases. This should be done for the different levels of testing: unit, integration, and system. In black-box testing, we are concerned with the relationship between the unit inputs and outputs. There should be test data to check for all expected inputs as well as unanticipated data. The test plan should also specify the expected unit behavior and outputs for each set of input data.

In white-box testing, we are concerned with exercising alternative paths through the code. Thus, the test data should be designed to ensure that all `if` statement conditions will evaluate to both `true` and `false`. For nested `if` statements, test different combinations of `true` and `false` values. For `switch` statements, make sure that the selector variable can take on all values listed as case labels and some that are not.

For loops, verify that the result is correct if an immediate exit occurs (zero repetitions). Also, verify that the result is correct if only one iteration is performed and if the maximum number of iterations is performed. Finally, verify that loop repetition can always terminate.

### Testing Boundary Conditions

When hand-tracing through an algorithm using white-box testing, you must exercise all paths through the algorithm. It is also important to check special cases called boundary conditions to make sure that the algorithm works for these cases as well as the more common ones. For example, if you are testing a method that searches for a particular target element in an array testing, the boundary conditions means that you should make sure that the method works for the following special cases:

- The target element is the first element in the array.
- The target element is the last element in the array.
- The target is somewhere in the middle.

- The target element is not in the array.
- There is more than one occurrence of the target element, and we find the first occurrence.
- The array has only one element and it is not the target.
- The array has only one element and it is the target.
- The array has no elements.

These boundary condition tests would be required in black-box testing too.

---

## EXERCISES FOR SECTION 3.2

### SELF-CHECK

1. List two boundary conditions that should be checked when testing method `readInt` below. The second and third parameters represent the upper and lower bounds for a range of valid integers.

```
/** Returns an integer data value within range minN and maxN inclusive
 * @param scan a Scanner object
 * @param minN smallest possible value to return
 * @param maxN largest possible value to return
 * @return the first value read between minN and maxN
 */
public static int readInt (Scanner scan, int minN, int maxN) {
    if (minN > maxN)
        throw new IllegalArgumentException ("In readInt, minN " + minN
                                         + " not <= maxN " + maxN) ;
    boolean inRange = false; // Assume no valid number read.
    int n = 0;
    while (!inRange) { // Repeat until valid number read.
        System.out.println("Enter an integer from " + minN + " to "
                           + maxN + ": ") ;
        try {
            n = scan.nextInt();
            inRange = (minN <= n & & n <= maxN) ;
        } catch (InputMismatchException ex) {
            scan.nextLine();
            System.out.println("not an integer - try again");
        }
    } // End while
    return n; // n is in range
}
```

2. Devise test data to test the method `readInt` using
  - a. white-box testing
  - b. black-box testing

### PROGRAMMING

1. Write a search method with four parameters: the search array, the target, the start subscript, and the finish subscript. The last two parameters indicate the part of the array that should be searched. Your method should catch or throw exceptions where warranted.

## 3.3 Stubs and Drivers

---

In this section, we describe two kinds of methods, stubs and drivers, that facilitate testing. We also show how to document the requirements that a method should meet using preconditions and postconditions.

### Stubs

Although we want to do unit testing as soon as possible, it may be difficult to test a method or a class that interacts with other methods or classes. The problem is that not all methods and not all classes will be completed at the same time. So if a method in class A calls a method defined in class B (not yet written), the unit test for class A can't be performed without the help of a replacement method for the one in class B. The replacement for a method that has not yet been implemented or tested is called a stub. A stub has the same header as the method it replaces, but its body only displays a message indicating that the stub was called.

---

**EXAMPLE 3.2** The following method is a stub for a void method `save`. The stub will enable a method that calls `save` to be tested, even though the real method `save` has not been written.

```
/** Stub for method save.
 * @pre the initial directory contents are read from a data file.
 * @post Writes the directory contents back to a data file.
 *       The boolean flag modified is reset to false.
 */
public void save() {
    System.out.println("Stub for save has been called");
    modified = false;
}
```

Besides displaying an identification message, a stub can print out the values of the inputs and can assign predictable values (e.g., 0 or 1) to any outputs to prevent execution errors caused by undefined values. Also, if a method is supposed to change the state of a data field, the stub can do so (`modified` is set to `false` by the stub just shown). If a client program calls one or more stubs, the message printed by each stub when it is executed provides a trace of the call sequence and enables the programmer to determine whether the flow of control within the client program is correct.

---

### Preconditions and Postconditions

In the comment for the method `save`, the lines

```
@pre the initial directory contents are read from a data file.
@post Writes the directory contents back to a data file.
The boolean flag modified is reset to false.
```

show the precondition (following `@pre`) and postcondition (following `@post`) for the method `save`. A *precondition* is a statement of any assumptions or constraints on the method data (input parameters) before the method begins execution. A *postcondition* describes the result of executing the method. A method's preconditions and postconditions serve as a contract between a method caller and the method programmer—if a caller satisfies the precondition, the method result should satisfy the postcondition. If the precondition is not satisfied, there is no guarantee that the method will do what is expected, and it may even fail. The preconditions and postconditions allow both a method user and a method implementer to proceed without further coordination.

We will use postconditions to describe the change in object state caused by executing a mutator method. As a general rule, you should write a postcondition comment for all void methods. If a method returns a value, you do not usually need a postcondition comment because the @return comment describes the effect of executing the method.

## Drivers

Another testing tool for a method is a driver program. A driver program declares any necessary object instances and variables, assigns values to any of the method's inputs (as specified in the method's preconditions), calls the method, and displays the values of any outputs returned by the method. Alternatively, driver methods can be written in a separate test class and executed under the control of a test framework such as JUnit, which we discuss in the next section.

## EXERCISES FOR SECTION 3.3

### SELF-CHECK

1. Can a main method be used as a stub or a driver? Explain your answer.

### PROGRAMMING

1. Write a driver program to the test method `readInt` in Self Check exercise 1 of Section 3.2 using the test data derived for Self-Check Exercise 2, part b in Section 3.2.
2. Write a stub to use in place of the method `readInt`.
3. Write a driver program to test method search in programming Exercise 1, Section 3.2.

## 3.4 The JUnit Test Framework

A *test harness* is a driver program written to test a method or class. It does this by providing known inputs for a series of tests, called a *test suite*, and then compares the expected and actual results of each test and an indication of pass or fail.

A *test framework* is a software product that facilitates writing test cases, organizing the test cases into test suites, running the test suites, and reporting the results. One test framework often used for Java projects is `JUnit`, an open-source product that can be used in a stand-alone mode and is available from [junit.org](http://junit.org). It is also bundled with at least two popular IDEs (NetBeans and Eclipse). In the next section, we show a test suite for the `ArraySearch` class constructed using the `JUnit` framework.

`JUnit` uses the term *test suite* to represent the collection of tests to be run at one time. A test suite may consist of one or more classes that contain the individual tests. These classes are called *test harnesses*. A test harness may also contain common code to be executed before/after each test so that the class being tested is in a known state.

Each test harness in a test suite that will be run by the `JUnit` main method (called a *test runner*) begins with the two import statements:

```
import org.junit.Test;
import static org.junit.Assert.*;
```

The first import makes the `Test` interface visible, which allows us to use the `@Test` attribute to identify test cases. Annotations such as `@Test` are directions to the compiler and other

language-processing tools; they do not affect the execution of the program. The JUnit main method (*test runner*) searches the classes that are listed in the args parameter for methods with the @Test annotation. When it executes them, it keeps track of the pass/fail results.

The second import statement makes the methods in the `Assert` class visible. The assert methods are used to determine pass/fail for a test. Table 3.2 describes the various assert methods that are defined in `org.junit.Assert`. If an assert method fails, then an exception is thrown causing an error to be reported as specified in the description for method `assertArrayEquals`. If one of the assertions fail, then the test fails; if none fails, then the test passes.

**TABLE 3.2**Methods Defined in `org.junit.Assert`

Method	Parameters	Description
<code>assertArrayEquals</code>	<code>[message,] expected, actual</code>	Tests to see whether the contents of the two array parameters <i>expected</i> and <i>actual</i> are equal. This method is overloaded for arrays of the primitive types and <code>Object</code> . Arrays of <code>Objects</code> are tested with the <code>.equals</code> method applied to the corresponding elements. The test fails if an unequal pair is found, and an <code>AssertionError</code> is thrown. If the optional message is included, the <code>AssertionError</code> is thrown with this message followed by the default message; otherwise it is thrown with a default message
<code>assertEquals</code>	<code>[message,] expected, actual</code>	Tests to see whether <i>expected</i> and <i>actual</i> are equal. This method is overloaded for the primitive types and <code>Object</code> . To test <code>Objects</code> , the <code>.equals</code> method is used
<code>assertFalse</code>	<code>[message,] condition</code>	Tests to see whether the <code>boolean</code> expression <i>condition</i> is false
<code>assertNotNull</code>	<code>[message,] object</code>	Tests to see if the <i>object</i> is not <code>null</code>
<code>assertNotSame</code>	<code>[message,] expected, actual</code>	Tests to see if <i>expected</i> and <i>actual</i> are not the same object. (Applies the <code>!=</code> operator.)
<code>assertNull</code>	<code>[message,] object</code>	Tests to see whether the <i>object</i> is <code>null</code>
<code>assertSame</code>	<code>[message,] expected, actual</code>	Tests to see whether <i>expected</i> and <i>actual</i> are the same object. (Applies the <code>==</code> operator.)
<code>assertTrue</code>	<code>[message,] condition</code>	Tests to see whether the <code>boolean</code> expression <i>condition</i> is true
<code>fail</code>	<code>[message]</code>	Always throws <code>AssertionError</code>

---

**EXAMPLE 3.3** Listing 3.1 shows a JUnit test harness for an array search method (`ArraySearch.search`) that returns the location of the first occurrence of a target value (the second parameter) in an array (the first parameter) or -1 if the target is not found. The test harness contains methods that implement the tests first described in Section 3.2 and repeated below.

- The target element is the first element in the array.
  - The target element is the last element in the array.
  - The target is somewhere in the middle.
  - The target element is not in the array.
  - There is more than one occurrence of the target element and we find the first occurrence.
  - The array has only one element and it is not the target.
  - The array has only one element and it is the target.
  - The array has no elements.
-

Method `firstElementTest` in Listing 3.1 implements the first test case. It tests to see whether the target is the first element in the 7-element array {5, 12, 15, 4, 8, 12, 7}. In the statement

```
assertEquals("5 is not found at position 0", 0, ArraySearch.search(x, 5));
```

the call to method `ArraySearch.search` returns the location of the target (5) in array `x`. The test passes (`ArraySearch.search` returns 0), and JUnit remembers the result. After all tests are run, JUnit displays a message such as

```
Testsuite: KW.CH03New.ArraySearchTest
```

```
Tests run: 9, Failures: 0, Errors: 0, Skipped: 0, Time elapsed: 0.111 sec
```

where 0.111 is the execution time in seconds. The Netbeans IDE also shows a Test Results window as shown in Figure 3.1. (The gray box at the end of Section 3.5 shows how to access JUnit in Netbeans.)

If instead we used the following statement that incorrectly searches for target 4 as the first element

```
assertEquals("4 is not found at position 0", 0, ArraySearch.search(x, 4));
```

the test would fail and `AssertionErrorException` would display the messages

```
Testcase: firstElementTest: FAILED
```

```
4 not found a position 0 expected:<0> but was:<3>
```

If we omitted the first argument in the call to `assertEquals`, the default message

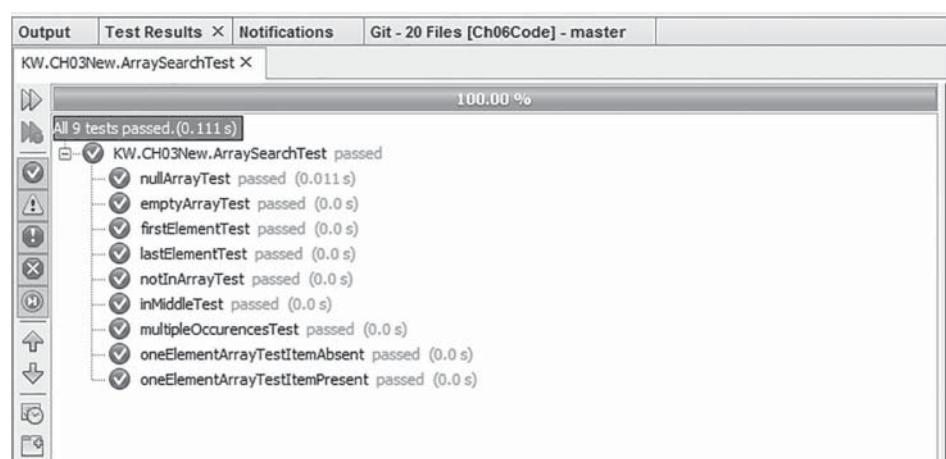
```
Testcase: firstElementTest: FAILED
```

```
expected:<0> but was:<3>
```

would be displayed instead where 3 is the position of the target 4.

**FIGURE 3.1**

Test Results



### LISTING 3.1

JUnit test of `ArraySearch.search`

```
import org.junit.Test;
import static org.junit.Assert.*;

/**
 * JUnit test of ArraySearch.search
 * @author Koffman and Wolfgang
 */
public class ArraySearchTest {

    // Common array to search for most of the tests
    private final int[] x = {5, 12, 15, 4, 8, 12, 7};
}
```

```
@Test
public void firstElementTest() {
    // Test for target as first element.
    assertEquals("5 not at position 0",
                0, ArraySearch.search(x, 5));
}

@Test
public void lastElementTest() {
    // Test for target as last element.
    assertEquals("7 not at position 6",
                6, ArraySearch.search(x, 7));
}

@Test
public void inMiddleTest() {
    // Test for target somewhere in middle.
    assertEquals("4 is not found at position 3",
                3, ArraySearch.search(x, 4));
}

@Test
public void notInArrayTest() {
    // Test for target not in array.
    assertEquals(-1, ArraySearch.search(x, -5));
}

@Test
public void multipleOccurrencesTest() {
    // Test for multiple occurrences of target.
    assertEquals(1, ArraySearch.search(x, 12));
}

@Test
public void oneElementArrayTestItemPresent() {
    // Test for 1-element array
    int[] y = {10};
    assertEquals(0, ArraySearch.search(y, 10));
}

@Test
public void oneElementArrayTestItemAbsent() {
    // Test for 1-element array
    int[] y = {10};
    assertEquals(-1, ArraySearch.search(y, -10));
}

@Test
public void emptyArrayTest() {
    // Test for an empty array
    int[] y = new int[0];
    assertEquals(-1, ArraySearch.search(y, 10));
}

@Test(expected = NullPointerException.class)
public void nullArrayTest() {
    int[] y = null;
    int i = ArraySearch.search(y, 10);
}
```

The last test case in Listing 3.1 does not implement one of the tests in our earlier list of test cases. Its purpose is to test that the `ArraySearch.search` method fails as it should when it is passed a `null` array. To tell JUnit that a test is expected to throw an exception, we added the parameter `expected = exception-class` to the `@Test` attribute. This parameter tells JUnit that the test should cause a `NullPointerException` (the result of calling `ArraySearch.search` with a `null` array). Without this parameter, JUnit would have indicated that the test case did not pass but reported an error instead. If the exception is not thrown as expected, the test will fail.

## EXERCISES FOR SECTION 3.4

### SELF-CHECK

1. Modify the test(s) in the list for Example 3.3 to verify a method that finds the last occurrence of a target element in an array?
2. List the boundary conditions and tests needed for a method with the following heading:

```
/***
 * Search an array to find the first occurrence of the
 * largest element
 * @param x Array to search
 * @return The subscript of the first occurrence of the
 *         largest element
 * @throws NullPointerException if x is null
 */
public static int findLargest(int[] x) {
```

### PROGRAMMING

1. Write the JUnit test harness for the method described in Self-Check Question 1.
2. Write the JUnit test harness for the tests listed in Self-Check Exercise 1.



## 3.5 Test-Driven Development

Rather than writing a complete method and then testing it, test-driven development involves writing the tests and the method in parallel. The sequence is as follows:

- Write a test case for a feature.
- Run the test and observe that it fails, but other tests still pass.
- Make the minimum change necessary to make the test pass.
- Revise the method to remove any duplication between the code and the test.
- Rerun the test to see that it still passes.

We then repeat these steps adding a new feature until all of the requirements for the method have been implemented.

We will use this approach to develop a method to find the first occurrence of a target in an array.

### Case Study: Test-Driven Development of `ArraySearch.search`

Write a program to search an array that performs the same way as Java method `ArraySearch.search`. This method should return the index of the first occurrence of a target in an array, or `-1` if the target is not present.

We start by creating a test list like that in the last section and then work through them one at a time. During this process, we may think of additional tests to add to the test list.

Our test list is as follows:

1. The target element is not in the list.
2. The target element is the first element in the list.
3. The target element is the last element in the list.
4. There is more than one occurrence of the target element and we find the first occurrence.
5. The target is somewhere in the middle.
6. The array has only one element.
7. The array has no elements.

We start by creating a stub for the method we want to code:

```
/**
 * Provides a static method search that searches an array
 * @author Koffman & Wolfgang
 */
public class ArraySearch {
    /**
     * Search an array to find the first occurrence of a target
     * @param x Array to search
     * @param target Target to search for
     * @return The subscript of the first occurrence if found:
     *         otherwise return -1
     * @throws NullPointerException if x is null
     */
    public static int search(int[] x, int target) {
        return Integer.MIN_VALUE;
    }
}
```

Now, we create the first test that combines tests 1 and 6 above. We will screen the test code in gray to distinguish it from the search method code.

```
/**
 * Test for ArraySearch class
 * @author Koffman & Wolfgang
 */
public class ArraySearchTest {
    @Test
    public void itemNotFirstElementInSingleElementArray() {
        int[] x = {5};
        assertEquals(-1, ArraySearch.search(x, 10));
    }
}
```

And when we run this test, we get the message:

```
Testcase: itemNotFirstElementInSingleElementArray: FAILED
expected:<-1> but was:<-2147483648>
```

The minimum change to enable method search to pass the test is

```
public static int search(int[] x, int target) {
    return -1; // target not found
}
```

Now, we can add a second test to see whether we find the target in the first element (tests 2 and 6 above).

```
@Test
public void itemFirstElementInSingleElementArray() {
```

```

    int[] x = new int[]{5};
    assertEquals(0, ArraySearch.search(x, 5));
}

```

As expected, this test fails because the search method returns  $-1$ . To make it pass, we modify our search method:

```

public static int search(int[] x, int target) {
    if (x[0] == target) {
        return 0; // target found at 0
    }
    return -1; // target not found
}

```

Both tests for a single element array now pass. Before moving on, let us see whether we can improve this. The process of improving code without changing its functionality is known as *refactoring*. Refactoring is an important step in test-driven development. It is also facilitated by TDD since having a working test suite gives you the confidence to make changes. (Kent Beck, a proponent of TDD says that TDD gives courage.<sup>1</sup>)

The statement:

```
return 0;
```

is a place for possible improvement. The value  $0$  is the index of the target. For a single element array this is obviously  $0$ , but for larger arrays it may be different. Thus, an improved version is

```

public static int search(int[] x, int target) {
    int index = 0;
    if (x[index] == target)
        return index; // target at 0
    return -1; // target not found
}

```

Now, let us see whether we can find an item that is last in a larger array (test 3 above). We start with a 2-element array:

```

@Test
public void itemSecondItemInTwoElementArray() {
    int[] x = {10, 20};
    assertEquals(1, ArraySearch.search(x, 20));
}

```

The first two tests still pass, but the new test fails. As expected, we get the message:

```
Testcase: itemSecondItemInTwoElementArray: FAILED
expected:<1> but was:<-1>
```

The test failed because we did not compare the second array element to the target. We can modify the method to do this as shown next.

```

public static int search(int[] x, int target) {
    int index = 0;
    if (x[index] == target)
        return index; // target at 0
    index = 1;
    if (x[index] == target)
        return index; // target at 1
    return -1; // target not found
}

```

However, this would result in an `ArrayOutOfBoundsException` error for test `itemNotFirstElementInSingleElementArray` because there is no second element in the array `{5}`. If we change the method to first test that there is a second element before comparing it to target, all tests will pass.

<sup>1</sup>Beck, Kent. *Test-Driven Development by Example*. Addison-Wesley, 2003.

```
public static int search(int[] x, int target) {
    int index = 0;
    if (x[index] == target)
        return index; // target at 0
    index = 1;
    if (index < x.length) {
        if (x[index] == target)
            return index; // target at 1
    }
    return -1; // target not found
}
```

However, what happens if we increase the number of elements beyond 2?

```
@Test
public void itemLastInMultiElementArray() {
    int[] x = new int[]{5, 10, 15};
    assertEquals(2, ArraySearch.search(x, 15));
}
```

This test would fail because the target is not at position 0 or 1. To make it pass, we could continue to add `if` statements to test more elements, but this is a fruitless approach. Instead, we should modify the code so that the value of `index` advances to the end of the array. We can change the second `if` to a `while` and add an increment of `index`.

```
public static int search(int[] x, int target) {
    int index = 0;
    if (x[index] == target)
        return index; // target at 0
    index = 1;
    while (index < x.length) {
        if (x[index] == target)
            return index; // target at index
        index++;
    }
    return -1; // target not found
}
```

At this point, we have a method that will pass all of the tests for any size array. We can group all the tests in a single testing method to verify this.

```
@Test
public void verificationTests() {
    int[] x = {5, 12, 15, 4, 8, 12, 7};
    // Test for target as first element
    assertEquals(0, ArraySearch.search(x, 5));
    // Test for target as last element
    assertEquals(6, ArraySearch.search(x, 7));
    // Test for target not in array
    assertEquals(-1, ArraySearch.search(x, -5));
    // Test for multiple occurrences of target
    assertEquals(1, ArraySearch.search(x, 12));
    // Test for target somewhere in middle
    assertEquals(3, ArraySearch.search(x, 4));
}
```

Although it may look like we are done, we are not finished because we also need to check that an empty array will always return `-1`:

```
@Test
public void itemNotInEmptyArray() {
    int[] x = new int[0];
    assertEquals(-1, ArraySearch.search(x, 5));
}
```

Unfortunately, this test does not pass because of an `ArrayIndexOutOfBoundsException` in the first `if` condition for method `search` (there is no element `x[0]` in an empty array). If we look closely at the code for `search`, we see that the initial test for when `index` is 0 is the same as for the other elements. So we can remove the first statement and start the loop at 0 instead of 1 (another example of refactoring). Our code becomes more compact and this test will also pass. A slight improvement would be to replace the `while` with a `for` statement.

```
public static int search(int[] x, int target) {
    int index = 0;
    while (index < x.length) {
        if (x[index] == target)
            return index; // target at index
        index++;
    }
    return -1; // target not found
}
```

Finally, if we pass a null pointer instead of a reference to an array, a `NullPointerException` should be thrown (an additional test not in our original list).

```
@Test(expected = NullPointerException.class)
public void nullValueOfXThrowsException() {
    assertEquals(0, ArraySearch.search(null, 5));
}
```

### JUnit in Netbeans

It is fairly easy to create a JUnit test harness in Netbeans. Once you have written class `ArraySearch.java`, right click on the class name in the Projects view and then select Tools → Create/Update Tests. A Create Tests window will pop up. Select OK and then a Select JUnit Version window will pop up: select the most recent version of JUnit (currently JUnit 4.x). At this point, a new class will be created (`ArraySearchTest.java`) that will contain prototype tests for all the public functions in class `ArraySearch`. You can replace the prototype tests with your own. To execute the tests, right click on class `ArraySearchTest`.

## EXERCISES FOR SECTION 3.5

### SELF-CHECK

1. Why did the first version of method `search` that passed the first test `itemNotFirstElementInSingleElementArray` contain only the statement `return -1`?
2. Assume the first JUnit test for the `findLargest` method described in Self-Check Exercise 2 in Section 3.4 is a test that determines whether the first item in a one element array is the largest. What would be the minimal code for a method `findLargest` that passed this test?

### PROGRAMMING

1. Write the `findLargest` method described in Self-Check Exercise 2 in Section 3.4 using Test-Driven Development.

## 3.6 Testing Interactive Programs in JUnit

In this section, we show how to use JUnit to test a method that gets an integer value in a specified range from the program user method `readInt` is defined in class `MyInput`:

```
/**
 * Method to return an integer data value between two
 * specified end points.
 * @pre minN <= maxN.
 * @param prompt Message
 * @param minN Smallest value in range
 * @param maxN Largest value in range
 * @throws IllegalArgumentException
 * @return The first data value that is in range
 */
public static int readInt(String prompt, int minN, int maxN) {
    if (minN > maxN) {
        throw new IllegalArgumentException("In readInt, minN " + minN +
                                           "not <= maxN " + maxN);
    }
    // Arguments are valid, read a number
    boolean inRange = false; //Assume no valid number read
    int n = 0;
    Scanner in = new Scanner(System.in);
    while (!inRange) {
        try {
            System.out.println(prompt + "\nEnter an integer between " +
                               + minN + " and " + maxN);
            String line = in.nextLine();
            n = Integer.parseInt(line);
            inRange = (minN <= n && n <= maxN);
        } catch (NumberFormatException ex) {
            System.out.println("Bad numeric string - Try again");
        }
    }
    return n;
}
```

The advantage of using a test framework such as JUnit is that tests are automated. They do not require any user input, and they always present the same test cases to the unit being tested. With JUnit, we can test that an `IllegalArgumentException` is thrown if the input parameters are not valid:

```
@Test(expected=IllegalArgumentException.class)
public void testForInvalidInput() {
    int n = MyInput.readInt("Enter weight", 5, 2);
}
```

We also need to test that the method works correctly for values that are within a valid range and for values that are outside without requiring the user to enter these data. So we need to provide repeatable user input, and we need to verify that the output displayed is correct.

`System.in` is a public static field in the `System` class. It is initialized to an `InputStream` that reads from the system-defined standard input. The method `System.setIn` can be used to change the value of `System.in`. Similarly, `System.out` is initialized to a `PrintStream` that writes to the operating system-defined standard output, and the method `System.setOut` can be used to change it.

## ByteArrayInputStream

The `ByteArrayInputStream` is an `InputStream` that provides input from a fixed array of bytes. The array is initialized by the constructor. Calls to the `read` method return successive bytes from the array until the end of the array is reached. Thus, if we wanted to test what `readInt` does when the string "3" is entered, we can do the following:

```
@Test
public void testForNormalInput() {
    ByteArrayInputStream testIn = new ByteArrayInputStream("3".getBytes());
    System.setIn(testIn);
    int n = MyInput.readInt("Enter weight", 2, 5);
    assertEquals(n, 3);
}
```

## ByteArrayOutputStream

The `ByteArrayOutputStream` is an `OutputStream` that collects each byte written to it into an internal byte array. The `toString` method will then convert the current contents into a `String`. To capture and verify output written to `System.out`, we need to create a `PrintStream` that writes to a `ByteArrayOutputStream` and then set `System.out` to this `PrintStream`.

To verify that the prompt is properly displayed for the normal case, we use the following test:

```
@Test
public void testThatPromptIsCorrectForNormalInput() {
    ByteArrayInputStream testIn = new ByteArrayInputStream("3".getBytes());
    System.setIn(testIn);
    ByteArrayOutputStream testOut = new ByteArrayOutputStream();
    System.setOut(new PrintStream(testOut));
    int n = MyInput.readInt("Enter weight", 2, 5);
    assertEquals(n, 3);
    String displayedPrompt = testOut.toString();
    String expectedPrompt = "Enter weight" +
        "\nEnter an integer between 2 and 5" + NL;
    assertEquals(expectedPrompt, displayedPrompt);
}
```

For the data string "3", the string formed by `testOut.toString()` should match the string in `expectedPrompt`. String `expectedPrompt` ends with the string constant `NL` instead of `\n`. This is because the `println` method ends the output with a system-dependent line terminator. On the Windows operating system, this is the sequence `\r\n`, and on the Linux operating system it is `\n`. The statement

```
private static final String NL = System.getProperty("line.separator");
```

initializes the `String` constant `NL` to the system-specific line terminator.

Finally, we need to write additional tests (4 in all) that verify that method `readInt` works properly when an invalid integer string is entered and when the integer string entered is not in range. Besides verifying that the value returned is correct, you should verify the prompts displayed. For each of these tests, your test input should include a valid input following the invalid input so that the method will return normally. These tests are left to programming exercises 1 and 2. For example, the statement

```
ByteArrayInputStream testIn = new ByteArrayInputStream("X\n 3".getBytes());
```

would provide a data sample for the first test case ("X" is an invalid integer string, "3" is valid).

## EXERCISES FOR SECTION 3.6

### SELF-CHECK

1. Explain why it is not necessary to write a test to verify that `readInt` works properly when the input consists of an invalid integer string, followed by an out-of-range integer, then followed by an integer that is in range.

### PROGRAMMING

1. Write separate tests to verify the result returned by `readInt` and the prompts displayed when the input consists of an invalid integer string followed by a valid integer. Verify that the expected error message is presented followed by a repeat of the prompt.
2. Write separate tests to verify the result returned by `readInt` and the prompts displayed when the input consists of an out-of-range integer followed by a valid integer. Verify that the expected error message is presented followed by a repeat of the prompt.



## 3.7 Debugging a Program

In this section, we will discuss the process of debugging (removing errors) both with and without the use of a debugger program. Debugging is the major activity performed by programmers during the testing phase. Testing determines whether you have an error; during debugging you determine the cause of run-time and logic errors and correct them, without introducing new ones. If you have followed the suggestions for testing described in the previous section, you will be well prepared to debug your program.

Debugging is like detective work. To debug a program, you must inspect carefully the information displayed by your program, starting at the beginning, to determine whether what you see is what you expect. For example, if the result returned by a method is incorrect but the arguments (if any) passed to the method had the correct values, then there is a problem inside the method. You can try to trace through the method to see whether you can find the source of the error and correct it. If you can't, you may need more information. One way to get that information is to insert additional diagnostic output statements in the method. For example, if the method contains a loop, you may want to display the values of loop control variables during loop execution.

---

**EXAMPLE 3.4** The loop in Listing 3.2 does not seem to terminate when the user enters the sentinel string ("\*\*\*"). The loop exits eventually after the user has entered 10 data items but the string returned contains the sentinel.

---

**LISTING 3.2**The Method `getSentence`

```
.....
LISTING 3.2
The Method getSentence

/*
 * Return the individual words entered by the user.
 * The user can enter the sentinel *** to terminate data entry.
 * @return A string with a maximum of 10 words
 */
public static String getSentence() {
    Scanner in = new Scanner(System.in);
    StringJoiner stb = new StringJoiner(" ");
    int count = 0;
    while (count < 10) {
        System.out.println("Enter a word or *** to quit");
        String word = in.next();
        if (word == "***") break;
        stb.add(word);
        count++;
    }
    return stb.toString();
}

```

To determine the source of the problem, you should insert a diagnostic output statement that displays the values of `word` and `count` to make sure that `word` is receiving the sentinel string ("\*\*\*"). You could insert the line

```
System.out.println("!!! Next word is " + word + ", count is " + count);
```

as the first statement in the loop body. If the third data item you enter is the sentinel string, you will get the output line:

```
!!! next word is ***, count is 2
```

This will show you that `word` does indeed receive the sentinel string, but the loop body continues to execute. Therefore, there must be something wrong with the `if` statement that tests for the sentinel. In fact, the `if` statement must be changed to

```
if (word == null || word.equals("***")) break;
```

because `word == "***"` compares the address of the string stored in `word` with the address of the literal string "\*\*\*", not the contents of the two strings as intended. The strings' addresses will always be different, even when their contents are the same. To compare their contents, the `equals` method must be used. Note that we needed to do the test `word == null` first because we would get a `NullPointerException` if `equals` was called when `word` was `null`.

## Using a Debugger

If you are using an IDE, you will most likely have a debugger program as part of the IDE. A debugger can execute your program incrementally rather than all at once. After each increment of the program executes, the debugger pauses, and you can view the contents of variables to determine whether the statement(s) executed as expected. You can inspect all the program variables without needing to insert diagnostic output statements. When you have finished examining the program variables, you direct the debugger to execute the next increment.

You can choose to execute in increments as small as one program statement (called *single-step execution*) to see the effect of each statement's execution. Another possibility is to set breakpoints in your program to divide it into sections. The debugger can execute all the statements from one breakpoint to the next as a group. For example, if you wanted to see the

effects of a loop's execution but did not want to step through every iteration, you could set breakpoints at the statements just before and just after the loop.

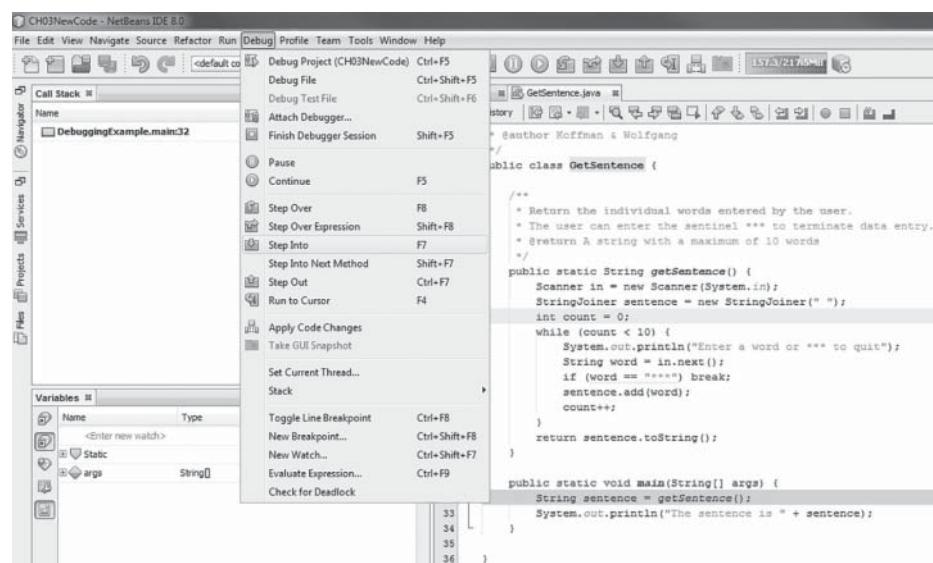
When your program pauses, if the next statement contains a call to a method, you can select single-step execution in the method being called (i.e., step into the method). Alternatively, you can execute all the method statements as a group and pause after the return from the method execution (i.e., step over the method).

The actual mechanics of using a debugger depends on the IDE that you are using. However, the process that you follow is similar among IDEs, and if you understand the process for one, you should be able to use any debugger. In this section, we demonstrate how to use the debugger in NetBeans, the IDE that is distributed by Sun along with the software development kit (SDK).

Before starting the debugger, you must set a breakpoint. Figure 3.2 is the display produced by the debugger at the beginning of debugging the `GetSentence.main` method. In NetBeans, you set a breakpoint by clicking in the vertical bar just to the left of the statement that you want to select as a breakpoint. The small squares and highlighted bars indicate the breakpoints. You can click again on a small square to remove the breakpoint. The source editor window displays the code to be debugged. The Debug pull-down menu shows the options for executing the code. The selected item, Step Into, is a common technique for starting single-step execution, as we have just described. A window (such as window Variables in the center left) typically shows the values of data fields and local variables. In this case, there is one local variable for method `main`: the String array `args`, which is empty. The arrow to the left of the highlighted line in the source editor window indicates the next step to execute (the call to method `getSentence`). Select Step Into again to execute the individual statements of method `getSentence`.

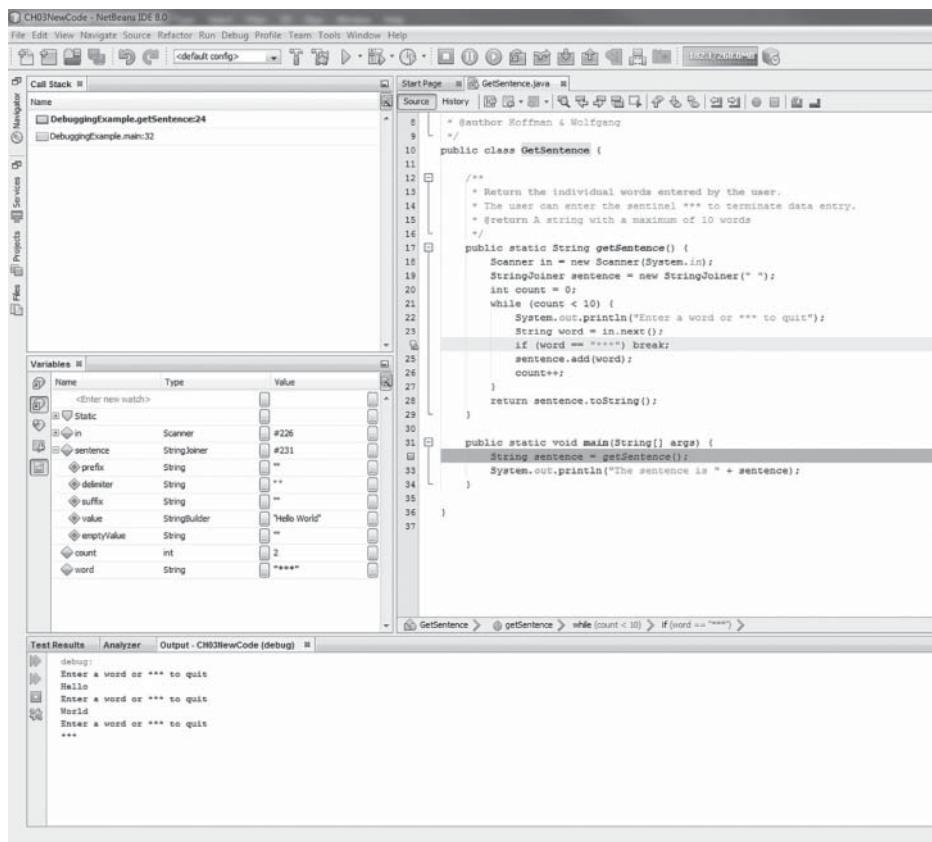
Figure 3.3 shows the editor and Variables windows after we have entered "Hello", "world", and "\*\*\*". The contents of `sentence` is "Hello world", the value of `count` is 2, and the contents of `word` is "\*\*\*". The next statement to execute is highlighted. It is the `if` statement, which tests for the sentinel. Although we expect the condition to be true, it is false (why?), so the loop continues to execute and "\*\*\*" will be appended to `sentence`.

**FIGURE 3.2**  
Using the Debugger  
for NetBeans



**FIGURE 3.3**

Editor and Debugging Windows



## EXERCISES FOR SECTION 3.7

### SELF-CHECK

- The following method does not appear to be working properly if all data are negative numbers. Explain where you would add diagnostic output statements to debug it, and give an example of each statement.

```

/** Finds the target value in array elements x[start] through x[last].
 * @param x array whose largest value is found
 * @param start first subscript in range
 * @param last last subscript in range
 * @return the largest value of x[start] through x[last]
 * @pre first <= last
 */
public int findMax(int[] x, int start, int last) {
    if (start > last)
        throw new IllegalArgumentException("Empty range");
    int maxSoFar = 0;
    for (int i = start; i < last; i++) {
        if (x[i] > maxSoFar)
            maxSoFar = i;
    }
    return maxSoFar;
}

```

- Explain the difference between selecting Step Into and Step Over during debugging.

3. Explain the rationale for the position of the breakpoints in method `getSentence`.
4. How would the execution of method `getSentence` change if the breakpoint were set at the statement just before the loop instead of at the loop heading?

#### PROGRAMMING

1. After debugging, provide a corrected version of the method in Self-Check Exercise 1. Leave the debugging statements in, but execute them only when the global constant `TESTING` is `true`.
2. Write and test a driver program to test method `findMax` in Self-Check exercise 1.



## Chapter Review

- ◆ Program testing is done at several levels starting with the smallest testable piece of the program, called a unit. A unit is either a method or a class, depending on the complexity.
- ◆ Once units are individually tested, they can be tested together; this level is called integration testing.
- ◆ Once the whole program is put together, it is tested as a whole; this level is called system testing.
- ◆ Finally, the program is tested in an operational manner demonstrating its functionality; this is called acceptance testing.
- ◆ Black-box (also called closed-box) testing tests the item (unit or system) based on its functional requirements without using any knowledge of the internal structure.
- ◆ White-box (also called glass-box or open-box) testing tests the item using knowledge of its internal structure. One of the goals of white-box testing is to achieve test coverage. This can range from testing every statement at least once, to testing each branch condition (`if` statements, `switch` statements, and loops) to verify each possible path through the program.
- ◆ Test drivers and stubs are tools used in testing. A test driver exercises a method or class and drives the testing. A stub stands in for a method that the unit being tested calls. This can be used to provide test results, and it can be used to enable a call of that method to be tested when the method being called is not yet coded.
- ◆ The JUnit test framework is a software product that facilitates writing test cases, organizing the test cases into test suites, running the test suites, and reporting the results.
- ◆ Test-Driven development is an approach to developing programs that has gained popularity among professional software developers. The approach is to write test cases one at a time and make the changes to the program to pass the test.
- ◆ Interactive programs can be tested by using the `ByteArrayInputStream` to provide known input and the `ByteArrayOutputStream` to verify output.
- ◆ We described the debugging process and showed an example of how a debugger can be used to obtain information about a program's state.

## Java API Classes Introduced in This Chapter

`ByteArrayInputStream`  
`ByteArrayOutputStream`

## User-Defined Interfaces and Classes in This Chapter

`class ArraySearch`  
`class MyInput`

### Quick-Check Exercises

1. \_\_\_\_\_ testing requires the use of test data that exercises each statement in a module.
2. \_\_\_\_\_ testing focuses on testing the functional characteristics of a module.
3. \_\_\_\_\_ determines whether a program has an error: \_\_\_\_\_ determines the \_\_\_\_\_ of the error and helps you \_\_\_\_\_ it.
4. A method's \_\_\_\_\_ and \_\_\_\_\_ serve as a \_\_\_\_\_ between a method caller and the method programmer—if a caller satisfies the, the method result should satisfy the \_\_\_\_\_.
5. Explain how the old adage “We learn from our mistakes” applies to test-driven development.

### Review Questions

1. Indicate in which state of testing (unit, integration, system) each of the following kinds of errors should be detected:
  - a. An array index is out of bounds.
  - b. A `FileNotFoundException` is thrown.
  - c. An incorrect value of withholding tax is being computed under some circumstances.
2. Describe the differences between stubs and drivers.
3. What is refactoring? How it is used in test-driven development?
4. Modify the list of tests for finding the first occurrence of a target in an array shown in Section 3.4 to finding the last occurrence.
5. Use the list of tests for review question 4. To develop a JUnit test harness.
6. Use test-driven development to code the method for finding the last occurrence of a target in an array.

### Programming

1. Design and code a JUnit test harness for a method that finds the smallest element in an array.
2. Develop the method in project 1 using test-driven development.
3. Design and code the JUnit test cases for a class that computes the sine and cosine functions in a specialized manner. This class is going to be part of an embedded system running on a processor that does not support floating-point arithmetic or the Java Math class. The class to be tested is shown in Listing 3.3. Your job is to test the methods `sin` and `cos`; you are to assume that the methods `sin0to90` and `sin45to90` have already been tested.

You need to design a set of test data that will exercise each of the `if` statements. To do this, look at the boundary conditions and pick values that are

- Exactly on the boundary
- Close to the boundary
- Between boundaries

**LISTING 3.3**

SinCos.java

```
.....
LISTING 3.3
SinCos.java

/** This class computes the sine and cosine of an angle
   expressed in degrees. The result will be an
   an integer representing the sine or cosine as
   ten-thousandths.
*/
public class SinCos {
    /** Compute the sine of an angle in degrees.
        @param x The angle in degrees
        @return the sine of x
    */
    public static int sin(int x) {
        if (x < 0) {
            x = -x;
        }
        x = x % 360;
        if (0 <= x && x <= 45) {
            return sin0to45(x);
        } else if (45 <= x && x <= 90) {
            return sin45to90(x);
        } else if (90 <= x && x <= 180) {
            return sin(180 - x);
        } else {
            return -sin(x - 180);
        }
    }

    /** Compute the cosine of an angle in degrees
        @param x The angle in degrees
        @return the cosine of x
    */
    public static int cos(int x) {
        return sin(x + 90);
    }

    /** Compute the sine of an angle in degrees
        between 0 and 45
        @param x The angle
        @return the sine of x
        @pre 0 <= x < 45
    */
    private static int sin0to45(int x) {
        // In a realistic program this method would
        // use a polynomial approximation that was
        // optimized for the input range
        // Insert code to compute sin(x) for x between 0 and 45 degrees
    }

    /** Compute the sine of an angle in degrees
        between 45 and 90.
        @param x - The angle
        @return the sine of x
        @pre 45 <= x <= 90
    */
    private static int sin45to90(int x) {
        // In a realistic program this method would
        // use a polynomial approximation that was
        // optimized for the input range
        // Insert code to compute sin(x) for x between 45 and 90 degrees
    }
}
```

## Answers to Quick-Check Exercises

1. *White-box* testing requires the use of test data that exercises each statement in a module.
2. *Black-box* testing focuses on testing the functional characteristics of a module.
3. *Testing* determines whether a program has an error; *debugging* determines the *cause* of the error and helps you *correct* it.
4. A method's *precondition* and *postcondition* serve as a *contract* between a method caller and the method programmer—if a caller satisfies the *precondition* the method result should satisfy the *postcondition*.
5. In test-driven development, failing a test informs the method coder that the method developed so far needs to be modified. By repeated failure and adaptation of the code to pass the test just failed while still passing the rest of the tests in the test harness, the method coder develops a correct version of the desired method.

# *Stacks and Queues*

## Chapter Objectives

- ◆ To learn about stack data type and how to use its four methods: push, pop, peek, and empty
- ◆ To understand how Java implements a stack
- ◆ To learn how to implement a stack using an underlying array or a linked list
- ◆ To see how to use a stack to perform various applications, including finding palindromes, testing for balanced (properly nested) parentheses, and evaluating arithmetic expressions
- ◆ To learn how to represent a waiting line (queue) and how to use the methods in the Queue interface for insertion (offer and add), for removal (remove and poll), and for accessing the element at the front (peek and element)
- ◆ To understand how to implement the Queue interface using a single-linked list, a circular array, and a double-linked list
- ◆ To become familiar with the Deque interface and how to use its methods to insert and remove items from both ends of a deque

In this chapter we study two abstract data types, the stack and queue, that are widely used. A stack is a LIFO (last-in, first-out) list because the last element pushed onto a stack will be the first element to be popped off. A queue, on the other hand, is a FIFO (first-in, first-out) list because the first element inserted in the queue will be the first element removed.

Stacks and queues are more restrictive than the list data type that we studied in Chapter 2. A client can access any element in a list and can insert elements at any location. However, a client can access only a single element in a stack or queue: the one that was most recently inserted in the stack or the oldest one in the queue. This may seem like a serious restriction that would make them not very useful, but it turns out that stacks and queues are actually two of the most commonly used data structures in computer science. For example, during program execution, a stack is used to store information about the parameters and return points for all the methods that are currently executing (you will see how this is done in Chapter 5, “Recursion”). Compilers also use stacks to store information while evaluating expressions.

Operating systems also make extensive use of queues. One application of queues is to manage process execution. In a modern operating system, several processes may be executing

at the same time and, therefore, may request the use of the same computer resource (for example, the CPU or a printer). The operating system stores these requests in a queue and then removes them so that the process that is waiting the longest will be the next one to get access to the desired resource.

We will discuss several applications of stacks and queues. We will also show how to implement them using both arrays and linked lists.

## Stacks and Queues

- 4.1** Stack Abstract Data Type
- 4.2** Stack Application
  - Case Study:* Finding Palindromes
- 4.3** Implementing a Stack
- 4.4** Additional Stack Applications
  - Case Study:* Evaluating Postfix Expressions
  - Case Study:* Converting from Infix to Postfix
  - Case Study:* Converting Expressions with Parentheses
- 4.5** Queue Abstract Data Type
- 4.6** Queue Applications
  - Case Study:* Maintaining a Queue
- 4.7** Implementing the Queue Interface
- 4.8** The Deque Interface

## 4.1 Stack Abstract Data Type

In a cafeteria you can see stacks of dishes placed in spring-loaded containers. Usually, several dishes are visible above the top of the container, and the rest are inside the container. You can access only the dish that is on top of the stack. If you want to place more dishes on the stack, you can place the dishes on top of those that are already there. The spring inside the stack container compresses under the weight of the additional dishes, adjusting the height of the stack so that only the top few dishes are always visible.

**FIGURE 4.1**  
A Pez Dispenser



Another physical example of a stack is a Pez® dispenser (see Figure 4.1). A Pez dispenser is a toy that contains candies. There is also a spring inside the dispenser. The top of the dispenser is a character's head. When you open the dispenser, a single candy *oops* out. You can only extract one candy at a time. If you want to eat more than one candy, you have to open the dispenser multiple times.

In programming, a stack is a data structure with the property that only the top element of the stack is accessible. In a stack, the top element is the data value that was most recently stored in the stack. Sometimes this storage policy is known as last-in, first-out, or *LIFO*.

Next, we specify some of the operations that we might wish to perform on a stack.

### Specification of the Stack Abstract Data Type

Because only the top element of a stack is visible, a stack performs just a few operations. We need to be able to inspect the top element (method `peek`), retrieve the top element (method `pop`), push a new element onto the stack (method `push`), and test for an empty stack (method

**TABLE 4.1**

Specification of StackInt&lt;E&gt;

Methods	Bhavior
boolean isEmpty()	Returns true if the stack is empty; otherwise, returns false
E peek()	Returns the object at the top of the stack without removing it
E pop()	Returns the object at the top of the stack and removes it
E push(E obj)	Pushes an item onto the top of the stack and returns the item pushed

isEmpty). Table 4.1 shows a specification for the Stack ADT that specifies the stack operations. We will write this as interface StackInt<E>.

Listing 4.1 shows the interface StackInt<E>, which declares the methods in the Stack ADT.

**LISTING 4.1**

StackInt.java

```
/** A Stack is a data structure in which objects are inserted into
 * and removed from the same end (i.e., Last-In, First-Out).
 * @param <E> The element type
 */
public interface StackInt<E> {

    /** Pushes an item onto the top of the stack and returns
     * the item pushed.
     * @param obj The object to be inserted
     * @return The object inserted
    */
    E push(E obj);

    /** Returns the object at the top of the stack
     * without removing it.
     * @post The stack remains unchanged.
     * @return The object at the top of the stack
     * @throws NoSuchElementException if stack is empty
    */
    E peek();

    /** Returns the object at the top of the stack
     * and removes it.
     * @post The stack is one item smaller.
     * @return The object at the top of the stack
     * @throws NoSuchElementException if stack is empty
    */
    E pop();

    /** Returns true if the stack is empty; otherwise,
     * returns false.
     * @return true (false) if the stack is empty (not empty)
    */
    boolean isEmpty();
}
```

**EXAMPLE 4.1** A stack names (type Stack<String>) contains five strings as shown in Figure 4.2(a). The name "Rich" was placed on the stack before the other four names; "Jonathan" was the last element placed on the stack.

For stack names in Figure 4.2(a), the value of names.isEmpty() is false. The statement

```
String last = names.peek();
```

stores "Jonathan" in last without changing names. The statement

```
String temp = names.pop();
```

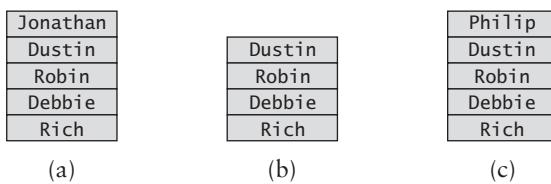
removes "Jonathan" from names and stores a reference to it in temp. The stack names now contains four elements and is shown in Figure 4.2(b). The statement

```
names.push("Philip");
```

pushes "Philip" onto the stack; the stack names now contains five elements and is shown in Figure 4.2(c).

**FIGURE 4.2**

Stack Names



## EXERCISES FOR SECTION 4.1

### SELF-CHECK

- Assume that the stack names is defined as in Figure 4.2(c) and perform the following sequence of operations. Indicate the result of each operation and show the new stack if it is changed.

```
names.push("Jane");
names.push("Joseph");
String top = names.pop();
String nextTop = names.peek();
```

- For the stack names in Figure 4.2 (c), what is the effect of the following:

```
while (!names.isEmpty()) {
    System.out.println(names.pop());
}
```

- What would be the effect of using peek instead of pop in Question 2?

### PROGRAMMING

- Write a main function that creates three stacks of Integer objects. Store the numbers -1, 15, 23, 44, 4, 99 in the first two stacks. The top of each stack should store 99.
- Write a loop to get each number from the first stack and store it into the third stack.
- Write a second loop to remove a value from the second and third stacks and display each pair of values on a separate output line. Continue until the stacks are empty. Show the output.



## 4.2 Stack Applications

In this section we will study a client program that uses a stack, a palindrome finder. The `java.util.Stack` class is part of the original Java API but is not recommended for new applications. Instead, the Java designers recommend that we use the `java.util.Deque` interface and the `java.util.ArrayDeque` class to provide the methods listed in Table 4.1. The `Deque` interface specifies the methods in our interface `StackInt` (see Table 4.1) and also those needed for a queue. We discuss the `Deque` interface and class `ArrayDeque` in more detail in Section 4.8.

### CASE STUDY Finding Palindromes

**Problem** A palindrome is a string that reads the same in either direction: left to right or right to left. For example, “kayak” is a palindrome, as is “I saw I was I.” A well-known palindrome regarding Napoleon Bonaparte is “Able was I ere I saw Elba” (the island where he was sent in exile). We would like a program that reads a string and determines whether it is a palindrome.

**Analysis** This problem can be solved in many different ways. For example, you could set up a loop in which you compare the characters at each end of a string as you work toward the middle. If any pair of characters is different, the string can’t be a palindrome. Another approach would be to scan a string backward (from right to left) and append each character to the end of a new string, which would become the reverse of the original string. Then you can see whether the strings are equal. The approach we will study here uses a stack to assist in forming the reverse of a string. It is not the most efficient way to solve the problem, but it makes good use of a stack.

If we scan the input string from left to right and push each character in the input string onto a stack, or we can form the reverse of the string by popping the characters and joining them together in the order that they come off the stack. For example, the stack at left contains the characters in the string “I saw”.

w
a
s
I

If we pop them off and join them together, we will get “w” + “a” + “s” + “ ” + “I”, or the string “was I”. When the stack is empty, we can compare the string we formed with the original. If they are the same, the original string is a palindrome. Because `char` is a primitive type, each character must be wrapped in a `Character` object before it can be pushed onto the stack.

#### Data Requirements

##### PROBLEM INPUTS

An input string to be tested

##### PROBLEM OUTPUTS

A message indicating whether the string is a palindrome

**TABLE 4.2**  
Class PalindromeFinder

Methods	Behavior
<code>private static Deque&lt;Character&gt; fillStack(String inputString)</code>	Returns a stack that is filled with the characters in <code>inputString</code>
<code>private String buildReverse(String inputString)</code>	Calls <code>fillStack</code> to fill the stack based on <code>inputString</code> and returns a new string formed by popping each character from this stack and joining the characters. Empties the stack
<code>public boolean isPalindrome(String inputString)</code>	Returns <code>true</code> if <code>inputString</code> and the string built by <code>buildReverse</code> have the same contents, except for case. Otherwise, returns <code>false</code>

**Design** We can define a class called `PalindromeFinder` (Table 4.2) with three static methods: `fillStack` pushes all characters from the input string onto a stack, `buildReverse` builds a new string by popping the characters off the stack and joining them, and `isPalindrome` compares the input string and new string to see whether they are palindromes. Method `isPalindrome` is the only public method and is called with the string to be tested as its argument.

**Implementation** Listing 4.2 shows the class. Method `isPalindrome` calls `buildReverse`, which calls `fillStack` to build the stack (an `ArrayDeque`). In `fillStack`, the statement

```
charStack.push(inputString.charAt(i));
```

autoboxes a character and pushes it onto the stack.

In method `buildReverse`, the loop

```
while (!charStack.isEmpty()) {
    // Remove top item from stack and append it to result.
    result.append(charStack.pop());
}
```

pops each object off the stack and appends it to the result string.

Method `isPalindrome` uses the `String` method `equalsIgnoreCase` to compare the original string with its reverse.

```
return inputString.equalsIgnoreCase(buildReverse(inputString));
```

#### **LISTING 4.2**

`PalindromeFinder.java`

```
import java.util.*;

/** Class with methods to check whether a string is a palindrome. */
public class PalindromeFinder {

    /** Fills a stack of characters from an input string.
     *  @param inputString the string to be checked
     *  @return a stack of characters in inputString
     */
    private static Deque<Character> fillStack(String inputString) {
        Deque<Character> charStack = new ArrayDeque<>();
}
```

```

        for (int i = 0; i < inputString.length(); i++) {
            charStack.push(inputString.charAt(i));
        }
        return charStack;
    }

    /**
     * Builds the reverse of a string by calling fillsStack
     * to push its characters onto a stack and then popping them
     * and appending them to a new string.
     * @post The stack is empty.
     * @return The string containing the characters in the stack
     */
    private static String buildReverse(String inputString) {
        Deque<Character> charStack = fillStack(inputString);
        StringBuilder result = new StringBuilder();
        while (!charStack.isEmpty()) {
            // Remove top item from stack and append it to result.
            result.append(charStack.pop());
        }
        return result.toString();
    }

    /**
     * Calls buildReverse and compares its result to inputString
     * @param inputString the string to be checked
     * @return true if inputString is a palindrome, false if not
     */
    public static boolean isPalindrome(String inputString) {
        return inputString.equalsIgnoreCase(buildReverse(inputString));
    }
}

```

**Testing** To test this class, you should run it with several different strings, including both palindromes and nonpalindromes, as follows:

- A single character (always a palindrome)
- Multiple characters in one word
- Multiple words
- Different cases
- Even-length strings
- Odd-length strings
- An empty string (considered a palindrome)

Listing 4.3 is a JUnit test suite for the PalindromeFinder class.

.....

**LISTING 4.3**

PalindromeFinderTest

```

import org.junit.Test;
import static org.junit.Assert.*;

/**
 * Test of the PalindromeFinder
 * @author Koffman & Wolfgang
 */

```

```

public class PalindromeFinderTest {
    public PalindromeFinderTest() {
    }

    @Test
    public void singleCharacterIsAlwaysAPalindrome() {
        assertTrue(PalindromeFinder.isPalindrome("x"));
    }

    @Test
    public void aSingleWordPalindrome() {
        assertTrue(PalindromeFinder.isPalindrome("kayak"));
    }

    @Test
    public void aSingleWordNonPalindrome() {
        assertFalse(PalindromeFinder.isPalindrome("foobar"));
    }

    @Test
    public void multipleWordsSameCase() {
        assertTrue(PalindromeFinder.isPalindrome("I saw I was I"));
    }

    @Test
    public void multipleWordsDifferentCase() {
        assertTrue(PalindromeFinder.isPalindrome(
            "Able was I ere I saw Elba"));
    }

    @Test
    public void anEmptyStringIsAPalindrome() {
        assertTrue(PalindromeFinder.isPalindrome(""));
    }

    @Test
    public void anEvenLengthStringPalindrome() {
        assertTrue(PalindromeFinder.isPalindrome("fooof"));
    }
}

```



## PITFALL

### Attempting to Pop an Empty Stack

If you attempt to pop an empty stack, your program will throw a `NoSuchElementException`. You can guard against this error by testing for a nonempty stack before popping the stack. Alternatively, you can catch the error if it occurs and handle it.

### Stack Application on Textbook Website

There is an additional case study on the textbook website that uses a stack to check that the parentheses in an expression are balanced. This can be found at the URL specified in the preface.

## EXERCISES FOR SECTION 4.2

### SELF-CHECK

1. The result returned by the palindrome finder depends on all characters in a string, including spaces and punctuation. Discuss how you would modify the palindrome finder so that only the letters in the input string are used to determine whether the input string is a palindrome. You should ignore any other characters.

### PROGRAMMING

1. Write a method that reads a line and reverses the words in the line (not the characters) using a stack. For example, given the following input:

The quick brown fox jumps over the lazy dog  
you should get the following output:  
dog lazy the over jumps fox brown quick The

2. Three different approaches to finding palindromes are discussed in the Analysis section of that case study. Code the first approach.
3. Code the second approach to find palindromes.



## 4.3 Implementing a Stack

This section discusses how to implement our stack interface (`StackInt`). We will show how to do this using class `ArrayList` and also using class `LinkedList`.

### Implementing a Stack with an `ArrayList` Component

You may have recognized that a stack is very similar to an `ArrayList`. In fact, in the Java Collections framework, the class `Stack` extends class `Vector`, which is the historical predecessor of `ArrayList`. Just as they suggest using class `Deque` instead of the `Stack` class in new applications, the Java designers recommend using class `ArrayList` instead of class `Vector`.

Next we show how to write a class, which we will call `ListStack`, that has an `ArrayList` component. We will call this component `theData`, and it will contain the stack data.

We code the `ListStack<E>.push` method as:

```
public E push(E obj) {
    theData.add(obj);
    return obj;
}
```

The `ListStack` class is said to be an *adapter class* because it adapts the methods available in another class (a `List`) to the interface its clients expect by giving different names to essentially the same operations (e.g., `push` instead of `add`). This is an example of *method delegation*.

Listing 4.4 shows the `ListStack` class. Note that the statements that manipulate the stack explicitly refer to data field `theData`. For example, in `ListStack.push` we use the statement

```
theData.add(obj);  
to push obj onto the stack as the new last element of ArrayList theData.
```

**LISTING 4.4**

ListStack.java

```

.....
LISTING 4.4
ListStack.java
import java.util.*;

/** Class ListStack<E> implements the interface StackInt<E> as
 * an adapter to the List.
 * @param <E> The type of elements in the stack.
 */
public class ListStack<E> implements StackInt<E> {

    /** The List containing the data */
    private List<E> theData;

    /** Construct an empty stack using an ArrayList as the container. */
    public ListStack() {
        theData = new ArrayList<>();
    }

    /** Push an object onto the stack.
     * @post The object is at the top of the stack.
     * @param obj The object to be pushed
     * @return The object pushed
     */
    @Override
    public E push(E obj) {
        theData.add(obj);
        return obj;
    }

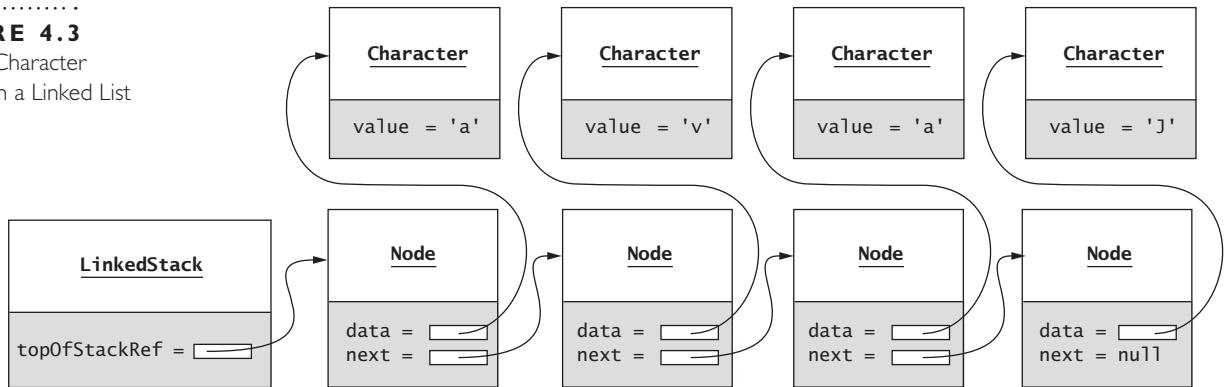
    /** Peek at the top object on the stack.
     * @return The top object on the stack
     * @throws NoSuchElementException if the stack is empty
     */
    @Override
    public E peek() {
        if (isEmpty()) {
            throw new NoSuchElementException();
        }
        return theData.get(theData.size() - 1);
    }

    /** Pop the top object off the stack.
     * @post The object at the top of the stack is removed.
     * @return The top object, which is removed
     * @throws NoSuchElementException if the stack is empty
     */
    @Override
    public E pop() {
        if (isEmpty()) {
            throw new NoSuchElementException();
        }
        return theData.remove(theData.size() - 1);
    }

    /** See whether the stack is empty.
     * @return true if the stack is empty
     */
    @Override
    public boolean isEmpty() {
        return theData.isEmpty();
    }
}

```

**FIGURE 4.3**  
Stack of Character Objects in a Linked List



## Implementing a Stack as a Linked Data Structure

We can also implement a stack using a single-linked list of nodes. We show the stack containing the characters in "Java" in Figure 4.3, with the last character in the string stored in the node at the top of the stack. Class `LinkedStack<E>` contains a collection of `Node<E>` objects (see Section 2.5). Recall that inner class `Node<E>` has attributes `data` (type `E`) and `next` (type `Node<E>`).

Reference variable `topOfStackRef` (type `Node<E>`) references the last element placed on the stack. Because it is easier to insert and delete from the head of a linked list, we will have `topOfStackRef` reference the node at the head of the list.

Method `push` inserts a node at the head of the list. The statement

```
topOfStackRef = new Node<E>(obj, topOfStackRef);
```

sets `topOfStackRef` to reference the new node; `topOfStackRef.next` references the old top of the stack. When the stack is empty, `topOfStackRef` is `null`, so the attribute `next` for the first object pushed onto the stack (the item at the bottom) will be `null`.

Method `peek` will be very similar to the `LinkedList` method `getFirst`. Method `isEmpty` tests for a value of `topOfStackRef` equal to `null`. Method `pop` simply resets `topOfStackRef` to the value stored in the `next` field of the list head and returns the old `topOfStackRef` data. Listing 4.5 shows class `LinkedStack`.

**LISTING 4.5**

Class `LinkedStack`

```
import java.util.NoSuchElementException;

/** Class to implement interface StackInt<E> as a linked list. */
public class LinkedStack<E> implements StackInt<E> {
    // Insert inner class Node<E> here. (See Listing 2.1)

    // Data Fields
    /** The reference to the first stack node. */
    private Node<E> topOfStackRef = null;

    /** Insert a new item on top of the stack.
     * @post The new item is the top item on the stack.
     *       All other items are one position lower.
     * @param obj The item to be inserted
     * @return The item that was inserted
    */
}
```

```

@Override
public E push(E obj) {
    topOfStackRef = new Node<E>(obj, topOfStackRef);
    return obj;
}

/** Remove and return the top item on the stack.
 * @pre The stack is not empty.
 * @post The top item on the stack has been
       removed and the stack is one item smaller.
 * @return The top item on the stack
 * @throws NoSuchElementException if the stack is empty
 */
@Override
public E pop() {
    if (isEmpty()) {
        throw new NoSuchElementException();
    }
    else {
        E result = topOfStackRef.data;
        topOfStackRef = topOfStackRef.next;
        return result;
    }
}

/** Return the top item on the stack.
 * @pre The stack is not empty.
 * @post The stack remains unchanged.
 * @return The top item on the stack
 * @throws NoSuchElementException if the stack is empty
 */
@Override
public E peek() {
    if (isEmpty()) {
        throw new NoSuchElementException();
    }
    else {
        return topOfStackRef.data;
    }
}

/** See whether the stack is empty.
 * @return true if the stack is empty
 */
@Override
public boolean isEmpty() {
    return topOfStackRef == null;
}
}

```

## Comparison of Stack Implementations

The easiest approach to implementing a stack in Java would be to give it a `List` component for storing the data. Since all insertions and deletions are at one end, the stack operations would all be  $O(1)$  operations. You could use an object of any class that implements the `List` interface to store the stack data, but the `ArrayList` is the simplest.

Finally, you could also use your own linked data structure. This has the advantage of using exactly as much storage as is needed for the stack. However, you would also need to allocate

storage for the links. Because all insertions and deletions are at one end, the flexibility provided by a linked data structure is not utilized. All stack operations using a linked data structure would also be  $O(1)$ .

## EXERCISES FOR SECTION 4.3

### SELF-CHECK

- For the implementation of stack `s` using an `ArrayList` as the underlying data structure, show how the underlying data structure changes after each statement below executes. Assume the characters in "Happy" are already stored on the stack (`H` pushed on first).

```
s.push('i');
s.push('s');
char ch1 = s.pop(); s.pop();
s.push(' ');
char ch2 = s.peek();
```

- How do your answers to Question 1 change if the initial capacity is 4 instead of 7?
- For the implementation of stack `s` using a linked list of nodes as the underlying data structure (see Figure 4.3), show how the underlying data structure changes after each of the following statements executes. Assume the characters in "Happy" are already stored on the stack (`H` pushed on first).

```
s.push('i');
s.push('s');
char ch1 = s.pop();
s.pop();
s.push(' ');
char ch2 = s.peek();
```

### PROGRAMMING

- Write a method `size` for class `LinkedStack<E>` that returns the number of elements currently on a `LinkedStack<E>`.
- Write method `size` for the `ArrayList` implementation.



## 4.4 Additional Stack Applications

In this section we consider two case studies that relate to evaluating arithmetic expressions. The first problem is slightly easier, and it involves evaluating expressions that are in postfix form. The second problem discusses how to convert from *infix notation* (common mathematics notation) to postfix form.

Normally we write expressions using infix notation, in which binary operators (`*`, `+`, etc.) are inserted between their operands. Infix expressions present no special problem to humans because we can easily scan left and right to find the operands of a particular operator. A calculator (or computer), however, normally scans an expression string in the order that it is input (left to right). Therefore, it is easier to evaluate an expression if the user types in the operands for each operator before typing the operator (*postfix notation*). Table 4.3 shows

**TABLE 4.3**  
Postfix Expressions

Postfix Expression	Infix Expression	Value
<u>4 7 *</u>	$4 * 7$	28
<u>4 7 2 + *</u>	$4 * (7 + 2)$	36
<u>4 7 * 20 -</u>	$(4 * 7) - 20$	8
<u>3 4 7 * 2 / +</u>	$3 + ((4 * 7) / 2)$	17

some examples of expressions in postfix and infix form. The braces under each postfix expression will help you visualize the operands for each operator.

The advantage of the postfix form is that there is no need to group subexpressions in parentheses or even to consider operator precedence. (We talk more about postfix form in the second case study in this section.) The next case study develops a program that evaluates a postfix expression.

## CASE STUDY Evaluating Postfix Expressions

**Problem** Write a class that evaluates a postfix expression. The postfix expression will be a string containing digit characters and operator characters from the set  $+, -, *, /$ . The space character will be used as a delimiter between tokens (integers and operators).

**Analysis** In a postfix expression, the operands precede the operators. A stack is the perfect place to save the operands until the operator is scanned. When the operator is scanned, its operands can be popped off the stack (the last operand scanned, i.e., the right operand, will be popped first). Therefore, our program will push each integer operand onto the stack. When an operator is read, the top two operands are popped, the operation is performed on its operands, and the result is pushed back onto the stack. The final result should be the only value remaining on the stack when the end of the expression is reached.

**Design** We will write class `PostfixEvaluator` to evaluate postfix expressions. The class should define a method `eval`, which scans a postfix expression and processes each of its tokens, where a token is either an operand (an integer) or an operator. We also need a method `evalOp`, which evaluates each operator when it is scanned, and a method `isOperator`, which determines whether a character is an operator. Table 4.4 describes the class.

The algorithm for `eval` follows. The stack operators perform algorithm steps 1, 5, 7, 8, 10, and 11.

Table 4.5 shows the evaluation of the third expression in Table 4.3 using this algorithm. The arrow under the expression points to the character being processed; the stack diagram shows the stack after this character is processed.

**TABLE 4.4**  
Class PostfixEvaluator

Method	Behavior
<code>public static int eval(String expression)</code>	Returns the value of expression
<code>private static int evalOp(char op, Deque&lt;Integer&gt; operandStack)</code>	Pops two operands and applies operator op to its operands, returning the result
<code>private static boolean isOperator(char ch)</code>	Returns true if ch is an operator symbol

**TABLE 4.5**  
Evaluating a Postfix Expression

Expression	Action	Stack
4 7 * 20 - ↑	Push 4	4
4 7 * 20 - ↑	Push 7	7     4
4 7 * 20 - ↑	Pop 7 and 4 Evaluate 4 * 7 Push 28	28
4 7 * 20 - ↑	Push 20	20     28
4 7 * 20 - ↑	Pop 20 and 28 Evaluate 28 – 20 Push 8	8
4 7 * 20 - ↑	Pop 8 Stack is empty Result is 8	

### Algorithm for method eval

1. Create an empty stack of integers.
2. while there are more tokens
  3. Get the next token.
  4. if the first character of the token is a digit.
    5. Push the integer onto the stack.
  6. else if the token is an operator
    7. Pop the right operand off the stack.
    8. Pop the left operand off the stack.
    9. Evaluate the operation.
    10. Push the result onto the stack.
  11. Pop the stack and return the result.

**Implementation** Listing 4.6 shows the implementation of class `PostfixEvaluator`. There is an inner class that defines the exception `SyntaxErrorException`.

Method `eval` implements the algorithm shown in the design section. To simplify the extraction of tokens, we will assume that there are spaces between operators and operands. As explained in Appendix A.5, the method call

```
String[] tokens = expression.split("\\s+");
```

stores in array `tokens` the individual tokens (operands and operators) of string `expression` where the argument string "`\\s+`" specifies that the delimiter is one or more white-space characters. (We will remove the requirement for spaces between tokens and consider parentheses in the last case study of this section.)

The enhanced for statement

```
for (String nextToken : tokens) {
```

ensures that each of the strings in `tokens` is processed, and the `if` statement in the loop tests the first character of each token to determine its category (number or operator). Therefore, the body of method `eval` is enclosed within a `try-catch` sequence. A `NoSuchElementException`, thrown either as a result of a `pop` operation in `eval` or by a `pop` operation in a method called by `eval`, will be caught by the `catch` clause. In either case, a `SyntaxErrorException` is thrown.

Private method `isOperator` determines whether a character is an operator. When an operator is encountered, private method `evalOp` is called to evaluate it. This method pops the top two operands from the stack. The first item popped is the right-hand operand, and the second is the left-hand operand.

```
int rhs = operandStack.pop();
int lhs = operandStack.pop();
```

A `switch` statement is then used to select the appropriate expression to evaluate for the given operator. For example, the following case processes the addition operator and saves the sum of `lhs` and `rhs` in `result`.

```
case '+': result = lhs + rhs; break;
```

#### LISTING 4.6

`PostfixEvaluator.java`

```
import java.util.*;

/** Class that can evaluate a postfix expression. */
public class PostfixEvaluator {

    // Nested Class
    /** Class to report a syntax error. */
    public static class SyntaxErrorException extends Exception {
        /** Construct a SyntaxErrorException with the specified message.
         * @param message The message
         */
        SyntaxErrorException(String message) {
            super(message);
        }
    }

    // Constant
    /** A list of operators. */
    private static final String OPERATORS = "+-*";
```

```

// Methods
/** Evaluates the current operation.
   This function pops the two operands off the operand
   stack and applies the operator.
   @param op A character representing the operator
   @param operandStack the current stack of operands
   @return The result of applying the operator
   @throws NoSuchElementException if pop is attempted on an empty stack
*/
private static int evalOp(char op, Deque<Integer> operandStack) {
    // Pop the two operands off the stack.
    int rhs = operandStack.pop();
    int lhs = operandStack.pop();
    int result = 0;
    // Evaluate the operator.
    switch (op) {
        case '+': result = lhs + rhs;
                     break;
        case '-': result = lhs - rhs;
                     break;
        case '/': result = lhs / rhs;
                     break;
        case '*': result = lhs * rhs;
                     break;
    }
    return result;
}

/** Determines whether a character is an operator.
   @param op The character to be tested
   @return true if the character is an operator
*/
private static boolean isOperator(char ch) {
    return OPERATORS.indexOf(ch) != -1;
}

/** Evaluates a postfix expression.
   @param expression The expression to be evaluated
   @return The value of the expression
   @throws SyntaxErrorException if a syntax error is detected
*/
public static int eval(String expression) throws SyntaxErrorException {
    // Create an empty stack.
    Deque<Integer> operandStack = new ArrayDeque<>();

    // Process each token.
    String[] tokens = expression.split("\\s+");
    try {
        for (String nextToken : tokens) {
            char firstChar = nextToken.charAt(0);
            // Does it start with a digit?
            if (Character.isDigit(firstChar)) {
                // Get the integer value.
                int value = Integer.parseInt(nextToken);
                // Push value onto operand stack.
                operandStack.push(value);
            } // Is it an operator?
        }
    }
}

```

```

        else if (isOperator(firstChar)) {
            // Evaluate the operator.
            int result = evalOp(firstChar, operandStack);
            // Push result onto the operand stack.
            operandStack.push(result);
        }
        else {
            // Invalid character.
            throw new SyntaxErrorException
                ("Invalid character encountered: " + firstChar);
        }
    } // End for.

    // No more tokens - pop result from operand stack.
    int answer = operandStack.pop();
    // Operand stack should be empty.
    if (operandStack.isEmpty()) {
        return answer;
    } else {
        // Indicate syntax error.
        throw new SyntaxErrorException
            ("Syntax Error: Stack should be empty");
    }
} catch (NoSuchElementException ex) {
    // Pop was attempted on an empty stack.
    throw new SyntaxErrorException("Syntax Error: Stack is empty");
}
}

```



## PROGRAM STYLE

# Creating Your Own Exception Class

The program would work just the same if we did not bother to declare the `SyntaxErrorException` class and just threw a new `Exception` object each time an error occurred. However, we feel that this approach gives the user a more meaningful description of the cause of an error. Also, if other errors are possible in a client of this class, any `SyntaxErrorException` can be caught and handled in a separate `catch` clause.

**Testing** You will need to write a JUnit test harness for the PostfixEvaluator class. Each test case should pass an expression to eval and compare the expected and actual results. A white-box approach to testing would lead you to consider the following test cases. First, you want to exercise each path in the evalOp method by entering a simple expression that uses each operator. Then you need to exercise the paths through eval by trying different orderings and multiple occurrences of the operators. These tests exercise the normal cases, so you next need to test for possible syntax errors. You should consider the following cases: an operator without any operands, a single operand, an extra operand, an extra operator, a variable name, and finally an empty string.

## CASE STUDY Converting From Infix To Postfix

We normally write expressions in infix notation. Therefore, one approach to evaluating expressions in infix notation is first to convert it to postfix and then to apply the evaluation technique just discussed. We will show in this case study how to accomplish this conversion using a stack. An infix expression can also be evaluated directly using two stacks. This is left as a programming project.

**Problem** To complete the design of an expression evaluator, we need a set of methods that convert infix expressions to postfix form. We will assume that the expression will consist only of spaces, operands, and operators, where the space is a delimiter character between tokens. All operands that are identifiers begin with a letter or underscore character; all operands that are numbers begin with a digit. (Although we are allowing for identifiers, our postfix evaluator can't really handle them.)

**Analysis** Table 4.3 showed the infix and postfix forms of four expressions. For each expression pair, the operands are in the same sequence; however, the placement of the operators changes in going from infix to postfix. For example, in converting

w - 5.1 / sum \* 2  
to its postfix form

w 5.1 sum / 2 \* -

we see that the four operands (the tokens w, 5.1, sum, 2) retain their relative ordering from the infix expression, but the order of the operators is changed. The first operator in the infix expression, -, is the last operator in the postfix expression. Therefore, we can insert the operands in the output expression (postfix) as soon as they are scanned in the input expression (infix), but each operator should be inserted in the postfix string after its operands and in the order in which they should be evaluated, not the order in which they were scanned. For expressions without parentheses, there are two criteria that determine the order of operator evaluation:

- Operators are evaluated according to their *precedence* or rank. Higher precedence operators are evaluated before lower precedence operators. **For example, \*, /, and % (the multiplicative operators) are evaluated before +, -.**
- Operators with the same precedence are evaluated in left-to-right order (left-associative rule).

If we temporarily store the operators on a stack, we can pop them whenever we need to and insert them in the postfix string in an order that indicates when they should be evaluated, rather than when they were scanned. For example, if we have the first two operators from the string "w - 5.1 / sum \* 2" stored on a stack as follows,



the operator / (scanned second) must come off the stack and be placed in the postfix string before the operator - (scanned first). If we have the stack as just shown and the next operator is \*, we need to pop the / off the stack and insert it in the postfix string before \*, because the multiplicative operator scanned earlier (/) should be evaluated before the multiplicative operator (\*) scanned later (the left-associative rule).

**TABLE 4.6**  
Class InfixToPostfix

Data Field	Attribute
private static final String OPERATORS	The operators
private static final int[] PRECEDENCE	The precedence of the operators, matches their order in OPERATORS
private Deque<Character> operatorStack	Stack of operators
private StringJoiner postfix	The postfix string being formed
Method	Behavior
public static String convert(String infix)	Instantiates an instance of the InfixToPostfix class, calls convertToPostfix and then returns the result of calling getPostfix
public void convertToPostfix(String infix)	Extracts and processes each token in infix and stores the result in StringJoiner postfix
private void processOperator(char op)	Processes operator op by updating operatorStack and postfix
private String getPostfix()	Returns the result of postfix.toString()
private static int precedence(char op)	Returns the precedence of operator op
private static boolean isOperator(char ch)	Returns true if ch is an operator symbol

**Design** Class InfixToPostfix contains methods needed for the conversion. The class should have a data field operatorStack, which stores the operators. It should also have a method convert, which does the initial processing of all tokens (operands and operators). Method convert needs to get each token and process it. Each token that is an operand should be appended to the postfix string. Method processOperator will process each operator token. Method isOperator determines whether a token is an operator, and method precedence returns the precedence of an operator. Table 4.6 describes class InfixToPostfix.

The algorithm for method convert follows. The while loop extracts and processes each token, calling processOperator to process each operator token. After all tokens are extracted from the infix string and processed, any operators remaining on the stack should be popped and appended to the postfix string. They are appended to the end because they have lower precedence than those operators inserted earlier.

#### Algorithm for Method convert

1. Initialize postfix to an empty StringJoiner.
2. Initialize the operator stack to an empty stack.
3. while there are more tokens in the infix string.
4.     Get the next token.
5.     if the next token is an operand.
6.         Append it to postfix.
7.     else if the next token is an operator.
8.         Call processOperator to process the operator.
9.     else

10. Indicate a syntax error.
11. Pop remaining operators off the operator stack and append them to `postfix`.

### Method `processOperator`

The real decision making happens in method `processOperator`. By pushing operators onto the stack or popping them off the stack (and into the postfix string), this method controls the order in which the operators will be evaluated.

Each operator will eventually be pushed onto the stack. However, before doing this, `processOperator` compares the operator's precedence with that of the stacked operators, starting with the operator at the top of the stack. If the current operator has higher precedence than the operator at the top of the stack, it is pushed onto the stack immediately. This will ensure that none of the stacked operators can be inserted into the postfix string before it.

However, if the operator at the top of the stack has higher precedence than the current operator, it is popped off the stack and inserted in the postfix string, because it should be performed before the current operator, according to the precedence rule. Also, if the operator at the top of the stack has the same precedence as the current operator, it is popped off the stack and inserted into the postfix string, because it should be performed before the current operator, according to the left-associative rule. After an operator is popped off the stack, we repeat the process of comparing the precedence of the operator currently at the top of the stack with the precedence of the current operator until the current operator is pushed onto the stack.

A special case is an empty operator stack. In this case, there are no stacked operators to compare with the new one, so we will simply push the current operator onto the stack. We use method `peek` to access the operator at the top of the stack without removing it.

### Algorithm for Method `processOperator`

1. if the operator stack is empty
2. Push the current operator onto the stack.
- else
3. Peek the operator stack and let `topOp` be the top operator.
4. if the precedence of the current operator is greater than the precedence of `topOp`
5. Push the current operator onto the stack.
- else
6. while the stack is not empty and the precedence of the current operator is less than or equal to the precedence of `topOp`
7. Pop `topOp` off the stack and append it to `postfix`.
8. if the operator stack is not empty
9. Peek the operator stack and let `topOp` be the top operator.
10. Push the current operator onto the stack.

Table 4.7 traces the conversion of the infix expression `w - 5.1 / sum * 2` to the postfix expression `w 5.1 sum / 2 * -`. The final value of `postfix` shows that `/` is performed first (operands `5.1` and `sum`), `*` is performed next (operands `5.1 / sum` and `2`), and `-` is performed last.

Although the algorithm will correctly convert a well-formed expression and will detect some expressions with invalid syntax, it doesn't do all the syntax checking required. For

**TABLE 4.7**

Conversion of w - 5.1 / sum \* 2

Next Token	Action	Effect on operatorStack	Effect on postfix
w	Append w to postfix		w
-	The stack is empty Push – onto the stack	-	w
5.1	Append 5.1 to postfix	-	w 5.1
/	precedence(⟨⟩) > precedence(–), Push / onto the stack	/ -	w 5.1
sum	Append sum to postfix	/ -	w 5.1 sum
*	precedence(*) equals precedence(⟨⟩) Pop / off of stack and append to postfix	-	w 5.1 sum /
*	precedence(*) > precedence(–), Push * onto the stack	* -	w 5.1 sum /
2	Append 2 to postfix	* -	w 5.1 sum / 2
End of input	Stack is not empty, Pop * off the stack and append to postfix	-	w 5.1 sum / 2 *
End of input	Stack is not empty, Pop – off the stack and append to postfix		w 5.1 sum / 2 * -

example, an expression with extra operands would not be detected. We discuss this further in the testing section.

### Implementation

Listing 4.7 shows the `InfixToPostfix` class. A client would call method `convert`, passing it an infix string. Method `convert` creates object `infixToPostfix` and calls `convertToPostfix` to convert the argument string to postfix and return the conversion result.

The `convertToPostfix` method begins by initializing `postfix` and the `operatorStack`. The tokens are extracted using `String.split` and processed within a try block. The condition

```
(Character.isJavaIdentifierStart(firstChar)
 || Character.isDigit(firstChar))
```

tests the first character (`firstChar`) of the next token to see whether the next token is an operand (identifier or number). Method `isJavaIdentifierStart` returns `true` if the next token is an identifier; method `isDigit` returns `true` if the next token is a number (starts with a digit). If this condition is true, the token is added to `postfix`. The next condition,

```
(isOperator(firstChar))
```

is true if `nextToken` is an operator. If so, method `processOperator` is called. If the next token is not an operand or an operator, the exception `SyntaxErrorException` is thrown.

Once the end of the expression is reached, the remaining operators are popped off the stack and appended to `postfix`.

Method `processOperator` uses private method `precedence` to determine the precedence of an operator (2 for `*`, `/`; 1 for `+`, `-`). If the stack is empty or the condition

```
(precedence(op) > precedence(topOp))
```

is true, the current operator, `op`, is pushed onto the stack. Otherwise, the while loop executes, popping all operators off the stack that have the same or greater precedence than `op` and appending them to the postfix string (a `StringJoiner`).

```
while (!operatorStack.isEmpty()
    && precedence(op) <= precedence(topOp)) {
    operatorStack.pop();
    postfix.add(new Character(topOp).toString())
```

After loop exit, the statement

```
operatorStack.push(op);
```

pushes the current operator onto the stack.

In method `precedence`, the statement

```
return PRECEDENCE[OPERATORS.indexOf(op)];
```

returns the element of `int[]` array `PRECEDENCE` selected by the method call `OPERATORS.indexOf(op)`. The precedence value returned will be 1 or 2.

#### **LISTING 4.7**

`InfixToPostfix.java`

```
import java.util.*;

/** Translates an infix expression to a postfix expression. */
public class InfixToPostfix {

    // Insert nested class SyntaxErrorException. See Listing 4.6.

    // Data Fields
    /** The operator stack */
    private final Deque<Character> operatorStack = new ArrayDeque<>();
    /** The operators */
    private static final String OPERATORS = "+-*/";
    /** The precedence of the operators matches order in OPERATORS. */
    private static final int[] PRECEDENCE = {1, 1, 2, 2};
    /** The postfix string */
    private final StringJoiner postfix = new StringJoiner(" ");

    /** Convert a string from infix to postfix. Public convert is called
     * by a client - Calls private method convertToPostfix to do the conversion.
     * @param infix The infix expression
     * @throws SyntaxErrorException
     * @return the equivalent postfix expression.
     */
    public static String convert(String infix)
        throws SyntaxErrorException {
        InfixToPostfix infixToPostfix = new InfixToPostfix();
        infixToPostfix.convertToPostfix(infix);
        return infixToPostfix.getPostfix();
    }
}
```

```

    /**
     * Return the final postfix string.
     */
    private String getPostfix() {
        return postfix.toString();
    }

    /**
     * Convert a string from infix to postfix. Public convert is called
     * by a client - Calls private method convertToPostfix to do the conversion.
     * Uses a stack to convert an infix expression to postfix
     * @pre operator stack is empty
     * @post postFix contains postfix expression and stack is empty
     * @param infix the string to convert to postfix
     * @throws SyntaxErrorException if argument is invalid
     */
    private void convertToPostfix(String infix) throws SyntaxErrorException {
        String[] tokens = infix.split("\\s+");
        try {
            // Process each token in the infix string.
            for (String nextToken : tokens) {
                char firstChar = nextToken.charAt(0);
                // Is it an operand?
                if (Character.isJavaIdentifierStart(firstChar)
                    || Character.isDigit(firstChar)) {
                    postfix.add(nextToken);
                } // Is it an operator?
                else if(isOperator(firstChar)) {
                    processOperator(firstChar);
                }
                else {
                    throw new SyntaxErrorException
                        ("Unexpected Character Encountered: " + firstChar);
                }
            } // end loop.
            // Pop any remaining operators and
            // append them to postfix.
            while (!operatorStack.isEmpty()) {
                char op = operatorStack.pop();
                postfix.add(new Character(op).toString());
            }
            // assert: Stack is empty, return.
        } catch (NoSuchElementException ex) {
            throw new SyntaxErrorException
                ("Syntax Error: The stack is empty");
        }
    }

    /**
     * Method to process operators.
     * @param op The operator
     * @throws NoSuchElementException
     */
    private void processOperator(char op) {
        if (operatorStack.isEmpty()) {
            operatorStack.push(op);
        } else {
            // Peek the operator stack and
            // let topOp be top operator.
        }
    }
}

```

```
char topOp = operatorStack.peek();
if (precedence(op) > precedence(topOp)) {
    operatorStack.push(op);
}
else {
    // Pop all stacked operators with equal
    // or higher precedence than op.
    while (!operatorStack.isEmpty() && precedence(op) <=
        precedence(topOp)) {
        operatorStack.pop();
        postfix.add(new Character(topOp).toString());
        if (!operatorStack.isEmpty()) {
            // Reset topOp.
            topOp = operatorStack.peek();
        }
    }
    // assert: Operator stack is empty or
    // current operator precedence >
    // top of stack operator precedence.
    operatorStack.push(op);
}
}

/** Determine whether a character is an operator.
 * @param ch The character to be tested
 * @return true if ch is an operator
 */
private static boolean isOperator(char ch) {
    return OPERATORS.indexOf(ch) != -1;
}

/** Determine the precedence of an operator.
 * @param op The operator
 * @return the precedence
 */
private static int precedence(char op) {
    return PRECEDENCE[OPERATORS.indexOf(op)] ;
}
```



## PROGRAM STYLE

### Updating a `StringJoiner` Is an Efficient Operation

We used a `StringJoiner` object for `postfix` because we knew that `postfix` was going to be continually updated and each token was separated by a space. Because `String` objects are immutable, it would have been less efficient to use a `String` object for `postfix`. A new `String` object would have to be allocated each time `postfix` changed.

**Testing** Listing 4.8 shows a JUnit test for the `InfixToPostfix` class. Note that we are careful to type a space character between operands and operators.

We use enough test expressions to satisfy ourselves that the conversion is correct for properly formed input expressions. For example, try different orderings and multiple occurrences of the operators. You should also try infix expressions where all operators have the same precedence (e.g., all multiplicative).

If `convert` detects a syntax error, it will throw the exception `InfixToPostfix.SyntaxErrorException`. The driver will catch this exception and display an error message. If an exception is not thrown, the driver will display the result. Unfortunately, not all possible errors are detected. For example, an adjacent pair of operators or operands is not detected. To detect this error, we would need to add a boolean flag whose value indicates whether the last token was an operand. If the flag is true, the next token must be an operator; if the flag is false, the next token must be an operand. This modification is left as an exercise.

#### LISTING 4.8

JUnit test for `InfixToPostfix`

```
/*
 * Test for InfoxToPostfix
 * @author Koffman & Wolfgang
 */
public class InfixToPostfixTest {

    public InfixToPostfixTest() {
    }

    @Test
    public void simpleExpressionWithSamePrecedence() throws Exception {
        String infix = "a + b";
        String expResult = "a b + ";
        String result = InfixToPostfix.convert(infix);
        assertEquals(expResult, result);
    }

    @Test
    public void simpleExpressionWithNumbersSamePrecedence() throws Exception {
        String infix = "2.5 * 6";
        String expResult = "2.5 6 *";
        String result = InfixToPostfix.convert(infix);
        assertEquals(expResult, result);
    }

    @Test
    public void expressionWithMixedPrecedence() throws Exception {
        String infix = "x1 - y / 2 + foo";
        String expResult = "x1 y 2 / - foo +";
        String result = InfixToPostfix.convert(infix);
        assertEquals(expResult, result);
    }

    @Test(expected = InfixToPostfix.SyntaxErrorException.class)
    public void expressionWithInvalidOperator() throws Exception {
        String infix = "x1 & 2";
        String expResult = "x1 2";
        String result = InfixToPostfix.convert(infix);
        assertEquals(expResult, result);
    }
}
```

## CASE STUDY Converting Expressions with Parentheses

**Problem** The ability to convert expressions with parentheses is an important (and necessary) addition. Parentheses are used to separate expressions into subexpressions.

**Analysis** We can think of an opening parenthesis on an operator stack as a boundary or fence between operators. Whenever we encounter an opening parenthesis, we want to push it onto the stack. A closing parenthesis is the terminator symbol for a subexpression. Whenever we encounter a closing parenthesis, we want to pop off all operators on the stack until we pop the matching opening parenthesis. Neither opening nor closing parentheses should appear in the postfix expression. Because operators scanned after the opening parenthesis should be evaluated before the opening parenthesis, the precedence of the opening parentheses must be smaller than any other operator. We also give a closing parenthesis the lowest precedence. This ensures that a "(" can only be popped by a ")".

**Design** We should modify method `processOperator` to push each opening parenthesis onto the stack as soon as it is scanned. Therefore, the method should begin as follows:

```
if (operatorStack.isEmpty() || op == '(') {
    operatorStack.push(op);
```

When a closing parenthesis is scanned, we want to pop all operators up to and including the matching opening parenthesis, inserting all operators popped (except for the opening parenthesis) in the postfix string. This will happen automatically in the `while` statement if the precedence of the closing parenthesis is smaller than that of any other operator except for the opening parenthesis:

```
while (!operatorStack.isEmpty() && precedence(op) <= precedence(topOp)) {
    operatorStack.pop();
    if (topOp == '(') {
        // Matching '(' popped - exit loop.
        break;
    }
    postfix.add(new Character(topOp).toString());
```

A closing parenthesis is considered processed when an opening parenthesis is popped from the stack and the closing parenthesis is not placed on the stack. The following `if` statement executes after the `while` loop exits:

```
if (op != ')')
    operatorStack.push(op);
```

**Implementation** Listing 4.9 shows class `InfixToPostfixParen`, modified to handle parentheses. The additions are shown in bold. We have omitted parts that do not change.

Rather than impose the requirement of spaces between delimiters, we will use the `Scanner` method `findInLine` to extract tokens. The statements

```
Scanner scan = new Scanner(infix);
while ((nextToken = scan.findInLine(PATTERN)) != null) {
```

create a `Scanner` object to scan the characters in `infix` (see Appendix A.10). The `while` loop repetition condition calls method `findInLine` to extract the next token from `infix`. The string constant `PATTERN` is a regular expression that describes the form of a token:

```
private static final String PATTERN =
    "\\\d+\\.\\d*|\\d+" + "\\p{L}[^\\p{L}\\p{N}]*" + "|" + OPERATORS + "]";
```

Each token can be an integer (a sequence of one or more digits), a double (a sequence of one or more digits followed by a decimal point followed by zero or more digits), an identifier (a letter followed by zero or more letters or digits), or an operator. The operators are processed in the same way as was done in Listing 4.7. Loop exit occurs after all tokens are extracted (`findInLine` returns `null`).

#### LISTING 4.9

InfixToPostfixParens.java

```
import java.util.*;

/** Translates an infix expression with parentheses
 *  to a postfix expression.
 */
public class InfixToPostfixParens {

    // Insert nested class SyntaxErrorException here. See Listing 4.6.

    // Data Fields
    /** The operators. */
    private static final String OPERATORS = "-+*/()";
    /** The precedence of the operators, matches order of OPERATORS. */
    private static final int[] PRECEDENCE = {1, 1, 2, 2, -1, -1};
    /** The Pattern to extract tokens.
     * A token is either a number, an identifier, or an operator */
    private static final String PATTERN =
        "\\\d+\\.\\d*|\\d+" + "\\p{L}[^\\p{L}\\p{N}]*" + "|" + OPERATORS + "]";
    /** The stack of characters. */
    private final Deque<Character> operatorStack = new ArrayDeque<>();
    /** The postfix string. */
    private final StringJoiner postfix = new StringJoiner(" ");

    /** Convert a string from infix to postfix. Public convert is called
     * by a client - Calls private method convertToPostfix to do the conversion.
     * @param infix The infix expression
     * @throws SyntaxErrorException
     * @return the equivalent postfix expression.
     */
    public static String convert(String infix) throws SyntaxErrorException {
        InfixToPostfixParens infixToPostfixParens = new InfixToPostfixParens();
        infixToPostfixParens.convertToPostfix(infix);
        return infixToPostfixParens.getPostfix();
    }

    /** Return the final postfix string.
     * @return The final postfix string
     */
    private String getPostfix() {
        return postfix.toString();
    }
}
```

```


    /**
     * Convert a string from infix to postfix.
     * Uses a stack to convert an infix expression to postfix
     * @pre operator stack is empty
     * @post postFix contains postfix expression and stack is empty
     * @param infix the string to convert to postfix
     * @throws SyntaxErrorException if argument is invalid
     */
    private void convertToPostfix(String infix) throws SyntaxErrorException {
        // Process each token in the infix string.
        try {
            String nextToken;
            Scanner scan = new Scanner(infix);
            while ((nextToken = scan.findInLine(PATTERN)) != null) {
                char firstChar = nextToken.charAt(0);
                // Is it an operand?
                if (Character.isLetter(firstChar) || Character.isDigit(firstChar)) {
                    postfix.add(nextToken);
                } // Is it an operator?
                else if(isOperator(firstChar)) {
                    processOperator(firstChar);
                }
                else {
                    throw new SyntaxErrorException
                        ("Unexpected Character Encountered: " + firstChar);
                }
            } // end loop.

            // Pop any remaining operators
            // and append them to postfix.
            while (!operatorStack.isEmpty()) {
                char op = operatorStack.pop();
                // Any '(' on the stack is not matched.
                if (op == '(') throw new SyntaxErrorException
                    ("Unmatched opening parenthesis");
                postfix.add(new Character(op).toString());
            }
            // assert: Stack is empty, return result.
            return postfix.toString();
        } catch (NoSuchElementException ex) {
            throw new SyntaxErrorException("Syntax Error: The stack is empty");
        }
    }

    /**
     * Method to process operators.
     * @param op The operator
     * @throws NoSuchElementException
     */
    private void processOperator(char op) {
        if (operatorStack.isEmpty() || op == '(') {
            operatorStack.push(op);
        }
        else {
            // Peek the operator stack and
            // let topOp be the top operator.
            char topOp = operatorStack.peek();


```

```

        if (precedence(op) > precedence(topOp)) {
            operatorStack.push(op);
        }
        else {
            // Pop all stacked operators with equal
            // or higher precedence than op.
            while (!operatorStack.isEmpty() && precedence(op) <=
                precedence(topOp)) {
                operatorStack.pop();
                if (topOp == '(') {
                    // Matching '(' popped - exit loop.
                    break;
                }
                postfix.add(new Character(topOp).toString());
                if (!operatorStack.isEmpty()) {
                    // Reset topOp.
                    topOp = operatorStack.peek();
                }
            }
            // assert: Operator stack is empty or
            // current operator precedence >
            // top of stack operator precedence.
            if (op != ')')
                operatorStack.push(op);
        }
    }
}

// Insert isOperator and precedence here. See Listing 4.7.
}

```

## Tying the Case Studies Together

You can use the classes developed for the previous case studies to evaluate infix expressions with integer operands and nested parentheses. The argument for `InfixToPostfix.convert` will be the infix expression. The result will be its postfix form. Next it will apply the method `PostfixEvaluator.eval`. The argument for `eval` will be the postfix expression returned by `convert`.

## EXERCISES FOR SECTION 4.4

### SELF-CHECK

1. Trace the evaluation of the following expressions using class `PostfixEvaluator`. Show the operand stack each time it is modified.

13 2 \* 5 / 6 2 5 \* - +  
 5 4 \* 6 7 + 4 2 / - \*

2. Trace the conversion of the following expressions to postfix using class `InfixToPostfix` or `InfixToPostfixParens`. Show the operator stack each time it is modified.

$$y - 7 / 35 + 4 * 6 - 10$$

$$(x + 15) * (3 * (4 - (5 + 7 / 2)))$$

### PROGRAMMING

1. Modify class `InfixToPostfix` to handle the exponentiation operator, indicated by the symbol `^`. The first operand is raised to the power indicated by the second operand. Assume that a sequence of `^` operators will not occur and that `precedence('^') > precedence('*')`.
2. Discuss how you would modify the infix-to-postfix convert method to detect a sequence of two operators or two operands.



## 4.5 Queue Abstract Data Type

You can think of a queue as a line of customers waiting for a scarce resource, such as a line waiting to buy tickets to an event. Figure 4.4 shows a line of “men” waiting to enter a restroom. The next one to enter the restroom is the one who has been waiting the longest, and latecomers are added to the end of the line. The Queue ADT gets its name from the fact that such a waiting line is called a “queue” in English-speaking countries other than the United States.

### A Print Queue

In computer science, queues are used in operating systems to keep track of tasks waiting for a scarce resource and to ensure that the tasks are carried out in the order that they were generated. One example is a print queue. A Web surfer may select several pages to be

**FIGURE 4.4**  
Co-author Elliot Koffman is the last person in the queue.



Caryn J. Koffman

**FIGURE 4.5**

A Print Queue in the Windows Operating System

HP LaserJet 4050 Series P5 - Use Printer Offline					
Document Name	Status	Owner	Pages	Size	Submitted
Microsoft Word - Queues_Paul_1007.doc	Paul Wolfgang	52	9.75 MB	1:53:18 PM 10/7/2003	
Microsoft Word - Stacks.doc	Paul Wolfgang	46	9.05 MB	1:53:57 PM 10/7/2003	
Microsoft Word - Trees2.doc	Paul Wolfgang	54	38.4 MB	1:54:41 PM 10/7/2003	

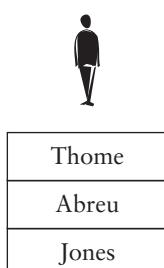
3 document(s) in queue

printed in a few seconds. Because a printer is a relatively slow device (approximately 10 pages/minute), you will often select new pages to print faster than they can be printed. Rather than require you to wait until the current page is finished before you can select a new one, the operating system stores documents to be printed in a print queue (see Figure 4.5). Because they are stored in a queue, the pages will be printed in the same order as they were selected (first-in, first-out). The document first inserted in the queue will be the first one printed.

**FIGURE 4.6**

A Queue of Customers

Ticket agent



## The Unsuitability of a “Print Stack”

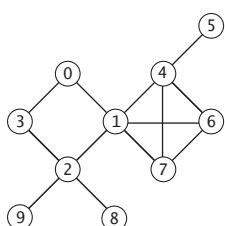
Suppose your operating system used a stack (last-in, first-out) instead of a queue to store documents waiting to be printed. Then the most recently selected Web page would be the next page to be printed. This may not matter if only one person is using the printer. However, if the printer is connected to a computer network, this would be a big problem. Unless the print queue was empty when you selected a page to print (and the page printed immediately), that page would not print until all pages selected after it (by yourself or any other person on the network) were printed. If you were waiting by the printer for your page to print before going to your next class, you would have no way of knowing how long your wait might be. You would also be very unhappy if people who started after you had their documents printed before yours. So a print queue is a much more sensible alternative than a print stack.

## A Queue of Customers

A queue of three customers waiting to buy concert tickets is shown in Figure 4.6. The name of the customer who has been waiting the longest is Thome; the name of the most recent arrival is Jones. Customer Thome will be the first customer removed from the queue (and able to buy tickets) when a ticket agent becomes available, and customer Abreu will then become the first one in the queue. Any new customers will be inserted in the queue after customer Jones.

**FIGURE 4.7**

A Network of Nodes



## Using a Queue for Traversing a Multi-Branch Data Structure

In Chapter 10, you will see a data structure, called a graph, that models a *network of nodes*, with many links connecting each node to other nodes in the network (see Figure 4.7). Unlike a linked list, in which each node has only one successor, a node in a graph may have several successors. For example, node 0 in Figure 4.7 has nodes 1 and 3 as its successors. Consequently, it is not a simple matter to visit the nodes in a systematic way and to ensure that each node is visited only once. Programmers often use a queue to ensure that nodes closer to the starting point are visited before nodes that are farther away. We will not go into the details here because we cover them later, but the idea is to put nodes that have not yet

been visited into the queue when they are first encountered. After visiting the current node, the next node to visit is taken from the queue. This ensures that nodes are visited in the same order that they were encountered. Such a traversal is called a *breadth-first traversal* because the nodes visited spread out from the starting point. If we use a stack to hold the new nodes that are encountered and take the next node to visit from the stack, we will follow one path to the end before embarking on a new path. This kind of traversal is called a *depth-first traversal*.

## Specification for a Queue Interface

The `java.util` API provides a `Queue` interface (Table 4.8) that extends the `Collection` interface and, therefore, the `Iterable` interface (see Table 2.34).

Method `offer` inserts an element at the rear of the queue and returns `true` if successful and `false` if unsuccessful. Methods `remove` and `poll` both remove and return the element at the front of the queue. The only difference in their behavior is when the queue happens to be empty: `remove` throws an exception and `poll` just returns `null`. Methods `peek` and `element` both return the element at the front of the queue without removing it. The difference is that `element` throws an exception when the queue is empty.

Because interface `Queue` extends interface `Collection`, a full implementation of the `Queue` interface must implement all required methods of the `Collection` interface. Classes that implement the `Queue` interface need to code the methods in Table 4.1 as well as methods `add`, `iterator`, `isEmpty`, and `size` declared in the `Collection` interface.

## Class `LinkedList` Implements the Queue Interface

Because the `LinkedList` class provides methods for inserting and removing elements at either end of a double-linked list, all the `Queue` methods can be easily implemented in class `LinkedList`. For this reason, the `LinkedList` class implements the `Queue` interface. The statement

```
Queue<String> names = new LinkedList<>();
```

creates a new `Queue` reference, `names`, that stores references to `String` objects. The actual object referenced by `names` is type `LinkedList<String>`. However, because `names` is a type `Queue<String>` reference, you can apply only the `Queue` methods to it.

**TABLE 4.8**

Specification of Interface `Queue<E>`

Method	Behavior
<code>boolean offer(E item)</code>	Inserts <code>item</code> at the rear of the queue. Returns <code>true</code> if successful; returns <code>false</code> if the item could not be inserted
<code>E remove()</code>	Removes the entry at the front of the queue and returns it if the queue is not empty. If the queue is empty, throws a <code>NoSuchElementException</code>
<code>E poll()</code>	Removes the entry at the front of the queue and returns it; returns <code>null</code> if the queue is empty
<code>E peek()</code>	Returns the entry at the front of the queue without removing it; returns <code>null</code> if the queue is empty
<code>E element()</code>	Returns the entry at the front of the queue without removing it. If the queue is empty, throws a <code>NoSuchElementException</code>

**EXAMPLE 4.2** The queue names created above contains five strings as shown in Figure 4.5(a). The name "Jonathan" was placed in the queue before the other four names; "Rich" was the last element placed in the queue.

For queue names in Figure 4.8(a), the value of `names.isEmpty()` is false. The statement

```
String first = names.peek();
```

or

```
String first = names.element();
```

stores "Jonathan" in `first` without changing `names`. The statement

```
String temp = names.remove();
```

or

```
String temp = names.poll();
```

removes "Jonathan" from `names` and stores a reference to it in `temp`. The queue names now contains four elements and is shown in Figure 4.8(b). The statement

```
names.offer("Philip");
```

or

```
names.add("Philip");
```

adds "Philip" to the rear of the queue; the queue names now contains five elements and is shown in Figure 4.8(c).

**FIGURE 4.8**  
Queue Names

<table border="1"><tr><td>Jonathan</td></tr><tr><td>Dustin</td></tr><tr><td>Robin</td></tr><tr><td>Debbie</td></tr><tr><td>Rich</td></tr></table>	Jonathan	Dustin	Robin	Debbie	Rich	<table border="1"><tr><td>Dustin</td></tr><tr><td>Robin</td></tr><tr><td>Debbie</td></tr><tr><td>Rich</td></tr></table>	Dustin	Robin	Debbie	Rich	<table border="1"><tr><td>Dustin</td></tr><tr><td>Robin</td></tr><tr><td>Debbie</td></tr><tr><td>Rich</td></tr><tr><td>Philip</td></tr></table>	Dustin	Robin	Debbie	Rich	Philip
Jonathan																
Dustin																
Robin																
Debbie																
Rich																
Dustin																
Robin																
Debbie																
Rich																
Dustin																
Robin																
Debbie																
Rich																
Philip																
(a)	(b)	(c)														

## EXERCISES FOR SECTION 4.5

### SELF-CHECK

1. Draw the queue in Figure 4.6 as it will appear after the insertion of customer Harris and the removal of one customer from the queue. Which customer is removed? How many customers are left?
2. Answer Question 1 for the queue in Figure 4.8(c).
3. Assume that `myQueue` is an instance of a class that implements `Queue<String>` and `myQueue` is an empty queue. Explain the effect of each of the following operations.

```
myQueue.offer("Hello");
myQueue.offer("Bye");
System.out.println(myQueue.peek());
myQueue.remove();
myQueue.offer("Welcome");
if (!myQueue.isEmpty()) {
    System.out.println(myQueue.remove() + ", new size is " + myQueue.size());
    System.out.println("Item in front is " + myQueue.peek());
}
```

4. For the queue `names` in Figure 4.6, what is the effect of the following?

```
while (!names.isEmpty()) {
    System.out.println(names.remove());
}
```

5. What would be the effect of using `peek` instead of `remove` in Question 4?

### PROGRAMMING

1. Write a main function that creates two stacks of `Integer` objects and a queue of `Integer` objects. Store the numbers  $-1, 15, 23, 44, 4, 99$  in the first stack. The top of the stack should store 99.
2. Write a loop to get each number from the first stack and store it in the second stack and in the queue.
3. Write a second loop to remove a value from the second stack and from the queue and display each pair of values on a separate output line. Continue until the data structures are empty. Show the output.



## 4.6 Queue Applications

In this section we present an application that maintains a queue of `Strings` representing the names of customers waiting for service. Our goal is just to maintain the list and ensure that customers are inserted and removed properly. We will allow a user to determine the queue size, the person at the front of the queue, and how many people are ahead of a particular person in the queue.

### CASE STUDY Maintaining a Queue

**Problem** Write a menu-driven program that maintains a list of customers waiting for service. The program user should be able to insert a new customer in the line, display the customer who is next in line, remove the customer who is next in line, display the length of the line, or determine how many people are ahead of a specified customer.

**Analysis** As discussed earlier, a queue is a good data structure for storing a list of customers waiting for service because they would expect to be served in the order in which they arrived. We can display the menu and then perform the requested operation by calling the appropriate `Queue` method to update the customer list.

#### Problem Inputs

The operation to be performed  
The name of a customer

#### Problem Outputs

The effect of each operation

**TABLE 4.9**  
Class MaintainQueue

Method	Behavior
public static void processCustomers()	Accepts and processes each user's selection

**Design** We will write a class `MaintainQueue` to store the queue and control its processing.

Class `MaintainQueue` has a `Queue<String>` component `customers`.

Method `processCustomers` displays a menu of choices and processes the user selection by calling the appropriate Queue method. Table 4.9 shows class `MaintainQueue`.

The algorithm for method `processCustomers` follows.

#### Algorithm for `processCustomers`

1. while the user is not finished
2.     Display the menu and get the operation selected.
3.     Perform the operation selected.

Each operation is performed by a call to one of the Queue methods, except for determining the position of a particular customer in the queue. The algorithm for this operation follows.

#### Algorithm for Determining the Position of a Customer

1. Get the customer name.
2. Set the count of customers ahead of this one to 0.
3. for each customer in the queue
  4.     if this customer is not the one sought
  5.         Increment the count.
  6.     else
  7.         Display the count of customers and exit the loop.
  8.     if all customers were examined without success
  9.         Display a message that the customer is not in the queue.

The loop that begins at step 3 requires us to access each element in the queue. However, only the element at the front of the queue is directly accessible using method `peek` or `element`. We will show how to get around this limitation by using an `Iterator` to access each element of the queue.

#### Implementation

Listing 4.10 shows the data field declarations and the constructor for class `MaintainQueue`. The constructor sets `customers` to reference an instance of class `LinkedList<String>`.

#### LISTING 4.10

Constructor for Class `MaintainQueue`

```
import java.util.Queue;
import java.util.LinkedList;
import java.util.NoSuchElementException;
import java.util.Scanner;
import java.util.Arrays;
```

```

/**
 * Class to maintain a queue of customers.
 * @author Koffman & Wolfgang
 */
public class MaintainQueue {

    // Data Field
    private final Queue<String> customers;
    private final Scanner in;

    // Constructor
    /** Create an empty queue. */
    public MaintainQueue() {
        customers = new LinkedList<>();
        in = new Scanner(System.in);
    }
}

```

In method `processCustomers` (Listing 4.11), the `while` loop executes until the user enters "quit". The user enters each desired operation into `choice`. The `switch` statement calls a `Queue` method to perform the selected operation. For example, if the user enters "add", the following statements read the customer name and insert it into the queue.

```

System.out.println("Enter new customer name");
name = in.nextLine();
customers.offer(name);

```

Case "position" finds the position of a customer in the queue. The enhanced `for` statement uses an `Iterator` to access each element of the queue and store it in `nextName`. The `if` condition compares `nextName` to the name of the customer being sought. The variable `countAhead` is incremented each time this comparison is unsuccessful.

```

int countAhead = 0;
for (String nextName : customers) {
    if (!nextName.equals(name)) {
        countAhead++;
    } else {
        System.out.println("The number of customers ahead of " + name
                           + " is " + countAhead);
        break; // Customer found, exit loop.
    }
}

```

If the desired name is accessed, its position is displayed and the loop is exited. If the name is not found, the loop is exited after the last name is processed. The `if` statement following the loop displays a message if the name was not found. This will be the case when `countAhead` is equal to the queue size.

The `switch` statement is inside a `try-catch` sequence that handles a `NoSuchElementException` (caused by an attempt to remove or retrieve an element from an empty queue) by displaying an error message.

.....  
**LISTING 4.11**

Method `processCustomers` in Class `MaintainQueue`

```

/**
 * Performs the operations selected on queue customers.
 * @pre customers has been created.
 * @post customers is modified based on user selections.
 */

```

```

public void processCustomers() {
    String choice = "";
    String[] choices =
        {"add", "peek", "remove", "size", "position", "quit"};
    // Perform all operations selected by user.
    while (!choice.equals("quit")) {
        // Process the current choice.
        try {
            String name;
            System.out.println("Choose from the list: "
                + Arrays.toString(choices));
            choice = in.nextLine();
            switch (choice) {
                case "add":
                    System.out.println("Enter new customer name");
                    name = in.nextLine();
                    customers.offer(name);
                    System.out.println("Customer " + name +
                        " added to the queue");
                    break;
                case "peek":
                    System.out.println("Customer " + customers.element() +
                        " is next in the queue");
                    break;
                case "remove":
                    System.out.println("Customer " + customers.remove() +
                        " removed from the queue");
                    break;
                case "size":
                    System.out.println("Size of queue is " + customers.size());
                    break;
                case "position":
                    System.out.println("Enter customer name");
                    name = in.nextLine();
                    int countAhead = 0;
                    for (String nextName : customers) {
                        if (!nextName.equals(name)) {
                            countAhead++;
                        } else {
                            System.out.println("The number of customers ahead of "
                                + name + " is " + countAhead);
                            break;
                            // Customer found, exit loop.
                        }
                    }
                    // Check whether customer was found.
                    if (countAhead == customers.size()) {
                        System.out.println(name + " is not in queue");
                    }
                    break;
                case "quit":
                    System.out.println("Leaving customer queue. "
                        + "\nNumber of customers in queue is " +
                        + customers.size());
                    break;
                default:
                    System.out.println("Invalid choice - try again");
            }
        }
    }
}

```

```

        } // end switch
    } catch (NoSuchElementException e) {
        System.out.println("The Queue is empty");
    } // end try-catch
} // end while
}

```

**Testing** You can use class `MaintainQueue` to test each of the different Queue implementations discussed in the next section. You should verify that all customers are stored and retrieved in FIFO order. You should also verify that a `NoSuchElementException` is thrown if you attempt to remove or retrieve a customer from an empty queue. Thoroughly test the queue by selecting different sequences of queue operations. Figure 4.9 shows a sample run.

**FIGURE 4.9**  
Sample Run of Client  
of `MaintainQueue`

```

Choose from the list: [add, peek, remove, size, position, quit]
add
Enter new customer name
koffman
Customer koffman added to the queue
Choose from the list: [add, peek, remove, size, position, quit]
add
Enter new customer name
wolfgang
Customer wolfgang added to the queue
Choose from the list: [add, peek, remove, size, position, quit]
remove
Customer koffman removed from the queue
Choose from the list: [add, peek, remove, size, position, quit]
peek
Invalid choice - try again
Choose from the list: [add, peek, remove, size, position, quit]
peek
Customer wolfgang is next in the queue
Choose from the list: [add, peek, remove, size, position, quit]
quit
Leaving customer queue.
Number of customers in queue is 1

```



## PROGRAM STYLE

### When to Use the Different Queue Methods

For a queue of unlimited size, `add` and `offer` are logically equivalent. Both will return `true` and never throw an exception. For a bounded queue, `add` will throw an exception if the queue is full, but `offer` will return `false`.

For `peek` versus `element` and `poll` versus `remove`, `peek` and `poll` don't throw exceptions, but the user should either check for a return value of `null`, or be sure that the calls are within an `if` or `while` block that tests for a nonempty queue before they are called.

### Using Queues for Simulation

*Simulation* is a technique used to study the performance of a physical system by using a physical, mathematical, or computer model of the system. Through simulation, the designers of a new system can estimate the expected performance of the system before they actually build it. The use of simulation can lead to changes in the design that will improve the expected performance of the new system. Simulation is especially useful when the actual system would be too expensive to build or too dangerous to experiment with after its construction.

The textbook website provides a case study of a computer simulation of an airline check-in counter in order to compare various strategies for improving service and reducing the waiting time for each passenger. It uses a queue to simulate the passenger waiting line. A special branch of mathematics called *queueing theory* has been developed to study these kinds of problems using mathematical models (systems of equations) instead of computer models.

---

## EXERCISES FOR SECTION 4.6

### SELF-CHECK

1. Write an algorithm to display all the elements in a queue using just the queue operations. How would your algorithm change the queue?
2. Trace the following fragment for a `Stack<String>` `s` and an empty queue `q` (type `Queue<String>`).

```
String item;
while (!s.empty()) {
    item = s.pop();
    q.offer(item);
}
while (!q.isEmpty()) {
    item = q.remove();
    s.push(item);
}
```

- a. What is stored in stack `s` after the first loop executes? What is stored in queue `q` after the first loop executes?
- b. What is stored in stack `s` after the second loop executes? What is stored in queue `q` after the second loop executes?

### PROGRAMMING

1. Write a `toString` method for class `MaintainQueue`.

## 4.7 Implementing the Queue Interface

In this section we discuss three approaches to implementing a queue: using a double-linked list, a single-linked list, and an array. We begin with a double-linked list.

### Using a Double-Linked List to Implement the Queue Interface

Insertion and removal from either end of a double-linked list is  $O(1)$ , so either end can be the front (or rear) of the queue. The Java designers decided to make the head of the linked list the front of the queue and the tail the rear of the queue. If you declare your queue using the statement:

```
Queue<String> myQueue = new LinkedList<>();
```

the fact that the actual class for `myQueue` is a `LinkedList` is not visible. The only methods available for `myQueue` are those declared in the `Queue` interface.

### Using a Single-Linked List to Implement the Queue Interface

We can implement a queue using a single-linked list like the one shown in Figure 4.10. Class `ListQueue` contains a collection of `Node<E>` objects (see Section 2.5). Recall that class `Node<E>` has attributes `data` (type `E`) and `next` (type `Node<E>`).

Insertions are at the rear of a queue, and removals are from the front. We need a reference to the last list node so that insertions can be performed in  $O(1)$  time; otherwise, we would have to start at the list head and traverse all the way down the list to do an insertion. There is a reference variable `front` to the first list node (the list head) and a reference variable `rear` to the last list node. There is also a data field `size`.

The number of elements in the queue is changed by methods `insert` and `remove`, so `size` must be incremented by one in `insert` and decremented by one in `remove`. The value of `size` is tested in `isEmpty` to determine the status of the queue. The method `size` simply returns the value of data field `size`.

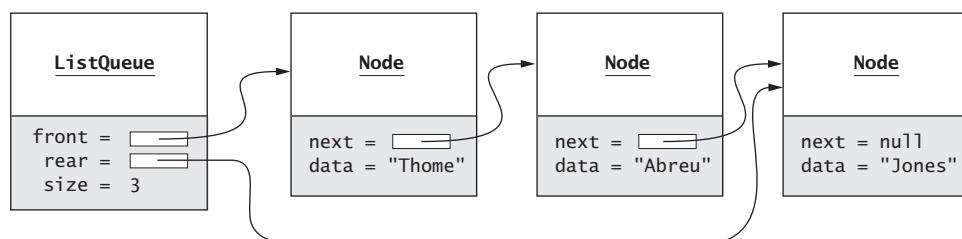
Listing 4.12 shows class `ListQueue<E>`. Method `offer` treats insertion into an empty queue as a special case because both `front` and `rear` should reference the new node after the insertion.

```
rear = new Node<>(item, null); front = rear;
```

If we insert into a queue that is not empty, the new node must be linked to the old rear of the queue, but `front` is unchanged.

```
rear.next = new Node<>(item, null);
rear = rear.next;
```

**FIGURE 4.10**  
A Queue as a Single-Linked List



If the queue is empty, method `peek` returns `null`. Otherwise, it returns the element at the front of the queue:

```
    return front.data;
```

Method `poll` calls method `peek` and returns its result. However, before returning, it disconnects the node at the front of a nonempty queue and decrements `size`.

```
    front = front.next;
    size--;
```

Listing 4.12 is incomplete. To finish it, you need to write methods `remove`, `element`, `size`, and `isEmpty`. You also need to code an `Iterator` method and a class `Iter` with methods `next`, `hasNext`, and `remove`. This class will be similar to class `KWLListIter` (see Section 2.6).

You can simplify this task by having `ListQueue<E>` extend class `java.util.AbstractQueue<E>`. This class implements `add`, `remove`, and `element` using `offer`, `poll`, and `peek`, and inherits from its superclass, `AbstractCollection<E>`, all methods needed to implement the `Collection<E>` interface (the superinterface of `Queue<E>`).

.....

#### **LISTING 4.12**

Class `ListQueue`

```
import java.util.*;
/** Implements the Queue interface using a single-linked list. */
public class ListQueue<E> extends AbstractQueue<E>
    implements Queue<E> {

    // Data Fields
    /** Reference to front of queue. */
    private Node<E> front;
    /** Reference to rear of queue. */
    private Node<E> rear;
    /** Size of queue. */
    private int size;

    // Insert inner class Node<E> for single-linked list here.
    // (See Listing 2.1.)
    // Methods
    /** Insert an item at the rear of the queue.
        @post item is added to the rear of the queue.
        @param item The element to add
        @return true (always successful)
    */
    @Override
    public boolean offer(E item) {
        // Check for empty queue.
        if (front == null) {
            rear = new Node<E>(item);
            front = rear;
        } else {
            // Allocate a new node at end, store item in it, and
            // link it to old end of queue.
            rear.next = new Node<E>(item);
            rear = rear.next;
        }
        size++;
        return true;
    }

    /** Remove the entry at the front of the queue and return it
        if the queue is not empty.
    }
```

```

    @post front references item that was second in the queue.
    @return The item removed if successful, or null if not
*/
@Override
public E poll() {
    E item = peek();
    // Retrieve item at front.
    if (item == null)
        return null;
    // Remove item at front.
    front = front.next; size--;
    return item;
    // Return data at front of queue.
}

/** Return the item at the front of the queue without removing it.
@return The item at the front of the queue if successful;
return null if the queue is empty
*/
@Override
public E peek() {
    if (size == 0)
        return null;
    else
        return front.data;
}

// Insert class Iter and other required methods.
. . .
}

```

## Using a Circular Array to Implement the Queue Interface

While the time efficiency of using a single- or double-linked list to implement the Queue is acceptable, there is some space inefficiency. Each node of a single-linked list contains a reference to its successor, and each node of a double-linked list contains references to its predecessor and successor. These additional references will increase the storage space required.

An alternative is to use an array. If we use an array, we can do an insertion at the rear of the array in  $O(1)$  time. However, a removal from the front will be an  $O(n)$  process if we shift all the elements that follow the first one over to fill the space vacated. Similarly, removal from the rear is  $O(1)$ , but insertion at the front is  $O(n)$ . We next discuss how to avoid this inefficiency.

### Overview of the Design

To represent a queue, we will use an object with four `int` data members (`front`, `rear`, `size`, `capacity`) and an array data member, `theData`, which provides storage for the queue elements.

```

/** Index of the front of the queue. */
private int front;
/** Index of the rear of the queue. */
private int rear;
/** Current number of elements in the queue. */
private int size;
/** Current capacity of the queue. */
private int capacity;
/** Default capacity of the queue. */
private static final int DEFAULT_CAPACITY = 10;

/** Array to hold the data. */
private E[] theData;

```

The `int` fields `front` and `rear` are indices to the queue elements at the front and rear of the queue, respectively. The `int` field `size` keeps track of the actual number of items in the queue and allows us to determine easily whether the queue is empty (`size` is 0) or full (`size` is `capacity`).

It makes sense to store the first queue item in element 0, the second queue item in element 1, and so on. So we should set `front` to 0 and `rear` to -1 when we create an initially empty queue. Each time we do an insertion, we should increment `size` and `rear` by 1 so that `front` and `rear` will both be 0 if a queue has one element. Figure 4.11 shows an instance of a queue that is filled to its capacity (`size` is `capacity`). The queue contains the symbols `&`, `*`, `+`, `/`, `-`, inserted in that order.

Because the queue in Figure 4.11 is filled to capacity, we cannot insert a new character without allocating more storage. However, we can remove a queue element by decrementing `size` and incrementing `front` to 1, thereby removing `theData[0]` (the symbol `&`) from the queue. Figure 4.12 shows the queue after removing the first element (it is still in the array, but not part of the queue). The queue contains the symbols `*`, `+`, `/`, `-` in that order.

**FIGURE 4.11**

A Queue Filled with Characters

**FIGURE 4.12**

The Queue after Deletion of the First Element



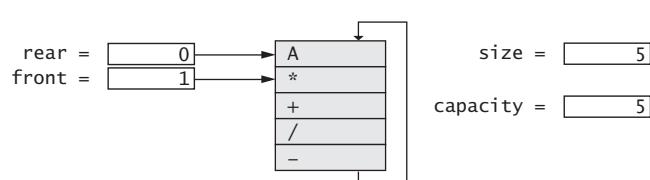
Although the queue in Figure 4.12 is no longer filled, we cannot insert a new character because `rear` is at its maximum value. One way to solve this problem is to

shift the elements in array `theData` so that the empty cells come after `rear` and then adjust `front` and `rear` accordingly. This array shifting must be done very carefully to avoid losing track of active array elements. It is also an  $O(n)$  operation.

A better way to solve this problem is to represent the array field `theData` as a *circular array*. In a circular array, the elements wrap around so that the first element actually follows the last. This is like counting modulo `size`; the array subscripts take on the values  $0, 1, \dots, size - 1, 0, 1, \dots$ . This allows us to “increment” `rear` to 0 and store a new character in `theData[0]`. Figure 4.13 shows the queue after inserting a new element (the character A). After the insertion, `front` is still 1 but `rear` becomes 0. The contents of `theData[0]` change from `&` to `A`. The queue now contains the symbols `*`, `+`, `/`, `-`, `A` in that order.

**FIGURE 4.13**

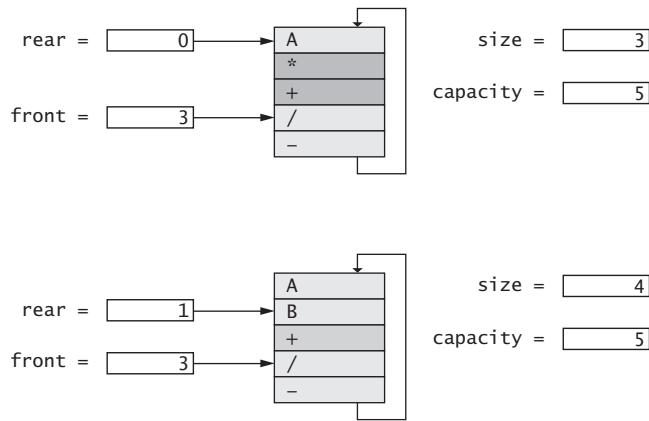
A Queue as a Circular Array



**EXAMPLE 4.3** The upper half of Figure 4.14 shows the effect of removing two elements from the queue just described. There are currently three characters in this queue (stored in `theData[3]`, `theData[4]`, and `theData[0]`). The queue now contains the symbols `/`, `-`, `A` in that order.

The lower half of Figure 4.14 shows the queue after insertion of a new character (`B`). The value of `rear` is incremented to 1, and the next element is inserted in `theData[1]`. This queue element follows the character `A` in `theData[0]`. The value of `front` is still 3 because the character `/` at `theData[3]` has been in the queue the longest. `theData[2]` is now the only queue element that is unused. The queue now contains the symbols `/`, `-`, `A`, `B` in that order.

**FIGURE 4.14**  
The Effect of  
Two Deletions . . . and  
One Insertion



### Implementing `ArrayQueue<E>`

Listing 4.13 shows the implementation of the class `ArrayQueue<E>`.

The constructors set `size` to 0 and `front` to 0 because array element `theData[0]` is considered the front of the empty queue, and `rear` is initialized to `capacity - 1` (instead of `-1`) because the queue is circular.

In method `offer`, the statement

```
rear = (rear + 1) % capacity;
```

is used to increment the value of `rear` modulo `capacity`. When `rear` is less than `capacity`, this statement simply increments its value by one. But when `rear` becomes equal to `capacity - 1`, the next value of `rear` will be 0 (capacity mod capacity is 0), thereby wrapping the last element of the queue around to the first element. Because the constructor initializes `rear` to `capacity - 1`, the first queue element will be placed in `theData[0]` as desired.

In method `poll`, the element currently stored in `theData[front]` is copied into `result` before `front` is incremented modulo `capacity`; `result` is then returned. In method `peek`, the element at `theData[front]` is returned, but `front` is not changed.

**LISTING 4.13**  
`ArrayQueue.java`

```
/** Implements the Queue interface using a circular array. */
public class ArrayQueue<E> extends AbstractQueue<E>
    implements Queue<E> {

    // Data Fields
    /** Index of the front of the queue. */
    int front = 0;
    int rear = capacity - 1;
    int size = 0;
    E[] theData;

    public ArrayQueue() {
        theData = (E[]) new Object[capacity];
    }

    public void offer(E e) {
        if (size == capacity)
            throw new IllegalStateException("Queue full");
        theData[++rear] = e;
        size++;
    }

    public E poll() {
        if (size == 0)
            throw new NoSuchElementException("Queue empty");
        E result = theData[front];
        theData[front] = null;
        front = (front + 1) % capacity;
        size--;
        return result;
    }

    public E peek() {
        if (size == 0)
            throw new NoSuchElementException("Queue empty");
        return theData[front];
    }

    public int size() {
        return size;
    }

    public boolean isEmpty() {
        return size == 0;
    }
}
```

```

private int front;
/** Index of the rear of the queue. */
private int rear;
/** Current size of the queue. */
private int size;
/** Current capacity of the queue. */
private int capacity;
/** Default capacity of the queue. */
private static final int DEFAULT_CAPACITY = 10;
/** Array to hold the data. */
private E[] theData;

// Constructors
/** Construct a queue with the default initial capacity. */
public ArrayQueue() {
    this(DEFAULT_CAPACITY);
}

@SuppressWarnings("unchecked")
/** Construct a queue with the specified initial capacity.
     @param initCapacity The initial capacity
 */
public ArrayQueue(int initCapacity) {
    capacity = initCapacity;
    theData = (E[]) new Object[capacity];
    front = 0;
    rear = capacity - 1;
    size = 0;
}

// Public Methods
/** Inserts an item at the rear of the queue.
     @post item is added to the rear of the queue.
     @param item The element to add
     @return true (always successful)
 */
@Override
public boolean offer(E item) {
    if (size == capacity) {
        reallocate();
    }
    size++;
    rear = (rear + 1) % capacity; theData[rear] = item;
    return true;
}

/** Returns the item at the front of the queue without removing it.
     @return The item at the front of the queue if successful; return null if
            the queue is empty
 */
@Override
public E peek() {
    if (size == 0)
        return null;
    else
        return theData[front];
}

/** Removes the entry at the front of the queue and returns it if the queue is
     not empty.
 */

```

```

    @post front references item that was second in the queue.
    @return The item removed if successful or null if not
*/
@Override
public E poll() {
    if (size == 0) {
        return null;
    }
    E result = theData[front];
    front = (front + 1) % capacity;
    size--;
    return result;
}

// Private Methods
/** Double the capacity and reallocate the data.
 * @pre The array is filled to capacity.
 * @post The capacity is doubled and the first half of the expanded array is
 *       filled with data.
*/
@SuppressWarnings("unchecked")
private void reallocate() {
    int newCapacity = 2 * capacity;
    E[] newData = (E[]) new Object[newCapacity];
    int j = front;
    for (int i = 0; i < size; i++) {
        newData[i] = theData[j];
        j = (j + 1) % capacity;
    }
    front = 0;
    rear = size - 1;
    capacity = newCapacity;
    theData = newData;
}
}

```

## Increasing Queue Capacity

When the capacity is reached, we double the capacity and copy the array into the new one, as was done for the `ArrayList`. However, we can't simply use the `reallocate` method we developed for the `ArrayList` because of the circular nature of the array. We can't copy over elements from the original array to the first half of the expanded array, maintaining their position. We must first copy the elements from position `front` through the end of the original array to the beginning of the expanded array; then copy the elements from the beginning of the original array through `rear` to follow those in the expanded array (see Figure 4.15).

We begin by creating an array `newData`, whose capacity is double that of `theData`. The loop

```

int j = front;
for (int i = 0; i < size; i++) {
    newData[i] = theData[j];
    j = (j + 1) % capacity;
}

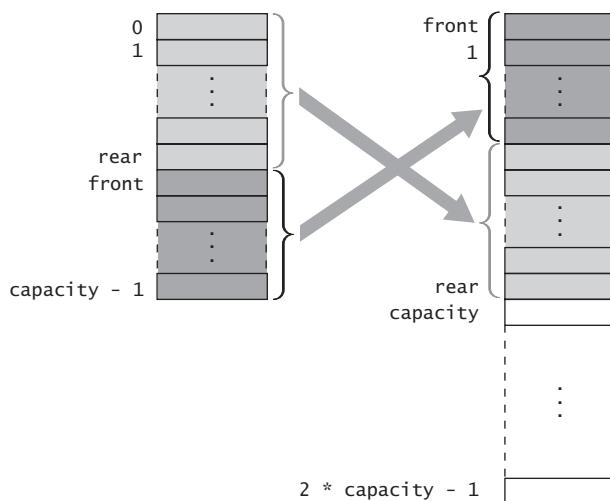
```

copies `size` elements over from `theData` to the first half of `newData`. In the copy operation  
`newData[i] = theData[j]`

`subscript i` for `newData` goes from 0 to `size - 1` (the first half of `newData`). `Subscript j` for `theData` starts at `front`. The statement

```
j = (j + 1) % capacity;
```

**FIGURE 4.15**  
Reallocating a Circular Array



increments the subscript for array `theData`. Therefore, subscript `j` goes from `front` to `capacity - 1` (in increments of 1) and then back to 0. So the elements are copied from `theData` in the sequence `theData[front], . . . , theData[capacity - 1], theData[0], . . . , theData[rear]`, where `theData[front]` is stored in `newData[0]` and `theData[rear]` is stored in `newData[size - 1]`. After the copy loop, `front` is reset to 0 and `rear` is reset to `size - 1` (see Figure 4.15).

By choosing a new capacity that is twice the current capacity, the cost of the reallocation is amortized across each insert, just as for an `ArrayList`. Thus, insertion is still considered an  $O(1)$  operation.



### PITFALL

#### Incorrect Use of `Arrays.copyOf` to Expand a Circular Array

You might consider using the following method to copy all of the elements over from the original array `theData` to the first half of the expanded array `theData`.

```
private void reallocate() {
    capacity = 2 * capacity;
    theData = Arrays.copyOf(theData, capacity);
}
```

The problem is that in the circular array before expansion, element `theData[0]` followed the last array element. However, after expansion, the element that was formerly in the last position would now be in the middle of the array, so `theData[0]` would not follow it.

### Implementing Class `ArrayQueue<E>.Iter`

Just as for class `ListQueue<E>`, we must implement the missing Queue methods and an inner class `Iter` to fully implement the Queue interface. Listing 4.14 shows inner class `Iter`.

Data field `index` stores the subscript of the next element to access. The constructor initializes `index` to `front` when a new `Iter` object is created. Data field `count` keeps track of the number of items accessed so far. Method `hasNext` returns `true` if `count` is less than the queue size.

**LISTING 4.14**

```

.....  

Class ArrayQueue<E>.Iter  

/** Inner class to implement the Iterator<E> interface. */  

private class Iter implements Iterator<E> {  

    // Data Fields  

    // Index of next element  

    private int index;  

    // Count of elements accessed so far  

    private int count = 0;  

    // Methods  

    // Constructor  

    /** Initializes the Iter object to reference the first queue element. */  

public Iter() {  

    index = front;  

}  

/** Returns true if there are more elements in the queue to access. */  

@Override  

public boolean hasNext() {  

    return count < size;  

}  

/** Returns the next element in the queue.  

 * @pre index references the next element to access.  

 * @post index and count are incremented.  

 * @return The element with subscript index  

 */  

@Override  

public E next() {  

    if (!hasNext()) {  

        throw new NoSuchElementException();  

    }  

    E returnValue = theData[index];  

    index = (index + 1) % capacity;  

    count++;  

    return returnValue;  

}  

/** Remove the item accessed by the Iter object - not implemented. */  

@Override  

public void remove() {  

    throw new UnsupportedOperationException();  

}  

}

```

Method `next` returns the element at position `index` and increments `index` and `count`. Method `Iter.remove` throws an `UnsupportedOperationException` because it would violate the contract for a queue to remove an item other than the first one.

### Comparing the Three Implementations

As mentioned earlier, all three implementations of the Queue interface are comparable in terms of computation time. All operations are  $O(1)$  regardless of the implementation. Although reallocating an array is an  $O(n)$  operation, it is amortized over  $n$  items, so the cost per item is  $O(1)$ .

In terms of storage requirements, both linked-list implementations require more storage because of the extra space required for links. To perform an analysis of the storage requirements, you need to know that Java stores a reference to the data for a queue element in each node in addition to the links. Therefore, each node for a single-linked list would store a total of two references (one for the data and one for the link), a node for a double-linked list would store a total of three references, and a node for a circular array would store just one reference. Therefore, a double-linked list would require 1.5 times the storage required for a single-linked list with the same number of elements. A circular array that is filled to capacity would require half the storage of a single-linked list to store the same number of elements. However, if the array were just reallocated, half the array would be empty, so it would require the same storage as a single-linked list.

## EXERCISES FOR SECTION 4.7

### SELF-CHECK

1. Show the new array for the queue in Figure 4.13 after the array size is doubled.
2. Provide the algorithm for the methods in Programming Exercise 1 below.
3. Redraw the queue in Figure 4.10 so that rear references the list head and front references the list tail. Show the queue after an element is inserted and an element is removed. Explain why the approach used in the book is better.

### PROGRAMMING

1. Write the missing methods required by the Queue interface and inner class `Iter` for class `ListQueue<E>`. Class `Iter` should have a data field `current` of type `Node<E>`. Data field `current` should be initialized to `first` when a new `Iter` object is created. Method `next` should return the value of `current` and advance `current`. Method `remove` should throw an `UnsupportedOperationException`.
2. Write the missing methods for class `ArrayList<E>` required by the Queue interface.
3. Replace the loop in method `reallocate` with two calls to `System.arraycopy`.

## 4.8 The Deque Interface

As we mentioned in Section 4.2, Java provides the `Deque` interface. The name `deque` (pronounced "deck") is short for double-ended queue, which means that it is a data structure that allows insertions and removals from both ends (front and rear). Methods are provided to insert, remove, and examine elements at both ends of the deque. Method names that end in *first* access the front of the deque, and method names that end in *last* access the rear of the deque. Table 4.10 shows some of the `Deque` methods.

As you can see from the table, there are two pairs of methods that perform each of the insert, remove, and examine operations. One pair returns a boolean value indicating the

**TABLE 4.10**

The Deque&lt;E&gt; Interface

Method	Behavior
boolean offerFirst(E item)	Inserts item at the front of the deque. Returns true if successful; returns false if the item could not be inserted
boolean offerLast(E item)	Inserts item at the rear of the deque. Returns true if successful; returns false if the item could not be inserted
void addFirst(E item)	Inserts item at the front of the deque. Throws an exception if the item could not be inserted
void addLast(E item)	Inserts item at the rear of the deque. Throws an exception if the item could not be inserted
E pollFirst( )	Removes the entry at the front of the deque and returns it; returns null if the deque is empty
E pollLast( )	Removes the entry at the rear of the deque and returns it; returns null if the deque is empty
E removeFirst( )	Removes the entry at the front of the deque and returns it if the deque is not empty. If the deque is empty, throws a NoSuchElementException
E removeLast( )	Removes the item at the rear of the deque and returns it. If the deque is empty, throws a NoSuchElementException
E peekFirst( )	Returns the entry at the front of the deque without removing it; returns null if the deque is empty
E peekLast( )	Returns the item at the rear of the deque without removing it; returns null if the deque is empty
E getFirst( )	Returns the entry at the front of the deque without removing it. If the deque is empty, throws a NoSuchElementException
E getLast( )	Returns the item at the rear of the deque without removing it. If the deque is empty, throws a NoSuchElementException
boolean removeFirstOccurrence (Object item)	Removes the first occurrence of item in the deque. Returns true if the item was removed
boolean removeLastOccurrence(Object item)	Removes the last occurrence of item in the deque. Returns true if the item was removed
Iterator<E> iterator( )	Returns an iterator to the elements of this deque in the proper sequence
Iterator<E> descendingIterator( )	Returns an iterator to the elements of this deque in reverse sequential order

method result, and the other pair throws an exception if the operation is unsuccessful. For example, offerFirst and offerLast return a value indicating the insertion result, whereas addFirst and addLast throw an exception if the insertion is not successful. Normally, you should use a method that returns a value. Table 4.11 shows the use of these methods.

**TABLE 4.11**Effect of Using Deque Methods on an Initially Empty `Deque<Character>` `d`.

Deque Method	Deque <code>d</code>	Effect
<code>d.offerFirst('b')</code>	<code>b</code>	'b' inserted at front
<code>d.offerLast('y')</code>	<code>by</code>	'y' inserted at rear
<code>d.addLast('z')</code>	<code>byz</code>	'z' inserted at rear
<code>d.addFirst('a')</code>	<code>abyz</code>	'a' inserted at front
<code>d.peekFirst()</code>	<code>abyz</code>	Returns 'a'
<code>d.peekLast()</code>	<code>abyz</code>	Returns 'z'
<code>d.pollLast()</code>	<code>aby</code>	Removes 'z'
<code>d.pollFirst()</code>	<code>by</code>	Removes 'a'

## Classes that Implement Deque

The Java Collections Framework provides four implementations of the `Deque` interface, including `ArrayDeque` and `LinkedList`. `ArrayDeque` utilizes a resizable circular array like our class `ArrayList` and is the recommended implementation because, unlike `LinkedList`, it does not support indexed operations.

## Using a Deque as a Queue

The `Deque` interface extends the `Queue` interface, which means that a class that implements `Deque` also implements `Queue`. The `Queue` methods are equivalent to `Deque` methods, as shown in Table 4.12. If elements are always inserted at the front of a deque and removed from the rear (FIFO), then the deque functions as a queue. In this case, you could use either method `add` or `addLast` to insert a new item.

**TABLE 4.12**

Equivalent Queue and Deque Methods

Queue Method	Equivalent Deque Method
<code>add(e)</code>	<code>addLast(e)</code>
<code>offer(e)</code>	<code>offerLast(e)</code>
<code>remove()</code>	<code>removeFirst()</code>
<code>poll()</code>	<code>pollFirst()</code>
<code>element()</code>	<code>getFirst()</code>
<code>peek()</code>	<code>peekFirst()</code>

## Using a Deque as a Stack

Earlier in this chapter, we used Deques as stacks (LIFO). When a deque is used as a stack, elements are always pushed and popped from the front of the deque. Using the `Deque` interface is preferable to using the legacy `Stack` class (based on the `Vector` class). Stack methods are equivalent to `Deque` methods, as shown in Table 4.13.

**TABLE 4.13**

Equivalent Stack and Deque Methods

Stack Method	Equivalent Deque Method
push(e)	addFirst(e)
pop()	removeFirst()
peek()	peekFirst()
isEmpty()	isEmpty()

The statement

```
Deque<String> stackOfStrings = new ArrayDeque<>();
```

creates a new Deque object called `stackOfStrings`. You can use methods `push`, `pop`, `peek`, and `isEmpty` in the normal way to manipulate `stackOfStrings`.

## EXERCISES FOR SECTION 4.8

### SELF-CHECK

1. For object `stackOfStrings` declared above, replace each stack operation with the appropriate Deque method and explain the effect of each statement in the following fragment.

```
stackOfStrings.push("Hello");
String one = stackOfStrings.pop();
if (!stackOfStrings.isEmpty())
    System.out.println(stackOfStrings.peek());
stackOfStrings.push("Good bye");
for (String two : stackOfStrings)
    System.out.println(two);
```

2. What would be the effect of omitting the conditional test before calling the `peek` method?

3. Would the following statements execute without error? If your answer is “yes,” what would their effect be? If “no,” why not?

```
stackOfStrings.offer("away");
String three = stackOfStrings.remove();
```

### PROGRAMMING

1. Write a fragment that reads a sequence of strings and inserts each string that is not numeric at the front of a deque and each string that is numeric at the rear of a deque. Your fragment should also count the number of strings of each kind.
2. For a deque that has the form required by Programming Exercise 1, display the message “Strings that are not numeric” followed by the nonnumeric strings and then the message “Strings that are numbers” followed by the numeric strings. Do not empty the deque.
3. Write a `Deque.addFirst` method for class `ArrayList`.



# Chapter Review

- ◆ A stack is a LIFO data structure. This means that the last item added to a stack is the first one removed.
- ◆ A stack is a simple but powerful data structure. It has only four operators: empty, peek, pop, and push.
- ◆ Stacks are useful when we want to process information in the reverse of the order that it is encountered. For this reason, a stack was used to implement the palindrome finder.
- ◆ `java.util.Stack` is implemented as an extension of the `Vector` class. The problem with this approach is that it allows a client to invoke other methods from the `Vector` class.
- ◆ We showed three different ways to implement stacks: using an object of a class that implements the `List` interface as a container; using an array as a container; and using a linked list as a container.
- ◆ Stacks can be applied in algorithms for evaluating arithmetic expressions. We showed how to evaluate postfix expressions and how to translate infix expressions with and without parentheses to postfix.
- ◆ The queue is an abstract data type with a FIFO structure. This means that the item that has been in the queue the longest will be the first one removed. Queues can be used to represent reservation lists and waiting lines (from which the data structure gets its name “queue”).
- ◆ The `Queue` interface declares methods `offer`, `remove`, `poll`, `peek`, and `element`.
- ◆ We discussed three ways to implement the `Queue` interface: as a double-linked list, as a single-linked list, and as a circular array. All three implementations support insertion and removal in  $O(1)$  time; however, there will be a need for reallocation in the circular array implementation (amortized  $O(1)$  time). The array implementation requires the smallest amount of storage when it is close to capacity. The `LinkedList` class requires the most storage but no implementation because it is part of `java.util`.
- ◆ We discussed the `Deque` interface and showed how its methods allow insertion and removal at either end of a deque. We showed the correspondence between its methods and methods found in the `Stack` class and `Queue` interface.

## Java API Classes Introduced in This Chapter

```
java.util.Stack
java.lang.UnsupportedOperationException
java.util.AbstractQueue
java.util.ArrayDeque
java.util.Deque
java.util.NoSuchElementException
java.util.Queue
```

## User-Defined Interfaces and Classes in This Chapter

<code>ArrayStack</code>	<code>PalindromeFinder</code>
<code>InfixToPostfix</code>	<code>PostfixEvaluator</code>
<code>InfixToPostfixParens</code>	<code>StackInt</code>
<code>IsPalindrome</code>	

LinkedStack	SyntaxErrorException
ListStack	ListQueue
ArrayQueue	MaintainQueue
ArrayQueue.Iter	KWQueue

## Quick-Check Exercises

1. A stack is a \_\_\_\_\_-in, \_\_\_\_\_-out data structure.
2. Draw this stack `s` as an object of type `ArrayList<Character>`. What is the value of data field `topOfStack`?

\$
*
&

3. What is the value of `s.empty()` for the stack shown in Question 2?
4. What is returned by `s.pop()` for the stack shown in Question 2?
5. Answer Question 2 for a stack `s` implemented as a linked list (type `LinkedStack<Character>`).
6. Why should the statement `s.remove(i)`, where `s` is of type `StackInt` and `i` is an integer index, not appear in a client program? Can you use this statement with an object of the `Stack` class defined in `java.util`? Can you use it with an object of class `ArrayList` or `LinkedStack`?
7. What would be the postfix form of the following expression?  
 $x + y - 24 * zone - ace / 25 + c1$   
 Show the contents of the operator stack just before each operator is processed and just after all tokens are scanned using method `InfixToPostfix.convert`.
8. Answer Question 7 for the following expression.  
 $(x + y - 24) * (zone - ace / (25 + c1))$
9. The value of the expression  $20\ 35 - 5 / 10\ 7^+ +$  is \_\_\_\_\_. Show the contents of the operand stack just before each operator is processed and just after all tokens are scanned.
10. A queue is a \_\_\_\_\_-in, \_\_\_\_\_-out data structure.
11. Would a compiler use a stack or a queue in a program that converts infix expressions to postfix?
12. Would an operating system use a stack or a queue to determine which print job should be handled next?
13. Assume that a queue `q` of capacity 6 (circular array representation) contains the five characters `+`, `*`, `-`, `&`, and `#` (all wrapped in `Character` objects), where `+` is the first character inserted. Assume that `+` is stored in the first position in the array. What is the value of `q.front`? What is the value of `q.rear`?
14. Remove the first element from the queue in Question 13 and insert the characters `\` then `%`. Draw the new queue. What is the value of `q.front`? What is the value of `q.rear`?
15. If a single-linked list were used to implement the queue in Question 13, the character \_\_\_\_\_ would be at the head of the list and the character \_\_\_\_\_ would be at the tail of the list.
16. For a nonempty queue implemented as a single-linked list, the statement \_\_\_\_\_ would be used inside method `offer` to store a new node whose data field is referenced by `item` in the queue; the statement \_\_\_\_\_ would be used to disconnect a node after its data was retrieved from the queue.
17. Pick the queue implementation (circular array, single-linked list, double-linked list) that is most appropriate for each of the following conditions.
- Storage must be reallocated when the queue is full.
  - This implementation is normally most efficient in use of storage.
  - This is an existing class in the Java API.

## Review Questions

1. Show the effect of each of the following operations on stack *s*. Assume that *y* (type Character) contains the character '&'. What are the final values of *x* and *success* and the contents of the stack *s*?

```
Deque<Character> s = new ArrayDeque<>();
char x;
s.push('+');
try {
    x = s.pop();
    success = true;
}
catch (NoSuchElementException e) {
    success = false;
}
try {
    x = s.pop();
    success = true;
}
catch (NoSuchElementException e) {
    success = false;
}
s.push('(');
s.push(y);
try {
    x = s.pop();
    success = true;
}
catch (NoSuchElementException e) {
    success = false;
}
```

2. Write a *toString* method for class *ArrayList<E>*.
3. Write a *toString* method for class *LinkedStack<E>*.
4. Write an infix expression that would convert to the postfix expression in Quick-Check Question 9.
5. Write a constructor for class *LinkedStack<E>* that loads the stack from an array parameter. The last array element should be at the top of the stack.
6. Write a client that removes all negative numbers from a stack of *Integer* objects. If the original stack contained the integers 30, -15, 20, -25 (top of stack), the new stack should contain the integers 30, 20.
7. Write a method *peekNextToTop* that allows you to retrieve the element just below the one at the top of the stack without removing it. Write this method for both *ArrayList<E>* and *LinkedStack<E>*. It should return *null* if the stack has just one element, and it should throw an exception if the stack is empty.
8. Show the effect of each of the following operations on queue *q*. Assume that *y* (type Character) contains the character '&'. What are the final values of *x* and *success* (type boolean) and the contents of queue *q*?

```
Queue<Character> q = new ArrayDeque<>();
boolean success = true;
char x;
q.offer('+');
try {
    x = q.remove();
    x = q.remove();
    success = true;
} catch(NoSuchElementException e) {
    success = false;
}
```

```

q.offer(')');
q.offer(y);
try {
    x = q.remove(); success = true;
} catch(NoSuchElementException e) {
    success = false;
}

```

9. Write a new queue method called `moveToRear` that moves the element currently at the front of the queue to the rear of the queue. The element that was second in line will be the new front element. Do this using methods `Queue.offer` and `Queue.remove`.
10. Answer Question 9 without using methods `Queue.offer` or `Queue.remove` for a single-linked list implementation of Queue. You will need to manipulate the queue internal data fields directly.
11. Answer Question 9 without using methods `Queue.offer` or `Queue.remove` for a circular array implementation of Queue. You will need to manipulate the queue internal data fields directly.
12. Write a new queue method called `moveToFront` that moves the element at the rear of the queue to the front of the queue, while the other queue elements maintain their relative positions behind the old front element. Do this using methods `Queue.offer` and `Queue.remove`.
13. Answer Question 12 without using `Queue.offer` and `Queue.remove` for a single-linked list implementation of Queue.
14. Answer Question 12 without using methods `Queue.offer` or `Queue.remove` for a circular array implementation of Queue.

## Programming Projects

1. Add a method `isPalindromeLettersOnly` to the `PalindromeFinder` class that bases its findings only on the letters in a string (ignoring spaces, digits, and other characters that are not letters).
2. Provide a complete implementation of class `LinkedStack` and test it on each of the applications in this chapter.
3. Provide a complete implementation of class `ArrayStack` and test it on each of the applications in this chapter.
4. Develop an Expression Manager that can do the following operations:

*Balanced Symbols Check*

- Read a mathematical expression from the user.
- Check and report whether the expression is balanced.
- {}, {}, (), [], [] are the only symbols considered for the check. All other characters can be ignored.

*Infix to Postfix Conversion*

- Read an infix expression from the user.
- Perform the Balanced Parentheses Check on the expression read.
- If the expression fails the Balanced Parentheses Check, report a message to the user that the expression is invalid.
- If the expression passes the Balanced Parentheses Check, convert the infix expression into a postfix expression and display it to the user.
- Operators to be considered are +, -, \*, /, %.

*Postfix to Infix Conversion*

- Read a postfix expression from the user.
- Convert the postfix expression into an infix expression and display it to the user.
- Display an appropriate message if the postfix expression is not valid.
- Operators to be considered are +, -, \*, /, %.

*Evaluating a Postfix Expression*

- Read the postfix expression from the user.
- Evaluate the postfix expression and display the result.

- Display an appropriate message if the postfix expression is not valid.
- Operators to be considered are  $+, -, *, /, \%$ .
- Operands should be only integers.

*Implementation*

- Design a menu that has buttons or requests user input to select from all the aforementioned operations.

5. Write a client program that uses the Stack abstract data type to simulate a session with a bank teller. Unlike most banks, this one has decided that the last customer to arrive will always be the first to be served. Create classes that represent information about a bank customer and a transaction. For each customer, you need to store a name, current balance, and a reference to the transaction. For each transaction, you need to store the transaction type (deposit or withdrawal) and the amount of the transaction. After every five customers are processed, display the size of the stack and the name of the customer who will be served next.

6. Write a program to handle the flow of widgets into and out of a warehouse. The warehouse will have numerous deliveries of new widgets and orders for widgets. The widgets in a filled order are billed at a profit of 50 percent over their cost. Each delivery of new widgets may have a different cost associated with it. The accountants for the firm have instituted a LIFO system for filling orders. This means that the newest widgets are the first ones sent out to fill an order. Also, the most recent orders are filled first. This method of inventory can be represented using two stacks: orders-to-be-filled and widgets-on-hand. When a delivery of new widgets is received, any unfilled orders (on the orders-to-be-filled stack) are processed and filled. After all orders are filled, if there are widgets remaining in the new delivery, a new element is pushed onto the widgets-on-hand stack. When an order for new widgets is received, one or more objects are popped from the widgets-on-hand stack until the order has been filled. If the order is completely filled and there are widgets left over in the last object popped, a modified object with the quantity updated is pushed onto the widgets-on-hand stack. If the order is not completely filled, the order is pushed onto the orders-to-be-filled stack with an updated quantity of widgets to be sent out later. If an order is completely filled, it is not pushed onto the stack.

Write a class with methods to process the shipments received and to process orders. After an order is filled, display the quantity sent out and the total cost for all widgets in the order. Also indicate whether there are any widgets remaining to be sent out at a later time. After a delivery is processed, display information about each order that was filled with this delivery and indicate how many widgets, if any, were stored in the object pushed onto the widgets-on-hand stack.

7. You can combine the algorithms for converting between infix to postfix and for evaluating postfix to evaluate an infix expression directly. To do so you need two stacks: one to contain operators and the other to contain operands. When an operand is encountered, it is pushed onto the operand stack. When an operator is encountered, it is processed as described in the infix to postfix algorithm. When an operator is popped off the operator stack, it is processed as described in the postfix evaluation algorithm: The top two operands are popped off the operand stack, the operation is performed, and the result is pushed back onto the operand stack. Write a program to evaluate infix expressions directly using this combined algorithm.

8. Write a client program that uses the Stack abstract data type to compile a simple arithmetic expression without parentheses. For example, the expression

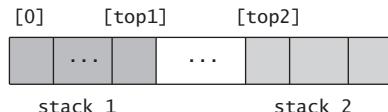
$a + b * c - d$

should be compiled according to the following table:

Operator	Operand 1	Operand 2	Result
*	b	c	z
+	a	z	y
-	y	d	x

The table shows the order in which the operations are performed (\*, +, -) and operands for each operator. The result column gives the name of an identifier (working backward from z) chosen to hold each result. Assume the operands are the letters a through m and the operators are (+, -, \*, /). Your program should read each character and process it as follows: If the character is blank, ignore it. If the character is neither blank nor an operand nor an operator, display an error message and terminate the program. If it is an operand, push it onto the operand stack. If it is an operator, compare its precedence to that of the operator on top of the operator stack. If the current operator has higher precedence than the one currently on top of the stack (or stack is empty), it should be pushed onto the operator stack. If the current operator has the same or lower precedence, the operator on top of the operator stack must be evaluated next. This is done by popping that operator off the operator stack along with a pair of operands from the operand stack and writing a new line in the output table. The character selected to hold the result should then be pushed onto the operand stack. Next, the current operator should be compared to the new top of the operator stack. Continue to generate output lines until the top of the operator stack has lower precedence than the current operator or until it is empty. At this point, push the current operator onto the top of the stack and examine the next character in the data string. When the end of the string is reached, pop any remaining operator along with its operand pair just described. Remember to push the result character onto the operand stack after each table line is generated.

9. Another approach to checking for palindromes would be to store the characters of the string being checked in a stack and then remove half of the characters, pushing them onto a second stack. When you are finished, if the two stacks are equal, then the string is a palindrome. This works fine if the string has an even number of characters. If the string has an odd number of characters, an additional character should be removed from the original stack before the two stacks are compared. It doesn't matter what this character is because it doesn't have to be matched. Design, code, and test a program that implements this approach.
10. Operating systems sometimes use a fixed array storage area to accommodate a pair of stacks such that one grows from the bottom (with its first item stored at index 0) and the other grows from the top (with its first item stored at the highest array index). As the stacks grow, the top of the stacks will move closer together.



The stacks are full when the two top elements are stored in adjacent array elements (`top2 == top1 + 1`). Design, code, and test a class `DoubleStack` that implements this data structure. `DoubleStack` should support the normal stack operations (push, pop, peek, empty, etc.). Each stack method should have an additional `int` parameter that indicates which of the stacks (1 or 2) is being processed. For example, `push(1, item)` will push item onto stack 1.

11. Redo Programming Project 6, assuming that widgets are shipped using a FIFO inventory system.
12. Write a class `MyArrayDeque` that extends class `ArrayList`. Class `MyArrayDeque` should implement the `Deque` interface. Test your new class by comparing its operation to that of the `ArrayDeque` class in the Java Collections Framework.
13. Write a program that reads in a sequence of characters and stores each character in a deque. Display the deque contents. Then use a second deque to store the characters in reverse order. When done, display the contents of both deques.
14. Write a program that simulates the operation of a busy airport that has only two runways to handle all takeoffs and landings. You may assume that each takeoff or landing takes 15 minutes to complete. One runway request is made during each 5-minute time interval, and the likelihood of a landing request is the same as for a takeoff request.  
Priority is given to planes requesting a landing. If a request cannot be honored, it is added to a takeoff or landing queue.

Your program should simulate 120 minutes of activity at the airport. Each request for runway clearance should be time-stamped and added to the appropriate queue. The output from your program should include the final queue contents, the number of take-offs completed, the number of landings completed, and the average number of minutes spent in each queue.

15. An operating system assigns jobs to print queues based on the number of pages to be printed (less than 10 pages, less than 20 pages, or more than 20 pages but less than 50 pages). You may assume that the system printers are able to print 10 pages per minute. Smaller print jobs are printed before larger print jobs, and print jobs of the same priority are queued up in the order in which they are received. The system administrators would like to compare the time required to process a set of print jobs using one, two, or three system printers.

Write a program that simulates processing 100 print jobs of varying lengths using one, two, or three printers. Assume that a print request is made every minute and that the number of pages to print varies from 1 to 50 pages.

The output from your program should indicate the order in which the jobs were received, the order in which they were printed, and the time required to process the set of print jobs. If more than one printer is being used, indicate which printer each job was printed on.

16. Write a menu-driven program that uses an array of queues to keep track of a group of executives as they are transferred from one department to another, get paid, or become unemployed. Executives within a department are paid based on their seniority, with the person who has been in the department the longest receiving the most money. Each person in the department receives \$1000 in salary for each person in her department having less seniority than she has. Persons who are unemployed receive no compensation.

Your program should be able to process the following set of commands:

Join <person> <department>	<person> is added to <department>
Quit <person>	<person> is removed from his or her department
Change <person> <department>	<person> is moved from old department to <department>
Payroll	Each executive's salary is computed and displayed by department in decreasing order of seniority

*Hint:* You might want to include a table that contains each executive's name and information and the location of the queue that contains his or her name, to make searching more efficient.

17. Simulate the operation of a bank. Customers enter the bank, and there are one or more tellers. If a teller is free, that teller serves the customer. Otherwise the customer enters the queue and waits until a teller is free. Your program should accept the following inputs:

- The arrival rate for the customers
- The average processing time
- The number of tellers

Use your program to determine how many tellers are required for a given arrival rate and average processing time.

18. Simulate a checkout area of a supermarket consisting of one superexpress counter, two express counters, and numStandLines standard counters. All customers with numSuper or fewer items proceed to a superexpress counter with the fewest customers, unless there is a free express or regular line, and those with between numSuper and numExp proceed to the express counter with the shortest line unless there is a free standard line. Customers with more than numExp go to the standard counter with the shortest standard line.

The number of items bought will be a random number in the range 1 to maxItems.

The time to process a customer is 5 seconds per item.

Calculate the following statistics:

- Average waiting time for each of the lines
- Overall average waiting time

- Maximum length of each line
- Number of customers per hour for each line and overall
- Number of items processed per hour for each line and overall
- Average free time of each counter
- Overall free time

*Note:* The average waiting time for a line is the total of the customer waiting times divided by the number of customers. A customer's waiting time is the time from when he (or she) enters the queue for a given checkout line until the checkout processing begins. If the customer can find a free line, then the wait time is zero.

Your program should read the following data:

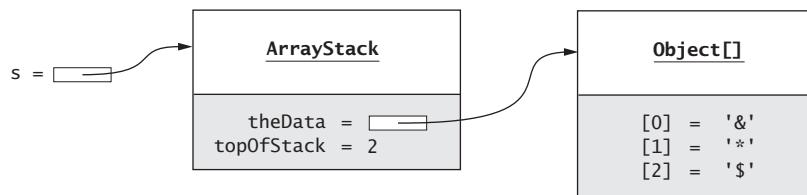
<code>numSuper</code>	The number of items allowed in the superexpress line
<code>numExp</code>	The number of items allowed in the express line
<code>numStandLines</code>	The number of regular lines
<code>arrivalRate</code>	The arrival rate of customers per hour
<code>maxItems</code>	The maximum number of items
<code>maxSimTime</code>	The simulation time

It may be that some lines do not get any business. In that case you must be sure, in calculating the average, not to divide by zero.

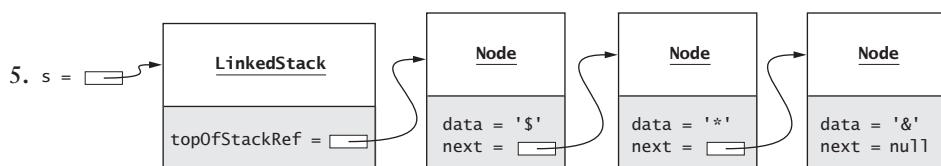
19. A *randomized queue* is similar to a queue, except that the item removed is chosen at random from the items in the queue. Create a `RandomizedQueue` that contains the normal queue methods except that the `remove` method will delete an item chosen using a uniform distribution. You should write this class as an extension of the `ArrayQueue` class.

## Answers to Quick-Check Exercises

1. A stack is a LIFO data structure.
2. Each character in array `theData` should be wrapped in a `Character` object. The value of `topOfStack` should be 2.



3. Method `empty` returns `false`.
4. `pop` returns a reference to the `Character` object that wraps '\$'.



6. Method `remove(int i)` is not defined for classes that implement interface `StackInt`. The `Stack` class defined in API `java.util` would permit its use. Classes `ArrayList` and `LinkedList` would not.

7. Infix: `x + y - 24 * zone - ace / 25 + c1`

Postfix: `x y + 24 zone * - ace 25 / - c1 +`  
 Operator stack before first + : | Empty stack (vertical bar is bottom of stack)  
 Operator stack before first - : | +  
 Operator stack before first \* : | -  
 Operator stack before second - : | -, \*  
 Operator stack before first / : | -  
 Operator stack before second + : | -, /  
 Operator stack after all tokens scanned: | +

8. Infix: `( x + y - 24 ) * ( zone - ace / ( 25 + c1 ) )`

Postfix: `x y + 24 - zone ace 25 c1 + / - *`  
 Operator stack before first ( : | Empty stack (vertical bar is bottom of stack)  
 Operator stack before first + : | (   
 Operator stack before first - : | (, +  
 Operator stack before first ) : | (, -  
 Operator stack before first \* : | Empty stack  
 Operator stack before second ( : | \*  
 Operator stack before second - : | \*, (   
 Operator stack before second / : | \*, (, -  
 Operator stack before third ( : | \*, (, -, /   
 Operator stack before second + : | \*, (, -, /, (   
 Operator stack before second ) : | \*, (, -, /, (, +  
 Operator stack before third ) : | \*, (, -, /  
 Operator stack after all tokens scanned: | \*

9. `20 35 - 5 / 10 7 * + is 67 (-3 + 70)`

Operand stack just before - : | 20, 35  
 Operand stack just before / : | -15, 5  
 Operand stack just before \* : | -3, 10, 7  
 Operand stack just before + : | -3, 70  
 Operand stack after all tokens scanned: | 67

10. `first, first`

11. `stack`

12. `queue`

13. `q.front is 0; q.rear is 4.`

14. `q.rear %`  
`q.front *`  
`---`  
`&`  
`#`  
`\`

`q.front is 1; q.rear is 0`

15. '\*', '%'
16. For insertion: `rear.next = new Node<E>(item);`  
To disconnect the node removed: `front = front.next;`
17. a. circular array  
b. single-linked list  
c. double-linked list (`class LinkedList`)



# Recursion

## Chapter Objectives

- ◆ To understand how to think recursively
- ◆ To learn how to trace a recursive method
- ◆ To learn how to write recursive algorithms and methods for searching arrays
- ◆ To learn about recursive data structures and recursive methods for a `LinkedList` class
- ◆ To understand how to use recursion to solve the Towers of Hanoi problem
- ◆ To understand how to use recursion to process two-dimensional images
- ◆ To learn how to apply backtracking to solve search problems such as finding a path through a maze

This chapter introduces a programming technique called recursion and shows you how to think recursively. You can use recursion to solve many kinds of programming problems that would be very difficult to conceptualize and solve without recursion. Computer scientists in the field of artificial intelligence (AI) often use recursion to write programs that exhibit intelligent behavior: playing games such as chess, proving mathematical theorems, recognizing patterns, and so on.

In the beginning of the chapter, you will be introduced to recursive thinking and how to design a recursive algorithm and prove that it is correct. You will also learn how to trace a recursive method and use activation frames for this purpose.

Recursive algorithms and methods can be used to perform common mathematical operations, such as computing a factorial or a greatest common divisor(gcd). Recursion can be used to process familiar data structures, such as strings, arrays, and linked lists, and to design a very efficient array search technique called binary search. You will also see that a linked list is a recursive data structure and learn how to write recursive methods that perform common list-processing tasks.

Recursion can be used to solve a variety of other problems. The case studies in this chapter use recursion to solve a game, to search for “blobs” in a two-dimensional image, and to find a path through a maze.

## Recursion

- 5.1** Recursive Thinking
- 5.2** Recursive Definitions of Mathematical Formulas
- 5.3** Recursive Array Search
- 5.4** Recursive Data Structures
- 5.5** Problem Solving with Recursion
  - Case Study:* Towers of Hanoi
  - Case Study:* Counting Cells in a Blob
- 5.6** Backtracking
  - Case Study:* Finding a Path through a Maze

## 5.1 Recursive Thinking

*Recursion* is a problem-solving approach that can be used to generate simple solutions to certain kinds of problems that would be difficult to solve in other ways. In a recursive algorithm, the original problem is split into one or more simpler versions of itself. For example, if the solution to the original problem involved  $n$  items, recursive thinking might split it into two problems: one involving  $n - 1$  items and one involving just a single item. Then the problem with  $n - 1$  items could be split again into one involving  $n - 2$  items and one involving just a single item, and so on. If the solution to all the one-item problems is “trivial,” we can build up the solution to the original problem from the solutions to the simpler problems.

As an example of how this might work, consider a collection of nested wooden figures as shown in Figure 5.1. If you wanted to write an algorithm to “process” this collection in some way (such as counting the figures or painting a face on each figure), you would have difficulty doing it because you don’t know how many objects are in the nest. But you could use recursion to solve the problem in the following way.

### Recursive Algorithm to Process Nested Figures

1. **if** there is one figure in the nest
  2. Do whatever is required to the figure.
- else**
  3. Do whatever is required to the outer figure in the nest.
  4. Process the nest of figures inside the outer figure in the same way.

**FIGURE 5.1**

A Set of Nested Wooden Figures



In this recursive algorithm, the solution is trivial if there is only one figure: perform Step 2. If there is more than one figure, perform Step 3 to process the outer figure. Step 4 is the recursive operation—recursively process the nest of figures inside the outer figure. This nest will, of course, have one less figure than before, so it is a simpler version of the original problem.

As another example, let's consider searching for a target value in an array. Assume that the array elements are sorted and are in increasing order. A recursive approach, which we will study in detail in Section 5.3, involves replacing the problem of searching an array of  $n$  elements with one of searching an array of  $n/2$  elements. How do we do that? We compare the target value to the value of the element in the middle of the sorted array. If there is a match, we have found the target. If not, based on the result of the comparison, we either search the elements that come before the middle one or the elements that come after the middle one. So we have replaced the problem of searching an array with  $n$  elements to one that involves searching a smaller array with only  $n/2$  elements. The recursive algorithm follows.

### Recursive Algorithm to Search an Array

1. **if** the array is empty
2.     Return  $-1$  as the search result.
3.     **else if** the middle element matches the target
4.         Return the subscript of the middle element as the result.
5.     **else if** the target is less than the middle element
6.         Recursively search the array elements before the middle element  
and return the result.
7.     **else**
8.         Recursively search the array elements after the middle element and  
return the result.

The condition in Step 1 is true when there are no elements left to search. Step 2 returns  $-1$  to indicate that the search failed. Step 3 executes when the middle element matches the target. Otherwise, we recursively apply the search algorithm (Steps 4 and 5), thereby searching a smaller array (approximately half the size), and return the result. For each recursive search, the region of the array being searched will be different, so the middle element will also be different.

The two recursive algorithms we showed so far follow this general approach:

### General Recursive Algorithm

1. **if** the problem can be solved for the current value of  $n$
2.     Solve it.
3.     **else**
4.         Recursively apply the algorithm to one or more problems involving  
smaller values of  $n$ .
5.         Combine the solutions to the smaller problems to get the solution to  
the original.

Step 1 involves a test for what is called the *base case*: the value of  $n$  for which the problem can be solved easily. Step 3 is the *recursive case* because we recursively apply the algorithm. Because the value of  $n$  for each recursive case is smaller than the original value of  $n$ , each recursive case makes progress toward the base case. Whenever a split occurs, we revisit Step 1 for each new problem to see whether it is a base case or a recursive case.

## Steps to Design a Recursive Algorithm

From what we have seen so far, we can summarize the characteristics of a recursive solution:

- There must be at least one case (the base case), for a small value of  $n$ , that can be solved directly.
- A problem of a given size (say,  $n$ ) can be split into one or more smaller versions of the same problem (the recursive case).

Therefore, to design a recursive algorithm, we must

- Recognize the base case and provide a solution to it.
- Devise a strategy to split the problem into smaller versions of itself. Each recursive case must make progress toward the base case.
- Combine the solutions to the smaller problems in such a way that each larger problem is solved correctly.

Next, we look at a recursive algorithm for a common programming problem. We will also provide a Java method that solves this problem. All of the methods in this section and in the next will be found in class `RecursiveMethods.java` on this textbook's Web site.

**EXAMPLE 5.1** Let's see how we could write our own recursive method for finding the string length. How would you go about doing this? If there is a special character that marks the end of a string, then you can count all the characters that precede this special character. But if there is no special character, you might try a recursive approach. The base case is an empty string—its length is 0. For the recursive case, consider that each string has two parts: the first character and the “rest of the string.” If you can find the length of the “rest of the string,” you can then add 1 (for the first character) to get the length of the larger string. For example, the length of “abcde” is 1 + the length of “bcde”.

### Recursive Algorithm for Finding the Length of a String

1. **if** the string is empty (has no characters)
2.     The length is 0.
3.     **else**
  - The length is 1 plus the length of the string that excludes the first character.

We can implement this algorithm as a `static` method with a `String` argument. The test for the base case is a string reference of `null` or a string that contains no characters (“”). In either case, the length is 0. In the recursive case,

```
return 1 + length(str.substring(1));
```

the method call `str.substring(1)` returns a reference to a string containing all characters in string `str` except for the character at position 0. Then we call method `length` again with this substring as its argument. The method result is one more than the value returned from the next call to `length`. Each time we reenter the method `length`, the **if** statement executes with `str` referencing a string containing all but the first character in the previous call. Method `length` is called a *recursive method* because it calls itself.

```
/** Recursive method length (in RecursiveMethods.java).
 * @param str The string
 * @return The length of the string
 */
```

```

public static int length(String str) {
    if (str == null || str.isEmpty())
        return 0;
    else
        return 1 + length(str.substring(1));
}

```

---

**EXAMPLE 5.2** Method `printChars` is a recursive method that displays each character in its string argument on a separate line. In the base case (an empty or nonexistent string), the method return occurs immediately and nothing is displayed. In the recursive case, `printChars` displays the first character of its string argument and then calls itself to display the characters in the rest of the string. If the initial call is `printChars("hat")`, the method will display the lines

```

h
a
t

```

Unlike the method `length` in Example 5.1, `printChars` is a `void` method. However, both methods follow the format for the general recursive algorithm shown earlier.

```

/** Recursive method printChars (in RecursiveMethods.java).
   post: The argument string is displayed, one character per line.
   @param str The string
*/
public static void printChars(String str) {
    if (str == null || str.isEmpty()) {
        return;
    } else {
        System.out.println(str.charAt(0));
        printChars(str.substring(1));
    }
}

```

You get an interesting result if you reverse the two statements in the recursive case.

```

/** Recursive method printCharsReverse (in RecursiveMethods.java).
   post: The argument string is displayed in reverse,
         one character per line.
   @param str The string
*/
public static void printCharsReverse(String str) {
    if (str == null || str.isEmpty()) {
        return;
    } else {
        printCharsReverse(str.substring(1));
        System.out.println(str.charAt(0));
    }
}

```

Method `printCharsReverse` calls itself to display the rest of the string before displaying the first character in the current string argument. The effect will be to delay displaying the first character in the current string until all characters in the rest of the string are displayed. Consequently, the characters in the string will be displayed in reverse order. If the initial call is `printCharsReverse("hat")`, the method will display the lines

```

t
a
h

```

---

# Proving that a Recursive Method Is Correct

To prove that a recursive method is correct, you must verify that you have performed correctly the design steps listed earlier. You can use a technique that mathematicians use to prove that a theorem is true for all values of  $n$ . A *proof by induction* works the following way:

- Prove the theorem is true for the base case of (usually)  $n = 0$  or  $n = 1$ .
  - Show that if the theorem is assumed true for  $n$ , then it must be true for  $n + 1$ .

We can extend the notion of an inductive proof and use it as the basis for proving that a recursive algorithm is correct. To do this:

- Verify that the base case is recognized and solved correctly.
  - Verify that each recursive case makes progress toward the base case.
  - Verify that if all smaller problems are solved correctly, then the original problem is also solved correctly.

If you can show that your algorithm satisfies these three requirements, then your algorithm will be correct.

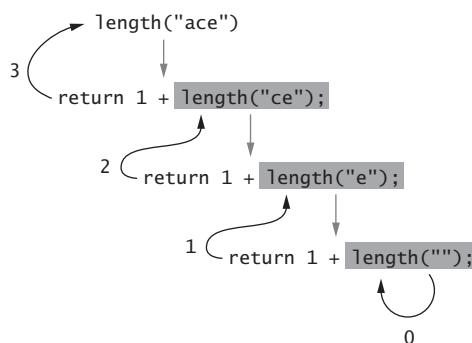
### EXAMPLE 5.3

To prove that the `length` method is correct, we know that the base case is an empty string and its length is correctly set at 0. The recursive case involves a call to `length` with a smaller string, so it is making progress toward the base case. Finally, if we know the length of the rest of the string, adding 1 gives us the length of the longer string consisting of the first character and the rest of the string.

# Tracing a Recursive Method

Figure 5.2 traces the execution of the method call `length("ace")`. The diagram shows a sequence of recursive calls to the method `length`. After returning from each call to `length`, we complete execution of the statement `return 1 + length(...)`; by adding 1 to the result so far and then returning from the current call. The final result, 3, would be returned from the original call. The arrow alongside each word `return` shows which call to `length` is associated with that result. For example, 0 is the result of the method call `length("")`. After adding 1, we return 1, which is the result of the call `length("e")`, and so on. This process of returning from the recursive calls and computing the partial results is called *unwinding the recursion*.

**FIGURE 5.2**  
Trace of  
`length("ace")`



## The Run-Time Stack and Activation Frames

You can also trace a recursive method by showing what Java does when one method calls another. Java maintains a run-time stack, on which it saves new information in the form of an *activation frame*. The activation frame contains storage for the method arguments and any local variables as well as the return address of the instruction that called the method. Whenever a method is called, Java pushes a new activation frame onto the run-time stack and saves this information on the stack. This is done whether or not the method is recursive.

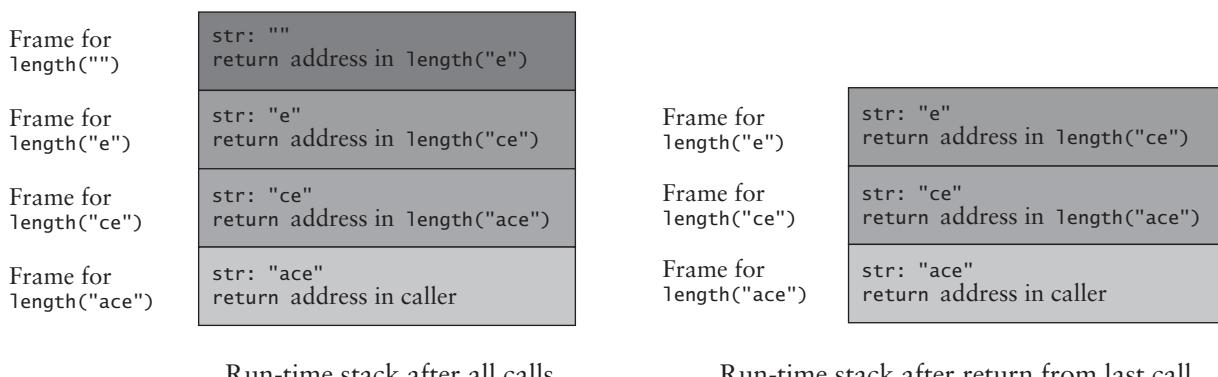
The left side of Figure 5.3 shows the activation frames on the run-time stack after the last recursive call (corresponding to `length("")`) resulting from an initial call to `length("ace")`. At any given time, only the frame at the top of the stack is accessible, so its argument values will be used when the method instructions execute. When the `return` statement executes, control will be passed to the instruction at the specified return address, and this frame will be popped from the stack (Figure 5.3, right). The activation frame corresponding to the next-to-last call (`length("e")`) is now accessible.

You can think of the run-time stack for a sequence of calls to a recursive method as an office tower in which an employee on each floor has the same list of instructions.<sup>1</sup> The employee in the bottom office carries out part of the instructions on the list, calls the employee in the office above, and is put on hold. The employee in the office above starts to carry out the list of instructions, calls the employee in the next higher office, is put on hold, and so on. When the employee on the top floor is called, that employee carries out the list of instructions to completion and then returns an answer to the employee below. The employee below then resumes carrying out the list of instructions and returns an answer to the employee on the next lower floor, and so on, until an answer is returned to the employee in the bottom office, who then resumes carrying out the list of instructions.

To make the flow of control easier to visualize, we will draw the activation frames from the top of the page down (see Figure 5.4). For example, the activation frame at the top, which would actually be at the bottom of the run-time stack, represents the first call to the recursive method. The downward-pointing arrows connect each statement that calls a method with the

**FIGURE 5.3**

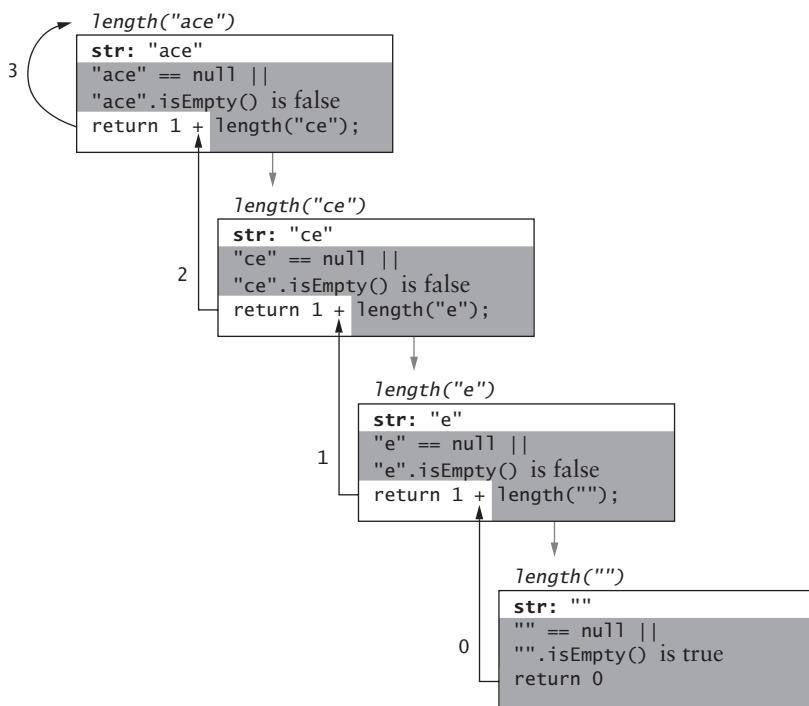
Run-Time Stack before and after Removal of Frame for `length("")`



<sup>1</sup>Analogy suggested by Rich Pattis, University of California, Irvine, CA.

**FIGURE 5.4**

Trace of  
`length("ace")`  
Using Activation  
Frames



frame for that particular execution of the method. The upward-pointing arrows show the return point from each lower-level call with the value returned alongside the arrow. For each frame, the return point is to the addition operator in the statement `return 1 + length(...);`. For each frame, the code in the gray screen is executed prior to the creation of the next activation frame; the rest of the code shown is executed after the return.

## EXERCISES FOR SECTION 5.1

### SELF-CHECK

1. Trace the execution of the call `mystery(4)` for the following recursive method using the technique shown in Figure 5.2. What does this method do?
 

```
public static mystery(int n) {
    if (n == 0)
        return 0;
    else
        return n * n + mystery(n - 1);
}
```
2. Answer Exercise 1 above using activation frames.
3. Trace the execution of `printChars("tic")` (Example 5.2) using activation frames.
4. Trace the execution of `printCharsReverse("toc")` using activation frames.
5. Prove that the `printChars` method is correct.
6. Trace the execution of `length("tictac")` using a diagram like Figure 5.2.

7. Write a recursive algorithm that determines whether a specified target character is present in a string. It should return `true` if the target is present and `false` if it is not. The stopping steps should be

- a string reference to null or a string of length 0, the result is `false`
- the first character in the string is the target, the result is `true`.

The recursive step would involve searching the rest of the string.

## PROGRAMMING

- Write a recursive method `toNumber` that forms the integer sum of all digit characters in a string. For example, the result of `toNumber("3ac4")` would be 7. *Hint:* If `next` is a digit character ('0' through '9'), `Character.isDigit(next)` is true and the numeric value of `next` is `Character.digit(next, 10)`.
- Write a recursive method `stutter` that returns a string with each character in its argument repeated. For example, if the string passed to `stutter` is "hello", `stutter` will return the string "hheelllloo".
- Write a recursive method that implements the recursive algorithm for searching a string in Self-Check Exercise 7. The method heading should be  
`public static boolean searchString(String str, char ch)`



## 5.2 Recursive Definitions of Mathematical Formulas

Mathematicians often use recursive definitions of formulas. These definitions lead very naturally to recursive algorithms.

**EXAMPLE 5.4** The factorial of  $n$ , or  $n!$ , is defined as follows:

$$\begin{aligned}0! &= 1 \\n! &= n \times (n - 1)!\end{aligned}$$

The first formula identifies the base case:  $n$  equal to 0. The second formula is a recursive definition. It leads to the following algorithm for computing  $n!$ .

### Recursive Algorithm for Computing $n!$

- if**  $n$  equals 0
- $n!$  is 1.
- else**
- $n! = n \times (n - 1)!$

To verify the correctness of this algorithm, we see that the base case is solved correctly ( $0!$  is 1). The recursive case makes progress toward the base case because it involves the calculation of a smaller factorial. Also, if we can calculate  $(n - 1)!$ , the recursive case gives us the correct formula for calculating  $n!$ .

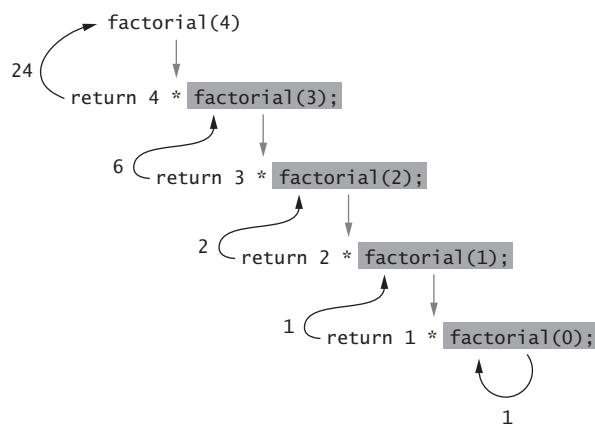
The recursive method follows. The statement

```
return n * factorial(n - 1);
```

implements the recursive case. Each time factorial calls itself, the method body executes again with a different argument value. An initial method call such as factorial(4) will generate four recursive calls, as shown in Figure 5.5.

```
/** Recursive factorial method (in RecursiveMethods.java).
  pre: n >= 0
  @param n The integer whose factorial is being computed
  @return n!
 */
public static int factorial(int n) {
    if (n == 0)
        return 1;
    else
        return n * factorial(n - 1);
}
```

**FIGURE 5.5**  
Trace of  
factorial(4)



## PITFALL

### Infinite Recursion and Stack Overflow

If you call method factorial with a negative argument, you will see that the recursion does not terminate. It will continue forever because the stopping case,  $n$  equals 0, can never be reached, as  $n$  gets more negative with each call. For example, if the original value of  $n$  is  $-4$ , you will make method calls  $\text{factorial}(-5)$ ,  $\text{factorial}(-6)$ ,  $\text{factorial}(-7)$ , and so on. You should make sure that your recursive methods are constructed so that a stopping case is always reached. One way to prevent the infinite recursion in this case would be to change the terminating condition to  $n \leq 0$ . However, this would incorrectly return a value of 1 for  $n!$  if  $n$  is negative. A better solution would be to throw an `IllegalArgumentException` if  $n$  is negative.

If your program does not terminate properly, you may see an extremely long display on the console (if the console is being used to display its results). Eventually the exception `StackOverflowError` will be thrown. This means that the memory area used to store information about method calls (the run-time stack) has been used up because there have been too many calls to the recursive method. Because there is no memory available for this purpose, your program can't execute any more method calls.

---

**EXAMPLE 5.5** Let's develop a recursive method that raises a number  $x$  to a power  $n$ , where  $n$  is positive or zero. You can raise a number to a power by repeatedly multiplying that number by itself. So if we know  $x^k$ , we can get  $x^{k+1}$  by multiplying  $x^k$  by  $x$ . The recursive definition is

$$x^n = x \times x^{n-1}$$

This gives us the recursive case. You should know that any number raised to the power 0 is 1, so the base case is

$$x^0 = 1$$

### Recursive Algorithm for Calculating $x^n$ ( $n \geq 0$ )

1. **if**  $n$  is 0
2.     The result is 1.
3.     **else**
3.         The result is  $x \times x^{n-1}$ .

We show the method next.

```
/** Recursive power method (in RecursiveMethods.java).
pre: n >= 0
@param x The number being raised to a power
@param n The exponent
@return x raised to the power n
*/
public static double power(double x, int n) {
    if (n == 0)
        return 1;
    else
        return x * power(x, n - 1);
}
```

---

**EXAMPLE 5.6** The greatest common divisor (gcd) of two numbers is the largest integer that divides both numbers. For example, the gcd of 20, 15 is 5; the gcd of 36, 24 is 12; the gcd of 36, 18 is 18. The mathematician Euclid devised an algorithm for finding the greatest common divisor (gcd) of two integers,  $m$  and  $n$ , based on the following definition.

### Definition of $\text{gcd}(m, n)$ for $m > n$

$\text{gcd}(m, n) = n$  if  $n$  is a divisor of  $m$

$\text{gcd}(m, n) = \text{gcd}(n, m \% n)$  if  $n$  isn't a divisor of  $m$

This definition states that  $\text{gcd}(m, n)$  is  $n$  if  $n$  divides  $m$ . This is correct because no number larger than  $n$  can divide  $n$ . Otherwise, the definition states that  $\text{gcd}(m, n)$  is the same as  $\text{gcd}(n, m \% n)$ , where  $m \% n$  is the integer remainder of  $m$  divided by  $n$ . Therefore,  $\text{gcd}(20, 15)$  is the same as  $\text{gcd}(15, 5)$ , or 5, because 5 divides 15. This recursive definition leads naturally to a recursive algorithm.

### Recursive Algorithm for Calculating $\text{gcd}(m, n)$ for $m > n$

1. **if**  $n$  is a divisor of  $m$
2.     The result is  $n$ .
3.     **else**
3.         The result is  $\text{gcd}(n, m \% n)$ .

To verify that this is correct, we need to make sure that there is a base case and that it is solved correctly. The base case is “ $n$  is a divisor of  $m$ .” If so, the solution is  $n$  ( $n$  is the gcd), which is correct. Does the recursive case make progress to the base case? It must because both arguments in each recursive call are smaller than in the previous call, and the new second argument is always smaller than the new first argument ( $m \% n$  must be less than  $n$ ). Eventually a divisor will be found, or the second argument will become 1. Since 1 is a base case (1 divides every integer), we have verified that the recursive case makes progress toward the base case.

Next, we show method `gcd`. Note that we do not need a separate clause to handle arguments that initially are not in the correct sequence. This is because if  $m < n$ , then  $m \% n$  is  $m$  and the recursive call will transpose the arguments so that  $m > n$  in the first recursive call.

```
/** Recursive gcd method (in RecursiveMethods.java).
pre: m > 0 and n > 0
@param m The larger number
@param n The smaller number
@return Greatest common divisor of m and n
*/
public static double gcd(int m, int n) {
    if (m % n == 0)
        return n;
    else
        return gcd(n, m % n);
}
```

---

## Tail Recursion versus Iteration

Method `gcd` above is an example of *tail recursion*. In tail recursion, the last thing a method does is to call itself. You may have noticed that there are some similarities between tail recursion and iteration. Both techniques enable us to repeat a compound statement. In iteration, a loop repetition condition in the loop header determines whether we repeat the loop body or exit from the loop. We repeat the loop body while the repetition condition is true. In tail recursion, the condition usually tests for a base case. In a recursive method, we stop the recursion when the base case is reached (the condition is true), and we execute the method body again when the condition is false.

We can always write an iterative solution to any problem that is solvable by recursion. However, the recursive solutions will be easier to conceptualize and should, therefore, lead to methods that are easier to write, read, and debug—all of which are very desirable attributes of code.

---

**EXAMPLE 5.7** In Example 5.4, we wrote the recursive method.

```
public static int factorial(int n) {
    if (n == 0)
        return 1;
    else
        return n * factorial(n - 1);
}
```

It is a straightforward process to turn this method into an iterative one, replacing the `if` statement with a loop, as we show next.

```

/** Iterative factorial method.
 * pre: n >= 0
 * @param n The integer whose factorial is being computed
 * @return n!
 */
public static int factorialIter(int n) {
    int result = 1;
    while (n > 0){
        result *= n;
        n = n - 1;
    }
    return result;
}

```

---

## Efficiency of Recursion

The iterative method `factorialIter` multiplies all integers between 1 and  $n$  to compute  $n!$ . It may be slightly less readable than the recursive method `factorial`, but not much. In terms of efficiency, both algorithms are  $O(n)$  because the number of loop repetitions or recursive calls increases linearly with  $n$ . However, the iterative version is probably faster because the overhead for a method call and return would be greater than the overhead for loop repetition (testing and incrementing the loop control variable). The difference, though, would not be significant. Generally, if it is easier to conceptualize an algorithm using recursion, then you should code it as a recursive method because the reduction in efficiency does not outweigh the advantage of readable code that is easy to debug.

---

**EXAMPLE 5.8** The Fibonacci numbers  $\text{fib}_n$  are a sequence of numbers that were invented to model the growth of a rabbit colony. Therefore, we would expect this sequence to grow very quickly, and it does. For example,  $\text{fib}_{10}$  is 55,  $\text{fib}_{15}$  is 610,  $\text{fib}_{20}$  is 6765, and  $\text{fib}_{25}$  is 75,025 (that's a lot of rabbits!). The definition of this sequence follows:

$$\begin{aligned}\text{fib}_0 &= 0 \\ \text{fib}_1 &= 1 \\ \text{fib}_n &= \text{fib}_{n-1} + \text{fib}_{n-2}\end{aligned}$$

Next, we show a method that calculates the  $n$ th Fibonacci number. The last line codes the recursive case.

```

/** Recursive method to calculate Fibonacci numbers
 * (in RecursiveMethods.java).
 * pre: n >= 0
 * @param n The position of the Fibonacci number being calculated
 * @return The Fibonacci number
 */
public static int fibonacci(int n) {
    if (n == 0)
        return 0;
    else if (n == 1)
        return 1;
    else
        return fibonacci(n - 1) + fibonacci(n - 2);
}

```

Unfortunately, this solution is very inefficient because of multiple calls to `fibonacci` with the same argument. For example, calculating `fibonacci(5)` results in calls to `fibonacci(4)` and `fibonacci(3)`. Calculating `fibonacci(4)` results in calls to `fibonacci(3)` (second call) and also `fibonacci(2)`. Calculating `fibonacci(3)` twice results in two more calls to `fibonacci(2)` (three calls total), and so on (see Figure 5.6).

Because of the redundant method calls, the time required to calculate `fibonacci(n)` increases exponentially with  $n$ . For example, if  $n$  is 100, there are approximately  $2^{100}$  activation frames. This number is approximately  $10^{30}$ . If you could process one million activation frames per second, it would still take  $10^{24}$  seconds, which is approximately  $3 \times 10^{16}$  years. However, it is possible to write recursive methods for computing Fibonacci numbers that have  $O(n)$  performance. We show one such method next.

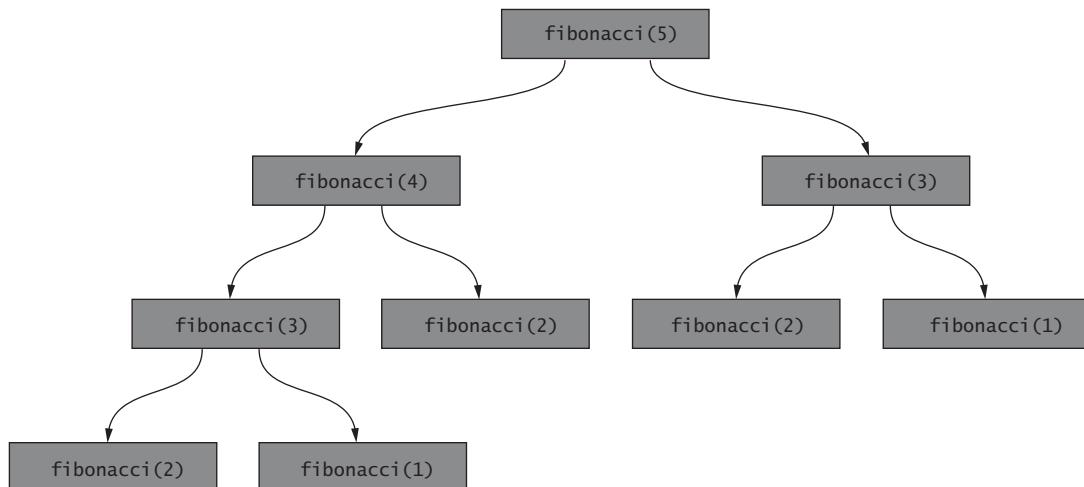
```
/** Recursive O(n) method to calculate Fibonacci numbers
 * in RecursiveMethods.java.
 * pre: n >= 1
 * @param fibCurrent The current Fibonacci number
 * @param fibPrevious The previous Fibonacci number
 * @param n The count of Fibonacci numbers left to calculate
 * @return The value of the Fibonacci number calculated so far
 */
private static int fibo(int fibCurrent, int fibPrevious, int n) {
    if (n == 1)
        return fibCurrent;
    else
        return fibo(fibCurrent + fibPrevious, fibCurrent, n - 1);
}
```

Unlike method `fibonacci`, method `fibo` does not follow naturally from the recursive definition of the Fibonacci sequence. In the method `fibo`, the first argument is always the current Fibonacci number and the second argument is the previous one. We update these values for each new call. When  $n$  is 1 (the base case), we have calculated the required Fibonacci number, so we return its value (`fibCurrent`). The recursive case

```
    return fibo(fibCurrent + fibPrevious, fibCurrent, n - 1);
```

passes the sum of the current Fibonacci number and the previous Fibonacci number to the first parameter (the new value of `fibCurrent`); it passes the current Fibonacci number to the second parameter (the new value of `fibPrevious`); and it decrements  $n$ , making progress toward the base case.

**FIGURE 5.6**  
Method Calls Resulting from `fibonacci(5)`

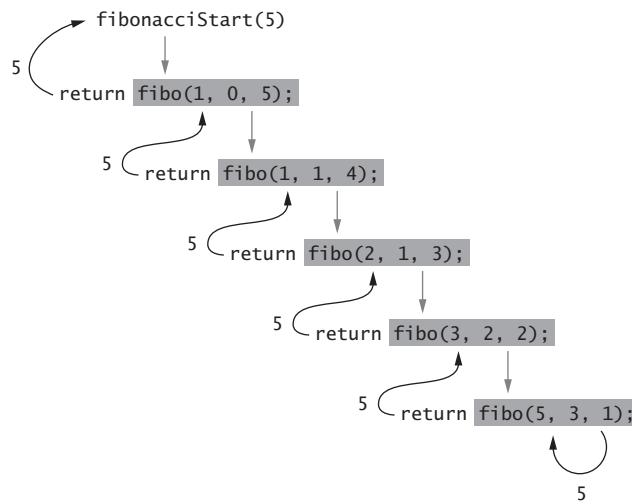


To start this method executing, we need the following *wrapper method*, which is not recursive. This method is called a wrapper method because its main purpose is to call the recursive method and return its result. Its parameter,  $n$ , specifies the position in the Fibonacci sequence of the number we want to calculate. After testing for the special case  $n$  equals 0, it calls the recursive method `fibo`, passing the first Fibonacci number as its first argument and  $n$  as its third.

```
/** Wrapper method for calculating Fibonacci numbers
 * (in RecursiveMethods.java).
 * pre: n >= 0
 * @param n The position of the desired Fibonacci number
 * @return The value of the nth Fibonacci number
 */
public static int fibonacciStart(int n) {
    if (n == 0)
        return 0;
    else
        return fibo(1, 0, n);
}
```

Figure 5.7 traces the execution of the method call `fibonacciStart(5)`. Note that the first arguments for the method calls to `fibo` form the sequence 1, 1, 2, 3, 5, which is the Fibonacci sequence. Also note that the result of the first return (5) is simply passed on by each successive return. That is because the recursive case does not specify any operations other than returning the result of the next call. Note that the method `fibo` is an example of tail recursion.

.....  
**FIGURE 5.7**  
Trace of  
`fibonacciStart(5)`



## EXERCISES FOR SECTION 5.2

### SELF-CHECK

1. Does the recursive algorithm for raising  $x$  to the power  $n$  work for negative values of  $n$ ? Does it work for negative values of  $x$ ? Indicate what happens if it is called for each of these cases.
2. Trace the execution of `fibonacciStart(5)` using activation frames.
3. Trace the execution of the following using activation frames.

```
gcd(33, 12)
gcd(12, 33)
gcd(11, 5)
```

4. For each of the following method calls, show the argument values in the activation frames that would be pushed onto the run-time stack.
  - a. `gcd(6, 21)`
  - b. `factorial(5)`
  - c. `gcd(31, 7)`
  - d. `fibonacci(6)`
  - e. `fibonacciStart(7)`
5. See for what value of  $n$  the method `fibonacci` begins to take a long time to run on your computer (over 1 minute). Compare the performance of `fibonacciStart` and `fibo` for this same value.

### PROGRAMMING

1. Write a recursive method for raising  $x$  to the power  $n$  that works for negative  $n$  as well as positive  $n$ . Use the fact that  $x^{-n} = \frac{1}{x^n}$ .
2. Modify the factorial method to throw an `IllegalArgumentException` if  $n$  is negative.
3. Modify the Fibonacci method to throw an illegal argument exception if its argument is less than or equal to zero.
4. Write a class that has an iterative method for calculating Fibonacci numbers. Use an array that saves each Fibonacci number as it is calculated. Your method should take advantage of the existence of this array so that subsequent calls to the method simply retrieve the desired Fibonacci number if it has been calculated. If not, start with the largest Fibonacci number in the array rather than repeating all calculations.



## 5.3 Recursive Array Search

Searching an array is an activity that can be accomplished using recursion. The simplest way to search an array is a *linear search*. In a linear search, we examine one array element at a time, starting with the first element or the last element, to see whether it matches the target. The array element we are seeking may be anywhere in the array, so on average we will examine  $\frac{n}{2}$  items to find the target if it is in the array. If it is not in the array, we will have to examine all  $n$  elements (the worst case). This means linear search is an  $O(n)$  algorithm.

### Design of a Recursive Linear Search Algorithm

Let's consider how we might write a recursive algorithm for an array search that returns the subscript of a target item.

The base case would be an empty array. If the array is empty, the target cannot be there, so the result should be  $-1$ . If the array is not empty, we will assume that we can examine just the first element of the array, so another base case would be when the first array element matches the target. If so, the result should be the subscript of the first array element.

The recursive step would be to search the rest of the array, excluding the first element. So our recursive step should search for the target starting with the current second array element, which will become the first element in the next execution of the recursive step. The algorithm follows.

## Algorithm for Recursive Linear Array Search

1. if the array is empty
2. The result is -1.
3. else if the first element matches the target  
The result is the subscript of the first element.
4. else  
Search the array excluding the first element and return the result.

## Implementation of Linear Search

The following method, `linearSearch` (part of class `RecursiveMethods`), shows the linear search algorithm.

```
/** Recursive linear search method (in RecursiveMethods.java).
 * @param items The array being searched
 * @param target The item being searched for
 * @param posFirst The position of the current first element
 * @return The subscript of target if found; otherwise -1
 */
private static int linearSearch(Object[] items,
                                Object target, int posFirst) {
    if (posFirst == items.length)
        return -1;
    else if (target.equals(items[posFirst]))
        return posFirst;
    else
        return linearSearch(items, target, posFirst + 1);
}
```

The method parameter `posFirst` represents the subscript of the current first element. The first condition tests whether the array left to search is empty. The condition (`posFirst == items.length`) is `true` when the subscript of the current first element is beyond the bounds of the array. If so, method `linearSearch` returns -1. The statement

```
return linearSearch(items, target, posFirst + 1);
```

implements the recursive step; it increments `posFirst` to exclude the current first element from the next search.

To search an array `x` for `target`, you could use the method call

```
RecursiveMethods.linearSearch(x, target, 0)
```

However, since the third argument would always be 0, we can define a nonrecursive wrapper method (also called `linearSearch`) that has just two parameters: `items` and `target`.

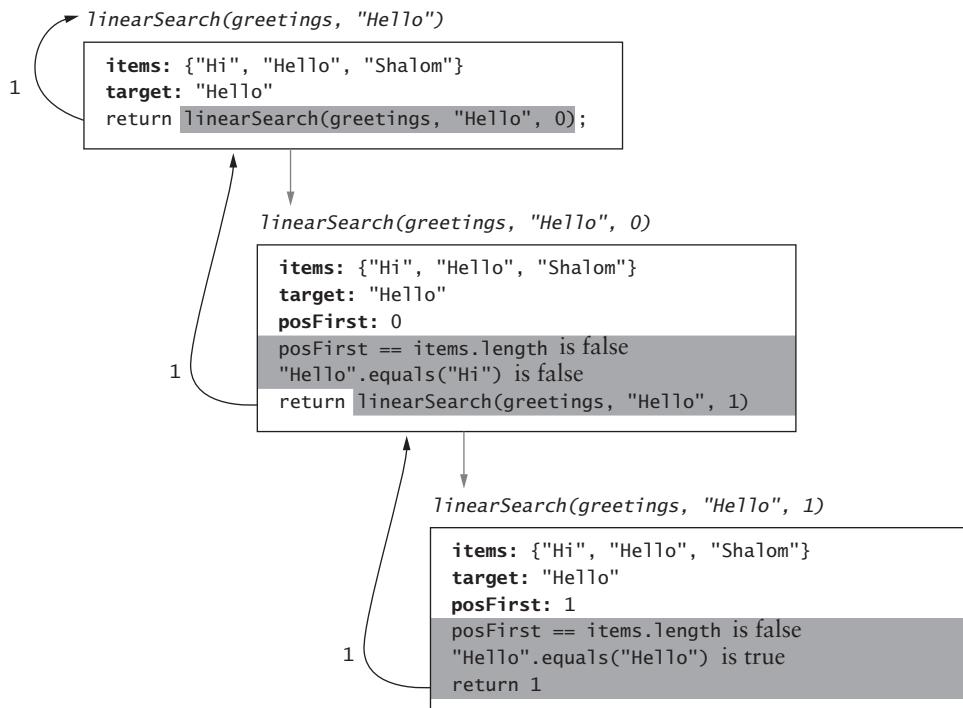
```
/** Wrapper for recursive linear search method (in
 * RecursiveMethods.java).
 * @param items The array being searched
 * @param target The object being searched for
 * @return The subscript of target if found; otherwise -1
 */
public static int linearSearch(Object[] items, Object target) {
    return linearSearch(items, target, 0);
}
```

The sole purpose of this method is to call the recursive method, passing on its arguments with 0 as a third argument, and return its result. This method definition overloads the previous one, which has private visibility.

Figure 5.8 traces the execution of the call to `linearSearch` in the second statement.

```
String[] greetings = {"Hi", "Hello", "Shalom"};
int posHello = linearSearch(greetings, "Hello");
The value returned to posHello will be 1.
```

**FIGURE 5.8**  
Trace of `linearSearch(greetings, "Hello")`



## Design of a Binary Search Algorithm

A second approach to searching an array is called *binary search*. Binary search can be performed only on an array that has been sorted. In binary search, the stopping cases are the same as for linear search:

- When the array is empty.
- When the array element being examined matches the target.

However, rather than examining the last array element, binary search compares the “middle” element of the array to the target. If there is a match, it returns the position of the middle element. Otherwise, because the array has been sorted, we know with certainty which half of the array must be searched to find the target. We then can exclude the other half of the array (not just one element as with linear search). The binary search algorithm (first introduced in Section 5.1) follows.

### Binary Search Algorithm

1. **if** the array is empty
2.     Return -1 as the search result.
- else if** the middle element matches the target

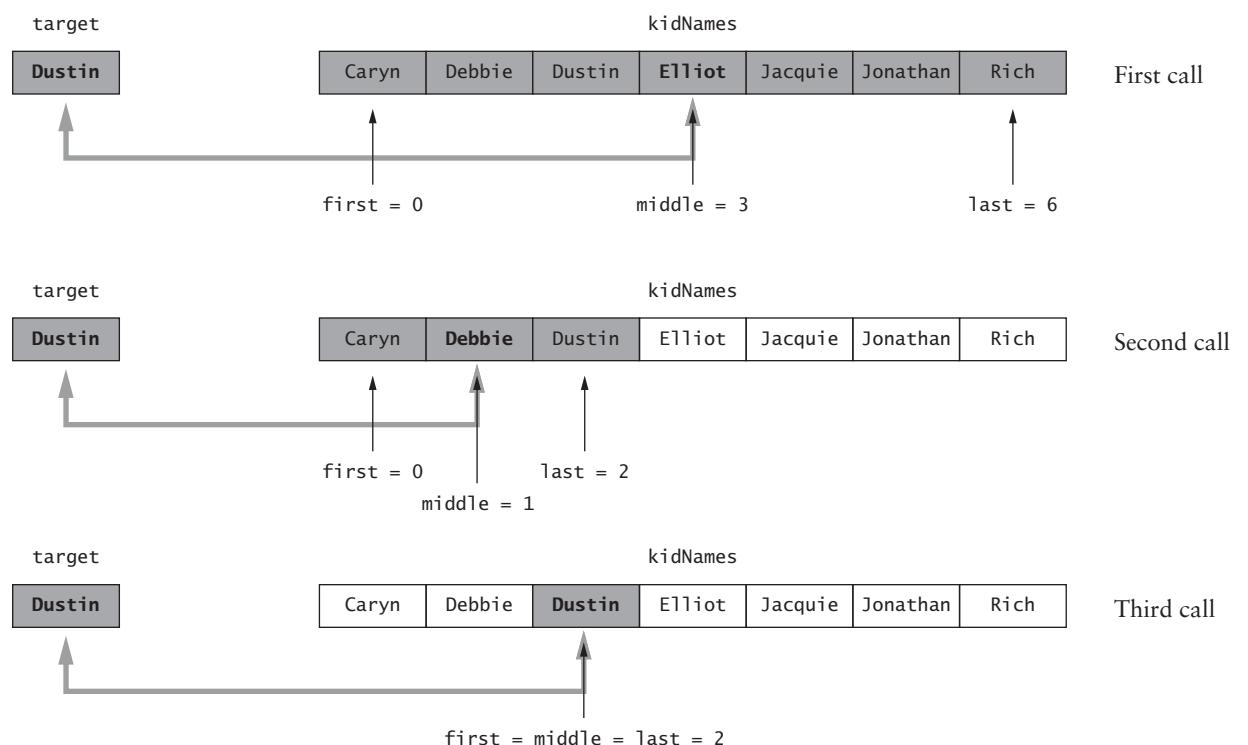
3. Return the subscript of the middle element as the result.
  - else if** the target is less than the middle element
  4. Recursively search the array elements before the middle element and return the result.
- else**
5. Recursively search the array elements after the middle element and return the result.

Figure 5.9 illustrates binary search for an array with seven elements. The shaded array elements are the ones that are being searched each time. The array element in bold is the one that is being compared to the target. In the first call, we compare "Dustin" to "Elliot". Because "Dustin" is smaller, we need to search only the part of the array before "Elliot" (consisting of just three candidates). In the second call, we compare "Dustin" to "Debbie". Because "Dustin" is larger, we need to search only the shaded part of the array after "Debbie" (consisting of just one candidate). In the third call, we compare "Dustin" to "Dustin", and the subscript of "Dustin" (2) is our result. If there were no match at this point (e.g., the array contained "Duncan" instead of "Dustin"), the array of candidates to search would become an empty array.

## Efficiency of Binary Search

Because we eliminate at least half of the array elements from consideration with each recursive call, binary search is an  $O(\log n)$  algorithm. To verify this, an unsuccessful search of an array of size 16 could result in our searching arrays of size 16, 8, 4, 2, and 1 to determine that the

**FIGURE 5.9**  
Binary Search for "Dustin"



target was not present. Thus, an array of size 16 requires a total of 5 probes in the worst case (16 is  $2^4$ , so 5 is  $\log_2 16 + 1$ ). If we double the array size, we would need to make only 6 probes for an array of size 32 in the worst case (32 is  $2^5$ , so 6 is  $\log_2 32 + 1$ ). The advantages of binary search become even more apparent for larger arrays. For an array with 32,768 elements, the maximum number of probes required would be 16 ( $\log_2 32,768$  is 15), and if we expand the array to 65,536 elements, we would increase the number of probes required only to 17.

## The Comparable Interface

We introduced the Comparable interface in Section 2.8. Classes that implement this interface must define a `compareTo` method that enables its objects to be compared in a standard way. The method `compareTo` returns an integer whose value indicates the relative ordering of the two objects being compared (as described in the `@return` tag below). If the target is type Comparable, we can apply its `compareTo` method to compare the target to the objects stored in the array. T represents the type of the object being compared.

```
/** Instances of classes that realize this interface can be
   compared.
  @param <T> The type of object this object can be compared to.
 */
public interface Comparable<T> {
    /** Method to compare this object to the argument object.
       @param obj The argument object
       @return Returns a negative integer if this object < obj;
               zero if this object equals obj;
               a positive integer if this object > obj
    */
    int compareTo(T obj);
}
```

## Implementation of Binary Search

Listing 5.1 shows a recursive implementation of the binary search algorithm and its nonrecursive wrapper method. The parameters `first` and `last` are the subscripts of the first element and last element in the array being searched. For the initial call to the recursive method from the wrapper method, `first` is 0 and `last` is `items.length - 1`. The parameter `target` is type Comparable.

The condition (`first > last`) becomes true when the list of candidates is empty. The statement

```
int middle = (first + last) / 2;
```

computes the subscript of the “middle” element in the current array (midway between `first` and `last`).

The statement

```
int compResult = target.compareTo(items[middle]);
```

saves the result of comparing the target to the middle element of the array. If the result is 0 (a match), the subscript `middle` is returned. If the result is negative, the recursive step

```
return binarySearch(items, target, first, middle - 1);
```

returns the result of searching the part of the current array before the middle item (with subscripts `first` through `middle - 1`). If the result is positive, the recursive step

```
return binarySearch(items, target, middle + 1, last);
```

returns the result of searching the part of the current array after the middle item (with subscripts `middle + 1` through `last`).

**LISTING 5.1**Method `binarySearch`

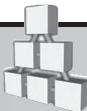
```
.....
LISTING 5.1
Method binarySearch

/** Recursive binary search method (in RecursiveMethods.java).
    @param <T> The item type
    @param items The array being searched
    @param target The object being searched for
    @param first The subscript of the first element
    @param last The subscript of the last element
    @return The subscript of target if found; otherwise -1.
 */
private static <T> int binarySearch(T[] items, Comparable<T> target,
                                    int first, int last) {
    if (first > last)
        return -1;      // Base case for unsuccessful search.
    else {
        int middle = (first + last) / 2; // Next probe index.
        int compResult = target.compareTo(items[middle]);
        if (compResult == 0)
            return middle; // Base case for successful search.
        else if (compResult < 0)
            return binarySearch(items, target, first, middle - 1);
        else
            return binarySearch(items, target, middle + 1, last);
    }
}

/** Wrapper for recursive binary search method (in RecursiveMethods.java).
    @param <T> The item type.
    @param items The array being searched
    @param target The object being searched for
    @return The subscript of target if found; otherwise -1.
 */
public static <T> int binarySearch(T[] items, Comparable<T> target) {
    return binarySearch(items, target, 0, items.length - 1);
}

```

Figure 5.10 traces the execution of `binarySearch` for the array shown in Figure 5.9. The parameter `items` always references the same array; however, the pool of candidates changes with each call.



## SYNTAX Declaring a Generic Method

### FORM:

`methodModifiers <genericParameters> returnType methodName(methodParameters)`

### EXAMPLE

`public static <T> int binarySearch(T[] items, Comparable<T> target)`

### MEANING

To declare a generic method, list the `genericParameters` inside the symbol pair `<>` and between the `methodModifiers` (e.g., `public static`) and the `returnType`. The `genericParameters` can then be used in the specification of the `methodParameters` and in the method body.

**FIGURE 5.10**Trace of `binarySearch(kidNames, "Dustin")`

## Testing Binary Search

To test the binary search algorithm, you must test arrays with an even number of elements and arrays with an odd number of elements. You must also test arrays that have duplicate items. Each array must be tested for the following cases:

- The target is the element at each position of the array, starting with the first position and ending with the last position.
- The target is less than the smallest array element.
- The target is greater than the largest array element.
- The target is a value between each pair of items in the array.

## Method Arrays.binarySearch

The Java API class `Arrays` contains a `binarySearch` method. It can be called with sorted arrays of primitive types or with sorted arrays of objects. If the objects in the array are not mutually comparable or if the array is not sorted, the results are undefined. If there are multiple copies of the target value in the array, there is no guarantee as to which one will be found. This is the same as for our `binarySearch` method. The method throws a `ClassCastException` if the target is not comparable to the array elements (e.g., if the target is type `Integer` and the array elements are type `String`).

---

## EXERCISES FOR SECTION 5.3

### SELF-CHECK

1. For the array shown in Figure 5.9, show the values of `first`, `last`, `middle`, and `compResult` in successive frames when searching for a target of "Rich"; when searching for a target of "Alice"; and when searching for a target of "Daryn".
2. How many elements will be compared to `target` for an unsuccessful binary search in an array of 1000 items? What is the answer for 2000 items?
3. If there are multiple occurrences of the target item in an array, what can you say about the subscript value that will be returned by `linearSearch`? Answer the same question for `binarySearch`.
4. Write a recursive algorithm to find the largest value in an array of integers.
5. Write a recursive algorithm that searches a string for a target character and returns the position of its first occurrence if it is present or `-1` if it is not.

### PROGRAMMING

1. Write a recursive method to find the sum of all values stored in an array of integers.
2. Write a recursive linear search method with a recursive step that finds the last occurrence of a target in an array, not the first. You will need to modify the linear search method so that the last element of the array is always tested, not the first. You will need to pass the current length of the array as an argument.
3. Implement the method for Self-Check Exercise 4. You will need to keep track of the largest value found so far through a method parameter.
4. Implement the method for Self-Check Exercise 5. You will need to keep track of the current position in the string through a method parameter.



## 5.4 Recursive Data Structures

Computer scientists often encounter data structures that are defined recursively. A *recursive data structure* is one that has another version of itself as a component. We will define the tree data structure as a recursive data structure in Chapter 6, but we can also define a linked list, described in Chapter 2, as a recursive data structure. In this section, we demonstrate that recursive methods provide a very natural mechanism for processing recursive data structures. The first language developed for artificial intelligence research was a recursive language designed expressly for LISP Processing and therefore called LISP.

## Recursive Definition of a Linked List

The following definition implies that a nonempty linked list is a collection of nodes such that each node references another linked list consisting of the nodes that follow it in the list. The last node references an empty list.

A linked list is empty, or it consists of a node, called the list head, that stores data and a reference to a linked list.

### Class LinkedListRec

We will define a class `LinkedListRec<E>` that implements several list operations using recursive methods. The class `LinkedListRec<E>` has a private inner class called `Node<E>`, which is defined in Listing 2.1. A `Node<E>` object has attributes `data` (type `E`) and `next` (type `Node`). Class `LinkedListRec<E>` has a single data field `head` (data type `Node<E>`).

```
/** A recursive linked list class with recursive methods. */
public class LinkedListRec<E> {
    /** The list head */
    private Node<E> head;
    // Insert inner class Node<E> here. See Listing 2.1.
    .
    .
}
```

We will write the following recursive methods: `size` (returns the size), `toString` (represents the list contents as a string), `add` (adds an element to the end of the list), and `replace` (replaces one object in a list with another). We code each operation using a pair of methods: a public wrapper method that calls a private recursive method. To perform a list operation, you apply a wrapper method to an instance of class `LinkedListRec`.

#### Method size

The method `size` returns the size of a linked list and is similar to the method `length` defined earlier for a string. The recursive method returns 0 if the list is empty (`head == null` is true). Otherwise, the statement

```
return 1 + size(head.next);
```

returns 1 plus the size of the rest of the list that is referenced by `head.next`.

The wrapper method calls the recursive method, passing the list head as an argument, and returns the value returned by the recursive method. In the initial call to the recursive method, `head` will reference the first list node. In each subsequent call, `head` will reference the successor of the node that it currently references.

```
/** Finds the size of a list.
 * @param head The head of the current list
 * @return The size of the current list
 */
private int size(Node<E> head) {
    if (head == null)
        return 0;
    else
        return 1 + size(head.next);
}

/** Wrapper method for finding the size of a list.
 * @return The size of the list
 */
public int size() {
    return size(head);
}
```

## Method `toString`

The method `toString` returns a string representation of a linked list. The recursive method is very similar to the method `size`. The statement

```
    return head.data + "\n" + toString(head.next);
```

appends the data in the current list `head` to the string representation of the rest of the list. The line space character is inserted after each list item. If the list contains the elements "hat", "55", and "dog", the string result would be "hat\n55\ndog\n".

```
/** Returns the string representation of a list.
 * @param head The head of the current list
 * @return The state of the current list
 */
private String toString(Node<E> head) {
    if (head == null)
        return "";
    else
        return head.data + "\n" + toString(head.next);
}

/** Wrapper method for returning the string representation of a list.
 * @return The string representation of the list
 */
public String toString() {
    return toString(head);
}
```

## Method `replace`

The method `replace` replaces each occurrence of an object in a list (parameter `oldObj`) with a different object (parameter `newObj`). The `if` statement in the recursive method is different from what we are used to. The method does nothing for the base case of an empty list. If the list is not empty, the `if` statement

```
if (oldObj.equals(head.data))
    head.data = newObj;
```

tests whether the item in the current list `head` matches `oldObj`. If so, it stores `newObj` in the current list `head`. Regardless of whether or not a replacement is performed, the method `replace` is called recursively to process the rest of the list.

```
/** Replaces all occurrences of oldObj with newObj.
 * post: Each occurrence of oldObj has been replaced by newObj.
 * @param head The head of the current list
 * @param oldObj The object being removed
 * @param newObj The object being inserted
 */
private void replace(Node<E> head, E oldObj, E newObj) {
    if (head != null) {
        if (oldObj.equals(head.data))
            head.data = newObj;
        replace(head.next, oldObj, newObj);
    }
}

/** Wrapper method for replacing oldObj with newObj.
 * post: Each occurrence of oldObj has been replaced by newObj.
 * @param oldObj The object being removed
 * @param newObj The object being inserted
 */
public void replace(E oldObj, E newObj) {
    replace(head, oldObj, newObj);
}
```

## Method add

You can use the `add` method to add nodes to an existing list. You can also use it to build a list by adding new nodes to the end of an initially empty list.

The `add` methods have two features that are different from what we have seen before. The wrapper method tests for an empty list (`head == null` is `true`), and it calls the recursive `add` method only if the list is not empty. If the list is empty, the wrapper `add` method creates a new node, which is referenced by the data field `head`, and stores the first list item in this node.

```
/** Adds a new node to the end of a list.
 * @param head The head of the current list
 * @param data The data for the new node
 */
private void add(Node<E> head, E data) {
    // If the list has just one element, add to it.
    if (head.next == null)
        head.next = new Node<E>(data);
    else
        add(head.next, data);           // Add to rest of list.
}

/** Wrapper method for adding a new node to the end of a list.
 * @param data The data for the new node
 */
public void add(E data) {
    if (head == null)
        head = new Node<E>(data);   // List has 1 node.
    else
        add(head, data);
}
```

For each node referenced by argument `head`, the recursive method tests to see whether the node referenced by argument `head` is the last node in the list (`head.next` is `null`). If so, the method `add` then resets `head.next` to reference a new node that contains the data being inserted.



### PITFALL

#### Testing for an Empty List Instead of Testing for the Last List Node

In the recursive method `add`, we test whether `head.next` is `null`. This condition is true when `head` references a list with just one node. We then reset its `next` field to reference a new node. If we tested whether `head` was `null` (an empty list) and then executed the statement

```
head = new Node<E>(data);
```

this would have no effect on the original list. The local reference `head` would be changed to reference the new node, but this node would not be connected to a node in the original list.

## Removing a List Node

One of the reasons for using linked lists is that they enable easy insertion and removal of nodes. We show how to do removal next and leave insertion as an exercise. In the following recursive method `remove`, the first base case returns `false` if the list is empty. The second base case determines whether the list `head` should be removed by comparing its data field to `outData`. If there is a match, the assignment statement removes the list `head` by connecting its

predecessor (referenced by `pred`) to the successor of the list head. For this case, method `remove` returns `true`. The recursive case applies `remove` to the rest of the list. In the next execution of the recursive method, the current list head will be referenced by `pred`, and the successor of the current list head will be referenced by `head`.

```
/** Removes a node from a list.
 * post: The first occurrence of outData is removed.
 * @param head The head of the current list
 * @param pred The predecessor of the list head
 * @param outData The data to be removed
 * @return true if the item is removed
 *         and false otherwise
 */
private boolean remove(Node<E> head, Node<E> pred, E outData) {
    if (head == null) // Base case - empty list.
        return false;
    else if (head.data.equals(outData)) { // 2nd base case.
        pred.next = head.next; // Remove head.
        return true;
    } else
        return remove(head.next, head, outData);
}
```

The following wrapper method takes care of the special case where the node to be removed is at the head of the list. The first condition returns `false` if the list is empty. The second condition removes the list head and returns `true` if the list head contains the data to be removed. The `else` clause calls the recursive `remove` method. In the first execution of the recursive method, `head` will reference the actual second node and `pred` will reference the actual first node.

```
/** Wrapper method for removing a node (in LinkedListRec).
 * post: The first occurrence of outData is removed.
 * @param outData The data to be removed
 * @return true if the item is removed,
 *         and false otherwise
 */
public boolean remove(E outData) {
    if (head == null)
        return false;
    else if (head.data.equals(outData)) {
        head = head.next;
        return true;
    } else
        return remove(head.next, head, outData);
}
```

---

## EXERCISES FOR SECTION 5.4

### SELF-CHECK

1. Describe the result of executing each of the following statements:

```
LinkedListRec<String> aList = new LinkedListRec<String>();
aList.add("bye");
aList.add("hello");
System.out.println(aList.size() + ", " + aList.toString());
aList.replace("hello", "welcome");
aList.add("OK");
aList.remove("bye");
aList.remove("hello");
System.out.println(aList.size() + ", " + aList.toString());
```

2. Trace each call to a `LinkedListRec` method in Exercise 1 above.
3. Write a recursive algorithm for method `insert(E obj, int index)` where `index` is the position of the insertion.
4. Write a recursive algorithm for method `remove(int index)` where `index` is the position of the item to be removed.

### PROGRAMMING

1. Write an `equals` method for the `LinkedListRec` class that compares this `LinkedListRec` object to one specified by its argument. Two lists are equal if they have the same number of nodes and store the same information at each node. Don't use the `size` method.
2. Write a `search` method that returns `true` if its argument is stored as the data field of a `LinkedListRec` node and returns `false` if its argument is not stored in any node.
3. Write a recursive method `insertBefore` that inserts a specified data object before the first occurrence of another specified data object. For example, the method call `aList.insertBefore(target, inData)` would insert the object referenced by `inData` in a new node just before the first node of `aList` that stores a reference to `target` as its data.
4. Write a recursive method `reverse` that reverses the elements in a linked list.
5. Code method `insert` in Self-Check Exercise 3.
6. Code method `remove` in Self-Check Exercise 4.



## 5.5 Problem Solving with Recursion

In this section, we discuss recursive solutions to two problems. Our recursive solutions will break each problem up into multiple smaller versions of the original problem. Both problems are easier to solve using recursion because recursive thinking enables us to split each problem into more manageable subproblems. They would both be much more difficult to solve without recursion.

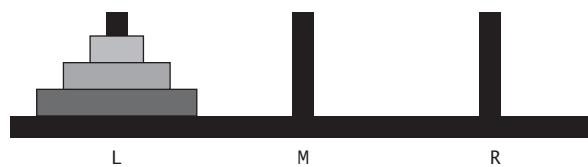
### CASE STUDY Towers of Hanoi

**Problem** You may be familiar with a version of this problem that is sold as a child's puzzle. There is a board with three pegs and three disks of different sizes (see Figure 5.11). The goal of the game is to move the three disks from the peg where they have been placed (largest disk on the bottom, smallest disk on the top) to one of the empty pegs, subject to the following constraints:

- Only the top disk on a peg can be moved to another peg.
- A larger disk cannot be placed on top of a smaller disk.

**Analysis** We can solve this problem by displaying a list of moves to be made. The problem inputs will be the number of disks to move, the starting peg, the destination peg, and the temporary peg. Table 5.1 shows the problem inputs and outputs. We will write a class `Tower` that contains a method `showMoves` that builds a string with all the moves.

**FIGURE 5.11**  
Children's Version of Towers of Hanoi



**TABLE 5.1**  
Inputs and Outputs for Towers of Hanoi Problem

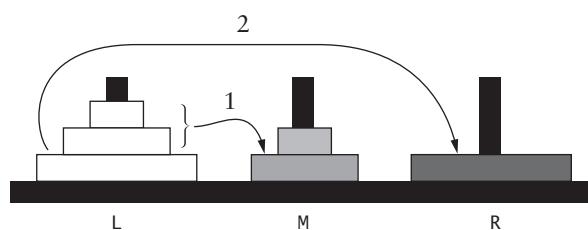
Problem Inputs
Number of disks (an integer)
Letter of starting peg: L (left), M (middle), or R (right)
Letter of destination peg: (L, M, or R), but different from starting peg
Letter of temporary peg: (L, M, or R), but different from starting peg and destination peg
Problem Outputs
A list of moves

**Design** We still need to determine a strategy for making a move. If we examine the situation in Figure 5.11 (all three disks on the L peg), we can derive a strategy to solve it. If we can figure out how to move the top two disks to the M peg (a two-disk version of the original problem), we can then place the bottom disk on the R peg (see Figure 5.12). Now all we need to do is move the two disks on the M peg to the R peg. If we can solve both of these two-disk problems, then the three-disk problem is also solved.

#### Solution to Two-Disk Problem: Move Three Disks from Peg L to Peg R

1. Move the top two disks from peg L to peg M.
2. Move the bottom disk from peg L to peg R.
3. Move the top two disks from peg M to peg R.

**FIGURE 5.12**  
Towers of Hanoi After the First Two Steps in Solution of the Three-Disk Problem



We can split the solution to each two-disk problem into three problems involving single disks. We solve the second two-disk problem next; the solution to the first one (move the top two disks from peg L to peg M) is quite similar.

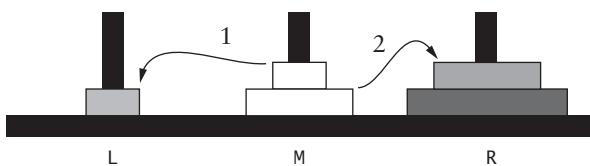
### Solution to Two-Disk Problem: Move Top Two Disks from Peg M to Peg R

1. Move the top disk from peg M to peg L.
2. Move the bottom disk from peg M to peg R.
3. Move the top disk from peg L to peg R.

In Figure 5.13, we show the pegs after Steps 1 and 2. When Step 3 is completed, the three pegs will be on peg R.

**FIGURE 5.13**

Towers of Hanoi after First Two Steps in Solution of the Two-Disk Problem



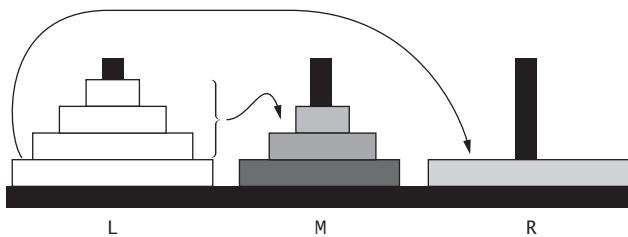
In a similar way, we can split a four-disk problem into two three-disk problems. Figure 5.14 shows the pegs after the top three disks have been moved from peg L to peg M. Because we know how to solve three-disk problems, we can also solve four-disk problems.

### Solution to Four-Disk Problem: Move Four Disks from Peg L to Peg R

1. Move the top three disks from peg L to peg M.
2. Move the bottom disk from peg L to peg R.
3. Move the top three disks from peg M to peg R.

**FIGURE 5.14**

Towers of Hanoi after the First Two Steps in Solution of the Four-Disk Problem



Next, we show a general recursive algorithm for moving  $n$  disks from one of the three pegs to a different peg.

### Recursive Algorithm for $n$ -Disk Problem: Move $n$ Disks from the Starting Peg to the Destination Peg

1. **if**  $n$  is 1
2.     Move disk 1 (the smallest disk) from the starting peg to the destination peg.
3. **else**
4.     Move the top  $n - 1$  disks from the starting peg to the temporary peg (neither starting nor destination peg).
5.     Move disk  $n$  (the disk at the bottom) from the starting peg to the destination peg.
6.     Move the top  $n - 1$  disks from the temporary peg to the destination peg.

The stopping case is the one-disk problem. The recursive step enables us to split the  $n$ -disk problem into two ( $n - 1$ ) disk problems and a single-disk problem. Each problem has a different starting peg and destination peg.

Our recursive solution method `showMoves` will display the solution as a list of disk moves. For each move, we show the number of the disk being moved and its starting and destination pegs. For example, for the two-disk problem shown earlier (move two disks from the middle peg, M, to the right peg, R), the list of moves would be

```
Move disk 1 from peg M to peg L
Move disk 2 from peg M to peg R
Move disk 1 from peg L to peg R
```

The method `showMoves` must have the number of disks, the starting peg, the destination peg, and the temporary peg as its parameters. If there are  $n$  disks, the bottom disk has number  $n$  (the top disk has number 1). Table 5.2 describes the method required for class `TowersOfHanoi`.

**Implementation** Listing 5.2 shows class `TowersOfHanoi`. In method `showMoves`, the recursive step

```
return showMoves(n - 1, startPeg, tempPeg, destPeg)
    + "Move disk " + n + " from peg " + startPeg
    + " to peg " + destPeg + "\n"
    + showMoves(n - 1, tempPeg, destPeg, startPeg);
```

**TABLE 5.2**  
Class `TowersOfHanoi`

Method	Behavior
<code>public String showMoves(int n, char startPeg, char destPeg, char tempPeg)</code>	Builds a string containing all moves for a game with $n$ disks on <code>startPeg</code> that will be moved to <code>destPeg</code> using <code>tempPeg</code> for temporary storage of disks being moved

returns the string formed by concatenating the list of moves for the first ( $n - 1$ )-disk problem (the recursive call after `return`), the move required for the bottom disk (disk  $n$ ), and the list of moves for the second ( $n - 1$ )-disk problem.

**LISTING 5.2**

```

Class TowersOfHanoi

/** Class that solves Towers of Hanoi problem. */
public class TowersOfHanoi {
    /** Recursive method for "moving" disks.
        pre: startPeg, destPeg, tempPeg are different.
        @param n is the number of disks
        @param startPeg is the starting peg
        @param destPeg is the destination peg
        @param tempPeg is the temporary peg
        @return A string with all the required disk moves
    */
    public static String showMoves(int n, char startPeg,
                                   char destPeg, char tempPeg) {
        if (n == 1) {
            return "Move disk 1 from peg " + startPeg +
                   " to peg " + destPeg + "\n";
        } else { // Recursive step
            return showMoves(n - 1, startPeg, tempPeg, destPeg)
                + "Move disk " + n + " from peg " + startPeg
                + " to peg " + destPeg + "\n"
                + showMoves(n - 1, tempPeg, destPeg, startPeg);
        }
    }
}

```

**Testing**

Figure 5.15 shows the result of executing the following `main` method for the data 3, L, and R (“move 3 disks from peg L to peg R”). The first three lines are the solution to the problem “move 2 disks from peg L to peg M,” and the last three lines are the solution to the problem “move 2 disks from peg M to peg R.”

```

public static void main(String[] args) {
    Scanner in = new Scanner(System.in);
    System.out.print("Enter number of disks ");
    int nDisks = in.nextInt();
    String moves = showMoves(nDisks, 'L', 'R', 'M');
    System.out.println(moves);
}

```

**FIGURE 5.15**

Solution to “Move 3 Disks from Peg L to Peg R”

```

Enter number of disks 3
Move disk 1 from peg L to peg R
Move disk 2 from peg L to peg M
Move disk 1 from peg R to peg M
Move disk 3 from peg L to peg R
Move disk 1 from peg M to peg L
Move disk 2 from peg M to peg R
Move disk 1 from peg L to peg R

```

**Visualization of Towers of Hanoi**

We have provided a graphical visualization that you can use to observe the movement of disks in a solution to the Towers of Hanoi. You can access it through the textbook Web site for this book.

## CASE STUDY Counting Cells in a Blob

In this case study, we consider how we might process an image that is presented as a two-dimensional array of color values. The information in the two-dimensional array might come from a variety of sources. For example, it could be an image of part of a person's body that comes from an X-ray or an MRI, or it could be a picture of part of the earth's surface taken by a satellite. Our goal in this case study is to determine the size of any area in the image that is considered abnormal because of its color values.

**Problem** You have a two-dimensional grid of cells, and each cell contains either a normal background color or a second color, which indicates the presence of an abnormality. The user wants to know the size of a *blob*, where a blob is a collection of contiguous abnormal cells. The user will enter the *x*, *y* coordinates of a cell in the blob, and the count of all cells in that blob will be determined.

### Analysis Data Requirements

#### PROBLEM INPUTS

- The two-dimensional grid of cells
- The coordinates of a cell in a blob

#### PROBLEM OUTPUTS

- The count of cells in the blob

### Classes

We will have two classes. Class `TwoDimGrid` will manage the two-dimensional grid of cells. You can find the discussion of the design and implementation of this class on the Web site for this book. Here we will focus on the design of class `Blob`, which contains the recursive method that counts the number of cells in a blob.

**Design** Table 5.3 describes the public methods of class `TwoDimGrid`, and Table 5.4 describes class `Blob`. Method `countCells` in class `Blob` is a recursive method that is applied to a `TwoDimGrid` object. Its parameters are the (*x*, *y*) position of a cell. The algorithm follows.

**TABLE 5.3**  
Class `TwoDimGrid`

Method	Behavior
<code>void recolor(int x, int y, Color aColor)</code>	Resets the color of the cell at position ( <i>x</i> , <i>y</i> ) to <i>aColor</i>
<code>Color getColor(int x, int y)</code>	Retrieves the color of the cell at position ( <i>x</i> , <i>y</i> )
<code>int getNRows()</code>	Returns the number of cells in the <i>y</i> -axis
<code>int getNCols()</code>	Returns the number of cells in the <i>x</i> -axis

**TABLE 5.4**  
Class `Blob`

Method	Behavior
<code>int countCells(int x, int y)</code>	Returns the number of cells in the blob at ( <i>x</i> , <i>y</i> )

**Algorithm for countCells(x, y)**

1. **if** the cell at (x, y) is outside the grid
2.     The result is 0.
3.     **else if** the color of the cell at (x, y) is not the abnormal color
4.         The result is 0.
5.     **else**
6.         Set the color of the cell at (x, y) to a temporary color.
7.         The result is 1 plus the number of cells in each piece of the blob that includes a nearest neighbor.

The two stopping cases are reached if the coordinates of the cell are out of bounds or if the cell does not have the abnormal color and, therefore, can't be part of a blob. The recursive step involves counting 1 for a cell that has the abnormal color and adding the counts for the blobs that include each immediate neighbor cell. Each cell has eight immediate neighbors: two in the horizontal direction, two in the vertical direction, and four in the diagonal directions.

If no neighbor has the abnormal color, then the result will be just 1. If any neighbor cell has the abnormal color, then it will be counted along with all its neighbor cells that have the abnormal color, and so on until no neighboring cells with abnormal color are encountered (or the edge of the grid is reached). The reason for setting the color of the cell at (x, y) to a temporary color is to prevent it from being counted again when its neighbors' blobs are counted.

**Implementation** Listing 5.3 shows class `Blob`. The interface `GridColors` defines the three constants: `BACKGROUND`, `ABNORMAL`, and `TEMPORARY`. We make these constants available by using the static import statement:

```
import static GridColors.*;
```

The first terminating condition,

```
(x < 0 || x >= grid.getNcols() || y < 0 || y >= grid.getNrows())
```

compares x to 0 and the value returned by `getNcols()`, the number of columns in the grid. Because x is plotted along the horizontal axis, it is compared to the number of columns, not the number of rows. The same test is applied to y and the number of rows. The second terminating condition,

```
(!grid.getColor(x, y).equals(ABNORMAL))
```

is true if the cell at (x, y) has either the background color or the temporary color.

The recursive step is implemented by the statement

```
return 1
+ countCells(x - 1, y + 1) + countCells(x, y + 1)
+ countCells(x + 1, y + 1) + countCells(x - 1, y)
+ countCells(x + 1, y) + countCells(x - 1, y - 1)
+ countCells(x, y - 1) + countCells(x + 1, y - 1);
```

Each recursive call to `countCells` has as its arguments the coordinates of a neighbor of the cell at (x, y). The value returned by each call will be the number of cells in the blob it belongs to, excluding the cell at (x, y) and any other cells that may have been counted already.

**LISTING 5.3**

```

Class Blob

import java.awt.*;
import static GridColors.*;

/** Class that solves problem of counting abnormal cells. */
public class Blob {

    /** The grid */
    private TwoDimGrid grid;

    /** Constructors */
    public Blob(TwoDimGrid grid) {
        this.grid = grid;
    }

    /** Finds the number of cells in the blob at (x,y).
     * pre: Abnormal cells are in ABNORMAL color;
     *      Other cells are in BACKGROUND color.
     * post: All cells in the blob are in the TEMPORARY color.
     * @param x The x-coordinate of a blob cell
     * @param y The y-coordinate of a blob cell
     * @return The number of cells in the blob that contains (x, y)
    */
    public int countCells(int x, int y) {
        int result;

        if (x < 0 || x >= grid.getNCols()
            || y < 0 || y >= grid.getNRows())
            return 0;
        else if (!grid.getColor(x, y).equals(ABNORMAL))
            return 0;
        else {
            grid.recolor(x, y, TEMPORARY);
            return 1
                + countCells(x - 1, y + 1) + countCells(x, y + 1)
                + countCells(x + 1, y + 1) + countCells(x - 1, y)
                + countCells(x + 1, y) + countCells(x - 1, y - 1)
                + countCells(x, y - 1) + countCells(x + 1, y - 1);
        }
    }
}

```

**SYNTAX Static Import****FORM:**

```
import static Class.*;
```

or

```
import static Class.StaticMember;
```

**EXAMPLE:**

```
import static GridColors.*;
```

**MEANING:**

The static members of the class *Class* or interface *Class* are visible in the file containing the import. If \* is specified, then all static members are imported, otherwise if a specific member is listed, then this member is visible.

**Testing** To test the recursive algorithm in this case study and the one in the next section, we will need to implement class `TwoDimGrid`. To make the program interactive and easy to use, we implemented `TwoDimGrid` as a two-dimensional grid of buttons placed in a panel. When the button panel is placed in a frame and displayed, the user can toggle the color of a button (from normal to abnormal and back to normal) by clicking it. Similarly, the program can change the color of a button by applying the `recolor` method to the button. Information about the design of class `TwoDimGrid` is on the textbook Web site for this book, as is the class itself.

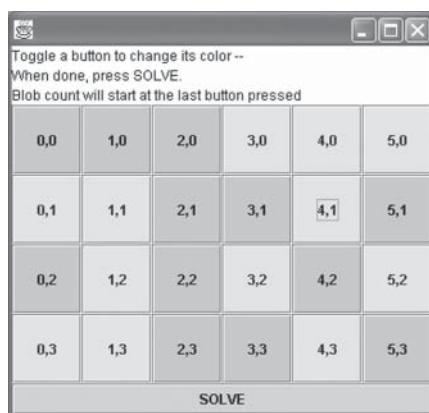
We also provide a class `BlobTest` on the textbook Web site. This class allows the user to load the colors for the button panel from a file that contains a representation of the image as lines of 0s and 1s, where 0 is the background color and 1 is the abnormal color. Alternatively, the user can set the dimensions of the grid and then enter the abnormal cells by clicking on each button that represents an abnormal cell. When the grid has been finalized, the user clicks twice on one of the abnormal cells (to change its color to normal and then back to abnormal) and then clicks the button labeled `Solve`. This invokes method `countCells` with the coordinates of the last button clicked as its arguments. Figure 5.16 shows a sample grid of buttons with the  $x, y$  coordinate of each button shown as the button label. The background cells are dark gray, and the abnormal cells are light gray. Invoking `countCells` with a starting point of  $(x = 4, y = 1)$  should return a count of 7. Figure 5.17 shows the blob cells in the temporary color (black) after the execution of method `countCells`.

When you test this program, make sure you verify that it works for the following cases:

- A starting cell that is on the edge of the grid.
- A starting cell that has no neighboring abnormal cells.
- A starting cell whose only abnormal neighbor cells are diagonally connected to it.
- A “bull’s-eye”: a starting cell whose neighbors are all normal but their neighbors are abnormal.
- A starting cell that is normal.
- A grid that contains all abnormal cells.
- A grid that contains all normal cells.

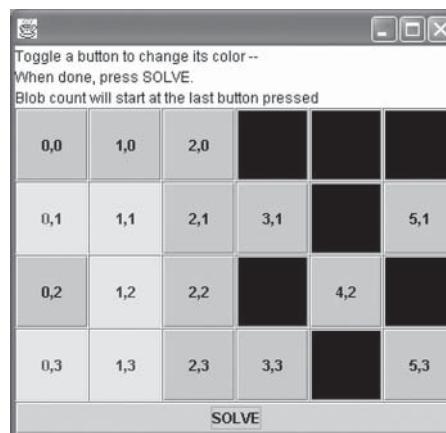
**FIGURE 5.16**

A Sample Grid for Counting Cells in a Blob



**FIGURE 5.17**

Blob Cells (in Black) after Execution of `countCells`



## EXERCISES FOR SECTION 5.5

### SELF-CHECK

- What is the big-O for the Towers of Hanoi as a function of  $n$ , where  $n$  represents the number of disks? Compare it to the function  $2^n$ .
- How many moves would be required to solve the five-disk problem?
- Provide a “trace” of the solution to a four-disk problem by showing all the calls to `showMoves` that would be generated.
- Explain why the first condition of method `countCells` must precede the second condition.

### PROGRAMMING

- Modify method `countCells`, assuming that cells must have a common side in order to be counted in the same blob. This means that they must be connected horizontally or vertically but not diagonally. Under this condition, the value of the method call `aBlob.countCells(4, 1)` would be 4 for the grid in Figure 5.16.
- Write a method `Blob.restore` that restores the grid to its original state. You will need to reset the color of each cell that is in the temporary color back to its original color.



## 5.6 Backtracking

In this section, we consider the problem-solving technique called *backtracking*. Backtracking is an approach to implementing systematic trial and error in a search for a solution. An application of backtracking is finding a path through a maze.

If you are attempting to walk through a maze, you will probably follow the general approach of walking down a path as far as you can go. Eventually either you will reach your destination and exit the maze, or you won’t be able to go any further. If you exit the maze, you are done. Otherwise, you need to retrace your steps (backtrack) until you reach a fork in the path. At each fork, if there is a branch you did not follow, you will follow that branch hoping to reach your destination. If not, you will retrace your steps again, and so on.

What makes backtracking different from random trial and error is that backtracking provides a systematic approach to trying alternative paths and eliminating them if they don’t work out. You will never try the exact same path more than once, and you will eventually find a solution path if one exists.

Problems that are solved by backtracking can be described as a set of choices made by some method. If, at some point, it turns out that a solution is not possible with the current set of choices, the most recent choice is identified and removed. If there is an untried alternative choice, it is added to the set of choices and search continues. If there is no untried alternative choice, then the next most recent choice is removed and an alternative is sought for it. This process continues until either we reach a choice with an untried alternative and can continue our search for a solution, or we determine that there are no more alternative choices to try.

Recursion allows us to implement backtracking in a relatively straightforward manner because we can use each activation frame to remember the choice that was made at that particular decision point.

We will show how to use backtracking to find a path through a maze, but it can be applied to many other kinds of problems that involve a search for a solution. For example, a program that plays chess may use a kind of backtracking. If a sequence of moves it is considering does not lead to a favorable position, it will backtrack and try another sequence.

## CASE STUDY Finding a Path through a Maze

**Problem** Use backtracking to find and display the path through a maze. From each point in a maze, you can move to the next cell in the horizontal or vertical direction, if that cell is not blocked. So there are at most four possible moves from each point.

**Analysis** Our maze will consist of a grid of colored cells like the grid used in the previous case study. The starting point is the cell at the top left corner (0, 0), and the exit point is the cell at the bottom right corner (`getNcols() - 1, getNrows() - 1`). All cells that can be part of a path will be in the BACKGROUND color. All cells that represent barriers and cannot be part of a path will be in the ABNORMAL color. To keep track of a cell that we have visited, we will set it to the TEMPORARY color. If we find a path, all cells on the path will be reset to the PATH color (a new color for a button defined in `GridColors`). So there are a total of four possible colors for a cell.

**Design** The following recursive algorithm returns `true` if a path is found. It changes the color of all cells that are visited but found not to be on the path to the temporary color. In the recursive algorithm, each cell (x, y) being tested is reachable from the starting point. We can use recursion to simplify the problem of finding a path from cell (x, y) to the exit. We know that we can reach any unblocked neighbor cell that is in the horizontal or vertical direction from cell (x, y). So a path exists from cell (x, y) to the maze exit if there is a path from a neighbor cell of (x, y) to the maze exit. If there is no path from any neighbor cell, we must backtrack and replace (x, y) with an alternative that has not yet been tried. That is done automatically through recursion. If there is a path, it will eventually be found and `findMazePath` will return `true`.

### Recursive Algorithm for `findMazePath(x, y)`

1. **if** the current cell is outside the maze  
**Return false** (you are out of bounds).
2. **else if** the current cell is part of the barrier or has already been visited  
**Return false** (you are off the path or in a cycle).
3. **else if** the current cell is the maze exit  
 Recolor it to the path color and return `true` (you have successfully completed the maze).
4. **else // Try to find a path from the current path to the exit:**  
 Mark the current cell as on the path by recoloring it to the path color.
5. **for** each neighbor of the current cell

7.           **if** a path exists from the neighbor to the maze exit  
8.            Return **true**.  
*// No neighbor of the current cell is on the path.*  
9.            Recolor the current cell to the temporary color (visited) and return  
              **false**.

If no stopping case is reached (Steps 2, 3, or 4), the recursive case (the **else** clause) marks the current cell as being on the path and then tests whether there is a path from any neighbor of the current cell to the exit. If a path is found, we return **true** and begin unwinding from the recursion. During the process of unwinding from the recursion, the method will continue to return **true**. However, if all neighbors of the current cell are tested without finding a path, this means that the current cell cannot be on the path, so we recolor it to the temporary color and return **false** (Step 9). Next, we backtrack to a previous call and try to find a path through a cell that is an alternative to the cell just tested. The cell just tested will have been marked as visited (the temporary color), so we won't try using it again.

Note that there is no attempt to find the shortest path through the maze. We just show the first path that is found.

## Implementation

Listing 5.4 shows class `Maze` with data field `maze` (type `TwoDimGrid`). There is a wrapper method that calls recursive method `findMazePath` with its argument values set to the coordinates of the starting point (0, 0). The wrapper method returns the result of this call (**true** or **false**).

The recursive version of `findMazePath` begins with three stopping cases: two unsuccessful and one successful [(x, y) is the exit point]. The recursive case contains an **if** condition with four recursive calls. Because of short-circuit evaluation, if any call returns **true**, the rest are not executed. The arguments for each call are the coordinates of a neighbor cell. If a path exists from a neighbor to the maze exit, then the neighbor is part of the solution path, so we return **true**. If a neighbor cell is not on the solution path, we try the next neighbor until all four neighbors have been tested. If there is no path from any neighbor, we recolor the current cell to the temporary color and return **false**.

.....  
**LISTING 5.4**

```
Class Maze
import java.awt.*;
import static GridColors.*;

/** Class that solves maze problems with backtracking. */
public class Maze {

    /** The maze */
    private TwoDimGrid maze;

    public Maze(TwoDimGrid m) {
        maze = m;
    }

    /** Wrapper method. */
    public boolean findMazePath() {
        return findMazePath(0, 0); // (0, 0) is the start point.
    }
}
```

```

/** Attempts to find a path through point (x, y).
pre: Possible path cells are in BACKGROUND color;
     barrier cells are in ABNORMAL color.
post: If a path is found, all cells on it are set to the
      PATH color; all cells that were visited but are
      not on the path are in the TEMPORARY color.
@param x The x-coordinate of current point
@param y The y-coordinate of current point
@return If a path through (x, y) is found, true;
         otherwise, false
*/
public boolean findMazePath(int x, int y) {
    if (x < 0 || y < 0
        || x >= maze.getNCols() || y >= maze.getNRows())
        return false;           // Cell is out of bounds.
    else if (!maze.getColor(x, y).equals(BACKGROUND))
        return false;           // Cell is on barrier or dead end.
    else if (x == maze.getNCols() - 1
             && y == maze.getNRows() - 1) {
        maze.recolor(x, y, PATH); // Cell is on path
        return true;              // and is maze exit.
    } else {                      // Recursive case.
        // Attempt to find a path from each neighbor.
        // Tentatively mark cell as on path.
        maze.recolor(x, y, PATH);
        if (findMazePath(x - 1, y)
            || findMazePath(x + 1, y)
            || findMazePath(x, y - 1)
            || findMazePath(x, y + 1) ) {
            return true;
        } else {
            maze.recolor(x, y, TEMPORARY); // Dead end.
            return false;
        }
    }
}

```

## The Effect of Marking a Cell as Visited

If a path can't be found from a neighbor of the current cell to the maze exit, the current cell is considered a "dead end" and is recolored to the temporary color. You may be wondering whether the program would still work if we just recolored it to the background color. The answer is "yes." In this case, cells that turned out to be dead ends or cells that were not visited would be in the background color after the program terminated. This would not affect the ability of the algorithm to find a path or to determine that none exists; however, it would affect the algorithm's efficiency. After backtracking, the method could try to place on the path a cell that had been found to be a dead end. The cell would be classified once again as a dead end. Marking it as a dead end (color TEMPORARY) the first time prevents this from happening.

To demonstrate the efficiency of this approach, we tested the program on a maze with four rows and six columns that had a single barrier cell at the maze exit. When we recolored each dead end cell in the TEMPORARY color, it took 93 recursive calls to `findMazePath` to

determine that a path did not exist. When we recolored each tested cell in the BACKGROUND color, it took 177,313 recursive calls to determine that a path did not exist.

**Testing** We will use class `TwoDimGrid` and class `MazeTest` (from the textbook Web site) to test the maze. The `MazeTest` class is very similar to `BlobTest`. The `main` method prompts for the grid dimensions and creates a new `TwoDimGrid` object with those dimensions. The class constructor builds the graphical user interface (GUI) for the maze solver, including the button panel, and registers a listener for each button. When the SOLVE button is clicked, method `MazeTest.actionPerformed` calls `findMazePath` and displays its result. Figure 5.18 shows the GUI before the SOLVE button is clicked (barrier cells are in light gray, and other cells are in dark gray), and Figure 5.19 shows it after the SOLVE button is clicked and the final path is displayed. In Figure 5.19, the barrier cells are in light gray (ABNORMAL color), the cells on the final path are in white (PATH color), and the cells that were visited but then rejected (not on the path) are in black (TEMPORARY color).

You should test this with a variety of mazes, some that can be solved and some that can't (no path exists). You should also try a maze that has no barrier cells and one that has a single barrier cell at the exit point. In the latter case, no path exists.

FIGURE 5.18

Maze as Grid of Buttons before SOLVE Is Clicked

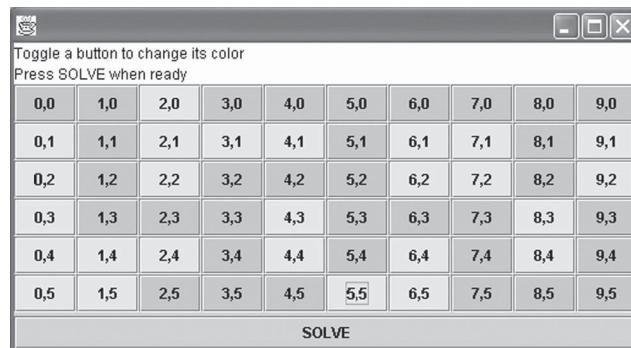
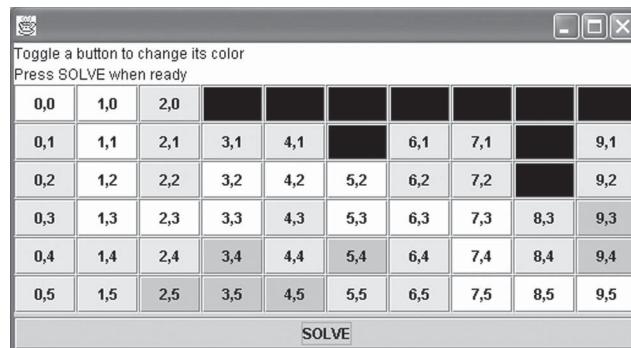


FIGURE 5.19

Maze as Grid of Buttons after SOLVE Is Clicked



## EXERCISES FOR SECTION 5.6

### SELF-CHECK

1. The terminating conditions in `findMazePath` must be performed in the order specified. What could happen if the second or third condition was evaluated before the first? If the third condition was evaluated before the second condition?
2. Does it matter in which order the neighbor cells are tested in `findMazePath`? How could this order affect the path that is found?
3. Is the path shown in Figure 5.19 the shortest path to the exit? If not, list the cells on the shortest path.

### PROGRAMMING

1. Show the interface `GridColors`.
2. Write a `Maze.resetTemp` method that recolors the cells that are in the `TEMPORARY` color to the `BACKGROUND` color.
3. Write a `Maze.restore` method that restores the maze to its initial state.



## Chapter Review

- ◆ A recursive method has the following form, where Step 2 is the base case, and Steps 3 and 4 are the recursive case:
  1. **`if`** the problem can be solved for the current value of  $n$
  2. Solve it.
  - `else`**
  3. Recursively apply the algorithm to one or more problems involving smaller values of  $n$ .
  4. Combine the solutions to the smaller problems to get the solution to the original.
- ◆ To prove that a recursive algorithm is correct, you must
  - Verify that the base case is recognized and solved correctly.
  - Verify that each recursive case makes progress toward the base case.
  - Verify that if all smaller problems are solved correctly, then the original problem must also be solved correctly.
- ◆ The run-time stack uses activation frames to keep track of argument values and return points during recursive method calls. Activation frames can be used to trace the execution of a sequence of recursive method calls.
- ◆ Mathematical sequences and formulas that are defined recursively can be implemented naturally as recursive methods.
- ◆ Recursive data structures are data structures that have a component that is the same data structure. A linked list can be considered a recursive data structure because each node

consists of a data field and a reference to a linked list. Recursion can make it easier to write methods that process a linked list.

- ◆ Two problems that can be solved using recursion were investigated: the Towers of Hanoi problem and counting cells in a blob.
- ◆ Backtracking is a technique that enables you to write programs that can be used to explore different alternative paths in a search for a solution.

## User-Defined Classes in This Chapter

Blob	MazeTest
BlobTest	RecursiveMethods
GridColors	TowersOfHanoi
LinkedListRec	TwoDimGrid
Maze	

## Quick-Check Exercises

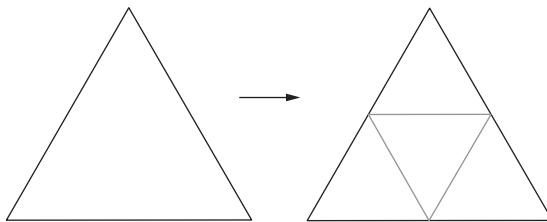
1. A recursive method has two cases: \_\_\_\_\_ and \_\_\_\_\_.
2. Each recursive call of a recursive method must lead to a situation that is \_\_\_\_\_ to the \_\_\_\_\_ case.
3. The control statement used in a recursive method is the \_\_\_\_\_ statement.
4. What three things are stored in an activation frame? Where are the activation frames stored?
5. You can sometimes substitute \_\_\_\_\_ for recursion.
6. Explain how a recursive method might cause a stack overflow exception.
7. If you have a recursive method and an iterative method that calculate the same result, which do you think would be more efficient? Explain your answer.
8. Binary search is an  $O(\_\_)$  algorithm and linear search is an  $O(\_\_)$  algorithm.
9. Towers of Hanoi is an  $O(\_\_)$  algorithm. Explain your answer.
10. Why did you need to provide a wrapper method for recursive methods `linearSearch` and `binarySearch`?
11. Why did you need to provide a wrapper method for recursive methods in the `LinkedListRec` class?

## Review Questions

1. Explain the use of the run-time stack and activation frames in processing recursive method calls.
2. What is a recursive data structure? Give an example.
3. For class `LinkedListRec`, write a recursive search method that returns `true` if its target argument is found and `false` otherwise. If you need a wrapper method, provide one.
4. For class `LinkedListRec`, write a recursive `replaceFirst` method that replaces the first occurrence of a reference to its first argument with a reference to its second argument. If you need a wrapper method, provide one.
5. For Towers of Hanoi, show the output string that would be created by the method call `showMoves(3, 'R', 'M', 'L')`. Also, show the sequence of method calls.
6. For the counting cells in a blob problem, show the activation frames in the first 10 recursive calls to `countCells` following `countCells(4, 1)`.
7. For the maze path found in Figure 5.19, explain why cells (3, 4), (2, 5), (3, 5), and (4, 5) were never visited and why cells (5, 1) and (3, 0) through (9, 0) were visited and rejected. Show the activation frames for the first 10 recursive calls in solving the maze.

## Programming Projects

- Download and run class `BlobTest`. Try running it with a data file made up of lines consisting of 0s and 1s with no spaces between them. Also run it without a data file.
- Download and run class `MazeTest`. Try running it with a data file made up of lines consisting of 0s and 1s with no spaces between them. Also run it without a data file.
- Write a recursive method that converts a decimal integer to a binary string. Write a recursive method that converts a binary string to a decimal integer.
- Write a `LinkedListRec` class that has the following methods: `size`, `empty`, `insertBefore`, `insertAfter`, `addAtHead`, `addAtEnd`, `remove`, `replace`, `peekFront`, `peekEnd`, `removeFront`, `removeEnd`, `toString`. Use recursion to implement most of these methods.
- As discussed in Chapter 3, a palindrome is a word that reads the same left to right as right to left. Write a recursive method that determines whether its argument string is a palindrome.
- Write a program that will read a list of numbers and a desired sum, then determine the subset of numbers in the list that yield that sum if such a subset exists.
- Write a recursive method that will dispense change for a given amount of money. The method will display all combinations of quarters, dimes, nickels, and pennies that equal the desired amount.
- Produce the Sierpinski fractal. Start by drawing an equilateral triangle that faces upward. Then draw an equilateral triangle inside it that faces downward.



Continue this process on each of the four smaller triangles. Stop when the side dimension for a triangle to be drawn is smaller than a specified minimum size.

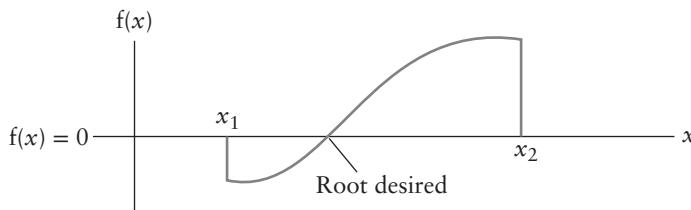
- Write a recursive method for placing eight queens on a chessboard. The eight queens should be placed so that no queen can capture another. Recall that a queen can move in the horizontal, vertical, or diagonal direction.
- Write a recursive method that will find and list all of the one-element sequences of letters in a `char[]` array, then all the two-element sequences, then all of the three element sequences, and so on such that the characters in each sequence appear in the same order as they are in the array. For example, for the following array:

```
char[] letters = {'A', 'C', 'E', 'G'};
```

the one-element sequences are "A", "C", "E", and "G"  
 the two-element sequences are "AC", "AE", "AG", "CE", "CG", "EG"  
 the three-element sequences are "ACE", "ACG", "AEG", and "CEG"  
 the four-element sequence is "ACEG"

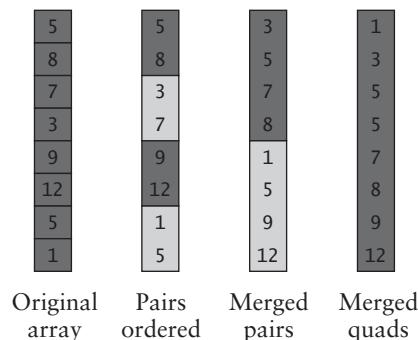
- One method of solving a continuous numerical function for a root implements a technique similar to the binary search. Given a numerical function, defined as  $f(x)$ , and two values of  $x$  that are known to bracket one of the roots, an approximation to this root can be determined through a method of repeated division of this bracket. For a set of values of  $x$  to bracket a root, the value of the function for one  $x$  must be negative and the other must be positive as illustrated below, which plots  $f(x)$  for values of  $x$  between  $x_1$  and  $x_2$ .

The algorithm requires that the midpoint between  $x_1$  and  $x_2$  be evaluated in the function, and if it equals zero the root is found; otherwise,  $x_1$  or  $x_2$  is set to this midpoint. To determine whether to replace  $x_1$  or  $x_2$ , the sign of the midpoint is compared against the signs of the values  $f(x_1)$  and  $f(x_2)$ . The midpoint replaces the  $x$  ( $x_1$  or  $x_2$ ) whose function value has the same sign as the function value at the midpoint.



This algorithm can be written recursively. The terminating conditions are true when either the midpoint evaluated in the function is zero or the absolute value of  $x_1 - x_2$  is less than some small predetermined value (e.g., 0.0005). If the second condition occurs, then the root is said to be approximately equal to the last midpoint.

12. We can use a merge technique to sort two arrays. The *mergesort* begins by taking adjacent pairs of array values and ordering the values in each pair. It then forms groups of four elements by merging adjacent pairs (first pair with second pair, third pair with fourth pair, etc.) into another array. It then takes adjacent groups of four elements from this new array and merges them back into the original array as groups of eight, and so on. The process terminates when a single group is formed that has the same number of elements as the array. The mergesort is illustrated here for an array with eight elements. Write a `mergeSort` method.



## Answers to Quick-Check Exercises

1. A recursive method has two cases: *base case* and *recursive case*.
2. Each recursive call of a recursive method must lead to a situation that is *closer* to the *base case*.
3. The control statement used in a recursive method is the `if` statement.
4. An activation frame stores the following information on the run-time stack: the method argument values, the method local variable values, and the address of the return point in the caller of the method.
5. You can sometimes substitute *iteration* for recursion.
6. A recursive method that doesn't stop would continue to call itself, eventually pushing so many activation frames onto the run-time stack that a stack overflow exception would occur.
7. An *iterative* method would generally be more efficient because there is more overhead associated with multiple method calls.
8. Binary search is an  $O(\log_2 n)$  algorithm and linear search is an  $O(n)$  algorithm.
9. Towers of Hanoi is an  $O(2^n)$  algorithm because each problem splits into two problems at the next lower level.
10. Both search methods should be called with the array name and target as arguments. However, the recursive linear search method needs the subscript of the element to be compared to the target. The binary search method needs the search array bounds.
11. The wrapper method should be applied to a `LinkedListRec` object. The recursive method needs the current list head as an argument.



# Trees

## Chapter Objectives

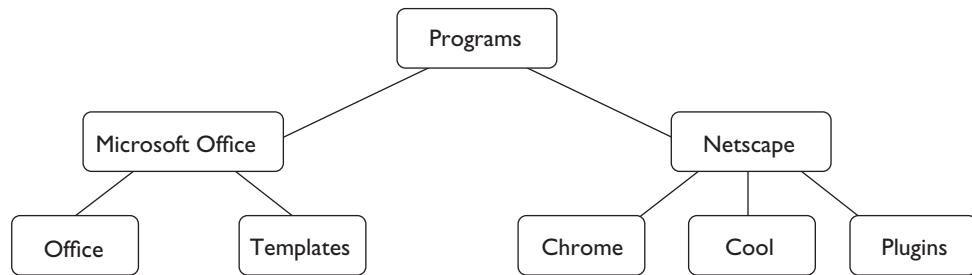
- ◆ To learn how to use a tree to represent a hierarchical organization of information
- ◆ To learn how to use recursion to process trees
- ◆ To understand the different ways of traversing a tree
- ◆ To understand the difference between binary trees, binary search trees, and heaps
- ◆ To learn how to implement binary trees, binary search trees, and heaps using linked data structures and arrays
- ◆ To learn how to use Java 8 Lambda Expressions and Functional Interfaces to simplify coding
- ◆ To learn how to use a binary search tree to store information so that it can be retrieved in an efficient manner
- ◆ To learn how to use a Huffman tree to encode characters using fewer bits than ASCII or Unicode, resulting in smaller files and reduced storage requirements

The data organizations you have studied so far are linear in that each element has only one predecessor or successor. Accessing all the elements in sequence is an  $O(n)$  process. In this chapter, we begin our discussion of a data organization that is nonlinear or hierarchical: the tree. Instead of having just one successor, a node in a tree can have multiple successors, but it has just one predecessor. A tree in computer science is like a natural tree, which has a single trunk that may split off into two or more main branches. The predecessor of each main branch is the trunk. Each main branch may spawn several secondary branches (successors of the main branches). The predecessor of each secondary branch is a main branch. In computer science, we draw a tree from the top down, so the root of the tree is at the top of the diagram instead of the bottom.

Because trees have a hierarchical structure, we use them to represent hierarchical organizations of information, such as a class hierarchy, a disk directory and its subdirectories (see Figure 6.1), or a family tree. You will see that trees are recursive data structures because they can be defined recursively. For this reason, many of the methods used to process trees are written as recursive methods.

**FIGURE 6.1**

Part of the Programs  
Directory



This chapter will focus on a restricted tree structure, a binary tree, in which each element has, at most, two successors. You will learn how to use linked data structures and arrays to represent binary trees. You will also learn how to use a special kind of binary tree called a binary search tree to store information (e.g., the words in a dictionary) in an ordered way. Because each element of a binary tree can have two successors, you will see that searching for an item stored in a binary search tree is much more efficient than searching for an item in a linear data structure: (generally  $O(\log n)$  for a binary tree versus  $O(n)$  for a list).

You also will learn about other kinds of binary trees. Expression trees are used to represent arithmetic expressions. The heap is an ordered tree structure that is used as the basis for a very efficient sorting algorithm and for a special kind of queue called the priority queue. The Huffman tree is used for encoding information and compressing files.

## Trees

- 6.1** Tree Terminology and Applications
- 6.2** Tree Traversals
- 6.3** Implementing a `BinaryTree` Class
- 6.4** Java 8 Lambda Expressions and Functional Interfaces
- 6.5** Binary Search Trees
  - Case Study:* Writing an Index for a Term Paper
- 6.6** Heaps and Priority Queues
- 6.7** Huffman Trees
  - Case Study:* Building a Custom Huffman Tree

## 6.1 Tree Terminology and Applications

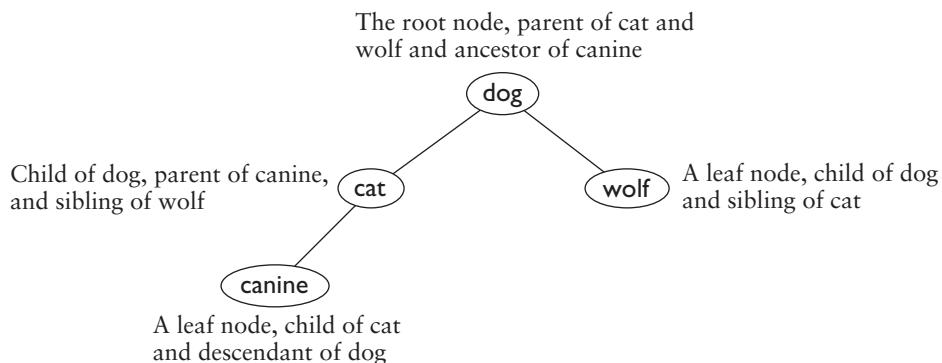
### Tree Terminology

We use the same terminology to describe trees in computer science as we do trees in nature. A computer science tree consists of a collection of elements or nodes, with each node linked to its successors. The node at the top of a tree is called its *root* because computer science trees grow from the top down. The links from a node to its successors are called *branches*. The successors of a node are called its *children*. The predecessor of a node is called its *parent*. Each node in a tree has exactly one parent except for the root node, which has no parent. Nodes that have the same parent are *siblings*. A node that has no children is a *leaf node*. Leaf nodes are also known as *external nodes*, and nonleaf nodes are known as *internal nodes*.

A generalization of the parent–child relationship is the *ancestor–descendant relationship*. If node A is the parent of node B, which is the parent of node C, node A is node C's *ancestor*, and node C is node A's *descendant*. Sometimes we say that node A and node C are a grandparent and grandchild, respectively. The root node is an ancestor of every other node in a tree, and every other node in a tree is a descendant of the root node.

Figure 6.2 illustrates these features in a tree that stores a collection of words. The branches are the lines connecting a parent to its children. In discussing this tree, we will refer to a node by the string that it stores. For example, we will refer to the node that stores the string "dog" as node *dog*.

**FIGURE 6.2**  
A Tree of Words



A *subtree of a node* is a tree whose root is a child of that node. For example, the nodes *cat* and *canine* and the branch connecting them are a subtree of node *dog*. The other subtree of node *dog* is the tree consisting of the single node *wolf*. The subtree consisting of the single node *canine* is a subtree of node *cat*.

The *level of a node* is a measure of its distance from the root. It is defined recursively as follows:

- If node *n* is the root of tree T, its level is 1.
- If node *n* is not the root of tree T, its level is  $1 + \text{the level of its parent}$ .

For the tree in Figure 6.2, node *dog* is at level 1, nodes *cat* and *wolf* are at level 2, and node *canine* is at level 3. Since nodes are below the root, we sometimes use the term *depth* as an alternative term for level. The two have the same meaning.

The *height of a tree* is the number of nodes in the longest path from the root node to a leaf node. The height of the tree in Figure 6.2 is 3 (the longest path goes through the nodes *dog*, *cat*, and *canine*). Another way of saying this is as follows:

- If T is empty, its height is 0.
- If T is not empty, its height is the maximum depth of its nodes.

An alternate definition of the height of a tree is the number of branches in the longest path from the root node to a leaf node +1.

## Binary Trees

The tree in Figure 6.2 is a *binary tree*. Informally, this is a binary tree because each node has at most two subtrees. A more formal definition for a binary tree follows.

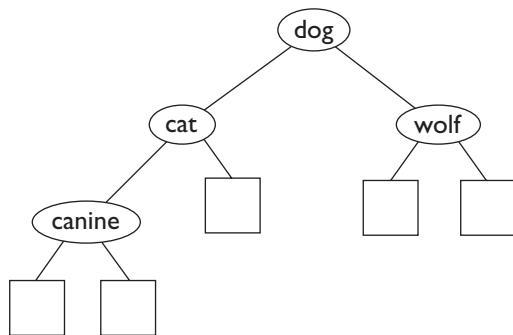
A set of nodes  $T$  is a binary tree if either of the following is true:

- $T$  is empty.
- If  $T$  is not empty, its root node has two subtrees,  $T_L$  and  $T_R$ , such that  $T_L$  and  $T_R$  are binary trees.

We refer to  $T_L$  as the left subtree and  $T_R$  as the right subtree. For the tree in Figure 6.2, the right subtree of node *cat* is empty. The leaf nodes (*wolf* and *canine*) have empty left and right subtrees. This is illustrated in Figure 6.3, where the empty subtrees are indicated by the squares. Generally, the empty subtrees are represented by `null` references, but another value may be chosen. From now on, we will consistently use a `null` reference and will not draw the squares for the empty subtrees.

FIGURE 6.3

A Tree of Words with Null Subtrees Indicated



## Some Types of Binary Trees

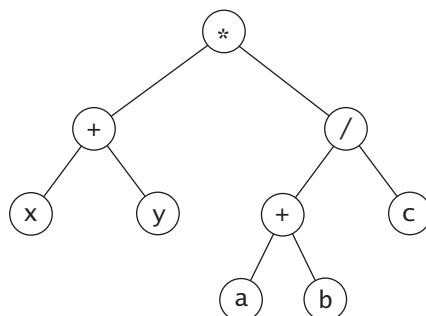
Next, we discuss three different types of binary trees that are common in computer science.

### An Expression Tree

Figure 6.4 shows a binary tree that stores an expression. Each node contains an operator (+, -, \*, /, %) or an operand. The expression in Figure 6.4 corresponds to  $(x + y) * ((a + b) / c)$ . Operands are stored in leaf nodes. Parentheses are not stored in the tree because the tree structure dictates the order of operator evaluation. Operators in nodes at higher levels are evaluated after operators in nodes at lower levels, so the operator \* in the root node is evaluated last. If a node contains a binary operator, its left subtree represents the operator's left operand and its right subtree represents the operator's right operand. The left subtree of the root represents the expression  $x + y$ , and the right subtree of the root represents the expression  $(a + b) / c$ .

FIGURE 6.4

Expression Tree



## A Huffman Tree

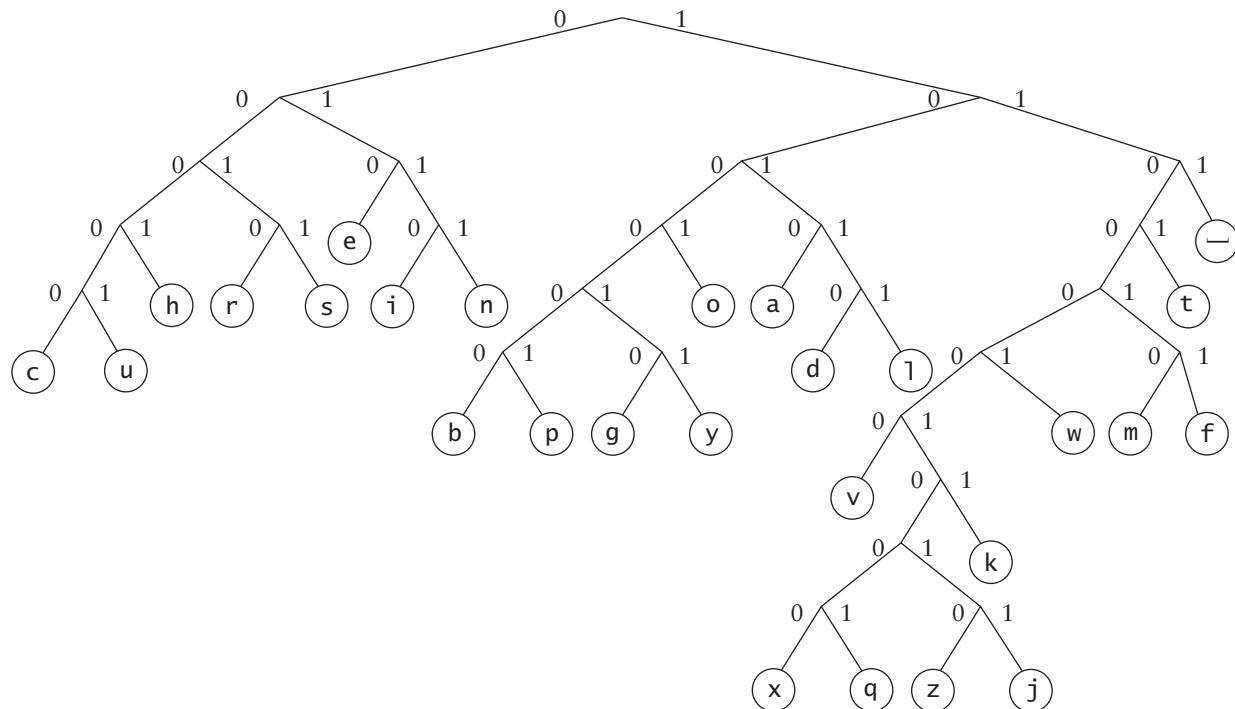
Another use of a binary tree is to represent *Huffman codes* for characters that might appear in a text file. Unlike ASCII or Unicode encoding, which use the same number of bits to encode each character, a Huffman code uses different numbers of bits to encode the letters. It uses fewer bits for the more common letters (e.g., space, *e*, *a*, and *t*) and more bits for the less common letters (e.g., *q*, *x*, and *z*). On average, using Huffman codes to encode text files should give you files with fewer bits than you would get using other codes. Many programs that compress files use Huffman encoding to generate smaller files in order to save disk space or to reduce the time spent sending the files over the Internet.

Figure 6.5 shows the Huffman encoding tree for an alphabet consisting of the lowercase letters and the space character. All the characters are at leaf nodes. The data stored at nonleaf nodes is not shown. To determine the code for a letter, you form a binary string by tracing the path from the root node to that letter. Each time you go left, append a 0, and each time you go right, append a 1. To reach the space character, you go right three times, so the code is 111. The code for the letter *d* is 10110 (right, left, right, right, left).

The two characters shown at level 4 of the tree (space, *e*) are the most common and, therefore, have the shortest codes (111, 010). The next most common characters (*a*, *o*, *i*, etc.) are at level 5 of the tree.

You can store the code for each letter in an array. For example, the code for the space ' ' would be at position 0, the letter '*a*' would be at position 1, and the code for letter '*z*' would be at position 26. You can *encode* each letter in a file by looking up its code in the array.

**FIGURE 6.5**  
Huffman Code Tree



However, to *decode* a file of letters and spaces, you walk down the Huffman tree, starting at the root, until you reach a letter and then append that letter to the output text. Once you have reached a letter, go back to the root. Here is an example. The substrings that represent the individual letters are shown in alternate shades of black to help you follow the process. The underscore in the second line represents a space character (code is 111).

```
100010100111101010100010101110100011
      g   o _ e   a   g   l   e   s
```

Huffman trees are discussed further in Section 6.7.

## A Binary Search Tree

The tree in Figure 6.2 is a *binary search tree* because, for each node, all words in its left subtree precede the word in that node, and all words in its right subtree follow the word in that node. For example, for the root node *dog*, all words in its left subtree (*cat*, *canine*) precede *dog* in the dictionary, and all words in its right subtree (*wolf*) follow *dog*. Similarly, for the node *cat*, the word in its left subtree (*canine*) precedes it. There are no duplicate entries in a binary search tree.

More formally, we define a binary search tree as follows:

A set of nodes  $T$  is a binary search tree if either of the following is true:

- $T$  is empty.
- If  $T$  is not empty, its root node has two subtrees,  $T_L$  and  $T_R$ , such that  $T_L$  and  $T_R$  are binary search trees and the value in the root node of  $T$  is greater than all values in  $T_L$  and is less than all values in  $T_R$ .

The order relations in a binary search tree expedite searching the tree. A recursive algorithm for searching a binary search tree follows:

1. **if** the tree is empty
2.     Return **null** (*target is not found*).
3.     **else if** the target matches the root node's data
4.     Return the data stored at the root node.
5.     **else if** the target is less than the root node's data
6.         Return the result of searching the left subtree of the root.
7.     **else**
8.         Return the result of searching the right subtree of the root.

The first two cases are base cases and self-explanatory. In the first recursive case, if the target is less than the root node's data, we search only the left subtree ( $T_L$ ) because all data items in  $T_R$  are larger than the root node's data and, therefore, larger than the target. Likewise, we execute the second recursive step (search the right subtree) if the target is greater than the root node's data.

Just as with a binary search of an array, each probe into the binary search tree has the potential of eliminating half the elements in the tree. If the binary search tree is relatively balanced (i.e., the depths of the leaves are approximately the same), searching a binary search tree is an  $O(\log n)$  process, just like a binary search of an ordered array.

What is the advantage of using a binary search tree instead of just storing elements in an array and then sorting it? A binary search tree never has to be sorted because its elements always satisfy the required order relations. When new elements are inserted (or removed), the binary search tree property can be maintained. In contrast, an array must be expanded whenever new elements are added, and it must be compacted whenever elements are removed. Both expanding and contracting involve shifting items and are thus  $O(n)$  operations.

## Full, Perfect, and Complete Binary Trees

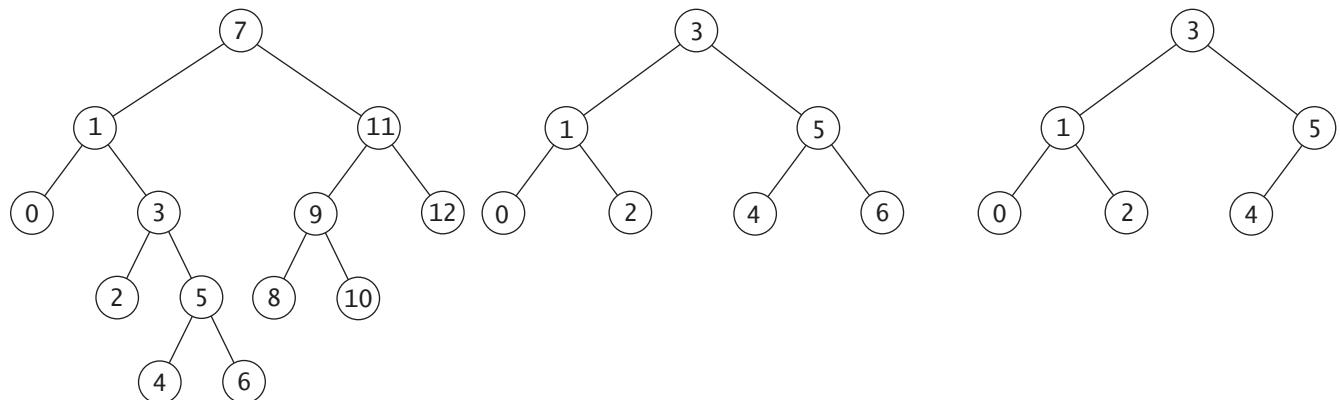
The tree on the left in Figure 6.6 is called a *full binary tree* because all nodes have either 2 children or 0 children (the leaf nodes). The tree in the middle is a *perfect binary tree*, which is defined as a full binary tree of height  $n$  ( $n$  is 3) with exactly  $2^n - 1$  (7) nodes. The tree on the right is a *complete binary tree*, which is a perfect binary tree through level  $n - 1$  with some extra leaf nodes at level  $n$  (the tree height), all toward the left.

## General Trees

A general tree is a tree that does not have the restriction that each node of a tree has at most two subtrees. So nodes in a general tree can have any number of subtrees. Figure 6.7 shows a general tree that represents a family tree showing the descendants of King William I (the Conqueror) of England.

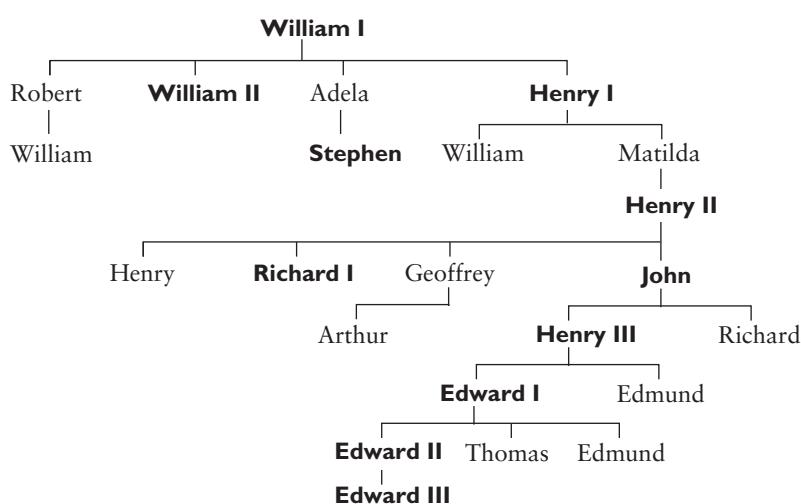
**FIGURE 6.6**

Full, Perfect, and Complete Binary Trees



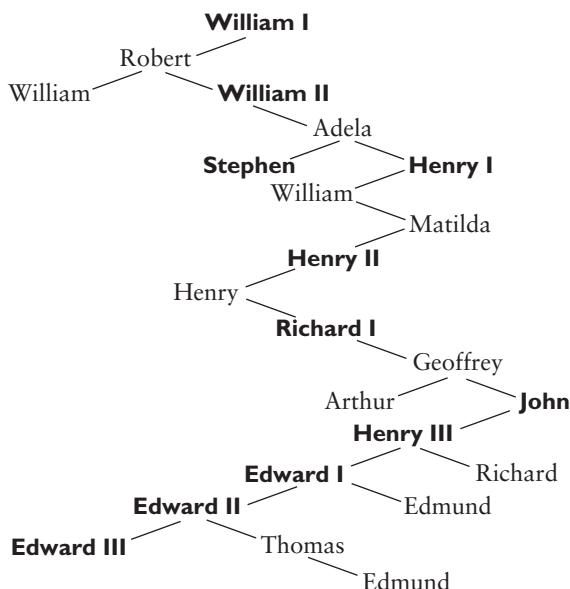
**FIGURE 6.7**

Family Tree for the Descendants of William I of England



**FIGURE 6.8**

Binary Tree Equivalent  
of King William's Family  
Tree



We will not discuss general trees in this chapter. However, it is worth mentioning that a general tree can be represented using a binary tree. Figure 6.8 shows a binary tree representation of the family tree in Figure 6.7. We obtained it by connecting the left branch from a node to the oldest child (if any). Each right branch from a node is connected to the next younger sibling (if any).

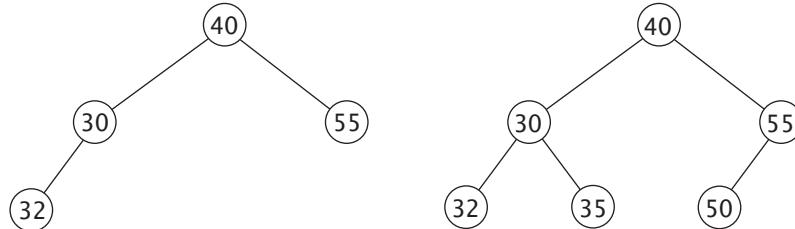
The names of the men who became kings are in boldface type. You would expect the eldest son to succeed his father as king; however, this would not be the case if the eldest male died before his father. For example, Robert died before William I, so William II became king instead. Starting with King John (near the bottom of the tree), the eldest son of each king did become the King of England.

## EXERCISES FOR SECTION 6.1

### SELF-CHECK

1. Draw binary expression trees for the following infix expressions. Your trees should enforce the Java rules for operator evaluation (higher-precedence operators before lower-precedence operators and left associativity).
  - a.  $x / y + a - b * c$
  - b.  $(x * a) - y / b * (c + d)$
  - c.  $(x + (a * (b - c))) / d$
2. Using the Huffman tree in Figure 6.5,
  - a. Write the binary string for the message “scissors cuts paper”.
  - b. Decode the following binary string using the tree in Figure 6.5:  
110001000101000100101110110001111110001101010111101101001

3. For each tree shown below, answer these questions. What is its height? Is it a full tree? Is it a complete tree? Is it a binary search tree? If not, make it a binary search tree.



4. For the binary trees in Figures 6.2–6.5, indicate whether each tree is full, perfect, complete, or none of the above.
5. Represent the general tree in Figure 6.1 as a binary tree.



## 6.2 Tree Traversals

Often we want to determine the nodes of a tree and their relationship. We can do this by walking through the tree in a prescribed order and visiting the nodes (processing the information in the nodes) as they are encountered. This process is known as *tree traversal*. We will discuss three kinds of traversal in this section: inorder, preorder, and postorder. These three methods are characterized by when they visit a node in relation to the nodes in its subtrees ( $T_L$  and  $T_R$ ).

- Preorder: Visit root node, traverse  $T_L$ , and traverse  $T_R$ .
- Inorder: Traverse  $T_L$ , visit root node, and traverse  $T_R$ .
- Postorder Traverse  $T_L$ , traverse  $T_R$ , and visit root node.

Because trees are recursive data structures, we can write similar recursive algorithms for all three techniques. The difference in the algorithms is whether the root is visited before the children are traversed (pre), in between traversing the left and right children (in), or after the children are traversed (post).

### Algorithm for Preorder Traversal

1. **if** the tree is empty
2.     Return.
- else**
3.     Visit the root.
4.     Preorder traverse the left subtree.
5.     Preorder traverse the right subtree.

### Algorithm for Inorder Traversal

1. **if** the tree is empty
2.     Return.
- else**
3.     Inorder traverse the left subtree.
4.     Visit the root.
5.     Inorder traverse the right subtree.

### Algorithm for Postorder Traversal

1. **if** the tree is empty
2.     Return.
- else**
3.     Postorder traverse the left subtree.
4.     Postorder traverse the right subtree.
5.     Visit the root.

## Visualizing Tree Traversals

You can visualize a tree traversal by imagining a mouse that walks along the edge of the tree. If the mouse always keeps the tree to the left (from the mouse's point of view), it will trace the route shown in gray around the tree shown in Figure 6.9. This is known as an *Euler tour*.

If we record each node as the mouse first encounters it (indicated by the arrows pointing down in Figure 6.9), we get the following sequence:

a b d g e h c f i j

This is a preorder traversal because the mouse visits each node before traversing its subtrees. The mouse also walks down the left branch (if it exists) of each node before going down the right branch, so the mouse visits a node, traverses its left subtree, and traverses its right subtree.

If we record each node as the mouse returns from traversing its left subtree (indicated by the arrows pointing to the right in Figure 6.9), we get the following sequence:

d g b h e a i f j c

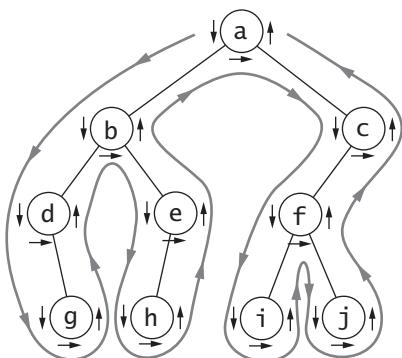
This is an inorder traversal. The mouse traverses the left subtree, visits the root, and then traverses the right subtree. Node *d* is visited first because it has no left subtree.

If we record each node as the mouse last encounters it (indicated by the arrows pointing up in Figure 6.9), we get the following sequence:

g d h e b i j f c a

This is a postorder traversal because we visit the node after traversing both its subtrees. The mouse traverses the left subtree, traverses the right subtree, and then visits the node.

**FIGURE 6.9**  
Traversal of a Binary Tree

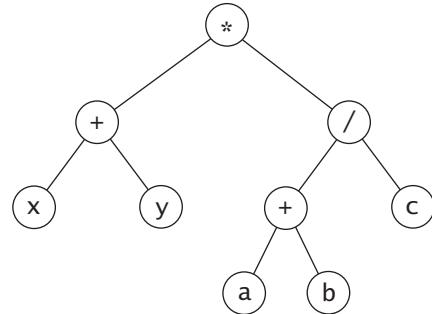


## Traversals of Binary Search Trees and Expression Trees

An inorder traversal of a binary search tree results in the nodes being visited in sequence by increasing data value. For example, for the binary search tree shown earlier in Figure 6.2, the inorder traversal would visit the nodes in the sequence:

*canine, cat, dog, wolf*

Traversals of expression trees give interesting results. If we perform an inorder traversal of the expression tree first shown in Figure 6.4 and repeated here, we visit the nodes in the sequence *x + y<sup>\*</sup> a + b / c*. If we insert parentheses where they belong, we get the infix expression

$$(x + y) * ((a + b) / c)$$


The postorder traversal of this tree would visit the nodes in the sequence

$$\underline{x \ y \ + \ a \ b \ + \ c \ / \ *} \quad *$$

which is the postfix form of the expression. To illustrate this, we show the *operator–operand–operator* groupings under the expression.

The preorder traversal visits the nodes in the sequence

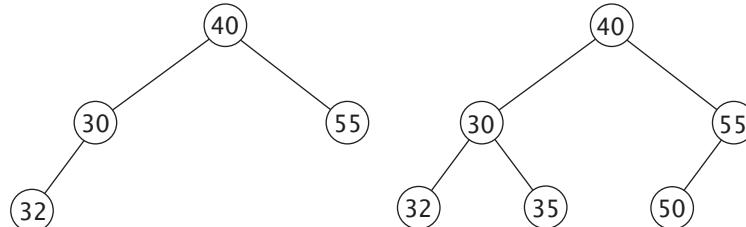
$$\underline{\underline{* \ + \ x \ y \ / \ + \ a \ b \ c}} \quad *$$

which is the prefix form of the expression. To illustrate this, we show the *operator–operand–operator* groupings under the expression.

## EXERCISES FOR SECTION 6.2

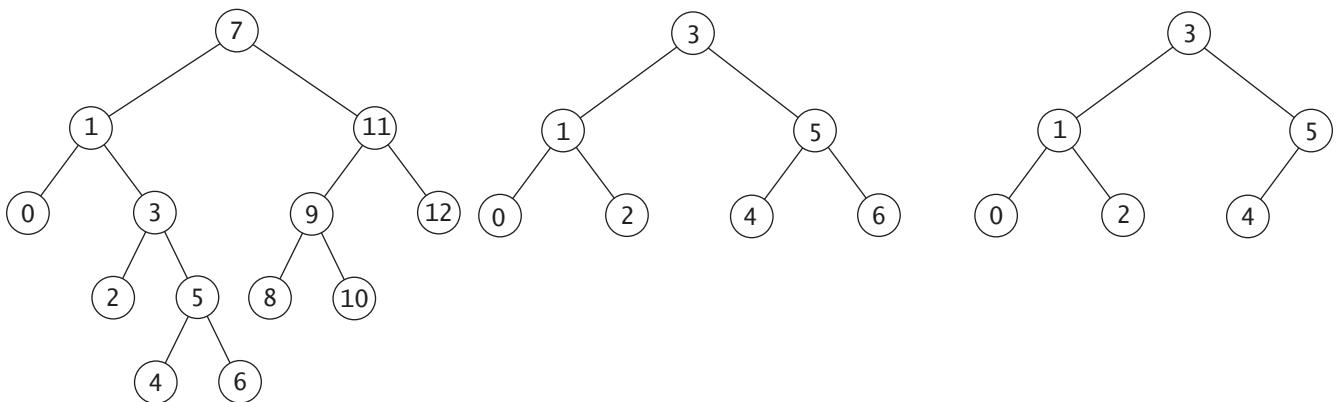
### SELF-CHECK

1. For the following trees:



If visiting a node displays the integer value stored, show the inorder, preorder, and postorder traversal of each tree.

2. Repeat Exercise 1 above for the trees in Figure 6.6, redrawn below.



3. Draw an expression tree corresponding to each of the following:
  - a. Inorder traversal is  $x / y + 3 * b / c$  (Your tree should represent the Java meaning of the expression.)
  - b. Postorder traversal is  $x y z + a b - c * / -$
  - c. Preorder traversal is  $* + a - x y / c d$
4. Explain why the statement “Your tree should represent the Java meaning of the expression” was not needed for parts b and c of Exercise 3 above.



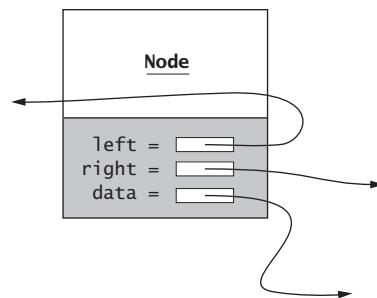
## 6.3 Implementing a BinaryTree Class

In this section, we show how to use linked data structures to represent binary trees and binary tree nodes. We begin by focusing on the structure of a binary tree node.

### The Node<E> Class

Just as for a linked list, a node consists of a data part and links (references) to successor nodes. So that we can store any kind of data in a tree node, we will make the data part a reference of type E. Instead of having a single link (reference) to a successor node as in a list, a binary tree node must have links (references) to both its left and right subtrees. Figure 6.10 shows the structure of a binary tree node; Listing 6.1 shows its implementation.

**FIGURE 6.10**  
Linked Structure to  
Represent a Node



**LISTING 6.1**  
Nested Class Node

```
/** Class to encapsulate a tree node. */
protected static class Node<E> implements Serializable {
    // Data Fields
    /** The information stored in this node. */
    protected E data;
    /** Reference to the left child. */
    protected Node<E> left;
    /** Reference to the right child. */
    protected Node<E> right;

    // Constructors
    /** Construct a node with given data and no children.
        @param data The data to store in this node
    */
    public Node(E data) {
        this.data = data;
        left = null;
        right = null;
    }
}
```

```

// Methods
/** Return a string representation of the node.
 * @return A string representation of the data fields
 */
public String toString () {
    return data.toString();
}
}

```

Class `Node<E>` is nested within class `BinaryTree<E>`. Note that it is declared `protected` and its data fields are all `protected`. Later, we will use the `BinaryTree<E>` and `Node<E>` classes as superclasses. By declaring the nested `Node<E>` class and its data fields protected, we make them accessible in the subclasses of `BinaryTree<E>` and `Node<E>`.

The constructor for class `Node<E>` creates a leaf node (both `left` and `right` are `null`). The `toString` method for the class just displays the data part of the node.

Both the `BinaryTree<E>` class and the `Node<E>` class are declared to implement the `Serializable` interface. The `Serializable` interface defines no methods; it is used to provide a marker for classes that can be written to a binary file using the `ObjectOutputStream` and read using the `ObjectInputStream`. We clarify what this means later in the section.

## The `BinaryTree<E>` Class

Table 6.1 shows the design of the `BinaryTree<E>` class. The single data field `root` references the root node of a `BinaryTree<E>` object. It has protected visibility because we will need to access it in subclass `BinarySearchTree`, discussed later in this chapter. In Figure 6.11, we draw

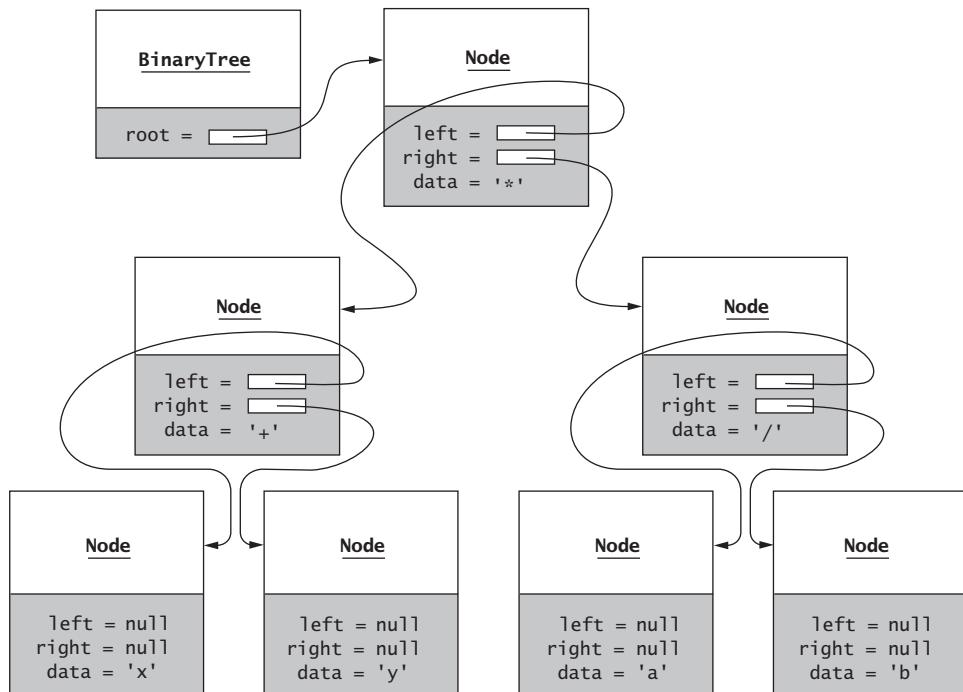
**TABLE 6.1**

Design of the `BinaryTree<E>` Class

Data Field	Attribute
<code>protected Node&lt;E&gt; root</code>	Reference to the root of the tree
Constructor	Behavior
<code>public BinaryTree()</code>	Constructs an empty binary tree
<code>protected BinaryTree(Node&lt;E&gt; root)</code>	Constructs a binary tree with the given node as the root
<code>public BinaryTree(E data, BinaryTree&lt;E&gt; leftTree, BinaryTree&lt;E&gt; rightTree)</code>	Constructs a binary tree with the given data at the root and the two given subtrees
Method	Behavior
<code>public BinaryTree&lt;E&gt; getLeftSubtree()</code>	Returns the left subtree
<code>public BinaryTree&lt;E&gt; getRightSubtree()</code>	Returns the right subtree
<code>public E getData()</code>	Returns the data in the root
<code>public boolean isLeaf()</code>	Returns <code>true</code> if this tree is a leaf, <code>false</code> otherwise
<code>public String toString()</code>	Returns a <code>String</code> representation of the tree
<code>public void preOrderTraverse (BiConsumer&lt;E, Integer&gt; consumer)</code>	Performs a preorder traversal of the tree. Each node and its depth are passed to the consumer function
<code>public static BinaryTree&lt;E&gt; readBinaryTree(Scanner scan)</code>	Constructs a binary tree by reading its data using <code>Scanner scan</code>

**FIGURE 6.11**

Linked Representation of Expression Tree  
 $((x + y) * (a / b))$



the expression tree for  $((x + y) * (a / b))$  using our **Node** representation. Each character shown as tree data would be stored in a **Character** object.

**EXAMPLE 6.1** Assume the tree drawn in Figure 6.11 is referenced by variable `bT` (type `BinaryTree`).

- `bT.root.data` references the **Character** object storing '\*'.
- `bT.root.left` references the left subtree of the root (the root node of tree  $x + y$ ).
- `bT.root.right` references the right subtree of the root (the root node of tree  $a / b$ ).
- `bT.root.right.data` references the **Character** object storing '/'.

The class heading and data field declarations follow:

```
import java.io.*;

/** Class for a binary tree that stores type E objects. */
public class BinaryTree<E> implements Serializable {

    // Insert inner class Node<E> here.

    // Data Field
    /** The root of the binary tree */
    protected Node<E> root;
    ...
}
```

## The Constructors

There are three constructors: a no-parameter constructor, a constructor that creates a tree with a given node as its root, and a constructor that builds a tree from a data value and two trees.

The no-parameter constructor merely sets the data field `root` to `null`.

```
public BinaryTree() {
    root = null;
}
```

The constructor that takes a `Node` as a parameter is a protected constructor. This is because client classes do not know about the `Node` class. This constructor can be used only by methods internal to the `BinaryTree` class and its subclasses.

```
protected BinaryTree(Node<E> root) {
    this.root = root;
}
```

The third constructor takes three parameters: data to be referenced by the root node and two `BinaryTrees` that will become its left and right subtrees.

```
/** Constructs a new binary tree with data in its root leftTree
     as its left subtree and rightTree as its right subtree.
 */
public BinaryTree(E data, BinaryTree<E> leftTree,
                  BinaryTree<E> rightTree) {
    root = new Node<E>(data);
    if (leftTree != null) {
        root.left = leftTree.root;
    } else {
        root.left = null;
    }
    if (rightTree != null) {
        root.right = rightTree.root;
    } else {
        root.right = null;
    }
}
```

If `leftTree` is not `null`, the statement

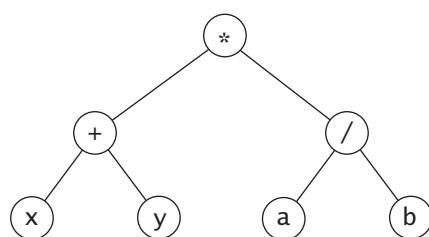
```
root.left = leftTree.root;
```

executes. After its execution, the root node of the tree referenced by `leftTree` (`leftTree.root`) is referenced by `root.left`, making `leftTree` the left subtree of the new root node. If `lT` and `rT` are type `BinaryTree<Character>` and `lT.root` references the root node of binary tree `x + y` and `rT.root` references the root node of binary tree `a / b`, the statement

```
BinaryTree<Character> bT = new BinaryTree<E> ('*', lT, rT);
```

would cause `bT` to reference the tree shown in Figure 6.12.

**FIGURE 6.12**  
The Expression Tree  
 $(x + y) * (a / b)$



### The `getLeftSubtree` and `getRightSubtree` Methods

The `getLeftSubtree` method returns a binary tree whose root is the left subtree of the object on which the method is called. It uses the protected constructor discussed before to construct

a new `BinaryTree<E>` object whose root references the left subtree of this tree. The `getRightSubtree` method is symmetric.

```
/** Return the left subtree.
 * @return The left subtree or null if either the root or
 *         the left subtree is null
 */
public BinaryTree<E> getLeftSubtree() {
    if (root != null && root.left != null) {
        return new BinaryTree<E>(root.left);
    } else {
        return null;
    }
}
```

### The `isLeaf` Method

The `isLeaf` method tests to see whether this tree has any subtrees. If there are no subtrees, then `true` is returned.

```
/** Determine whether this tree is a leaf.
 * @return true if the root has no children
 */
public boolean isLeaf() {
    return (root.left == null && root.right == null);
}
```

### The `toString` Method

The `toString` method generates a string representation of the `BinaryTree` for display purposes. The string representation is a preorder traversal in which each local root is indented a distance proportional to its depth. If a subtree is empty, the string "null" is displayed. The tree in Figure 6.12 would be displayed as follows:

```

*
+
x
  null
  null
y
  null
  null
/
a
  null
  null
b
  null
  null
```

The `toString` method creates a `StringBuilder` and then calls the recursive `toString` method (described next) passing the `root` and 1 (depth of root node) as arguments.

```
public String toString() {
    StringBuilder sb = new StringBuilder();
    toString(root, 1, sb);
    return sb.toString();
}
```

### The Recursive `toString` Method

This method follows the preorder traversal algorithm given in Section 6.2. It begins by appending a string of spaces proportional to the level so that all nodes at a particular level will be

indented to the same point in the tree display. Then, if the node is `null`, the string "null\n" is appended to the `StringBuilder`. Otherwise the string representation of the node is appended to the `StringBuilder` and the method is recursively called on the left and right subtrees.

```
/** Converts a sub-tree to a string.
   Performs a preorder traversal.
   @param node The local root
   @param depth The depth
   @param sb The StringBuilder to save the output
*/
private void toString(Node<E> node, int depth,
                      StringBuilder sb) {
    for (int i = 1; i < depth; i++) {
        sb.append("  ");
    }
    if (node == null) {
        sb.append("null\n");
    } else {
        sb.append(node.toString());
        sb.append("\n");
        toString(node.left, depth + 1, sb);
        toString(node.right, depth + 1, sb);
    }
}
```

## Reading a Binary Tree

If we use a `Scanner` to read the individual lines created by the `toString` method previously discussed, we can reconstruct the binary tree using the algorithm:

1. Read a line that represents information at the root.
2. Remove the leading and trailing spaces using the `String.trim` method.
3. **if** it is "null"
4.     Return `null`.
- else**
5.     Recursively read the left child.
6.     Recursively read the right child.
7.     Return a tree consisting of the root and the two children.

The tree that is constructed will be type `BinaryTree<String>`. The code for a method that implements this algorithm is shown in Listing 6.2.

---

### LISTING 6.2

Method to Read a Binary Tree

```
/** Method to read a binary tree.
   pre: The input consists of a preorder traversal
        of the binary tree. The line "null" indicates a null tree.
   @param scan the Scanner attached to the input file.
   @return The binary tree
*/
public static BinaryTree<String> readBinaryTree(Scanner scan) {
    // Read a line and trim leading and trailing spaces.
    String data = scan.nextLine().trim();
    if (data.equals("null")) {
        return null;
    } else {
        BinaryTree<String> leftTree = readBinaryTree(scan);
```

```

        BinaryTree<String> rightTree = readBinaryTree(scan);
        return new BinaryTree<>(data, leftTree, rightTree);
    }
}

```

## Using an `ObjectOutputStream` and `ObjectInputStream`

The Java API includes the class `ObjectOutputStream` that will write any object that is declared to be `Serializable`. You declare that an object is `Serializable` by adding the declaration

```
implements Serializable
```

to the class declaration. The `Serializable` interface (in `java.io`) contains no methods, but it serves to mark the class as being `Serializable`. This gives you control over whether or not you want your class to be written to an external file. Generally, you will want to have this capability.

To write an object of a `Serializable` class to a file, you do the following:

```

try {
    ObjectOutputStream out =
        new ObjectOutputStream(new FileOutputStream(nameOfFile));
    out.writeObject(nameOfObject);
} catch (Exception ex) {
    ex.printStackTrace();
    System.exit(1);
}

```

The `writeObject` method performs a traversal of whatever data structure is referenced by the object being written. Thus, if the object is a binary tree, a deep copy of all of the nodes of the binary tree will be written to the file.

To read a `Serializable` class from a file, you do the following:

```

try {
    ObjectInputStream in =
        new ObjectInputStream(new FileInputStream(nameOfFile));
    objectName = (objectClass) in.readObject();
} catch (Exception ex) {
    ex.printStackTrace();
    System.exit(1);
}

```

This code will reconstruct the object that was saved to the file, including any referenced objects. Thus, if a `BinaryTree` is written to an `ObjectOutputStream`, this method will read it back and restore it completely.



### PITFALL

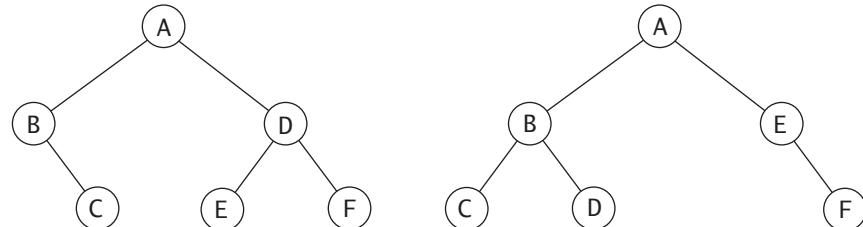
#### Modifying the Class File of a Serialized Object

When an object is serialized, a unique class signature is recorded with the data. If you recompile the Java source file for the class to re-create the `.class` file, even though you did not make any changes, the resulting `.class` file will have a different class signature. When you attempt to read the object, you will get an exception.

## EXERCISES FOR SECTION 6.3

### SELF-CHECK

1. Draw the linked representation of the following two trees.



2. Show the tree that would be built by the following input string:

```

30
15
4
null
null
20
18
null
19
null
null
null
35
32
null
null
38
null
null
  
```

3. What can you say about this tree?

4. Write the strings that would be displayed for the two binary trees in Figure 6.6.

### PROGRAMMING

1. Write a method for the `BinaryTree` class that returns the preorder traversal of a binary tree as a sequence of strings each separated by a space.
2. Write a method to display the postorder traversal of a binary tree in the same form as Programming Exercise 1.
3. Write a method to display the inorder traversal of a binary tree in the same form as Programming Exercise 1, except place a left parenthesis before each subtree and a right parenthesis after each subtree. Don't display anything for an empty subtree. For example, the expression tree shown in Figure 6.12 would be represented as `((x) + (y)) * ((a) / (b))`.



## 6.4 Java 8 Lambda Expressions and Functional Interfaces

Java 8 introduces new features that enable functional programming. In *functional programming*, you can assign functions (methods) to variables or pass them as arguments to another function. The behavior of the function called with a function argument will vary, depending on its function argument. Assume you wrote a function called `plot` that cycled through angles from 0 to 360° in 5° increments and produced a graph showing a particular function value for each angle. If `plot` took a function argument, you could pass it a function such as `sin` or `cosin`. If you passed it `sin`, function `plot` would show a graph of sine values; if you passed it `cosin`, it would show a graph of cosine values.

Java can't really pass methods as arguments, but it accomplishes the same thing using lambda expressions and functional interfaces. A *lambda expression* is a method without a declaration—sometimes called an *anonymous method* because it doesn't have a name. A lambda expression is a shorthand notation that allows you to write a method where you are going to use it. This is useful when a method is going to be used only once and it saves you the effort of writing a separate method declaration. A lambda expression consists of a parameter list and either an expression or a statement block separated by the symbol `->`.

---

**EXAMPLE 6.2** The following lambda expression has no arguments as indicated by the empty parentheses. It simply displays the message shown in the `println`.

```
( ) -> System.out.println("That's all folks") // displays message
```

---



---

**EXAMPLE 6.3** The next lambda expression has the value of `m` cubed. Because it has a single untyped argument `m`, the parentheses are omitted.

```
m -> m * m * m
```

---



---

**EXAMPLE 6.4** The next lambda expression represents a method that returns the larger of its two arguments. Because the method body has multiple statements, it is enclosed in braces. The argument types could be omitted but the parentheses are required.

```
(int x, int y) -> {
    if (x >= y)
        return x;
    return y;
}
```

---

We will see how to execute lambda expressions (anonymous methods) in the next section.



### SYNTAX Lambda Expression

#### FORM:

*(parameter list)* `->` *expression*

or

*(parameter list)* `->` *{statement list}*

**EXAMPLE:**

```
x -> x * x
(d, e) -> {
    sb.append(d.toString());
    sb.append(" : ");
    sb.append(e.toString());
}
```

**INTERPRETATION**

The *parameter list* for the anonymous method has the same syntax as the parameter list for a method—a comma separated list of type identifier sequences enclosed in parentheses. If the compiler can infer the types, they can be omitted. If there is only one parameter, then the parentheses may also be omitted. An empty parameter list is denoted by (). The method body (following the ->) may be an expression or a statement block enclosed in curly braces.

## Functional Interfaces

To cause a lambda expression to be executed, we must first assign it to a variable or pass it as an argument to another method. The data type of the variable being assigned to, or the corresponding parameter, must be a functional interface. A *functional interface* is an interface that declares exactly one abstract method. Java provides a set of functional interfaces, but you can also create your own.

**EXAMPLE 6.5** The following is an example of a custom functional interface called `MyFunction`. It has a single abstract method `apply` that accepts two integer arguments and returns an integer result.

```
@FunctionalInterface
interface MyFunctionType {
    public int apply(int x, int y);
}
```

Listing 6.3 shows a class that uses the lambda expression from Example 6.4 that returns the larger of its two arguments. The statement

```
MyFunctionType mFT = (x, y) -> {
    if (x > y)
        return x;
    return y;
};
```

creates an object of an anonymous class type that implements interface `MyFunctionType`. The statement block to the right of -> implements method `apply`. Therefore, the statement

```
System.out.println("The larger number is : " + mFT.apply(m, n));
```

causes the statement block above to execute with `m` and `n` as its arguments. The larger of the two data values entered will be returned as the method result and displayed.

.....  
**LISTING 6.3**

Using Functional Interface `MyFunctionType` with a Lambda Expression

```
import java.util.Scanner;
@FunctionalInterface
interface MyFunctionType {
    public int apply(int x, int y);
}
```

```

public class TestMyFunctionType {
    public static void main(String[] args) {
        Scanner sc = new Scanner(System.in);
        System.out.println("Enter 2 integers: ");
        int m = sc.nextInt();
        int n = sc.nextInt();

        MyFunctionType mFT = (x, y) -> {
            if (x > y)
                return x;
            return y;
        };

        System.out.println("The larger number is : " + mFT.apply(m, n));
    }
}

```

---

The Java developers created a set of functional interfaces in the `java.util.function` package. Table 6.2 lists some of these; there are many more. The type parameters T and U represent input types and R the result type. Each functional interface has a single abstract method; you need to check the Java documentation to see what it is.

Since our lambda expression in Listing 6.3 takes two arguments and returns a result of the same type, we can insert the statement

```

import java.util.function.BinaryOperator;
in our program and delete our custom interface. Object mFT should implement the
BinaryOperator<T> interface (declared as type BinaryOperator<Integer> instead of

```

**TABLE 6.2**

Selected Java Functional Interfaces

Interface	Abstract Method	Description
<code>BiConsumer&lt;T, U&gt;</code>	<code>void accept(T t, U u)</code>	Represents an operation that accepts two input arguments and returns no result
<code>BiFunction&lt;T, U, R&gt;</code>	<code>R apply(T t, U u)</code>	Represents an operation that accepts two arguments of different types and produces a result
<code>BinaryOperator&lt;T&gt;</code>	<code>T apply(T t1, T t2)</code>	Represents an operation upon two operands of the same type, producing a result of the operand type
<code>Comparator&lt;T&gt;</code>	<code>int compare(T t1, T t2)</code>	Represents an operation that accepts two arguments of the same type and returns a positive, zero, or negative result based on their ordering (negative if $t1 < t2$ , zero if $t1$ equals $t2$ , positive if $t1 > t2$ )
<code>Consumer&lt;T&gt;</code>	<code>void accept(T t)</code>	Represents an operation that accepts one input argument and returns no result
<code>Function&lt;T, R&gt;</code>	<code>R apply(T t)</code>	Represents a function that accepts one argument and produces a result
<code>Predicate&lt;T&gt;</code>	<code>boolean test(T t)</code>	Represents a predicate (boolean-valued function) of one argument

`MyFunctionType`) and the method specified in the lambda expression will still implement abstract method `apply`. The revised declaration for *function object* `mFT` will start with

```
BiConsumer<Integer, Integer> mFT = (x, y) -> {
```

## Passing a Lambda Expression as an Argument

We started our discussion of lambda expressions by stating that we wanted to be able to write a function that could plot the values of another function passed to it. The plot would vary depending on the function argument. To accomplish this in Java, we need to pass a lambda expression as an argument to a method.

**EXAMPLE 6.6** The following method displays the values of a function (its last argument) in the range represented by `low` to `high` in increments of `step`. The function represented by `f` takes an `int` argument and returns a `double` result.

```
/** Displays values associated with function f in the range specified.
 * @param low the lower bound
 * @param high the upper bound
 * @param step the increment
 * @param f the function object
 */
public static void show(int low, int high, int step,
                       Function<Integer, Double> f) {
    for (int i = low; i <= high; i += step) {
        System.out.println(i + " : " + f.apply(i));
    }
}
```

We can call function `show` using the statements

```
Function<Integer, Double> f;
f = angle -> Math.cos(Math.toRadians(angle));
show(0, 360, 30, f);
```

The first statement declares `f` to be an object of type `Function<Integer, Double>`. The second statement assigns `f` to a lambda expression with an integer argument `angle`. The method body implements abstract method `Function.apply`. Therefore, `f.apply(angle)` in `show` will calculate and return the value of the cosine after first converting an angle value to radians. The last statement calls `show`, passing it the range boundaries and increment and function `f`. The for loop body in method `show` will display a table of angle and cosine values for  $0^\circ$  through  $360^\circ$ ,

```
0 : 1.0
30 : 0.8660254037844387
60 : 0.5000000000000001
...
```

A more compact way of doing this would be to replace the three statements above that declare and initialize `f` and call `show` with

```
show(0, 360, 30, angle -> Math.cos(Math.toRadians(angle)));
```

This statement passes the lambda expression directly to method `show` as an anonymous `Function` object.

## A General Preorder Traversal Method

The recursive `toString` method in the previous section performs a preorder traversal to generate the string representation of a tree. There are other applications that require a preorder traversal. Thus, we want to separate the traversal from the action performed on each node. We will specify the action in a lambda interface.

At each node, we need to know the current node and its depth to perform an action, so we will use a functional interface that has parameter types `Node<E>` for the current node and `Integer` for the depth. Table 6.2 shows functional interface `BiConsumer<T, U>` that takes two arguments and has an abstract method `accept` whose return type is `void`. We will specify the action in a lambda expression that instantiates this interface. The preorder traversal starter method is passed a lambda expression with the action to be performed (referenced by `consumer`), and it calls the recursive preorder traversal passing it the root node, 1 for its level, and the object referenced by `consumer`.

```
/** Starter method for preorder traversal
 * @param consumer an object that instantiates
 *                 the BiConsumer interface. Its method implements
 *                 abstract method apply.
 */
public void preOrderTraverse(BiConsumer<E, Integer> consumer) {
    preOrderTraverse(root, 1, consumer);
}
```

The private `preOrderTraverse` method performs the actual traversal.

```
/**
 * Performs a recursive pre-order traversal of the tree,
 * applying the action specified in the consumer object.
 * @param consumer object whose accept method specifies
 *                 the action to be performed on each node
 */
private void preOrderTraverse(Node<E> node, int depth,
                             BiConsumer<E, Integer> consumer) {
    if (node == null) {
        consumer.accept(null, depth);
    } else {
        consumer.accept(node.data, depth);
        preOrderTraverse(node.left, depth + 1, consumer);
        preOrderTraverse(node.right, depth + 1, consumer);
    }
}
```

## Using `preOrderTraverse`

We can rewrite the `toString` method to use the `preOrderTraverse` method. The `preOrderTraverse` method visits each node in preorder applying the statement block specified in the lambda expression passed as an argument to the preorder traversal methods. The lambda expression passed by `toString` to `PreOrderTraverse` has arguments representing the node and its depth. When `PreOrderTraverse` applies its statement block to a node, it creates a new `StringBuilder` object consisting of `d` (the node depth) blanks followed by the string representation of the current node and "`\n`".

```
public String toString() {
    final StringBuilder sb = new StringBuilder();
    preOrderTraverse((e, d) -> {
        for (int i = 1; i < d; i++) {
```

```

        sb.append(" ");
    }
    sb.append(e);
    sb.append("\n");
});
return sb.toString();
}

```

## EXERCISES FOR SECTION 6.4

### SELF-CHECK

1. Describe the effect of each lambda expression below that is valid. Correct the expression if it is not a valid lambda expression and then describe its effect.
  - a. `(int m, n) -> 2 * m + 3 * n`
  - b. `(int m, double x) -> m * (int) x`
  - c. `msg -> return "****" + msg + "****"`
  - d. `-> System.out.println("Hello")`
  - e. `(x, y) -> {System.out.println(x);  
                System.out.println("_____");  
                System.out.println(y);  
                System.out.println("_____");  
                System.out.println("_____");  
};`
  - f. `(x, y) -> Math.sqrt(x * x + y * y)`
  - g. `(x) -> x > 0`
2. Select a Functional Interface appropriate for each of the expressions in Self-Check Exercise 1 from Table 6.2 or the `java.util.function` library.
3. Declare a function object of the correct type for each of the lambda expressions in Self-Check Exercise 1 and assign the lambda expression to it.

### PROGRAMMING

1. Write a lambda expression that takes a double argument (`x`) and an integer argument (`n`). The method result should be the double value raised to the power `n`. Do this using a Java API method and also using a loop.
2. Write and test a Java class that enters two numbers and displays the result of calling each lambda expression in Programming Exercise 1. Also, compare the results to make sure that they are the same.
3. Modify the program in Example 6.6 to use two function objects that calculate trigonometric values and display the angle and corresponding values for each function object on the same line.
4. Write a general `postOrderTraverse` method for the `BinaryTree` class similar to the `preOrderTraverse` method.
5. Write a general `inOrderTraverse` method for the `BinaryTree` class similar to the `preOrderTraverse` method.

## 6.5 Binary Search Trees

### Overview of a Binary Search Tree

In Section 6.1, we provided the following recursive definition of a binary search tree:

A set of nodes  $T$  is a binary search tree if either of the following is true:

- $T$  is empty.
- If  $T$  is not empty, its root node has two subtrees,  $T_L$  and  $T_R$ , such that  $T_L$  and  $T_R$  are binary search trees and the value in the root node of  $T$  is greater than all values in  $T_L$  and is less than all values in  $T_R$ .

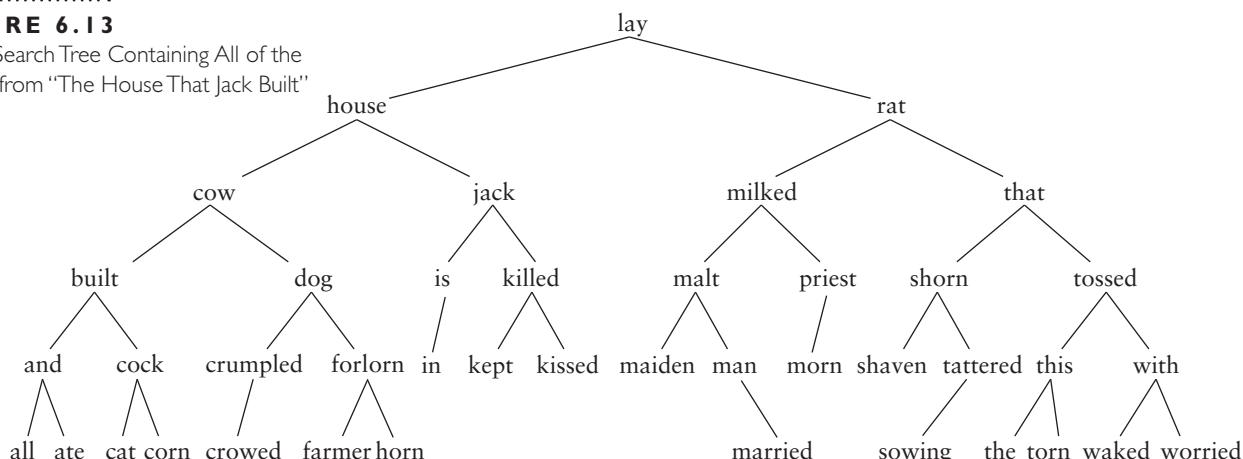
Figure 6.13 shows a binary search tree that contains the words in lowercase from the nursery rhyme “The House That Jack Built.” We can use the following algorithm to find an object in a binary search tree.

### Recursive Algorithm for Searching a Binary Search Tree

1. **if** the root is **null**  
2.     The item is not in the tree; return **null**.  
3.     Compare the value of **target**, the item being sought, with **root.data**.  
4.     **if** they are equal  
5.         The target has been found, return the data at the root.  
    **else if** target is less than **root.data**  
6.         Return the result of searching the left subtree.  
    **else**  
7.         Return the result of searching the right subtree.

**FIGURE 6.13**

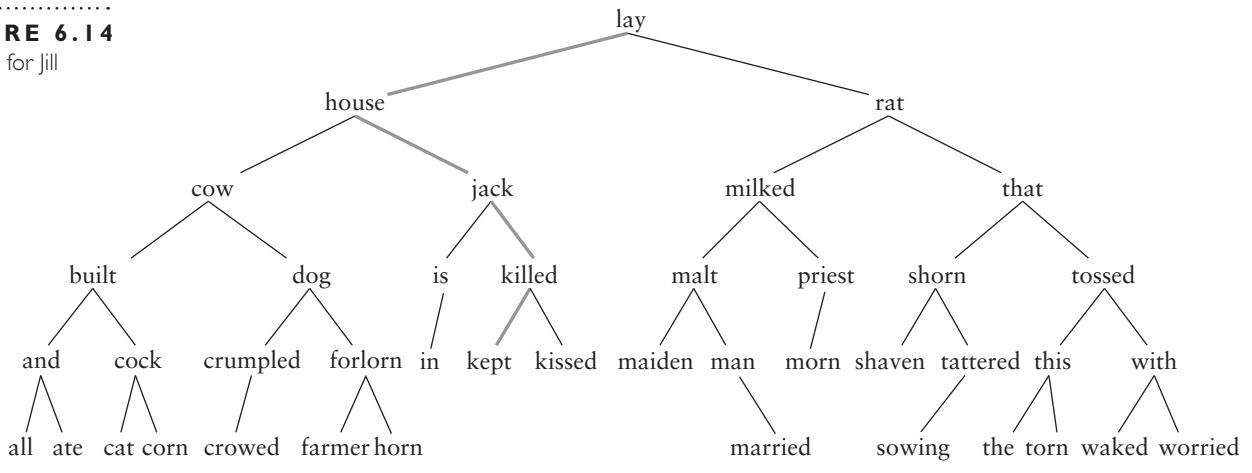
Binary Search Tree Containing All of the Words from “The House That Jack Built”



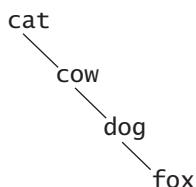
**EXAMPLE 6.7** Suppose we wish to find *jill* in Figure 6.13. We first compare *jill* with *lay*. Because *jill* is less than *lay*, we continue the search with the left subtree and compare *jill* with *house*. Because *jill* is greater than *house*, we continue with the right subtree and compare *jill* with *jack*. Because *jill* is greater than *jack*, we continue with *killed* followed by *kept*. Now, *kept* has no left child, and *jill* is less than *kept*, so we conclude that *jill* is not in this binary search tree. (She's in a different nursery rhyme.) Follow the path shown in gray in Figure 6.14.

**FIGURE 6.14**

Looking for Jill

**TABLE 6.3**The `SearchTree<E>` Interface

Method	Behavior
<code>boolean add(E item)</code>	Inserts <code>item</code> where it belongs in the tree. Returns <code>true</code> if item is inserted; <code>false</code> if it isn't (already in tree)
<code>boolean contains(E target)</code>	Returns <code>true</code> if <code>target</code> is found in the tree
<code>E find(E target)</code>	Returns a reference to the data in the node that is equal to <code>target</code> . If no such node is found, returns <code>null</code>
<code>E delete(E target)</code>	Removes <code>target</code> (if found) from tree and returns it; otherwise, returns <code>null</code>
<code>boolean remove(E target)</code>	Removes <code>target</code> (if found) from tree and returns <code>true</code> ; otherwise, returns <code>false</code>



## Performance

Searching the tree in Figure 6.14 is  $O(\log n)$ . However, if a tree is not very full, performance will be worse. The tree in the figure at left has only right subtrees, so searching it is  $O(n)$ .

## Interface `SearchTree`

As described, the binary search tree is a data structure that enables efficient insertion, search, and retrieval of information (best case is  $O(\log n)$ ). Table 6.3 shows a `SearchTree<E>` interface for a class that implements the binary search tree. The interface includes methods for insertion (`add`), search (`boolean contains` and `E find`), and removal (`E delete` and `boolean remove`). Next, we discuss a class `BinarySearchTree<E>` that implements this interface.

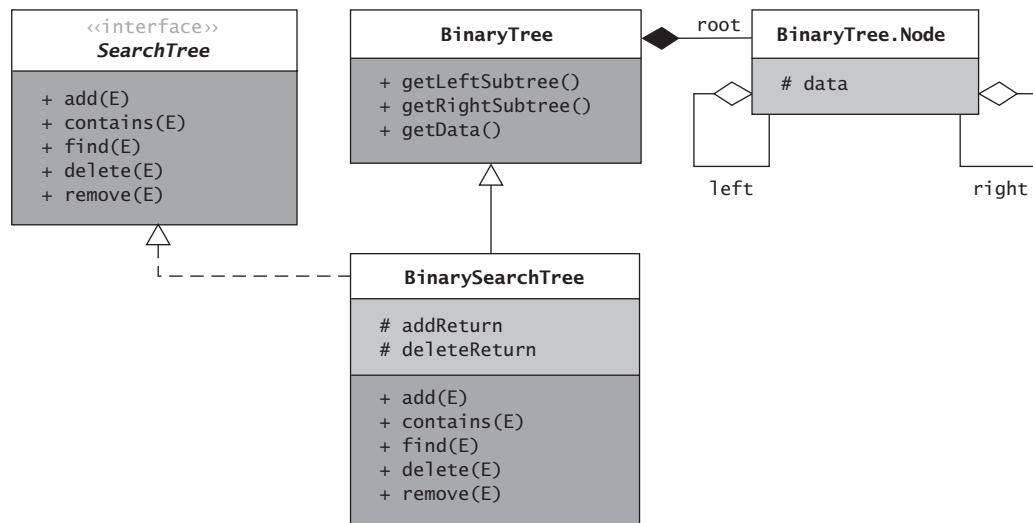
## The `BinarySearchTree` Class

Next, we implement class `BinarySearchTree<E>` that extends `Comparable<E>`. The type parameter specified when we create a new `BinarySearchTree` must implement the `Comparable` interface.

Table 6.4 shows the data fields declared in the class. These data fields are used to store a second result from the recursive add and delete methods that we will write for this class. Neither result can be returned directly from the recursive add or delete method because they return a reference to a tree node affected by the insertion or deletion operation. The interface for method `add` in Table 6.3 requires a `boolean` result (stored in `addReturn`) to indicate success

**TABLE 6.4**Data Fields of Class `BinarySearchTree<E extends Comparable<E>>`

Data Field	Attribute
<code>protected boolean addReturn</code>	Stores a second return value from the recursive <code>add</code> method that indicates whether the item has been inserted
<code>protected E deleteReturn</code>	Stores a second return value from the recursive <code>delete</code> method that references the item that was stored in the tree

**FIGURE 6.15**UML Diagram of `BinarySearchTree`

or failure. Similarly, the interface for `delete` requires a type `E` result (stored in `deleteReturn`) that is either the item deleted or `null`.

The class heading and data field declarations follow. Note that class `BinarySearchTree` extends class `BinaryTree` and implements the `SearchTree` interface (see Figure 6.15). Besides the data fields shown, class `BinarySearchTree` inherits the data field `root` from class `BinaryTree` (declared as `protected`) and the inner class `Node<E>`.

```

public class BinarySearchTree<E extends Comparable<E>>
    extends BinaryTree<E> implements SearchTree<E> {
    // Data Fields
    /** Return value from the public add method. */
    protected boolean addReturn;
    /** Return value from the public delete method. */
    protected E deleteReturn;
    ...
}
  
```

### Implementing the `find` Methods

Earlier, we showed a recursive algorithm for searching a binary search tree. Next, we show how to implement this algorithm and a nonrecursive starter method for the algorithm. Our method `find` will return a reference to the node that contains the information we are seeking.

Listing 6.4 shows the code for method `find`. The starter method calls the recursive method with the tree root and the object being sought as its parameters. If `bST` is a reference to a `BinarySearchTree`, the method call `bST.find(target)` invokes the starter method.

The recursive method first tests the local root for `null`. If it is `null`, the object is not in the tree, so `null` is returned. If the local root is not `null`, the statement

```
int compResult = target.compareTo(localRoot.data);
```

compares `target` to the data at the local root. Recall that method `compareTo` returns an `int` value that is negative, zero, or positive depending on whether the object (`target`) is less than, equal to, or greater than the argument (`localRoot.data`).

If the objects are equal, we return the data at the local root. If `target` is smaller, we recursively call the method `find`, passing the left subtree root as the parameter.

```
return find(localRoot.left, target);
```

Otherwise, we call `find` to search the right subtree.

```
return find(localRoot.right, target);
```

#### **LISTING 6.4**

##### `BinarySearchTree` `find` Method

```
/** Starter method find.
   pre: The target object must implement
         the Comparable interface.
   @param target The Comparable object being sought
   @return The object, if found, otherwise null
 */
public E find(E target) {
    return find(root, target);
}

/** Recursive find method.
   @param localRoot The local subtree's root
   @param target The object being sought
   @return The object, if found, otherwise null
 */
private E find(Node<E> localRoot, E target) {
    if (localRoot == null)
        return null;

    // Compare the target with the data field at the root.
    int compResult = target.compareTo(localRoot.data);
    if (compResult == 0)
        return localRoot.data;
    else if (compResult < 0)
        return find(localRoot.left, target);
    else
        return find(localRoot.right, target);
}
```

## **Insertion into a Binary Search Tree**

Inserting an item into a binary search tree follows a similar algorithm as searching for the item because we are trying to find where in the tree the item would be, if it were there. In searching, a result of `null` is an indicator of failure; in inserting, we replace this `null` with a new leaf that contains the new item. If we reach a node that contains the object we are trying to insert, then we can't insert it (duplicates are not allowed), so we return `false` to indicate that we were unable to perform the insertion. The insertion algorithm follows.

### Recursive Algorithm for Insertion in a Binary Search Tree

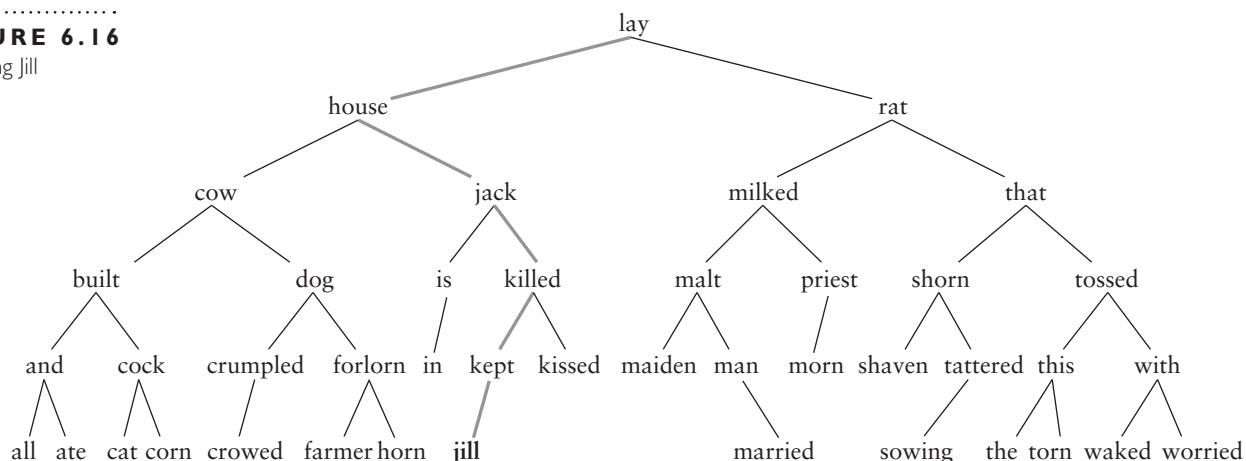
1. if the root is `null`
2. Replace empty tree with a new tree with the item at the root and return `true`.
3. else if the item is equal to `root.data`
4. The item is already in the tree; return `false`.
5. else if the item is less than `root.data`
6. Recursively insert the item in the left subtree.
7. else
8. Recursively insert the item in the right subtree.

The algorithm returns `true` when the new object is inserted and `false` if it is a duplicate (the second stopping case). The first stopping case tests for an empty tree. If so, a new `BinarySearchTree` is created and the new item is stored in its root node (Step 2).

### EXAMPLE 6.8

To insert *jill* into Figure 6.13, we would follow the steps shown in Example 6.7 except that when we reached *kept*, we would insert *jill* as the left child of the node that contains *kept* (see Figure 6.16).

**FIGURE 6.16**  
Inserting Jill



### Implementing the add Methods

Listing 6.5 shows the code for the starter and recursive add methods. The recursive add follows the algorithm presented earlier, except that the return value is the new (sub)tree that contains the inserted item. The data field `addReturn` is set to `true` if the item is inserted and to `false` if the item already exists. The starter method calls the recursive method with the root as its argument. The root is set to the value returned by the recursive method (the modified tree). The value of `addReturn` is then returned to the caller.

In the recursive method, the statements

```

addReturn = true;
return new Node<>(item);
  
```

execute when a `null` branch is reached. The first statement sets the insertion result to `true`; the second returns a new node containing `item` as its data.

The statements

```
addReturn = false;
return localRoot;
```

execute when `item` is reached. The first statement sets the insertion result to `false`; the second returns a reference to the subtree that contains `item` in its root.

If `item` is less than the root's data, the statement

```
localRoot.left = add(localRoot.left, item);
```

attempts to insert `item` in the left subtree of the local root. After returning from the call, this left subtree is set to reference the modified subtree, or the original subtree if there is no insertion. The statement

```
localRoot.right = add(localRoot.right, item);
```

affects the right subtree of `localRoot` in a similar way.

.....

#### **LISTING 6.5**

BinarySearchTree add Methods

```
/** Starter method add.
   pre: The object to insert must implement the
        Comparable interface.
   @param item The object being inserted
   @return true if the object is inserted, false
           if the object already exists in the tree
 */
public boolean add(E item) {
    root = add(root, item);
    return addReturn;
}

/** Recursive add method.
   post: The data field addReturn is set true if the item is added to
         the tree, false if the item is already in the tree.
   @param localRoot The local root of the subtree
   @param item The object to be inserted
   @return The new local root that now contains the
           inserted item
 */
private Node<E> add(Node<E> localRoot, E item) {
    if (localRoot == null) {
        // item is not in the tree - insert it.
        addReturn = true;
        return new Node<E>(item);
    } else if (item.compareTo(localRoot.data) == 0) {
        // item is equal to localRoot.data
        addReturn = false;
        return localRoot;
    } else if (item.compareTo(localRoot.data) < 0) {
        // item is less than localRoot.data
        localRoot.left = add(localRoot.left, item);
        return localRoot;
    } else {
        // item is greater than localRoot.data
        localRoot.right = add(localRoot.right, item);
        return localRoot;
    }
}
```



## PROGRAM STYLE

### Comment on Insertion Algorithm and add Methods

Note that as we return along the search path, the statement

```
localRoot.left = add(localRoot.left, item);  
or
```

```
localRoot.right = add(localRoot.right, item);
```

resets each local root to reference the modified tree below it. You may wonder whether this is necessary. The answer is “No.” In fact, it is only necessary to reset the reference from the parent of the new node to the new node; all references above the parent remain the same. We can modify the insertion algorithm to do this by checking for a leaf node before making the recursive call to add:

- 5.1. **else if** the item is less than `root.data`
- 5.2.     **if** the local root is a leaf node.
- 5.3.         Reset the left subtree to reference a new node with the item as its data.
- else**
- 5.4.         Recursively insert the item in the left subtree.

A similar change should be made for the case where `item` is greater than the local root’s data. You would also have to modify the starter `add` method to check for an empty tree and insert the new item in the root node if the tree is empty instead of calling the recursive `add` method.

One reason we did not write the algorithm this way is that we want to be able to adjust the tree if the insertion makes it unbalanced. This involves resetting one or more branches above the insertion point. We discuss how this is done in Chapter 9.



## PROGRAM STYLE

### Multiple Calls to compareTo

Method `add` has two calls to method `compareTo`. We wrote it this way so that the code mirrors the algorithm. However, it would be more efficient to call `compareTo` once and save the result in a local variable as we did for method `find`. Depending on the number and type of data fields being compared, the extra call to method `compareTo` could be costly.

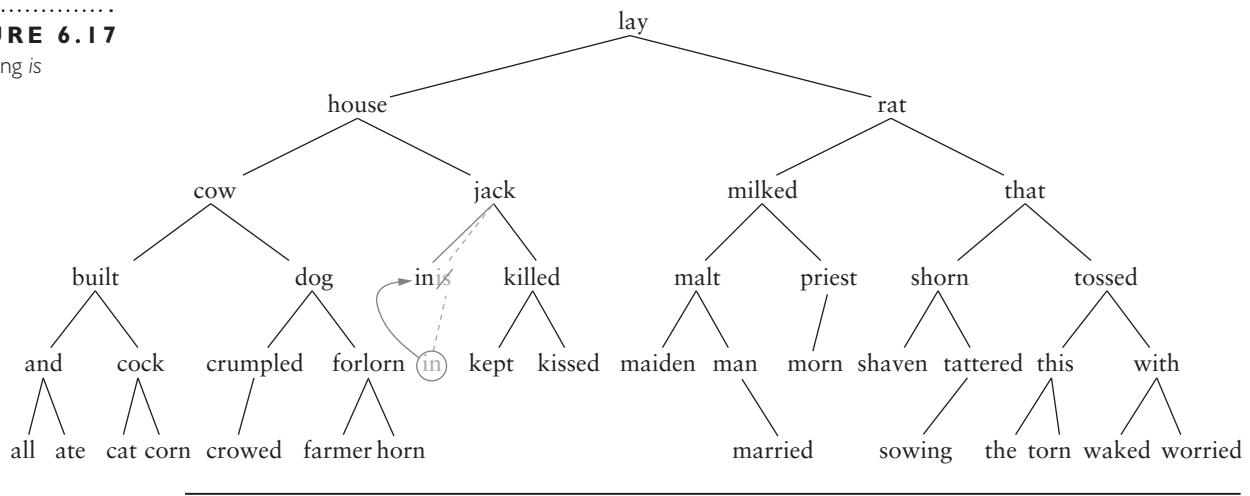
## Removal from a Binary Search Tree

Removal also follows the search algorithm except that when the item is found, it is removed. If the item is a leaf node, then its parent’s reference to it is set to `null`, thereby removing the leaf node. If the item has only a left or right child, then the grandparent references the remaining child instead of the child’s parent (the node we want to remove).

**EXAMPLE 6.9** If we remove *is* from Figure 6.13, we can replace it with *in*. This is accomplished by changing the left child reference in *jack* (the grandparent) to reference *in* (see Figure 6.17).

FIGURE 6.17

Removing *is*

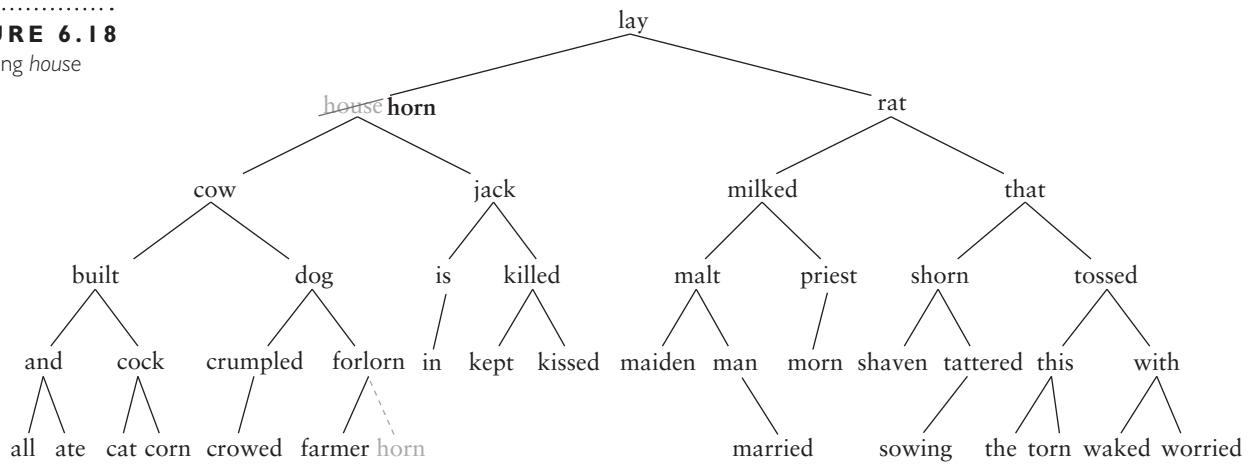


A complication arises when the item we wish to remove has two children. In this case, we need to find a replacement parent for the children. Remember that the parent must be larger than all of the data fields in the left subtree and smaller than all of the data fields in the right subtree. If we take the largest item in the left subtree and promote it to be the parent, then all of the remaining items in the left subtree will be smaller. This item is also less than the items in the right subtree. This item is also known as the *inorder predecessor* of the item being removed. (We could use the inorder successor instead; this is discussed in the exercises.)

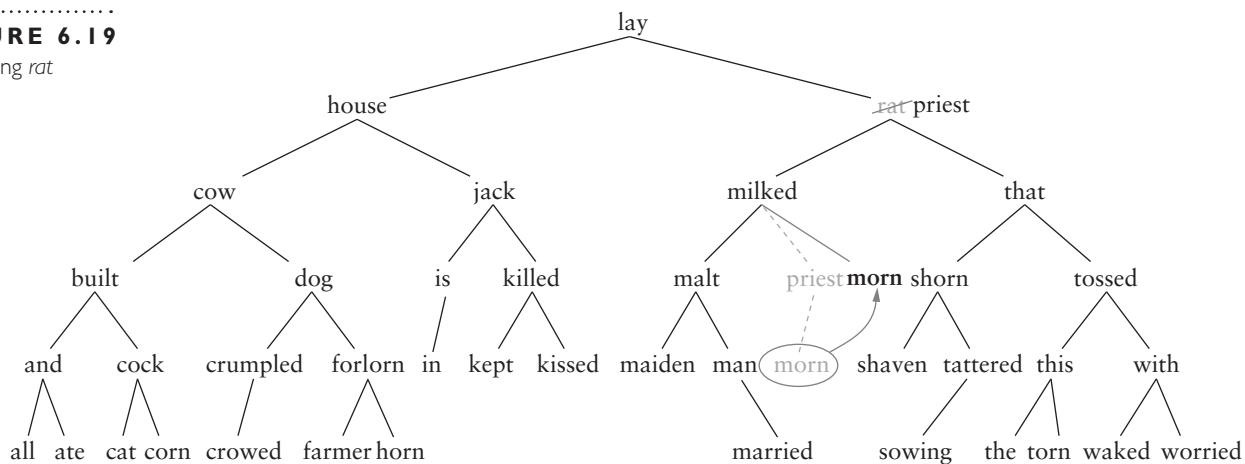
**EXAMPLE 6.10** If we remove *house* from Figure 6.13, we look in the left subtree (root contains *cow*) for the largest item, *horn*. We then replace *house* with *horn* and remove the node containing *horn* (see Figure 6.18).

FIGURE 6.18

Removing *house*



**EXAMPLE 6.11** If we want to remove *rat* from the tree in Figure 6.13, we would start the search for the inorder successor at *milked* and see that it has a right child, *priest*. If we now look at *priest*, we see that it does not have a right child, but it does have a left child. We would then replace *rat* with *priest* and replace the reference to *priest* in *milked* with a reference to *morn* (the left subtree of the node containing *priest*). See Figure 6.19.

**FIGURE 6.19**Removing *rat*

### Recursive Algorithm for Removal from a Binary Search Tree

1. **if** the root is **null**  
The item is not in tree – return **null**.
2. Compare the item to the data at the local root.
3. **if** the item is less than the data at the local root  
Return the result of deleting from the left subtree.
4. **else if** the item is greater than the local root  
Return the result of deleting from the right subtree.
5. **else // The item is in the local root**  
Store the data in the local root in **deleteReturn**.
6. **if** the local root has no children  
Set the parent of the local root to reference **null**.
7. **else if** the local root has one child  
Set the parent of the local root to reference that child.
8. **else // Find the inorder predecessor**  
**if** the left child has no right child it is the inorder predecessor  
Set the parent of the local root to reference the left child.
9. **else**  
Find the rightmost node in the right subtree of the left child.  
Copy its data into the local root's data and remove it by setting its parent to reference its left child.

## Implementing the delete Methods

Listing 6.6 shows both the starter and the recursive delete methods. As with the add method, the recursive delete method returns a reference to a modified tree that, in this case, no longer contains the item. The public starter method is expected to return the item removed. Thus, the recursive method saves this value in the data field `deleteReturn` before removing it from the tree. The starter method then returns this value.

---

### LISTING 6.6

#### BinarySearchTree delete Methods

```
/** Starter method delete.
   post: The object is not in the tree.
   @param target The object to be deleted
   @return The object deleted from the tree
           or null if the object was not in the tree
   @throws ClassCastException if target does not implement
           Comparable
 */
public E delete(E target) {
    root = delete(root, target);
    return deleteReturn;
}
/** Recursive delete method.
   post: The item is not in the tree;
         deleteReturn is equal to the deleted item
         as it was stored in the tree or null
         if the item was not found.
   @param localRoot The root of the current subtree
   @param item The item to be deleted
   @return The modified local root that does not contain
           the item
 */
private Node<E> delete(Node<E> localRoot, E item) {
    if (localRoot == null) {
        // item is not in the tree.
        deleteReturn = null;
        return localRoot;
    }

    // Search for item to delete.
    int compResult = item.compareTo(localRoot.data);
    if (compResult < 0) {
        // item is smaller than localRoot.data.
        localRoot.left = delete(localRoot.left, item);
        return localRoot;
    } else if (compResult > 0) {
        // item is larger than localRoot.data.
        localRoot.right = delete(localRoot.right, item);
        return localRoot;
    } else {
        // item is at local root.
        deleteReturn = localRoot.data;
        if (localRoot.left == null) {
            // If there is no left child, return right child
            // which can also be null.
            return localRoot.right;
        }
    }
}
```

```

} else if (localRoot.right == null) {
    // If there is no right child, return left child.
    return localRoot.left;
} else {
    // Node being deleted has 2 children, replace the data
    // with inorder predecessor.
    if (localRoot.left.right == null) {
        // The left child has no right child.
        // Replace the data with the data in the
        // left child.
        localRoot.data = localRoot.left.data;
        // Replace the left child with its left child.
        localRoot.left = localRoot.left.left;
        return localRoot;
    } else {
        // Search for the inorder predecessor (ip) and
        // replace deleted node's data with ip.
        localRoot.data = findLargestChild(localRoot.left);
        return localRoot;
    }
}
}

```

For the recursive method, the two stopping cases are an empty tree and a tree whose root contains the item being removed. We first test to see whether the tree is empty (local root is `null`). If so, then the item sought is not in the tree. The `deleteReturn` data field is set to `null`, and the local root is returned to the caller.

Next, `localRoot.data` is compared to the item to be deleted. If the item to be deleted is less than `localRoot.data`, it must be in the left subtree if it is in the tree at all, so we set `localRoot.left` to the value returned by recursively calling this method.

```
localRoot.left = delete(localRoot.left, item);
```

If the item to be deleted is greater than `localRoot.data`, the statement

```
localRoot.right = delete(localRoot.right, item);
```

affects the right subtree of `localRoot` in a similar way.

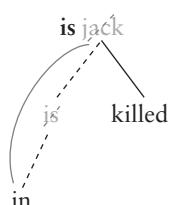
If `localRoot.data` is the item to be deleted, we have reached the second stopping case, which begins with the lines

```
    } else {
        // item is at local root.
        deleteReturn = localRoot.data;
```

The value of `localRoot.data` is saved in `deleteReturn`. If the node to be deleted has one child (or zero children), we return a reference to the only child (or `null`), so the parent of the deleted node will reference its only grandchild (or `null`).

If the node to be deleted (*jack* in the figure at left) has two children, we need to find the replacement for this node. If its left child has no right subtree, the left child (*is*) is the inorder predecessor. The first statement below

```
localRoot.data = localRoot.left.data;  
// Replace the left child with its left child.  
localRoot.left = localRoot.left.left;
```



copies the left child's data into the local node's data (*is* to *jack*); the second resets the local node's left branch to reference its left child's left subtree (*in*).

If the left child of the node to be deleted has a right subtree, the statement

```
localRoot.data = findLargestChild(localRoot.left);
```

calls `findLargestChild` to find the largest child and to remove it. The largest child's data is referenced by `localRoot.data`. This is illustrated in Figure 6.19. The left child *milked* of the node to be deleted (*rat*) has a right child *priest*, which is its largest child. Therefore, *priest* becomes referenced by `localRoot.data` (replacing *rat*) and *morn* (the left child of *priest*) becomes the new right child of *milked*.

## Method `findLargestChild`

Method `findLargestChild` (see Listing 6.7) takes the parent of a node as its argument. It then follows the chain of rightmost children until it finds a node whose right child does not itself have a right child. This is done via tail recursion.

When a parent node is found whose right child has no right child, the right child is the inorder predecessor of the node being deleted, so the data value from the right child is saved.

```
E returnValue = parent.right.data;
parent.right = parent.right.left;
```

The right child is then removed from the tree by replacing it with its left child (if any).

.....

### LISTING 6.7

`BinarySearchTree` `findLargestChild` Method

```
/** Find the node that is the
   inorder predecessor and replace it
   with its left child (if any).
   post: The inorder predecessor is removed from the tree.
   @param parent The parent of possible inorder
              predecessor (ip)
   @return The data in the ip
*/
private E findLargestChild(Node<E> parent) {
    // If the right child has no right child, it is
    // the inorder predecessor.
    if (parent.right.right == null) {
        E returnValue = parent.right.data;
        parent.right = parent.right.left;
        return returnValue;
    } else {
        return findLargestChild(parent.right);
    }
}
```

## Testing a Binary Search Tree

To test a binary search tree, you need to verify that an inorder traversal will display the tree contents in ascending order after a series of insertions (to build the tree) and deletions are performed. You need to write a `toString` method for a `BinarySearchTree` that returns the `String` built from an inorder traversal (see Programming Exercise 3).

## CASE STUDY Writing an Index for a Term Paper

**Problem** You would like to write an index for a term paper. The index should show each word in the paper followed by the line number on which it occurred. The words should be displayed in alphabetical order. If a word occurs on multiple lines, the line numbers should be listed in ascending order. For example, the three lines

```
a, 3
a, 13
are, 3
```

show that the word *a* occurred on lines 3 and 13 and the word *are* occurred on line 3.

**Analysis** A binary search tree is an ideal data structure to use for storing the index entries. We can store each word and its line number as a string in a tree node. For example, the two occurrences of the word *Java* on lines 5 and 10 could be stored as the strings "java, 005" and "java, 010". Each word will be stored in lowercase to ensure that it appears in its proper position in the index. The leading zeros are necessary so that the string "java, 005" is considered less than the string "java, 010". If the leading zeros were removed, this would not be the case ("java, 5" is greater than "java, 10"). After all the strings are stored in the search tree, we can display them in ascending order by performing an inorder traversal. Storing each word in a search tree is an  $O(\log n)$  process where  $n$  is the number of words currently in the tree. Storing each word in an ordered list would be an  $O(n)$  process.

**Design** We can represent the index as an instance of the `BinarySearchTree` class just discussed or as an instance of a binary search tree provided in the Java API. The Java API provides a class `TreeSet<E>` (discussed further in Section 7.1) that uses a binary search tree as its basis. Class `TreeSet<E>` provides three of the methods in interface `SearchTree`: insertion (`add`), search (`boolean contains`), and removal (`boolean remove`). It also provides an iterator that enables inorder access to the elements of a tree. Because we are only doing tree insertion and inorder access, we will use class `TreeSet<E>`.

We will write a class `IndexGenerator` (see Table 6.5) with a `TreeSet<String>` data field. Method `buildIndex` will read each word from a data file and store it in the search tree. Method `showIndex` will display the index.

**TABLE 6.5**

Data Fields and Methods of Class `IndexGenerator`

Data Field	Attribute
<code>private TreeSet&lt;String&gt; index</code>	The search tree used to store the index
<code>private static final String PATTERN</code>	Pattern for extracting words from a line. A word is a string of one or more letters or numbers or characters
Method	Behavior
<code>public void buildIndex(Scanner scan)</code>	Reads each word from the file scanned by <code>scan</code> and stores it in tree <code>index</code>
<code>public void showIndex()</code>	Performs an inorder traversal of tree <code>index</code>

**Implementation** Listing 6.8 shows class IndexGenerator. In method `buildIndex`, the repetition condition for the outer while loop calls method `hasNextLine`, which scans the next data line into a buffer associated with Scanner `scan` or returns `null` (causing loop exit) if all lines were scanned. If the next line is scanned, the repetition condition for the inner while loop below

```
while ((token = scan.findInLine(PATTERN)) != null) {
    token = token.toLowerCase();
    index.add(String.format("%s, %3d", token, lineNumber));
}
```

calls Scanner method `findInLine` to extract a token from the buffer (a sequence of letters, digits, and the apostrophe character). Next, it inserts in `index` a string consisting of the next token in lowercase followed by a comma, a space, and the current line number formatted with leading spaces so that it occupies a total of three columns. This format is prescribed by the first argument "`%s, %3d`" passed to method `String.format` (see Appendix A.5). The inner loop repeats until `findInLine` returns `null`, at which point the inner loop is exited, the buffer is emptied by the statement

```
scan.nextLine(); // Clear the scan buffer
```

and the outer loop is repeated.

.....

**LISTING 6.8**

Class `IndexGenerator.java`

```
import java.io.*;
import java.util.*;

/** Class to build an index. */
public class IndexGenerator {

    // Data Fields
    /** Tree for storing the index. */
    private final TreeSet<String> index;

    /** Pattern for extracting words from a line. A word is a string of
        one or more letters or numbers or ' characters */
    private static final String PATTERN =
        "[\\p{L}\\p{N}']+";

    // Methods
    public IndexGenerator() {
        index = new TreeSet<>();
    }

    /** Reads each word in a data file and stores it in an index
        along with its line number.
        post: Lowercase form of each word with its line
              number is stored in the index.
        @param scan A Scanner object
    */
    public void buildIndex(Scanner scan) {
        int lineNumber = 0; // line number

        // Keep reading lines until done.
        while (scan.hasNextLine()) {
            lineNumber++;
            while ((token = scan.findInLine(PATTERN)) != null) {
                token = token.toLowerCase();
                index.add(String.format("%s, %3d", token, lineNumber));
            }
            scan.nextLine(); // Clear the scan buffer
        }
    }
}
```

```

        // Extract each token and store it in index.
        String token;
        while ((token = scan.findInLine(PATTERN)) != null) {
            token = token.toLowerCase();
            index.add(String.format("%s, %3d", token, lineNumber));
        }
        scan.nextLine(); // Clear the scan buffer
    }

/** Displays the index, one word per line. */
public void showIndex() {
    index.forEach(next -> System.out.println(next));
}
}

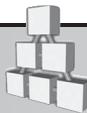
```

Method `showIndex` at the end of Listing 6.8 uses the the default method `forEach` to display each line of the index. We describe the `forEach` in the next syntax box. Without the `forEach`, we could use the enhanced for loop below with an iterator.

```

public void showIndex() {
    // Use an iterator to access and display tree data.
    for (String next : index) {
        System.out.println(next);
    }
}

```



## SYNTAX Using The Java 8 forEach statement

### FORM

`iterable.forEach(lambda expression);`

### EXAMPLE

`index.forEach(next -> System.out.println(next));`

### INTERPRETATION

Java 8 added the default method `forEach` to the `Iterable` interface. A *default method* enables you to add new functionality to an interface while still retaining compatibility with earlier implementations that did not provide this method. The `forEach` method applies a method (represented by `lambda expression`) to each item of an `Iterable` object. Since the `Set` interface extends the `Iterable` interface and `TreeSet` implements `Set`, we can use the `forEach` method on the `index` as shown in the example above.

**Testing** To test class `IndexGenerator`, write a `main` method that declares new `Scanner` and `IndexGenerator<String>` objects. The `Scanner` can reference any text file stored on your hard drive. Make sure that duplicate words are handled properly (including duplicates on the same line), that words at the end of each line are stored in the index, that empty lines are processed correctly, and that the last line of the document is also part of the index.

## EXERCISES FOR SECTION 6.5

### SELF-CHECK

1. Show the tree that would be formed for the following data items. Exchange the first and last items in each list, and rebuild the tree that would be formed if the items were inserted in the new order.
  - a. happy, depressed, manic, sad, ecstatic
  - b. 45, 30, 15, 50, 60, 20, 25, 90
2. Explain how the tree shown in Figure 6.13 would be changed if you inserted *mother*. If you inserted *jane*? Does either of these insertions change the height of the tree?
3. Show or explain the effect of removing the nodes *kept*, *cow* from the tree in Figure 6.13.
4. In Exercise 3 above, a replacement value must be chosen for the node *cow* because it has two children. What is the relationship between the replacement word and the word *cow*? What other word in the tree could also be used as a replacement for *cow*? What is the relationship between that word and the word *cow*?
5. The algorithm for deleting a node does not explicitly test for the situation where the node being deleted has no children. Explain why this is not necessary.
6. In Step 19 of the algorithm for deleting a node, when we replace the reference to a node that we are removing with a reference to its left child, why is it not a concern that we might lose the right subtree of the node that we are removing?

### PROGRAMMING

1. Write methods `contains` and `remove` for the `BinarySearchTree` class. Use methods `find` and `delete` to do the work.
2. Self-Check Exercise 4 indicates that two items can be used to replace a data item in a binary search tree. Rewrite method `delete` so that it retrieves the leftmost element in the right subtree instead. You will also need to provide a method `findSmallestChild`.
3. Write a `main` method to test a binary search tree. Write a `toString` method that returns the tree contents in ascending order (using an inorder traversal) with newline characters separating the tree elements.
4. Write a `main` method for the index generator that declares new `Scanner` and `IndexGenerator` objects. The `Scanner` can reference any text file stored on your hard drive.

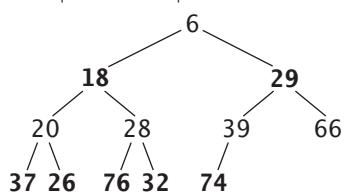


## 6.6 Heaps and Priority Queues

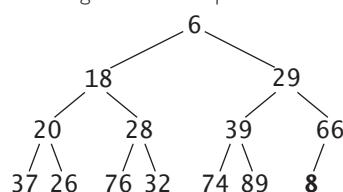
In this section, we discuss a binary tree that is ordered but in a different way from a binary search tree. At each level of a heap, the value in a node is less than all values in its two subtrees. Figure 6.20 shows an example of a heap. Observe that 6 is the smallest value. Observe that each parent is smaller than its children and that each parent has two children, with the exception of node 39 at level 3 and the leaves. Furthermore, with the exception of 66, all leaves are at the lowest level. Also, 39 is the next-to-last node at level 3, and 66 is the last (rightmost) node at level 3.

**FIGURE 6.20**

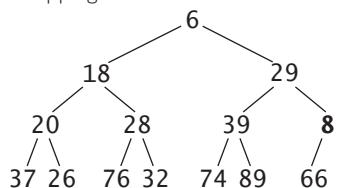
Example of a Heap

**FIGURE 6.21**

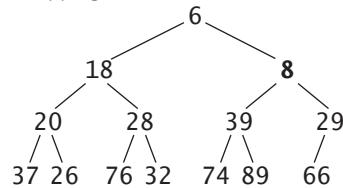
Inserting 8 into a Heap

**FIGURE 6.22**

Swapping 8 and 66

**FIGURE 6.23**

Swapping 8 and 29



More formally, a heap is a complete binary tree with the following properties:

- The value in the root is the smallest item in the tree.
- Every subtree is a heap.

## Inserting an Item into a Heap

We use the following algorithm for inserting an item into a heap. Our approach is to place each item initially in the bottom row of the heap and then move it up until it reaches the position where it belongs.

### Algorithm for Inserting in a Heap

1. Insert the new item in the next position at the bottom of the heap.
2. **while** new item is not at the root and new item is smaller than its parent
3.     Swap the new item with its parent, moving the new item up the heap.

New items are added to the last row (level) of a heap. If a new item is larger than or equal to its parent, nothing more need be done. If we insert 89 in the heap in Figure 6.20, 89 would become the right child of 39 and we are done. However, if the new item is smaller than its parent, the new item and its parent are swapped. This is repeated up the tree until the new item is in a position where it is no longer smaller than its parent. For example, let's add 8 to the heap shown in Figure 6.21. Since 8 is smaller than 66, these values are swapped as shown in Figure 6.22. Also, 8 is smaller than 29, so these values are swapped resulting in the updated heap shown in Figure 6.23. But 8 is greater than 6, so we are done.

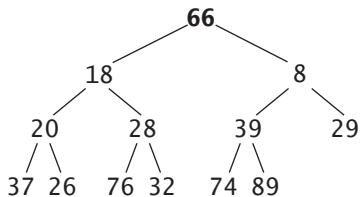
## Removing an Item from a Heap

Removal from a heap is always from the top. The top item is first replaced with the last item in the heap (at the lower right-hand position) so that the heap remains a complete tree. If we used any other value, there would be a “hole” in the tree where that value used to be. Then the new item at the top is moved down the heap until it is in its proper position.

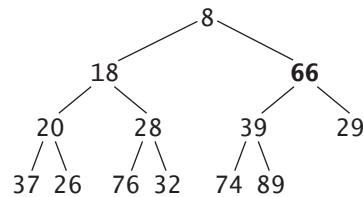
### Algorithm for Removal from a Heap

1. Remove the item in the root node by replacing it with the last item in the heap (LIH).
2. **while** item LIH has children, and item LIH is larger than either of its children
3.     Swap item LIH with its smaller child, moving LIH down the heap.

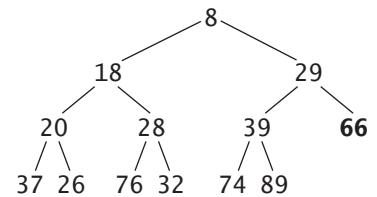
**FIGURE 6.24**  
After Removal of 6



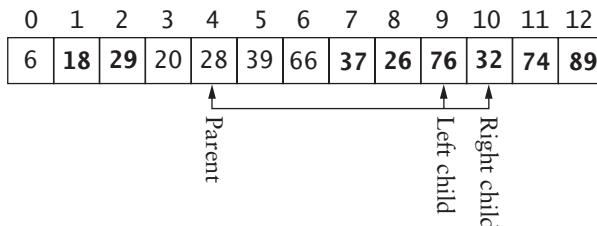
**FIGURE 6.25**  
Swapping 66 and 8



**FIGURE 6.26**  
Swapping 66 and 29



**FIGURE 6.27**  
Internal Representation  
of the Heap



As an example, if we remove 6 from the heap shown in Figure 6.23, 66 replaces it as shown in Figure 6.24. Since 66 is larger than both of its children, it is swapped with the smaller of the two, 8, as shown in Figure 6.25. The result is still not a heap because 66 is larger than both its children. Swapping 66 with its smaller child, 29, restores the heap as shown in Figure 6.26.

## Implementing a Heap

Because a heap is a complete binary tree, we can implement it efficiently using an array (or `ArrayList`) instead of a linked data structure. We can use the first element (subscript 0) for storing a reference to the root data. We can use the next two elements (subscripts 1 and 2) for storing the two children of the root. We can use elements with subscripts 3, 4, 5, and 6 for storing the four children of these two nodes, and so on. Therefore, we can view a heap as a sequence of rows; each row is twice as long as the previous row. The first row (the root) has one item, the second row two, the third four, and so on. All of the rows are full except for the last one (see Figure 6.27).

Observe that the root, 6, is at position 0. The root's two children, 18 and 29, are at positions 1 and 2. For a node at position  $p$ , the left child is at  $2p + 1$  and the right child is at  $2p + 2$ . A node at position  $c$  can find its parent at  $(c - 1) / 2$ . Thus, as shown in Figure 6.27, children of 28 (at position 4) are at positions 9 and 10.

## Insertion into a Heap Implemented as an `ArrayList`

We will use an `ArrayList` for storing our heap because it is easier to expand and contract than an array. Figure 6.28 shows the heap after inserting 8 into position 13. This corresponds to inserting the new value into the lower right position as shown in the figure, right. Now we need to move 8 up the heap, by comparing it to the values stored in its ancestor nodes. The parent (66) is in position 6 (13 minus 1 is 12, divided by 2 is 6). Since 66 is larger than 8, we need to swap as shown in Figure 6.29.

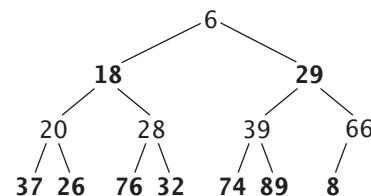
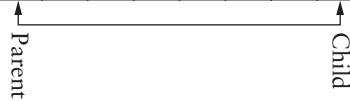
Now the child is at position 6 and the parent is at position 2 (6 minus 1 is 5, divided by 2 is 2). Since the parent, 29, is larger than the child, 8, we must swap again as shown in Figure 6.30.

The child is now at position 2 and the parent is at position 0. Since the parent is smaller than the child, the heap property is restored. In the heap insertion and removal algorithms that follow, we will use `table` to reference the `ArrayList` that stores the heap. We will use `table[index]` to represent the element at position `index` of `table`. In the actual code, a subscript cannot be used with an `ArrayList`.

**FIGURE 6.28**

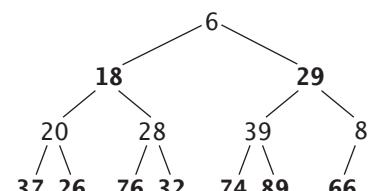
Internal Representation  
of Heap after Insertion

0	1	2	3	4	5	6	7	8	9	10	11	12	13
6	18	29	20	28	39	66	37	26	76	32	74	89	8

**FIGURE 6.29**

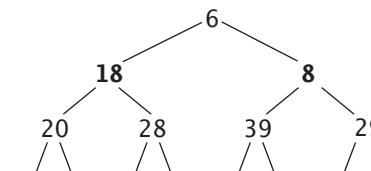
Internal Representation  
of Heap after First  
Swap

0	1	2	3	4	5	6	7	8	9	10	11	12	13
6	18	29	20	28	39	8	37	26	76	32	74	89	66

**FIGURE 6.30**

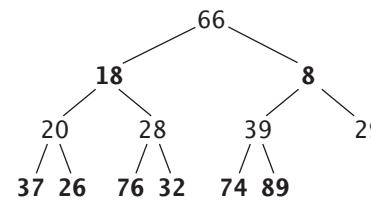
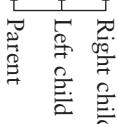
Internal Representation  
of Heap after Second  
Swap

0	1	2	3	4	5	6	7	8	9	10	11	12	13
6	18	8	20	28	39	29	37	26	76	32	74	89	66

**FIGURE 6.31**

Internal Representation  
of Heap after 6 Is  
Removed

0	1	2	3	4	5	6	7	8	9	10	11	12
66	18	8	20	28	39	29	37	26	76	32	74	89



### Insertion of an Element into a Heap Implemented as an ArrayList

1. Insert the new element at the end of the ArrayList and set child to `table.size() - 1`.
2. Set parent to  $(\text{child} - 1) / 2$ .
3. **while** (`parent >= 0` and `table[parent] > table[child]`)
  4. Swap `table[parent]` and `table[child]`.
  5. Set `child` equal to `parent`.
  6. Set `parent` equal to  $(\text{child} - 1) / 2$ .

### Removal from a Heap Implemented as an ArrayList

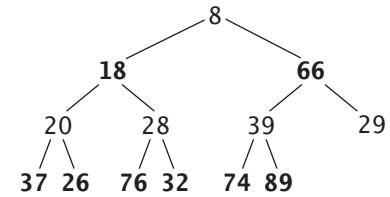
In removing elements from a heap, we must always remove and save the element at the top of the heap, which is the smallest element. We start with an ArrayList that has been organized to form a heap. To remove the first item (6), we begin by replacing the first item with the last item and then removing the last item. This is illustrated in Figure 6.31. The new value of the root (position 0) is larger than both of its children (18 in position 1 and 8 in position 2). The smaller of the two children (8 in position 2) is swapped with the parent as shown in

**FIGURE 6.32**

Internal Representation  
of Heap after 8 and 66  
Are Swapped

0	1	2	3	4	5	6	7	8	9	10	11	12
8	18	66	20	28	39	29	37	26	76	32	74	89

↑ Parent      ↑ Left child      ↑ Right child

**FIGURE 6.33**

Internal Representation  
of Heap after Swap of  
66 and 29

0	1	2	3	4	5	6	7	8	9	10	11	12
8	18	29	20	28	39	66	37	26	76	32	74	89

↑ Parent      ↑ Left child      ↑ Right child

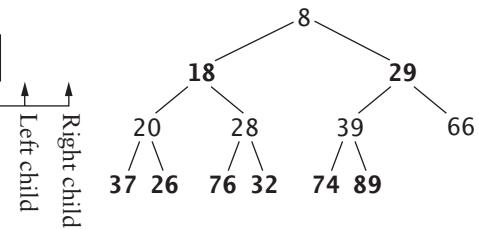


Figure 6.32. Next, 66 is swapped with the smaller of its two new children (29), and the heap is restored (Figure 6.33).

The algorithm for removal from a heap implemented as an `ArrayList` follows.

### Removing an Element from a Heap Implemented as an `ArrayList`

1. Remove the last element (i.e., the one at `size() - 1`) and set the item at 0 to this value.
2. Set `parent` to 0.
3. **while (true)**
  4. Set `leftChild` to `(2 * parent) + 1` and `rightChild` to `leftChild + 1`.
  5. **if** `leftChild >= table.size()`
  6.     Break out of loop.
  7.     Assume `minChild` (the smaller child) is `leftChild`.
  8.     **if** `rightChild < table.size()` and  
`table[rightChild] < table[leftChild]`
  9.         Set `minChild` to `rightChild`.
  10.     **if** `table[parent] > table[minChild]`
  11.         Swap `table[parent]` and `table[minChild]`.
  12.         Set `parent` to `minChild`.
  13.     Break out of loop.

The loop (Step 3) is terminated under one of two circumstances: either the item has moved down the tree so that it has no children (line 5 is true), or it is smaller than both its children (line 10 is false). In these cases, the loop terminates (line 6 or 13). This is shown in Figure 6.33. At this point the heap property is restored, and the next smallest item can be removed from the heap.

### Performance of the Heap

Method `remove` traces a path from the root to a leaf, and method `insert` traces a path from a leaf to the root. This requires at most  $h$  steps, where  $h$  is the height of the tree. The largest heap of height  $h$  is a full tree of height  $h$ . This tree has  $2^h - 1$  nodes. The smallest heap of

height  $h$  is a complete tree of height  $h$ , consisting of a full tree of height  $h - 1$ , with a single node as the left child of the leftmost child at height  $h - 1$ . Thus, this tree has  $2^{(h-1)}$  nodes. Therefore, both `insert` and `remove` are  $O(\log n)$  where  $n$  is the number of items in the heap.

## Priority Queues

In computer science, a heap is used as the basis of a very efficient algorithm for sorting arrays, called heapsort, which you will study in Chapter 8. The heap is also used to implement a special kind of queue called a priority queue. However, the heap is not very useful as an abstract data type (ADT) on its own. Consequently, we will not create a `Heap` interface or code a class that implements it. Instead we will incorporate its algorithms when we implement a priority queue class and heapsort.

Sometimes a FIFO (first-in-first-out) queue may not be the best way to implement a waiting line. In a print queue, you might want to print a short document before some longer documents that were ahead of the short document in the queue. For example, if you were waiting by the printer for a single page to print, it would be very frustrating to have to wait until several documents of 50 pages or more were printed just because they entered the queue before yours did. Therefore, a better way to implement a print queue would be to use a priority queue. A *priority queue* is a data structure in which only the highest priority item is accessible. During insertion, the position of an item in the queue is based on its priority relative to the priorities of other items in the queue. If a new item has higher priority than all items currently in the queue, it will be placed at the front of the queue and, therefore, will be removed before any of the other items inserted in the queue at an earlier time. This violates the FIFO property of an ordinary queue.

**EXAMPLE 6.12** Figure 6.34 sketches a print queue that at first (top of diagram) contains two documents. We will assume that each document's priority is inversely proportional to its page count (priority is  $\frac{1}{\text{page count}}$ ). The middle queue shows the effect of inserting a document three pages long. The bottom queue shows the effect of inserting a second one-page document. It follows the earlier document with that page length.

FIGURE 6.34

Insertion into a Priority Queue

```
pages = 1
title = "web page 1"
```

```
pages = 4
title = "history paper"
```

After inserting document with 3 pages

```
pages = 1
title = "web page 1"
```

```
pages = 3
title = "Lab1"
```

```
pages = 4
title = "history paper"
```

After inserting document with 1 page

```
pages = 1
title = "web page 1"
```

```
pages = 1
title = "receipt"
```

```
pages = 3
title = "Lab1"
```

```
pages = 4
title = "history paper"
```

**TABLE 6.6**Methods of the `PriorityQueue<E>` Class

Method	Behavior
<code>boolean offer(E item)</code>	Inserts an item into the queue. Returns <code>true</code> if successful; returns <code>false</code> if the item could not be inserted
<code>E remove()</code>	Removes the smallest entry and returns it if the queue is not empty. If the queue is empty, throws a <code>NoSuchElementException</code>
<code>E poll()</code>	Removes the smallest entry and returns it. If the queue is empty, returns <code>null</code>
<code>E peek()</code>	Returns the smallest entry without removing it. If the queue is empty, returns <code>null</code>
<code>E element()</code>	Returns the smallest entry without removing it. If the queue is empty, throws a <code>NoSuchElementException</code>

## The `PriorityQueue` Class

Java provides a `PriorityQueue<E>` class that implements the `Queue<E>` interface given in Chapter 4. The differences are in the specification for the `peek`, `poll`, and `remove` methods. These are defined to return the smallest item in the queue rather than the oldest item in the queue. Table 6.6 summarizes the methods of the `PriorityQueue<E>` class.

## Using a Heap as the Basis of a Priority Queue

The smallest item is always removed first from a priority queue (the smallest item has the highest priority) just as it is for a heap. Because insertion into and removal from a heap is  $O(\log n)$ , a heap can be the basis for an efficient implementation of a priority queue. We will call our class `KWPriorityQueue` to differentiate it from class `PriorityQueue` in the `java.util` API, which also uses a heap as the basis of its implementation.

A key difference is that class `java.util.PriorityQueue` class uses an array of type `Object[]` for heap storage. We will use an `ArrayList` for storage in `KWPriorityQueue` because the size of an `ArrayList` automatically adjusts as elements are inserted and removed. To insert an item into the priority queue, we first insert the item at the end of the `ArrayList`. Then, following the algorithm described earlier, we move this item up the heap until it is smaller than its parent.

To remove an item from the priority queue, we take the first item from the `ArrayList`; this is the smallest item. We then remove the last item from the `ArrayList` and put it into the first position of the `ArrayList`, overwriting the value currently there. Then, following the algorithm described earlier, we move this item down until it is smaller than its children or it has no children.

## Design of `KWPriorityQueue` Class

The design of the `KWPriorityQueue<E>` class is shown in Table 6.7. The data field `theData` is used to store the heap. We discuss the purpose of data field `comparator` shortly. We have added methods `compare` and `swap` to those shown earlier in Table 6.6. Method `compare` compares its two arguments and returns a type `int` value indicating their relative ordering. The class heading and data field declarations follow.

```
import java.util.*;

/** The KWPriorityQueue implements the Queue interface
 * by building a heap in an ArrayList. The heap is structured
 * so that the "smallest" item is at the top.
 */
```

```

public class KWPriorityQueue<E> extends AbstractQueue<E>
    implements Queue<E> {

    // Data Fields
    /** The ArrayList to hold the data. */
    private ArrayList<E> theData;
    /** An optional reference to a Comparator object. */
    Comparator<E> comparator = null;

    // Methods
    // Constructor
    public KWPriorityQueue() {
        theData = new ArrayList<>();
    }
    ...
}

```

**TABLE 6.7**Design of `KWPriorityQueue<E>` Class

Data Field	Attribute
<code>ArrayList&lt;E&gt; theData</code>	An <code>ArrayList</code> to hold the data
<code>Comparator&lt;E&gt; comparator</code>	An optional object that implements the <code>Comparator&lt;E&gt;</code> interface by providing a <code>compare</code> method
Method	Behavior
<code>KWPriorityQueue()</code>	Constructs a heap-based priority queue that uses the elements' natural ordering
<code>KWPriorityQueue(Comparator&lt;E&gt; comp)</code>	Constructs a heap-based priority queue that uses the <code>compare</code> method of <code>Comparator comp</code> to determine the ordering of the elements
<code>private int compare(E left, E right)</code>	Compares two objects and returns a negative number if object <code>left</code> is less than object <code>right</code> , zero if they are equal, and a positive number if object <code>left</code> is greater than object <code>right</code>
<code>private void swap(int i, int j)</code>	Exchanges the object references in <code>theData</code> at indexes <code>i</code> and <code>j</code>

## The offer Method

The `offer` method appends the new item to the `ArrayList theData`. It then moves this item up the heap until the `ArrayList` is restored to a heap.

```

/** Insert an item into the priority queue.
  pre: The ArrayList theData is in heap order.
  post: The item is in the priority queue and
        theData is in heap order.
  @param item The item to be inserted
  @throws NullPointerException if the item to be inserted is null.
 */
@Override
public boolean offer(E item) {
    // Add the item to the heap.
    theData.add(item);
    // child is newly inserted item.
    int child = theData.size() - 1;
    int parent = (child - 1) / 2; // Find child's parent.
    // Reheap
    while (parent >= 0 && compare(theData.get(parent),
        theData.get(child)) > 0) {

```

```

        swap(parent, child);
        child = parent;
        parent = (child - 1) / 2;
    }
    return true;
}

```

## The poll Method

The `poll` method first saves the item at the top of the heap. If there is more than one item in the heap, the method removes the last item from the heap and places it at the top. Then it moves the item at the top down the heap until the heap property is restored. Next it returns the original top of the heap.

```

/** Remove an item from the priority queue
 * pre: The ArrayList theData is in heap order.
 * post: Removed smallest item, theData is in heap order.
 * @return The item with the smallest priority value or null if empty.
 */
@Override
public E poll() {
    if (isEmpty()) {
        return null;
    }
    // Save the top of the heap.
    E result = theData.get(0);
    // If only one item then remove it.
    if (theData.size() == 1) {
        theData.remove(0);
        return result;
    }

    /* Remove the last item from the ArrayList and place it into
     * the first position. */
    theData.set(0, theData.remove(theData.size() - 1));
    // The parent starts at the top.
    int parent = 0;
    while (true) {
        int leftChild = 2 * parent + 1;
        if (leftChild >= theData.size()) {
            break; // Out of heap.
        }
        int rightChild = leftChild + 1;
        int minChild = leftChild; // Assume leftChild is smaller.
        // See whether rightChild is smaller.
        if (rightChild < theData.size())
            && compare(theData.get(leftChild),
                       theData.get(rightChild)) > 0) {
            minChild = rightChild;
        }
        // assert: minChild is the index of the smaller child.
        // Move smaller child up heap if necessary.
        if (compare(theData.get(parent),
                    theData.get(minChild)) > 0) {
            swap(parent, minChild);
            parent = minChild;
        } else { // Heap property is restored.
            break;
        }
    }
    return result;
}

```

## The Other Methods

The `iterator` and `size` methods are implemented via delegation to the corresponding `ArrayList` methods. Method `isEmpty` tests whether the result of calling method `size` is 0 and is inherited from class `AbstractCollection` (a super interface to `AbstractQueue`). Methods `peek` and `remove` (based on `poll`) must also be implemented; they are left as exercises. Methods `add` and `element` are inherited from `AbstractQueue` where they are implemented by calling methods `offer` and `peek`, respectively.

## Using a Comparator

How do we compare elements in a `PriorityQueue`? In many cases, we will insert objects that implement `Comparable<E>` and use their natural ordering as specified by method `compareTo`. However, we may need to insert objects that do not implement `Comparable<E>`, or we may want to specify a different ordering from that defined by the object's `compareTo` method. For example, files to be printed may be ordered by their name using the `compareTo` method, but we may want to assign priority based on their length. The Java API contains the `Comparator<E>` interface, which allows us to specify alternative ways to compare objects. An implementer of the `Comparator<E>` interface must define a `compare` method that is similar to `compareTo` except that it has two parameters (see Table 6.7).

To indicate that we want to use an ordering that is different from the natural ordering for the objects in our heap, we will provide a constructor that has a `Comparator<E>` parameter. The constructor will set data field `comparator` to reference this parameter. Otherwise, `comparator` will remain `null`. To match the form of this constructor in the `java.util.PriorityQueue` class, we provide a first parameter that specifies the initial capacity of `ArrayList` `theData`.

```
/** Creates a heap-based priority queue with the specified initial
   capacity that orders its elements according to the specified
   comparator.
   @param cap The initial capacity for this priority queue
   @param comp The comparator used to order this priority queue
   @throws IllegalArgumentException if cap is less than 1
 */
public KWPriorityQueue(int cap, Comparator<E> comp) {
    if (cap < 1)
        throw new IllegalArgumentException();
    theData = new ArrayList<E>();
    comparator = comp;
}
```

## The compare Method

If data field `comparator` references a `Comparator<E>` object, method `compare` will delegate the task of comparing its argument objects to that object's `compare` method. If `comparator` is `null`, the natural ordering of the objects should be used, so method `compare` will delegate to method `compareTo`. Note that parameter `left` is cast to type `Comparable<E>` in this case. In the next example, we show how to write a `Comparator` class.

```
/** Compare two items using either a Comparator object's compare method
   or their natural ordering using method compareTo.
   @pre: If comparator is null, left and right implement Comparable<E>.
   @param left One item
   @param right The other item
   @return Negative int if left less than right,
           0 if left equals right,
           positive int if left > right
   @throws ClassCastException if items are not Comparable
 */

```

```

@SuppressWarnings("unchecked")
private int compare(E left, E right) {
    if (comparator != null) { // A Comparator is defined.
        return comparator.compare(left, right);
    } else { // Use left's compareTo method.
        return ((Comparable<E>) left).compareTo(right);
    }
}

```

**EXAMPLE 6.13** The class `PrintDocument` is used to define documents to be printed on a printer. This class implements the `Comparable` interface, but the result of its `compareTo` method is based on the name of the file being printed. The class also has a `getSize` method that gives the number of bytes to be transmitted to the printer and a `getTimestamp` method that gets the time that the print job was submitted. Instead of basing the ordering on file names, we want to order the documents by a value that is a function of both size and the waiting time of a document. If we were to use either time or size alone, small documents could be delayed while big ones are printed, or big documents would never be printed. By using a priority value that is a combination, we achieve a balanced usage of the printer.

In Java 8, the `Comparator` interface is also defined as a functional interface (see Table 6.2). Its abstract method `compare` takes two arguments of the same type and returns an integer indicating their ordering. We can pass a lambda expression as a function parameter to the constructor that creates a `KWPriorityQueue` object. This method will implement the abstract `compare` method and determine the ordering of objects in the print queue.

The `compare` method for `printQueue` specified in the following fragment uses the weighted sum of the size and time stamp for documents `left` and `right` using the weighting factors `P1` and `P2`. Method `Double.compare` in this fragment compares two double values and returns a negative value, 0, or a positive value depending on whether `leftValue` is `<`, equal to, or `>` `rightValue`.

```

final double P1 = 0.8;
final double P2 = 0.2;
Queue<PrintDocument> printQueue =
    new KWPriorityQueue<>(25, (left, right) -> {
        double leftValue = P1 * left.getSize() + P2 * left.getTimestamp();
        double rightValue = P1 * right.getSize() + P2 * right.getTimestamp();
        return Double.compare(leftValue, rightValue);
});

```

Earlier versions of Java could not implement the `Comparator` object by passing a lambda expression to a constructor. The textbook website discusses how to define the `Comparator` object before Java 8.

## EXERCISES FOR SECTION 6.5

### SELF-CHECK

1. Show the heap that would be used to store the words *this, is, the, house, that, jack, built*, assuming they are inserted in that sequence. Exchange the order of arrival of the first and last words and build the new heap.
2. Draw the heaps for Exercise 1 above as arrays.

3. Show the result of removing the number 18 from the heap in Figure 6.26. Show the new heap and its array representation.
4. The heaps in this chapter are called min heaps because the smallest key is at the top of the heap. A max heap is a heap in which each element has a key that is smaller than its parent, so the largest key is at the top of the heap. Build the max heap that would result from the numbers 15, 25, 10, 33, 55, 47, 82, 90, 18 arriving in that order.
5. Show the printer queue after receipt of the following documents:

time stamp	size
1100	256
1101	512
1102	64
1103	96

### PROGRAMMING

1. Complete the implementation of the `KWPriorityQueue` class. Write method `swap`. Also write methods `peek`, `remove`, `isEmpty`, and `size`.

## 6.7 Huffman Trees

In Section 6.1, we showed the Huffman coding tree and how it can be used to decode a message. We will now implement some of the methods needed to build a tree and decode a message. We will do this using a binary tree and a `PriorityQueue` (which also uses a binary tree).

A straight binary coding of an alphabet assigns a unique binary number  $k$  to each symbol in the alphabet  $a_k$ . An example of such a coding is Unicode, which is used by Java for the `char` data type. There are 65,536 possible characters, and they are assigned a number between 0 and 65,535, which is a string of 16 binary digit ones. Therefore, the length of a message would be  $16 \times n$ , where  $n$  is the total number of characters in the message. For example, the message “go eagles” contains 9 characters and would require  $9 \times 16$  or 144 bits. As shown in the example in Section 6.1, a Huffman coding of this message requires just 38 bits.

Table 6.8, based on data published in Donald Knuth, *The Art of Computer Programming, Vol 3: Sorting and Searching* (Addison-Wesley, 1973), p. 441, represents the relative frequencies of the letters in English text and is the basis of the tree shown in Figure 6.35. The letter *e* occurs an average of 103 times every 1000 letters, or 10.3 percent of the letters are *es*. (This is a useful table to know if you are a fan of *Wheel of Fortune*.) We can use this Huffman tree to encode and decode a file of English text. However, files may contain other symbols or may contain these symbols in different frequencies from what is found in normal English. For this reason, you may want to build a custom Huffman tree based on the contents of the file you are encoding. You would attach this tree to the encoded file so that it can be used to decode the file. We discuss how to build a Huffman tree in the next case study.

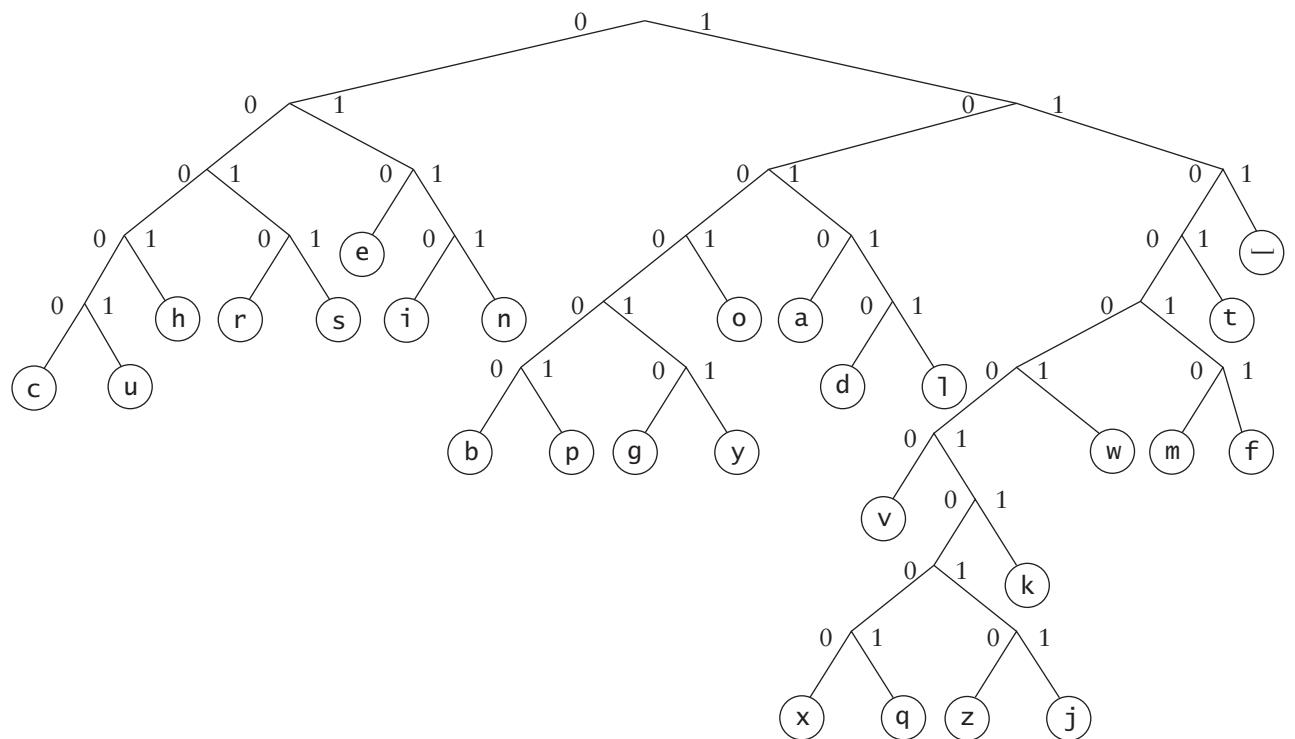
**TABLE 6.8**

## Frequency of Letters in English Text

Symbol	Frequency	Symbol	Frequency	Symbol	Frequency
—	186	h	47	g	15
e	103	d	32	p	15
t	80	l	32	b	13
a	64	u	23	v	8
o	63	c	22	k	5
i	57	f	21	j	1
n	57	m	20	q	1
s	51	w	18	x	1
r	48	y	16	z	1

**FIGURE 6.35**

## Huffman Tree Based on Frequency of Letters in English Text



## CASE STUDY Building a Custom Huffman Tree

### Problem

You want to build a custom Huffman tree for a particular file. Your input will consist of an array of objects such that each object contains a reference to a symbol occurring in that file and the frequency of occurrence (weight) for the symbol in that file.

### Analysis

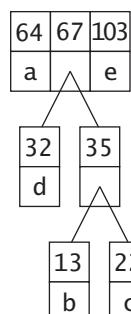
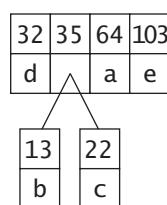
Each node of a Huffman tree has storage for two data items: the weight of the node and the symbol associated with that node. All symbols will be stored at leaf nodes. For nodes that are not leaf nodes, the symbol part has no meaning. The weight of a leaf node will be the frequency of the symbol stored at that node. The weight of an interior node will be the sum of frequencies of all nodes in the subtree rooted at the interior node. For example, the interior node with leaf nodes *c* and *u* (on the left of Figure 6.35) would have a weight of 45 ( $22 + 23$ ).

We will use a priority queue as the key data structure in constructing the Huffman tree. We will store individual symbols and subtrees of multiple symbols in order by their priority (frequency of occurrence). We want to remove symbols that occur less frequently first because they should be lower down in the Huffman tree we are constructing. We discuss how this is done next.

**FIGURE 6.36**

Priority Queue with the Symbols *a*, *b*, *c*, *d*, and *e*

13	22	32	64	103
<i>b</i>	<i>c</i>	<i>d</i>	<i>a</i>	<i>e</i>

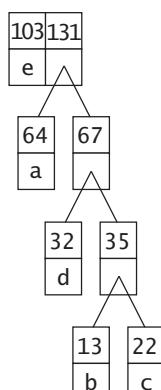


To build a Huffman tree, we start by inserting references to trees with just leaf nodes in a priority queue. Each leaf node will store a symbol and its weight. The queue elements will be ordered so that the leaf node with the smallest weight (lowest frequency) is removed first. Figure 6.36 shows a priority queue, containing just the symbols *a*, *b*, *c*, *d*, and *e*, that uses the weights shown in Table 6.8. The item at the front of the queue stores a reference to a tree with a root node that is a leaf node containing the symbol *b* with a weight (frequency) of 13. To represent the tree referenced by a queue element, we list the root node information for that tree. The queue elements are shown in priority order.

Now we start to build the Huffman tree. We build it from the bottom up. The first step is to remove the first two tree references from the priority queue and combine them to form a new tree. The weight of the root node for this tree will be the sum of the weights of its left and right subtrees. We insert the new tree back into the priority queue. The priority queue now contains references to four binary trees instead of five. The tree referenced by the second element of the queue has a combined weight of 35 ( $13 + 22$ ) as shown on the left.

Again we remove the first two tree references and combine them. The new binary tree will have a weight of 67 in its root node. We put this tree back in the queue, and it will be referenced by the second element of the queue.

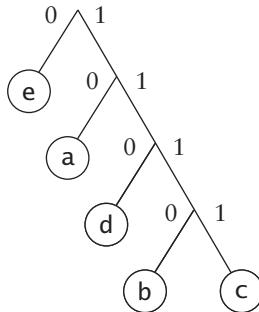
We repeat this process again. The new queue follows:



Finally, we combine the last two elements into a new tree and put a reference to it in the priority queue. Now there is only one tree in the queue, so we have finished building the Huffman tree (see Figure 6.37). Table 6.9 shows the codes for this tree.

**FIGURE 6.37**

Huffman Tree of *a*, *b*, *c*, *d*, and *e*

**TABLE 6.9**

Huffman Code for *a*, *b*, *c*, *d*, and *e*

Symbol	Code
<i>a</i>	10
<i>b</i>	1110
<i>c</i>	1111
<i>d</i>	110
<i>e</i>	0

### Design

The class `HuffData` will represent the data to be stored in each node of the Huffman binary tree. For a leaf, a `HuffData` object will contain the symbol and the weight.

Our class `HuffmanTree` will have the methods and attributes listed in Table 6.10.

### Algorithm for Building a Huffman Tree

1. Construct a set of trees with root nodes that contain each of the individual symbols and their weights.
2. Place the set of trees into a priority queue.
3. **while** the priority queue has more than one item
  4. Remove the two trees with the smallest weights.
  5. Combine them into a new binary tree in which the weight of the tree root is the sum of the weights of its children.
  6. Insert the newly created tree back into the priority queue.

Each time through the **while** loop, two nodes are removed from the priority queue and one is inserted. Thus, effectively one tree is removed, and the queue gets smaller with each pass through the loop.

**TABLE 6.10**

Data Fields and Methods of Class `HuffmanTree`

Data Field	Attribute
<code>BinaryTree&lt;HuffData&gt; huffTree</code>	A reference to the Huffman tree
Method	Behavior
<code>buildTree(HuffData[] input)</code>	Builds the Huffman tree using the given alphabet and weights
<code>String decode(String message)</code>	Decodes a message using the generated Huffman tree
<code>printCode(PrintStream out)</code>	Outputs the resulting code

### Implementation

Listing 6.9 shows the data field declarations for class `HuffmanTree`. Method `buildTree` and the comparator are discussed in the next section.

**LISTING 6.9**

```

Class HuffmanTree

import java.util.*;
import java.io.*;

/** Class to represent and build a Huffman tree. */
public class HuffmanTree implements Serializable {

    // Nested Classes
    /** A datum in the Huffman tree. */
    public static class HuffData implements Serializable {
        // Data Fields
        /** The weight or probability assigned to this HuffData. */
        private double weight;
        /** The alphabet symbol if this is a leaf. */
        private char symbol;

        public HuffData(double weight, Character symbol) {
            this.weight = weight;
            this.symbol = symbol;
        }
    }

    // Data Fields
    /** A reference to the completed Huffman tree. */
    private BinaryTree<HuffData> huffTree;
}

```

**The buildTree Method**

Method `buildTree` (see Listing 6.10) takes an array of `HuffData` objects as its parameter. The statement

```

Queue<BinaryTree<HuffData>> theQueue
    = new PriorityQueue<>(symbols.length,
                           (lt, rt) -> Double.compare(lt.getData().weight,
                                                       rt.getData().weight)
                           );

```

creates a new priority queue for storing `BinaryTree<HuffData>` objects using the `PriorityQueue` class in the `java.util` API. The constructor above takes two arguments representing the initial capacity and comparator for `theQueue`. The lambda expression passed as the constructor's second argument is the comparator:

```

(lt, rt) -> Double.compare(lt.getData().weight,
                           rt.getData().weight)

```

The enhanced `for` loop loads the priority queue with trees consisting just of leaf nodes. Each leaf node contains a `HuffData` object with the weight and alphabet symbol.

The `while` loop builds the tree. Each time through this loop, the trees with the smallest weights are removed and referenced by `left` and `right`. The statements

```

HuffData sum = new HuffData(wl + wr, '\u0000');
BinaryTree<HuffData> newTree
    = new BinaryTree<>(sum, left, right);

```

combine them to form a new `BinaryTree` with a root node whose weight is the sum of the weights of its children and whose symbol is the null character '`\u0000`'. This new tree is then inserted into the priority queue. The number of trees in the queue decreases by 1 each time we do this. Eventually there will only be one tree in the queue, and that will be the completed Huffman tree. The last statement sets the variable `huffTree` to reference this tree.

**LISTING 6.10**

The buildTree Method (HuffmanTree.java)

```
/** Builds the Huffman tree using the given alphabet and weights.
 * post: huffTree contains a reference to the Huffman tree.
 * @param symbols An array of HuffData objects
 */
public void buildTree(HuffData[] symbols) {
    Queue<BinaryTree<HuffData>> theQueue
        = new PriorityQueue<>(symbols.length,
            (lt, rt) -> Double.compare(lt.getData().weight,
                rt.getData().weight));
    // Load the queue with the leaves.
    for (HuffData nextSymbol : symbols) {
        BinaryTree<HuffData> aBinaryTree =
            new BinaryTree<>(nextSymbol, null, null);
        theQueue.offer(aBinaryTree);
    }

    // Build the tree.
    while (theQueue.size() > 1) {
        BinaryTree<HuffData> left = theQueue.poll();
        BinaryTree<HuffData> right = theQueue.poll();
        double wl = left.getData().weight;
        double wr = right.getData().weight;
        HuffData sum = new HuffData(wl + wr, '\u0000');
        BinaryTree newTree =
            new BinaryTree<>(sum, left, right);
        theQueue.offer(newTree);
    }

    // The queue should now contain only one item.
    huffTree = theQueue.poll();
}
```

The textbook Web site shows how to write method `buildTree` and a comparator without using the new features of Java 8.

**Testing**

Methods `printCode` and `decode` can be used to test the custom Huffman tree. Method `printCode` displays the tree, so you can examine it and verify that the Huffman tree that was built is correct based on the input data.

Method `decode` will decode a message that has been encoded using the code stored in the Huffman tree and displayed by `printCode`, so you can pass it a message string that consists of binary digits only and see whether it can be transformed back to the original symbols.

We will discuss testing the Huffman tree further in the next chapter when we continue the case study.

**The printCode Method**

To display the code for each alphabet symbol, we perform a preorder traversal of the final tree. The code so far is passed as a parameter along with the current node. If the current node is a leaf, as indicated by the symbol not being `null`, then the code is output. Otherwise the left and right subtrees are traversed. When we traverse the left subtree, we append a 0 to the code, and when we traverse the right subtree, we append a 1 to the code. Recall that at each level in the recursion, there is a new copy of the parameters and local variables.

```

/** Outputs the resulting code.
 * @param out A PrintStream to write the output to
 * @param code The code up to this node
 * @param tree The current node in the tree
 */
private void printCode(PrintStream out, String code,
    BinaryTree<HuffData> tree) {
    HuffData theData = tree.getData();
    if (theData.symbol != '\u0000') {
        if (theData.symbol.equals(" ")) {
            out.println("space: " + code);
        } else {
            out.println(theData.symbol + ": " + code);
        }
    } else {
        printCode(out, code + "0", tree.getLeftSubtree());
        printCode(out, code + "1", tree.getRightSubtree());
    }
}

```

### The decode Method

To illustrate the decode process, we will show a method that takes a `String` that contains a sequence of the digit characters '`0`' and '`1`' and decodes it into a message that is also a `String`. Method `decode` starts by setting `currentTree` to the Huffman tree. It then loops through the coded message one character at a time. If the character is a '`1`', then `currentTree` is set to the right subtree; otherwise, it is set to the left subtree. If the `currentTree` is now a leaf, the symbol is appended to the result and `currentTree` is reset to the Huffman tree (see Listing 6.11). Note that this method is for testing purposes only. In actual usage, a message would be encoded as a string of bits (not digit characters) and would be decoded one bit at a time.

.....

#### **LISTING 6.11**

The `decode` Method (`HuffmanTree.java`)

```

/** Method to decode a message that is input as a string of
 * digit characters '0' and '1'.
 * @param codedMessage The input message as a String of
 * zeros and ones.
 * @return The decoded message as a String
 */
public String decode(String codedMessage) {
    StringBuilder result = new StringBuilder();
    BinaryTree<HuffData> currentTree = huffTree;
    for (int i = 0; i < codedMessage.length(); i++) {
        if (codedMessage.charAt(i) == '1') {
            currentTree = currentTree.getRightSubtree();
        } else {
            currentTree = currentTree.getLeftSubtree();
        }
        if (currentTree.isLeaf()) {
            HuffData theData = currentTree.getData();
            result.append(theData.symbol);
            currentTree = huffTree;
        }
    }
    return result.toString();
}

```



## PROGRAM STYLE

### A Generic HuffmanTree Class

We chose to implement a nongeneric `HuffmanTree` class to simplify the coding. However, it may be desirable to build a Huffman tree for storing `Strings` (e.g., to encode words in a document instead of the individual letters) or for storing groups of pixels in an image file. A generic `HuffmanTree<T>` class would define a generic inner class `HuffData<T>` where the `T` is the data type of data field `symbol`. Each parameter type `<HuffData>` in our class `HuffmanTree` would be replaced by `<HuffData<T>>`, which indicates that `T` is a type parameter for class `HuffData`.

## EXERCISES FOR SECTION 6.6

### SELF-CHECK

1. What is the Huffman code for the letters *a*, *j*, *k*, *l*, *s*, *t*, and *v* using Figure 6.35?
2. Trace the execution of method `printCode` for the Huffman tree in Figure 6.37.
3. Trace the execution of method `decode` for the Huffman tree in Figure 6.37 and the encoded message string "11101011011001111".
4. Create the Huffman code tree for the following frequency table. Show the different states of the priority queue as the tree is built (see Figure 6.36).

Symbol	Frequency
*	50
+	30
-	25
/	10
%	5

5. What would the Huffman code look like if all symbols in the alphabet had equal frequency?

### PROGRAMMING

1. Write a method `encode` for the `HuffmanTree` class that encodes a `String` of letters that is passed as its first argument. Assume that a second argument, `codes` (type `String[]`), contains the code strings (binary digits) for the symbols (space at position 0, *a* at position 1, *b* at position 2, etc.).



# Chapter Review

- ◆ A tree is a recursive, nonlinear data structure that is used to represent data that is organized as a hierarchy.
- ◆ A binary tree is a collection of nodes with three components: a reference to a data object, a reference to a left subtree, and a reference to a right subtree. A binary tree object has a single data field, which references the root node of the tree.
- ◆ In a binary tree used to represent arithmetic expressions, the root node should store the operator that is evaluated last. All interior nodes store operators, and the leaf nodes store operands. An inorder traversal (traverse left subtree, visit root, traverse right subtree) of an expression tree yields an infix expression, a preorder traversal (visit root, traverse left subtree, traverse right subtree) yields a prefix expression, and a postorder traversal (traverse left subtree, traverse right subtree, visit root) yields a postfix expression.
- ◆ Java 8 lambda expressions enable a programmer to practice functional programming in Java. A lambda expression is an anonymous method with a special shorthand notation to specify its arguments and method body. A lambda interface may be assigned to an object that instantiates a functional interface. The method body of the lambda expression will implement the single abstract method of the functional interface and will execute when applied to the functional object. Java 8 provides a set of functional interfaces in library `java.util.function`. A lambda expression may also be passed to a method with a parameter that is a function object.
- ◆ A binary search tree is a tree in which the data stored in the left subtree of every node is less than the data stored in the root node, and the data stored in the right subtree of every node is greater than the data stored in the root node. The performance depends on the fullness of the tree and can range from  $O(n)$  (for trees that resemble linked lists) to  $O(\log n)$  if the tree is full. An inorder traversal visits the nodes in increasing order.
- ◆ A heap is a complete binary tree in which the data in each node is less than the data in both its subtrees. A heap can be implemented very effectively as an array. The children of the node at subscript  $p$  are at subscripts  $2p + 1$  and  $2p + 2$ . The parent of child  $c$  is at  $(c - 1) / 2$ . The item at the top of a heap is the smallest item.
- ◆ Insertion and removal in a heap are both  $O(\log n)$ . For this reason, a heap can be used to efficiently implement a priority queue. A priority queue is a data structure in which the item with the highest priority (indicated by the smallest value) is removed next. The item with the highest priority is at the top of a heap and is always removed next.
- ◆ A Huffman tree is a binary tree used to store a code that facilitates file compression. The length of the bit string corresponding to a symbol in the file is inversely proportional to its frequency, so the symbol with the highest frequency of occurrence has the shortest length. In building a Huffman tree, a priority queue is used to store the symbols and trees formed so far. Each step in building the Huffman tree consists of removing two items and forming a new tree with these two items as the left and right subtrees of the new tree's root node. A reference to each new tree is inserted in the priority queue.

## Java API Interfaces and Classes Introduced in This Chapter

`java.text.DecimalFormat`  
`java.util.Comparator`  
`java.util.function.BiConsumer`  
`java.util.function.Consumer`  
`java.util.function.IntPredicate`

`java.util.PriorityQueue`  
`java.util.TreeSet`  
`java.util.function.BinaryOperator`  
`java.util.function.Function`

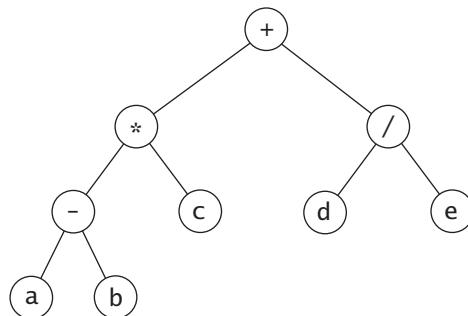
## User-Defined Interfaces and Classes in This Chapter

BinarySearchTree  
BinaryTree  
CompareHuffmanTrees  
ComparePrintDocuments  
HuffData  
HuffmanTree

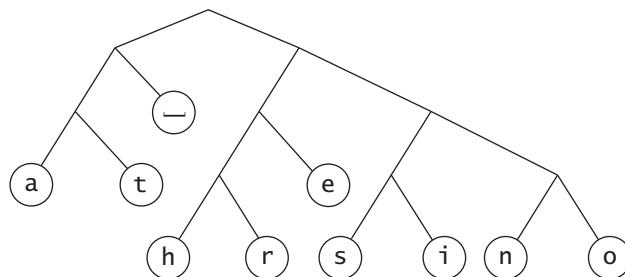
IndexGenerator  
KWPriorityQueue  
Node  
PrintDocument  
PriorityQueue  
SearchTree

## Quick-Check Exercises

1. For the following expression tree



- Is the tree full? \_\_\_\_ Is the tree complete? \_\_\_\_
  - List the order in which the nodes would be visited in a preorder traversal.
  - List the order in which the nodes would be visited in an inorder traversal.
  - List the order in which the nodes would be visited in a postorder traversal.
- Searching a full binary search tree is  $O(\underline{\hspace{2cm}})$ .
  - A heap is a binary tree that is a (full / complete) tree.
  - Write a lambda expression that can be used as a comparator that compares two objects by weight  
Assume there is method `getWeight()` that returns a double value.
  - Show the binary search tree that would result from inserting the items 35, 20, 30, 50, 45, 60, 18, 25 in this sequence.
  - Show the binary search tree in Exercise 4 after 35 is removed.
  - Show the heap that would result from inserting the items from Exercise 4 in the order given.
  - Draw the heap from Exercise 6 as an array.
  - Show the heap in Exercise 7 after 18 is removed.
  - In a Huffman tree, the item with the highest frequency of occurrence will have the \_\_\_\_ code.
  - List the code for each symbol shown in the following Huffman tree.



## Review Questions

1. Draw the tree that would be formed by inserting the words in this question into a binary search tree. Use lowercase letters.
2. Show all three traversals of this tree.
3. Show the tree from Question 1 after removing *draw*, *by*, and *letters* in that order.
4. Answer Question 1, but store the words in a heap instead of a binary search tree.
5. Write a lambda expression that can be used as a predicate that returns true if object's color is red. Assume there is a method `getColor` that returns the color as a string.
6. Given the following frequency table, construct a Huffman code tree. Show the initial priority queue and all changes in its state as the tree is constructed.

Symbol	Frequency
x	34
y	28
w	20
a	10
b	8
c	5

## Programming Projects

1. Assume that a class `ExpressionTree` has a data field that is a `BinaryTree`. Write an instance method to evaluate an expression stored in a binary tree whose nodes contain either integer values (stored in `Integer` objects) or operators (stored in `Character` objects). Your method should implement the following algorithm.

### Algorithm to Evaluate an Expression Tree

1. **if** the root node is an `Integer` object  
2.     Return the integer value.
3. **else if** the root node is a `Character` object  
4.     Let `leftVal` be the value obtained by recursively applying this algorithm to the left subtree.  
5.     Let `rightVal` be the value obtained by recursively applying this algorithm to the right subtree.  
6.     Return the value obtained by applying the operator in the root node to `leftVal` and `rightVal`.

Use method `readBinaryTree` to read the expression tree.

2. Write an application to test the `HuffmanTree` class. Your application will need to read a text file and build a frequency table for the characters occurring in that file. Once that table is built, create a Huffman code tree and then a string consisting of '0' and '1' digit characters that represents the code string for that file. Read that string back in and re-create the contents of the original file.
3. Solve Programming Project 4 in Chapter 4, "Queues," using the class `PriorityQueue`.
4. Build a generic `HuffmanTree<T>` class such that the symbol type `T` is specified when the tree is created. Test this class by using it to encode the words in your favorite nursery rhyme.
5. Write `clone`, `size`, and `height` methods for the `BinaryTree` class.
6. In a breadth-first traversal of a binary tree, the nodes are visited in an order prescribed by their level. First visit the node at level 1, the root node. Then visit the nodes at level 2, in left-to-right order, and so on. You can use a queue to implement a breadth-first traversal of a binary tree.

## Algorithm for Breadth-First Traversal of a Binary Tree

1. Insert the root node in the queue.
2. **while** the queue is not empty
  3. Remove a node from the queue and visit it.
  4. Place references to its left and right subtrees in the queue.

Code this algorithm and test it on several binary trees.

7. Define an `IndexTree` class such that each node has data fields to store a word, the count of occurrences of that word in a document file, and the line number for each occurrence. Use an `ArrayList` to store the line numbers. Use an `IndexTree` object to store an index of words appearing in a text file, and then display the index by performing an inorder traversal of this tree.
8. Extend the `BinaryTreeClass` to implement the `Iterable` interface by providing an iterator. The iterator should access the tree elements using an inorder traversal. The iterator is implemented as a nested private class. (Note: Unlike `Node`, this class should not be static.)

Design hints:

You will need a stack to hold the path from the current node back to the root. You will also need a reference to the current node (`current`) and a variable that stores the last item returned.

To initialize `current`, the constructor should start at the root and follow the left links until a node is reached that does not have a left child. This node is the initial current node.

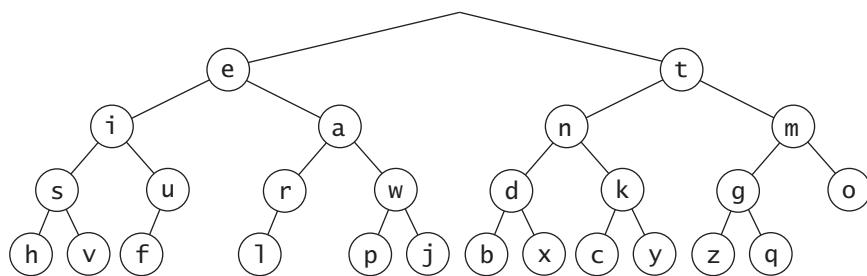
The `remove` method can throw an `UnsupportedOperationException`. The `next` method should use the following algorithm:

1. Save the contents of the current node.
2. **If** the current node has a right child
  3. push the current node onto the stack
  4. set the current node to the right child
  5. **while** the current node has a left child
    6. push the current node onto the stack
    7. set the current node to the left child
  8. **else** the current node does not have a right child
    9. **while** the stack is not empty and
      - the top node of the stack's right child is equal to the current node
    10. set the current node to the top of the stack and pop the stack
  11. **if** the stack is empty
    12. set the current node to null indicating that iteration is complete
  13. **else**
    14. set the current node to the top of the stack and pop the stack
  15. return the saved contents of the initial current node
9. The Morse code (see Table 6.11) is a common code that is used to encode messages consisting of letters and digits. Each letter consists of a series of dots and dashes; for example, the code for the letter *a* is `--` and the code for the letter *b* is `-•••`. Store each letter of the alphabet in a node of a binary tree of level 5. The root node is at level 1 and stores no letter. The left node at level 2 stores the letter *e* (code is `•`), and the right node stores the letter *t* (code is `-`). The four nodes at level 3 store the letters with codes `(••, •-, -•, --)`. To build the tree (see Figure 6.38), read a file in which each line consists of a letter followed by its code. The letters should be ordered by tree level. To find the position for a letter in the tree, scan the code and branch left for a dot and branch right for a dash. Encode a message by replacing each letter by its code symbol. Then decode the message using the Morse code tree. Make sure you use a delimiter symbol between coded letters.

**TABLE 6.11**  
Morse Code for Letters

a	•—	b	—•••	c	—•—•	d	—••	e	•	f	••—•
g	——•	h	••••	i	••	j	•---	k	—•—	l	•—••
m	——	n	—•	o	---	p	•—••	q	——•	r	•••
s	•••	t	—	u	••—	v	•••—	w	•——	x	—••—
y	—•——	z	——••								

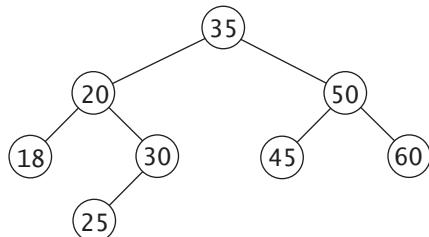
**FIGURE 6.38**  
Morse Code Tree



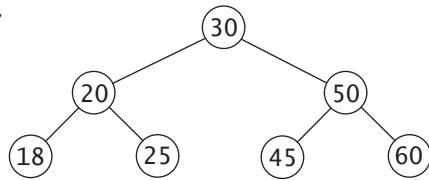
10. Create an abstract class `Heap` that has two subclasses, `MinHeap` and `MaxHeap`. Each subclass should have two constructors, one that takes no parameters and the other that takes a `Comparator` object. In the abstract class, the `compare` method should be abstract, and each subclass should define its own `compare` method to ensure that the ordering of elements corresponds to that required by the heap. For a `MinHeap`, the key in each node should be greater than the key of its parent; the ordering is reversed for a `MaxHeap`.
11. A right-threaded tree is a binary search tree in which each right link that would normally be null is a “thread” that links that node to its inorder successor. The thread enables nonrecursive algorithms to be written for search and inorder traversals that are more efficient than recursive ones. Implement a `RightThreadTree` class as an extension of a `BinarySearchTree`. You will also need an `RTNode` that extends the `Node` class to include a flag that indicates whether a node’s right link is a real link or a thread.

## Answers to Quick-Check Exercises

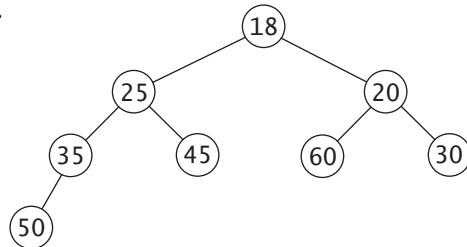
1. a. Not full, complete  
b. + \* — a b c / d e  
c. a — b \* c + d / e  
d. a b — c \* d e / +
2.  $O(\log n)$
3. A heap is a binary tree that is a *complete* tree.
4. `(o1, o2) -> Double.compare(o1.getWeight(), o2.getWeight())`
- 5.



6.

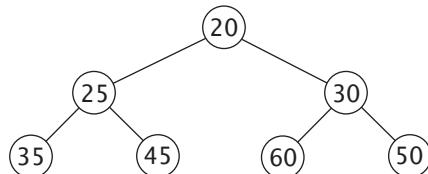


7.



8. 18, 25, 20, 35, 45, 60, 30, 50, where 18 is at position 0 and 50 is at position 7.

9.

10. In a Huffman tree, the item with the highest frequency of occurrence will have the *shortest* code.

11.

<b>Symbol</b>	<b>Code</b>	<b>Symbol</b>	<b>Code</b>
Space	01	n	1110
a	000	o	1111
e	101	r	1001
h	1000	s	1100
i	1101	t	001



# *Sets and Maps*

## Chapter Objectives

- ◆ To understand the Java Map and Set interfaces and how to use them
- ◆ To learn about hash coding and its use to facilitate efficient search and retrieval
- ◆ To study two forms of hash tables—open addressing and chaining—and to understand their relative benefits and performance tradeoffs
- ◆ To learn how to implement both hash table forms
- ◆ To be introduced to the implementation of Maps and Sets
- ◆ To see how two earlier applications can be implemented more easily using Map objects for data storage

In Chapter 2, we introduced the Java Collections Framework, focusing on the `List` interface and the classes that implement it (`ArrayList` and `LinkedList`). The classes that implement the `List` interface are all indexed collections. That is, there is an index or a subscript associated with each member (element) of an object of these classes. Often an element's index reflects the relative order of its insertion in the `List` object. Searching for a particular value in a `List` object is generally an  $O(n)$  process. The exception is a binary search of a sorted object, which is an  $O(\log n)$  process.

In this chapter, we consider the other part of the `Collection` hierarchy: the `Set` interface and the classes that implement it. `Set` objects are not indexed, and the order of insertion of items is not known. Their main purpose is to enable efficient search and retrieval of information. It is also possible to remove elements from these collections without moving other elements around. By contrast, if an element is removed from the collection in an `ArrayList` object, the elements that follow it are normally shifted over to fill the vacated space.

A second, related interface is the `Map`. `Map` objects provide efficient search and retrieval of entries that consist of pairs of objects. The first object in each pair is the key (a unique value), and the second object is the information associated with that key. You retrieve an object from a `Map` by specifying its key.

We also study the hash table data structure. The hash table is a very important data structure that has been used very effectively in compilers and in building dictionaries. It can be

used as the underlying data structure for a Map or Set implementation. It stores objects at arbitrary locations and offers an average constant time for insertion, removal, and searching.

We will see two ways to implement a hash table and how to use it as the basis for a class that implements the Map or Set. We will not show you the complete implementation of an object that implements Map or Set because we expect that you will use the ones provided by the Java API. However, we will certainly give you a head start on what you need to know to implement these interfaces.

## Sets and Maps

- 7.1** Sets and the Set Interface
- 7.2** Maps and the Map Interface
- 7.3** Hash Tables
- 7.4** Implementing the Hash Table
- 7.5** Implementation Considerations for Maps and Sets
- 7.6** Additional Applications of Maps
  - Case Study:* Implementing a Cell Phone Contact List
  - Case Study:* Completing the Huffman Coding Problem
- 7.7** Navigable Sets and Maps

## 7.1 Sets and the Set Interface

We introduced the Java Collections Framework in Chapter 2. We covered the part of that framework that focuses on the List interface and its implementers. In this section, we explore the Set interface and its implementers.

Figure 7.1 shows the part of the Collections Framework that relates to sets. It includes interfaces Set, SortedSet, and NavigableSet; abstract class AbstractSet; and actual classes HashSet, TreeSet, and ConcurrentSkipListSet. The HashSet is a set that is implemented using a hash table (discussed in Section 7.3). The TreeSet is implemented using a special kind of binary search tree, called the Red–Black tree (discussed in Chapter 9). The ConcurrentSkipListSet is implemented using a skip list (discussed in Chapter 9). In Section 6.5, we showed how to use a TreeSet to store an index for a term paper.

### The Set Abstraction

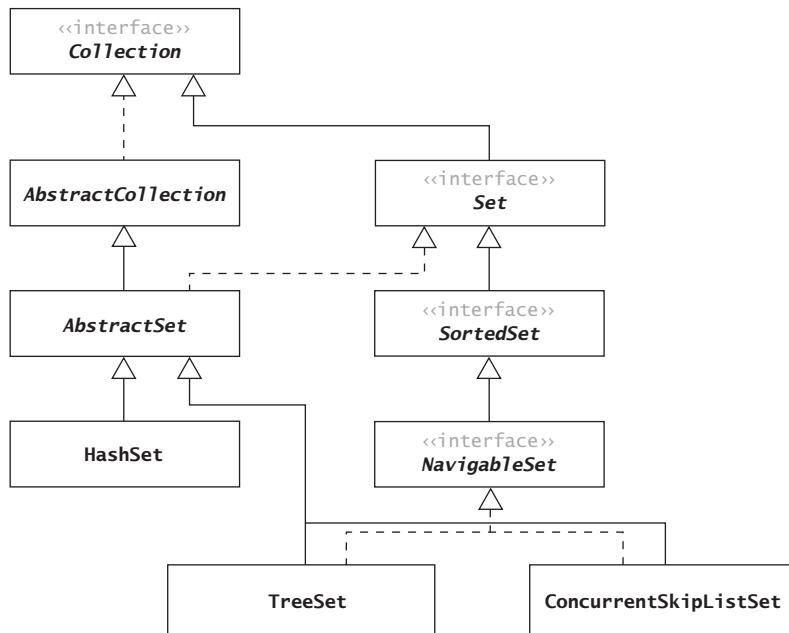
The Java API documentation for the interface java.util.Set describes the Set as follows:

A collection that contains no duplicate elements. More formally, sets contain no pair of elements e<sub>1</sub> and e<sub>2</sub> such that e<sub>1</sub>.equals(e<sub>2</sub>), and at most one null element. As implied by its name, this interface models the mathematical *set* abstraction.

What mathematicians call a *set* can be thought of as a collection of objects. There is the additional requirement that the elements contained in the set are unique. For example, if we have the set of fruits {"apples", "oranges", and "pineapples"} and add "apples" to it, we still have the same set. Also, we usually want to know whether or not a particular object is a member of the set rather than where in the set it is located. Thus, if s is a set, we would be interested in the expression

```
s.contains("apples")
```

**FIGURE 7.1**  
The Set Hierarchy



which returns the value `true` if "apples" is in set `s` and `false` if it is not. We would not have a need to use a method such as

```
s.indexOf("apples")
```

which might return the location or position of "apples" in set `s`. Nor would we have a need to use the expression

```
s.get(i)
```

where `i` is the position (index) of an object in set `s`.

We assume that you are familiar with sets from a course in discrete mathematics. Just as a review, however, the operations that are performed on a mathematical set are testing for membership (method `contains`), adding elements, and removing elements. Other common operations on a mathematical set are *set union* ( $A \cup B$ ), *set intersection* ( $A \cap B$ ), and *set difference* ( $A - B$ ). There is also a *subset operator* ( $A \subset B$ ). These operations are defined as follows:

- The union of two sets  $A, B$  is a set whose elements belong either to  $A$  or  $B$  or to both  $A$  and  $B$ .  
Example:  $\{1, 3, 5, 7\} \cup \{2, 3, 4, 5\}$  is  $\{1, 2, 3, 4, 5, 7\}$
- The intersection of sets  $A, B$  is the set whose elements belong to both  $A$  and  $B$ .  
Example:  $\{1, 3, 5, 7\} \cap \{2, 3, 4, 5\}$  is  $\{3, 5\}$
- The difference of sets  $A, B$  is the set whose elements belong to  $A$  but not to  $B$ .  
Examples:  $\{1, 3, 5, 7\} - \{2, 3, 4, 5\}$  is  $\{1, 7\}$ ;  $\{2, 3, 4, 5\} - \{1, 3, 5, 7\}$  is  $\{2, 4\}$
- Set  $A$  is a subset of set  $B$  if every element of set  $A$  is also an element of set  $B$ .  
Example:  $\{1, 3, 5, 7\} \subset \{1, 2, 3, 4, 5, 7\}$  is true

## The Set Interface and Methods

A Set has required methods for testing for set membership (`contains`), testing for an empty set (`isEmpty`), determining the set size (`size`), and creating an iterator over the set (`iterator`). It has optional methods for adding an element (`add`) and removing an element (`remove`). It

**TABLE 7.1**Some `java.util.Set<E>` Methods (with Mathematical Set Operations in Italics)

Method	Behavior
<code>boolean add(E obj)</code>	Adds item <code>obj</code> to this set if it is not already present (optional operation) and returns <code>true</code> . Returns false if <code>obj</code> is already in the set
<code>boolean addAll(Collection&lt;E&gt; coll)</code>	Adds all of the elements in collection <code>coll</code> to this set if they're not already present (optional operation). Returns <code>true</code> if the set is changed. Implements <i>set union</i> if <code>coll</code> is a Set
<code>boolean contains(Object obj)</code>	Returns <code>true</code> if this set contains an element that is equal to <code>obj</code> . Implements a test for <i>set membership</i>
<code>boolean containsAll(Collection&lt;E&gt; coll)</code>	Returns <code>true</code> if this set contains all of the elements of collection <code>coll</code> . If <code>coll</code> is a set, returns <code>true</code> if this set is a subset of <code>coll</code>
<code>boolean isEmpty()</code>	Returns <code>true</code> if this set contains no elements
<code>Iterator&lt;E&gt; iterator()</code>	Returns an iterator over the elements in this set
<code>boolean remove(Object obj)</code>	Removes the set element equal to <code>obj</code> if it is present (optional operation). Returns <code>true</code> if the object was removed
<code>boolean removeAll(Collection&lt;E&gt; coll)</code>	Removes from this set all of its elements that are contained in collection <code>coll</code> (optional operation). Returns <code>true</code> if this set is changed. If <code>coll</code> is a set, performs the <i>set difference</i> operation
<code>boolean retainAll(Collection&lt;E&gt; coll)</code>	Retains only the elements in this set that are contained in collection <code>coll</code> (optional operation). Returns <code>true</code> if this set is changed. If <code>coll</code> is a set, performs the <i>set intersection</i> operation
<code>int size()</code>	Returns the number of elements in this set (its cardinality)

provides the additional restriction on constructors that all sets they create must contain no duplicate elements. It also puts the additional restriction on the `add` method that a duplicate item cannot be inserted. Table 7.1 shows the commonly used methods of the `Set` interface. The `Set` interface also has methods that support the mathematical set operations. The required method `containsAll` tests the subset relationship. There are optional methods for set union (`addAll`), set intersection (`retainAll`), and set difference (`removeAll`). We show the methods that are used to implement the mathematical set operations in italics in Table 7.1.

Calling a method “optional” means just that an implementer of the `Set` interface is not required to provide it. However, a method that matches the signature must be provided. This method should throw the `UnsupportedOperationException` whenever it is called. This gives the class designer some flexibility. For example, if a class instance is intended to provide efficient search and retrieval of the items stored, the class designer may decide to omit the optional mathematical set operations.



## FOR PYTHON PROGRAMMERS

The Python `Set` class is similar to the Java `HashSet` class. Both have operations for creating sets, adding and removing objects, and forming union, intersection, and difference.

---

**EXAMPLE 7.1** Listing 7.1 contains a `main` method that creates three sets: `setA`, `setAcopy`, and `setB`. It loads these sets from two arrays and then forms their union in `setA` and their intersection in `setAcopy`, using the statements

```
setA.addAll(setB);           // Set union
setAcopy retainAll(setB);   // Set intersection
```

Running this method generates the output lines below. The brackets and commas are inserted by method `toString`.

```
The 2 sets are:
[Jill, Ann, Sally]
[Bill, Jill, Ann, Bob]
Items in set union are: [Bill, Jill, Ann, Sally, Bob]
Items in set intersection are: [Jill, Ann]
```

---

### LISTING 7.1

Illustrating the Use of Sets

```
public static void main(String[] args) {

    // Create the sets.
    String[] listA = {"Ann", "Sally", "Jill", "Sally"};
    String[] listB = {"Bob", "Bill", "Ann", "Jill"};
    Set<String> setA = new HashSet<>();
    Set<String> setAcopy = new HashSet<>(); // Copy of setA
    Set<String> setB = new HashSet<>();

    // Load sets from arrays.
    for (String s : listA) {
        setA.add(s);
        setAcopy.add(s);
    }
    for (String s : listB) {
        setB.add(s);
    }
    System.out.println("The 2 sets are: " + "\n" + setA
        + "\n" + setB);
    // Display the union and intersection.
    setA.addAll(setB);           // Set union
    setAcopy retainAll(setB);   // Set intersection
    System.out.println("Items in set union are: " + setA);
    System.out.println("Items in set intersection are: "
        + setAcopy);
}
```

---

## Comparison of Lists and Sets

Collections implementing the `Set` interface must contain unique elements. Unlike the `List.add` method, the `Set.add` method will return `false` if you attempt to insert a duplicate item.

Unlike a `List`, a `Set` does not have a `get` method. Therefore, elements cannot be accessed by index. So if `setA` is a `Set` object, the method call `setA.get(0)` would cause the syntax error `method get(int) not found`.

Although you can't reference a specific element of a `Set`, you can iterate through all its elements using an `Iterator` object. The loop below accesses each element of `Set` object `setA`.

However, the elements will be accessed in arbitrary order. This means that they will not necessarily be accessed in the order in which they were inserted.

```
// Create an iterator to setA.
Iterator<String> setAIter = setA.iterator();
while (setAIter.hasNext()) {
    String nextItem = setAIter.next();
    // Do something with nextItem
    . . .
}
```

We can simplify the task of accessing each element in a Set using the Java 5.0 enhanced **for** statement.

```
for (String nextItem : setA) {
    // Do something with nextItem
    . . .
}
```

## EXERCISES FOR SECTION 7.1

### SELF-CHECK

1. Explain the effect of the following method calls.

```
Set<String> s = new HashSet<String>();
s.add("hello");
s.add("bye");
s.addAll(s);
Set<String> t = new TreeSet<String>();
t.add("123");
s.addAll(t);
System.out.println(s.containsAll(t));
System.out.println(t.containsAll(s));
System.out.println(s.contains("ace"));
System.out.println(s.contains("123"));
sretainAll(t);
System.out.println(s.contains("123"));
t.retainAll(s);
System.out.println(t.contains("123"));
```

2. What is the relationship between the Set interface and the Collection interface?
3. What are the differences between the Set interface and the List interface?
4. In Example 7.1, why is `setAcopy` needed? What would happen if you used the statement  
`setAcopy = setA;`  
to define `setAcopy`?

### PROGRAMMING

1. Assume you have declared three sets `a`, `b`, and `c` and that sets `a` and `b` store objects. Write statements that use methods from the Set interface to perform the following operations:
  - $c = (a \cup b)$
  - $c = (a \cap b)$
  - $c = (a - b)$

- d. if ( $a \subset b$ )  
      $c = a$ ;  
 else  
      $c = b$ ;
2. Write a `toString` method for a class that implements the `Set` interface and displays the set elements in the form shown in Example 9.1.



## 7.2 Maps and the Map Interface

The `Map` is related to the `Set`. Mathematically, a `Map` is a set of ordered pairs whose elements are known as the key and the value. The key is required to be unique, as are the elements of a set, but the value is not necessarily unique. For example, the following would be a map:

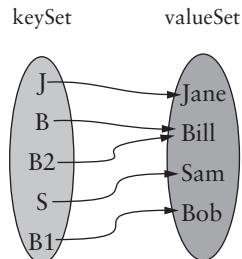
`{(J, Jane), (B, Bill), (S, Sam), (B1, Bob), (B2, Bill)}`

The keys in this example are strings consisting of one or two characters, and each value is a person's name. The keys are unique but not the values (there are two Bills). The key is based on the first letter of the person's name. The keys `B1` and `B2` are the keys for the second and third person whose name begins with the letter `B`.

You can think of each key as "mapping" to a particular value (hence the name *map*). For example, the key `J` maps to the value `Jane`. The keys `B` and `B2` map to the value `Bill`. You can also think of the keys as forming a set (`keySet`) and the values as forming a set (`valueSet`). Each element of `keySet` maps to a particular element of `valueSet`, as shown in Figure 7.2. In mathematical set terminology, this is a *many-to-one mapping* (i.e., more than one element of `keySet` may map to a particular element of `valueSet`). For example, both keys `B` and `B2` map to the value `Bill`. This is also an *onto mapping* in that all elements of `valueSet` have a corresponding member in `keySet`.

A `Map` can be used to enable efficient storage and retrieval of information in a table. The key is a unique identification value associated with each item stored in a table. As you will see, each key value has an easily computed numeric code value.

**FIGURE 7.2**  
Example of Mapping



### EXAMPLE 7.2

When information about an item is stored in a table, the information stored may consist of a unique ID (identification code, which may or may not be a number) as well as descriptive data. The unique ID would be the key, and the rest of the information would represent the value associated with that key. Some examples follow.

Type of Item	Key	Value
University student	Student ID number	Student name, address, major, grade-point average
Customer for online store	E-mail address	Customer name, address, credit card information, shopping cart
Inventory item	Part ID	Description, quantity, manufacturer, cost, price

In the above examples, the student ID number may be assigned by the university, or it may be the student's social security number. The e-mail address is a unique address for each customer, but it is not numeric. Similarly, a part ID could consist of a combination of letters and digits.

In comparing maps to indexed collections, you can think of the keys as selecting the elements of a map, just as indexes select elements in a `List` object. The keys for a map, however, can have arbitrary values (not restricted to 0, 1, etc., as for indexes). As you will see later, an implementation of the `Map` interface should have methods of the form

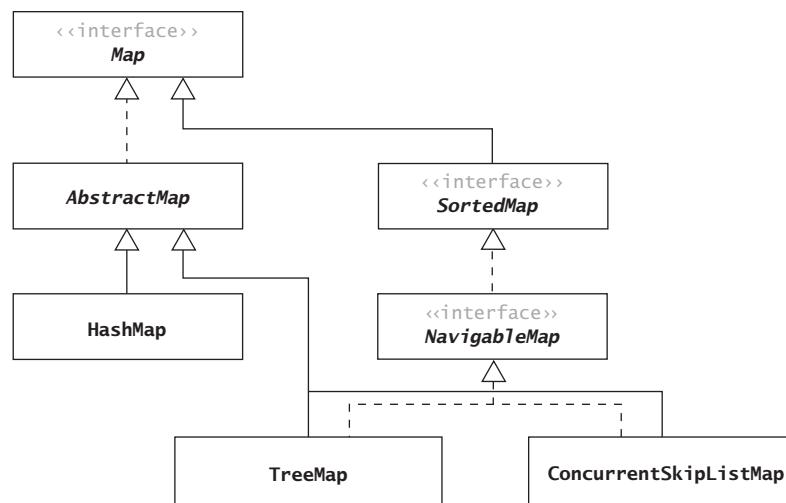
```
V get(Object key)
V put(K key, V value)
```

The `get` method retrieves the value corresponding to a specified key; the `put` method stores a key-value pair in a map.

## The Map Hierarchy

Figure 7.3 shows part of the `Map` hierarchy in the Java API. Although not strictly part of the `Collection` hierarchy, the `Map` interface defines a structure that relates elements in one set to elements in another set. The first set, called the *keys*, must implement the `Set` interface; that is, the *keys* are unique. The second set is not strictly a `Set` but an arbitrary `Collection` known as the *values*. These are not required to be unique. The `Map` is a more useful structure than the `Set`. In fact, the Java API implements the `Set` using a `Map`.

**FIGURE 7.3**  
The Map Hierarchy



The `TreeMap` uses a Red–Black binary search tree (discussed in Chapter 9) as its underlying data structure, and the `ConcurrentSkipListMap` uses a skip list (also discussed in Chapter 9) as its underlying data structure. We will focus on the `HashMap` and show how to implement it later in the chapter.

## The Map Interface

Methods of the `Map` interface (in Java API `java.util`) are shown in Table 7.2. The `put` method either inserts a new mapping or changes the value associated with an existing mapping. The `get` method returns the current value associated with a given key or null if there is none. The `getOrDefault` method returns the provided `defaultValue` instead of null if the key is not present. The `remove` method deletes an existing mapping.

The `getOrDefault` method is a default method, which means that the implementation of this method is defined in the interface. The code for `getOrDefault` is equivalent to the following:

```
default V getOrDefault(Object key, V defaultValue) {
    V value = get(key);
    if (value != null) return value;
    return defaultValue;
}
```

**TABLE 7.2**Some `java.util.Map<K, V>` Methods

Method	Behavior
<code>V get(Object key) default</code>	Returns the value associated with the specified key. Returns <code>null</code> if the key is not present
<code>V getOrDefault(Object key, V default)</code>	Returns the value associated with the specified key. Returns <code>default</code> if the key is not present
<code>boolean isEmpty()</code>	Returns <code>true</code> if this map contains no key-value mappings
<code>V put(K key, V value)</code>	Associates the specified value with the specified key in this map (optional operation). Returns the previous value associated with the specified key, or <code>null</code> if there was no mapping for the key
<code>V remove(Object key)</code>	Removes the mapping for this key from this map if it is present (optional operation). Returns the previous value associated with the specified key, or <code>null</code> if there was no mapping for the key
<code>void forEach(BiConsumer&lt;K, V&gt;)</code>	Performs the action given by the BiConsumer to each entry in the map, binding the key to the first parameter and the value to the second
<code>int size()</code>	Returns the number of key-value mappings in this map

Both `put` and `remove` return the previous value (or `null`, if there was none) of the mapping that is changed or deleted. There are two type parameters, `K` and `V`, and they represent the data type of the key and value, respectively.

---

**EXAMPLE 7.3** The following statements build a `Map` object that contains the mapping shown in Figure 7.2.

```
Map<String, String> aMap = new HashMap<>();
// HashMap implements Map
aMap.put("J", "Jane");
aMap.put("B", "Bill");
aMap.put("S", "Sam");
aMap.put("B1", "Bob");
aMap.put("B2", "Bill");
```

The statement

```
System.out.println("B1 maps to " + aMap.get("B1"));
would display "B1 maps to Bob". The statement
```

```
System.out.println("Bill maps to " + aMap.get("Bill"));
would display "Bill maps to null" because "Bill" is a value, not a key.
```

---

**EXAMPLE 7.4** In Section 6.5, we used a binary search tree to store an index of words occurring in a term paper. Each data element in the tree was a string consisting of a word followed by a three-digit line number.

Although this is one approach to storing an index, it would be more useful to store each word and all the line numbers for that word as a single index entry. We could do this by storing the index in a `Map` in which each word is a key and its associated value is a list of all the line numbers at which the word occurs. While building the index, each time a word is encountered, its list of line numbers would be retrieved (using the word as a key) and the most recent line number would be appended to this list (a `List<Integer>`). For example, if the word `fire` has

already occurred on lines 4 and 8 and we encounter it again on line 20, the `List<Integer>` associated with `line` would reference three `Integer` objects wrapping the numbers 4, 8, and 20.

Listing 7.2 shows method `buildIndex` (adapted from `buildIndex` in Listing 6.8). Data field `index` is a Map with key type `String` and value type `List<Integer>`.

```
private Map<String, List<Integer>> index;
```

The statement

```
List<Integer> lines = index.getOrDefault(token, new ArrayList<>());
```

retrieves the value (an `ArrayList<Integer>`) associated with the next token or an empty `ArrayList` if this is the first occurrence of `token`. The statements

```
lines.add(lineNum);
index.put(token, lines);           // Store the list.
```

add the new line number to the `ArrayList` `lines` and store it back in the `Map`. In Section 7.5, we show how to display the final index.

---

### LISTING 7.2

Method `buildIndexAllLines`

```
/** Reads each word in a data file and stores it in an index
 * along with a list of line numbers where it occurs.
 * @post Lowercase form of each word with its line
 *       number is stored in the index.
 * @param scan A Scanner object
 */
public void buildIndex(Scanner scan) {
    int lineNum = 0;      // Line number

    // Keep reading lines until done.
    while (scan.hasNextLine()) {
        lineNum++;

        // Extract each token and store it in index.
        String token;
        while ((token = scan.findInLine(PATTERN)) != null) {
            token = token.toLowerCase();
            // Get the list of line numbers for token
            List<Integer> lines = index.getOrDefault(token, new ArrayList<>());
            lines.add(lineNum);
            index.put(token, lines);    // Store new list of line numbers
        }
        scan.nextLine();    // Clear the scan buffer
    }
}
```

---

## EXERCISES FOR SECTION 7.2

### SELF-CHECK

1. If you were using a `Map` to store the following lists of items, which data field would you select as the key, and why?
  - a. textbook title, author, ISBN (International Standard Book Number), year, publisher
  - b. player's name, uniform number, team, position
  - c. computer manufacturer, model number, processor, memory, disk size
  - d. department, course title, course ID, section number, days, time, room

2. For the Map `index` in Example 7.4, what key–value pairs would be stored for each token in the following data file?

```
this line is first
and line 2 is second
followed by the third line
```

3. Explain the effect of each statement in the following fragment on the index built in Self-Check Exercise 2.

```
lines = index.get("this");
lines = index.get("that");
lines = index.get("line");
lines.add(4);
index.put("is", lines);
```

### PROGRAMMING

1. Write statements to create a Map object that will store each word occurring in a term paper along with the number of times the word occurs.
2. Write a method `buildWordCounts` (based on `buildIndex`) that builds the Map object described in Programming Exercise 1.



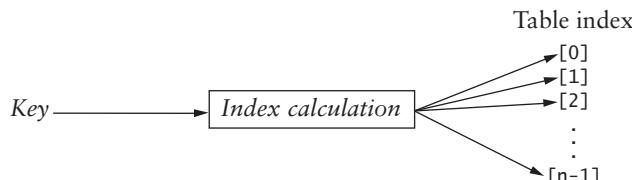
## 7.3 Hash Tables

Before we discuss the details of implementing the required methods of the Set and Map interfaces, we will describe a data structure, the *hash table*, that can be used as the basis for such an implementation. The goal behind the hash table is to be able to access an entry based on its key value, not its location. In other words, we want to be able to access an element directly through its key value rather than having to determine its location first by searching for the key value in an array. (This is why the Set interface has method `contains(obj)` instead of `get(index)`.) Using a hash table enables us to retrieve an item in constant time (expected  $O(1)$ ). We say expected  $O(1)$  rather than just  $O(1)$  because there will be some cases where the performance will be much worse than  $O(1)$  and may even be  $O(n)$ , but on the average, we expect that it will be  $O(1)$ . Contrast this with the time required for a linear search of an array,  $O(n)$ , and the time to access an element in a binary search tree,  $O(\log n)$ .

### Hash Codes and Index Calculation

The basis of hashing (and hash tables) is to transform the item’s key value to an integer value (its *hash code*) that will then be transformed into a table index. Figure 7.4 illustrates this process for a table of size  $n$ . We discuss how this might be done in the next few examples.

**FIGURE 7.4**  
Index Calculation  
for a Key



---

**EXAMPLE 7.5** Consider the Huffman code problem discussed in Section 6.7. To build the Huffman tree, you needed to know the number of occurrences of each character in the text being encoded. Let's assume that the text contained only the ASCII characters (the first 128 Unicode values starting with \u0000). We could use a table of size 128, one element for each possible character, and let the Unicode for each character be its location in the table. Using this approach, table element 65 would give us the number of occurrences of the letter A, table element 66 would give us the number of occurrences of the letter B, and so on. The hash code for each character is its Unicode value (a number), which is also its index in the table. In this case, we could calculate the table index for character `asciiChar` using the following assignment statement, where `asciiChar` represents the character we are seeking in the table:

```
int index = asciiChar;
```

---

**EXAMPLE 7.6** Let's consider a slightly harder problem: assume that any of the Unicode characters can occur in the text, and we want to know the number of occurrences of each character. There are over 65,000 Unicode characters, however. For any file, let's assume that at most 100 different characters actually appear. So, rather than use a table with 65,536 elements, it would make sense to try to store these items in a much smaller table (say, 200 elements). If the hash code for each character is its Unicode value, we need to convert this value (between 0 and 65,536) to an array index between 0 and 199. We can calculate the array index for character `uniChar` as

```
int index = uniChar % 200
```

Because the range of Unicode values (the key range) is much larger than the `index` range, it is likely that some characters in our text will have the same index value. Because we can store only one key-value pair in a given array element, a situation known as a *collision* results. We discuss how to deal with collisions shortly.

---

## Methods for Generating Hash Codes

In most applications, the keys that we will want to store in a table will consist of strings of letters or digits rather than a single character (e.g., a social security number, a person's name, or a part ID). We need a way to map each string to a particular table index. Again, we have a situation in which the number of possible key values is much larger than the table size. For example, if a string can store up to 10 letters or digits, the number of possible strings is  $36^{10}$  (approximately  $3.7 \times 10^{15}$ ), assuming the English alphabet with 26 letters.

Generating good hash codes for arbitrary strings or arbitrary objects is somewhat of an experimental process. Simple algorithms tend to generate a lot of collisions. For example, simply summing the `int` values for all characters in a string would generate the same hash code for words that contained the same letters but in different orders, such as "sign" and "sing", which would have the same hash code using this algorithm ('s' + 'i' + 'n' + 'g'). The algorithm used by the Java API accounts for the position of the characters in the string as well as the character values.

The `String.hashCode()` method returns the integer calculated by the formula:

$$s_0 \times 31^{(n-1)} + s_1 \times 31^{(n-2)} + \dots + s_{n-1}$$

where  $s_i$  is the  $i$ th character of the string and  $n$  is the length of the string. For example, the string "Cat" would have a hash code of 'C' × 31<sup>2</sup> + 'a' × 31 + 't'. This is the number 67,510. (The number 31 is a prime number that generates relatively few collisions.)

As previously discussed, the integer value returned by method `String.hashCode` can't be unique because there are too many possible strings. However, the probability of two strings having the same hash code value is relatively small because the `String.hashCode` method distributes the hash code values fairly evenly throughout the range of `int` values.

Because the hash codes are distributed evenly throughout the range of `int` values, method `String.hashCode` will appear to produce a random value, as will the expressions `s.hashCode() % table.length`, which selects the initial value of `index` for `String s`. If the object is not already present in the table, the probability that this expression does not yield an empty slot in the table is proportional to how full the table is.

One additional criterion for a good hash function, besides a random distribution for its values, is that it be relatively simple and efficient to compute. It doesn't make much sense to use a hash function whose computation is an  $O(n)$  process to avoid doing an  $O(n)$  search.

## Open Addressing

Next, we consider two ways to organize hash tables: open addressing and chaining. In open addressing, each hash table element (type `Object`) references a single key–value pair. We can use the following simple approach (called *linear probing*) to access an item in a hash table. If the index calculated for an item's key is occupied by an item with that key, we have found the item. If that element contains an item with a different key, we increment the index by 1. We keep incrementing the index (modulo the table length) until either we find the key we are seeking or we reach a `null` entry. A `null` entry indicates that the key is not in the table.

### Algorithm for Accessing an Item in a Hash Table

1. Compute the index by taking the item's `hashCode() % table.length`.
2. **if** `table[index]` is `null`
  3. The item is not in the table.
  4. **else if** `table[index]` is equal to the item
    5. The item is in the table.
  6. Continue to search the table by incrementing the index until either the item is found or a `null` entry is found.

Step 1 ensures that the `index` is within the table range (0 through `table.length - 1`). If the condition in Step 2 is true, the table index does not reference an object, so the item is not in the table. The condition in Step 4 is true if the item being sought is at position `index`, in which case the item is located. Steps 1 through 5 can be done in  $O(1)$  expected time.

Step 6 is necessary for two reasons. The values returned by method `hashCode` are not unique, so the item being sought can have the same hash code as another one in the table. Also, the remainder calculated in Step 1 can yield the same index for different hash code values. Both of these cases are examples of collisions.

## Table Wraparound and Search Termination

Note that as you increment the table index, your table should wrap around (as in a circular array) so that the element with subscript 0 “follows” the element with subscript `table.length - 1`. This enables you to use the entire table, not just the part with subscripts larger than the hash code value, but it leads to the potential for an infinite loop in Step 6 of the algorithm. If the

table is full and the objects examined so far do not match the one you are seeking, how do you know when to stop? One approach would be to stop when the index value for the next probe is the same as the hash code value for the object. This means that you have come full circle to the starting value for the index. A second approach would be to ensure that the table is never full by increasing its size after an insertion if its occupancy rate exceeds a specified threshold. This is the approach that we take in our implementation.

**EXAMPLE 7.7** We illustrate insertion of five names in a table of size 5 and in a table of size 11. Table 7.3 shows the names, the corresponding hash code, the hash code modulo 5 (in column 3), and the hash code modulo 11 (in column 4). We picked prime numbers (5 and 11) because empirical tests have shown that hash tables with a size that is a prime number often give better results.

For a table of size 5 (an occupancy rate of 100 percent), "Tom", "Dick", and "Sam" have hash indexes of 4, and "Harry" and "Pete" have hash indexes of 3; for a table length of 11 (an occupancy rate of 45 percent), "Dick" and "Sam" have hash indexes of 5, but the others have hash indexes that are unique. We see how the insertion process works next.

For a table of size 5, if "Tom" and "Dick" are the first two entries, "Tom" would be stored at the element with index 4, the last element in the table. Consequently, when "Dick" is inserted, because element 4 is already occupied, the hash index is incremented to 0 (the table wraps around to the beginning), where "Dick" is stored.

.....  
**TABLE 7.3**

Names and hashCode Values for Table Sizes 5 and 11

Name	hashCode()	hashCode()%5	hashCode()%11
"Tom"	84274	4	3
"Dick"	2129869	4	5
"Harry"	69496448	3	10
"Sam"	82879	4	5
"Pete"	2484038	3	7

[0]	"Dick"
[1]	null
[2]	null
[3]	null
[4]	"Tom"

"Harry" is stored in position 3 (the hash index), and "Sam" is stored in position 1 because its hash index is 4 but the elements at 4 and 0 are already filled.

[0]	"Dick"
[1]	"Sam"
[2]	null
[3]	"Harry"
[4]	"Tom"

Finally, "Pete" is stored in position 2 because its hash index is 3 but the elements at positions 3, 4, 0, 1 are filled.

[0]	"Dick"
[1]	"Sam"
[2]	"Pete"
[3]	"Harry"
[4]	"Tom"

For the table of size 11, the entries would be stored as shown in the following table, assuming that they were inserted in the order "Tom", "Dick", "Harry", "Sam", and finally "Pete". Insertions go more smoothly for the table of size 11. The first collision occurs when "Sam" is stored, so "Sam" is stored at position 6 instead of position 5.

[0]	<b>null</b>
[1]	<b>null</b>
[2]	<b>null</b>
[3]	"Tom"
[4]	<b>null</b>
[5]	"Dick"
[6]	"Sam"
[7]	"Pete"
[8]	<b>null</b>
[9]	<b>null</b>
[10]	"Harry"

For the table of size 5, retrieval of "Tom" can be done in one step. Retrieval of all of the others would require a linear search because of collisions that occurred when they were inserted. For the table of size 11, retrieval of all but "Sam" can be done in one step, and retrieval of "Sam" requires only two steps. This example illustrates that the best way to reduce the probability of a collision is to increase the table size.

---

## Traversing a Hash Table

One thing that you cannot do is traverse a hash table in a meaningful way. If you visit the hash table elements in sequence and display the objects stored, you would display the strings "Dick", "Sam", "Pete", "Harry", and "Tom" for the table of length 5 and the strings "Tom", "Dick", "Sam", "Pete", and "Harry" for a table of length 11. In either case, the list of names is in arbitrary order.

## Deleting an Item Using Open Addressing

When an item is deleted, we cannot just set its table entry to **null**. If we do, then when we search for an item that may have collided with the deleted item, we may incorrectly conclude that the item is not in the table. (Because the item that collided was inserted after the deleted item, we will have stopped our search prematurely.) By storing a dummy value when an item is deleted, we force the search algorithm to keep looking until either the desired item is found or a **null** value, representing a free cell, is located.

Although the use of a dummy value solves the problem, keep in mind that it can lead to search inefficiency, particularly when there are many deletions. Removing items from the table does not reduce the search time because the dummy value is still in the table and is part of a search chain. In fact, you cannot even replace a deleted value with a new item because you still need to go to the end of the search chain to ensure that the new item is not already present in the table. So deleted items waste storage space and reduce search efficiency. In the worst case, if the table is almost full and then most of the items are deleted, you will have  $O(n)$  performance when searching for the few items remaining in the table.

## Reducing Collisions by Expanding the Table Size

Even with a good hashing function, it is still possible to have collisions. The first step in reducing these collisions is to use a prime number for the size of the table.

In addition, the probability of a collision is proportional to how full the table is. Therefore, when the hash table becomes sufficiently full, a larger table should be allocated and the entries reinserted.

We previously saw examples of expanding the size of an array. Generally, what we did was to allocate a new array with twice the capacity of the original, copy the values in the original array to the new array, and then reference the new array instead of the original. This approach will not work with hash tables. If you use it, some search chains will be broken because the new table does not wrap around in the same way as the original table. The last element in the original table will be in the middle of the new table, and it does not wrap around to the first element of the new table. Therefore, you expand a hash table (called *rehashing*) using the following algorithm.

### Algorithm for Rehashing

1. Allocate a new hash table with twice the capacity of the original.
2. Reinsert each old table entry that has not been deleted into the new hash table.
3. Reference the new table instead of the original.

Step 2 reinserts each item from the old table into the new table instead of copying it over to the same location. We illustrate this in the hash table implementation. Note that deleted items are not reinserted into the new table, thereby saving space and reducing the length of some search chains.

## Reducing Collisions Using Quadratic Probing

The problem with linear probing is that it tends to form clusters of keys in the table, causing longer search chains. For example, if the table already has keys with hash codes of 5 and 6, a new item that collides with either of these keys will be placed at index 7. An item that collides with any of these three items will be placed at index 8, and so on. Figure 7.5 shows a hash table of size 11 after inserting elements with hash codes in the sequence 5, 6, 5, 6, 7. Each new collision expands the cluster by one element, thereby increasing the length of the search chain for each element in that cluster. For example, if another element is inserted with any hash code in the range 5 through 9, it will be placed at position 10, and the search chain for items with hash codes of 5 and 6 would include the elements at indexes 7, 8, 9, and 10.

**FIGURE 7.5**  
Clustering with  
Linear Probing

[0]	
[1]	
[2]	
[3]	
[4]	
[5]	1 <sup>st</sup> item with hash code 5
[6]	1 <sup>st</sup> item with hash code 6
[7]	2 <sup>nd</sup> item with hash code 5
[8]	2 <sup>nd</sup> item with hash code 6
[9]	1 <sup>st</sup> item with hash code 7
[10]	

One approach to reduce the effect of clustering is to use *quadratic probing* instead of linear probing. In quadratic probing, the increments form a quadratic series ( $1 + 2^2 + 3^2 + \dots$ ). Therefore, the next value of index is calculated using the steps:

```
probeNum++;
index = (startIndex + probeNum * probeNum) % table.length
```

where `startIndex` is the index calculated using method `hashCode` and `probeNum` starts at 0. Ignoring wraparound, if an item has a hash code of 5, successive values of `index` will be 6 ( $5 + 1$ ), 9 ( $5 + 4$ ), 14 ( $5 + 9$ ), . . . , instead of 6, 7, 8, . . . . Similarly, if the hash code is 6, successive values of `index` will be 7, 10, 15, and so on. Unlike linear probing, these two search chains have only one table element in common (at index 6).

Figure 7.6 illustrates the hash table after elements with hash codes in the same sequence as in the preceding table (5, 6, 5, 6, 7) have been inserted with quadratic probing. Although the cluster of elements looks similar, their search chains do not overlap as much as before. Now the search chain for an item with a hash code of 5 consists of the elements at 5, 6, and 9, and the search chain for an item with a hash code of 6 consists of the elements at positions 6 and 7.

## Problems with Quadratic Probing

One disadvantage of quadratic probing is that the next index calculation is a bit time-consuming as it involves a multiplication, an addition, and a modulo division. A more efficient way to calculate the next index follows:

```
k += 2;
index = (index + k) % table.length;
```

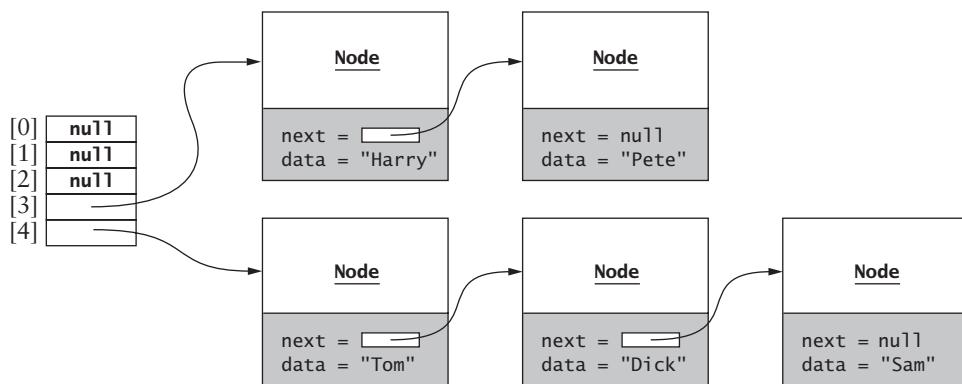
which replaces the multiplication with an addition. If the initial value of `k` is -1, successive values of `k` will be 1, 3, 5, 7, . . . . If the hash code is 5, successive values of `index` will be 5, 6 ( $5 + 1$ ), 9 ( $5 + 1 + 3$ ), 14 ( $5 + 1 + 3 + 5$ ), . . . . The proof of the equality of these two approaches to calculating `index` is based on the following mathematical series:

$$n^2 = 1 + 3 + 5 + \dots + 2n - 1$$

**FIGURE 7.6**  
Insertion with  
Quadratic Probing

[0]	
[1]	
[2]	
[3]	
[4]	
[5]	1 <sup>st</sup> item with hash code 5
[6]	1 <sup>st</sup> item with hash code 6
[7]	2 <sup>nd</sup> item with hash code 6
[8]	1 <sup>st</sup> item with hash code 7
[9]	2 <sup>nd</sup> item with hash code 5
[10]	

**FIGURE 7.7**  
Example of Chaining



A more serious problem with quadratic probing is that not all table elements are examined when looking for an insertion index, so it is possible that an item can't be inserted even when the table is not full. It is also possible that your program can get stuck in an infinite loop while searching for an empty slot. It can be proved that if the table size is a prime number and the table is never more than half full, this can't happen. However, requiring that the table be half empty at all times wastes quite a bit of memory. For these reasons, we will use linear probing in our implementation.

## Chaining

An alternative to open addressing is a technique called *chaining*, in which each table element references a linked list that contains all the items that hash to the same table index. This linked list is often called a *bucket*, and this approach is sometimes called *bucket hashing*. Figure 7.7 shows the result of chaining for our earlier example with a table of size 5. Each new element with a particular hash index can be placed at the beginning or the end of the associated linked list. The algorithm for accessing such a table is the same as for open addressing, except for the step for resolving collisions. Instead of incrementing the table index to access the next item with a particular hash code value, you traverse the linked list referenced by the table element with index `hashCode() % table.length`.

One advantage of chaining is that only items that have the same value for `hashCode() % table.length` will be examined when looking for an object. In open addressing, search chains can overlap, so a search chain may include items in the table that have different starting index values.

A second advantage is that you can store more elements in the table than the number of table slots (indexes), which is not the case for open addressing. If each table index already references a linked list, additional items can be inserted in an existing list without increasing the table size (number of indexes).

Once you have determined that an item is not present, you can insert it either at the beginning or at the end of the list. To delete an item, simply remove it from the list. In contrast to open addressing, removing an item actually deletes it, so it will not be part of future search chains.

## Performance of Hash Tables

The *load factor* for a hash table is the number of filled cells divided by table size. The load factor has the greatest effect on hash table performance. The lower the load factor, the better the performance because there is less chance of a collision when a table is sparsely populated. If there are no collisions, the performance for search and retrieval is  $O(1)$ , regardless of the table size.

## Performance of Open Addressing Versus Chaining

Donald E. Knuth (*Searching and Sorting*, vol. 3 of *The Art of Computer Programming*, Addison-Wesley, 1973) derived the following formula for the expected number of comparisons,  $c$ , required for finding an item that is in a hash table using open addressing with linear probing and a load factor  $L$ :

$$c = \frac{1}{2} \left( 1 + \frac{1}{1-L} \right)$$

**TABLE 7.4**

Number of Probes for Different Values of Load Factor ( $L$ )

<b><math>L</math></b>	<b>Number of Probes with Linear Probing</b>	<b>Number of Probes with Chaining</b>
0.0	1.00	1.00
0.25	1.17	1.13
0.5	1.50	1.25
0.75	2.50	1.38
0.85	3.83	1.43
0.9	5.50	1.45
0.95	10.50	1.48

Table 7.4 (second column) shows the value of  $c$  for different values of load factor ( $L$ ). It shows that if  $L$  is 0.5 (half full), the expected number of comparisons required is 1.5. If  $L$  increases to 0.75, the expected number of comparisons is 2.5, which is still very respectable. If  $L$  increases to 0.9 (90 percent full), the expected number of comparisons is 5.5. This is true regardless of the size of the table.

Using chaining, if an item is in the table, on average we have to examine the table element corresponding to the item's hash code and then half of the items in each list. The average number of items in a list is  $L$ , the number of items divided by the table size. Therefore, we get the formula

$$c = 1 + \frac{L}{2}$$

for a successful search. Table 7.4 (third column) shows the results for chaining. For values of  $L$  between 0.0 and 0.75, the results are similar to those of linear probing, but chaining gives better performance than linear probing for higher load factors. Quadratic probing (not shown) gives performance that is between those of linear probing and chaining.

## Performance of Hash Tables versus Sorted Arrays and Binary Search Trees

If we compare hash table performance with binary search of a sorted array, the number of comparisons required by binary search is  $O(\log n)$ , so the number of comparisons increases with table size. A sorted array of size 128 would require up to 7 probes ( $2^7$  is 128), which is more than for a hash table of any size that is 90 percent full. A sorted array of size 1024 would require up to 10 probes ( $2^{10}$  is 1024). A binary search tree would yield the same results.

You can insert into or remove elements from a hash table in  $O(1)$  expected time. Insertion or removal from a binary search tree is  $O(\log n)$ , but insertion or removal from a sorted array is  $O(n)$  (you need to shift the larger elements over). (Worst-case performance for a hash table or a binary search tree is  $O(n)$ .)

### Storage Requirements for Hash Tables, Sorted Arrays, and Trees

The performance of hashing is certainly preferable to that of binary search of an array (or a binary search tree), particularly if  $L$  is less than 0.75. However, the tradeoff is that the lower the load factor, the more unfilled storage cells there are in a hash table, whereas there are no empty cells in a sorted array. Because a binary search tree requires three references per node (the item, the left subtree, and the right subtrees), more storage would be required for a binary search tree than for a hash table with a load factor of 0.75.

**EXAMPLE 7.8**

A hash table of size 100 with open addressing could store 75 items with a load factor of 0.75. This would require storage for 100 references. This would require storage for 100 references (25 references would be `null`).

### Storage Requirements for Open Addressing and Chaining

Next, we consider the effect of chaining on storage requirements. For a table with a load factor of  $L$ , the number of table elements required is  $n$  (the size of the table). For open addressing, the number of references to an item (a key-value pair) is  $n$ . For chaining, the average number of nodes in a list is  $L$ . If we use the Java API `LinkedList`, there will be three references in each node (the item, the next list element, and the previous element). However, we could use our own single-linked list and eliminate the previous-element reference (at some time cost for deletions). Therefore, we will require storage for  $n + 2L$  references.

**EXAMPLE 7.9**

If we have 60,000 items in our hash table and use open addressing, we would need a table size of 80,000 to have a load factor of 0.75 and an expected number of comparisons of 2.5. Next, we calculate the table size,  $n$ , needed to get similar performance using chaining.

$$\begin{aligned} 2.5 &= 1 + \frac{L}{2} \\ 5.0 &= 2 + L \\ 3.0 &= \frac{60,000}{n} \\ n &= 20,000 \end{aligned}$$

A hash table of size 20,000 requires storage space for 20,000 references to lists. There will be 60,000 nodes in the table (one for each item). If we use linked lists of nodes, we will need storage for 140,000 references (2 references per node plus the 20,000 table references). This is almost twice the storage needed for open addressing.

## EXERCISES FOR SECTION 7.3

### SELF-CHECK

- For the hash table search algorithm shown in this section, why was it unnecessary to test whether all table entries had been examined as part of Step 5?
- For the items in the five-element table of Table 7.3, compute `hashCode() % table.length` for lengths of 7 and 13. What would be the position of each word in tables of

these sizes using open addressing and linear probing? Answer the same question for chaining.

- The following table stores `Integer` keys with the `int` values shown. Show one sequence of insertions that would store the keys as shown. Which elements were placed in their current position because of collisions? Show the table that would be formed by chaining.

Index	Key
[0]	24
[1]	6
[2]	20
[3]	
[4]	14

- For Table 7.3 and the table size of 5 shown in Example 7.7, discuss the effect of deleting the entry for Dick and replacing it with a `null` value. How would this affect the search for Sam, Pete, and Harry? Answer both questions if you replace the entry for Dick with the string "deleted" instead of `null`.
- Explain what is wrong with the following strategy to reclaim space that is filled with deleted items in a hash table: when attempting to insert a new item in the table, if you encounter an item that has been deleted, replace the deleted item with the new item.
- Compare the storage requirement for a hash table with open addressing, a table size of 500, and a load factor of 0.5 with a hash table that uses chaining and gives the same performance.
- One simple hash code is to use the sum of the ASCII codes for the letters in a word. Explain why this is not a good hash code.
- If  $p_i$  is the position of a character in a string and  $c_i$  is the code for that character, would  $c_1p_1 + c_2p_2 + c_3p_3 + \dots$  be a better hash code? Explain why or why not.
- Use the hash code in Self-Check Exercise 7 to store the words "cat", "hat", "tac", and "act" in a hash table of size 10. Show this table using open hashing and chaining.

## PROGRAMMING

- Code the following algorithm for finding the location of an object as a static method. Assume a hash table array and an object to be located in the table are passed as arguments. Return the object's position if it is found; return -1 if the object is not found.
  - Compute the index by taking the `hashCode() % table.length`.
  - `if table[index] is null`
  - The object is not in the table.
  - `else if table[index] is equal to the object`
  - The object is in the table.
  - `else`
  - Continue to search the table (by incrementing `index`) until either the object is found or a `null` entry is found.

## 7.4 Implementing the Hash Table

In this section, we discuss how to implement a hash table. We will show implementations for hash tables using open addressing and chaining.

### Interface KWHashMap

Because we want to show more than one way to implement a hash table, we introduce an interface `KWHashMap<K, V>` in Table 7.5. The methods for interface `KWHashMap<K, V>` (`get`, `put`, `isEmpty`, `remove`, and `size`) are similar to the ones shown earlier for the `Map` interface (see Table 7.2). There is a class `Hashtable` in the Java API `java.util`; however, it has been superseded by the class `HashMap`. Our interface `KWHashMap` doesn't include all the methods of interface `Map`.

### Class Entry

A hash table stores key–value pairs, so we will use an inner class `Entry` in each hash table implementation with data fields `key` and `value` (see Table 7.6). The implementation of inner class `Entry` is straightforward, and we show it in Listing 7.3.

**TABLE 7.5**

Interface `KWHashMap<K, V>`

Method	Behavior
<code>V get(Object key)</code>	Returns the value associated with the specified key. Returns <code>null</code> if the key is not present
<code>boolean isEmpty()</code>	Returns <code>true</code> if this table contains no key–value mappings
<code>V put(K key, V value)</code>	Associates the specified value with the specified key. Returns the previous value associated with the specified key, or <code>null</code> if there was no mapping for the key
<code>V remove(Object key)</code>	Removes the mapping for this key from this table if it is present (optional operation). Returns the previous value associated with the specified key, or <code>null</code> if there was no mapping
<code>int size()</code>	Returns the size of the table

**TABLE 7.6**

Inner Class `Entry<K, V>`

Data Field	Attribute
<code>private final K key</code>	The key
<code>private V value</code>	The value
Constructor	Behavior
<code>public Entry(K key, V value)</code>	Constructs an Entry with the given values
Method	Behavior
<code>public K getKey()</code>	Retrieves the key
<code>public V getValue()</code>	Retrieves the value
<code>public void setValue(V val)</code>	Sets the value

**LISTING 7.3**

Inner Class Entry

```
.....
LISTING 7.3
Inner Class Entry

/** Contains key-value pairs for a hash table. */
private static class Entry<K, V> {

    /** The key */
    private final K key;
    /** The value */
    private V value;

    /** Creates a new key-value pair.
     * @param key The key
     * @param value The value
    */
    public Entry(K key, V value) {
        this.key = key;
        this.value = value;
    }

    /** Retrieves the key.
     * @return The key
    */
    public K getKey() {
        return key;
    }

    /** Retrieves the value.
     * @return The value
    */
    public V getValue() {
        return value;
    }

    /** Sets the value.
     * @param val The new value
     * @return The old value
    */
    public V setValue(V val) {
        V oldVal = value;
        value = val;
        return oldVal;
    }
}
```

**Class HashtableOpen**

In a hash table that uses open addressing, we represent the hash table as an array of `Entry` objects (initial size is `START_CAPACITY`). We describe the data fields in Table 7.7. The `Entry` object `DELETED` is used to indicate that the `Entry` at a particular table element has been deleted; a `null` reference indicates that a table element was never occupied.

The data field declarations and constructor for `HashtableOpen` follow. Because generic arrays are not permitted, the constructor creates an `Entry[]` array, which is referenced by `table` (type `Entry<K, V>[]`).

```
/** Hash table implementation using open addressing. */
public class HashtableOpen<K, V> implements KWHashMap<K, V> {

    // Insert inner class Entry<K, V> here.
    // Data Fields
    private Entry<K, V>[] table;
    private static final int START_CAPACITY = 101;
```

```

private double LOAD_THRESHOLD = 0.75;
private int numKeys;
private int numDeletes;
private final Entry<K, V> DELETED =
    new Entry<>(null, null);

// Constructor
public HashTableOpen() {
    table = new Entry[START_CAPACITY];
}

...

```

**TABLE 7.7**Data Fields for Class `HashtableOpen<K, V>`

Data Field	Attribute
<code>private Entry&lt;K, V&gt;[] table</code>	The hash table array
<code>private static final int START_CAPACITY</code>	The initial capacity
<code>private double LOAD_THRESHOLD</code>	The maximum load factor
<code>private int numKeys</code>	The number of keys in the table excluding keys that were deleted
<code>private int numDeletes</code>	The number of deleted keys
<code>private final Entry&lt;K, V&gt; DELETED</code>	A special object to indicate that an entry has been deleted

Several methods for class `HashtableOpen` use a private method `find` that searches the table (using linear probing) until it finds either the target key or an empty slot. By expanding the table when its load factor exceeds the `LOAD_THRESHOLD`, we ensure that there will always be an empty slot in the table. Table 7.8 summarizes these private methods.

**TABLE 7.8**Private Methods for Class `HashtableOpen`

Method	Behavior
<code>private int find(Object key)</code>	Returns the index of the specified key if present in the table; otherwise, returns the index of the first available slot
<code>private void rehash()</code>	Doubles the capacity of the table and permanently removes deleted items

The algorithm for method `find` follows. Listing 7.4 shows the method.

### Algorithm for `HashtableOpen.find(Object key)`

1. Set `index` to `key.hashCode() % table.length`.
2. **if** `index` is negative, add `table.length`.
3. **while** `table[index]` is not empty and the key is not at `table[index]`
4.     Increment `index`.
5.     **if** `index` is greater than or equal to `table.length`
6.         Set `index` to 0.
7.     Return the `index`.

**LISTING 7.4**

Method HashtableOpen.find

```
.....  

LISTING 7.4  

Method HashtableOpen.find  

/** Finds either the target key or the first empty slot in the
   search chain using linear probing.
@pre The table is not full.
@param key The key of the target object
@return The position of the target or the first empty slot if
       the target is not in the table.
*/
private int find(Object key) {
    // Calculate the starting index.
    int index = key.hashCode() % table.length;
    if (index < 0)
        index += table.length;    // Make it positive.

    // Increment index until an empty slot is reached
    // or the key is found.
    while ((table[index] != null)
           && (!key.equals(table[index].getKey())))
    {
        index++;
        // Check for wraparound.
        if (index >= table.length)
            index = 0;    // Wrap around.
    }
    return index;
}
```

Note that the method call `key.hashCode()` calls key's `hashCode`. The condition `(!key.equals(table[index].getKey()))` compares the key at `table[index]` with the key being sought (the method parameter).

Next, we discuss the public methods: `get` and `put`. Listing 7.5 shows the code. The `get` algorithm follows.

**Algorithm for get(Object key)**

1. Find the first table element that is empty or the table element that contains the key.
2. **if** the table element found contains the key
  - Return the value at this table element.
3. **else**
  - Return `null`.

**LISTING 7.5**

Method HashtableOpen.get

```
.....  

LISTING 7.5  

Method HashtableOpen.get  

/** Method get for class HashtableOpen.
@param key The key being sought
@return the value associated with this key if found;
       otherwise, null
*/
@Override
public V get(Object key) {
    // Find the first table element that is empty
    // or the table element that contains the key.
    int index = find(key);
```

```

    // If the search is successful, return the value.
    if (table[index] != null)
        return table[index].getValue();
    else
        return null; // key not found.
}

```

Next, we write the algorithm for method `put`. After inserting a new entry, the method checks to see whether the load factor exceeds the `LOAD_THRESHOLD`. If so, it calls method `rehash` to expand the table and reinsert the entries. Listing 7.6 shows the code for method `put`.

### Algorithm for `HashtableOpen.put(K key, V value)`

1. Find the first table element that is empty or the table element that contains the key.
2. **if** an empty element was found
  3. Insert the new item and increment `numKeys`.
  4. Check for need to rehash.
  5. Return `null`.
6. The key was found. Replace the value associated with this table element and return the old value.

---

#### LISTING 7.6

Method `HashtableOpen.put`

```

/** Method put for class HashtableOpen.
 * @post This key-value pair is inserted in the
 *       table and numKeys is incremented. If the key is already
 *       in the table, its value is changed to the argument
 *       value and numKeys is not changed. If the LOAD_THRESHOLD
 *       is exceeded, the table is expanded.
 * @param key The key of item being inserted
 * @param value The value for this key
 * @return Old value associated with this key if found;
 *         otherwise, null
 */
@Override
public V put(K key, V value) {
    // Find the first table element that is empty
    // or the table element that contains the key.
    int index = find(key);

    // If an empty element was found, insert new entry.
    if (table[index] == null) {
        table[index] = new Entry<>(key, value);
        numKeys++;
        // Check whether rehash is needed.
        double loadFactor =
            (double) (numKeys + numDeletes) / table.length;
        if (loadFactor > LOAD_THRESHOLD)
            rehash();
        return null;
    }

    // assert: table element that contains the key was found.
    // Replace value for this key.
    V oldVal = table[index].getValue();
    table[index].setValue(value);
    return oldVal;
}

```



## PITFALL

### Integer Division for Calculating Load Factor

Before calling method `rehash`, method `put` calculates the load factor by dividing the number of filled slots by the table size. This is a simple computation, but if you forget to cast the numerator or denominator to `double`, the load factor will be zero (because of integer division), and the table will not be expanded. This will slow down the performance of the table when it becomes nearly full, and it will cause an infinite loop (in method `find`) when the table is completely filled.

Next, we write the algorithm for method `remove`. Note that we “remove” a table element by setting it to reference object `DELETED`. We leave the implementation as an exercise.

### Algorithm for `remove(Object key)`

1. Find the first table element that is empty or the table element that contains the key.
2. **if** an empty element was found
3.     Return `null`.
4. Key was found. Remove this table element by setting it to reference `DELETED`, increment `numDeletes`, and decrement `numKeys`.
5. Return the value associated with this key.

Finally, we write the algorithm for private method `rehash`. Listing 7.7 shows the method. Although we do not take the effort to make the table size a prime number, we do make it an odd number.

### Algorithm for `HashtableOpen.rehash`

1. Allocate a new hash table that is double the size and has an odd length.
2. Reset the number of keys and number of deletions to 0.
3. Reinsert each table entry that has not been deleted in the new hash table.

#### LISITING 7.7

Method `HashtableOpen.rehash`

```
/** Expands table size when loadFactor exceeds LOAD_THRESHOLD
 * @post The size of the table is doubled and is an odd integer.
 *       Each nondeleted entry from the original table is
 *       reinserted into the expanded table.
 *       The value of numKeys is reset to the number of items
 *       actually inserted; numDeletes is reset to 0.
 */
private void rehash() {
    // Save a reference to oldTable.
    Entry<K, V>[] oldTable = table;
    // Double capacity of this table.
    table = new Entry[2 * oldTable.length + 1];

    // Reinsert all items in oldTable into expanded table.
    numKeys = 0;
    numDeletes = 0;
    for (int i = 0; i < oldTable.length; i++) {
        if (oldTable[i] != null) && (oldTable[i] != DELETED) {
            // Insert entry in expanded table
            put(oldTable[i].getKey(), oldTable[i].getValue());
        }
    }
}
```

**TABLE 7.9**Data Fields for Class `HashtableChain<K, V>`

Data Field	Attribute
<code>private LinkedList&lt;Entry&lt;K, V&gt;&gt;[] table</code>	A table of references to linked lists of <code>Entry&lt;K, V&gt;</code> objects
<code>private int numKeys</code>	The number of keys (entries) in the table
<code>private static final int CAPACITY</code>	The size of the table
<code>private static final int LOAD_THRESHOLD</code>	The maximum load factor

## Class `HashtableChain`

Next, we turn our attention to class `HashtableChain`, which implements `KWHashMap` using chaining. We will represent the hash table as an array of linked lists as shown in Table 7.9. Even though a hash table that uses chaining can store any number of elements in the same slot, we will expand the table if the number of entries becomes three times the number of slots (`LOAD_THRESHOLD` is 3.0) to keep the performance at a reasonable level.



### PROGRAM STYLE

It is generally preferred that data fields be defined as interfaces and that implementing classes are assigned by the constructor. However, in this case, we define the data field `table` to be a `LinkedList`. This is because we want a linked list and use a method (`addFirst`) that is defined in the `LinkedList` class, but not in the `List` interface.

Listing 7.8 shows the data fields and the constructor for class `HashtableChain`.

#### LISTING 7.8

Data Fields and Constructor for `HashtableChain.java`

```
import java.util.*;

/** Hash table implementation using chaining. */
public class HashtableChain<K, V> implements KWHashMap<K, V> {

    // Insert inner class Entry<K, V> here.
    /** The table */
    private LinkedList<Entry<K, V>>[] table;
    /** The number of keys */
    private int numKeys;
    /** The capacity */
    private static final int CAPACITY = 101;
    /** The maximum load factor */
    private static final double LOAD_THRESHOLD = 3.0;

    // Constructor
    public HashtableChain() {
        table = new LinkedList[CAPACITY];
    }
    .
}
```

Next, we discuss the three methods `get`, `put`, and `remove`. Instead of introducing a `find` method to search a list for the key, we will include a search loop in each method. We will create a `ListIterator` object and use that object to access each list element.

We begin with the algorithm for `get`. Listing 7.9 shows its code. We didn't use methods `getKey` and `getValue` to access an item's key and value because those private data fields of class `Entry` are visible in the class that contains it.

### Algorithm for `HashtableChain.get(Object key)`

1. Set index to `key.hashCode() % table.length`.
  2. **if** index is negative
  3.     Add `table.length`.
  4. **if** `table[index]` is `null`
  5.     key is not in the table; return `null`.
  6. For each element in the list at `table[index]`
  7.     **if** that element's key matches the search key
  8.         Return that element's value.
  9. key is not in the table; return `null`.
- .....

#### **LISTING 7.9**

Method `HashtableChain.get`

```
/** Method get for class HashtableChain.
   @param key The key being sought
   @return The value associated with this key if found;
           otherwise, null
 */
@Override
public V get(Object key) {
    int index = key.hashCode() % table.length;
    if (index < 0)
        index += table.length;
    if (table[index] == null)
        return null; // key is not in the table.

    // Search the list at table[index] to find the key.
    for (Entry<K, V> nextItem : table[index]) {
        if (nextItem.getKey().equals(key))
            return nextItem.getValue();
    }

    // assert: key is not in the table.
    return null;
}
```

Next, we write the algorithm for method `put`. Listing 7.10 shows its code.

### Algorithm for `HashtableChain.put(K key, V value)`

1. Set index to `key.hashCode() % table.length`.
2. **if** index is negative, add `table.length`.
3. **if** `table[index]` is `null`
4.     Create a new linked list at `table[index]`.
5. Search the list at `table[index]` to find the key.
6. **if** the search is successful
7.     Replace the value associated with this key.
8.     Return the old value.
9. **else**
10.     Insert the new key-value pair in the linked list at `table[index]`.

11. Increment numKeys.
  12. **if** the load factor exceeds the LOAD\_THRESHOLD
  13. Rehash.
  14. Return **null**.
- .....

**LISTING 7.10**

Method HashtableChain.put

```
/** Method put for class HashtableChain.
 * @post This key-value pair is inserted in the
 *        table and numKeys is incremented. If the key is already
 *        in the table, its value is changed to the argument
 *        value and numKeys is not changed.
 * @param key The key of item being inserted
 * @param value The value for this key
 * @return The old value associated with this key if
 *         found; otherwise, null
 */
@Override
public V put(K key, V value) {
    int index = key.hashCode() % table.length;
    if (index < 0)
        index += table.length;
    if (table[index] == null) {
        // Create a new linked list at table[index].
        table[index] = new LinkedList<>();
    }

    // Search the list at table[index] to find the key.
    for (Entry<K, V> nextItem : table[index]) {
        // If the search is successful, replace the old value.
        if (nextItem.getKey().equals(key)) {
            // Replace value for this key.
            V oldVal = nextItem.getValue();
            nextItem.setValue(value);
            return oldVal;
        }
    }

    // assert: key is not in the table, add new item.
    table[index].addFirst(new Entry<>(key, value));
    numKeys++;
    if (numKeys > (LOAD_THRESHOLD * table.length))
        rehash();
    return null;
}
```

Last, we write the algorithm for method `remove`. We leave the implementation of `rehash` and `remove` as an exercise.

**Algorithm for HashtableChain.remove(Object key)**

1. Set index to `key.hashCode() % table.length`.
2. **if** index is negative, add `table.length`.
3. **if** `table[index]` is `null`
4.     key is not in the table; return `null`.
5. Search the list at `table[index]` to find the key.

6. **if** the search is successful
7. Remove the entry with this key and decrement numKeys.
8. **if** the list at table[index] is empty
9. Set table[index] to **null**.
10. Return the value associated with this key.
11. The key is not in the table; return **null**.

## Testing the Hash Table Implementations

We discuss two approaches to testing the hash table implementations. One way is to create a file of key–value pairs and then read each key–value pair and insert it in the hash table, observing how the table is filled. To do this, you need to write a `toString` method for the table that captures the index of each table element that is not **null** and then the contents of that table element. For open addressing, the contents would be the string representation of the key–value pair. For chaining, you could use a list iterator to traverse the linked list at that table element and append each key–value pair to the result string (see the Programming exercises for this section).

If you use a data file, you can carefully test different situations. The following are some of the cases you should examine:

- Does the array index wrap around as it should?
- Are collisions resolved correctly?
- Are duplicate keys handled appropriately? Is the new value retrieved instead of the original value?
- Are deleted keys retained in the table but no longer accessible via a `get`?
- Does rehashing occur when the load factor reaches 0.75 (3.0 for chaining)?

By stepping through the `get` and `put` methods, you can observe how the table is probed and examine the search chain that is followed to access or retrieve a key.

An alternative to creating a data file is to insert randomly generated integers in the hash table. This will allow you to create a very large table with little effort. The following loop generates SIZE key–value pairs. Each key is an integer between 0 and 32,000 and is autoboxed in an `Integer` object. For each table entry, the value is the same as the key. The `Integer.hashCode` method returns the `int` value of the object to which it is applied.

```
for (int i = 0; i < SIZE; i++) {
    Integer nextInt = (int) (32000 * Math.random());
    hashTable.put(nextInt, nextInt);
}
```

Because the keys are generated randomly, you can't investigate the effect of duplicate keys as you can with a data file. However, you can build arbitrarily large tables and observe how the elements are placed in the tables. After the table is complete, you can interactively enter items to retrieve, delete, and insert and verify that they are handled properly.

If you are using open addressing, you can add statements to count the number of items probed each time an insertion is made. You can accumulate these totals and display the average search chain length. If you are using chaining, you can also count the number of probes made and display the average. After all items have been inserted, you can calculate the average length of each linked list and compare that with the number predicted by the formula provided in the discussion of performance in Section 7.3.

## EXERCISES FOR SECTION 7.4

### SELF-CHECK

1. The following table stores `Integer` keys with the `int` values shown. Where would each key be placed in the new table resulting from rehashing the current table?

Index	Key
0	24
1	6
2	20
3	
4	14

### PROGRAMMING

1. Write a `remove` method for class `HashtableOpen`.
2. Write `rehash` and `remove` methods for class `HashtableChain`.
3. Write a `toString` method for class `HashtableOpen`.
4. Write a `toString` method for class `HashtableChain`.
5. Write a method `size` for both hash table implementations.
6. Modify method `find` to count and display the number of probes made each time it is called. Accumulate these in a data field `numProbes` and count the number of times `find` is called in another data field. Provide a method that returns the average number of probes per call to `find`.



## 7.5 Implementation Considerations for Maps and Sets

### Methods `hashCode` and `equals`

Class `Object` implements methods `hashCode` and `equals`, so every class can access these methods unless it overrides them. Method `Object.equals` compares two objects based on their addresses, not their contents. Similarly, method `Object.hashCode` calculates an object's hash code based on its address, not its contents. If you want to compare two objects for equality, you must implement an `equals` method for that class. In doing so, you should override the `equals` method for class `Object` by providing an `equals` method with the form

```
public boolean equals(Object obj) { . . . }
```

Most predefined classes (e.g., `String` and `Integer`) override method `equals` and method `hashCode`. If you override the `equals` method, Java recommends you also override the `hashCode` method. Otherwise, your class will violate the Java contract for `hashCode`, which states:

If `obj1.equals(obj2)` is `true`, then `obj1.hashCode() == obj2.hashCode()`.

Consequently, you should make sure that your `hashCode` method uses the same data field(s) as your `equals` method. We provide an example next.

---

**EXAMPLE 7.10** Class Person has data field IDNumber, which is used to determine whether two Person objects are equal. The equals method returns true only if the objects' IDNumber fields have the same contents.

```
public boolean equals(Object obj) {
    if (obj instanceof Person)
        return IDNumber.equals(((Person) obj).IDNumber);
    else
        return false;
}
```

To satisfy its contract, method Object.hashCode must also be overridden as follows. Now two objects that are considered equal will also have the same hash code.

```
public int hashCode() {
    return IDNumber.hashCode();
}
```

---

## Implementing HashSetOpen

We can modify the hash table methods from Section 7.4 to implement a hash set. Table 7.10 compares corresponding Map and Set methods.

The Set contains method performs a test for set membership instead of retrieving a value, so it is type **boolean**. Similarly, each of the other Set methods in Table 7.10 returns a **boolean** value that indicates whether the method was able to perform its task. The process of searching the hash table elements would be done the same way in each Set method as it is done in the corresponding Map method.

.....  
**TABLE 7.10**

Corresponding Map and Set Methods

Map Method	Set Method
V get(Object key)	boolean contains(Object key)
V put(K key, V value)	boolean add(K key)
V remove(Object key)	boolean remove(Object key)

For open addressing, method put uses the statement

```
table[index] = new Entry<>(key, value);
```

to store a reference to a new Entry object in the hash table. The corresponding statement in method add would be

```
table[index] = new Entry<>(key);
```

because the key is the only data that is stored.

## Writing HashSetOpen as an Adapter Class

Instead of writing new methods from scratch, we can implement HashSetOpen as an adapter class with the data field

```
private final KWHHashMap<K, V> setMap = new HashtableOpen<>();
```

We can write methods contains, add, and remove as follows. Because the map stores key-value pairs, we will have each set element reference an Entry object with the same key and value.

```
/** A hash table for storing set elements using open addressing. */
public class HashSetOpen<K> {
    private final KWHHashMap<K, V> setMap = new HashtableOpen<>();
```

```

    /**
     * Adapter method contains.
     * @return true if the key is found in setMap
     */
    public boolean contains(Object key) {
        // HashtableOpen.get returns null if the key is not found.
        return (setMap.get(key) != null);
    }

    /**
     * Adapter method add.
     * @post Adds a new Entry object (key, key)
     *       if key is not a duplicate.
     * @return true if the key is not a duplicate
     */
    public boolean add(K key) {
        /* HashtableOpen.put returns null if the
           key is not a duplicate. */
        return (setMap.put(key, key) == null);
    }

    /**
     * Adapter method remove.
     * @post Removes the key-value pair (key, key).
     * @return true if the key is found and removed
     */
    public boolean remove(Object key) {
        /* HashtableOpen.remove returns null if the
           key is not removed. */
        return (setMap.remove(key) != null);
    }
}

```

## Implementing the Java Map and Set Interfaces

Our goal in this chapter was to show you how to implement the operators in our hash table interface, not to implement the Map or Set interface fully. However, the Java API uses a hash table to implement both the Map and Set interfaces (class `HashMap` and class `HashSet`). You may be wondering what additional work would be required to implement the Map and Set interfaces using the classes we have developed so far.

The task of implementing these interfaces is simplified by the inclusion of abstract classes `AbstractMap` and `AbstractSet` in the Collections framework (see Figures 7.1 and 7.3). These classes provide implementations of several methods for the Map and Set interfaces. So if class `HashtableOpen` extends class `AbstractMap`, we can reduce the amount of additional work we need to do. We should also replace `KWHashMap` with `Map`. Thus, the declaration for `HashtableOpen` would be `class HashtableOpen<K, V> extends AbstractMap<K, V> implements Map<K, V>`.

The `AbstractMap` provides relatively inefficient ( $O(n)$ ) implementations of the `get` and `put` methods. Because we overrode these methods in both our implementations (`HashtableOpen` and `HashtableChain`), we will get  $O(1)$  expected performance. There are other, less critical methods that we don't need to provide because they are implemented in `AbstractMap` or its superclasses, such as `clear`, `isEmpty`, `putAll`, `equals`, `hashCode`, and `toString`.

## Interface `Map.Entry` and Class `AbstractMap.SimpleEntry`

One requirement on the key–value pairs for a `Map` object is that they implement the interface `Map.Entry<K, V>`, which is an inner interface of interface `Map`. This may sound a bit confusing, but what it means is that an implementer of the `Map` interface must contain an inner class that provides code for the methods described in Table 7.11. The `AbstractMap` includes the inner class `SimpleEntry` that implements the `Map.Entry` interface. We can remove the inner class `Entry<K, V>` (Listing 7.3) and replace `new Entry` with `new SimpleEntry`.

**TABLE 7.11**The `java.util.Map.Entry<K, V>` Interface

Method	Behavior
<code>K getKey()</code>	Returns the key corresponding to this entry
<code>V getValue()</code>	Returns the value corresponding to this entry
<code>V setValue(V val)</code>	Resets the value field for this entry to <code>val</code> . Returns its previous value field

## Creating a Set View of a Map

Method `entrySet` creates a set view of the entries in a `Map`. This means that method `entrySet` returns an object that implements the `Set` interface—that is, a set. The members of the set returned are the key–value pairs defined for that `Map` object. For example, if a key is "0123" and the corresponding value is "Jane Doe", the pair ("0123", "Jane Doe") would be an element of the set view. This is called a view because it provides an alternative way to access the contents of the `Map`, but there is only a single copy of the underlying `Map` object.

We usually call method `entrySet` via a statement of the form:

```
Iterator<Map.Entry<K, V>> iter = myMap.entrySet().iterator();
```

The method call `myMap.entrySet()` creates a set view of `myMap`; next, we apply method `iterator` to that set, thereby returning an `Iterator` object for it. We can access all the elements in the set through `Iterator` `iter`'s methods `hasNext` and `next`, but the elements are in arbitrary order. The objects returned by the iterator's `next` method are `Map.Entry<K, V>` objects. We show an easier way to do this using the enhanced `for` statement in Example 7.11.

## Method `entrySet` and Classes `EntrySet` and `SetIterator`

Method `entrySet` returns a set view of the underlying hash table (its key–value pairs) by returning an instance of inner class `EntrySet`. We define method `entrySet` next and then class `EntrySet`.

```
/** Creates a set view of a map.
 * @return a set view of all key-value pairs in this map
 */
public Set<Map.Entry<K, V>> entrySet() {
    return new EntrySet();
}
```

We show the inner class `EntrySet` in Listing 7.11. This class is an extension of the `AbstractSet`, which provides a complete implementation of the `Set` interface except for the `size` and `iterator` methods. The other methods required by the `Set` interface are defined using these methods. Most methods are implemented by using the `Iterator` object that is returned by the `EntrySet.iterator` method to access the contents of the hash table through its set view. You can also use such an `Iterator` object to access the elements of the set view.

**LISTING 7.11**The Inner Class `EntrySet`

```
/** Inner class to implement the set view. */
private class EntrySet<K, V> extends AbstractSet<Map.Entry<K, V>> {

    /** Return the size of the set. */
    @Override
    public int size() {
        return numKeys;
    }
}
```

```

    /** Return an iterator over the set. */
    @Override
    public Iterator<Map.Entry<K, V>> iterator() {
        return new SetIterator<>();
    }
}

```

The final step is to write class `SetIterator`, which implements the `Iterator` interface. The inner class `SetIterator` enables access to the entries in the hash table. The `SetIterator` class implements the `java.util.Iterator` interface and provides methods `hasNext`, `next`, and `remove`. Its implementation is left as a Programming Project (see Project 6).

## Classes `TreeMap` and `TreeSet`

Besides `HashMap` and `HashSet`, the Java Collections Framework provides classes `TreeMap` and `TreeSet` that implement the `Map` and `Set` interfaces. These classes use a Red–Black tree (Section 9.3), which is a balanced binary search tree. We discussed earlier that the performances for search, retrieval, insertion, and removal operations are better for a hash table than for a binary search tree (expected  $O(1)$  versus  $O(\log n)$ ). However, the primary advantage of a binary search tree is that it can be traversed in sorted order. Hash tables, however, can't be traversed in any meaningful way. Also, subsets based on a range of key values can be selected using a `TreeMap` but not by using a `HashMap`.

**EXAMPLE 7.11** In Example 7.4 we showed how to use a `Map` to build an index for a term paper. Because we want to display the words of the index in alphabetical order, we must store the index in a `SortedMap`. Method `showIndex` below displays the string representation of each index entry in the form

*key : value*

If the word *fire* appears on lines 4, 8, and 20, the corresponding output line would be

fire : [4, 8, 20]

It would be relatively easy to display this in the more common form: `fire 4, 8, 20` (see Programming Exercise 4).

```

    /** Displays the index, one word per line */
    public void showIndex() {
        index.forEach((k, v) -> System.out.println(k + " : " + v));
    }
}

```

The `Map.forEach` method applies a `BiConsumer` (See Table 6.2) to each map key–value pair. In the lambda expression

(k, v) -> System.out.println(k + " : " + v)

the parameter `k` is bound to the key, and the parameter `v` is bound to the value.

Without Java 8, the enhanced `for` loop could be used but the code would be more cumbersome:

```

    /** Displays the index, one word per line */
    public void showIndex() {
        for (Map.Entry<String, ArrayList<Integer>> entry
             : index.entrySet()) {
            System.out.println(entry);
        }
    }
}

```

## EXERCISES FOR SECTION 7.5

### SELF-CHECK

1. Explain why the nested interface `Map.Entry` is needed.

### PROGRAMMING

1. Write statements to display all key-value pairs in `Map` object `m`, one pair per line. You will need to create an iterator to access the map entries.
2. Assume that a `Person` has data fields `lastName` and `firstName`. Write an `equals` method that returns true if two `Person` objects have the same first and last names. Write a `hashCode` method that satisfies the `hashCode` contract. Make sure that your `hashCode` method does not return the same value for Henry James and James Henry. Your `equals` method should return a value of `false` for these two people.
3. Assume class `HashSetOpen` is written using an array `table` for storage instead of a `HashMap` object. Write method `contains`.
4. Modify method `showIndex` so each output line displays a word followed by a comma and a list of line numbers separated by commas. You can either edit the string corresponding to each `Map` entry before displaying it or use methods `Map.Entry.getKey` and `Map.Entry.getValue` to build a different string.



## 7.6 Additional Applications of Maps

In this section, we will consider two case studies that use a `Map` object. The first is the design of a contact list for a cell phone, and the second involves completing the Huffman Coding Case Study started in Section 6.7.

### CASE STUDY Implementing a Cell Phone Contact List

**Problem** A cell phone manufacturer would like a Java program that maintains the list of contacts for a cell phone owner. For each contact, a person's name, there should be a list of phone numbers that can be changed. The manufacturer has provided the interface for the software (see Table 7.12).

**Analysis** A map is an ideal data structure for the contact list. It should associate names (which must be unique) to lists of phone numbers. Therefore, the name should be the key field, and the list of phone numbers should be the value field. A sample entry would be:

*name: Jane Smith phone numbers: 215-555-1234, 610-555-4820*

Thus, we can implement `ContactListInterface` by using a `Map<String, List<String>>` object for the contact list. For the sample entry above, this object would contain the key-value pair ("Jane Smith", ["215-555-1234", "610-555-4820"] ).

**TABLE 7.12**

Methods of ContactListInterface

Method	Behavior
List<String> addOrChangeEntry(String name, List<String> numbers)	Changes the numbers associated with the given name or adds a new entry with this name and list of numbers. Returns the old list of numbers or null if this is a new entry
List<String> lookupEntry(String name)	Searches the contact list for the given name and returns its list of numbers or null if the name is not found
List<String> removeEntry(String name)	Removes the entry with the specified name from the contact list and returns its list of numbers or null if the name is not in the contact list
void display();	Displays the contact list in order by name

**Design** We need to design the class `MapContactList`, which implements `ContactListInterface`. The contact list can be stored in the data field declared as follows:

```
public class MapContactList implements ContactListInterface {

    /** The contact list */
    Map<String, List<String>> contacts =
        new TreeMap<>();
```

Writing the required methods using the Map methods is a straightforward task.

**Implementation** We begin with method `addOrChangeEntry`:

```
public List<String> addOrChangeEntry(String name,
                                      List<String> newNumbers) {
    List<String> oldNumbers = contacts.put(name, newNumbers);
    return oldNumbers;
}
```

Method `put` inserts the new name and list of numbers (method arguments) for a Map entry and returns the old value for that name (the list of numbers) if it was previously stored. If an entry with the given name was not previously stored, `null` is returned.

The `lookupEntry` method uses the `Map.get` method to retrieve the directory entry. The entry key field (`name`) is passed as an argument.

```
public List<String> lookupEntry(String name) {
    return contacts.get(name);
}
```

The `removeEntry` method uses the `Map.remove` method to delete a contact list entry.

```
public List<String> removeEntry(String name) {
    return contacts.remove(name);
}
```

To display the contact list in order by name, we need to retrieve each name and its associated list of numbers from the contact list. We can use the `Map.entrySet()` method to obtain a view of the map's contents as a set of `Map.Entry<String, List<String>>` objects.

We can use the for-each construct to write out the contents of the map as a sequence of consecutive lines containing the name–number pairs. The iterator accesses the entries in order by key field, which is what we desire.

```
public void display() {
    for (Map.Entry<String, List<String>> current :
        contacts.entrySet()) {
        // Display the name.
        println(current.getKey());
        // Display the list of numbers.
        println(current.getValue());
    }
}
```

Each entry is stored in `current`. Then method `getKey` returns the key field (the person's name), and method `getValue` returns the value field (the person's list of phone numbers).

### Testing

To test class `MapContactList`, you can write a main function that creates a new `MapContactList` object. Then apply the `addOrChangeEntry` method several times to this object, using new names and phone number lists, to build the initial contact list. For example, the following sequence of statements stores two entries in `contactList`.

```
MapContactList contactList = new MapContactList();
List<String> nums = new ArrayList<>();
nums.add("123"); nums.add("345");
contactList.addOrChangeEntry("Jones", nums);
nums = new ArrayList<>();
nums.add("135"); nums.add("357");
contactList.addOrChangeEntry("King", nums);
```

Once you have created an initial contact list, you can display it and then update it to verify that all the methods are correct.

## CASE STUDY Completing the Huffman Coding Problem

### Problem

In Section 6.7 we showed how to compress a file by using a Huffman tree to encode the symbols occurring in the file so that the most frequently occurring characters had the shortest binary codes. Recall that the custom Huffman tree shown in Fig. 6.37 was built for a data file in which the letter `e` occurs most often, then `a` and then `d`. The letters `b` and `c` occur least often. The letters are stored in leaf nodes. We used a priority queue to build the Huffman tree.

In this case study, we discuss how to create the frequency table for the symbols in a data file. We use a `Map` for storing each symbol and its corresponding count. We also show how to encode the data file using a second `Map` for storing the code table.

**Analysis** In Section 6.7, method `buildTree` of class `HuffmanTree` built the Huffman tree. The input to method `buildTree` was a frequency table (array `HuffData`) consisting of (weight, symbol) pairs, where the weight in each pair was the frequency of occurrence of the corresponding symbol in a data file to be encoded.

To encode our data file, we used a code table consisting of (symbol, code) pairs for each symbol in the data file. For both situations, we need to look up a symbol in a table. Using a Map ensures that the table lookup is an expected O(1) process. We will also need to write a representation of the Huffman tree to the data file so that we can decode it. We discuss this later.

**Design** To build the frequency table, we need to read a file and count the number of occurrences of each symbol in the file. The symbol will be the key for each entry in a `Map<Character, Integer>` object, and the corresponding value will be the count of occurrences so far. As each symbol is read, we retrieve its Map entry and increment the corresponding count. If the symbol is not yet in the frequency table, we insert it with a count of 1.

The algorithm for building the frequency table follows. After all characters are read, we create a set view of the map and traverse it using an iterator. We retrieve each `Map.Entry` and transpose its fields to create the corresponding `HuffData` array element, a (weight, symbol) pair.

### Algorithm for `buildFreqTable`

1. **while** there are more characters in the input file
2.     Read a character and retrieve its corresponding entry in Map frequencies.
3.     Increment value.
4.     Create a set view of frequencies.
5.     **for** each entry in the set view
6.         Store its data as a weight–symbol pair in the `HuffData` array.
7.     Return the `HuffData` array.

Once we have the frequency table, we can construct the Huffman tree using a priority queue as explained in Section 6.7. Then we need to build a code table that stores the bit string code associated with each symbol to facilitate encoding the data file. Storing the code table in a `Map<Character, BitString>` object makes the encoding process more efficient because we can look up the symbol and retrieve its bit string code (expected O(1) process). The code table should be declared as:

```
private Map<Character, BitString> codeTable;
```

Method `buildCodeTable` builds the code table by performing a preorder traversal of the Huffman tree. As we traverse the tree, we keep track of the bit code string so far. When we traverse left, we append a 0 to the bit string, and when we traverse right, we append a 1 to the bit string. If we encounter a symbol in a node, we insert that symbol along with a copy of the code so far as a new entry in the code table. Because all symbols are stored in leaf nodes, we return immediately without going deeper in the tree.

### Algorithm for Method buildCodeTable

1. Get the data at the current root.
2. **if** a symbol is stored in the current root (reached a leaf node)
  3. Insert the symbol and bit string code so far as a new code table entry.
4. **else**
  5. Append a 0 to the bit string code so far.
  6. Apply the method recursively to the left subtree.
  7. Append a 1 to the bit string code.
  8. Apply the method recursively to the right subtree.

Finally, to encode the file, we read each character, look up its bit string code in the code table Map, and then append it to the output `BitStringBuilder` (described after Listing 7.13).

### Algorithm for Method encode

1. **while** there are more characters in the input file
2. Read a character and get its corresponding bit string code.
3. Append its bit string to the output `BitStringBuilder`.

## Implementation

Listing 7.12 shows the code for method `buildFreqTable`. The `while` loop inside the `try` block builds the frequency table (Map `frequencies`). Each character is stored as an `int` in `nextChar` and then cast to type `char`. Because a Map stores references to objects, each character is autoboxed in a `Character` object (the key) and its count in an `Integer` object (the value). Once the table is built, we use an enhanced `for` loop to traverse the set view, retrieving each entry from the map and using its data to create a new `HuffData` element for array `freqTable`. When we finish, we return `freqTable` as the method result.

---

#### LISTING 7.12

Method `buildFreqTable`

```
public static HuffData[] buildFreqTable(BufferedReader ins) {
    // Map of frequencies.
    Map<Character, Integer> frequencies =
        new HashMap<>();
    try {
        int nextChar;    // For storing the next character as an int
        while ((nextChar = ins.read()) != -1) { // Test for more data
            // Get the current count and increment it.
            Integer count = frequencies.getOrDefault((char) nextChar, 0);
            count++;

            // Store updated count.
            frequencies.put((char) nextChar, count);
        }
        ins.close();
    } catch (IOException ex) {
        ex.printStackTrace();
        System.exit(1);
    }
    // Copy Map entries to a HuffData[] array.
    HuffData[] freqTable = new HuffData[frequencies.size()];
    int i = 0;      // Start at beginning of array.
```

```

    // Get each map entry and store it in the array
    // as a weight-symbol pair.
    for (Map.Entry<Character, Integer> entry : frequencies.entrySet()) {
        freqTable[i] =
            new HuffData(entry.getValue(), entry.getKey());
        i++;
    }
    return freqTable;    // Return the array.
}

```

Next, we show method `buildCodeTable`. We provide a starter method that initializes `codeMap` to an empty `HashMap` and calls the recursive method that implements the algorithm discussed in the Design section.

```

/** Starter method to build the code table.
 * @post The table is built.
 */
public void buildCodeTable() {
    // Initialize the code map.
    codeMap = new HashMap<>();
    // Call recursive method with empty bit string for code so far.
    buildCodeTable(huffTree, new BitString());
}

/** Recursive method to perform breadth-first traversal
 * of the Huffman tree and build the code table.
 * @param tree The current tree root
 * @param code The code string so far
 */
private void buildCodeTable(BinaryTree<HuffData> tree, BitString code) {
    // Get data at local root.
    HuffData datum = tree.getData();
    if (datum.symbol != '\u0000') { // Test for leaf node.
        // Found a symbol, insert its code in the map.
        codeMap.put(datum.symbol, code);
    } else {
        // Append 0 to code so far and traverse left.
        BitString leftCode = code.append(0);
        buildCodeTable(tree.getLeftSubtree(), leftCode);
        // Append 1 to code so far and traverse right
        BitString rightCode = code.append(1);
        buildCodeTable(tree.getRightSubtree(), rightCode);
    }
}

```

The goal of Huffman coding is to produce the smallest possible representation of the text. If we were to write the file as a String of 0 and 1 characters, the resulting file would be larger than the original, not smaller! Thus, we need to output a sequence of individual bits, packed into 8-bit bytes. The class `BitString` encapsulates a sequence of bits much like the `String` class encapsulates a sequence of characters. Like the `String`, `BitStrings` are immutable. The `BitString.append` method is equivalent to the `+` operator in the `String` class in that it creates a new `BitString` with the argument appended to the original. To facilitate appending individual bits or sequences of bits to a `BitString`, the `BitStringBuilder` provides methods analogous to the `StringBuilder`. Classes `BitString` and `BitStringBuilder` may be downloaded from the Web site for this textbook.

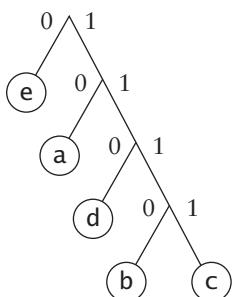
Method encode reads each character again, looks up its bit code string, and writes it to the output file. We assume that the code table is in Map `codeTable` (a data field).

```
/** The Map to store the code table. */
private Map<Character, BitString> codeTable
    = new HashMap<Character, BitString>();
```

Following is the encode method.

```
/** Encodes a data file by writing it in compressed bit string form.
 * @param ins The input stream
 * @return The coded file as a BitString
 */
public BitString encode(BufferedReader ins) {
    BitStringBuilder result = new BitStringBuilder(); // The complete bit string.
    try {
        int nextChar;
        while ((nextChar = ins.read()) != -1) { // More data?
            // Get bit string corresponding to symbol nextChar.
            BitString nextChunk = codeMap.get(char)(nextChar);
            result.append(nextChunk); // Append to result string.
        }
    } catch (IOException ex) {
        ex.printStackTrace();
        System.exit(1);
    }
    return result.toBitString();
}
```

**Testing** To test these methods completely, you need to download class `BitString` and `BitStringBuilder` (see Project 1) and write a `main` method that calls them in the proper sequence. For interim testing, you can read a data file and display the frequency table that is constructed to verify that it is correct. You can also use the `StringBuilder` class instead of class `BitString` in methods `buildCodeTable` and `encode`. The resulting code string would consist of a sequence of digit characters '0' and '1' instead of a sequence of 0 and 1 bits. But this would enable you to verify that the program works correctly.



## Encoding the Huffman Tree

Method decode in Listing 6.12 used the Huffman tree to decode the encoded file. This tree must be available to the receiver of the encoded file. Therefore, we must encode the Huffman tree and write it to the encoded file.

We can encode the Huffman tree by performing a preorder traversal and appending a 0 bit when we encounter an internal node and a 1 bit when we encounter a leaf. Also, when we reach a leaf, we append the 16-bit Unicode representation of the character. The Huffman tree in Figure 6.37 (reproduced here) is encoded as follows:

01e01a01d01b1c

where the italic letters are replaced with their Unicode representations.

**Algorithm for encodeHuffmanTree**

1. if the current root is not a leaf
  2. Append a 0 to the output `BitStringBuilder`.
  3. Recursively call `encodeHuffmanTree` on the left subtree.
  4. Recursively call `encodeHuffmanTree` on the right subtree.
- else**
5. Append a 1 to the output `BitStringBuilder`.
  6. Append the 16-bit Unicode of the symbol to the `BitStringBuilder`.

Reconstructing the Huffman tree is done recursively. If a 1 bit is encountered, construct a leaf node using the next 16 bits in the `BitString`, and return this node. If a zero bit is encountered, recursively decode the left subtree and the right subtree, then create an internal node with the resulting subtrees.

---

**EXERCISES FOR SECTION 7.6****PROGRAMMING**

1. Write a class to complete the test of the `MapContactList` class.



---

## 7.7 Navigable Sets and Maps

The `SortedSet` interface (part of Java 5.0) extends the `Set` by enabling the user to get an ordered view of the elements with the ordering defined by a `compareTo` method or by means of a `Comparator`. Because the items have an ordering, additional methods can be defined, which return the first and last elements and define subsets over a specified range. However, the ability to define subsets was limited. In particular, subsets were defined always to include the starting element and to exclude the ending element. The Java 5.0 `SortedMap` interface provides an ordered view of a map with the elements ordered by key value. Because the elements of a submap are ordered, submaps can also be defined.

In Java 6, the `NavigableSet` and `NavigableMap` interfaces were introduced as an extension to `SortedSet` and `SortedMap`. These interfaces allow the user to specify whether the start or end items are included or excluded. They also enable the user to specify a subset or submap that is traversable in the reverse order. As they are more general, we will discuss the `NavigableSet` and `NavigableMap` interfaces. Java retains the `SortedSet` and `SortedMap` interfaces for compatibility with existing software. Table 7.13 shows some methods of the `NavigableSet` interface.

---

**EXAMPLE 7.12** Listing 7.13 illustrates the use of a `NavigableSet`. The output of this program consists of the lines:

```
The original set odds is [1, 3, 5, 7, 9]
The ordered set b is [3, 5, 7]
Its first element is 3
Its smallest element >= 6 is 7
```

**LISTING 7.13**

Using a NavigableSet

```

public static void main(String[] args) {
    // Create and fill the sets
    NavigableSet<Integer> odds = new TreeSet<>();
    odds.add(5); odds.add(3); odds.add(7); odds.add(1); odds.add(9);
    System.out.println("The original set odds is " + odds);
    NavigableSet b = odds.subSet(1, false, 7, true);
    System.out.println("The ordered set b is " + b);
    System.out.println("Its first element is " + b.first());
    System.out.println("Its smallest element >= 6 is " + b.ceiling(6));
}

```

**TABLE 7.13**

NavigableSet Interface

Method	Behavior
E ceiling(E e)	Returns the smallest element in this set that is greater than or equal to e, or <code>null</code> if there is no such element
Iterator<E> descendingIterator()	Returns an iterator that traverses the Set in descending order
NavigableSet<E> descendingSet()	Returns a reverse order view of this set
E first()	Returns the smallest element in the set
E floor(E e)	Returns the largest element that is less than or equal to e, or <code>null</code> if there is no such element
NavigableSet<E> headSet(E toEl, boolean incl)	Returns a view of the subset of this set whose elements are less than toEl. If incl is <code>true</code> , the subset includes the element toEl if it exists
E higher(E e)	Returns the smallest element in this set that is strictly greater than e, or <code>null</code> if there is no such element
Iterator<E> iterator()	Returns an iterator to the elements in the set that traverses the set in ascending order
E last()	Returns the largest element in the set
E lower(E e)	Returns the largest element in this set that is strictly less than e, or <code>null</code> if there is no such element
E pollFirst()	Retrieves and removes the first element. If the set is empty, returns <code>null</code>
E pollLast()	Retrieves and removes the last element. If the set is empty, returns <code>null</code>
NavigableSet<E> subSet(E fromEl, boolean fromIncl, E toEl, boolean toIncl)	Returns a view of the subset of this set that ranges from fromEl to toEl. If the corresponding fromIncl or toIncl is <code>true</code> , then the fromEl or toEl elements are included
NavigableSet<E> tailSet(E fromEl, boolean incl)	Returns a view of the subset of this set whose elements are greater than fromEl. If incl is <code>true</code> , the subset includes the element fromEl if it exists

The methods defined by the `NavigableMap` interface are similar to those defined in `NavigableSet` except that the parameters are keys and the results are keys, `Map.Entry`s, or submaps. For example, the `NavigableSet` has a method `ceiling` that returns a single set element or `null` while the `NavigableMap` has two similar methods: `ceilingEntry(K key)`, which returns the `Map.Entry` that is associated with the smallest key greater than or equal to the given key, and `ceilingKey(K key)`, which returns just the key of that entry.

Table 7.14 shows some methods of the `NavigableMap` interface. Not shown are methods `firstKey`, `firstEntry`, `floorKey`, `floorEntry`, `lowerKey`, `lowerEntry`, `lastKey`, `lastEntry`, `higherKey`, `higherEntry`, which have the same relationship to their `NavigableSet` counterparts (methods `first`, `floor`, etc.) as do `ceilingKey` and `ceilingEntry`.

The entries in a `NavigableMap` can be processed in key–value order. This is sometimes a desirable feature, which is not available in a `HashMap` (or hash table). Class `TreeMap` and a Java 6 class, `ConcurrentSkipListMap`, implement the `NavigableMap` interface. To use the class `ConcurrentSkipListSet` or `ConcurrentSkipListMap`, you must insert both of the following statements.

```
import java.util.*;
import java.util.concurrent.*;
```

## Application of a `NavigableMap`

Listing 7.14 shows methods `computeAverage` and `computeSpans`. The method `computeAverage` computes the average of the values defined in a `Map`. Method `computeSpans` creates a group of submaps of a `NavigableMap` and passes each submap to `computeAverage`. Given a `NavigableMap` in which the keys represent years and the values some statistic for that year, we can generate a table of averages covering different periods. For example, if we have a map

**TABLE 7.14**  
NavigableMap Interface

Method	Behavior
<code>Map.Entry&lt;K, V&gt; ceilingEntry(K key)</code>	Returns a key–value mapping associated with the least key greater than or equal to the given key, or <code>null</code> if there is no such key
<code>K ceilingKey(K key)</code>	Returns the least key greater than or equal to the given key, or <code>null</code> if there is no such key
<code>NavigableSet&lt;K&gt; descendingKeySet()</code>	Returns a reverse-order <code>NavigableSet</code> view of the keys contained in this map
<code>NavigableMap&lt;K, V&gt; descendingMap()</code>	Returns a reverse-order view of this map
<code>NavigableMap&lt;K, V&gt; headMap(K toKey, boolean incl)</code>	Returns a view of the submap of this map whose keys are less than <code>toKey</code> . If <code>incl</code> is <code>true</code> , the submap includes the entry with key <code>toKey</code> if it exists
<code>NavigableMap&lt;K, V&gt; subMap(K fromKey, boolean fromIncl, K toKey, boolean toIncl)</code>	Returns a view of the submap of this map that ranges from <code>fromKey</code> to <code>toKey</code> . If the corresponding <code>fromIncl</code> or <code>toIncl</code> is <code>true</code> , then the entries with key <code>fromKey</code> or <code>toKey</code> are included
<code>NavigableSet&lt;E&gt; tailMap(K fromKey, boolean fromIncl)</code>	Returns a view of the submap of this map whose elements are greater than <code>fromKey</code> . If <code>fromIncl</code> is <code>true</code> , the submap includes the entry with key <code>fromKey</code> if it exists
<code>NavigableSet&lt;K&gt; navigableKeySet()</code>	Returns a <code>NavigableSet</code> view of the keys contained in this map

of storms that represents the number of tropical storms in a given year for the period 1960–1969, the method call

```
List<Number> stormAverage = computeSpans(storms, 2);
```

will calculate the average number of storms for the years 1960–1961, 1962–1963, and so on and store these values in the List `stormAverage`. The value passed to `delta` is 2, so within method `computeSpans`, the first statement below

```
double average =
    computeAverage(valueMap.subMap(index, true,
                                   index+delta, false));
result.add(average);
```

will create a submap for a pair of years (years `index` and `index+1`) and then compute the average of the two values in this submap. The second statement appends the average to the List `result`. This would be repeated for each pair of years starting with the first pair (1960–1961). For the method call

```
List<Number> stormAverage = computeSpans(storms, 3);
```

a submap would be created for each nonoverlapping group of 3 years: 1960–1962, 1963–1965, 1966–1968, and then 1969 by itself. The average of the three values in each submap (except for the last, which contains just one entry) would be calculated and stored in List `result`.

#### LISTING 7.14

Methods `computeAverage` and `computeSpans`

```
/** Returns the average of the numbers in its Map argument.
 * @param valueMap The map whose values are averaged
 * @return The average of the map values
 */
public static double computeAverage(Map<Integer, Double> valueMap) {
    int count = 0;
    double sum = 0;
    for (Map.Entry<Integer, Double> entry : valueMap.entrySet()) {
        sum += entry.getValue().doubleValue();
        count++;
    }
    return (double) sum / count;
}

/** Returns a list of the averages of nonoverlapping spans of
 * values in its NavigableMap argument.
 * @param valueMap The map whose values are averaged
 * @param delta The number of map values in each span
 * @return An ArrayList of average values for each span
 */
public static List<Double> computeSpans(NavigableMap valueMap, int delta)
{
    List<Double> result = new ArrayList<>();
    Integer min = (Integer) valueMap.firstEntry().getKey();
    Integer max = (Integer) valueMap.lastEntry().getKey();
    for (int index = min; index <= max; index += delta) {
        double average =
            computeAverage(valueMap.subMap(index, true,
                                           index+delta, false));
        result.add(average);
    }
    return result;
}
```

## EXERCISES FOR SECTION 7.7

### SELF-CHECK

1. What is displayed by the execution of the following program?

```
public static void main(String[] args) {
    NavigableSet<Integer> s = new TreeSet<>();
    s.add(5); s.add(6); s.add(3); s.add(2); s.add(9);
    NavigableSet<Integer> a = s.subSet(1, true, 9, false);
    NavigableSet<Integer> b = s.subSet(4, true, 9, true);
    System.out.println(a);
    System.out.println(b);
    System.out.println(s.higher(5));
    System.out.println(s.lower(5));
    System.out.println(a.first());
    System.out.println(b.lower(4));
    int sum = 0;
    for (int i : a) {
        System.out.println(i);
        sum += i;
    }
    System.out.println(sum);
}
```

2. Trace the execution of methods `computeSpans` and `computeAverage` for the call `computeSpans(storms, 4)` where `NavigableMap<Integer, Double> storms` has the following entries: `{(1960, 10), (1961, 5), (1962, 20), (1963, 8), (1964, 16), (1965, 50), (1966, 25), (1967, 15), (1968, 21), (1969, 13)}`.
3. Write an algorithm for method `computeGaps` that has two parameters like `computeSpans`, except the `int` parameter represents the number of years between entries in the `NavigableMap` that are being averaged together. For `NavigableMap` `storms` defined in the previous exercise, the call `computeGaps(storms, 2)` would first compute the average for the values in `{(1960, 10), (1962, 20), (1964, 16) ... }` and then compute the average for the values in `{(1961, 5), (1963, 8), (1965, 50) ... }`.

### PROGRAMMING

1. Write a program fragment to display the elements of a set `s` in normal order and then in reverse order.
2. Write a main method that tests method `computeSpans`.
3. Code method `computeGaps` from Self-Check Exercise 3.



## Chapter Review

- ◆ The `Set` interface describes an abstract data type that supports the same operations as a mathematical set. We use `Set` objects to store a collection of elements that are not ordered by position. Each element in the collection is unique. Sets are useful for determining whether a particular element is in the collection, not its position or relative order.

- ◆ The `Map` interface describes an abstract data type that enables a user to access information (a value) corresponding to a specified key. Each key is unique and is mapped to a value that may or may not be unique. Maps are useful for retrieving or updating the value corresponding to a given key.
- ◆ A hash table uses hashing to transform an item's key into a table index so that insertions, retrievals, and deletions can be performed in expected  $O(1)$  time. When the `hashCode` method is applied to a key, it should return an integer value that appears to be a random number. A good `hashCode` method should be easy to compute and should distribute its values evenly throughout the range of `int` values. We use modulo division to transform the hash code value to a table index. Best performance occurs when the table size is a prime number.
- ◆ A collision occurs when two keys hash to the same table index. Collisions are expected, and hash tables utilize either open addressing or chaining to resolve collisions. In open addressing, each table element references a key-value pair, or `null` if it is empty. During insertion, a new entry is stored at the table element corresponding to its hash index if it is empty; otherwise, it is stored in the next empty location following the one selected by its hash index. In chaining, each table element references a linked list of key-value pairs with that hash index or `null` if none does. During insertion, a new entry is stored in the linked list of key-value pairs for its hash index.
- ◆ In open addressing, linear probing is often used to resolve collisions. In linear probing, finding a target or an empty table location involves incrementing the table index by 1 after each probe. This approach may cause clusters of keys to occur in the table, leading to overlapping search chains and poor performance. To minimize the harmful effect of clustering, quadratic probing increments the index by the square of the probe number. Quadratic probing can, however, cause a table to appear to be full when there is still space available, and it can lead to an infinite loop.
- ◆ The best way to avoid collisions is to keep the table load factor relatively low by rehashing when the load factor reaches a value such as 0.75 (75 percent full). To rehash, you increase the table size and reinsert each table element.
- ◆ In open addressing, you can't remove an element from the table when you delete it, but you must mark it as deleted. In chaining, you can remove a table element when you delete it. In either case, traversal of a hash table visits its entries in an arbitrary order.
- ◆ A set view of a `Map` can be obtained through method `entrySet`. You can create an `Iterator` object for this set view and use it to access the elements in the set view in order by key value.
- ◆ Two Java API implementations of the `Map` (Set) interface are `HashMap` (`HashSet`) and `TreeMap` (`TreeSet`). The `HashMap` (and `HashSet`) implementation uses an underlying hash table; the `TreeMap` (and `TreeSet`) implementations use a Red–Black tree (discussed in Chapter 9). Search and retrieval operations are more efficient using the underlying hash table (expected  $O(1)$  versus  $O(\log n)$ ). The tree implementation, however, enables you to traverse the key-value pairs in a meaningful way and allows for subsets based on a range of key values.
- ◆ The Java 6 `NavigableSet` and `NavigableMap` interfaces enable the creation of subsets and submaps that may or may not include specified boundary items. The elements in a `NavigableSet` (or `NavigableMap`) are in increasing order by element value (or by key value for a `NavigableMap`).
- ◆ The Java 6 `ConcurrentSkipListSet` and `ConcurrentSkipListMap` are classes that implement these interfaces.

## Java API Interfaces and Classes Introduced in This Chapter

```
java.util.AbstractMap
java.util.AbstractSet
java.util.concurrent.ConcurrentSkipListMap
java.util.concurrent.ConcurrentSkipListSet
java.util.HashMap
java.util.HashSet
java.util.Map
java.util.Map.Entry
java.util.NavigableMap
java.util.NavigableSet
java.util.Set
java.util.TreeMap
java.util.TreeSet
```

## User-Defined Interfaces and Classes in This Chapter

BitString	HashtableChain
ContactListInterface	HashtableOpen
Entry	KWHashMap
EntrySet	MapContactList
HashSetOpen	SetIterator

## Quick-Check Exercises

1. If *s* is a set that contains the characters 'a', 'b', 'c', write a statement to insert the character 'd'.
2. What is the effect of each of the following method calls, given the set in Exercise 1, and what does it return?
 

```
s.add('a');
s.add('A');
next = 'b';
s.contains(next);
```
3. For Questions 3–7, a Map, *m*, contains the following entries: (1234, "Jane Doe"), (1999, "John Smith"), (1250, "Ace Ventura"), (2000, "Bill Smythe"), (2999, "Nomar Garciaparra").
4. What is the effect of the statement *m.put(1234, "Jane Smith");*? What is returned?
5. What is returned by *m.get(1234)*? What is returned by *m.get(1500)*?
6. If the entries for Map *m* are stored in a hash table of size 1000 with open addressing and linear probing, where would each of the items be stored?
7. Answer Question 5 for the case where the entries were stored using quadratic probing.
8. Answer Question 5 for the case where the entries were stored using chaining.
9. What class does the Java API provide that facilitates coding an implementer of the Map interface? Of the Set interface?
10. List two classes that the Java API provides that implement the Map interface. List two that implement the Set interface.
11. You apply method \_\_\_\_\_ to a Map to create a set view. You apply method \_\_\_\_\_ to this set view to get an object that facilitates sequential access to the Map elements.

## Review Questions

1. Show where the following keys would be placed in a hash table of size 5 using open addressing: 1000, 1002, 1007, 1003. Where would these keys be after rehashing to a table of size 11?
2. Answer Question 1 for a hash table that uses chaining.

3. Write a `toString` method for class `HashtableOpen`. This method should display each table element that is not `null` and is not deleted.
4. Class `HashtableChain` uses the class `LinkedList`, which is implemented as a double-linked list. Write the `put` method using a single-linked list to hold elements that hash to the same index.
5. Write the `get` method for the class in Question 4.
6. Write the `remove` method for the class in Question 4.
7. Write inner class `EntrySet` for the class in Question 4 (see Listing 7.11).

## Programming Projects

1. Complete all methods of class `HuffmanTree` and test them out using a document file and a Java source file on your computer. You can download class `BitString` from the Web site for this textbook.
2. Use a `HashMap` to store the frequency counts for all the words in a large text document. When you are done, display the contents of this `HashMap`. Next, create a set view of the `Map` and store its contents in an array. Then sort the array based on key value and display it. Finally, sort the array in decreasing order by frequency and display it.
3. Solve Project 2 using a `TreeMap`. You can display the words in key sequence without performing a sort.
4. Modify Project 2 to save the line numbers for every occurrence of a word as well as the count.
5. Based on an example in Brian W. Kernighan and Rob Pike, *The Practice of Programming*, Addison-Wesley, 1999, we want to generate “random text” in the style of another author. Your first task is to collect a group of prefix strings of two words that occur in a text file and associate them with a list of suffix strings using a `Map`. For example, the text for Charles Dickens’ *A Christmas Carol* contains the four phrases:

Marley was dead: to begin with.

Marley was as dead as a door-nail.

Marley was as dead as a door-nail.

Marley was dead.

The prefix string “Marley was” would be associated with the `ArrayList` containing the four suffix strings “dead:”, “as”, “as”, “dead.”. You must go through the text and examine each successive pair of two-word strings to see whether that pair is already in the map as a key. If so, add the next word to the `ArrayList` that is the value for that prefix string. For example, in examining the first two sentences shown, you would first add to the entry (“Marley was”, `ArrayList` “dead:”). Next you would add the entry (“was dead”, `ArrayList` “as”). Next you would add the entry (“dead as”, `ArrayList` “a”), and so on. When you retrieve the prefix “Marley was” again, you would modify the `ArrayList` that is its value, and the entry would become (“Marley was”, `ArrayList` “dead:”, “as”). When you are all finished, add the entry “THE\_END” to the suffix list for the last prefix placed in the `Map`.

Once you have scanned the complete text, it is time to use the `Map` to begin generating new text that is in the same style as the old text. Output the first prefix you placed in the `Map`: “Marley was”. Then retrieve the `ArrayList` that is the value for this prefix. Randomly select one of the suffixes and then output the suffix. For example, the output text so far might be “Marley was dead” if the suffix “dead” was selected from the `ArrayList` of suffixes for “Marley was”. Now continue with the two-word sequence consisting of the second word from the previous prefix and the suffix (that would be the string “was dead”). Look it up in the map, randomly select one of the suffixes and output it. Continue this process until the suffix “THE\_END” is selected.

6. Complete class `HashtableOpen` so that it fully implements the `Map` interface described in Section 7.2. As part of this, write method `entrySet` and classes `EntrySet` and `SetIterator` as described in Section 7.5. Class `SetIterator` provides methods `hasNext` and `next`. Use data field `index` to keep track of the next value of the iterator (initially 0). Data field `lastItemReturned` keeps track of the index of the last item returned by `next`; this is used by the `remove`

method. The `remove` method removes the last item returned by the `next` method from the Set. It may only be called once for each call to `next`. Thus, the `remove` method checks to see that `LastItemReturned` has a valid value (not `-1`) and then sets it to an invalid value (`-1`) just before returning to the caller.

7. Complete class `HashtableChain` so that it fully implements the `Map` interface, and test it out. Complete class `SetIterator` as described in Project 6.
8. Complete the implementation of class `HashSetOpen`, writing it as an adapter class of `HashtableOpen`.
9. Complete the implementation of class `HashSetChain`, writing it as an adapter class of `HashtableChain`.
10. Revise method `put` for `HashtableOpen` to place a new item into an already deleted spot in the search chain. Don't forget to check the scenario where the key has already been inserted.

## Answers to Quick-Check Exercises

1. `s.add('d');`
2. `s.add('a'); // add 'a', duplicate – returns false`  
`s.add('A'); // add 'A', returns true`  
`next = 'b';`  
`s.contains(next); // 'b' is in the set, returns true`
3. The value associated with key 1234 is changed to "Jane Smith". The string "Jane Doe" is returned.
4. The string "Jane Doe" and then `null`.
5. 1234 at 234, 1999 at 999, 1250 at 250, 2000 at 000, 3999 at 001.
6. 1234 at 234, 1999 at 999, 1250 at 250, 2000 at 000, 3999 at 003.
7. 2000 in a linked list at 000, 1234 in a linked list at 234, 1250 in a linked list at 250, 1999 and 3999 in a linked list at 999.
8. `AbstractMap`, `AbstractSet`
9. `HashMap` and `TreeMap`, `HashSet` and `TreeSet`
10. `entrySet`, `iterator`

# *Sorting*

## Chapter Objectives

- ◆ To learn how to use the standard sorting methods in the Java API
- ◆ To learn how to implement the following sorting algorithms: selection sort, insertion sort, Shell sort, merge sort, Timsort, heapsort, and quicksort
- ◆ To understand the difference in performance of these algorithms, and which to use for small arrays, which to use for medium arrays, and which to use for large arrays

Sorting is the process of rearranging the data in an array or a list so that it is in increasing (or decreasing) order. Because sorting is done so frequently, computer scientists have devoted much time and effort to developing efficient algorithms for sorting arrays. Even though many languages (including Java) provide sorting utilities, it is still very important to study these algorithms because they illustrate several well-known ways to solve the sorting problem, each with its own merits. You should know how they are written so that you can duplicate them if you need to use them with languages that don't have sorting utilities.

Another reason for studying these algorithms is that they illustrate some very creative approaches to problem solving. For example, the insertion sort algorithm adapts an approach used by card players to arrange a hand of cards; the merge sort algorithm builds on a technique used to sort external data files. Several algorithms use divide-and-conquer to break a larger problem into more manageable subproblems. The Shell sort is a very efficient sort that works by sorting many small sub-arrays using insertion sort, which is a relatively inefficient sort when used by itself. The merge sort and quicksort algorithms are both recursive. Method heapsort uses a heap as its underlying data structure. The final reason for studying sorting is to learn how computer scientists analyze and compare the performance of several different algorithms that perform the same operation.

We will cover two quadratic ( $O(n^2)$ ) sorting algorithms that are fairly simple and appropriate for sorting small arrays but are not recommended for large arrays. We will also discuss four sorting algorithms that give improved performance ( $O(n \log n)$ ) on large arrays and one that gives performance that is much better than  $O(n^2)$  but not as good as  $O(n \log n)$ .

Our goal is to provide a sufficient selection of quadratic sorts and faster sorts. A few other sorting algorithms are described in the programming projects. Our expectation is that your instructor will select which algorithms you should study.

## Sorting

- 8.1** Using Java Sorting Methods
- 8.2** Selection Sort
- 8.3** Insertion Sort
- 8.4** Comparison of Quadratic Sorts
- 8.5** Shell Sort: A Better Insertion Sort
- 8.6** Merge Sort
- 8.7** Timsort
- 8.8** Heapsort
- 8.9** Quicksort
- 8.10** Testing the Sort Algorithms
- 8.11** The Dutch National Flag Problem (Optional Topic)  
*Case Study:* The Problem of the Dutch National Flag

## 8.1 Using Java Sorting Methods

The Java API `java.util` provides a class `Arrays` with several overloaded `sort` methods for different array types. In addition, the class `Collections` (also part of the API `java.util`) contains similar sorting methods for `Lists`. The methods for arrays of primitive types are based on the quicksort algorithm (Section 8.9), and the methods for arrays of `Objects` and for `Lists` are based on the Timsort algorithm (Section 8.7). Both algorithms are  $O(n \log n)$ .

Method `Arrays.sort` is defined as a `public static void` method and is overloaded (see Table 8.1). The first argument in a call can be an array of any primitive type (although we have just shown `int[]`) or an array of objects. If the first argument is an array of objects, then either the class type of the array must implement the `Comparable` interface or a `Comparator` object must be passed as the last argument (see Section 6.6). A class that implements the `Comparable` interface must define a `compareTo` method that determines the natural ordering of its objects. If a `Comparator` is passed, its `compare` method will be used to determine the ordering.

For method `Collections.sort` (see Table 8.1), the first argument must be a collection of objects that implement the `List` interface (e.g., an `ArrayList` or a `LinkedList`). If only one argument is provided, the objects in the `List` must implement the `Comparable` interface. Method `compareTo` is called by the sorting method to determine the relative ordering of two objects.

Optionally, a `Comparator` can be passed as a second argument. Using a `Comparator`, you can compare objects based on some other information rather than using their natural ordering (as determined by method `compareTo`). The `Comparator` object must be the last argument in the call to the sorting method. Rather than rearranging the elements in the `List`, method `sort` first copies the `List` elements to an array, sorts the array using `Arrays.sort`, and then copies them back to the `List`.

**TABLE 8.1**Methods `sort` in Classes `java.util.Arrays` and `java.util.Collections`

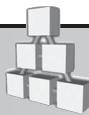
<b>Method sort in Class Arrays</b>	<b>Behavior</b>
<code>public static void sort(int[] items)</code>	Sorts the array <code>items</code> in ascending order
<code>public static void sort(int[] items, int fromIndex, int toIndex)</code>	Sorts array elements <code>items[fromIndex]</code> to <code>items[toIndex]</code> in ascending order
<code>public static void sort(Object[] items)</code>	Sorts the objects in array <code>items</code> in ascending order using their natural ordering (defined by method <code>compareTo</code> ). All objects in <code>items</code> must implement the <code>Comparable</code> interface and must be mutually comparable
<code>public static void sort(Object[] items, int fromIndex, int toIndex)</code>	Sorts array elements <code>items[fromIndex]</code> to <code>items[toIndex]</code> in ascending order using their natural ordering (defined by method <code>compareTo</code> ). All objects must implement the <code>Comparable</code> interface and must be mutually comparable
<code>public static &lt;T&gt; void sort(T[] items, Comparator&lt;? super T&gt; comp)</code>	Sorts the objects in <code>items</code> in ascending order as defined by method <code>comp.compare</code> . All objects in <code>items</code> must be mutually comparable using method <code>comp.compare</code>
<code>public static &lt;T&gt; void sort(T[] items, int fromIndex, int toIndex, Comparator&lt;? super T&gt; comp)</code>	Sorts the objects in <code>items[fromIndex]</code> to <code>items[toIndex]</code> in ascending order as defined by method <code>comp.compare</code> . All objects in <code>items</code> must be mutually comparable using method <code>comp.compare</code>
<b>Method sort in Class Collections</b>	<b>Behavior</b>
<code>public static &lt;T extends Comparable&lt;T&gt;&gt; void sort(List&lt;T&gt; list)</code>	Sorts the objects in <code>list</code> in ascending order using their natural ordering (defined by method <code>compareTo</code> ). All objects in <code>list</code> must implement the <code>Comparable</code> interface and must be mutually comparable
<code>public static &lt;T&gt; void sort (List&lt;T&gt; list, Comparator&lt;? super T&gt; comp)</code>	Sorts the objects in <code>list</code> in ascending order as defined by method <code>comp.compare</code> . All objects must be mutually comparable
<b>Method sort in Interface List</b>	<b>Behavior</b>
<code>default void sort(Comparator&lt;? super E&gt; comp)</code>	Sorts the objects in the list in ascending order as defined by method <code>comp.compare</code> . All objects must be mutually comparable

In Java 8, the `List` interface now contains a `sort` method. This method passes the list (`this`) to `Collections.sort`.

In class `Arrays`, the two methods that use a `Comparator` are *generic methods*. Generic methods, like generic classes, have parameters. The generic parameter(s) precede the method type. For example, in the declaration

```
public static <T> void sort(T[] items, Comparator<? super T> comp)
```

`T` represents the generic parameter for the `sort` method and should appear in the method parameter list (e.g., `T[] items`). For the second method parameter, the notation `Comparator<? super T>` means that `comp` must be an object that implements the `Comparator` interface for type `T` or for a superclass of type `T`. For example, you can define a class that implements `Comparator<Number>` and use it to sort an array of `Integer` objects or an array of `Double` objects.



## SYNTAX Declaring a Generic Method

### FORM:

*methodModifiers <genericParameters> returnType methodName(methodParameters)*

### EXAMPLE:

```
public static <T extends Comparable<T>> int binarySearch(T[] items, T target)
```

### MEANING:

To declare a generic method, list the *genericParameters* inside the symbol pair `<>` and between the *methodModifiers* (e.g., `public static`) and the return type. The *genericParameters* can then be used in the specification of the *methodParameters*.

Both methods in class `Collections` are generic.

```
public static <T extends Comparable<T>> void sort(List<T> list)
```

The notation `<T extends Comparable<T>>` means that generic parameter `T` must implement the interface `Comparable<T>`. The method parameter `list` (the object being sorted) is of type `List<T>`.

### EXAMPLE 8.1

If array `items` stores a collection of integers, the method call

```
Arrays.sort(items, 0, items.length / 2);
```

sorts the integers in the first half of the array, leaving the second half of the array unchanged.

### EXAMPLE 8.2

Let's assume class `Person` is defined as follows:

```
public class Person implements Comparable<Person> {
    // Data Fields
    /* The last name */
    private String lastName;
    /* The first name */
    private String firstName;
    /* Birthday represented by an integer from 1 to 366 */
    private int birthDay;

    // Methods
    /** Compares two Person objects based on names. The result
     *  is based on the last names if they are different;
     *  otherwise, it is based on the first names.
     * @param obj The other Person
     * @return A negative integer if this person's name
     *         precedes the other person's name;
     *         0 if the names are the same;
     *         a positive integer if this person's name follows
     *         the other person's name.
     */
    @Override
    public int compareTo(Person other) {
```

```

    // Compare this Person to other using last names.
    int result = lastName.compareTo(other.lastName);
    // Compare first names if last names are the same.
    if (result == 0)
        return firstName.compareTo(other.firstName);
    else
        return result;
}

// Other methods
. . .
}

```

Method `Person.compareTo` compares two `Person` objects based on their names using the last name as the primary key and the first name as the secondary key (the natural ordering). If `people` is an array of `Person` objects, the statement

```
Arrays.sort(people);
```

places the elements in array `people` in ascending order based on their names. Although the sort operation is  $O(n \log n)$ , the comparison of two names is  $O(k)$  where  $k$  is the length of the shorter name.

---

**EXAMPLE 8.3** You can also use a class that implements `Comparator<Person>` to compare `Person` objects. As an example, method `compare` in class `ComparePerson` compares two `Person` objects based on their birthdays, not their names.

```

import java.util.Comparator;

public class ComparePerson implements Comparator<Person> {
    /** Compare two Person objects based on birth date.
     * @param left The left-hand side of the comparison
     * @param right The right-hand side of the comparison
     * @return A negative integer if the left person's birthday
     *         precedes the right person's birthday;
     *         0 if the birthdays are the same;
     *         a positive integer if the left person's birthday
     *         follows the right person's birthday.
     */
    @Override
    public int compare(Person left, Person right) {
        return left.getBirthDay() - right.getBirthDay();
    }
}

```

If `peopleList` is a `List` of `Person` objects, the statement

```
Collections.sort(peopleList, new ComparePerson());
```

places the elements in `peopleList` in ascending order based on their birthdays. Comparing two birthdays is an  $O(1)$  operation.

In Java 8, you can pass a lambda expression as a `Comparator` object to the `List.sort` method instead of writing a class that implements `Comparator`. We can sort `peopleList` with the following statement, where the lambda expression specifies the `compare` method to be used with items of `peopleList`.

```
peopleList.sort((p1, p2) -> p1.getBirthDay() - p2.getBirthDay());
```

---

## EXERCISES FOR SECTION 8.1

### SELF-CHECK

1. Indicate whether each of the following method calls is valid. Describe why it isn't valid or, if it is valid, describe what it does. Assume `people` is an array of `Person` objects and `peopleList` is a `List` of `Person` objects.
  - a. `people.sort();`
  - b. `Arrays.sort(people, 0, people.length - 3);`
  - c. `Arrays.sort(peopleList, 0, peopleList.length - 3);`
  - d. `Collections.sort(people);`
  - e. `Collections.sort(peopleList, new ComparePerson());`
  - f. `Collections.sort(peopleList, 0, peopleList.size() - 3);`

### PROGRAMMING

1. Write a method call to sort the last half of array `people` using the natural ordering.
2. Write a method call to sort the last half of array `people` using the ordering determined by class `ComparePerson`.
3. Write a method call to sort `peopleList` using the natural ordering.



## 8.2 Selection Sort

*Selection sort* is a relatively easy-to-understand algorithm that sorts an array by making several passes through the array, *selecting* the next smallest item in the array each time, and placing it where it belongs in the array. We illustrate all sorting algorithms using an array of integer values for simplicity. However, each algorithm sorts an array of `Comparable` objects, so the `int` values must be wrapped in `Integer` objects.

We show the algorithm next, where  $n$  is the number of elements in an array with subscripts 0 through  $n - 1$  and `fill` is the subscript of the element that will store the next smallest item in the array.

### Selection Sort Algorithm

1. `for fill = 0 to n - 2 do`
2.     Set `posMin` to the subscript of the smallest item in the subarray starting at subscript `fill`.
3.     Exchange the item at `posMin` with the one at `fill`.

Step 2 involves a search for the smallest item in each subarray. It requires a loop in which we compare each element in the subarray, starting with the one at position `fill + 1`, with the smallest value found so far. In the refinement of Step 2 shown in the following algorithm (Steps 2.1 through 2.4), we use `posMin` to store the subscript of the smallest value found so far. We assume that its initial position is `fill`.

### Refinement of Selection Sort Algorithm (Step 2)

- 2.1 Initialize `posMin` to `fill`.
- 2.2 `for next = fill + 1 to n - 1`

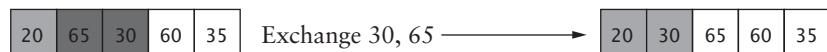
2.3      if the item at next is less than the item at posMin

2.4            Reset posMin to next.

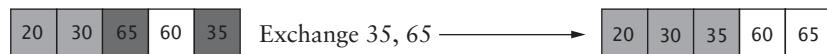
First the selection sort algorithm finds the smallest item in the array (smallest is 20) and moves it to position 0 by exchanging it with the element currently at position 0. At this point, the sorted part of the array consists of the new element at position 0. The values to be exchanged are shaded dark in all diagrams. The sorted elements are in light gray.



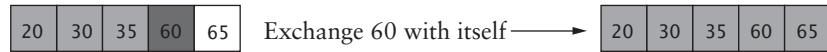
Next, the algorithm finds the smallest item in the subarray starting at position 1 (next smallest is 30) and exchanges it with the element currently at position 1:



At this point, the sorted portion of the array consists of the elements at positions 0 and 1. Next, the algorithm selects the smallest item in the subarray starting at position 2 (next smallest is 35) and exchanges it with the element currently at position 2:



At this point, the sorted portion of the array consists of the elements at positions 0, 1, and 2. Next, the algorithm selects the smallest item in the subarray starting at position 3 (next smallest is 60) and exchanges it with the element currently at position 3:



The element at position 4, the last position in the array, must store the largest value (largest is 65), so the array is sorted.

## Analysis of Selection Sort

Steps 2 and 3 are performed  $n - 1$  times. Step 3 performs an exchange of items; consequently, there are  $n - 1$  exchanges.

Step 2.3 involves a comparison of items and is performed  $(n - 1 - \text{fill})$  times for each value of *fill*. Since *fill* takes on all values between 0 and  $n - 2$ , the following series computes the number of executions of Step 2.3:

$$(n-1) + (n-2) + \dots + 3 + 2 + 1$$

This is a well-known series that can be written in closed form as

$$\frac{n \times (n-1)}{2} = \frac{n^2}{2} - \frac{n}{2}$$

For very large  $n$ , we can ignore all but the most significant term in this expression, so the number of comparisons is  $O(n^2)$  and the number of exchanges is  $O(n)$ . Because the number of comparisons increases with the square of  $n$ , the selection sort is called a *quadratic sort*.

## Code for Selection Sort

Listing 8.1 shows the code for selection sort, which follows the algorithm above.

.....  
**LISTING 8.1**  
 SelectionSort.java

```
/** Implements the selection sort algorithm. */
public class SelectionSort {

    /** Sort the array using selection sort algorithm.
     * @param table contains Comparable objects.
     * @post table is sorted.
     * @param table The array to be sorted
     */
    public static void sort(Comparable[] table) {
        int n = table.length;
        for (int fill = 0; fill < n - 1; fill++) {
            // Invariant: table[0 . . . fill - 1] is sorted.
            int posMin = fill;
            for (int next = fill + 1; next < n; next++) {
                // Invariant: table[posMin] is the smallest item in
                // table[fill . . . next - 1].
                if (table[next].compareTo(table[posMin]) < 0) {
                    posMin = next;
                }
            }
            // assert: table[posMin] is the smallest item in
            // table[fill . . . n - 1].
            // Exchange table[fill] and table[posMin].
            Comparable temp = table[fill];
            table[fill] = table[posMin];
            table[posMin] = temp;
            // assert: table[fill] is the smallest item in
            // table[fill . . . n - 1].
        }
        // assert: table[0 . . . n - 1] is sorted.
    }
}
```



## PROGRAM STYLE

### Making Sort Methods Generic

The code in Listing 8.1 will compile, but it will generate a warning message regarding an unchecked call to `compareTo`. You can eliminate this warning message by making the sort a generic sort. To accomplish this for the sort above, change the method heading to

```
public static <T extends Comparable<T>> void sort(T[] table) {
```

where the generic type parameter, `T`, must implement the `Comparable<T>` interface. Also, change the data type of variable `temp` from `Comparable` to type `T`, the data type of the array elements.

```
T temp = table[fill];
```

We will code the other sorting algorithms in this chapter as generic methods.

## EXERCISES FOR SECTION 8.2

### SELF-CHECK

1. Show the progress of each pass of the selection sort for the following array. How many passes are needed? How many comparisons are performed? How many exchanges? Show the array after each pass.

40 35 80 75 60 90 70 75 50 22

2. How would you modify selection sort to arrange an array of values in decreasing sequence?
3. It is not necessary to perform an exchange if the next smallest element is already at position `fill`. Modify the selection sort algorithm to eliminate the exchange of an element with itself. How does this affect big-O for exchanges? Discuss whether the time saved by eliminating unnecessary exchanges would exceed the cost of these extra steps.

### PROGRAMMING

1. Modify the selection sort method to sort the elements in decreasing order and to incorporate the change in Self-Check Exercise 3.
2. Add statements to trace the progress of selection sort. Display the array contents after each exchange.



## 8.3 Insertion Sort

Our next quadratic sorting algorithm, *insertion sort*, is based on the technique used by card players to arrange a hand of cards. The player keeps the cards that have been picked up so far in sorted order. When the player picks up a new card, the player makes room for the new card and then *inserts* it in its proper place.

The left diagram of Figure 8.1 shows a hand of cards (ignoring suits) after three cards have been picked up. If the next card is an 8, it should be inserted between the 6 and 10, maintaining the numerical order (middle diagram). If the next card is a 7, it should be inserted between the 6 and 8 as shown on the right in Figure 8.1.

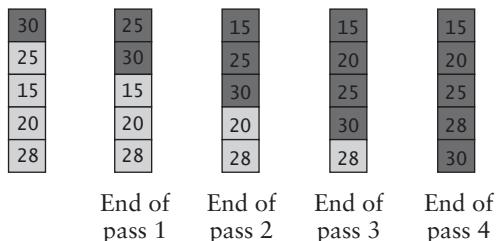
To adapt this insertion algorithm to an array that has been filled with data, we start with a sorted subarray consisting of the first element only. For example, in the leftmost array of Figure 8.2, the initial sorted subarray consists of only the first value 30 (in element 0). The array element(s) that are in order after each pass are shaded dark, and the elements waiting to be inserted are in gray. We first *insert* the second element (25). Because it is smaller than the

**FIGURE 8.1**  
Picking Up a Hand of Cards



Dr. Elliot Koffman

**FIGURE 8.2**  
An Insertion Sort



element in the sorted subarray, we insert it before the old first element (30), and the sorted subarray has two elements (25, 30 in second diagram). Next, we *insert* the third element (15). It is also smaller than all the elements in the sorted subarray, so we insert it before the old first element (25), and the sorted subarray has three elements (15, 25, 30 in third diagram). Next, we *insert* the fourth element (20). It is smaller than the second and third elements in the sorted subarray, so we insert it before the old second element (25), and the sorted subarray has four elements (15, 20, 25, 30 in the fourth diagram). Finally, we insert the last element (28). It is smaller than the last element in the sorted subarray, so we insert it before the old last element (30), and the array is sorted. The algorithm follows.

### Insertion Sort Algorithm

1. **for** each array element from the second (`nextPos = 1`) to the last
2.     Insert the element at `nextPos` where it belongs in the array, increasing the length of the sorted subarray by 1 element.

To accomplish Step 2, the insertion step, we need to make room for the element to be inserted (saved in `nextVal`) by shifting all values that are larger than it, starting with the last value in the sorted subarray.

### Refinement of Insertion Sort Algorithm (Step 2)

- 2.1     `nextPos` is the position of the element to insert.
- 2.2     Save the value of the element to insert in `nextVal`.
- 2.3     **while** `nextPos > 0` and the element at `nextPos - 1 > nextVal`
- 2.4         Shift the element at `nextPos - 1` to position `nextPos`.
- 2.5         Decrement `nextPos` by 1.
- 2.6     Insert `nextVal` at `nextPos`.

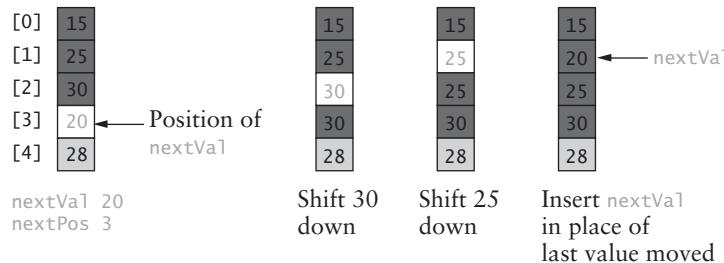
We illustrate these steps in Figure 8.3. For the array shown on the left, the first three elements (positions 0, 1, and 2) are in the sorted subarray, and the next element to insert is 20. First we save 20 in `nextVal` and 3 in `nextPos`. Next we shift the value in position 2 (30) down one position (see the second array in Figure 8.3), and then we shift the value in position 1 (25) down one position (see third array in Figure 8.3). After these shifts (third array), there will temporarily be two copies of the last value shifted (25). The first of these (white background in Figure 8.3) is overwritten when the value in `nextVal` is moved into its correct position (`nextPos` is 1). The four-element sorted subarray is shaded dark on the right of Figure 8.3.

### Analysis of Insertion Sort

The insertion step is performed  $n - 1$  times. In the worst case, all elements in the sorted subarray are compared to `nextVal` for each insertion, so the maximum number of comparisons is represented by the series

$$1 + 2 + 3 + \dots + (n - 2) + (n - 1)$$

**FIGURE 8.3**  
Inserting the Fourth  
Array Element



which is  $O(n^2)$ . In the best case (when the array is already sorted), only one comparison is required for each insertion, so the number of comparisons is  $O(n)$ . The number of shifts performed during an insertion is one less than the number of comparisons or, when the new value is the smallest so far, the same as the number of comparisons. However, a shift in an insertion sort requires the movement of only one item, whereas in a selection sort, an exchange involves a temporary item and requires the movement of three items. A Java array of objects contains references to the actual objects, and it is these references that are changed. The actual objects remain in the physical locations where they were first created.

## Code for Insertion Sort

Listing 8.2 shows the `InsertionSort`. We use method `insert` to perform the insertion step shown earlier. It would be more efficient to insert this code inside the `for` statement; however, using a method will make it easier to implement the Shell sort algorithm later.

The `while` statement in method `insert` compares and shifts all values greater than `nextVal` in the subarray `table[0 . . . nextPos - 1]`. The `while` condition

```
((nextPos > 0) && (nextVal.compareTo(table[nextPos - 1]) < 0))
```

causes loop exit if the first element has been moved or if `nextVal` is not less than the next element to move. It could lead to an out-of-range subscript error if the order of the conditions were reversed. Recall that Java performs short-circuit evaluation. If the left-hand operand of an `&&` operation is false, the right-hand operand is not evaluated. If this were not the case, when `nextPos` becomes 0, the array subscript would be `-1`, which is outside the subscript range. Because `nextPos` is a value parameter, variable `nextPos` in `sort` is unchanged.

**LISTING 8.2**  
`InsertionSort.java`

```
/** Implements the insertion sort algorithm. */
public class InsertionSort {
    /** Sort the table using insertion sort algorithm.
     * @pre table contains Comparable objects.
     * @post table is sorted.
     * @param table The array to be sorted
     */
    public static <T extends Comparable<T>> void sort(T[] table) {
        for (int nextPos = 1; nextPos < table.length; nextPos++) {
            // Invariant: table[0 . . . nextPos - 1] is sorted.
            // Insert element at position nextPos
            // in the sorted subarray.
            insert(table, nextPos);
        } // End for.
    } // End sort.
```

```

    /** Insert the element at nextPos where it belongs
     * in the array.
     * @pre table[0 . . . nextPos - 1] is sorted.
     * @post table[0 . . . nextPos] is sorted.
     * @param table The array being sorted
     * @param nextPos The position of the element to insert
    */
private static <T extends Comparable<T>> void insert(T[] table,
                                                       int nextPos) {
    T nextVal = table[nextPos];
    // Element to insert.
    while (nextPos > 0 && nextVal.compareTo(table
                                              [nextPos - 1]) < 0) {
        table[nextPos] = table[nextPos - 1];
        // Shift down.
        nextPos--;
        // Check next smaller element.
    }
    // Insert nextVal at nextPos.
    table[nextPos] = nextVal;
}
}

```

---

## EXERCISES FOR SECTION 8.3

### SELF-CHECK

- Sort the following array using insertion sort. How many passes are needed? How many comparisons are performed? How many exchanges? Show the array after each pass.

40 35 80 75 60 90 70 75 50 22

### PROGRAMMING

- Eliminate method `insert` in Listing 8.2 and write its code inside the `for` statement.
- Add statements to trace the progress of insertion sort. Display the array contents after the insertion of each value.



## 8.4 Comparison of Quadratic Sorts

Table 8.2 summarizes the performance of two quadratic sorts. To give you some idea as to what these numbers mean, Table 8.3 shows some values of  $n$  and  $n^2$ . If  $n$  is small (say, 100 or less), it really doesn't matter which sorting algorithm you use. Insertion sort gives the best performance for larger arrays. Insertion sort is better because it takes advantage of any partial sorting that is in the array and uses less costly shifts instead of exchanges to rearrange array elements. In the next section, we discuss a variation on insertion sort, known as Shell sort, that has  $O(n^{3/2})$  or better performance.

Since the time to sort an array of  $n$  elements is proportional to  $n^2$ , none of these algorithms is particularly good for large arrays (i.e.,  $n > 100$ ). The best sorting algorithms provide  $n \log n$  average-case behavior and are considerably faster for large arrays. In fact, one of the

**TABLE 8.2**  
Comparison of Quadratic Sorts

	Number of Comparisons		Number of Exchanges	
	Best	Worst	Best	Worst
Selection sort	$O(n^2)$	$O(n^2)$	$O(n)$	$O(n)$
Insertion sort	$O(n)$	$O(n^2)$	$O(1)$	$O(n^2)$

**TABLE 8.3**  
Comparison of Rates of Growth

$n$	$n^2$	$n \log n$
8	64	24
16	256	64
32	1024	160
64	4096	384
128	16,384	896
256	65,536	2048
512	262,144	4608

algorithms that we will discuss has  $n \log n$  worst-case behavior. You can get a feel for the difference in behavior by comparing the last column of Table 8.3 with the middle column.

Recall from Section 2.1 that big-O analysis ignores any constants that might be involved or any overhead that might occur from method calls needed to perform an exchange or a comparison. However, the tables give you an estimate of the relative performance of the different sorting algorithms.

We haven't talked about storage usage for these algorithms. Both quadratic sorts require storage for the array being sorted. However, there is only one copy of this array, so the array is sorted in place. There are also requirements for variables that store references to particular elements, loop control variables, and temporary variables. However, for large  $n$ , the size of the array dominates these other storage considerations, so the extra space usage is proportional to  $O(1)$ .

## Comparisons versus Exchanges

We have analyzed comparisons and exchanges separately, but you may be wondering whether one is more costly (in terms of computer time) than the other. In Java, an exchange requires your computer to switch two object references using a third object reference as an intermediary. A comparison requires your computer to execute a `compareTo` method. The cost of a comparison depends on its complexity, but it will probably be more costly than an exchange because of the overhead to call and execute method `compareTo`. In some programming languages (but not Java), an exchange may require physically moving the information in each object rather than simply swapping object references. For these languages, the cost of an exchange would be proportional to the size of the objects being exchanged and may be more costly than a comparison.

## EXERCISES FOR SECTION 8.4

### SELF-CHECK

1. Complete Table 8.3 for  $n = 1024$  and  $n = 2048$ .
2. What do the new rows of Table 8.3 tell us about the increase in time required to process an array of 1024 elements versus an array of 2048 elements for  $O(n)$ ,  $O(n^2)$ , and  $O(n \log n)$  algorithms?



## 8.5 Shell Sort: A Better Insertion Sort

Next, we describe the *Shell sort*, which is a type of insertion sort but with  $O(n^{3/2})$  or better performance. Unlike the other algorithms, Shell sort is named after its discoverer, Donald L. Shell (“A High-Speed Sorting Procedure,” *Communications of the ACM*, Vol. 2, No. 7 [1959], pp. 30–32). You can think of the Shell sort as a divide-and-conquer approach to insertion sort. Instead of sorting the entire array at the start, the idea behind Shell sort is to sort many smaller subarrays using insertion sort before sorting the entire array. The initial subarrays will contain two or three elements, so the insertion sorts will go very quickly. After each collection of subarrays is sorted, a new collection of subarrays with approximately twice as many elements as before will be sorted. The last step is to perform an insertion sort on the entire array, which has been presorted by the earlier sorts.

As an example, let’s sort the following array using initial subarrays with only two and three elements. We determine the elements in each subarray by setting a gap value between the subscripts in each subarray. We will explain how we pick the gap values later. We will use an initial gap of 7.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
40	35	80	75	60	90	70	75	55	90	85	34	45	62	57	65

A gap of 7 means the first subarray has subscripts 0, 7, 14 (element values 40, 75, 57, medium shade); the second subarray has subscripts 1, 8, 15 (element values 35, 55, 65, darkest shade); the third subarray has subscripts 2, 9 (element values 80, 90, lightest shade); and so on. There are seven subarrays. We start the process by inserting the value at position 7 (value of gap) into its subarray (elements at 0 and 7). Next, we insert the element at position 8 into its subarray (elements at 1 and 8). We continue until we have inserted the last element (at position 15) in its subarray (elements at 1, 8, and 15). The result of performing insertion sort on all seven subarrays with two or three elements follows:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
40	35	80	75	34	45	62	57	55	90	85	60	90	70	75	65

Next, we use a gap of 3. There are only three subarrays, and the longest one has six elements. The first subarray has subscripts 0, 3, 6, 9, 12, 15; the second subarray has subscripts 1, 4, 7, 10, 13; the third subarray has subscripts 2, 5, 8, 11, 14.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
40	35	80	75	34	45	62	57	55	90	85	60	90	70	75	65

We start the process by inserting the element at position 3 (value of gap) into its subarray. Next, we insert the element at position 4, and so on. The result of all insertions follows:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
40	34	45	62	35	55	65	57	60	75	70	75	90	85	80	90

Finally, we use a gap of 1, which performs an insertion sort on the entire array. Because of the presorting, it will require 1 comparison to insert 34, 1 comparison to insert 45 and 62, 4 comparisons to insert 35, 2 comparisons to insert 55, 1 comparison to insert 65, 3 comparisons to insert 57, 1 comparison to insert 60, and only 1 or 2 comparisons to insert each of the remaining values except for 80 (3 comparisons).

The algorithm for Shell sort follows. Steps 2 through 4 correspond to the insertion sort algorithm shown earlier. Because the elements with subscripts 0 through  $\text{gap} - 1$  are the first elements in their subarrays, we begin Step 4 by inserting the element at position  $\text{gap}$  instead of at position 1 as we did for the insertion sort. Step 1 sets the initial gap between subscripts to  $n / 2$ , where  $n$  is the number of array elements. To get the next gap value, Step 6 divides the current gap value by 2.2 (chosen by experimentation). We want the gap to be 1 during the last insertion sort so that the entire array will be sorted. Step 5 ensures this by resetting gap to 1 if it is 2.

### Shell Sort Algorithm

1. Set the initial value of  $\text{gap}$  to  $n / 2$ .
2. **while**  $\text{gap} > 0$
3.     **for** each array element from position  $\text{gap}$  to the last element
4.         Insert this element where it belongs in its subarray.
5.         **if**  $\text{gap}$  is 2, set it to 1.
6.         **else**  $\text{gap} = \text{gap} / 2.2$ .

### Refinement of Step 4, the Insertion Step

- 4.1  $\text{nextPos}$  is the position of the element to insert.
- 4.2 Save the value of the element to insert in  $\text{nextVal}$ .
- 4.3 **while**  $\text{nextPos} > \text{gap}$  and the element at  $\text{nextPos} - \text{gap} > \text{nextVal}$
- 4.4     Shift the element at  $\text{nextPos} - \text{gap}$  to position  $\text{nextPos}$ .
- 4.5     Decrement  $\text{nextPos}$  by  $\text{gap}$ .
- 4.6     Insert  $\text{nextVal}$  at  $\text{nextPos}$ .

### Analysis of Shell Sort

You may wonder why Shell sort is an improvement over regular insertion sort, because it ends with an insertion sort of the entire array. Each later sort (including the last one) will be performed on an array whose elements have been presorted by the earlier sorts. Because the behavior of insertion sort is closer to  $O(n)$  than  $O(n^2)$  when an array is nearly sorted, the presorting will make the later sorts, which involve larger subarrays, go more quickly. As a result of presorting, only 19 comparisons were required to perform an insertion sort on the last 15-element array shown in the previous section. This is critical because it is precisely for

larger arrays where  $O(n^2)$  behavior would have the most negative impact. For the same reason, the improvement of Shell sort over insertion sort is much more significant for large arrays.

A general analysis of Shell sort is an open research problem in computer science. The performance depends on how the decreasing sequence of values for gap is chosen. It is known that Shell sort is  $O(n^2)$  if successive powers of 2 are used for gap (i.e., 32, 16, 8, 4, 2, 1). If successive values for gap are of the form  $2^k - 1$  (i.e., 31, 15, 7, 3, 1), however, it can be proven that the performance is  $O(n^{3/2})$ . This sequence is known as *Hibbard's sequence*. There are other sequences that give similar or better performance.

We have presented an algorithm that selects the initial value of gap as  $n/2$  and then divides by 2.2 and truncates to the next lowest integer. Empirical studies of this approach show that the performance is  $O(n^{5/4})$  or maybe even  $O(n^{7/6})$ , but there is no theoretical basis for this result (M. A. Weiss, *Data Structures and Problem Solving Using Java* [Addison-Wesley, 1998], p. 230).

## Code for Shell Sort

Listing 8.3 shows the code for Shell sort. Method `insert` has a third parameter, `gap`. The expression after `&&`

`((nextPos > gap - 1) && (nextVal.compareTo(table[nextPos - gap]) < 0))`

compares elements that are separated by the value of `gap` instead of by 1. The expression before `&&` is false if `nextPos` is the subscript of the first element in a subarray. The statements in the `while` loop shift the element at `nextPos` down by `gap` (one position in the subarray) and reset `nextPos` to the subscript of the element just moved.

---

### LISTING 8.3

ShellSort.java

```
/** Implements the Shell sort algorithm. */
public class ShellSort {
    /** Sort the table using Shell sort algorithm.
        @pre table contains Comparable objects.
        @post table is sorted.
        @param table The array to be sorted
    */
    public static <T extends Comparable<T>> void sort(T[] table) {
        // Gap between adjacent elements.
        int gap = table.length / 2;
        while (gap > 0) {
            for (int nextPos = gap; nextPos < table.length; nextPos++) {
                // Insert element at nextPos in its subarray.
                insert(table, nextPos, gap);
            } // End for.

            // Reset gap for next pass.
            if (gap == 2) {
                gap = 1;
            } else {
                gap = (int) (gap / 2.2);
            }
        } // End while.
    } // End sort.

    /** Inserts element at nextPos where it belongs in array.
        @pre Elements through nextPos - gap in subarray are sorted.
        @post Elements through nextPos in subarray are sorted.
    */
}
```

```

@param table The array being sorted
@param nextPos The position of element to insert
@param gap The gap between elements in the subarray
*/
private static <T extends Comparable<T>> void insert(T[] table,
                                                       int nextPos, int gap) {
    T nextVal = table[nextPos];
    // Element to insert.
    // Shift all values > nextVal in subarray down by gap.
    while ((nextPos > gap - 1) && (nextVal.compareTo
        (table[nextPos - gap]) < 0)) {
        // First element not shifted.
        table[nextPos] = table[nextPos - gap];
        // Shift down.
        nextPos -= gap;
        // Check next position in subarray.
    }
    table[nextPos] = nextVal;
    // Insert nextVal.
}
}

```

## EXERCISES FOR SECTION 8.5

### SELF-CHECK

- Trace the execution of Shell sort on the following array. Show the array after all sorts when the gap is 5, the gap is 2, and after the final sort when the gap is 1. List the number of comparisons and exchanges required when the gap is 5, the gap is 2 and when the gap is 1. Compare this with the number of comparisons and exchanges that would be required for a regular insertion sort.

40 35 80 75 60 90 70 65 50 22

- For the example of Shell sort shown in this section, determine how many comparisons and exchanges are required to insert all the elements for each gap value. Compare this with the number of comparisons and exchanges that would be required for a regular insertion sort.

### PROGRAMMING

- Eliminate method `insert` in Listing 8.3 and write its code inside the `for` statement.
- Add statements to trace the progress of Shell sort. Display each value of gap, and display the array contents after all subarrays for that gap value have been sorted.

## 8.6 Merge Sort

The next algorithm that we will consider is called *merge sort*. A *merge* is a common data processing operation that is performed on two sequences of data (or data files) with the following characteristics:

- Both sequences contain items with a common `compareTo` method.
- The objects in both sequences are ordered in accordance with this `compareTo` method (i.e., both sequences are sorted).

The result of the merge operation is to create a third sequence that contains all of the objects from the first two sorted sequences. For example, if the first sequence is 3, 5, 8, 15 and the second sequence is 4, 9, 12, 20, the final sequence will be 3, 4, 5, 8, 9, 12, 15, 20. The algorithm for merging the two sequences follows.

### Merge Algorithm

1. Access the first item from both sequences.
2. **while** not finished with either sequence
3. Compare the current items from the two sequences, copy the smaller current item to the output sequence, and access the next item from the input sequence whose item was copied.
4. Copy any remaining items from the first sequence to the output sequence.
5. Copy any remaining items from the second sequence to the output sequence.

The **while** loop (Step 2) merges items from both input sequences to the output sequence. The current item from each sequence is the one that has been most recently accessed but not yet copied to the output sequence. Step 3 compares the two current items and copies the smaller one to the output sequence. If input sequence A's current item is the smaller one, the next item is accessed from sequence A and becomes its current item. If input sequence B's current item is the smaller one, the next item is accessed from sequence B and becomes its current item. After the end of either sequence is reached, Step 4 or Step 5 copies the items from the other sequence to the output sequence. Note that either Step 4 or Step 5 is executed, but not both.

As an example, consider the sequences shown in Figure 8.4. Steps 2 and 3 will first copy the items from sequence A with the values 244 and 311 to the output sequence; then items from sequence B with values 324 and 415 will be copied; and then the item from sequence A with value 478 will be copied. At this point, we have copied all items in sequence A, so we exit the **while** loop and copy the remaining items from sequence B (499, 505) to the output (Steps 4 and 5).

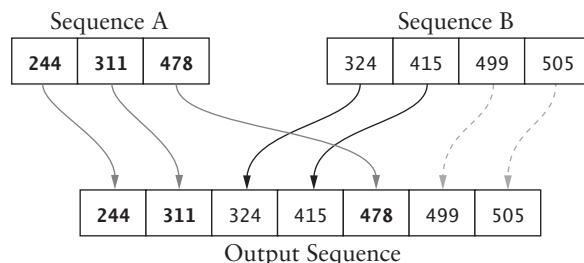
### Analysis of Merge

For two input sequences that contain a total of  $n$  elements, we need to move each element from its input sequence to its output sequence, so the time required for a merge is  $O(n)$ . How about the space requirements? We need to be able to store both initial sequences and the output sequence. So the array cannot be merged in place, and the *additional* space usage is  $O(n)$ .

### Code for Merge

Listing 8.4 shows the merge algorithm applied to arrays of `Comparable` objects. Algorithm Steps 4 and 5 are implemented as **while** loops at the end of the method.

**FIGURE 8.4**  
Merge Operation



**LISTING 8.4**

## Merge Method

```
.....  

LISTING 8.4  

Merge Method  

/** Merge two sequences.  

 * @pre leftSequence and rightSequence are sorted.  

 * @post outputSequence is the merged result and is sorted.  

 * @param outputSequence The destination  

 * @param leftSequence The left input  

 * @param rightSequence The right input  

 */  

private static <T extends Comparable<T>> void merge(T[] outputSequence,  

                                                 T[] leftSequence,  

                                                 T[] rightSequence) {  

    int i = 0;  

    // Index into the left input sequence.  

    int j = 0;  

    // Index into the right input sequence.  

    int k = 0;  

    // Index into the output sequence.  

    // While there is data in both input sequences  

    while (i < leftSequence.length && j < rightSequence.length) {  

        // Find the smaller and  

        // insert it into the output sequence.  

        if (leftSequence[i].compareTo(rightSequence[j]) < 0) {  

            outputSequence[k++] = leftSequence[i++];  

        } else {  

            outputSequence[k++] = rightSequence[j++];  

        }
    }
    // assert: one of the sequences has more items to copy.  

    // Copy remaining input from left sequence into the output.  

    while (i < leftSequence.length) {
        outputSequence[k++] = leftSequence[i++];
    }
    // Copy remaining input from right sequence into output.  

    while (j < rightSequence.length) {
        outputSequence[k++] = rightSequence[j++];
    }
}
```

**PROGRAM STYLE**

By using the postincrement operator on the index variables, you can both extract the current item from one sequence and append it to the end of the output sequence in one statement. The statement:

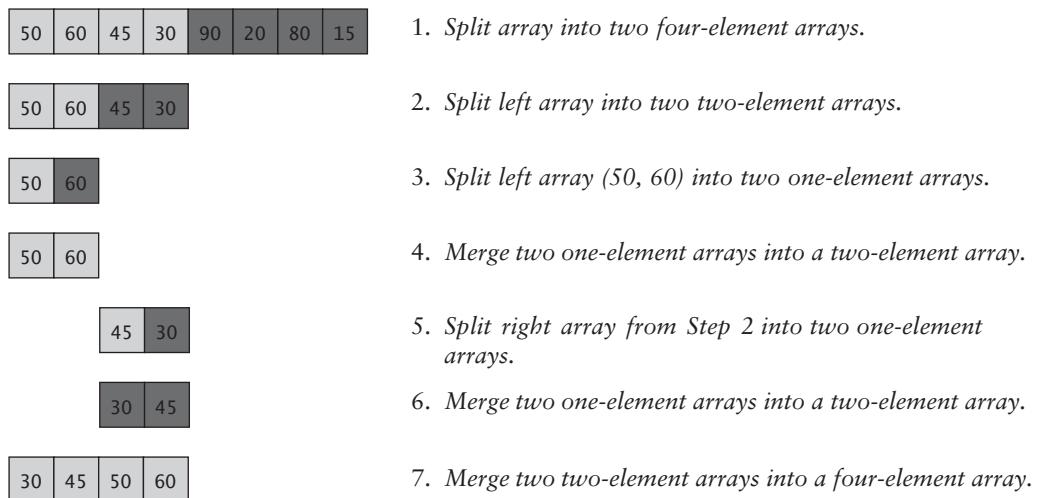
```
outputSequence[k++] = leftSequence[i++];
```

is equivalent to the following three statements, executed in the order shown:

```
outputSequence[k] = leftSequence[i];
k++;
i++;
```

Both the single statement and the group of three statements maintain the invariant that the indexes reference the current item.

**FIGURE 8.5**  
Trace of Merge Sort



## Algorithm for Merge Sort

We can modify merging to serve as an approach to sorting a single, unsorted array as follows:

1. Split the array into two halves.
2. Sort the left half.
3. Sort the right half.
4. Merge the two.

What sort algorithm should we use to do Steps 2 and 3? We can use the merge sort algorithm we are developing! The base case will be a table of size 1, which is already sorted, so there is nothing to do for the base case. We write the algorithm next, showing its recursive step.

### Algorithm for Merge Sort

1. **if** the tableSize is > 1
2. Set halfSize to tableSize divided by 2.
3. Allocate a table called **leftTable** of size halfSize.
4. Allocate a table called **rightTable** of size tableSize – halfSize.
5. Copy the elements from table[0 ... halfSize – 1] into **leftTable**.
6. Copy the elements from table[halfSize ... tableSize] into **rightTable**.
7. Recursively apply the merge sort algorithm to **leftTable**.
8. Recursively apply the merge sort algorithm to **rightTable**.
9. Apply the merge method using **leftTable** and **rightTable** as the input and the original table as the output.

## Trace of Merge Sort Algorithm

Figure 8.5 illustrates the merge sort. Each recursive call to method `sort` with an array argument that has more than one element splits the array argument into a left array and a right array, where each new array is approximately half the size of the array argument. We then

sort each of these arrays, beginning with the left half, by recursively calling method `sort` with the left array and right array as arguments. After returning from the sort of the left array and right array at each level, we merge these two halves together back into the space occupied by the array that was split. The left subarray in each recursive call (in gray) will be sorted before the processing of its corresponding right subarray (shaded dark) begins. Lines 4 and 6 merge two one-element arrays to form a sorted two-element array. At line 7, the two sorted two-element arrays (50, 60 and 30, 45) are merged into a sorted four-element array. Next, the right subarray shaded dark on line 1 will be sorted in the same way. When done, the sorted subarray (15, 20, 80, 90) will be merged with the sorted subarray on line 7.

## Analysis of Merge Sort

In Figure 8.5, the size of the arrays being sorted decreases from 8 to 4 (line 1) to 2 (line 2) to 1 (line 3). After each pair of subarrays is sorted, the pair will be merged to form a larger sorted array. Rather than showing a time sequence of the splitting and merging operations, we summarize them as follows:

50	60	45	30	90	20	80	15
50	60	45	30	90	20	80	15
50	60	45	30	90	20	80	15
50	60	30	45	20	90	15	80
30	45	50	60	15	20	80	90
15	20	30	45	50	60	80	90

1. *Split the eight-element array.*
2. *Split the four-element arrays.*
3. *Split the two-element arrays.*
4. *Merge the one-element arrays into two-element arrays.*
5. *Merge the two-element arrays into four-element arrays.*
6. *Merge the four-element arrays into an eight-element array.*

Lines 1 through 3 show the splitting operations, and lines 4 through 6 show the merge operations. Line 4 shows the two-element arrays formed by merging two-element pairs, line 5 shows the four-element arrays formed by merging two-element pairs, and line 6 shows the sorted array. Because each of these lines involves a movement of  $n$  elements from smaller-size arrays to larger arrays, the effort to do each merge is  $O(n)$ . The number of lines that require merging (three in this case) is  $\log n$  because each recursive step splits the array in half. So the total effort to reconstruct the sorted array through merging is  $O(n \log n)$ .

Recall from our discussion of recursion that whenever a recursive method is called, a copy of the local variables is saved on the run-time stack. Thus, as we go down the recursion chain sorting the `leftTables`, a sequence of `rightTables` of size  $\frac{n}{2}, \frac{n}{4}, \dots, \frac{n}{2^k}$  is allocated. Since  $\frac{n}{2} + \frac{n}{4} + \dots + 2 + 1 = n - 1$ , a total of  $n$  additional storage locations are required.

## Code for Merge Sort

Listing 8.5 shows the `MergeSort` class.

**LISTING 8.5**

MergeSort.java

```

.....
LISTING 8.5
MergeSort.java

/** Implements the recursive merge sort algorithm. In this version, copies
 * of the subtables are made, sorted, and then merged.
 */
public class MergeSort {
    /** Sort the array using the merge sort algorithm.
     * pre: table contains Comparable objects.
     * post: table is sorted.
     * @param table The array to be sorted
    */
    public static <T extends Comparable<T>> void sort(T[] table) {
        // A table with one element is sorted already.
        if (table.length > 1) {
            // Split table into halves.
            int halfSize = table.length / 2;
            T[] leftTable = (T[]) new Comparable[halfSize];
            T[] rightTable = (T[]) new Comparable[table.length - halfSize];
            System.arraycopy(table, 0, leftTable, 0, halfSize);
            System.arraycopy(table, halfSize, rightTable, 0,
                table.length - halfSize);

            // Sort the halves.
            sort(leftTable);
            sort(rightTable);

            // Merge the halves.
            merge(table, leftTable, rightTable);
        }
    }
    // See Listing 8.4 for the merge method.
    . . .
}

```

**EXERCISES FOR SECTION 8.6****SELF-CHECK**

1. Trace the execution of the merge sort on the following array, providing a figure similar to Figure 8.5.  
55 50 10 40 80 90 60 100 70 80 20 50 22
2. For the array in Question 1 above, show the value of `halfSize` and arrays `leftTable` and `rightTable` for each recursive call to method `sort` in Listing 8.4 and show the array elements after returning from each call to `merge`. How many times is `sort` called, and how many times is `merge` called?

**PROGRAMMING**

1. Add statements that trace the progress of method `sort` by displaying the array `table` after each merge operation. Also display the arrays referenced by `leftTable` and `rightTable`.



## 8.7 Timsort

---

Timsort was developed by Tim Peters in 2002 as the library sort algorithm for the Python language. Timsort is a modification of the merge sort algorithm that takes advantage of sorted subsets that may exist in the data being sorted. The Java 8 API replaced merge sort with Timsort as the sort algorithm to sort lists of objects.

Recall that merge sort recursively divides the array in half until there are two sequences of length one. It then merges the adjacent sequences. This is depicted in Figure 8.5. As the algorithm progresses, the run-time stack holds the left-hand sequences that are waiting to be merged with the right-hand sequences. Merge sort starts with two sequences of length 1 and merges them to form a sequence of length 2. It then takes the next two sequences of length 1 and merges them. It then merges the two sequences of length 2 to form a sequence of length 4. The process then repeats building two more sequences of length 2 to create a new sequence of length 4, which is then merged with the previous sequence of length 4 to form a sequence of length 8, and so on. If the data contains subsequences that are already sorted, they are still broken down into sequences of length 1 and then merged back together.

Timsort starts by looking for sequences that are already sorted in either ascending or descending order (called a *run*). A descending sequence is in-place converted to an ascending sequence. After a sequence is identified, it is placed onto a stack. In merge sort, the stack contains sequences of decreasing size such that the sequence at position  $(i - 2)$  is twice the length of the sequence at position  $(i - 1)$  and four times the length of the sequence at position  $(i)$ . Thus, the sequence at  $(i - 2)$  is longer than the sum of the lengths of the sequences at positions  $i$  and  $(i - 1)$ , and the sequence at position  $(i - 1)$  is longer than the sequence at position  $i$ . Timsort maintains this same invariant. If the new sequence is at position  $i$  on the stack, then if it is shorter than the one at  $i - 1$  and if the sum of the lengths of the new item and the one at  $i - 1$  is smaller than the one at  $i - 2$ , then we leave the new one on the stack and look for the next sequence. However, if this invariant is violated by the addition of the new sequence, the invariant is restored by merging the shorter of the sequence at  $i$  or  $i - 2$  with the sequence at  $i - 1$ . After the merge is completed, the invariant is again checked and the process repeats until the invariant is restored. Once all of the runs have been identified, the stack is collapsed by repeatedly merging the top two items on the stack.

For example, consider the following input data:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
1	3	5	7	9	8	6	4	12	14	15	16	17	11	10

The stack will contain the start position of the first item in a run and its length. The run from 0 to 4 is identified and is placed onto the stack at stack index 0.

Index	Start	Length
0	0	5

The next run (from 5 to 7) shown earlier is a descending sequence. It is in-place converted to an ascending sequence as shown next

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
1	3	5	7	9	4	6	8	12	14	15	16	17	11	10

and it is placed on the stack at stack index 1.

Index	Start	Length
0	0	5
1	5	3

Since the new sequence is shorter than the sequence below it in the stack, we look for the next sequence. This sequence is from 8 to 12. It is an ascending sequence and is placed onto the stack.

Index	Start	Length
0	0	5
1	5	3
2	8	5

This sequence is longer than the one below it. The sequence at 0 is shorter than the combined length of the sequences at 1 and 2. Since the length of the sequence at 0 on the stack is less than or equal to the length of the sequence at 2, we merge the sequences at 0 and 1.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
1	3	4	5	6	7	8	9	12	14	15	16	17	11	10

The stack now is

Index	Start	Length
0	0	8
1	8	5

and the invariant is restored.

Now the next sequence is identified from 13 to 14. It is a descending sequence, so it is in-place converted to an ascending sequence.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
1	3	4	5	6	7	8	9	12	14	15	16	17	10	11

The stack is now

Index	Start	Length
0	0	8
1	8	5
2	13	2

There are no more sequences, so we finish the merge process by merging the sequences on the stack, starting with the last two sequences placed on the stack, until there is only one sequence left.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
1	3	4	5	6	7	8	9	10	11	12	14	15	16	17

Index	Start	Length
0	0	8
1	8	7

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
1	3	4	5	6	7	8	9	10	11	12	14	15	16	17

Index	Start	Length
0	0	15

Observe that the last two sequences were already sorted (the numbers at array element [7] is smaller than the number at array element [8]), so there was no actual merging. The Timsort merge algorithm also takes advantage of this by finding the first item in the left sequence that is greater than the first item in the right sequence. It also finds the last item in the right sequence that is less than the last item in the right sequence. The resulting shortened sequences are then merged.

### Algorithm for Timsort

1. Set `lo` to zero.
2. **while** `lo` is less than the length of the array
  3. Find the length of the run starting at `lo`.
  4. If this is a decreasing run, reverse it.
  5. Add this run to the stack.
  6. Set `lo` to `lo` + length of the run.
  7. Collapse-merge the stack.
  8. Force-merge the stack.

### Algorithm to Find a Run

1. Set `hi` to `lo` + 1.
2. **if** `a[hi] < a[lo]`
  3. **while** `hi < a.length` and `a[hi] < a[lo]`
    4. increment `hi`
    - else**
    5. **while** `hi < a.length` and `a[hi] ≥ a[lo]`
      6. increment `hi`
  7. Return `hi - lo` as the length of the run.

### Algorithm to Reverse a Run

1. Set  $i$  to the beginning of the run.
2. Set  $j$  to the end of the run ( $i + \text{length}-1$ ).
3. **while**  $i < j$
4.     swap  $a[i]$  and  $a[j]$
5.     increment  $i$
6.     decrement  $j$

### Algorithm for Collapse-Merge

1. **while** the stack size is  $> 1$
2.     Let  $\text{top}$  be the index of the top of the stack.
3.     **if**  $\text{top} > 1$  and  $\text{stack}[\text{top}-2].\text{length} \leq \text{stack}[\text{top}-1].\text{length} + \text{stack}[\text{top}].\text{length}$
4.         **if**  $\text{stack}[\text{top}-2].\text{length} < \text{stack}[\text{top}].\text{length}$
5.             Merge the runs at  $\text{stack}[\text{top}-2]$  and  $\text{stack}[\text{top}-1]$ .
6.         **else**
7.             Merge the runs at  $\text{stack}[\text{top}-1]$  and  $\text{stack}[\text{top}]$ .
8.     **else if**  $\text{stack}[\text{top}-1].\text{length} \leq \text{stack}[\text{top}].\text{length}$
9.         Merge the runs at  $\text{stack}[\text{top}-1]$  and  $\text{stack}[\text{top}]$ .
10.     **else**
11.         Exit the **while** loop.

### Algorithm for Force-Merge

1. **while** the stack size is  $> 1$
2.     Let  $\text{top}$  be the index of the top of the stack.
3.     **if**  $\text{top} > 1$  **and**  $\text{stack}[\text{top}-2].\text{length} < \text{stack}[\text{top}].\text{length}$
4.         Merge the runs at  $\text{stack}[\text{top}-2]$  and  $\text{stack}[\text{top}-1]$ .
5.         **else**
6.         Merge the runs at  $\text{stack}[\text{top}-1]$  and  $\text{stack}[\text{top}]$ .

### Merging Adjacent Sequences

The actual merging of the sequences is the same as in merge sort. However, to avoid unnecessary copying of data to the temporary arrays, the start of the left sequence is adjusted to the first item that is greater than the first item of the right sequence. Likewise, the end of the right sequence is adjusted to be the first item that is less than the last item in the left sequence.

### Implementation

Listing 8.6 shows the implementation of Timsort. An internal class, `TS`, is used to hold the references to the array being sorted and the stack. An `ArrayList` is used for the stack because we need to access the top three items. The internal class `run` is used to represent the runs.

**LISTING 8.6**

Timsort.java

```

.....
LISTING 8.6
Timsort.java

/** A simplified version of the Timsort algorithm.
 * @author Koffman & Wolfgang
 */
public class TimSort implements SortAlgorithm {

    /** Sort the array using the Timsort algorithm
     * @pre table contains Comparable objects.
     * @post table is sorted.
     * @param table The array to be sorted
     */
    @Override
    public <T extends Comparable<T>> void sort(T[] table) {
        new TS(table).sort();
    }

    /** Private inner class to hold the working state of
     * the algorithm.
     */
    private static class TS<T extends Comparable<T>> {

        /** Private inner class to hold definitions
         * of the runs
         */
        private static class Run {
            int startIndex;
            int length;

            Run(int startIndex, int length) {
                this.startIndex = startIndex;
                this.length = length;
            }
        }
        // Array of runs that are pending merging.
        List<Run> runStack;
        // Reference to the array being sorted.
        T[] table;

        /** constructor
         * @param table array to be sorted
         */
        public TS(T[] table) {
            this.table = table;
            runStack = new ArrayList<>();
        }

        /** Sort the array using the Timsort algorithm.
         * @pre table contains Comparable objects.
         * @post table is sorted.
         */
        public void sort() {
            int nRemaining = table.length;
            if (nRemaining < 2) {
                // Single item array is already sorted.
                return;
            }
        }
    }
}

```

```

        int lo = 0;
        do {
            int runLength = nextRun(lo);
            runStack.add(new Run(lo, runLength));
            mergeCollapse();
            lo += runLength;
            nRemaining -= runLength;
        } while (nRemaining != 0);
        mergeForce();
    }

    /**
     * Method to find the length of the next run. A
     * run is a sequence of ascending items such that
     * a[i] <= a[i+1] or descending items such
     * that a[i] > a[i+1].
     * If a descending sequence is found, it is turned
     * into an ascending sequence.
     * @param table The table being sorted
     * @param lo The index where the sequence starts
     * @return the length of the sequence.
     */
    private int nextRun(int lo) {
        if (lo == table.length - 1) {
            return 1;
        }
        int hi = lo + 1;
        if (table[hi - 1].compareTo(table[hi]) <= 0) {
            while (hi < table.length &&
                   table[hi - 1].compareTo(table[hi]) <= 0) {
                hi++;
            }
        } else {
            while (hi < table.length &&
                   table[hi - 1].compareTo(table[hi]) > 0) {
                hi++;
            }
            swapRun(lo, hi - 1);
        }
        return hi - lo;
    }

    /**
     * Method to convert a descending sequence into
     * an ascending sequence.
     * @param table The table being sorted
     * @param lo The start index
     * @param hi The end index
     */
    private void swapRun(int lo, int hi) {
        while (lo < hi) {
            swap(lo++, hi--);
        }
    }

    /**
     * Swap the items in table[i] and table[j].
     * @param table The array that contains the items
     * @param i The index of one item
     * @param j The index of the other item
     */
    private void swap(int i, int j) {

```

```

        T temp = table[i];
        table[i] = table[j];
        table[j] = temp;
    }

/** Merge adjacent runs until the following
 * invariant is established.
 * 1. runLength[top - 2] > runLength[top - 1] + runLength[top]
 * 2. runLength[top - 1] > runLength[top]
 * This method is called each time a new run is
 * added to the stack.
 * Invariant is true before a new run is added to
 * the stack.
 */
private void mergeCollapse() {
    while (runStack.size() > 1) {
        int top = runStack.size() - 1;
        if (top > 1 && runStack.get(top - 2).length <=
            runStack.get(top - 1).length + runStack.get(top).length) {
            if (runStack.get(top - 2).length <
                runStack.get(top).length) {
                mergeAt(top - 2);
            } else {
                mergeAt(top - 1);
            }
        } else if (runStack.get(top - 1).length <=
                   runStack.get(top).length) {
            mergeAt(top - 1);
        } else {
            break;
        }
    }
}

/** Merge all remaining runs. This method is called
 * to complete the sort.
 */
private void mergeForce() {
    while (runStack.size() > 1) {
        int top = runStack.size() - 1;
        if (top > 1 && runStack.get(top - 2).length <
            runStack.get(top).length) {
            mergeAt(top - 2);
        } else {
            mergeAt(top - 1);
        }
    }
}

/* Merge the two adjacent runs at i and i+1. i must
 * be equal to runStack.size() - 2 or runStack.size() - 3.
 */
private void mergeAt(int i) {
    int base1 = runStack.get(i).startIndex;
    int len1 = runStack.get(i).length;
    int base2 = runStack.get(i + 1).startIndex;
    int len2 = runStack.get(i + 1).length;
    runStack.set(i, new Run(base1, len1 + len2));
}

```

```

        if (i == runStack.size() - 3) {
            runStack.set(i + 1, runStack.get(i + 2));
        }
        runStack.remove(runStack.size() - 1);
        int newBase1 = reduceLeft(base1, base2);
        len1 = len1 - (newBase1 - base1);
        if (len1 > 0) {
            len2 = reduceRight(newBase1, len1, base2, len2);
            if (len2 > 0) {
                T[] run1 = (T[]) (new Comparable[len1]);
                T[] run2 = (T[]) (new Comparable[len2]);
                System.arraycopy(table, newBase1, run1, 0, len1);
                System.arraycopy(table, base2, run2, 0, len2);
                merge(newBase1, run1, run2);
            }
        }
    }

    /**
     * Adjust the start of run1 so that its first
     * element is greater than or equal the first element of run2
     * @param base1 The index of the start of run1
     * @param base2 The index of the start of run2
     * @return the new start of run 1
     */
    int reduceLeft(int base1, int base2) {
        T base2Start = table[base2];
        while (table[base1].compareTo(base2Start) < 0) {
            base1++;
        }
        return base1;
    }

    /**
     * Adjust the end of run2 so that its last element
     * is less than or equal to the last element of run1
     * @param base1 The start of run 1
     * @param len1 The length of run 1
     * @param base2 The start of run 2
     * @param len2 The length of run 2
     * @return the new length of run 2
     */
    int reduceRight(int base1, int len1, int base2, int len2) {
        T run1End = table[base1 + len1 - 1];
        while (table[base2 + len2 - 1].compareTo(run1End) > 0) {
            len2--;
        }
        return len2;
    }

    /**
     * Merge two runs into the table
     * @param destIndex Index where the merged run is to be inserted
     * @param run1 Array containing one run
     * @param run2 Array containing the other run
     */
    private void merge(int destIndex, T[] run1, T[] run2) {
        int i = 0;
        int j = 0;
        while (i < run1.length && j < run2.length) {
            if (run1[i].compareTo(run2[j]) < 0) {

```

```
        table[destIndex++] = run1[i++];
    } else {
        table[destIndex++] = run2[j++];
    }
}
while (i < run1.length) {
    table[destIndex++] = run1[i++];
}
while (j < run2.length) {
    table[destIndex++] = run2[j++];
}
}
```

## 8.8 Heapsort

The merge sort algorithm has the virtue that its time is  $O(n \log n)$ , but it still requires, at least temporarily,  $n$  extra storage locations. This next algorithm can be implemented without requiring any additional storage. It uses a heap to store the array and so is called *heapsort*.

# First Version of a Heapsort Algorithm

We introduced the heap in Section 6.6. When used as a priority queue, a heap is a data structure that maintains the smallest value at the top. The following algorithm first places an array's data into a heap. Then it removes each heap item (an  $O(\log n)$  process) and moves it back into the array.

## Heapsort Algorithm: First Version

1. Insert each value from the array to be sorted into a priority queue (heap).
  2. Set  $i$  to 0.
  3. **while** the priority queue is not empty
  4.     Remove an item from the queue and insert it back into the array at position  $i$ .
  5.     Increment  $i$ .

Although this algorithm can be shown to be  $O(n \log n)$ , it does require  $n$  extra storage locations (the array and heap are both size  $n$ ). We address this problem next.

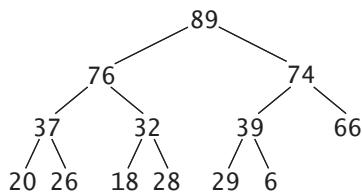
## Revising the Heapsort Algorithm

In the heaps we have used so far, each parent node value was less than the values of its children. We can also build the heap so that each parent is larger than its children. Figure 8.6 shows an example of such a heap.

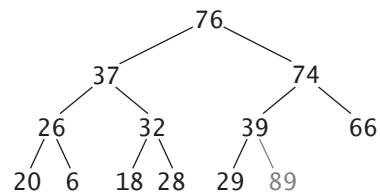
Once we have such a heap, we can remove one item at a time from the heap. The item removed is always the top element, and it will end up at the bottom of the heap. When we reheap, we move the larger of a node's two children up the heap, instead of the smaller, so the next largest item is then at the top of the heap. Figure 8.7 shows the heap after we have

**FIGURE 8.6**

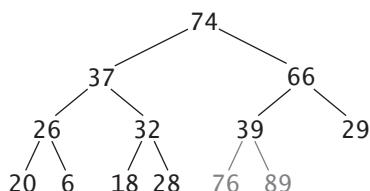
Example of a Heap with Largest Value in Root

**FIGURE 8.7**

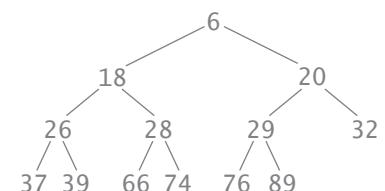
Heap after Removal of Largest Item

**FIGURE 8.8**

Heap after Removal of Two Largest Items

**FIGURE 8.9**

Heap after Removal of All Its Items



removed one item, and Figure 8.8 shows the heap after we have removed two items. In both figures, the items in bold have been removed from the heap. As we continue to remove items from the heap, the heap size shrinks as the number of the removed items increases. Figure 8.9 shows the heap after we have emptied it.

If we implement the heap using an array, each element removed will be placed at the end of the array but in front of the elements that were removed earlier. After we remove the last element, the array will be sorted. We illustrate this next.

Figure 8.10 shows the array representation of the original heap. As before, the root, 89, is at position 0. The root's two children, 76 and 74, are at positions 1 and 2. For a node at position  $p$ , the left child is at  $2p + 1$  and the right child is at  $2p + 2$ . A node at position  $c$  can find its parent at  $(c - 1) / 2$ .

Figure 8.11 shows the array representation of the heaps in Figures 8.7 through 8.9. The items in gray have been removed from the heap and are sorted. Each time an item is removed, the heap part of the array decreases by one element and the sorted part of the array increases by one element. In the array at the bottom of Figure 8.11, all items have been removed from the heap and the array is sorted.

From our foregoing observations, we can sort the array that represents a heap in the following way.

### Algorithm for In-Place Heapsort

1. Build a heap by rearranging the elements in an unsorted array.
2. **while** the heap is not empty
3. Remove the first item from the heap by swapping it with the last item in the heap and restoring the heap property.

**FIGURE 8.10**

Internal Representation  
of the Heap Shown in  
Figure 8.6

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
89	76	74	37	32	39	66	20	26	18	28	29	6

**FIGURE 8.11**

Internal Representation  
of the Heaps Shown in  
Figures 8.7 through 8.9

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
76	37	74	26	32	39	66	20	6	18	28	29	89
:												
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
74	37	66	26	32	39	29	20	6	18	28	76	89
:												
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
6	18	20	26	28	29	32	37	39	66	74	76	89

Each time through the loop (Steps 2 and 3), the largest item remaining in the heap is placed at the end of the heap, just before the previously removed items. Thus, when the loop terminates, the items in the array are sorted. In Section 6.6, we discussed how to remove an item from a heap and restore the heap property. We also implemented a `remove` method for a heap in an `ArrayList`.

## Algorithm to Build a Heap

Step 1 of the algorithm builds a heap. If we start with an array, `table`, of length `table.length`, we can consider the first item (index 0) to be a heap of one item. We now consider the general case where the items in array `table` from 0 through  $n - 1$  form a heap; the items from  $n$  through `table.length - 1` are not in the heap. As each is inserted, we must “reheap” to restore the heap property.

### Refinement of Step 1 for In-Place Heapsort

- 1.1   **while**  $n$  is less than `table.length`
- 1.2       Increment  $n$  by 1. This inserts a new item into the heap.
- 1.3       Restore the heap property.

## Analysis of Revised Heapsort Algorithm

From our knowledge of binary trees, we know that a heap of size  $n$  has  $\log n$  levels. Building a heap requires finding the correct location for an item in a heap with  $\log n$  levels. Because we have  $n$  items to insert and each insert (or remove) is  $O(\log n)$ , the `buildHeap` operation is  $O(n \log n)$ . Similarly, we have  $n$  items to remove from the heap, so that is also  $O(n \log n)$ . Because we are storing the heap in the original array, no extra storage is required.

## Code for Heapsort

Listing 8.7 shows the `HeapSort` class. The `sort` method merely calls the `buildHeap` method followed by the `shrinkHeap` method, which is based on the `remove` method shown in Section 6.6. Method `swap` swaps the items in the table.

**LISTING 8.7**

HeapSort.java

```

.....
LISTING 8.7
HeapSort.java

/** Implementation of the heapsort algorithm. */
public class HeapSort {
    /** Sort the array using heapsort algorithm.
     * @pre table contains Comparable items.
     * @post table is sorted.
     * @param table The array to be sorted
    */
    public static <T extends Comparable<T>> void sort(T[] table) {
        buildHeap(table);
        shrinkHeap(table);
    }

    /** buildHeap transforms the table into a heap.
     * @pre The array contains at least one item.
     * @post All items in the array are in heap order.
     * @param table The array to be transformed into a heap
    */
    private static <T extends Comparable<T>> void buildHeap(T[] table) {
        int n = 1;
        // Invariant: table[0 . . . n - 1] is a heap.
        while (n < table.length) {
            n++; // Add a new item to the heap and reheap.
            int child = n - 1;
            int parent = (child - 1) / 2; // Find parent.
            while (parent >= 0
                    && table[parent].compareTo(table[child]) < 0) {
                swap(table, parent, child);
                child = parent;
                parent = (child - 1) / 2;
            }
        }
    }

    /** shrinkHeap transforms a heap into a sorted array.
     * @pre All items in the array are in heap order.
     * @post The array is sorted.
     * @param table The array to be sorted
    */
    private static <T extends Comparable<T>> void shrinkHeap(T[] table) {
        int n = table.length;
        // Invariant: table[0 . . . n - 1] forms a heap.
        // table[n . . . table.length - 1] is sorted.
        while (n > 0) {
            n--;
            swap(table, 0, n);
            // table[1 . . . n - 1] form a heap.
            // table[n . . . table.length - 1] is sorted.
            int parent = 0;
            while (true) {
                int leftChild = 2 * parent + 1;
                if (leftChild >= n) {
                    break; // No more children.
                }
                int rightChild = leftChild + 1;
                // Find the larger of the two children.
                int maxChild = leftChild;
                if (rightChild < n // There is a right child.
                    && table[leftChild].compareTo(table[rightChild]) < 0) {

```

```

        maxChild = rightChild;
    }
    // If the parent is smaller than the larger child,
    if (table[parent].compareTo(table[maxChild]) < 0) {
        // Swap the parent and child.
        swap(table, parent, maxChild);
        // Continue at the child level.
        parent = maxChild;
    } else { // Heap property is restored.
        break; // Exit the loop.
    }
}
}

/** Swap the items in table[i] and table[j].
 * @param table The array that contains the items
 * @param i The index of one item
 * @param j The index of the other item
 */
private static <T extends Comparable<T>> void swap(T[] table,
                                                int i, int j) {
    T temp = table[i];
    table[i] = table[j];
    table[j] = temp;
}
}

```

## EXERCISES FOR SECTION 8.8

### SELF-CHECK

1. Build the heap from the numbers in the following list. How many exchanges were required? How many comparisons?  
55 50 10 40 80 90 60 100 70 80 20 50 22
2. Shrink the heap from Question 1 to create the array in sorted order. How many exchanges were required? How many comparisons?



## 8.9 Quicksort

The next algorithm we will study is called *quicksort*. Developed by C. A. R. Hoare in 1962, it works in the following way: given an array with subscripts `first . . . last` to sort, quicksort rearranges this array into two parts so that all the elements in the left subarray are less than or equal to a specified value (called the *pivot*) and all the elements in the right subarray are greater than the pivot. The pivot is placed between the two parts. Thus, all of the elements on the left of the pivot value are smaller than all elements on the right of the pivot value, so the pivot value is in its correct position. By repeating this process on the two halves, the whole array becomes sorted.

As an example of this process, let's sort the following array:

44	75	23	43	55	12	64	77	33
----	----	----	----	----	----	----	----	----

We will assume that the first array element (44) is arbitrarily selected as the pivot value. A possible result of rearranging, or *partitioning*, the element values follows:

12	33	23	43	44	55	64	77	75
----	----	----	----	----	----	----	----	----

After the partitioning process, the pivot value, 44, is at its correct position. All values less than 44 are in the left subarray, and all values larger than 44 are in the right subarray, as desired. The next step would be to apply quicksort recursively to the two subarrays on either side of the pivot value, beginning with the left subarray (12, 33, 23, 43). Here is the result when 12 is the pivot value:

12	33	23	43
----	----	----	----

The pivot value is in the first position. Because the left subarray does not exist, the right subarray (33, 23, 43) is sorted next, resulting in the following situation:

12	23	33	43
----	----	----	----

The pivot value 33 is in its correct place, and the left subarray (23) and right subarray (43) have single elements, so they are sorted. At this point, we are finished sorting the left part of the original subarray, and quicksort is applied to the right subarray (55, 64, 77, 75). In the following array, all the elements that have been placed in their proper position are shaded dark.

12	23	33	43	44	55	64	77	75
----	----	----	----	----	----	----	----	----

If we use 55 for the pivot, its left subarray will be empty after the partitioning process and the right subarray 64, 77, 75 will be sorted next. If 64 is the pivot, the situation will be as follows, and we sort the right subarray (77, 75) next.

55	64	77	75
----	----	----	----

If 77 is the pivot and we move it where it belongs, we end up with the following array. Because the left subarray (75) has a single element, it is sorted and we are done.

75	77
----	----

## Algorithm for Quicksort

The algorithm for quicksort follows. We will describe how to do the partitioning later. We assume that the indexes `first` and `last` are the endpoints of the array being sorted and that the index of the pivot after partitioning is `pivIndex`.

### Algorithm for Quicksort

1. **if** `first < last` **then**
2.     Partition the elements in the subarray `first . . . last` so that the pivot value is in its correct place (subscript `pivIndex`).
3.     Recursively apply quicksort to the subarray `first . . . pivIndex - 1`.
4.     Recursively apply quicksort to the subarray `pivIndex + 1 . . . last`.

## Analysis of Quicksort

If the pivot value is a random value selected from the current subarray, then statistically it is expected that half of the items in the subarray will be less than the pivot and half will be greater than the pivot. If both subarrays always have the same number of elements (the best case), there will be  $\log n$  levels of recursion. At each level, the partitioning process involves moving every element into its correct partition, so quicksort is  $O(n \log n)$ , just like merge sort.

But what if the split is not 50–50? Let us consider the case where each split is 90–10. Instead of a 100-element array being split into two 50-element arrays, there will be one array with 90 elements and one with just 10. The 90-element array may be split 50–50, or it may also be split 90–10. In the latter case, there would be one array with 81 elements and one with just 9 elements. Generally, for random input, the splits will not be exactly 50–50, but neither will they all be 90–10. An exact analysis is difficult and beyond the scope of this book, but the running time will be bound by a constant  $\times n \log n$ .

There is one situation, however, where quicksort gives very poor behavior. If, each time we partition the array, we end up with a subarray that is empty, the other subarray will have one less element than the one just split (only the pivot value will be removed). Therefore, we will have  $n$  levels of recursive calls (instead of  $\log n$ ), and the algorithm will be  $O(n^2)$ . Because of the overhead of recursive method calls (versus iteration), quicksort will take longer and require more extra storage on the run-time stack than any of the earlier quadratic algorithms. We will discuss a way to handle this situation later.

## Code for Quicksort

Listing 8.8 shows the QuickSort class. The public method `sort` calls the recursive `quickSort` method, giving it the bounds of the table as the initial values of `first` and `last`. The two recursive calls in `quickSort` will cause the procedure to be applied to the subarrays that are separated by the value at `pivIndex`. If any subarray contains just one element (or zero elements), an immediate return will occur.

.....  
**LISTING 8.8**  
 QuickSort.java

```
/** Implements the quicksort algorithm. */
public class QuickSort {

    /** Sort the table using the quicksort algorithm.
     * @pre table contains Comparable objects.
     * @post table is sorted.
     * @param table The array to be sorted
     */
    public static <T extends Comparable<T>> void sort(T[] table) {
        // Sort the whole table.
        quickSort(table, 0, table.length - 1);
    }

    /** Sort a part of the table using the quicksort algorithm.
     * @post The part of table from first through last is sorted.
     * @param table The array to be sorted
     * @param first The index of the low bound
     * @param last The index of the high bound
     */
}
```

```

private static <T extends Comparable<T>> void quickSort(T[] table,
    int first, int last) {
    if (first < last) { // There is data to be sorted.
        // Partition the table.
        int pivIndex = partition(table, first, last);
        // Sort the left half.
        quickSort(table, first, pivIndex - 1);
        // Sort the right half.
        quickSort(table, pivIndex + 1, last);
    }
}
// Insert partition method. See Listing 8.9
. . .
}

```

## Algorithm for Partitioning

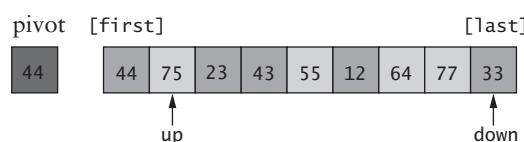
The partition method selects the pivot and performs the partitioning operation. When we are selecting the pivot, it does not really matter which element is the pivot value (if the arrays are randomly ordered to begin with). For simplicity we chose the element with subscript `first`. We then begin searching for the first value at the left end of the subarray that is greater than the pivot value. When we find it, we search for the first value at the right end of the subarray that is less than or equal to the pivot value. These two values are exchanged, and we repeat the search and exchange operations. This is illustrated in Figure 8.12, where `up` points to the first value greater than the pivot and `down` points to the first value less than or equal to the pivot value. The elements less than the pivot are shaded dark, and the elements greater than the pivot are in gray.

The value 75 is the first value at the left end of the array that is larger than 44, and 33 is the first value at the right end that is less than or equal to 44, so these two values are exchanged. The indexes `up` and `down` are advanced again, as shown in Figure 8.13.

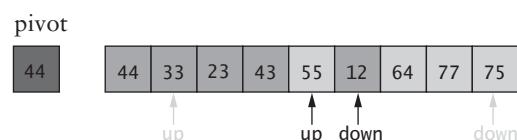
The value 55 is the next value at the left end that is larger than 44, and 12 is the next value at the right end that is less than or equal to 44, so these two values are exchanged, and `up` and `down` are advanced again, as shown in Figure 8.14.

After the second exchange, the first five array elements contain the pivot value and all values less than or equal to the pivot; the last four elements contain all values larger than the pivot. The value 55 is selected once again by `up` as the next element larger than the pivot; 12 is selected by `down` as the next element less than or equal to the pivot. Since `up` has now “passed” `down`, these values are not exchanged. Instead, the pivot value (subscript `first`) and the value at position `down` are exchanged. This puts the pivot value in its proper position (the new subscript is `down`) as shown in Figure 8.15.

**FIGURE 8.12**  
Locating First Values  
to Exchange

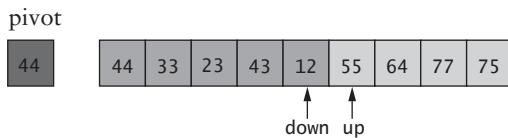


**FIGURE 8.13**  
Array after the  
First Exchange

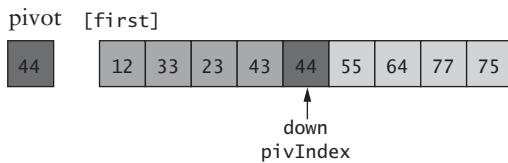


**FIGURE 8.14**

Array after the Second Exchange

**FIGURE 8.15**

Array after the Pivot Is Inserted



The partition process is now complete, and the value of `down` is returned to the pivot index `pivIndex`. Method `quickSort` will be called recursively to sort the left subarray and the right subarray. The algorithm for `partition` follows:

### Algorithm for `partition` Method

1. Define the pivot value as the contents of `table[first]`.
2. Initialize `up` to `first` and `down` to `last`.
3. **do**
4.     Increment `up` until `up` selects the first element greater than the pivot value or `up` has reached `last`.
5.     Decrement `down` until `down` selects the first element less than or equal to the pivot value or `down` has reached `first`.
6.     **if** `up < down` then
7.         Exchange `table[up]` and `table[down]`.
8.     **while** `up` is to the left of `down`
9.         Exchange `table[first]` and `table[down]`.
10.      Return the value of `down` to `pivIndex`.

### Code for `partition`

The code for `partition` is shown in Listing 8.9. The **while** statement:

```
while ((up < last) && (pivot.compareTo(table[up]) >= 0)) {
    up++;
}
```

advances the index `up` until it is equal to `last` or until it references an item in `table` that is greater than the pivot value. Similarly, the **while** statement:

```
while (pivot.compareTo(table[down]) < 0) {
    down--;
}
```

moves the index `down` until it references an item in `table` that is less than or equal to the pivot value. The **do-while** condition

```
(up < down)
```

ensures that the partitioning process will continue while `up` is to the left of `down`.

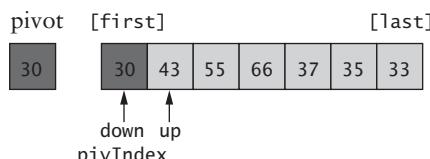
What happens if there is a value in the array that is the same as the pivot value? The index `down` will stop at such a value. If `up` has stopped prior to reaching that value, `table[up]` and `table[down]` will be exchanged, and the value equal to the pivot will be in the left partition. If `up` has passed this value and therefore passed `down`, `table[first]` will be exchanged with `table[down]` (same value as `table[first]`), and the value equal to the pivot will still be in the left partition.

What happens if the pivot value is the smallest value in the array? Since the pivot value is at `table[first]`, the loop will terminate with `down` equal to `first`. In this case, the left partition is empty. Figure 8.16 shows an array for which this is the case.

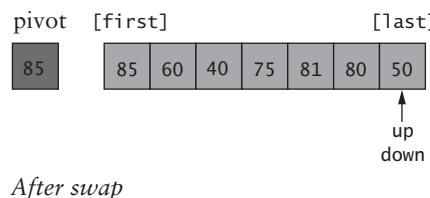
By similar reasoning, we can show that `up` will stop at `last` if there is no element in the array larger than the pivot. In this case, `down` will also stay at `last`, and the pivot value (`table[first]`) will be swapped with the last value in the array, so the right partition will be empty. Figure 8.17 shows an array for which this is the case.

**FIGURE 8.16**

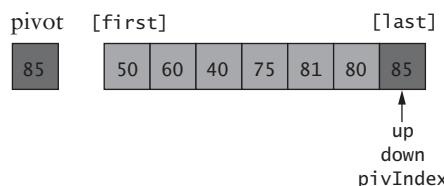
Values of `up`, `down`, and `pivIndex` If the Pivot Is the Smallest Value

**FIGURE 8.17**

Values of `up`, `down`, and `pivIndex` If the Pivot Is the Largest Value



*After swap*

**LISTING 8.9**

Quicksort `partition` Method (First Version)

```
/** Partition the table so that values from first to pivIndex
 *  are less than or equal to the pivot value, and values from
 *  pivIndex to last are greater than the pivot value.
 * @param table The table to be partitioned
 * @param first The index of the low bound
 * @param last The index of the high bound
 * @return The location of the pivot value
 */
private static <T extends Comparable<T>> int partition(T[] table,
                                                       int first, int last) {
    // Select the first item as the pivot value.
    T pivot = table[first];
    int up = first;
    int down = last;
    do {
        /* Invariant:
         * All items in table[first . . . up - 1] <= pivot
         * All items in table[down + 1 . . . last] > pivot
         */
        while ((up < last) && (pivot.compareTo(table[up]) >= 0)) {
            up++;
        }
        // assert: up equals last or table[up] > pivot.
        while (pivot.compareTo(table[down]) < 0) {
            down--;
        }
    }
}
```

```

    }
    // assert: down equals first or table[down] <= pivot.
    if (up < down) {   // if up is to the left of down.
        // Exchange table[up] and table[down].
        swap(table, up, down);
    }
} while (up < down); // Repeat while up is left of down.
// Exchange table[first] and table[down] thus putting the
// pivot value where it belongs.
swap(table, first, down);
// Return the index of the pivot value.
return down;
}

```

## A Revised partition Algorithm

We stated earlier that quicksort is  $O(n^2)$  when each split yields one empty subarray. Unfortunately, that would be the case if the array was sorted. So the worst possible performance occurs for a sorted array, which is not very desirable.

A better solution is to pick the pivot value in a way that is less likely to lead to a bad split. One approach is to examine the first, middle, and last elements in the array and select the median of these three values as the pivot. We can do this by sorting the three-element subarray (shaded dark in Figure 8.18). After sorting, the smallest of the three values is in position `first`, the median is in position `middle`, and the largest is in position `last`.

At this point, we can exchange the first element with the middle element (the median) and use the partition algorithm shown earlier, which uses the first element (now the median) as the pivot value. When we exit the partitioning loop, `table[first]` and `table[down]` are exchanged, moving the pivot value where it belongs (back to the middle position). This revised partition algorithm follows.

.....  
**FIGURE 8.18**  
 Sorting First, Middle,  
 and Last Elements in  
 Array

[first] [middle] [last]

44	75	23	43	55	12	64	77	33
----	----	----	----	----	----	----	----	----

After sorting, median is in table[middle]

[first] [middle] [last]

33	75	23	43	44	12	64	77	55
----	----	----	----	----	----	----	----	----

## Algorithm for Revised partition Method

1. Sort `table[first]`, `table[middle]`, and `table[last]`.
2. Move the median value to `table[first]` (the pivot value) by exchanging `table[first]` and `table[middle]`.
3. Initialize `up` to `first` and `down` to `last`.
4. **do**
5.     Increment `up` until `up` selects the first element greater than the pivot value or `up` has reached `last`.
6.     Decrement `down` until `down` selects the first element less than or equal to the pivot value or `down` has reached `first`.

7. if up < down then
8.     Exchange table[up] and table[down].
9. while up is to the left of down.
10. Exchange table[first] and table[down].
11. Return the value of down to pivIndex.

You may be wondering whether you can avoid the double shift (Steps 2 and 10) and just leave the pivot value at table[middle], where it belongs. The answer is “yes,” but you would also need to modify the partition algorithm further if you did this. Programming Project 6 addresses this issue and the construction of an industrial-strength quicksort method.

## Code for Revised partition Method

Listing 8.10 shows the revised version of method partition with method sort3, which uses three pairwise comparisons to sort the three selected items in table so that

```
table[first] <= table[middle] <= table[last]
```

Method partition begins with a call to method sort3 and then calls swap to make the median the pivot. The rest of the method is unchanged.

.....

### LISTING 8.10

Revised partition Method and sort3

```
private static <T extends Comparable<T>> int partition(T[] table,
                                                       int first, int last) {
    /* Put the median of table[first], table[middle], table[last]
       into table[first], and use this value as the pivot.
    */
    sort3(table, first, last);
    // Swap first element with median.
    swap(table, first, (first + last) / 2);

    // Continue as in Listing 8.9
    // . . .

}

/** Sort table[first], table[middle], and table[last].
 * @param table The table to be sorted
 * @param first Index of the first element
 * @param last Index of the last element
 */
private static <T extends Comparable<T>> sort3(T[] table,
                                                int first, int last) {
    int middle = (first + last) / 2;
    /* Sort table[first], table[middle],
       table[last]. */
    if (table[middle].compareTo(table[first]) < 0) {
        swap(table, first, middle);
    }
    // assert: table[first] <= table[middle]
    if (table[last].compareTo(table[middle]) < 0) {
        swap(table, middle, last);
    }
    // assert: table[last] is the largest value of the three.
    if (table[middle].compareTo(table[first]) < 0) {
        swap(table, first, middle);
    }
    // assert: table[first] <= table[middle] <= table[last].
}
```



## PITFALL

### Falling Off Either End of the Array

A common problem when incrementing up or down during the partition process is falling off either end of the array. This will be indicated by an `ArrayIndexOutOfBoundsException`. We used the condition

```
((up < last) && (pivot.compareTo(table[up]) >= 0))
```

to keep up from falling off the right end of the array. Self-Check Exercise 3 asks why we don't need to write similar code to avoid falling off the left end of the array.

## EXERCISES FOR SECTION 8.9

### SELF-CHECK

1. Trace the execution of quicksort on the following array, assuming that the first item in each subarray is the pivot value. Show the values of `first` and `last` for each recursive call and the array elements after returning from each call. Also, show the value of `pivot` during each call and the value returned through `pivIndex`. How many times is `sort` called, and how many times is `partition` called?  
55 50 10 40 80 90 60 100 70 80 20 50 22
2. Redo Question 1 above using the revised `partition` algorithm, which does a preliminary sort of three elements and selects their median as the pivot value.
3. Explain why the condition (`down > first`) is not necessary in the loop that decrements `down`.

### PROGRAMMING

1. Insert statements to trace the quicksort algorithm. After each call to `partition`, display the values of `first`, `pivIndex`, and `last` and the array.



## 8.10 Testing the Sort Algorithms

To test the sorting algorithms, we need to exercise them with a variety of test cases. We want to make sure that they work and also want to get some idea of their relative performance when sorting the same array. We should test the methods with small arrays, large arrays, arrays whose elements are in random order, arrays that are already sorted, and arrays with duplicate copies of the same value. For performance comparisons to be meaningful, the methods must sort the same arrays.

Listing 8.11 shows a driver program that tests methods `Arrays.Sort` (from the API `java.util`) and `QuickSort.sort` on the same array of random integer values. Method `System.currentTimeMillis` returns the current time in milliseconds. This method is called just before a sort begins and just after the return from a sort. The elapsed time between calls

**LISTING 8.11**

Driver to Test Sort Algorithms

```
.....
Driver program to test sorting methods.
@param args Not used
*/
public static void main(String[] args) {
    int size = Integer.parseInt(JOptionPane.showInputDialog("Enter Array size:"));
    Integer[] items = new Integer[size]; // Array to sort.
    Integer[] copy = new Integer[size]; // Copy of array.
    Random rInt = new Random(); // For random number generation

    // Fill the array and copy with random Integers.
    for (int i = 0; i < items.length; i++) {
        items[i] = rInt.nextInt();
        copy[i] = items[i];
    }

    // Sort with utility method.
    long startTime = System.currentTimeMillis();
    Arrays.sort(items);
    System.out.println("Utility sort time is "
        + (System.currentTimeMillis()
        - startTime) + "ms");
    System.out.println("Utility sort successful (true/false): "
        + verify(items));

    // Reload array items from array copy.
    for (int i = 0; i < items.length; i++) {
        items[i] = copy[i];
    }

    // Sort with quicksort.
    startTime = System.currentTimeMillis();
    QuickSort.sort(items);
    System.out.println("QuickSort time is "
        + (System.currentTimeMillis()
        - startTime) + "ms");
    System.out.println("QuickSort successful (true/false): "
        + verify(items));

    dumpTable(items); // Display part of the array.
}

/** Verifies that the elements in array test are
 * in increasing order.
 * @param test The array to verify
 * @return true if the elements are in increasing order;
 *         false if any 2 elements are not in increasing order
 */
private static boolean verify(Comparable[] test) {
    boolean ok = true;
    int i = 0;
    while (ok && i < test.length - 1) {
        ok = test[i].compareTo(test[i + 1]) <= 0;
        i++;
    }
    return ok;
}

```

is displayed in the console window. Although the numbers shown will not be precise, they give a good indication of the relative performance of two sorting algorithms if this is the only application currently executing.

Method `verify` verifies that the array elements are sorted by checking that each element in the array is not greater than its successor. Method `dumpTable` (not shown) should display the first 10 elements and last 10 elements of an array (or the entire array if the array has 20 or fewer elements).

## EXERCISES FOR SECTION 8.10

### SELF-CHECK

1. Explain why method `verify` will always determine whether an array is sorted. Does `verify` work if an array contains duplicate values?
2. Explain the effect of removing the second `for` statement in the `main` method.

### PROGRAMMING

1. Write method `dumpTable`.
2. Modify the driver method to fill array `items` with a collection of integers read from a file when `args[0]` is not `null`.
3. Extend the driver to test all  $O(n \log n)$  sorts and collect statistics on the different sorting algorithms. Test the sorts using an array of random numbers and also a data file processed by the solution to Programming Exercise 2.



## 8.11 The Dutch National Flag Problem (Optional Topic)

A variety of partitioning algorithms for quicksort have been published. Most are variations on the one presented in this text. There is another popular variation that uses a single left-to-right scan of the array (instead of scanning left and scanning right as we did). The following case study illustrates a partitioning algorithm that combines both scanning techniques to partition an array into three segments. The famous computer scientist Edsger W. Dijkstra described this problem in his book *A Discipline of Programming* (Prentice-Hall, 1976).

### CASE STUDY The Problem of the Dutch National Flag

**Problem** The Dutch national flag consists of three stripes that are colored (from top to bottom) red, white, and blue. In Figure 8.19 we use dark gray for blue and light gray for red. Unfortunately, when the flag arrived, it looked like Figure 8.20; threads of each of the colors were all scrambled together! Fortunately, we have a machine that can unscramble it, but it needs software.

**Analysis** Our unscrambling machine has the following abilities:

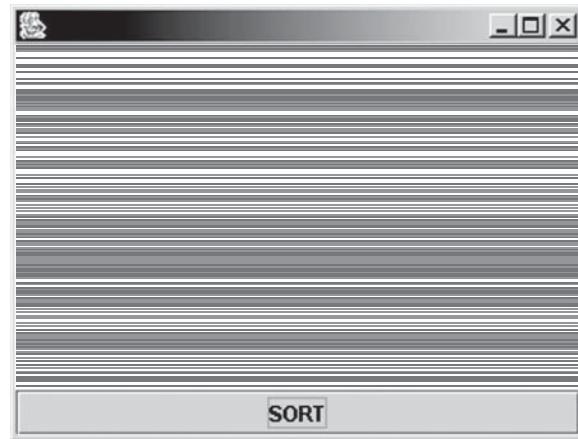
- It can look at one thread in the flag and determine its color.
- It can swap the position of two threads in the flag.

Our machine can also execute **while** loops and **if** statements.

**FIGURE 8.19**  
The Dutch National Flag

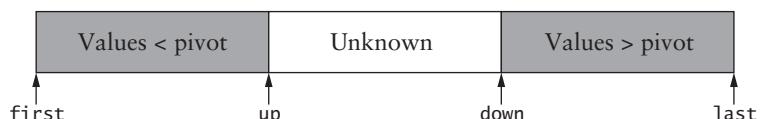


**FIGURE 8.20**  
Scrambled Dutch National Flag



### Design Loop Invariant

When we partitioned the array in quicksort, we split the array into three regions. Values between `first` and `up` were less than or equal to the pivot; values between `down` and `last` were greater than the pivot; and values between `up` and `down` were unknown. We started with the unknown region containing the whole array (`first == up`, and `down == last`). The partitioning algorithm preserves this invariant while shrinking the unknown region. The loop terminates when the unknown region becomes empty (`up > down`).



Since our goal is to have three regions when we are done, let us define four regions: the red region, the white region, the blue region, and the unknown region. Now, initially the whole flag is unknown. When we get done, however, we would like the red region on top, the white region in the middle, and the blue region on the bottom. The unknown region must be empty.

Let us assume that the threads are stored in an array `threads` and that the total number of threads is `HEIGHT`. Let us define `red` to be the upper bound of the red region, `white` to be the lower bound of the white region, and `blue` to be the lower bound of the blue region. Then, if our flag is complete, we can say the following:

- If  $0 \leq i < \text{red}$ , then `threads[i]` is red.
- If  $\text{white} < i \leq \text{blue}$ , then `threads[i]` is white.
- If  $\text{blue} < i < \text{HEIGHT}$ , then `threads[i]` is blue.

What about the case where  $\text{red} \leq i \leq \text{white}$ ? When the flag is all sorted, `red` should equal `white`, so this region should not exist. However, when we start, everything is in this region, so a thread in that region can have any color.

Thus, we can define the following loop invariant:

- If  $0 \leq i < \text{red}$ , then `threads[i]` is red.
- If  $\text{red} \leq i \leq \text{white}$ , then the color is unknown.
- If  $\text{white} < i \leq \text{blue}$ , then `threads[i]` is white.
- If  $\text{blue} < i < \text{HEIGHT}$ , then `threads[i]` is blue.

This is illustrated in Figure 8.21.

**FIGURE 8.21**

Dutch National Flag Loop Invariant



### Algorithm

We can solve our problem by establishing the loop invariant and then executing a loop that both preserves the loop invariant and shrinks the unknown region.

1. Set `red` to 0, `white` to `HEIGHT - 1`, and `blue` to `HEIGHT - 1`. This establishes our loop invariant with the unknown region the whole flag and the red, white, and blue regions empty.
2. **while** `red < white`
3.     Shrink the distance between `red` and `white` while preserving the loop invariant.

### Preserving the Loop Invariant

Let us assume that we now know the color of `threads[white]` (the thread at position `white`). Our goal is to either leave `threads[white]` where it is (in the white region if it is white) or “move it” to the region where it belongs. There are three cases to consider:

*Case 1:* The color of `threads[white]` is white. In this case, we merely decrement the value of `white` to restore the invariant. By doing so, we increase the size of the white region by one thread.

*Case 2:* The color of `threads[white]` is red. We know from our invariant that the color of `threads[red]` is unknown. Therefore, if we swap the thread at `threads[red]` with the one at `threads[white]`, we can then increment the value of `red` and preserve the invariant. By doing this, we add the thread to the end of the red region and reduce the size of the unknown region by one thread.

*Case 3:* The color of `threads[white]` is blue. We know from our invariant that the color of `threads[blue]` is white. Thus, if we swap the thread at `threads[white]` with the thread at `threads[blue]` and then decrement both `white` and `blue`, we preserve the invariant.

By doing this, we insert the thread at the beginning of the blue region and reduce the size of the unknown region by one thread.

### Implementation

A complete implementation of this program is left as a programming project. We show the coding of the sort algorithm in Listing 8.12.

.....

#### LISTING 8.12

Dutch National Flag Sort

```
public void sort() {
    int red = 0;
    int white = height - 1;
    int blue = height - 1;
    /* Invariant:
       0 <= i < red      ==> threads[i].getColor() == Color.RED
       red <= i <= white ==> threads[i].getColor() is unknown
       white < i < blue   ==> threads[i].getColor() == Color.WHITE
       blue < i < height ==> threads[i].getColor() == Color.BLUE
    */
    while (red <= white) {
        if (threads[white].getColor() == Color.WHITE) {
            white--;
        } else if (threads[white].getColor() == Color.RED) {
            swap(red, white, g);
            red++;
        } else { // threads[white].getColor() == Color.BLUE
            swap(white, blue, g);
            white--;
            blue--;
        }
    }
    // assert: red > white so unknown region is now empty.
}
```

---

## EXERCISES FOR SECTION 8.11

### PROGRAMMING

1. Adapt the Dutch National Flag algorithm to do the quicksort partitioning. Consider the red region to be those values less than the pivot, the white region to be those values equal to the pivot, and the blue region to be those values equal to the pivot. You should initially sort the first, middle, and last items and use the middle value as the pivot value.



# Chapter Review

- ◆ We analyzed several sorting algorithms; their performance is summarized in Table 8.4.
- ◆ Two quadratic algorithms,  $O(n^2)$ , are selection sort and insertion sort. They give satisfactory performance for small arrays (up to 100 elements). Generally, insertion sort is considered to be the best of the quadratic sorts.
- ◆ Shell sort,  $O(n^{5/4})$ , gives satisfactory performance for arrays up to 5000 elements.
- ◆ Quicksort has average-case performance of  $O(n \log n)$ , but if the pivot is picked poorly, the worst-case performance is  $O(n^2)$ .
- ◆ Merge sort and heapsort have  $O(n \log n)$  performance.
- ◆ The Java API contains “industrial-strength” sort algorithms in the classes `java.util.Arrays` and `java.util.Collections`. The methods in `Arrays` use a mixture of quicksort and insertion sort for sorting arrays of primitive-type values and merge sort for sorting arrays of objects. For primitive types, quicksort is used until the size of the subarray reaches the point where insertion sort is quicker (seven elements or less). The `sort` method in `Collections` merely copies the list into an array and then calls `Arrays.sort`.

**TABLE 8.4**

Comparison of Sort Algorithms

	Number of Comparisons		
	Best	Average	Worst
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
Timsort	$O(n)$	$O(n \log n)$	$O(n \log n)$
Shell sort	$O(n^{7/6})$	$O(n^{5/4})$	$O(n^2)$
Merge sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Heapsort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quicksort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$

## Java Classes Introduced in This Chapter

`java.util.Arrays`  
`java.util.Collections`

## User-Defined Interfaces and Classes in This Chapter

<code>ComparePerson</code> <code>InsertionSort</code> <code>MergeSort</code> <code>Person</code>	<code>QuickSort</code> <code>SelectionSort</code> <code>ShellSort</code> <code>Timsort</code>
---	--

## Quick-Check Exercises

1. Name two quadratic sorts.
2. Name two sorts with  $n \log n$  worst-case behavior.
3. Which algorithm is particularly good for an array that is already sorted? Which is particularly bad? Explain your answers.
4. What determines whether you should use a quadratic sort or a logarithmic sort?
5. Which quadratic sort's performance is least affected by the ordering of the array elements? Which is most affected?
6. What is a good all-purpose sorting algorithm for medium-size arrays?

## Review Questions

1. When does quicksort work best, and when does it work worst?
2. Write a recursive procedure to implement the insertion sort algorithm.
3. What is the purpose of the pivot value in quicksort? How did we first select it in the text, and what is wrong with that approach for choosing a pivot value?
4. For the following array  
 30 40 20 15 60 80 75 4 20  
 show the new array after each pass of insertion sort and selection sort. How many comparisons and exchanges are performed by each?
5. For the array in Question 4, trace the execution of Shell sort.
6. For the array in Question 4, trace the execution of merge sort.
7. For the array in Question 4, trace the execution of quicksort.
8. For the array in Question 4, trace the execution of heapsort.

## Programming Projects

1. Use the random number function to store a list of 1000 pseudorandom integer values in an array. Apply each of the sort classes described in this chapter to the array and determine the number of comparisons and exchanges. Make sure the same array is passed to each sort method.
2. Investigate the effect of array size and initial element order on the number of comparisons and exchanges required by each of the sorting algorithms described in this chapter. Use arrays with 100 and 10,000 integers. Use three initial orderings of each array (randomly ordered, inversely ordered, and ordered). Be certain to sort the same six arrays with each sort method.
3. A variation of the merge sort algorithm can be used to sort large sequential data files. The basic strategy is to take the initial data file, read in several (say, 10) data records, sort these records using an efficient array-sorting algorithm, and then write these sorted groups of records (runs) alternately to one of two output files. After all records from the initial data file have been distributed to the two output files, the runs on these output files are merged one pair of runs at a time and written to the original data file. After all runs from the output file have been merged, the records on the original data file are redistributed to the output files, and the merging process is repeated. Runs no longer need to be sorted after the first distribution to the temporary output files.

Each time runs are distributed to the output files, they contain twice as many records as the time before. The process stops when the length of the runs exceeds the number of records in the data file. Write a program that implements merge sort for sequential data files. Test your program on a file with several thousand data values.

4. Write a method that sorts a linked list.
5. Write an industrial-strength quicksort method with the following enhancements:
  - a. If an array segment contains 20 elements or fewer, sort it using insertion sort.
  - b. After sorting the first, middle, and last elements, use the median as the pivot instead of swapping the median with the first element. Because the first and last elements are in the correct partitions, it is not necessary to test them before advancing up and down. This is also the case after each exchange, so increment up and decrement down at the beginning of the **do-while** loop. Also, it is not necessary to test whether up is less than last before incrementing up because the condition `pivot.compareTo(last) > 0` is false when up equals last (the median must be  $\leq$  the last element in the array).
6. In the early days of data processing (before computers), data was stored on punched cards. A machine to sort these cards contained 12 bins (one for each digit value and + and -). A stack of cards was fed into the machine, and the cards were placed into the appropriate bin depending on the value of the selected column. By restacking the cards so that all 0s were first, followed by the 1s, followed by the 2s, and so forth, and then sorting on the next column, the whole deck of cards could be sorted. This process, known as *radix sort*, requires  $c \times n$  passes, where  $c$  is the number of columns and  $n$  is the number of cards.

We can simulate the action of this machine using an array of queues. During the first pass, the least-significant digit (the ones digit) of each number is examined and the number is added to the queue whose subscript matches that digit. After all numbers have been processed, the elements of each queue are added to an 11th queue, starting with `queue[0]`, followed by `queue[1]`, and so forth. The process is then repeated for the next significant digit, taking the numbers out of the 11th queue. After all the digits have been processed, the 11th queue will contain the numbers in sorted order.

Write a program that implements radix sort on an array of `int` values. You will need to make 10 passes because an `int` can store numbers up to 2,147,483,648.

7. Complete the Dutch National Flag case study. You will need to develop the following classes:
  - a. A main class that extends `JFrame` to contain the flag and a control button (`Sort`).
  - b. A class to represent the flag; an extension of `JPanel` is suggested. This class will contain the array of threads and the sort method.
  - c. A class to represent a thread. Each thread should have a color and a method to draw the thread.

## Answers to Quick-Check Exercises

1. Selection sort, insertion sort
2. Merge sort, heapsort
3. Insertion sort—it requires  $n - 1$  comparisons with no exchanges. Quicksort can be bad if the first element is picked as the pivot value because the partitioning process always creates one subarray with a single element.
4. Array size
5. Selection sort
6. Shell sort or any  $O(n \log n)$  sort



# *Self-Balancing Search Trees*

## Chapter Objectives

- ◆ To understand the impact that balance has on the performance of binary search trees
- ◆ To learn about the AVL tree for storing and maintaining a binary search tree in balance
- ◆ To learn about the Red–Black tree for storing and maintaining a binary search tree in balance
- ◆ To learn about 2–3 trees, 2–3–4 trees, and B-trees and how they achieve balance
- ◆ To learn about skip-lists and how they have properties similar to balanced search trees
- ◆ To understand the process of search and insertion in each of these trees and to be introduced to removal

In Chapter 6 we introduced the binary search tree. The performance (time required to find, insert, or remove an item) of a binary search tree is proportional to the total *height of the tree*, where we define the height of a tree as the maximum number of nodes along a path from the root to a leaf. A full binary tree of height  $k$  can hold  $2^k - 1$  items. Thus, if the binary search tree were full and contained  $n$  items, the expected performance would be  $O(\log n)$ .

Unfortunately, if we build the binary search tree as described in Chapter 6, the resulting tree is not necessarily full or close to being full. Thus, the actual performance is worse than expected. In this chapter, we explore two algorithms for building binary search trees so that they are as full as possible. We call these trees *self-balancing* because they attempt to achieve a balance so that the height of each left subtree and right subtree is equal or nearly equal.

Next, we look at the B-tree and its specializations, the 2–3 and 2–3–4 trees. These are not binary search trees, but they achieve and maintain balance.

Finally, we look at the skip-list. The skip-list is not a tree structure, but it allows for search, insertion, and removal in  $O(\log n)$  time.

In this chapter, we focus on algorithms and methods for search and insertion. We also discuss removing an item, but we have left the details of removal to the programming projects.

## Self-Balancing Search Trees

- 9.1** Tree Balance and Rotation
- 9.2** AVL Trees
- 9.3** Red–Black Trees
- 9.4** 2–3 Trees
- 9.5** B-Trees and 2–3–4 Trees
- 9.6** Skip-Lists

## 9.1 Tree Balance and Rotation

### Why Balance Is Important

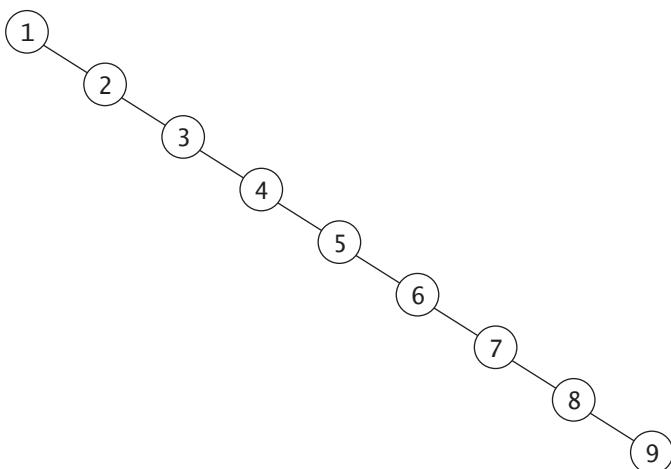
Figure 9.1 shows an example of a valid, but extremely unbalanced, binary search tree. Searches or inserts into this tree would be  $O(n)$ , not  $O(\log n)$ . Figure 9.2 shows the binary search tree resulting from inserting the words of the sentence “The quick brown fox jumps over the lazy dog”. It too is not well balanced, having a height of 7 but containing only nine words. (Note that the string “The” is the smallest because it begins with an uppercase letter.)

### Rotation

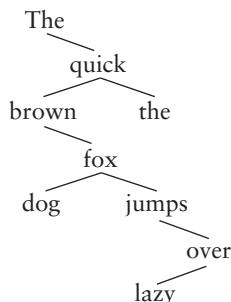
To achieve self-adjusting capability, we need an operation on a binary search tree that will change the relative heights of left and right subtrees but preserve the binary search tree property—that is, the items in each left subtree are less than the item at the root, and the items in each right subtree are greater than the item in the root. In Figure 9.3, we show an unbalanced binary search tree with a height of 4 right after the insertion of node 7. The height of the left subtree of the root (20) is 3, and the height of the right subtree is 1.

We can transform the tree in Figure 9.3 by doing a *right rotation* around node 20, making 10 the root and 20 the root of the right subtree of the new root (10). Because 20 is now the right subtree of 10, we need to move node 10’s old right subtree (root is 15). We will make it the left subtree of 20, as shown in Figure 9.4.

**FIGURE 9.1**  
Very Unbalanced  
Binary Search Tree



**FIGURE 9.2**  
Realistic Example of  
an Unbalanced Binary  
Search Tree



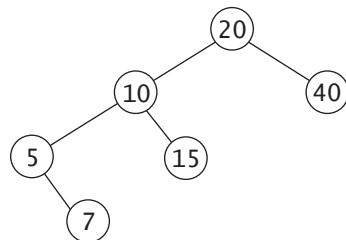
After these changes, the new binary search tree has a height of 3 (one less than before), and the left and right subtrees of the new root (10) have a height of 2, as shown in Figure 9.5. Note that the binary search tree property is maintained for all the nodes of the tree.

This result can be generalized. If node 15 had children, its children would have to be greater than 10 and less than 20 in the original tree. The left and right subtrees of node 15 would not change when node 15 was moved, so the binary search tree property would still be maintained for all children of node 15 in the new tree ( $> 10$  and  $< 20$ ). We can make a similar statement for any of the other leaf nodes in the original tree.

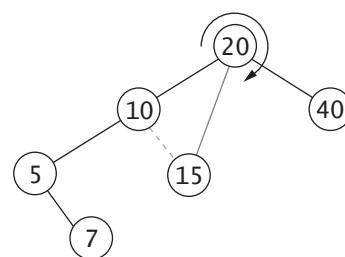
## Algorithm for Rotation

Figure 9.6 illustrates the internal representation of the nodes of our original binary search tree whose three branches (shown in gray) will be changed by rotation. Initially, root references node 20. Rotation right is achieved by the following algorithm.

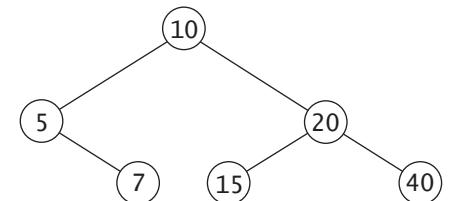
**FIGURE 9.3**  
Unbalanced Tree before Rotation



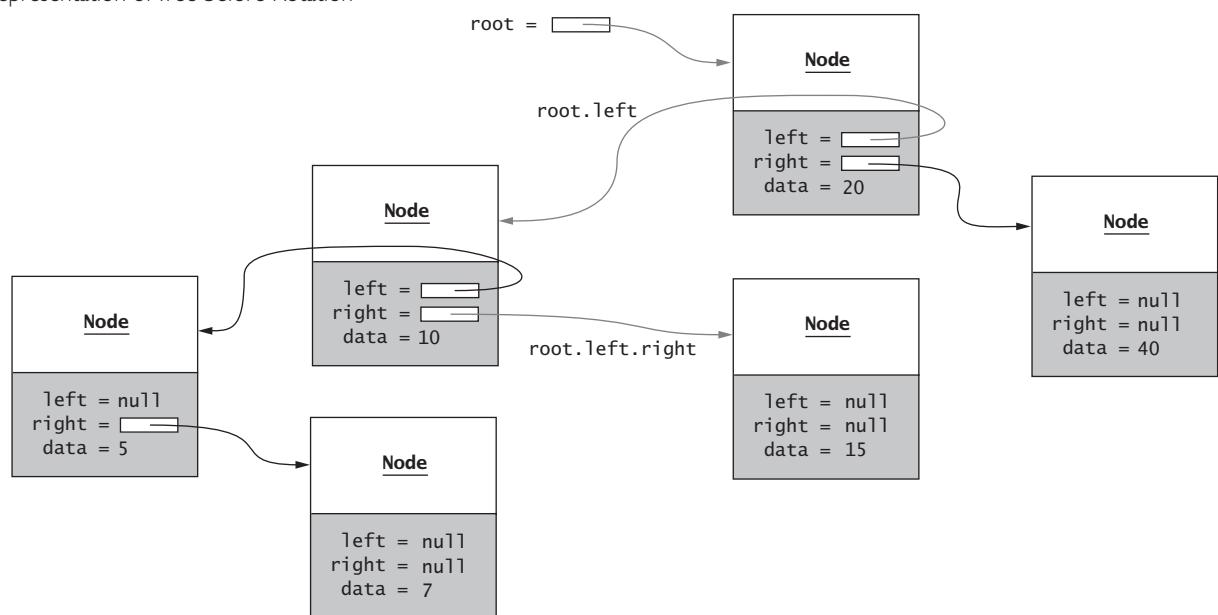
**FIGURE 9.4**  
Right Rotation



**FIGURE 9.5**  
More Balanced Tree after Rotation



**FIGURE 9.6**  
Internal Representation of Tree before Rotation



### Algorithm for Rotation Right

1. Remember the value of `root.left` (`temp = root.left`).
2. Set `root.left` to the value of `temp.right`.
3. Set `temp.right` to `root`.
4. Set `root` to `temp`.

Figure 9.7 shows the rotated tree. Step 1 sets `temp` to reference the left subtree (node 10) of the original root. Step 2 resets the original root's left subtree to reference node 15. Step 3 resets node `temp`'s right subtree to reference the original root. Then Step 4 sets `root` to reference node `temp`. The internal representation corresponds to the tree shown in Figure 9.5.

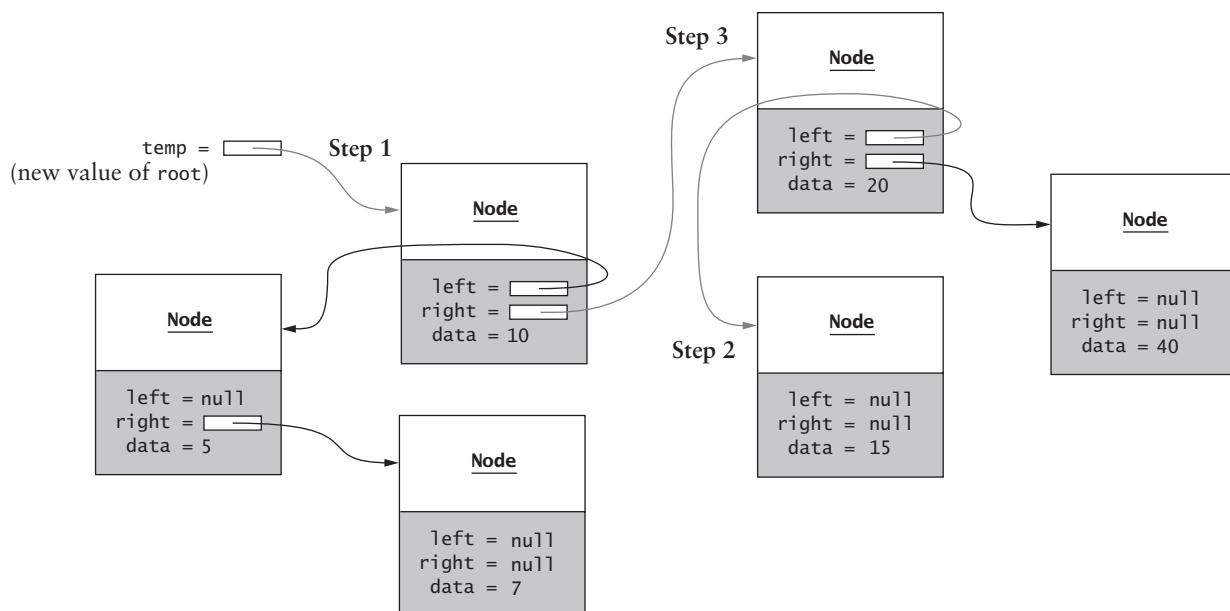
The algorithm for rotation left is symmetric to rotation right and is left as an exercise.

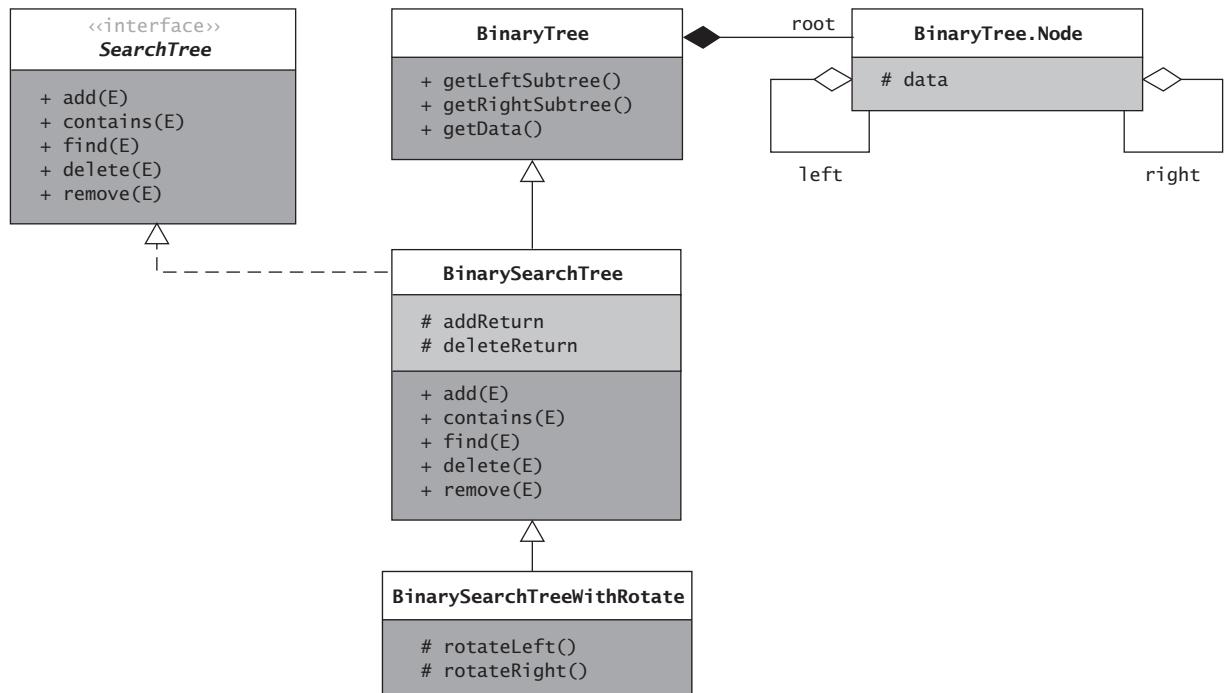
### Implementing Rotation

Listing 9.1 shows class `BinarySearchTreeWithRotate`. This class is an extension of the `BinarySearchTree` class described in Chapter 6, and it will be used as the base class for the other search trees discussed in this chapter. Like class `BinarySearchTree`, class `BinarySearchTreeWithRotate` must be declared as a generic class with type parameter `<E extends Comparable<E>`. It contains the methods `rotateLeft` and `rotateRight`. These methods take a reference to a `Node` that is the root of a subtree and return a reference to the root of the rotated tree. Figure 9.8 is a UML (Unified Modeling Language) class diagram that shows the relationships between `BinarySearchTreeWithRotate` and the other classes in the hierarchy. `BinarySearchTreeWithRotate` is a subclass of `BinaryTree` as well as `BinarySearchTree`. Class `BinaryTree` has the static inner class `Node` and the data field `root`, which references the `Node` that is the root of the tree. The figure shows that a `Node` contains a data field named `data` and two references (as indicated by the open diamond) to a `Node`. The names of the reference are `left` and `right`, as shown on the line from the `Node` to itself. We cover UML in Appendix B.

**FIGURE 9.7**

Effect of Rotation Right on Internal Representation



**FIGURE 9.8**UML Diagram of `BinarySearchTreeWithRotate`**LISTING 9.1**`BinarySearchTreeWithRotate.java`

```


/**
 * This class extends the BinarySearchTree by adding the rotate operations.
 * Rotation will change the balance of a search tree while preserving the
 * search tree property.
 * Used as a common base class for self-balancing trees.
 */
public class BinarySearchTreeWithRotate<E extends Comparable<E>>
    extends BinarySearchTree<E> {
    // Methods
    /**
     * Method to perform a right rotation.
     * @pre root is the root of a binary search tree.
     * @post root.right is the root of a binary search tree,
     *       root.right.right is raised one level,
     *       root.right.left does not change levels,
     *       root.left is lowered one level,
     *       the new root is returned.
     * @param root The root of the binary tree to be rotated
     * @return The new root of the rotated tree
     */
    protected Node<E> rotateRight(Node<E> root) {
        Node<E> temp = root.left;
        root.left = temp.right;
        temp.right = root;
        return temp;
    }
}


```

```

    /** Method to perform a left rotation (rotateLeft).
       // See Programming Exercise 1
    */
}

```

## EXERCISES FOR SECTION 9.1

### SELF-CHECK

1. Draw the binary search tree that results from inserting the words of the sentence “Now is the time for all good men to come to the aid of the party.” What is its height? Compare this with 4, the smallest integer greater than  $\log_2 13$ , where 13 is the number of distinct words in this sentence.
2. Try to construct a binary search tree that contains the same words as in Exercise 1 but has a maximum height of 4.
3. Describe the algorithm for rotation left.

### PROGRAMMING

1. Add the `rotateLeft` method to the `BinarySearchTreeWithRotate` class.



## 9.2 AVL Trees

Two Russian mathematicians, G. M. Adel'son-Vel'skiî and E. M. Landis, published a paper in 1962 that describes an algorithm for maintaining overall balance of a binary search tree. Their algorithm keeps track of the difference in height of each subtree. As items are added to (or removed from) the tree, the balance (i.e., the difference in the heights of the subtrees) of each subtree from the insertion point up to the root is updated. If the balance ever gets out of the range  $-1 \dots +1$ , the subtree is rotated to bring it back into balance. Trees using this approach are known as *AVL trees* after the initials of the inventors. As before, we define the height of a tree as the number of nodes in the longest path from the root to a leaf node, including the root.

### Balancing a Left-Left Tree

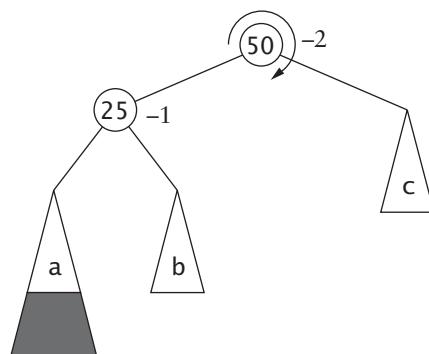
Figure 9.9 shows a binary search tree with a balance of  $-2$  caused by an insert into its left-left subtree. Each white triangle with label *a*, *b*, or *c* represents a tree of height  $k$ ; the shaded area at the bottom of the left-left triangle (*tree a*) indicates an insertion into this tree (its height is now  $k+1$ ). We use the formula

$$h_R - h_L$$

to calculate the balance for each node, where  $h_L$  and  $h_R$  are the heights of the left and right subtrees, respectively. The actual heights are not important; it is their relative difference that matters. The right subtree (*b*) of node 25 has a height of  $k$ ; its left subtree (*a*) has a height of  $k+1$ , so its balance is  $-1$ . The right subtree (of node 50) has a height of  $k$ ; its left subtree has a height of  $k+2$ , so its factor is  $-2$ . Such a tree is called a *Left-Left tree* because its root and the left subtree of the root are both left-heavy.

**FIGURE 9.9**

Left-Heavy Tree

**FIGURE 9.10**

Left-Heavy Tree after Rotation Right

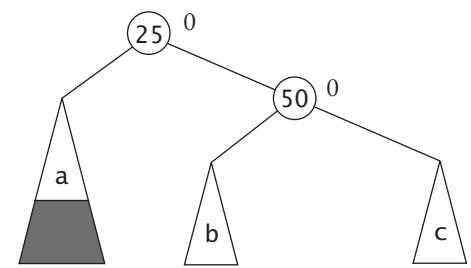


Figure 9.10 shows this same tree after a rotation right. The new tree root is node 25. Its right subtree (root 50) now has tree b as its left subtree. Note that balance has now been achieved. Also, the overall height has not increased. Before the insertion, the tree height was  $k + 2$ ; after the rotation, the tree height is still  $k + 2$ .

### Balancing a Left–Right Tree

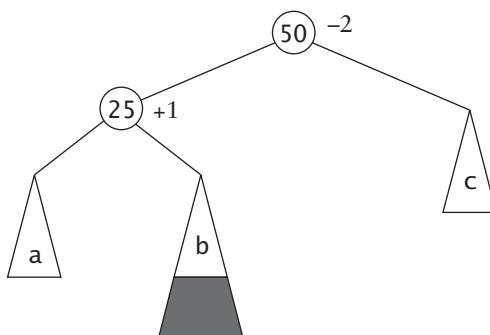
Figure 9.11 shows a left-heavy tree caused by an insert into the Left–Right subtree. This tree is called a Left–Right tree because its root is left-heavy but the left subtree of the root is right-heavy. We cannot fix this with a simple rotation right as in the Left–Left case. (See Self-Check Exercise 2 at the end of this section.)

Figure 9.12 shows a general Left–Right tree. Node 40, the root of the Left–Right subtree, is expanded into its subtrees  $b_L$  and  $b_R$ . Figure 9.12 shows the effect of an insertion into  $b_L$ , making node 40 left-heavy. If the left subtree is rotated left, as shown in Figure 9.13, the overall tree is now a Left–Left tree, similar to the case of Figure 9.9. Now if the modified tree is rotated right, overall balance is achieved, as shown in Figure 9.14. Figures 9.15–9.17 illustrate the effect of these double rotations after insertion into  $b_R$ .

In both cases, the new tree root is 40; its left subtree has node 25 as its root, and its right subtree has node 50 as its root. The balance of the root is 0. If the critically unbalanced situation was due to an insertion into subtree  $b_L$ , the balance of the root's left child is 0, and the balance of the root's right child is +1 (Figure 9.14). For insertion into subtree  $b_R$ , the balance of the root's left child is -1, and the balance of the root's right child is 0 (Figure 9.17).

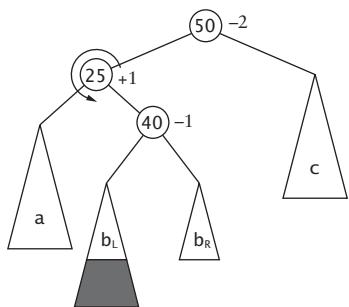
**FIGURE 9.11**

Left–Right Tree

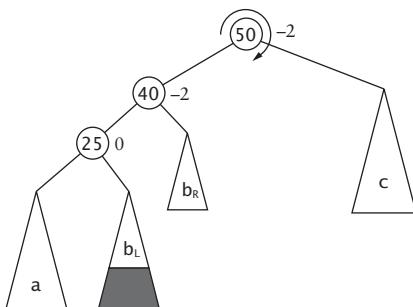


$$\text{Balance } 50 = (k - (k + 2))$$

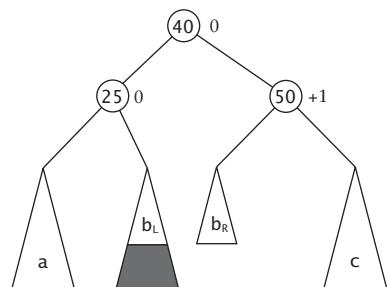
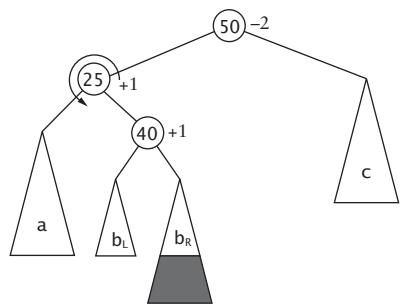
$$\text{Balance } 25 = ((k + 1) - k)$$

**FIGURE 9.12**Insertion into  $b_L$ **FIGURE 9.13**

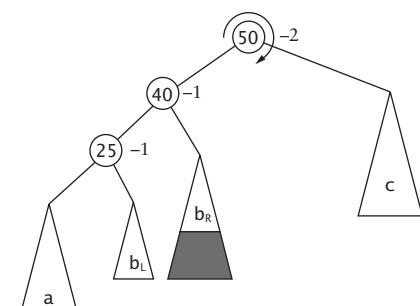
Left Subtree after Rotate Left

**FIGURE 9.14**

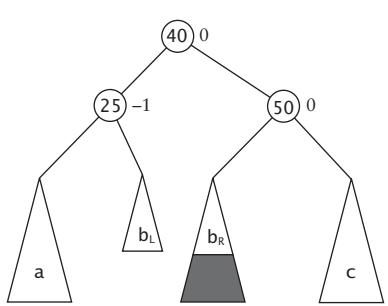
Tree after Rotate Right

**FIGURE 9.15**Insertion into  $b_R$ **FIGURE 9.16**

Left Subtree after Rotate Left

**FIGURE 9.17**

Tree after Rotate Right



## Four Kinds of Critically Unbalanced Trees

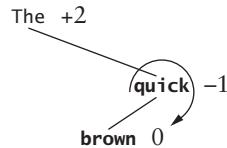
How do we recognize unbalanced trees and determine what to do to balance them? For the Left–Left tree shown in Figure 9.9 (parent and child nodes are both left-heavy, parent balance is  $-2$ , child balance is  $-1$ ), the remedy is to rotate right around the parent.

For the Left–Right example shown in Figure 9.11 (parent is left-heavy with balance  $-2$ , child is right-heavy with balance  $+1$ ), the remedy is to rotate left around the child and then rotate right around the parent. We list the four cases that need rebalancing and their remedies next.

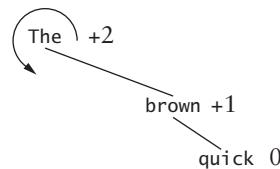
- Left–Left (parent balance is  $-2$ , left child balance is  $-1$ ): rotate right around parent.
- Left–Right (parent balance is  $-2$ , left child balance is  $+1$ ): rotate left around child, then rotate right around parent.
- Right–Right (parent balance is  $+2$ , right child balance is  $+1$ ): rotate left around parent.
- Right–Left (parent balance is  $+2$ , right child balance is  $-1$ ): rotate right around child, then rotate left around parent.

**EXAMPLE 9.1** We will build an AVL tree from the words in the sentence “The quick brown fox jumps over the lazy dog”.

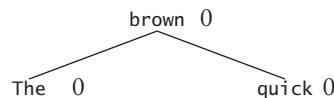
After inserting the words *The*, *quick*, and *brown*, we get the following tree.



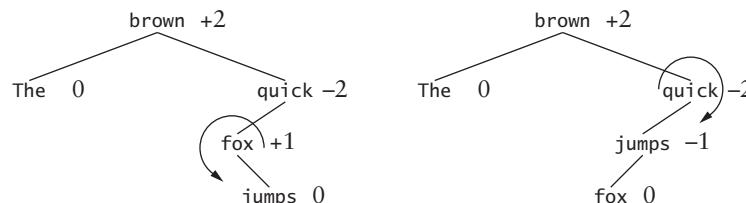
The subtree with the root *quick* is left-heavy by 1, but the overall tree with the root of *The* is right-heavy by 2 (Right–Left case). We must first rotate the subtree around *quick* to the right:



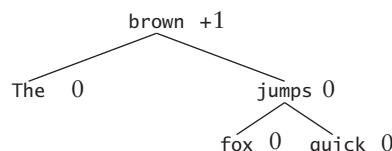
Then rotate left about *The*:



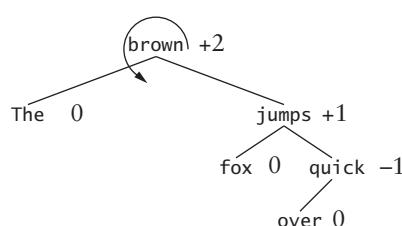
We now proceed to insert *fox* and *jumps*:



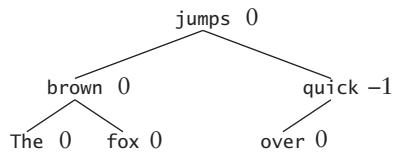
The subtree rooted about *quick* is now left-heavy by 2 (Left–Right case). Because this case is symmetric with the previous one, we rotate left about *fox* and then right about *quick*, giving the following result.



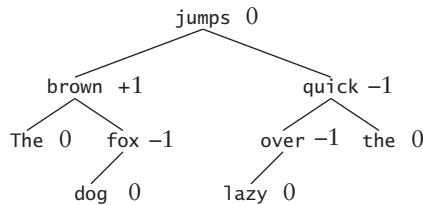
We now insert *over*.



The subtrees at *quick* and *jumps* are unbalanced by 1. The subtree at *brown*, however, is right-heavy by 2 (Right–Right case), so a rotation left solves the problem.



We can now insert *the*, *lazy*, and *dog* without any additional rotations being necessary.

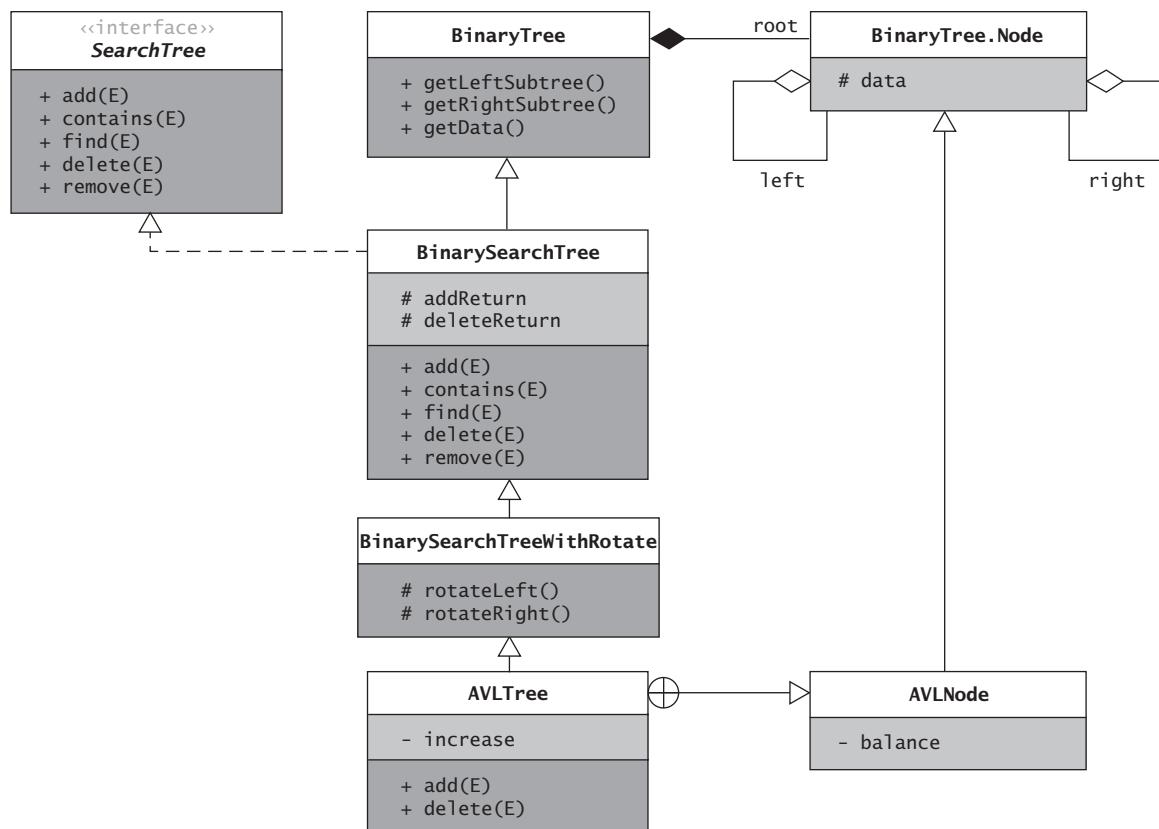


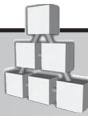
## Implementing an AVL Tree

We begin by deriving the class `AVLTree` from `BinarySearchTreeWithRotate` (see Listing 9.1). Figure 9.18 is a UML class diagram showing the relationship between `AVLTree` and `BinarySearchTreeWithRotate`. The `AVLTree` class contains the `boolean` data field `increase`,

**FIGURE 9.18**

UML Class Diagram of `AVLTree`





## SYNTAX UML SYNTAX

The line from the `AVLTree` class to the `AVLNode` class in the diagram in Figure 9.18 indicates that methods in the `AVLTree` class can access the private data field `balance`. The symbol  $\oplus$  next to the `AVLTree` class indicates that the `AVLNode` class is an inner class of `AVLTree`. The arrow pointing to `AVLNode` indicates that methods in `AVLTree` access the contents of `AVLNode`, but methods in `AVLNode` do not access the contents of `AVLTree`.

Note that the `Node` class is an inner class of the `BinaryTree` class, but we do not show the  $\oplus$ . This is because an object of type `Node`, called `root`, is a component of the `BinaryTree` class, as indicated by the filled diamond next to the `BinaryTree` class. Showing both the  $\oplus$  and the filled diamond would clutter the diagram, so only the filled diamond is shown.

which indicates whether the current subtree height has increased as a result of the insertion. We override the methods `add` and `delete` but inherit method `find` because searching a balanced tree is not different from searching an unbalanced tree. We also extend the inner class `BinaryTree.Node` with `AVLNode`. Within this class we add the additional field `balance`.

```
/** Self-balancing binary search tree using the algorithm defined
   by Adelson-Velskii and Landis.
*/
public class AVLTree<E extends Comparable<E>>
    extends BinarySearchTreeWithRotate<E> {
    // Insert nested class AVLNode<E> here.

    // Data Fields
    /** Flag to indicate that height of tree has increased. */
    private boolean increase;
    ...
}
```

## The `AVLNode` Class

The `AVLNode` class is shown in Listing 9.2. It is an extension of the `BinaryTree.Node` class. It adds the data field `balance` and the constants `LEFT_HEAVY`, `BALANCED`, and `RIGHT_HEAVY`.

### LISTING 9.2

The `AVLNode` Class

```
/** Class to represent an AVL Node. It extends the
BinaryTree.Node by adding the balance field.
*/
private static class AVLNode<E> extends Node<E> {
    /** Constant to indicate left-heavy */
    public static final int LEFT_HEAVY = -1;
    /** Constant to indicate balanced */
    public static final int BALANCED = 0;
    /** Constant to indicate right-heavy */
    public static final int RIGHT_HEAVY = 1;
    /** balance is right subtree height - left subtree height */
    private int balance;
```

```

// Methods
/** Construct a node with the given item as the data field.
 * @param item The data field

*/
public AVLNode(E item) {
    super(item);
    balance = BALANCED;
}

/** Return a string representation of this object.
 * The balance value is appended to the contents.
 * @return String representation of this object
*/
@Override
public String toString() {
    return balance + ": " + super.toString();
}
}

```

## Inserting into an AVL Tree

The easiest way to keep a tree balanced is never to let it become unbalanced. If any node becomes critical and needs rebalancing, rebalance immediately. You can identify critical nodes by checking the balance at the root node of a subtree as you return to each parent node along the insertion path. If the insertion was in the left subtree and the left subtree height has increased, you must check to see whether the balance for the root node of the left subtree has become critical (-2 or +2). If so, you need to fix it by calling `rebalanceLeft` (rebalance a left-heavy tree when balance is -2) or `rebalanceRight` (rebalance a right-heavy tree when balance is +2). A symmetric strategy should be followed after returning from an insertion into the right subtree. The `boolean` variable `increase` is set before return from recursion to indicate to the next higher level that the height of the subtree has increased. This information is then used to adjust the balance of the next level in the tree. The following algorithm is based on the algorithm for inserting into a binary search tree, described in Chapter 6.

### Algorithm for Insertion into an AVL Tree

1. **if** the root is `null`  
    Create a new tree with the item at the root and return `true`.
2.     **else if** the item is equal to `root.data`  
        The item is already in the tree; return `false`.
3.     **else if** the item is less than `root.data`  
        Recursively insert the item in the left subtree.
4.     **if** the height of the left subtree has increased (`increase` is `true`)  
        Decrement balance.
5.         **if** balance is zero, reset `increase` to `false`.
6.         **if** balance is less than -1  
           Reset `increase` to `false`.
7.         Perform a `rebalanceLeft`.
8.     **else if** the item is greater than `root.data`  
        The processing is symmetric to Steps 4 through 10. Note that balance is incremented if `increase` is true.

After returning from the recursion (Step 4), examine the global data field `increase` to see whether the left subtree has increased in height. If it did, then decrement the balance. If the balance had been +1 (current subtree was right-heavy), it is now zero, so the overall height of the current subtree is not changed. Therefore, reset `increase` to `false` (Steps 5–7).

If the balance was -1 (current subtree was left-heavy), it is now -2, and a `rebalanceLeft` must be performed. The rebalance operation reduces the overall height of the tree by 1, so `increase` is reset to `false`. Therefore, no more rebalancing operations will occur, so you can fix the tree by either a single rotation (Left–Left case) or a double rotation (Left–Right case) (Steps 8–10).

## add Starter Method

We are now ready to implement the insertion algorithm. The `add` starter method merely calls the recursive `add` method with the root as its argument. The returned `AVLNode` is the new root.

```
/** add starter method.
 * @pre the item to insert implements the Comparable interface.
 * @param item The item being inserted.
 * @return true if the object is inserted; false
 *         if the object already exists in the tree
 * @throws ClassCastException if item is not Comparable
 */
@Override
public boolean add(E item) {
    increase = false;
    root = add((AVLNode<E>) root, item);
    return addReturn;
}
```

As for the `BinarySearchTree` in Chapter 6, the recursive `add` method will set the data field `addReturn` to `true` (inherited from class `BinarySearchTree`) if the item is inserted and `false` if the item is already in the tree.

## Recursive add Method

The declaration for the recursive `add` method begins as follows:

```
/** Recursive add method. Inserts the given object into the tree.
 * @post addReturn is set true if the item is inserted,
 *       false if the item is already in the tree.
 * @param localRoot The local root of the subtree
 * @param item The object to be inserted
 * @return The new local root of the subtree with the item inserted
 */
private AVLNode<E> add(AVLNode<E> localRoot, E item)
```

We begin by seeing whether the `localRoot` is `null`. If it is, then we set `addReturn` and `increase` to `true` and return a new `AVLNode`, which contains the item to be inserted.

```
if (localRoot == null) {
    addReturn = true;
    increase = true;
    return new AVLNode<E>(item);
}
```

Next, we compare the inserted item with the data field of the current node. If it is equal, we set `addReturn` and `increase` to `false` and return the `localRoot` unchanged.

```

if (item.compareTo(localRoot.data) == 0) {
    // Item is already in the tree.
    increase = false;
    addReturn = false;
    return localRoot;
}

```

If it is less than this value, we recursively call the add method (Step 4 of the insertion algorithm), passing `localRoot.left` as the parameter and replacing the value of `localRoot.left` with the returned value.

```

else if (item.compareTo(localRoot.data) < 0) {
    // item < data
    localRoot.left = add((AVLNode<E>) localRoot.left, item);
    ...
}

```

Upon return from the recursion, we examine the global data field `increase`. If `increase` is **true**, then the height of the left subtree has increased, so we decrement the balance by calling the `decrementBalance` method. If the balance is now less than  $-1$ , we reset `increase` to **false** and call the `rebalanceLeft` method. The return value from the `rebalanceLeft` method is the return value from this call to `add`. If the balance is not less than  $-1$ , or if the left subtree height did not increase, then the return from this recursive call is the same local root that was passed as the parameter.

```

if (increase) {
    decrementBalance(localRoot);
    if (localRoot.balance < AVLNode.LEFT_HEAVY) {
        increase = false;
        return rebalanceLeft(localRoot);
    }
}
return localRoot;
// Rebalance not needed.

```

If the `item` is not equal to `localRoot.data` and not less than `localRoot.data`, then it must be greater than `localRoot.data`. The processing is symmetric with the less-than case and is left as an exercise.

### Initial Algorithm for `rebalanceLeft`

Method `rebalanceLeft` rebalances a left-heavy tree. Such a tree can be a Left–Left tree (fixed by a single right rotation) or a Left–Right tree (fixed by a left rotation followed by a right rotation). If its left subtree is right-heavy, we have a Left–Right case, so we first rotate left around the left subtree. Finally, we rotate the tree right.

1. **if** the left subtree has positive balance (Left–Right case)
2.     Rotate left around left subtree root.
3.     Rotate right.

The algorithm for `rebalanceRight` is left as an exercise.

### The Effect of Rotations on Balance

The rebalancing algorithm just presented is incomplete. So far we have focused on changes to the root reference and to the internal branches of the tree being balanced, but we have not adjusted the balances of the nodes. In the beginning of this section, we showed that for a

Left–Left tree, the balances of the new root node and of its right child are 0 after a right rotation; the balances of all other nodes are unchanged (see Figure 9.10).

The Left–Right case is more complicated. We made the following observation after studying the different cases.

The balance of the root is 0. If the critically unbalanced situation was due to an insertion into subtree  $b_L$ , the balance of the root’s left child is 0 and the balance of the root’s right child is +1 (Figure 9.14). For insertion into subtree  $b_R$ , the balance of the root’s left child is -1, and the balance of the root’s right child is 0 (Figure 9.17). So we need to change the balances of the new root node and both its left and right children; all other balances are unchanged. We will call insertion into subtree  $b_L$  the Left–Right–Left case and insertion into subtree  $b_R$  the Left–Right–Right case.

There is a third case where the left-right subtree is balanced; this occurs when a left-right leaf is inserted into a subtree that has only a left child. In this case after the rotates are performed, the root, left child, and right child are all balanced.

### Revised Algorithm for rebalanceLeft

Based on the foregoing discussion, we can now develop the complete algorithm for `rebalanceLeft`, including the required balance changes. It is easier to store the new balance for each node before the rotation than after.

1.     **if** the left subtree has a positive balance (Left–Right case)
2.         **if** the left–left subtree has a negative balance (Left–Right–Left case)
  3.             Set the left subtree (new left subtree) balance to 0.
  4.             Set the left–left subtree (new root) balance to 0.
  5.             Set the local root (new right subtree) balance to +1.
  6.         **else** (Left–Right–Right case)
    6.             Set the left subtree (new left subtree) balance to -1.
    7.             Set the left–left subtree (new root) balance to 0.
    8.             Set the local root (new right subtree) balance to 0.
    9.         Rotate the left subtree left.
    10.         **else** (Left–Left case)
      10.             Set the left subtree balance to 0.
      11.             Set the local root balance to 0.
      12.         Rotate the local root right.

The algorithm for `rebalanceRight` is left as an exercise.

### Method rebalanceLeft

The code for `rebalanceLeft` is shown in Listing 9.3. First, we test to see whether the left subtree is right-heavy (Left–Right case). If so, the Left–Right subtree is examined.

Depending on its balance, the balances of the left subtree and local root are set as previously described in the algorithm. The rotations will reduce the overall height of the tree by 1, so `increase` is now set to `false`. The left subtree is then rotated left, and the tree is rotated right.

**LISTING 9.3**

The rebalanceLeft Method

```

.....  

LISTING 9.3  

The rebalanceLeft Method  

/** Method to rebalance left.  

 * @pre localRoot is the root of an AVL subtree that is critically left-heavy.  

 * @post Balance is restored.  

 * @param localRoot Root of the AVL subtree that needs rebalancing  

 * @return a new localRoot  

 */  

private AVLNode<E> rebalanceLeft(AVLNode<E> localRoot) {  

    // Obtain reference to left child.  

    AVLNode<E> leftChild = (AVLNode<E>) localRoot.left;  

    // See whether left-right heavy.  

    if (leftChild.balance > AVLNode.BALANCED) {  

        // Obtain reference to left-right child.  

        AVLNode<E> leftRightChild = (AVLNode<E>) leftChild.right;  

        /** Adjust the balances to be their new values after  

         * the rotations are performed.  

        */  

        if (leftRightChild.balance < AVLNode.BALANCED) {  

            leftChild.balance = AVLNode.BALANCED;  

            leftRightChild.balance = AVLNode.BALANCED;  

            localRoot.balance = AVLNode.RIGHT_HEAVY;  

        } else {  

            leftChild.balance = AVLNode.LEFT_HEAVY;  

            leftRightChild.balance = AVLNode.BALANCED;  

            localRoot.balance = AVLNode.BALANCED;  

        }  

        // Perform left rotation.  

        localRoot.left = rotateLeft(leftChild);  

    } else { // Left-Left case  

        /** In this case the leftChild (the new root) and the root  

         * (new right child) will both be balanced after the rotation.  

        */  

        leftChild.balance = AVLNode.BALANCED;  

        localRoot.balance = AVLNode.BALANCED;  

    }  

    // Now rotate the local root right.  

    return (AVLNode<E>) rotateRight(localRoot);  

}

```

If the left child is LEFT\_HEAVY, the rotation process will restore the balance to both the tree and its left subtree and reduce the overall height by 1; the balance for the left subtree and local root are both set to BALANCED, and increase is now set to **false**. The tree is then rotated right to correct the imbalance.

We also need a rebalanceRight method that is symmetric with rebalanceLeft (i.e., all lefts are changed to rights and all rights are changed to lefts). Coding of this method is left as an exercise.

### The decrementBalance Method

As we return from an insertion into a node's left subtree, we need to decrement the balance of the node. We also need to indicate whether the subtree height at that node has not increased, by setting increase (currently **true**) to **false**. There are two cases to consider: a node that is balanced and a node that is right-heavy. If a node is balanced, insertion into its left subtree will cause it to become left-heavy, and its height will also increase by 1 (see Figure 9.19). If a node is right-heavy, insertion into its left subtree will cause it to become balanced, and its height will not increase (see Figure 9.20).

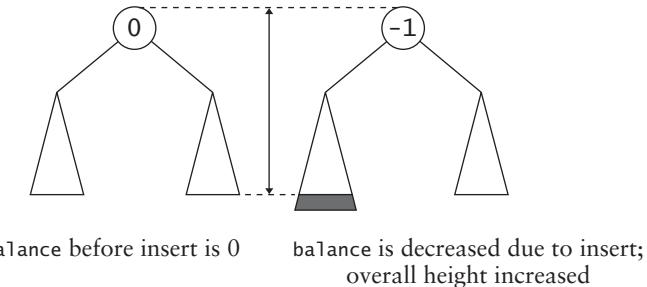
```

private void decrementBalance(AVLNode<E> node) {
    // Decrement the balance.
    node.balance--;
    if (node.balance == AVLNode.BALANCED) {
        /** If now balanced, overall height has not increased. */
        increase = false;
    }
}

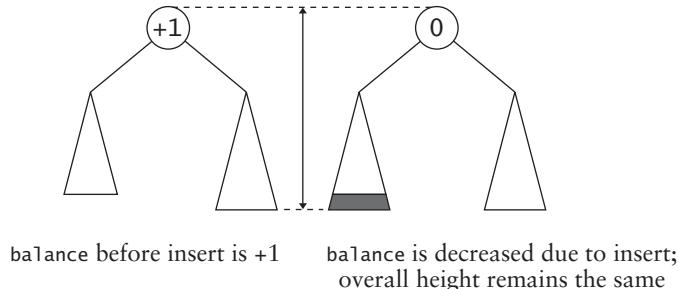
```

Step 11 of the insertion algorithm performs insertion into a right subtree. This can cause the height of the right subtree to increase, so we will also need an `incrementBalance` method that increments the balance and resets `increase` to `false` if the balance changes from left-heavy to balanced. Coding this method is left as an exercise.

**FIGURE 9.19**  
Decrement of  
balance by Insert on  
Left (Height Increases)



**FIGURE 9.20**  
Decrement of  
balance by Insert on  
Left (Height Does Not  
Change)



## Removal from an AVL Tree

When we remove an item from a left subtree, the balance of the local root is increased, and when we remove an item from the right subtree, the balance of the local root is decreased. We can adapt the algorithm for removal from a binary search tree to become an algorithm for removal from an AVL tree. We need to maintain a data field `decrease` that tells the previous level in the recursion that there was a decrease in the height of the subtree that was just returned from. (This data field is analogous to the data field `increase`, which is used in the insertion to indicate that the height of the subtree has increased.) We can then increment or decrement the local root balance. If the balance is outside the threshold, then the rebalance methods (`rebalanceLeft` or `rebalanceRight`) are used to restore the balance.

We need to modify methods `decrementBalance`, `incrementBalance`, `rebalanceLeft`, and `rebalanceRight` so that they set the value of `decrease` (as well as `increase`) after a node's

balance has been decremented. When a subtree changes from either left-heavy or right-heavy to balanced, then the height has decreased, and decrease should be set **true**; when the subtree changes from balanced to either left-heavy or right-heavy, then decrease should be reset to **false**. We also need to provide methods similar to the ones needed for removal in a binary search tree. Implementing removal is left as a programming project.

Also, observe that the effect of rotations is not only to restore balance but to decrease the height of the subtree being rotated. Thus, while only one `rebalanceLeft` or `rebalanceRight` was required for insertion, during removal each recursive return could result in a further need to rebalance.

## Performance of the AVL Tree

Since each subtree is kept as close to balanced as possible, one would expect that the AVL tree provides the expected  $O(\log n)$  performance. Each subtree is allowed to be out of balance by  $\pm 1$ . Thus, the tree may contain some holes.

It can be shown that in the worst case, the height of an AVL tree can be 1.44 times the height of a full binary tree that contains the same number of items. However, this would still yield  $O(\log n)$  performance because we ignore constants.

The worst-case performance is very rare. Empirical tests (see, e.g., Donald Knuth, *The Art of Computer Programming, Vol 3: Searching and Sorting* [Addison-Wesley, 1973], p. 460) show that, on the average,  $\log_2 n + 0.25$  comparisons are required to insert the  $n$ th item into an AVL tree. Thus, the average performance is very close to that of the corresponding complete binary search tree.

---

## EXERCISES FOR SECTION 9.2

### SELF-CHECK

1. Show how the final AVL tree for the “The quick brown fox” changes as you insert “apple”, “cat”, and “hat” in that order.
2. Show the effect of just rotating right on the tree in Figure 9.11. Why doesn’t this fix the problem?
3. Build an AVL tree that inserts the integers 30, 40, 15, 25, 90, 80, 70, 85, 15, 72 in the given order.
4. Build the AVL tree from the sentence “Now is the time for all good men to come to the aid of the party”.

### PROGRAMMING

1. Program the `rebalanceRight` method.
2. Program the code in the `add` method for the case where `item.compareTo(localRoot.data) > 0`.
3. Program the `incrementBalance` method.



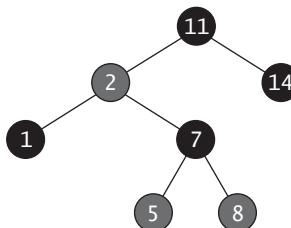
## 9.3 Red–Black Trees

We will now discuss another approach to keeping a tree balanced, called the *Red–Black tree*. Rudolf Bayer developed the Red–Black tree as a special case of his B-tree (the topic of Section 9.5); Leo Guibas and Robert Sedgewick refined the concept and introduced the color convention. A Red–Black tree maintains the following invariants:

1. A node is either red or black.
2. The root is always black.
3. A red node always has black children. (A `null` reference is considered to refer to a black node.)
4. The number of black nodes in any path from the root to a leaf is the same.

Figure 9.21 shows an example of a Red–Black tree. Invariant 4 states that a Red–Black tree is always balanced because the root node's left and right subtrees must be the same height where the height is determined by counting just black nodes. Note that by the standards of the AVL tree, this tree is out of balance and would be considered a Left–Right tree. However, by the standards of the Red–Black tree, it is balanced because there are two black nodes (counting the root) in any path from the root to a leaf. (We use gray to indicate a red node.)

**FIGURE 9.21**  
Red–Black Tree

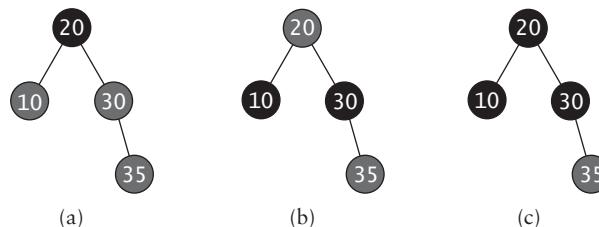


### Insertion into a Red–Black Tree

The algorithm for insertion follows the same recursive search process used for all binary search trees to reach the insertion point. When a leaf is found, the new item is inserted, and it is initially given the color red, so invariant 4 will be maintained. If the parent is black, we are done.

However, if the parent is also red, then invariant 3 has been violated. Figure 9.22(a) shows the insertion of 35 as a red child of 30. If the parent's sibling is also red, then we can change the grandparent's color to red and change both the parent and parent's sibling to black. This restores invariant 3 but does not violate invariant 4 (see Figure 9.22(b)). If the root of the overall tree is now red, we can change it to black to restore invariant 2 and still maintain invariant 4 (the heights of all paths to a leaf are increased by 1) (see Figure 9.22(c)).

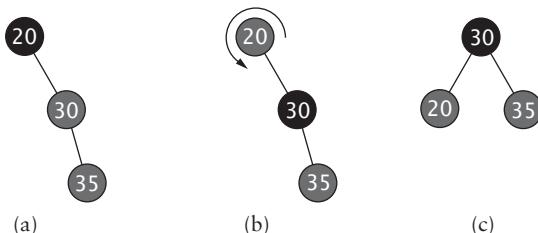
**FIGURE 9.22**  
Insertion into a Red–Black Tree, Case 1



If we insert a value with a red parent, but that parent does not have a red sibling (see Figure 9.23(a)), then we change the color of the grandparent to red and the parent to black (see Figure 9.23(b)). Now we have violated invariant 4, as there are more black nodes on the side of the parent. We correct this by rotating about the grandparent so that the parent moves into the position where the grandparent was, thus restoring invariant 4 (see Figure 9.23(c)).

**FIGURE 9.23**

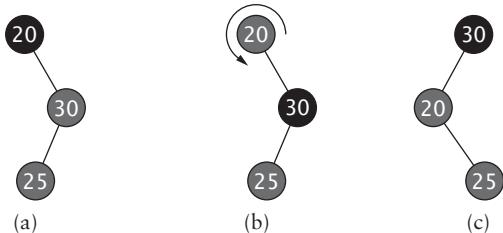
Insertion into a Red–Black Tree, Case 2



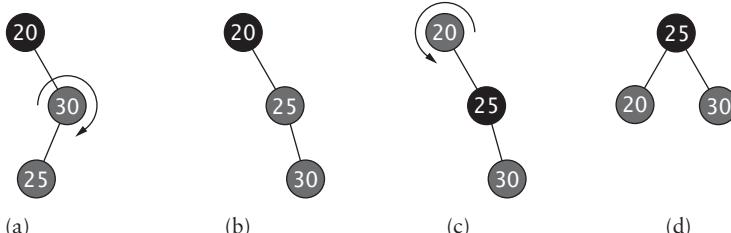
The preceding maneuver works only if the inserted value is on the same side of its parent as the parent is to the grandparent. Figure 9.24(a) shows 25 inserted as the left child of 30, which is the right child of 20. If we change the color of the grandparent (20) to red and the parent (30) to black (see Figure 9.24(b)) and then rotate (see Figure 9.24(c)), we are still left with a red parent–red child combination. Before changing the color and rotating about the grandparent level, we must first rotate about the parent so that the red child is on the same side of its parent as the parent is to the grandparent (see Figure 9.25(b)). We can then change the colors (see Figure 9.25(c)) and rotate (see Figure 9.25(d)).

**FIGURE 9.24**

Insertion into a Red–Black Tree, Case 3  
(Single Rotation Doesn't Work)

**FIGURE 9.25**

Insertion into a Red–Black Tree, Case 3  
(Double Rotation)

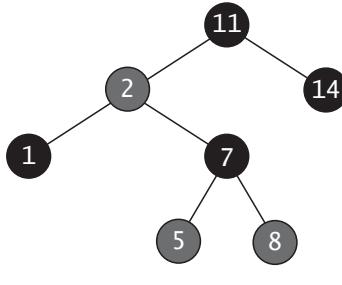


More than one of these cases can occur. Figure 9.26 shows the insertion of the value 4 into the Red–Black tree of Figure 9.21. Upon return from the insertion to the parent (node 5), it may be discovered that a red node now has a red child, which is a violation of invariant 3. If this node's sibling (node 8) is also red (Case 1), then they must have a black parent. If we make the parent red (node 7) and both of the parent's children black, invariant 4 is preserved, and the problem is shifted up, as shown in Figure 9.27.

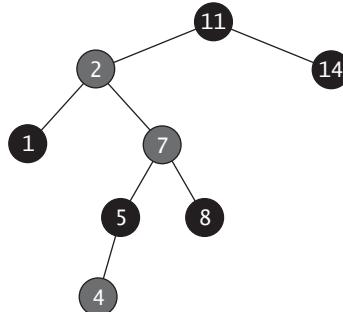
Looking at Figure 9.27, we see that 7 is red and that its parent, 2, is also red. However, we can't simply change 2's color as we did before because 2's sibling, 14, is black. This problem will require one or two rotations to correct.

Because the red child (7) is not on the same side of its parent (2) as the parent is to the grandparent (11), this is an example of Case 3. We rotate the tree left (around node 2) so that the red node 2 is on the same side of red node 7 as node 7 is to the grandparent (11) (see Figure 9.28). We now change node 7 to black and node 11 to red (Figure 9.29) and rotate right around node 11, restoring the balance of black nodes as shown in Figure 9.30.

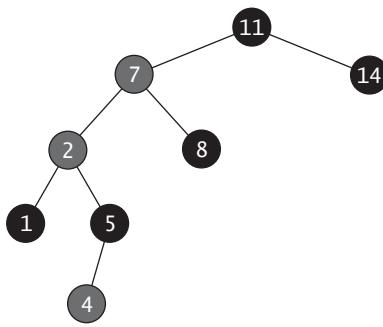
**FIGURE 9.26**  
Red–Black Tree after Insertion of 4



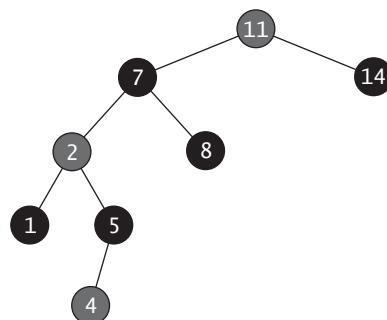
**FIGURE 9.27**  
Moving Black Down and Red Up



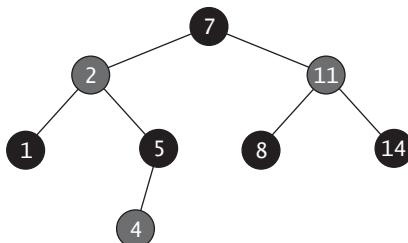
**FIGURE 9.28**  
Rotating Red Node to Outside



**FIGURE 9.29**  
Changing Colors of Parent and Grandparent Nodes

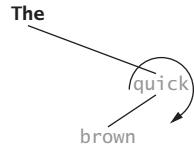


**FIGURE 9.30**  
Final Red–Black Tree after Insert

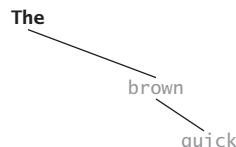


**EXAMPLE 9.2** We will now build the Red–Black tree for the sentence “The quick brown fox jumps over the lazy dog”.

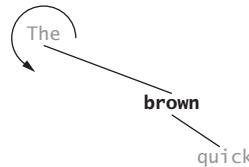
We start by inserting *The*, *quick*, and *brown*.



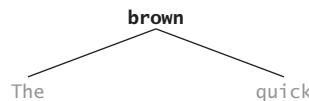
The parent of *brown* (*quick*) is red, but the sibling of *quick* is black (null nodes are considered black), so we have an example of Case 2 or Case 3. Because the child is not on the same side of the parent as the parent is to the grandparent, this is Case 3. We first rotate right about *quick* to get the child on the same side of the parent as the parent is to the grandparent.



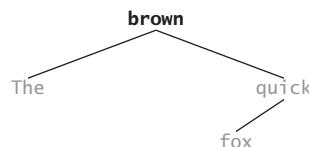
We then change the colors of *The* and *brown*.



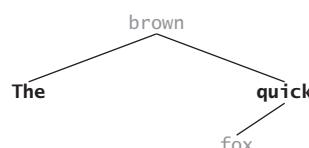
Then we rotate left about *The*.



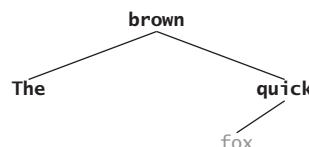
Next, we insert *fox*.



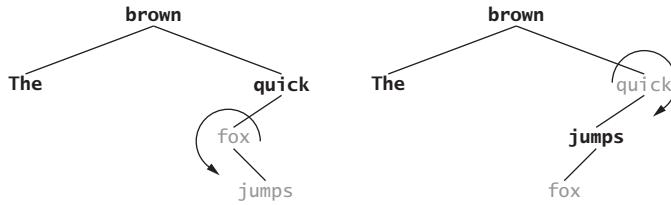
We see that *fox* has a red parent (*quick*) whose sibling is also red (*The*). This is a Case 1 insertion, so we can change the color of the parent and its sibling to black and the grandparent to red.



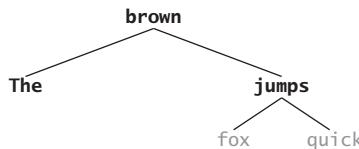
Since the root is red, we can change it to black without violating the rule of balanced black nodes.



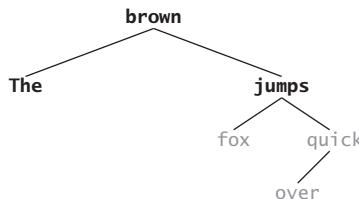
Now we add *jumps*, which gives us another Case 3 insertion.



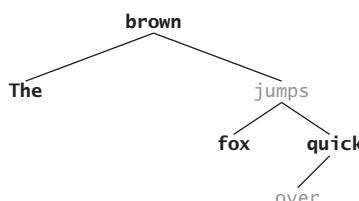
This triggers a double rotation. First, rotate left about *fox* and change the color of its parent *jumps* to black and its grandparent *quick* to red. Next, rotate right about *quick*.



Next, we insert *over*.

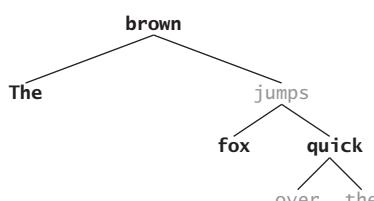


Because *quick* and *fox* are red, we have a Case 1 insertion, so we can move the black in *jumps* down, changing the color of *jumps* to red and *fox* and *quick* to black.

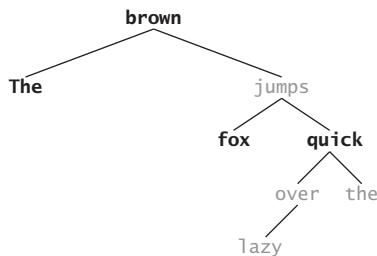


Next, we add *the*. No changes are required because its parent is black.

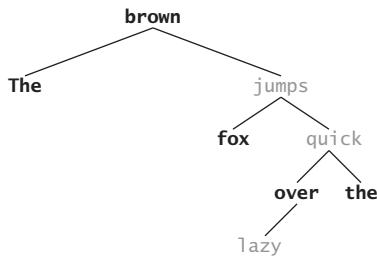
When compared to the corresponding AVL tree, this tree looks out of balance. But the black nodes are in balance (two in each path).



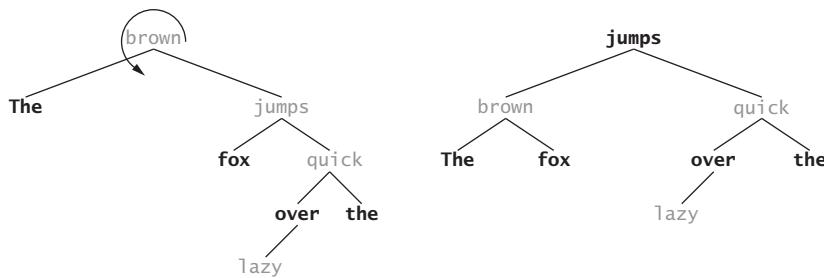
Now we insert *lazy*.



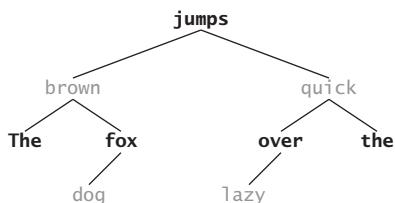
Because *over* and *the* are both red, we can move the black at *quick* down (Case 1).



But now *quick* is a red node with a red parent (*jumps*) whose sibling is black (*The*). Because *quick* and *jumps* are both right children, this is an example of Case 2. This triggers a rotate left around *brown*.



Finally, we can insert *dog*.



Surprisingly, the result is identical to the AVL tree for the same input, but the intermediate steps were very different.

## Implementation of Red–Black Tree Class

We begin by deriving the class `RedBlackTree` from `BinarySearchTreeNodeWithRotate` (see Listing 9.1). Figure 9.31 is a UML class diagram showing the relationship between `RedBlackTree` and `BinarySearchTreeNodeWithRotate`. The `RedBlackTree` class overrides the `add` and `delete` methods. The nested class `BinaryTreeNode` is extended with the `RedBlackNode` class. This class has the additional data field `isRed` to indicate red nodes. Listing 9.4 shows the `RedBlackNode` class.

### LISTING 9.4

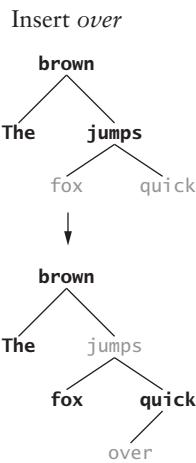
The `RedBlackTree` and `RedBlackNode` Classes

```
/** Class to represent Red–Black tree. */
public class RedBlackTree<E extends Comparable<E>>
    extends BinarySearchTreeNodeWithRotate<E> {

    /** Nested class to represent a Red–Black node. */
    private static class RedBlackNode<E> extends Node<E> {
        // Additional data members
        /** Color indicator. True if red, false if black. */
        private boolean isRed;

        // Constructor
        /** Create a RedBlackNode with the default color of red
            and the given data field.
            @param item The data field
        */
        public RedBlackNode(E item) {
            super(item);
            isRed = true;
        }

        // Methods
        /** Return a string representation of this object.
            The color (red or black) is appended to the node's contents.
            @return String representation of this object
        */
        @Override
        public String toString() {
            if (isRed) {
                return "Red : " + super.toString();
            } else {
                return "Black: " + super.toString();
            }
        }
    }
    . .
}
```



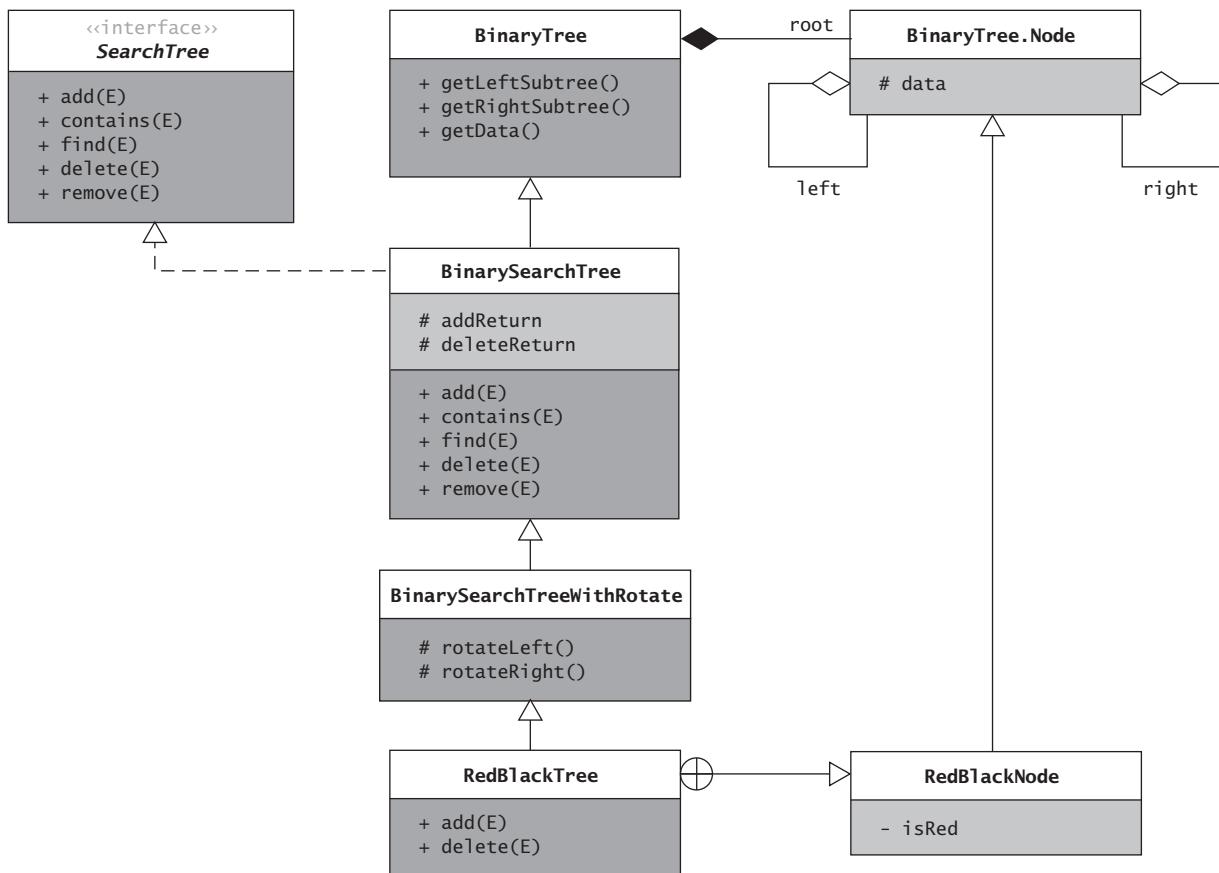
## Algorithm for Red–Black Tree Insertion

The foregoing outline of the Red–Black tree insertion algorithm is from the point of view of the node being inserted. It can be, and has been, implemented using a data structure that has a reference to the parent of each node stored in it so that, given a reference to a node, one can access the parent, grandparent, and the parent's sibling (the node's aunt or uncle).

We are going to present a recursive algorithm where the need for fix-ups is detected from the grandparent level. This algorithm has one additional difference from the algorithm as presented in the foregoing examples: whenever a black node with two red children is detected on the way down the tree, it is changed to red and the children are changed to black (e.g., *jumps* and its children in the figure at left). If this change causes a problem, it is fixed on the

**FIGURE 9.31**

UML Class Diagram of RedBlackTree



way back up. This modification simplifies the logic a bit and improves the performance of the algorithm. This algorithm is also based on the algorithm for inserting into a binary search tree that was described in Chapter 6.

### Algorithm for Red–Black Tree Insertion

1. **if** the root is **null**
  2. Insert a new Red–Black node and color it black.
  3. Return **true**.
4. **else if** the item is equal to root.data
  5. The item is already in the tree; return **false**.
6. **else if** the item is less than root.data
  7. **if** the left subtree is **null**
    8. Insert a new Red–Black node as the left subtree and color it red.
    9. Return **true**.
  10. **else**
    11. **if** both the left child and the right child are red
      12. Change the color of the children to black and change local root to red.
      13. Recursively insert the item into the left subtree.

14.           **if** the left child is now red
15.            **if** the left grandchild is now red (grandchild is an “outside node”)
16.            Change the color of the left child to black and change the local root to red.
17.            Rotate the local root right.
18.           **else if** the right grandchild is now red (grandchild is an “inside” node)
19.            Rotate the left child left.
20.            Change the color of the left child to black and change the local root to red.
21.            Rotate the local root right.
22.    **else**
23.           Item is greater than `root.data`; process is symmetric and is left as an exercise.
24.    **if** the local root is the root of the tree
25.           Force its color to be black.

Because Java passes the value of a reference, we have to work with a node that is a local root of a Red–Black tree. Thus, in Step 8, we replace the `null` reference to the left subtree with the inserted node.

If the left subtree is not `null` (Step 10), we recursively apply the algorithm (Step 13). But before we do so, we see whether both children are red. If they are, we change the local root to red and change the children to black (Steps 11 and 12). (If the local root’s parent was red, this condition will be detected at that level during the return from the recursion.)

Upon return from the recursion (Step 14), we see whether the local root’s left child is now red. If it is, we need to check its children (the local root’s grandchildren). If one of them is red, then we have a red parent with a red child, and a rotation is necessary. If the left grandchild is red, a single rotation will solve the problem (Steps 15 through 17). If the right grandchild is red, a double rotation is necessary (Steps 18 through 21). Note that there may be only one grandchild or no grandchildren. However, if there are two grandchildren, they cannot both be red because they would have been changed to black by Steps 11 and 12, as described in the previous paragraph.

### The add Starter Method

As with the other binary search trees we have studied, the add starter method checks for a `null` root and inserts a single new node. Since the root of a Red–Black tree is always black, we set the newly inserted node to black. The cast is necessary because `root` is a data field that was inherited from `BinaryTree` and is therefore of type `Node`.

```
public boolean add(E item) {
    if (root == null) {
        root = new RedBlackNode<>(item);
        ((RedBlackNode<E>) root).isRed = false;
        // root is black.
        return true;
    }
    . .
}
```

Otherwise the recursive add method is called. This method takes two parameters: the node that is the local root of the subtree into which the item is to be inserted and the item to be inserted. The return value is the node that is the root of the subtree that now contains the inserted item. The data field `addReturn` is set to `true` if the insert method succeeded and to `false` if the item is already in the subtree.

The root is replaced by the return value from the recursive add method, the color of the root is set to black, and the data field addReturn is returned to the caller of the add starter method.

```

    else {
        root = add((RedBlackNode<E>) root, item);
        ((RedBlackNode<E>) root).isRed = false;
        // root is always black.
        return addReturn;
    }
}

```

## The Recursive add Method

The recursive add method begins by comparing the item to be inserted with the data field of the local root. If they are equal, then the item is already in the tree; addReturn is set to **false** and the localRoot is returned (algorithm Step 5).

```

private Node<E> add(RedBlackNode<E> localRoot, E item) {
    if (item.compareTo(localRoot.data) == 0) {
        // item already in the tree.
        addReturn = false;
        return localRoot;
    }
    . .
}

```

If it is less, then localRoot.left is checked to see whether it is **null**. If so, then we insert a new node and return (Steps 7–9).

```

else if (item.compareTo(localRoot.data) < 0) {
    // item < localRoot.data.
    if (localRoot.left == null) {
        // Create new left child.
        localRoot.left = new RedBlackNode<E>(item);
        addReturn = true;
        return localRoot;
    }
    . .
}

```

Otherwise, check to see whether both children are red. If so, we make them black and change the local root to red. This is done by the method moveBlackDown. Then we recursively call the add method, using root.left as the new local root (Steps 11–13).

```

else { // Need to search.
    // Check for two red children, swap colors if found.
    moveBlackDown(localRoot);
    // Recursively add on the left.
    localRoot.left = add((RedBlackNode<E>) localRoot.left, item);
    . .
}

```

It is upon return from the recursive add that things get interesting. Upon return from the recursive call, localRoot.left refers to the parent of a Red–Black subtree that may be violating the rule against adjacent red nodes. Therefore, we check the left child to see whether it is red (Step 14).

```

// See whether the left child is now red
if (((RedBlackNode<E>) localRoot.left).isRed) {
    . .
}

```

If the left child is red, then we need to check its two children. First, we check the left grandchild (Step 15).

```

if (localRoot.left.left != null && ((RedBlackNode<E>)
    localRoot.left.left).isRed) {
    // Left-left grandchild is also red.
    . .
}

```

If the left–left grandchild is red, we have detected a violation of invariant 3 (no consecutive red children), and we have a left–left case. Thus, we change colors and perform a single rotation, returning the resulting local root to the caller (Steps 16–17).

```
// Single rotation is necessary.
((RedBlackNode<E>) localRoot.left).isRed = false;
localRoot.isRed = true;
return rotateRight(localRoot);
```

If the left grandchild is not red, we then check the right grandchild. If it is red, the process is symmetric to the preceding case, except that a double rotation will be required (Steps 18–21).

```
else if (localRoot.left.right != null && ((RedBlackNode<E>)
    localRoot.left.right).isRed) {
    // Left-right grandchild is also red.
    // Double rotation is necessary.
    localRoot.left = rotateLeft(localRoot.left);
    ((RedBlackNode<E>) localRoot.left).isRed = false;
    localRoot.isRed = true;
    return rotateRight(localRoot);
}
```

If upon return from the recursive call the left child is black, the return is immediate, and all of this complicated logic is skipped. Similarly, if neither the left nor the right grandchild is also red, nothing is done.

If the item is greater than `root.data`, the process is symmetric and is left as an exercise (Step 23 and Programming Exercise 1).

## Removal from a Red–Black Tree

Removal follows the algorithm for a binary search tree that was described in Chapter 6. Recall that we remove a node only if it is a leaf or if it has only one child. Otherwise, the node that contains the inorder predecessor of the value being removed is the one that is removed. If the node that is removed is red, nothing further must be done because red nodes do not affect a Red–Black tree’s balance. If the node to be removed is black and has a red child, then the red child takes its place, and we color it black. However, if we remove a black leaf, then the black height is now out of balance. There are several cases that must be considered. We will describe them in Programming Project 6 at the end of this chapter.

## Performance of a Red–Black Tree

It can be shown that the upper limit in the height for a Red–Black tree is  $2 \log_2 n + 2$ , which is still  $O(\log n)$ . As with the AVL tree, the average performance is significantly better than the worst-case performance. Empirical studies (see Robert Sedgewick, *Algorithms in C++*, 3rd ed. [Addison-Wesley, 1998], p. 570) show that the average cost of a search in a Red–Black tree built from random values is  $1.002 \log_2 n$ . Thus, both the AVL and Red–Black trees give performance that is close to that of a complete binary search tree.

## The TreeMap and TreeSet Classes

The Java API has a `TreeMap` class (part of the package `java.util`) that implements a Red–Black tree. The `TreeMap` class implements the `SortedMap` interface, so it defines methods `get`, `put` (a tree insertion), `remove`, and `containsKey`, among others. Because a Red–Black tree is used, these are all  $O(\log n)$  operations. There is also a `TreeSet` class (introduced in Section 7.5) that implements the `SortedSet` interface. This class is implemented as an adapter of the `TreeMap` class using a technique similar to what was described in Chapter 7 to implement the `HashSet` as an adapter of the `HashMap`.

## EXERCISES FOR SECTION 9.3

### SELF-CHECK

1. Show how the final AVL tree for the “The quick brown fox” changes as you insert “apple”, “cat”, and “hat” in that order.
2. Insert the numbers 6, 3, and 0 in the Red–Black tree in Figure 9.21.
3. Build the Red–Black tree from the sentence “Now is the time for all good men to come to the aid of the party”. Is it the same as the AVL tree?

### PROGRAMMING

1. Program the case where the item is greater than `root.data`.



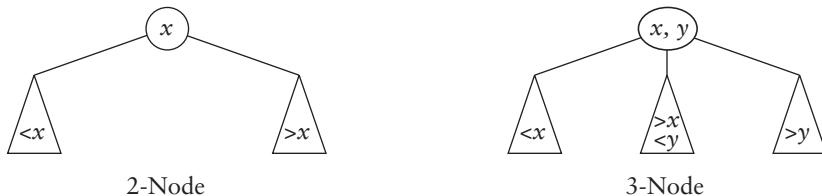
## 9.4 2–3 Trees

In this section, we begin our discussion of three nonbinary trees. We begin with the *2–3 tree*, named for the number of possible children from each node (either 2 or 3). A 2–3 tree is made up of nodes designated as *2-nodes* and *3-nodes*. A 2-node is the same as a binary search tree node: it consists of a data field and references to two children, one child containing values less than the data field and the other child containing values greater than the data field. A 3-node contains two data fields, ordered so that the first is less than the second, and references to three children: one child containing values less than the first data field, one child containing values between the two data fields, and one child containing values greater than the second data field.

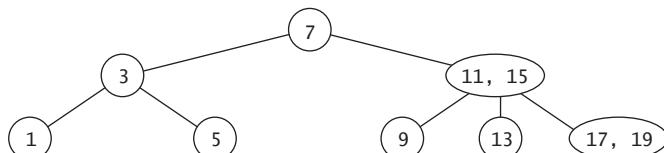
Figure 9.32 shows the general forms of a 2-node (data item is  $x$ ) and a 3-node (data items are  $x$  and  $y$ ). The children are represented as subtrees. Figure 9.33 shows an example of a 2–3 tree. There are only two 3-nodes in this tree (the right and right-right nodes); the rest are 2-nodes.

A 2–3 tree has the additional property that all of the leaves are at the lowest level. This is how the 2–3 tree maintains balance. This will be further explained when we study the insertion and removal algorithms.

**FIGURE 9.32**  
2-Node and 3-Node



**FIGURE 9.33**  
Example of a 2–3 Tree



## Searching a 2–3 Tree

Searching a 2–3 tree is very similar to searching a binary search tree.

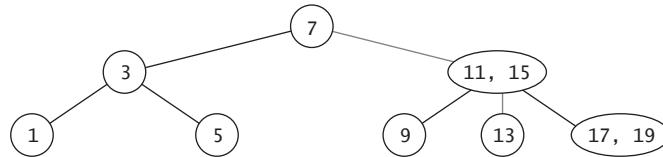
1. **if** the local root is **null**
2.     Return **null**; the item is not in the tree.
3. **else if** this is a 2-node
4.     **if** the item is equal to the **data1** field
5.         Return the **data1** field.
6.     **else if** the item is less than the **data1** field
7.         Recursively search the left subtree.
8.     **else**
9.         Recursively search the right subtree.
10. **else** // This is a 3-node
11.     **if** the item is equal to the **data1** field
12.         Return the **data1** field.
13.     **else if** the item is equal to the **data2** field
14.         Return the **data2** field.
15.     **else if** the item is less than the **data1** field
16.         Recursively search the left subtree.
17.     **else if** the item is less than the **data2** field
18.         Recursively search the middle subtree.
19.     **else**
20.         Recursively search the right subtree.

### EXAMPLE 9.3

To search for 13 in Figure 9.33, we would compare 13 with 7 and see that it is greater than 7, so we would search the node that contains 11 and 15. Because 13 is greater than 11 but less than 15, we would next search the middle child, which contains 13: success! The search path is shown in gray in Figure 9.34.

**FIGURE 9.34**

Searching a 2–3 Tree



## Inserting an Item into a 2–3 Tree

A 2–3 tree maintains balance by being built from the bottom up, not the top down. Instead of hanging a new node onto a leaf, we insert the new node into a leaf, as discussed in the following paragraphs. We search for the insertion node using the normal process for a 2–3 tree.

### Inserting into a 2-Node Leaf

Figure 9.35 (left) shows a 2–3 tree with three 2-nodes. We want to insert 15. Because the leaf we are inserting into is a 2-node, we can insert 15 directly, creating a new 3-node (Figure 9.35 right).

**FIGURE 9.35**

Inserting into a Tree with All 2-Nodes

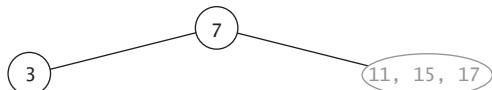


### Inserting into a 3-Node Leaf with a 2-Node Parent

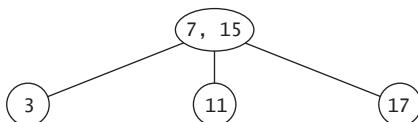
If we want to insert a number larger than 7 (say, 17), that number will be virtually inserted into the 3-node at the bottom right of the tree, giving the virtual node in gray in Figure 9.36. Because a node can't store three values, the middle value will propagate up to the 2-node parent, and the virtual node will be split into two new 2-nodes containing the smallest and largest values. Because the parent is a 2-node, it will be changed to a 3-node, and it will reference the three 2-nodes, as shown in Figure 9.37.

**FIGURE 9.36**

A Virtual Insertion

**FIGURE 9.37**

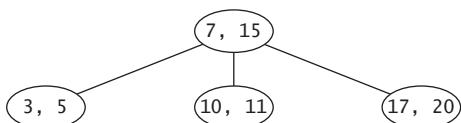
Result of Propagating 15 to 2-Node Parent



Let's now insert the numbers 5, 10, and 20. Each of these would go into one of the leaf nodes (all 2-nodes), changing them to 3-nodes, as shown in Figure 9.38.

**FIGURE 9.38**

Inserting 5, 10, and 20



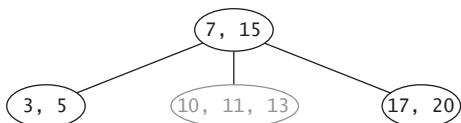
### Inserting into a 3-Node Leaf with a 3-Node Parent

In the tree in Figure 9.39, all the leaf nodes are full, so if we insert any other number, one of the leaf nodes will need to be virtually split, and its middle value will propagate to the parent. Because the parent is already a 3-node, it will also need to be split.

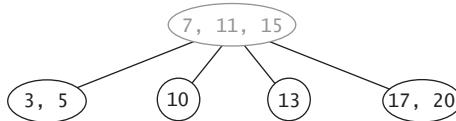
For example, if we were to insert 13, it would be virtually inserted into the leaf node with values 10 and 11 (see Figure 9.39). This would result in two new 2-nodes with values 10 and 13, and 11 would propagate up to be virtually inserted in the 3-node at the root (see Figure 9.40). Because the root is full, it would split into two new 2-nodes with values 7 and 15, and 11 would propagate up to be inserted in a new root node. The net effect is an increase in the overall height of the tree, as shown in Figure 9.41.

**FIGURE 9.39**

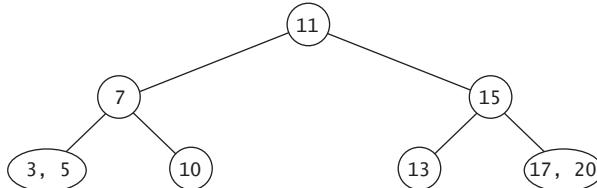
Virtually Inserting 13



**FIGURE 9.40**  
Virtually Inserting 11



**FIGURE 9.41**  
Result of Making 11 the New Root

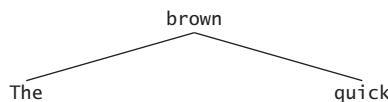


We summarize these observations in the following insertion algorithm.

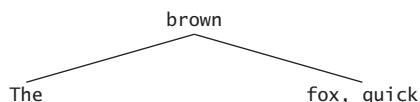
### Algorithm for Insertion

1. **if** the root is null
2.     Create a new 2-node that contains the new item.
3. **else if** the item is in the local root
4.     Return **false**.
5. **else if** the local root is a leaf
6.     **if** the local root is a 2-node
7.         Expand the 2-node to a 3-node and insert the item.
8.     **else**
9.         Split the 3-node (creating two 2-nodes) and pass the new parent and right child back up the recursion chain.
10. **else**
11.     **if** the item is less than the smaller item in the local root
12.         Recursively insert into the left child.
13.     **else if** the local root is a 2-node
14.         Recursively insert into the right child.
15.     **else if** the item is less than the larger item in the local root
16.         Recursively insert into the middle child.
17.     **else**
18.         Recursively insert into the right child.
19.     **if** a new parent was passed up from the previous level of recursion
20.         **if** the new parent will be the tree root
21.             Create a 2-node whose data item is the passed-up parent, left child is the old root, and right child is the passed-up child. This 2-node becomes the new root.
22.     **else**
23.         Recursively insert the new parent at the local root.
24.     Return **true**.

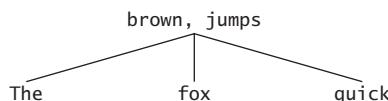
**EXAMPLE 9.4** We will create a 2–3 tree using “The quick brown fox jumps over the lazy dog.” The initial root contains *The, quick*. If we insert *brown*, we will split the root. Because *brown* is between *The* and *quick*, it gets passed up and will become the new root.



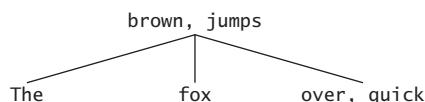
We now insert *fox* as the left neighbor of *quick*, creating a new 3-node.



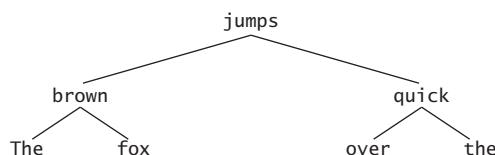
Next, *jumps* is inserted between *fox* and *quick*, thus splitting this 3-node, and *jumps* gets passed up and inserted next to *brown*.



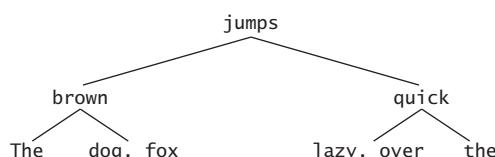
Then *over* is inserted next to *quick*.



Now we insert *the*. It will be inserted to the right of *over, quick*, splitting that node, and *quick* will be passed up. It will be inserted to the right of *brown, jumps*, splitting that node as well, causing *jumps* to be passed up to the new root.



Finally, *lazy* and *dog* are inserted next to *over* and *fox*, respectively.



## Analysis of 2–3 Trees and Comparison with Balanced Binary Trees

The 2–3 tree resulting from the preceding example is a balanced tree of height 3 that requires fewer complicated manipulations. There were no rotations, as were needed to build the AVL and Red–Black trees, which were both of height 4. The number of items that a 2–3 tree of height  $h$  can hold is between  $2^h - 1$  (all 2-nodes) and  $3^h - 1$  (all 3-nodes). Therefore, the height of a 2–3 tree is between  $\log_3 n$  and  $\log_2 n$ . Thus, the search time is  $O(\log n)$ , since logarithms are all related by a constant factor, and constant factors are ignored in big-O notation.

### Removal from a 2–3 Tree

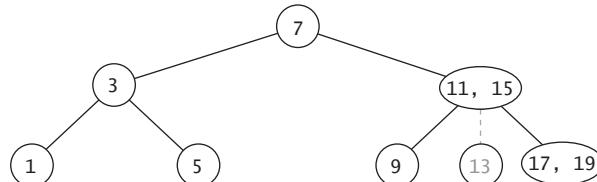
Removing an item from a 2–3 tree is somewhat the reverse of the insertion process. To remove an item, we must first search for it. If the item to be removed is in a leaf, we simply delete it. However, if the item to be removed is not in a leaf, we remove it by swapping it with its inorder predecessor in a leaf node and deleting it from the leaf node. If removing a node from a leaf causes the leaf to become empty, items from the sibling and parent can be redistributed into that leaf, or the leaf can be merged with its parent and sibling nodes. In the latter case, the height of the tree may decrease. We illustrate these cases next.

If we remove item 13 from the tree shown in Figure 9.42, its node would become empty, and item 15 in the parent node would have no left child. We can merge 15 and its right child to form the virtual leaf node {15, 17, 19}. Item 17 moves up to the parent node; item 15 is the new left child of 17 (see Figure 9.43).

We next remove 11 from the 2–3 tree. Because this is not a leaf, we replace it with its predecessor, 9, as shown in Figure 9.44. We now have the case where the left leaf node of 9 has become empty. So we merge 9 into its right leaf node as shown in Figure 9.45.

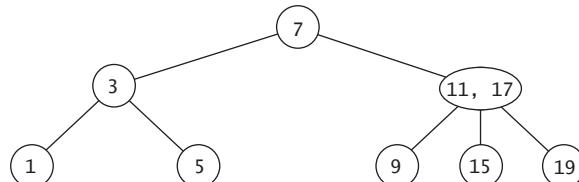
**FIGURE 9.42**

Removing 13 from a 2–3 Tree



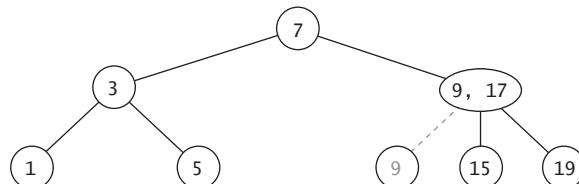
**FIGURE 9.43**

2–3 Tree after Redistribution of Nodes Resulting from Removal



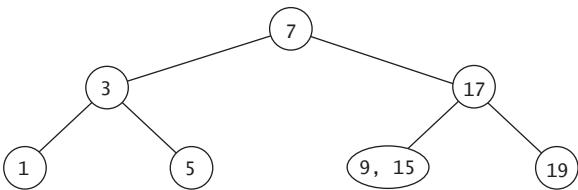
**FIGURE 9.44**

Removing 11 from the 2–3 Tree (Step 1)



**FIGURE 9.45**

2–3 Tree after  
Removing 11

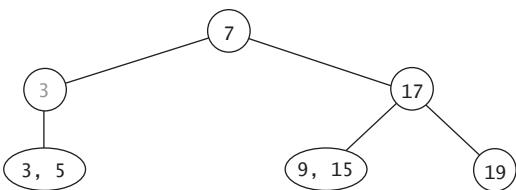


Finally, let's consider the case in which we remove the value 1 from Figure 9.45. First, 1's parent (3) and its right sibling (5) are merged to form a 3-node, as shown in Figure 9.46. This has the effect of deleting 3 from the next higher level. Therefore, the process repeats, and 3's parent (7) and 7's right child (17) are merged, as shown in Figure 9.47. The merged node becomes the root.

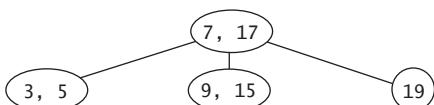
The 2–3 tree served as an inspiration for the more general B-tree and 2–3–4 tree. Rather than show an implementation of the 2–3 tree, which has some rather messy complications, we will describe and implement the more general B-tree in the next section.

**FIGURE 9.46**

After Removing 1  
(Intermediate Step)

**FIGURE 9.47**

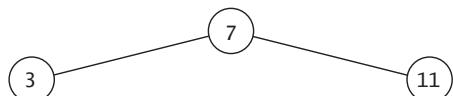
After Removing 1  
(Final Form)



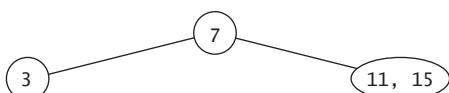
## EXERCISES FOR SECTION 9.4

### SELF-CHECK

1. Show the tree after inserting each of the following values one at a time: 1, 4, 9.



2. Show the tree after inserting each of the following one at a time: 9, 13.



3. Show the 2–3 tree that would be built for the sentence “Now is the time for all good men to come to the aid of their party”.



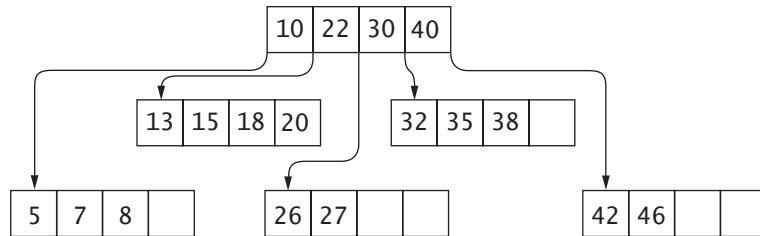
## 9.5 B-Trees and 2–3–4 Trees

The 2–3 tree was the inspiration for the more general B-tree, which allows up to  $n$  children per node, where  $n$  may be a very large number. The B-tree was designed for building indexes to very large databases stored on a hard disk. The 2–3–4 tree is a special case of a B-tree.

### B-Trees

In the 2–3 tree, a 2-node has two children and a 3-node has three children. In the B-tree, the maximum number of children is the order of the B-tree, which we will represent by the variable `order`. Other than the root, each node has between `order/2` and `order` children. The number of data items in a node is 1 less than the number of children. The data items in each node are in increasing order. Figure 9.48 shows an example of a B-tree with `order` equal to 5. The first link from a node connects to a subtree with values smaller than the parent's smallest value (10 for the root node); the last link from a node connects to a subtree with values greater than the parent's largest value (40 for the root node); the other links are to subtrees with values between each pair of consecutive values in the parent node (e.g.,  $> 10$  and  $< 22$  or  $> 22$  and  $< 30$ ).

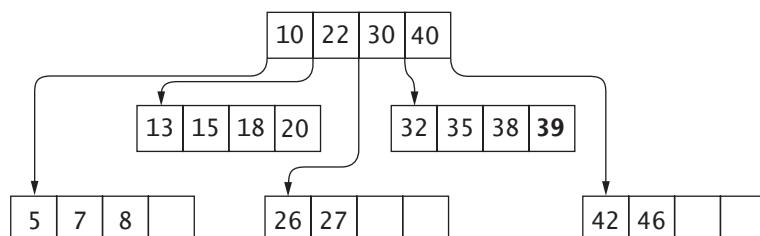
**FIGURE 9.48**  
Example of a B-Tree



B-trees were developed to store indexes to databases on disk storage. Disk storage is broken into blocks, and the nodes of a B-tree are sized to fit in a block, so each disk access to the index retrieves exactly one B-tree node. The time to retrieve a block is large compared to the time required to process it in memory, so by making the tree nodes as large as possible, we reduce the number of disk accesses required to find an item in the index. Assuming a block can store a node for a B-tree of order 200, each node would store at least 100 items. This would enable  $100^4$  or 100 million items to be accessed in a B-tree of height 4.

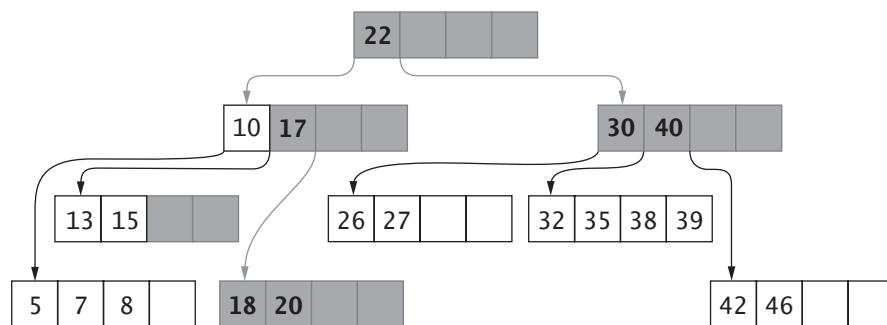
The insertion process for a B-tree is similar to that of a 2–3 tree, and each insertion is into a leaf. For Figure 9.48, a number less than 10 would be inserted into the leftmost leaf; a number greater than 40 would be inserted into the rightmost leaf; and numbers between 11 and 39 would be inserted into one of the interior leaves. A simple case is insertion of the number 39 into the fourth child of the root node (shown in bold in Figure 9.49).

**FIGURE 9.49**  
B-Tree after Inserting 39



However, if the leaf being inserted into is full, it is split into two nodes, each containing approximately half the items, and the middle item is passed up to the split node's parent. If the parent is full, it is split and its middle item is passed up to its parent, and so on. If a node being split is the root of the B-tree, a new root node is created, thereby increasing the height of the B-tree. The children of the new root will be the two nodes that resulted from splitting the old root. Figure 9.50 shows the B-tree after inserting 17. Because 17 is between 10 and 22, it should be inserted into the second leaf node {13, 15, 18, 20}, with 17 as the new third or middle item {13, 15, 17, 18, 20}. However, the original leaf node was full, so it is split, and its new middle item, 17, is passed up to the root node {10, 17, 22, 30, 40}. Because the original root node was full, it is also split and its new middle item, 22, is passed up as the first item in a new root node, thereby increasing the height of the tree. It is interesting that the B-tree grows by adding nodes to the top of the tree instead of adding them at the bottom.

**FIGURE 9.50**  
Inserting into a B-Tree



## Implementing the B-Tree

The `Node` class holds up to order-1 data items and order references to children. The array `data` stores the data, and the array `children` stores the references to the children. The number of data items currently stored is indicated by `size`.

```

/** A Node represents a node in a B-tree. */
private static class Node<E> {
    // Data Fields

    /** The number of data items in this node */
    private int size = 0;
    /** The information */
    private E[] data;
    /** The links to the children. child[i] refers to the subtree of
        children < data[i] for i < size and to the subtree
        of children > data[size-1] for i == size
    */
    private Node<E>[] child;

    /** Create an empty node of size order
        @param order The size of a node
    */
    @SuppressWarnings("unchecked")
    public Node(int order) {
        data = (E[]) new Comparable[order-1];
        child = (Node<E>[]) new Node[order];
        size = 0;
    }
}

```

The declaration for the B-tree class begins as follows:

```
/** An implementation of the B-tree. A B-tree is a search tree
 * in which each node contains n data items where n is between
 * (order-1)/2 and order-1. (For the root, n may be between 1
 * and order-1.) Each node not a leaf has n+1 children. The tree is
 * always balanced in that all leaves are on the same level, i.e.,
 * of the path from the root to a leaf is constant.
 * the length @author Koffman and Wolfgang
 */
public class BTTree<E extends Comparable<E>>
    // Nested class
    /** A Node represents a node in a B-tree. */
    private static class Node<E> {
        . .
    }

    /** The root node. */
    private Node<E> root = null;
    /** The maximum number of children of a node */
    private int order;

    /** Construct a B-tree with node size order
     * @param order the size of a node
     */
    public BTTree(int order) {
        this.order = order;
        root = null;
    }
}
```

The `insert` method is very similar to that for the 2–3 tree. Method `insert` searches the current node for the item. If the item is found, insertion is not possible, so it returns `false`. If the item is not found in the current node, it follows the appropriate link until it reaches a leaf and then inserts the item into that leaf. If the leaf is full, it is split. In the 2–3 tree, we described this process as a virtual insertion into the full leaf, and then the middle data value is used as the parent of the split-off node. This parent value was then inserted into the parent node during the return process of the recursion.

### Algorithm for insertion

1. **if** the root is `null`
2.     Create a new Node that contains the inserted item.
3. **else** search the local root for the item
4.     **if** the item is in the local root
5.         return `false`
6.     **else**
7.         **if** the local root is a leaf
8.             **if** the local root is not full
9.                 insert the new item
10.                 return `null` as the `new_child` and `true` to indicate successful insertion
11.     **else**
12.         split the local root
13.         return the `newParent` and a `newChild` and `true` to indicate successful insertion
14. **else** (*note: this else goes with if on line 7*)

15. recursively call the `insert` method
16. **if** the returned `newChild` is not `null`
17.     **if** the local root is not full
18.         insert the `newParent` and `newChild` into the local root
19.         return `null` as the `newChild` and `true` to indicate successful insertion
20.     **else**
21.         split the local root
22.         return the `newParent` and the `newChild` and `true` to indicate successful insertion
23.     **else**
24.         return the success/fail indicator for the insertion

In this algorithm, we showed multiple return values. There is the boolean return value that indicates success or failure of the insertion. There is the `newParent` of the split-off node, and there is the split-off node, which we call the `newChild`. We implement this by using the return value from the `insert` function as the success/fail indicator, and `newParent` and `newChild` are private data fields in the `BTree` class. If there is no new child, `newChild` is set to `null`.

## Code for the `insert` Method

The code for the `insert` method is shown in Listing 9.5. We use a binary search to locate the item in the local root. The `binarySearch` method returns the index of the item if it is present or the index of the position where the item should be inserted.

We need to test to see whether the local root is full. If it is not full, we can insert the item into the local root; otherwise, we need to split the local root. In either case, we return `true`.

```
if (root.size < order-1) {
    insertIntoNode(root, index, item, null);
    newChild = null;
} else {
    splitNode(root, index, item, null);
}
return true;
```

If the local root is not a leaf, then we recursively call the `insert` method using `local.child[index]` as the root argument. Upon return from the recursion, we test the value of `newChild`. If it is `null`, we return the result of the insertion. If it is not `null` and the local root is not full, we insert the `newParent` and `newChild` into the local root; otherwise, we split the local root.

```
boolean result = insert(root.child[index], item);
if (newChild != null) {
    if (root.size < order-1) {
        insertIntoNode(root, index, newParent, newChild);
        newChild = null;
    } else {
        splitNode(root, index, newParent, newChild);
    }
}
return result;
```

The `insertIntoNode` and `splitNode` methods are described next.

**LISTING 9.5**The `insert` Function from `BTTree.java`

```

.....
LISTING 9.5
The insert Function from BTTree.java

/** Recursively insert an item into the B-tree. Inserted
 * item must implement the Comparable interface
 * @param root The local root
 * @param item The item to be inserted
 * @return true if the item was inserted,
 *         false if the item is already in the tree
 */
private boolean insert(Node<E> root, E item) {
    int index = binarySearch(item, root.data, 0, root.size);
    if (index != root.size && item.compareTo(root.data[index]) == 0) {
        return false;
    }
    if (root.child[index] == null) {
        if (root.size < order-1) {
            insertIntoNode(root, index, item, null);
            newChild = null;
        } else {
            splitNode(root, index, item, null);
        }
        return true;
    } else {
        boolean result = insert(root.child[index], item);
        if (newChild != null) {
            if (root.size < order-1) {
                insertIntoNode(root, index, newParent, newChild);
                newChild = null;
            } else {
                splitNode(root, index, newParent, newChild);
            }
        }
        return result;
    }
}

```

**The `insertIntoNode` Method**

The `insertIntoNode` method shifts the data and child values to the right and inserts the new value and child at the indicated index.

```

/** Method to insert a new value into a node
 * @pre node.data[index-1] < item < node.data[index];
 * @post node.data[index] == item and old values are moved right one position
 * @param node The node to insert the value into
 * @param index The index where the inserted item is to be placed
 * @param item The value to be inserted
 * @param child The right child of the value to be inserted
 */
private void insertIntoNode(Node<E> node, int index, E obj, Node<E> child) {
    for (int i = node.size; i > index; i--) {
        node.data[i] = node.data[i - 1];
        node.child[i + 1] = node.child[i];
    }
}

```

```

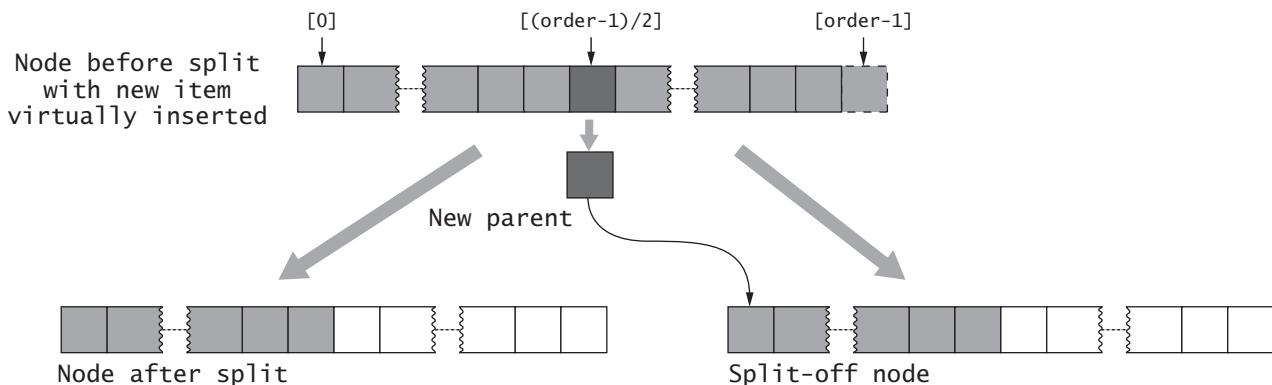
        node.data[index] = obj;
        node.child[index + 1] = child;
        node.size++;
    }
}

```

## The splitNode Method

The `splitNode` method will perform the virtual insertion of the new item (and its child) into the node and split it so that half of the items remain in the node being split and the rest are moved to the split-off node. The middle value becomes the parent of the split-off node. The middle value is passed up to the parent node, which still links to the node that was split. This is illustrated in Figure 9.51.

**FIGURE 9.51**  
Splitting the Node



The code for the `splitNode` method is shown in Listing 9.6. Since we cannot insert the new item into the node before it is split, we need to do the split first in such a way that space is available in either the original node or the split-off node for the new item. After the split, we keep half of the items in the original node and move the other half to the split-off node. The number of items to keep is `order-1`; thus half of them is  $(\text{order}-1)/2$ , and the number of items to move is  $(\text{order}-1) - (\text{order}-1)/2$ . The reason that this is not simply  $(\text{order}-1)/2$  is that `order-1` may be an odd number. Thus, we move  $(\text{order}-1) - (\text{order}-1)/2$  items, unless the new item is to be inserted into the split-off node, in which case we move one fewer item. The number of items to be moved is computed using the following statements:

```

// Determine number of items to move
int numToMove = (order-1) - ((order-1) / 2);
// If insertion point is in the right half, move one less item
if (index > (order-1) / 2) {
    numToMove--;
}

```

The `System.arraycopy` method is then used to move the data and the corresponding children.

```

// Move items and their children
System.arraycopy(node.data, order - numToMove - 1,
                newChild.data, 0, numToMove);

```

```

System.arraycopy(node.child, order - numToMove,
                 newChild.child, 1, numToMove);
node.size = order - numToMove - 1;
newChild.size = numToMove;

```

Now we are ready to insert the new item and set the `newChild.child[0]` reference. There are three cases: the item is to be inserted as the middle item, the item is to be inserted into the original node, and the item is to be inserted into the `newChild`. If the item is to be inserted into the middle, then it becomes the `newParent`, and its child becomes `newChild.child[0]`.

```

if (index == ((order-1) / 2)) { // Insert as middle item
    newParent = item;
    newChild.child[0] = child;
}

```

Otherwise we can use the `insertIntoNode` method to insert the item into either the original node or the `newChild` node.

```

if (index < ((order-1) / 2)) { // Insert into the left
    insertIntoNode(node, index, item, child);
} else {
    insertIntoNode(newChild, index - ((order-1) / 2) - 1, item, child);
}

```

In either case, after the insert, the last item in the original node becomes the `newParent` and its child becomes `newChild.child[0]`

```

// The rightmost item of the node is the new parent
newParent = node.data[node.size - 1];
// Its child is the left child of the split-off node
newChild.child[0] = node.child[node.size];
node.size--;

```

.....

#### **LISTING 9.6**

Function `splitNode` from `BTree.java`

```

private void splitNode(Node<E> node, int index, E item, Node<E> child) {
    // Create new child
    newChild = new Node<E>(order);
    // Determine number of items to move
    int numToMove = (order-1) - ((order-1) / 2);
    // If insertion point is in the right half, move one less item
    if (index > (order-1) / 2) {
        numToMove--;
    }

    // Move items and their children
    System.arraycopy(node.data, order - numToMove - 1,
                    newChild.data, 0, numToMove);
    System.arraycopy(node.child, order - numToMove,
                    newChild.child, 1, numToMove);
    node.size = order - numToMove - 1;
    newChild.size = numToMove;

    // Insert new item
    if (index == ((order-1) / 2)) { // Insert as middle item
        newParent = item;
        newChild.child[0] = child;
    } else {
        if (index < ((order-1) / 2)) { // Insert into the left
            insertIntoNode(node, index, item, child);
        } else {

```

```

        insertIntoNode(newChild, index - ((order-1) / 2) - 1, item, child);
    }
    // The rightmost item of the node is the new parent
    newParent = node.data[node.size - 1];
    // Its child is the left child of the split-off node
    newChild.child[0] = node.child[node.size];
    node.size--;
}

// Remove items and references to moved items
for (int i = node.size; i < node.data.length; i++) {
    node.data[i] = null;
    node.child[i + 1] = null;
}
}
}

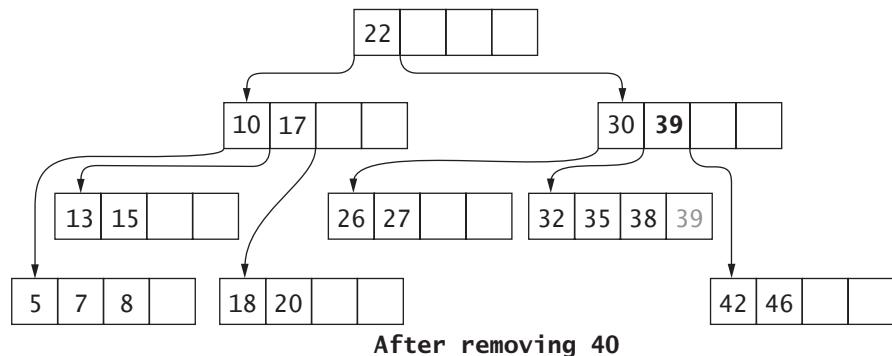
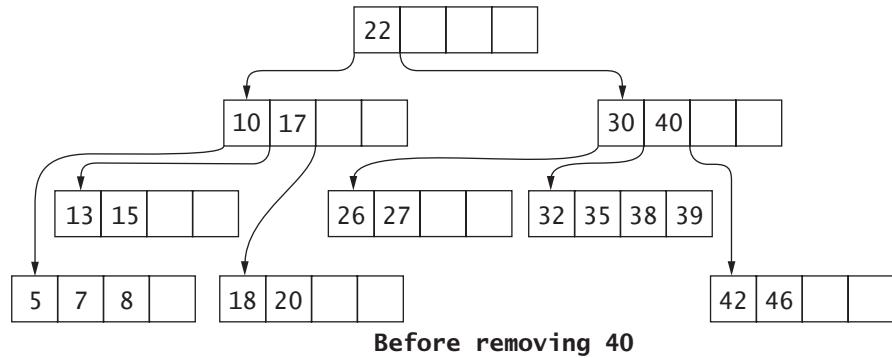
```

## Removal from a B-Tree

Removing an item from a B-tree is a generalization of removing an item from a 2–3 tree. The simpler case occurs when the item to be removed is in a leaf; in this case, it is deleted from the leaf. However, if the item to be removed is in an interior node, it can't be deleted simply because that would damage the B-tree. To retain the B-tree property, the item must be replaced by its inorder predecessor (or its inorder successor), which is in a leaf. As an example, Figure 9.52 shows the tree that would be formed when 40 is removed from the tree in Figure 9.50 and is replaced with its inorder predecessor (39).

**FIGURE 9.52**

Removing 40 from B-Tree of Figure 9.50



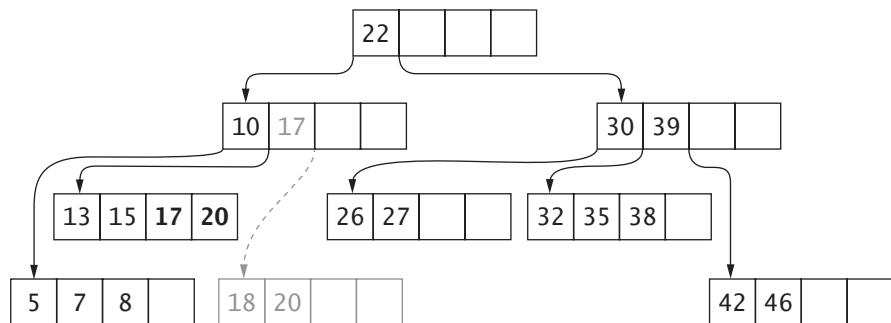
In both cases described above, if the removal of an item from a leaf node that is less than half full, this would violate a property of the B-tree (only the root node can be less than half full). To correct this, items from a sibling node and parent are redistributed into that leaf. However, if the sibling is itself exactly half full, the leaf, its parent item, and sibling are merged into a single node, deleting the parent item from the parent node. If the parent node is now half full, the process of node redistribution or merging is repeated during the recursive return process. The merging process may reduce the height of the B-tree.

We illustrate this process by deleting item 18 from the bottom B-tree in Figure 9.52. The leaf node that contained 18 would have only one item (20), so we merge it with its parent and left sibling into a new full node {13, 15, 17, 20} as shown in Figure 9.53.

The problem is that the parent of {13, 15, 17, 20} has only one item (10), so it is merged with its parent and right sibling to form a new root node {10, 22, 30, 40} as shown in Figure 9.54. Note that the height of the resulting B-tree has also been reduced by 1.

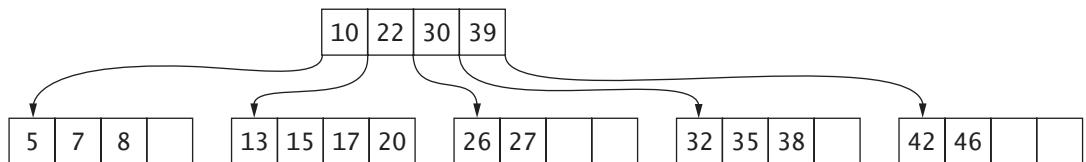
**FIGURE 9.53**

Step 1 of Removing 18 from Bottom B-Tree of Figure 9.52



**FIGURE 9.54**

Step 2 of Removing 18 from B-Tree of Figure 9.52



## B+ Trees

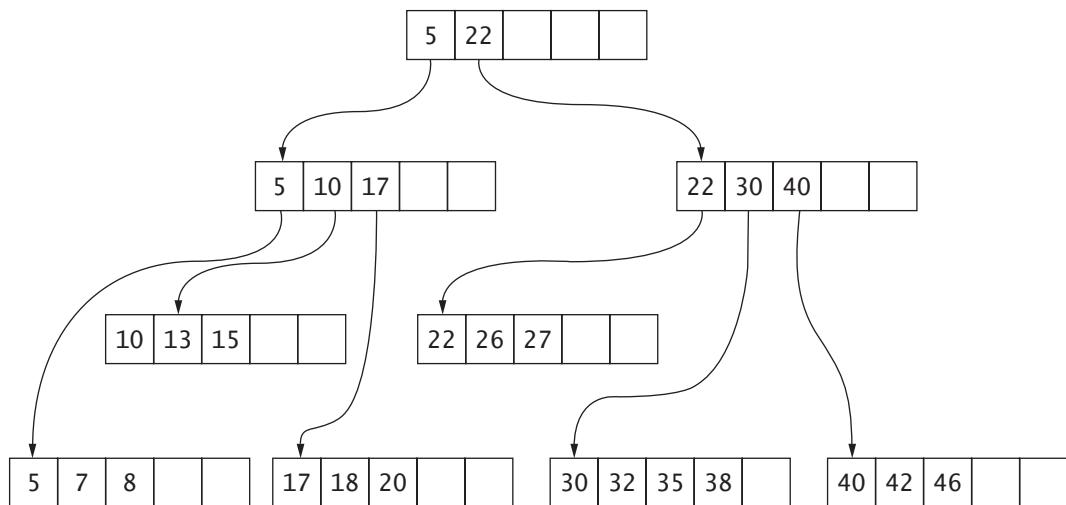
We stated earlier that B-trees were developed and are still used to create indexes for databases. The Node is stored on a disk block, and the pointers are pointers to disk blocks instead of being memory addresses. The E is a key-value pair, where the value is also a pointer to a disk block. Since in the leaf nodes all of the child pointers are `null`, there is a significant amount of wasted space. A modification to the B-tree, known as the B+ tree, was developed to reduce this wasted space. In the B+ tree, the leaves contain the keys and pointers to the corresponding values. The internal nodes only contain keys and pointers to children. In the B-tree there are order pointers to children and order-1 values. In the B+ tree, the parent's value is repeated as the first value; thus there are order pointers and order keys. An example of a B+ tree is shown in Figure 9.55.

## 2–3–4 Trees

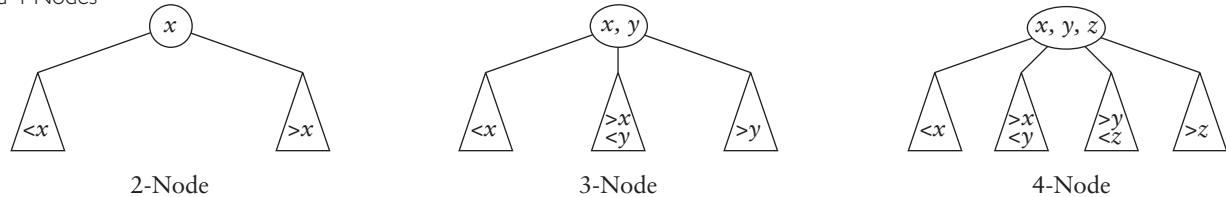
2–3–4 trees are a special case of the B-tree where order is fixed at 4. We refer to such a node as a 4-node (see Figure 9.56). This is a node with three data items and four children. Figure 9.57 shows an example of a 2–3–4 tree.

**FIGURE 9.55**

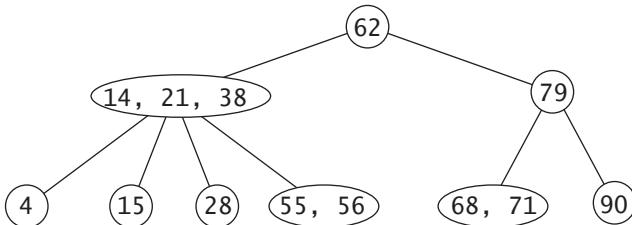
Example of a B+ Tree

**FIGURE 9.56**

2-, 3-, and 4-Nodes

**FIGURE 9.57**

Example of a 2-3-4 Tree



Fixing the capacity of a node at three data items simplifies the insertion logic. We can search for a leaf in the same way as for a 2–3 tree or a B-tree. If a 4-node is encountered at any point, we will split it, as discussed subsequently. Therefore, when we reach a leaf, we are guaranteed that there will be room to insert the item.

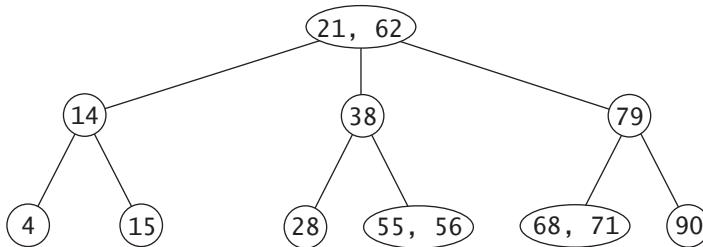
For the 2–3–4 tree shown in Figure 9.57, a number larger than 62 would be inserted in a leaf node in the right subtree. A number between 63 and 78, inclusive, would be inserted in the 3-node (68, 71), making it a 4-node. A number larger than 79 would be inserted in the 2-node (90), making it a 3-node.

When inserting a number smaller than 62 (say, 25), we would encounter the 4-node (14, 21, 38). We would immediately split it into two 2-nodes and insert the middle value (21) into the parent (62) as shown in Figure 9.58. Doing this guarantees that there will be room to insert the new item. We perform the split from the parent level and immediately insert the middle item from the split child into the parent node. Because we are guaranteed that the parent is not a 4-node, we will always have room to do this. We do not need to propagate a child or its parent back up the recursion chain. Consequently, the recursion becomes tail recursion.

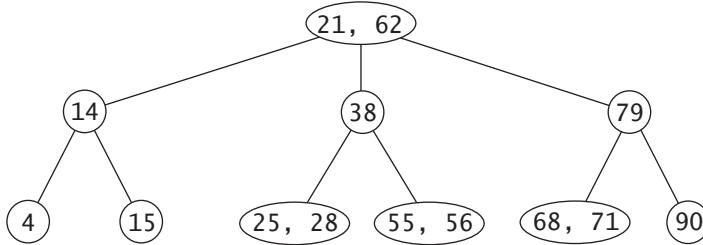
Now we can insert 25 as the left neighbor of 28 as shown in Figure 9.59.

**FIGURE 9.58**

Result of Splitting  
a 4-Node

**FIGURE 9.59**

2-3-4 Tree after  
Inserting 25



In this example, splitting the 4-node was not necessary. We could have merely inserted 25 as the left neighbor of 28. However, if the leaf being inserted into was a 4-node, we would have had to split it and propagate the middle item back up the recursion chain, just as we did for the 2–3 tree. Always splitting a 4-node when it is encountered results in prematurely splitting some nodes, but it simplifies the algorithm and has minimal impact on the overall performance.

## Relating 2–3–4 Trees to Red–Black Trees

A Red–Black tree is a binary-tree equivalent of a 2–3–4 tree. A 2-node is a black node (see Figure 9.60). A 4-node is a black node with two red children (see Figure 9.61). A 3-node can be represented as either a black node with a left red child or a black node with a right red child (see Figure 9.62).

Suppose we want to insert a value  $z$  that is greater than  $y$  into the 3-node shown at the top of Figure 9.62 (tree with black root  $y$ ). Node  $z$  would become the red right child of black node  $y$ , and the subtree with label  $>y$  would be split into two parts, giving Red–Black tree and the 4-node shown in Figure 9.61.

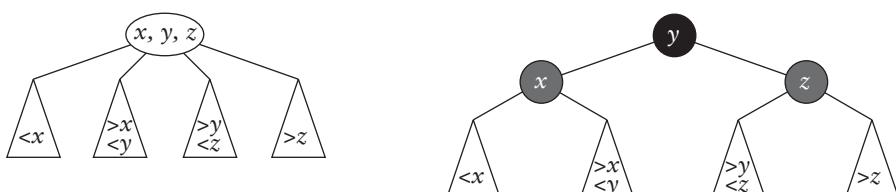
Suppose, on the other hand, we want to insert a value  $z$  that is between  $x$  and  $y$  into the 3-node shown at the bottom of Figure 9.62 (tree with black root  $x$ ). Node  $z$  would become

**FIGURE 9.60**

A 2-Node as a Black  
Node in a Red–Black  
Tree

**FIGURE 9.61**

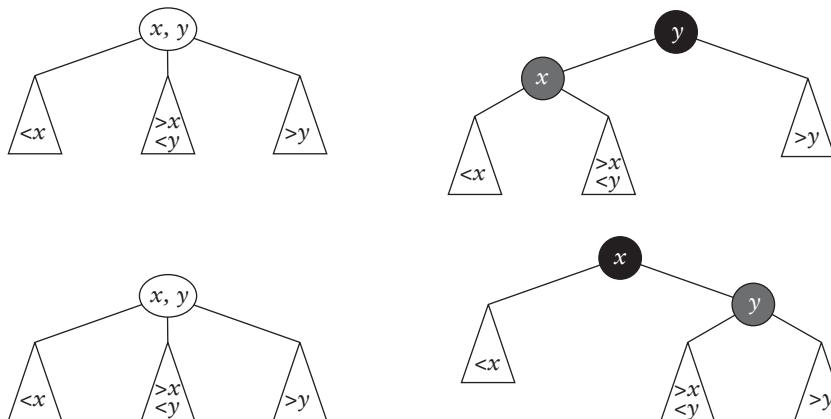
A 4-Node as a Black  
Node with Two Red  
Children in a Red–Black  
Tree



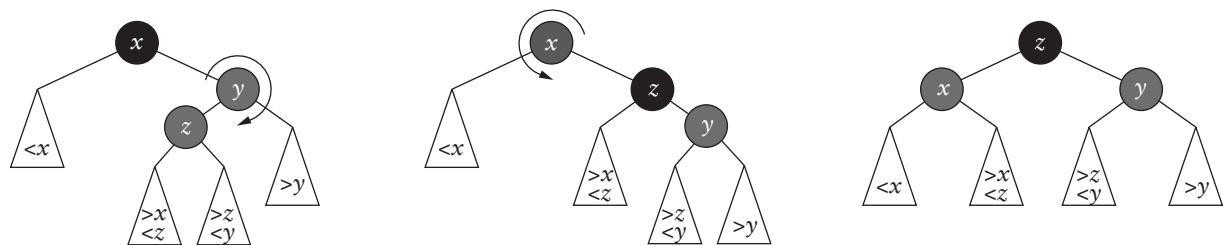
the red left child of red node  $y$  (see the left diagram in Figure 9.63), and a double rotation would be required. First rotate right around  $y$  (the middle diagram) and then rotate left around  $x$  (the right diagram). This corresponds to the situation shown in Figure 9.64 (a 4-node with  $x, z, y$ ).

**FIGURE 9.62**

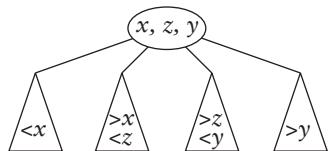
A 3-Node as a Black Node with One Red Child in a Red–Black Tree

**FIGURE 9.63**

Inserting into the Middle of a 3-Node (Red–Black Tree Equivalent)

**FIGURE 9.64**

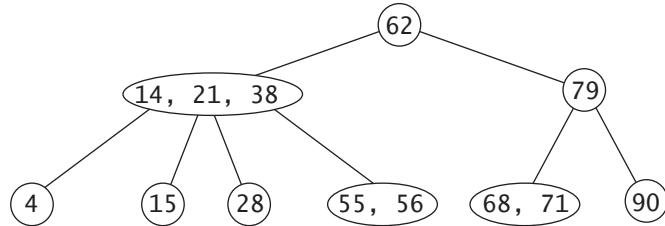
Inserting into the Middle of a 3-Node (2–3–4 Tree)



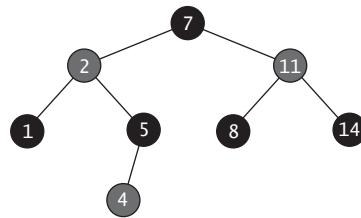
## EXERCISES FOR SECTION 9.5

### SELF-CHECK

1. Draw a B-tree with order 5 that stores the sequence of integers: 20, 30, 8, 10, 15, 18, 44, 26, 28, 23, 25, 43, 55, 36, 44, 39.
2. Remove items 30, 26, 15, and 17 from the B-tree in Figure 9.50.
3. Draw the B+ tree that would be formed by inserting the integers shown in Exercise 1.
4. Show the tree after inserting each of the following values one at a time: 1, 5, 9, and 13.



5. Build a 2–3–4 tree to store the words in the sentence “Now is the time for all good men to come to the aid of their party”.
6. Draw the Red–Black tree equivalent of the 2–3–4 tree shown in Exercise 5.
7. Draw the 2–3–4 tree equivalent to the following Red–Black tree.



#### PRO GRAMMING

1. Code the `binarySearch` method of the `BTree`.
2. Code the `insert` method for the 2–3–4 tree. The rest of the 2–3–4 tree implementation can be done by taking the B-tree implementation and fixing order at 4.



## 9.6 Skip-Lists

The skip-list is another data structure that can be used as the basis for the `NavigableSet` or `NavigableMap` and as a substitute for a balanced tree. Like a balanced tree, it provides for  $O(\log n)$  search, insert, and remove. It has the additional advantage over the Red–Black tree-based `TreeSet` in that concurrent references are easier to achieve. With the `TreeSet` class, if two threads have iterators to the set and one thread makes a modification to the set, then the iterators are invalid and will throw the `ConcurrentModificationException` when next referenced. The `ConcurrentSkipListSet` and `ConcurrentSkipListMap` were introduced in Java 6. The concurrency features are beyond the scope of this text, but we will describe the basic structure of the skip-list and the algorithms for search, insertion, and removal.

### Skip-List Structure

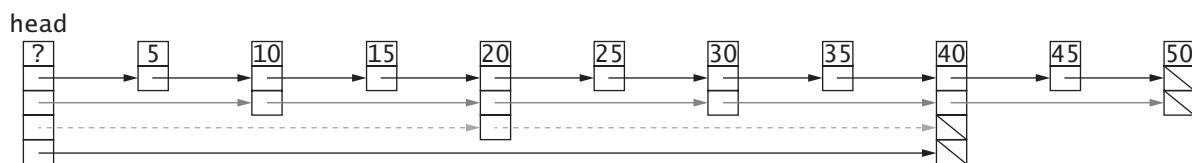
A skip-list is a list of lists. Each node in a list contains a data element with a key, and the elements in each list are in increasing order by key. Unlike the usual list node, which has a single forward link to the next node, the nodes in a skip list have a varying number of forward links. The number of such links is determined by the level of a node. A level- $m$  node has  $m$  forward links. When a new data element is inserted in a skip-list, a new node is inserted to store the element. The node's level is chosen randomly in such a way that approximately 50 percent are level 1 (one forward link), 25 percent are level 2 (two forward links), 12.5 percent are level 3, and so on.

Figure 9.65 shows a skip-list with 10 data elements (only the keys are shown). In this skip-list, there are five level-1 nodes (5, 15, 25, 35, and 45), three level-2 nodes (10, 30, and 50), one level-3 node (20), and one level-4 node (40). If we look at node-20, we see that its level-1 link (the top one) is to node-25, its level-2 link is to node-30, and its level-3 link (the bottom one) is to node-40. The last node in the skip-list, node-50, is a level-2 node, and both links are **null**.

The *level* of a skip-list is defined as its highest node level, or 4 for Figure 9.65. A level-4 skip-list has individual lists of level 4, level 3, level 2, and level 1. The level-1 list consists of every node (5, 10, 15, etc.); the level-2 list (in gray) consists of every other node (10, 20, 30, 40, 50); the level-3 list (in dashed gray) consists of nodes-20 and 40; and the level-4 list consists of node-40. This is an *ideal skip-list*; most skip-lists will not have this exact structure, but their behavior will be similar.

**FIGURE 9.65**

Ideal Skip-List



## Searching a Skip-List

To search a skip-list, we start by looking for our target in the highest-level list. This list always has the fewest elements. If the target is in this list, the search is successful. If not, we stop the search in the current list at the element that is the predecessor of the target. Then we continue the search in the list with level one less than the current list, starting where we left off (the predecessor to the target). We continue this process until we either find the target or reach the level-1 list. If the target is not in the level-1 list (the list of all elements), then it is not present.

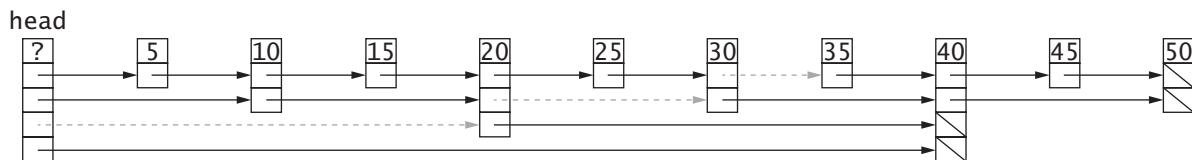
Figure 9.66 shows the search path for item 35. We start by searching the level-4 list. Its first node is 40 ( $>35$ ), so we move to the level-3 list. Its first node is 20 and its second node is 40 ( $>35$ ), so we stop at node-20. We then follow node-20's level-2 link, which points to a node whose value is 30. Advancing to the 30-node, we see that its level-2 link is to 40 ( $>35$ ), so we stop at node-30. We then follow its level-1 link, which points to 35, our target. The dashed gray links point to the predecessor of 35 in each list.

The algorithm for searching a skip-list follows:

1. Let  $m$  be the highest-level node.
2. **while**  $m > 0$

**FIGURE 9.66**

Searching for 35 in a Skip-List



3. Following the level- $m$  links, find the node with the largest value that is less than or equal to the target.
4. If it is equal to the target, the target has been found—exit loop.
5. Set  $m$  to  $m - 1$
6. If  $m = 0$ , the target is not in the list.

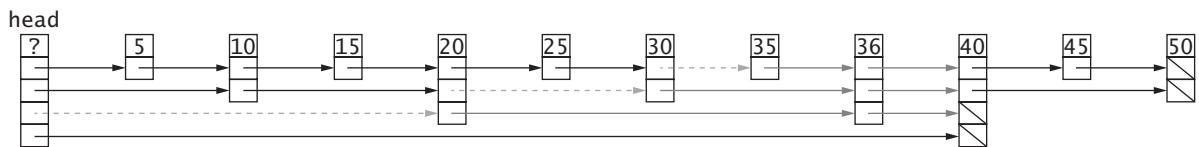
## Performance of a Skip-List Search

Because the first list we search has the fewest elements (generally one or two) and each lower-level list we search has approximately half as many elements as the current list, the search performance is similar to that of a binary search:  $O(\log n)$ , where  $n$  is the number of nodes in the skip-list. We discuss performance further at the end of this section.

## Inserting into a Skip-List

If the search algorithm fails to find the target, it will find its predecessor in the level-1 list, which is the target's insertion point. Therefore, if we keep track of the last node we visited at each level, we know where to insert a new node containing the target. The question then is “What level should this new node be?” The answer is it is chosen at random, based on the number of items currently in the skip-list. The random number is chosen with a logarithmic distribution. Half the time a level-1 node is chosen; a quarter of the time a level-2 node is chosen, and  $1/2^m$  time a level- $m$  node is chosen. To insert 36 into the skip-list shown in Figure 9.66, we would follow the same path as to locate 35. Along the way we would have recorded the last node visited at each level: the 20 node at level 3, the 30 node at level 2, and the 35 node at level 1. If the random number generator returns a 3, the new node will be a level-3 node. The level-3 link in the 20 node will be set to point to the new 36 node, and the level-3 link in the new 36 node will be set to point to node 40. The level-2 link in the 30 node will be set to point to the new 36 node, and the level-2 link in the new 36 node will be set to point to node 40. Finally, the level-1 link in the 35 node will be set to point to the new 36 node, and the level-1 link in the new 36 node will be set to point to the 40 node. The result is shown in Figure 9.67. The gray links show the new entries in the skip list.

**FIGURE 9.67**  
After Insertion of 36



## Increasing the Height of a Skip-List

The skip-list shown in the figures is a level-4 skip-list. Such a skip-list can efficiently hold up to 15 items. When a 16th item is inserted, the level is increased by 1. A level- $m$  skip-list can hold between  $2^{m-1}$  and  $2^m - 1$  items.

## Implementing a Skip-List

Next, we show how to implement a skip-list. We start with the `SLNode` class. When a new level- $m$  node is created, the declaration for links allocates array links with subscripts 0 through  $m-1$ . We use the same kind of node for the head node as the rest of the nodes in the skip-list. However, its data field value is not defined (indicated by ? in the figures).

```

/** Static class to contain the data and the links */
static class SLNode<E> {
    SLNode<E>[] links;
    E data;

    /** Create a node of level m */
    SLNode (int m, E data) {
        links = (SLNode<E>[]) new SLNode[m]; // create links
        this.data = data; // store item
    }
}

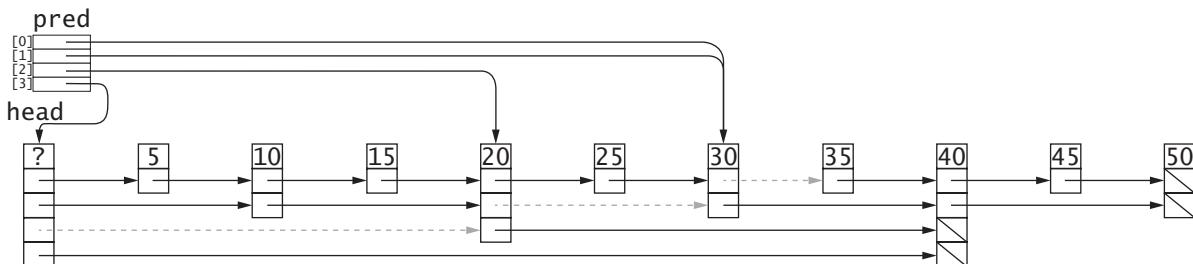
```

## Searching a Skip-List

Since insertion involves the same algorithm as searching to find the insertion point, we define a common method, `search`, which will return an array `pred` of references to the `SLNodes` at each level that were last examined in the search. Because array subscripts start at 0, `pred[i]` references the predecessor in the level-(*i*+1) list of the target. The level-(*i*+1) link for the node referenced by `pred[i]` is greater than or equal to the target or is `null`. The result of searching for 35 is shown in Figure 9.68. Array element `pred[3]` references the head node since the level-4 link references the node 40, which is greater than 35. Array element `pred[2]` references node 20 whose level-3 link references node 40. Element `pred[1]` references node 30 whose level-2 link references 40 and whose level-1 link references 35. Finally, `pred[0]` also references node 30. By examining `links[0]` of the node referenced by `pred[0]`, we can determine whether the target is in the skip-list.

**FIGURE 9.68**

Result of Search for 35



Listing 9.7 shows the code for searching a skip-list. Two methods are shown: `search` and `find`. Method `find` calls `search` to perform the search. The result returned by `search` is the array of references to the predecessor of `target` in each list. The `SkipList` data field `head` references an array of links to the first element of each list in the skip-list, where `head.links[i]` references the first node in the level-(*i*+1) list. Method `search` begins by setting `current` to `head`. The `for` loop ensures that each list is processed beginning with the highest-level list. The `while` loop advances `current` down the level-*i* list until `current` references the last node in the list or `current` references a node that is linked to either `target` or to the first element greater than `target`. The last value of `current` is saved in `pred[i]`, and the `for` loop sets *i* to *i*-1, causing the next list to be searched.

After `search` returns the array of predecessor references, method `find` examines the level-1 link of the predecessor saved for the level-1 list. If this link is `null` or if it references a node greater than the `target`, `null` is returned (`target` is not in the list); otherwise, the data stored in the node referenced by the level-1 link is returned as the search result.

**LISTING 9.7**

Methods for Searching a Skip-List

```

.....
LISTING 9.7
Methods for Searching a Skip-List

@SuppressWarnings("unchecked")
/** Search for an item in the list
 * @param item The item being sought
 * @return A SLNode array which references the predecessors
 *         of the target at each level.
 */
private SLNode<E>[] search (E target) {
    SLNode<E>[] pred = (SLNode<E>[][]) new SLNode[maxLevel];
    SLNode<E> current = head;
    for (int i = current.links.length-1; i >= 0; i--) {
        while (current.links[i] != null
            && current.links[i].data.compareTo(target) < 0) {
            current = current.links[i];
        }
        pred[i] = current;
    }
    return pred;
}

/** Find an object in the skip-list
 * @param target The item being sought
 * @return A reference to the object in the skip-list that matches
 *         the target. If not found, null is returned.
 */
public E find(E target) {
    SLNode<E>[] pred = search(target);
    if (pred[0].links[0] != null &
        pred[0].links[0].data.compareTo(target) == 0) {
        return pred[0].links[0].data;
    } else {
        return null;
    }
}

```

## Insertion

The result of calling search for 36 is shown in Figure 9.69. It is the same as the search for 35 except that pred[0] references 35 since 36 will follow 35. At each level ( $i+1$ ), the new node will be inserted between the node referenced by pred[i] and the node referenced by the predecessor node's links[i]. If the result of search is saved in the array pred and newNode is the new node, the code to splice newNode into the linked list is

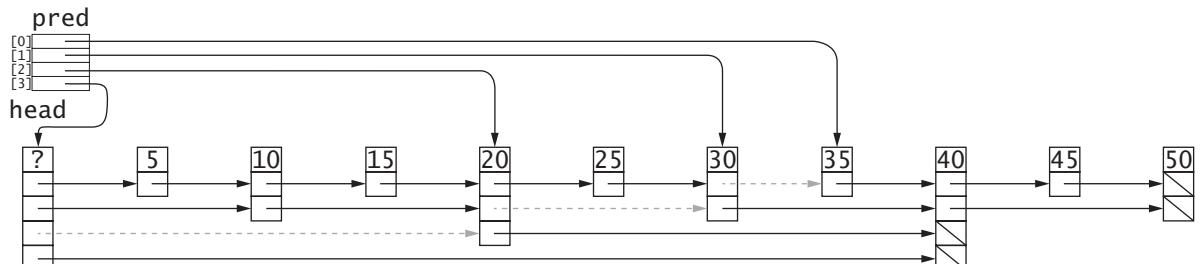
```

newNode.links[i] = pred[i].links[i];
pred[i].links[i] = newNode;

```

**FIGURE 9.69**

Result of Search for 36



## Determining the Size of the Inserted Node

We define `maxCap` as the smallest power of 2 that is greater than the current skip-list size. Therefore, `maxLevel`, the skip-list level is  $\log_2 \text{maxCap}$ . The random number class, `Random`, has a method `nextInt(int n)`, which returns a uniformly distributed random integer from 0 up to, but not including,  $n$ . If we compute the  $\log_2$  of this number plus 1, we get a logarithmically distributed random number between 1 and `maxLevel` (the skip-list level). This number has the opposite distribution of what we desire: 1/2 the numbers will be `maxLevel-1`, 1/4 of the numbers will be `maxLevel-2`, and so on. Thus, we subtract the result from `maxLevel`.

```
/** Natural Log of 2 */
static final double LOG2 = Math.log(2.0);

/** Method to generate a logarithmic distributed integer between
1 and maxLevel. i.e., 1/2 of the values returned are 1, 1/4
are 2, 1/8 are 3, etc.
@return a random logarithmic distributed int between 1 and
maxLevel
*/
private int logRandom() {
    int r = rand.nextInt(maxCap);
    int k = (int) (Math.log(r + 1) / LOG2);
    if (k > maxLevel - 1) {
        k = maxLevel - 1;
    }
    return maxLevel - k;
}
```

## Completing the Insertion Process

Whenever a new item is inserted, the `size` is compared to `maxCap`. Recall that `maxCap` is  $2^{\text{maxLevel}} - 1$ . If `size` is now greater than `maxCap`, `maxLevel` is incremented and a new value of `maxCap` is computed. The head node's link array needs to be expanded to accommodate nodes at the increased level.

```
if (size > maxCap) {
    maxLevel++;
    maxCap = computeMaxCap(maxLevel);
    head.links = Arrays.copyOf(head.links, maxLevel);
    pred = Arrays.copyOf(update, maxLevel);
    pred[maxLevel - 1] = head;
}
```

## Performance of a Skip-List

In an ideal skip-list (see Figure 9.65), every other node is at level 1, and every  $2^m$ th node is at least level  $m$ . With this ideal structure, searching is the same as a binary search; each repetition reduces the search population by 1/2, and thus the search is  $O(\log n)$ . By randomly choosing the levels of inserted nodes to have an exponential distribution, the skip-list will have the desired distribution of nodes. However, they will be randomly positioned through the skip-list. Therefore, on the average, the time for search and insertion will be  $O(\log n)$ .

---

## EXERCISES FOR SECTION 9.6

### SELF-CHECK

1. Show the skip-list after inserting the values 11, 12, 22, and 33 into the skip-list shown in Figure 9.65. Assume that the random number generator returned 2, 1, 3, and 1 for the new node levels.

2. Draw the ideal skip-list for storing the numbers 5, 10, 15, 20, 25, 30, 36, 42, 45, 50, 55, 60, 68, 72, 86, 93.

### PROGRAMMING

1. Complete the code for the add method.
2. To remove a node from a skip-list, you need to update all references to the node being deleted to reference its successors. Code the remove method.



## Chapter Review

- ◆ Tree balancing is necessary to ensure that a search tree has  $O(\log n)$  behavior. Tree balancing is done as part of an insertion or removal.
- ◆ An AVL tree is a balanced binary tree in which each node has a balance value that is equal to the difference between the heights of its right and left subtrees ( $h_R - h_L$ ). A node is balanced if it has a balance value of 0; a node is left(right)-heavy if it has a balance of -1 (+1). Tree balancing is done when a node along the insertion (or removal) path becomes critically out of balance; that is, the absolute value of the difference of the height of its two subtrees is 2. The rebalancing is done after returning from a recursive call in the add or delete method.
  - ◆ For an AVL tree, there are four kinds of imbalance and a different remedy for each.
    - Left-Left (parent balance is -2, left child balance is -1): rotate right around parent.
    - Left-Right (parent balance is -2, left child balance is +1): rotate left around child, then rotate right around parent.
    - Right-Right (parent balance is +2, right child balance is +1): rotate left around parent.
    - Right-Left (parent balance is +2, right child balance is -1): rotate right around child, then rotate left around parent.
- ◆ A Red-Black tree is a balanced tree with red and black nodes. After an insertion or removal, the following invariants must be maintained for a Red-Black tree:
  - A node is either red or black.
  - The root is always black.
  - A red node always has black children. (A `null` reference is considered to refer to a black node.)
  - The number of black nodes in any path from the root to a leaf is the same.
- ◆ To maintain tree balance in a Red-Black tree, it may be necessary to recolor a node and also to rotate around a node. The rebalancing is done inside the add or delete method, right after returning from a recursive call.
- ◆ Trees whose nodes have more than two children are an alternative to balanced binary search trees. These include 2-3 and 2-3-4 trees. A 2-node has two children, a 3-node has three children, and a 4-node has four children. The advantage of these trees is that keeping the trees balanced is a simpler process. Also, the tree may be less deep because a 3-node can have three children and a 4-node can have four children, but they still have  $O(\log n)$  behavior.

- ◆ A B-tree of order  $n$  is a tree whose nodes can store up to  $n-1$  items and have  $n$  children and is a generalization of a 2–3 tree. B-trees are used as indexes to large databases stored on disk. The value of  $n$  is chosen so that each node is as large as it can be and still fit in a disk block. The time to retrieve a block is large compared to the time required to process it in memory. By making the tree nodes as large as possible, we reduce the number of disk accesses required to find an item in the index.
- ◆ A 2–3–4 tree can be balanced on the way down the insertion path by splitting a 4-node into two 2-nodes before inserting a new item. This is easier than splitting nodes and rebalancing after returning from an insertion.

## Java Classes Introduced in This Chapter

`java.util.TreeMap`

## User-Defined Interfaces and Classes in This Chapter

`AVLTree`

`RedBlackTree.RedBlackNode`

`AVLTree.AVLNode`

`TwoThreeFourTree`

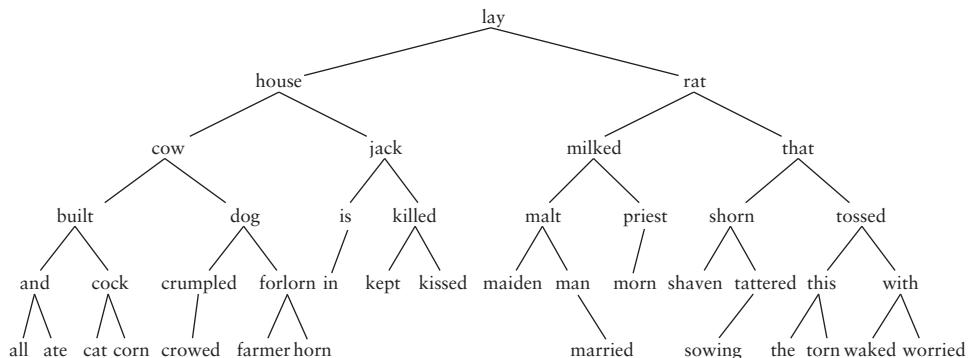
`BinarySearchTreeWithRotate`

`TwoThreeFourTree.Node`

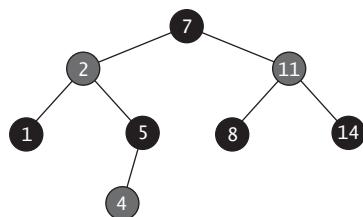
`RedBlackTree`

## Quick-Check Exercises

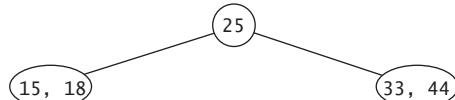
1. Show the following AVL tree after inserting *mouse*. What kind of imbalance occurs, and what is the remedy?



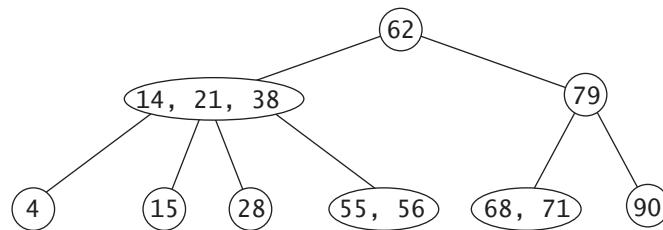
2. Show the following Red–Black tree after inserting 12 and then 13. What kind of rotation, if any, is performed?



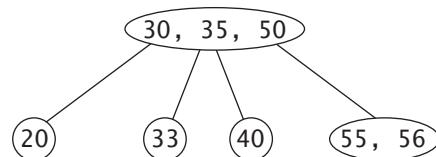
3. Show the following 2–3 tree after inserting 45 and then 20.



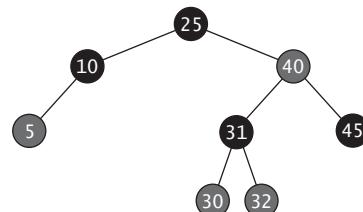
4. Show the following 2–3–4 tree after inserting 40 and then 50.



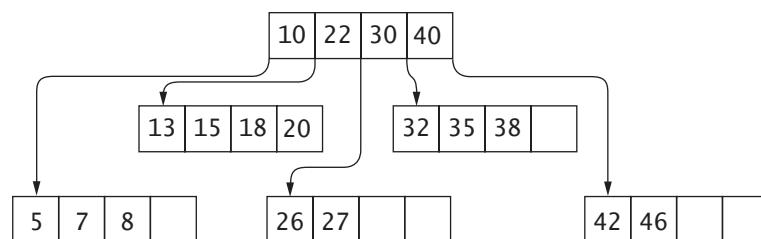
5. Draw the Red–Black tree equivalent to the following 2–3–4 tree.



6. Draw the 2–3–4 tree equivalent to the following Red–Black tree.



7. Show the following B-tree after inserting 45 and 21.



## Review Questions

1. Draw the mirror images of the three cases for insertion into a Red–Black tree and explain how each situation is resolved.
2. Show the AVL tree that would be formed by inserting the month names (12 strings) into a tree in their normal calendar sequence.
3. Show the Red–Black tree that would be formed by inserting the month names into a tree in their normal calendar sequence.
4. Show the 2–3 tree that would be formed by inserting the month names into a tree in their normal calendar sequence.
5. Show the 2–3–4 tree that would be formed by inserting the month names into a tree in their normal calendar sequence.
6. Show a B-tree of capacity 5 that would be formed by inserting the month names into a tree in their normal calendar sequence.

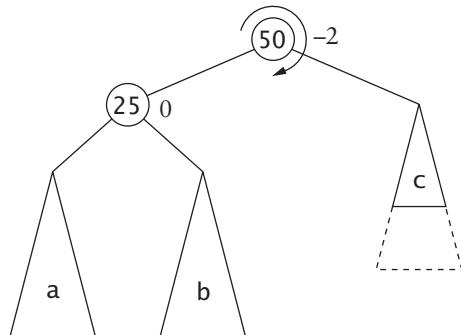
## Programming Projects

1. Complete the `AVLTree` class by coding the missing methods for insertion only. Use it to insert a collection of randomly generated numbers. Insert the same numbers in a binary search tree that is not balanced. Verify that each tree is correct by performing an inorder traversal. Also, display the format of each tree that was built and compare their heights.
2. Code the `RedBlackTree` class by coding the missing methods for insertion. Redo Project 1 using this class instead of the `AVLTree` class.
3. Code the `TwoThreeFourTree` class by coding the missing methods. Redo Project 1 using this class instead of the `AVLTree` class.
4. Code the `TwoThreeTree` class. Redo Project 1 using this class instead of the `AVLTree` class.
5. Complete the `AVLTree` class by providing the missing methods for removal. Demonstrate that these methods work.

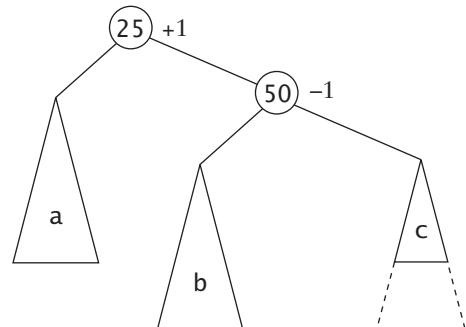
Review the changes required for methods `decrementBalance`, `incrementBalance`, `rebalanceLeft`, and `rebalanceRight` discussed at the end of Section 9.2. Also, modify `rebalanceLeft` (and `rebalanceRight`) to consider the cases where the left (right) subtree is balanced. This case can result when there is a removal from the right (left) subtree that causes the critical imbalance to occur. This is still a Left–Left (Right–Right) case, but after the rotation the overall balances are not zero. This is illustrated in Figures 9.70 and 9.71 where an item is removed from subtree c.

**FIGURE 9.70**

Left–Left Imbalance with Left Subtree Balanced

**FIGURE 9.71**

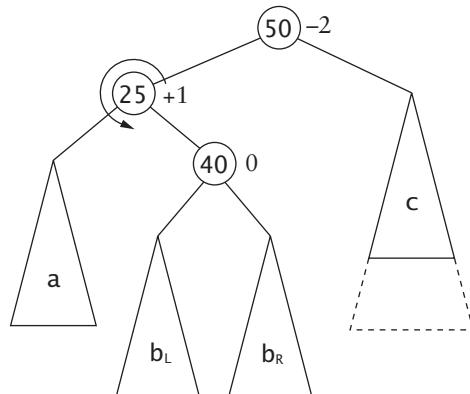
All Trees Unbalanced after Rotation



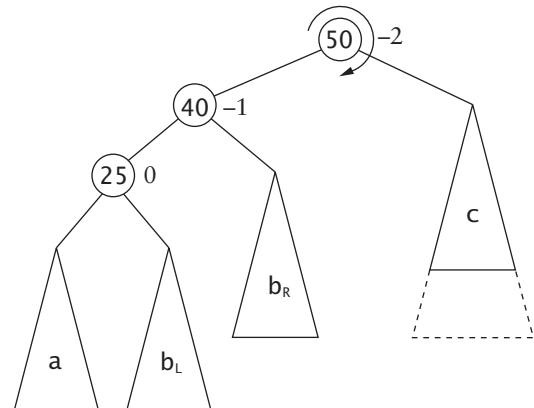
In addition, the Left–Right (or Right–Left) case can have a case in which the Left–Right (Right–Left) subtree is balanced. In this case, after the double rotation is performed, all balances are zero. This is illustrated in Figures 9.72 through 9.74.

**FIGURE 9.72**

Left–Right Case with Left–Right Subtree Balanced

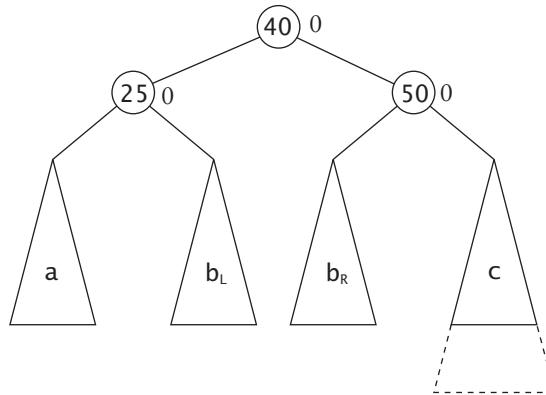
**FIGURE 9.73**

Imbalance after Single Rotation



**FIGURE 9.74**

Complete Balance after Double Rotation



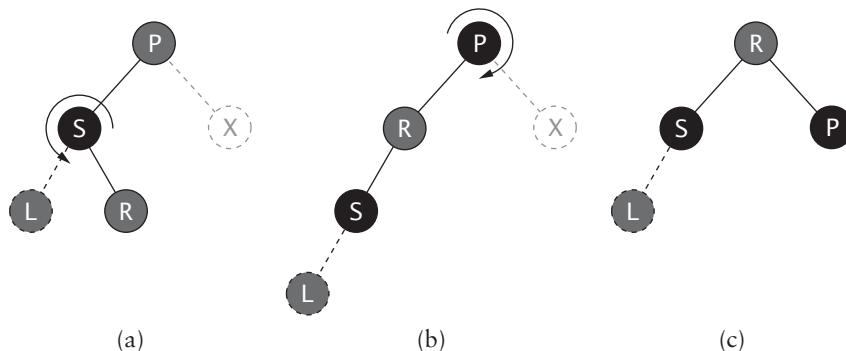
6. Complete the `RedBlackTree` class by coding the missing methods for removal. The methods `remove` and `findLargestChild` are adapted from the corresponding methods of the `BinarySearchTree` class. These adaptations are similar to those done for the AVL tree. A data field `fixupRequired` performs a role analogous to the `decrease` data field in the AVL tree. It is set when a black node is removed. Upon return from a method that can remove a node, this variable is tested. If the removal is from the right, then a new method `fixupRight` is called. If the removal is from the left, then a new method `fixupLeft` is called.

The `fixupRight` method must consider five cases, as follows:

- Case 1: Parent is red and sibling has a red right child. Figure 9.75(a) shows a red node P that is the root of a subtree that has lost a black node X from its right subtree. The root of the left subtree is S, and it must be a black node. If this subtree has a red right child, as shown in the figure, we can restore the black balance. First we rotate the left subtree left and change the color of the parent to black (see Figure 9.75(b)). Then we rotate right about the parent as shown in Figure 9.75(c). This restores the black balance. As shown in the figure, the node S may also have a left child. This does not affect the results.

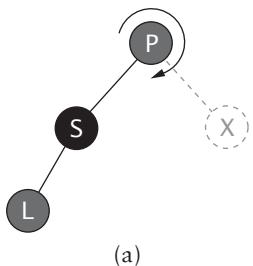
**FIGURE 9.75**

Red–Black Removal  
Case 1

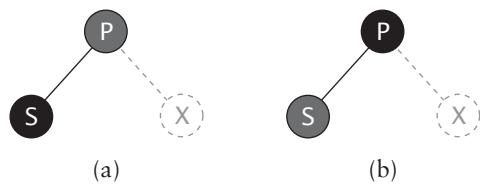


- Case 2: Parent is red, and sibling has only a left red child. Figure 9.76(a) shows the case where the red parent P has a left child S that has a red left child L. In this case, we change the color of S to red and the color of P to black. Then we rotate right as shown in Figure 9.76(b).
- Case 3: Parent is red, and the left child has no red children. Figure 9.77(a) shows the case where the red parent P has a left child S that has no children. As in the next two cases, this fixup process is started at the bottom of the tree but can move up the tree. In this case, S may have black children, and X may represent the root of a subtree whose black height is one less than the black height of S. The correction is quite easy. We change P to black and S to red (see Figure 9.77(b)). Now the balance is restored, and the black height at P remains the same as it was before the black height at X was reduced.

**FIGURE 9.76**  
Red–Black Removal Case 2

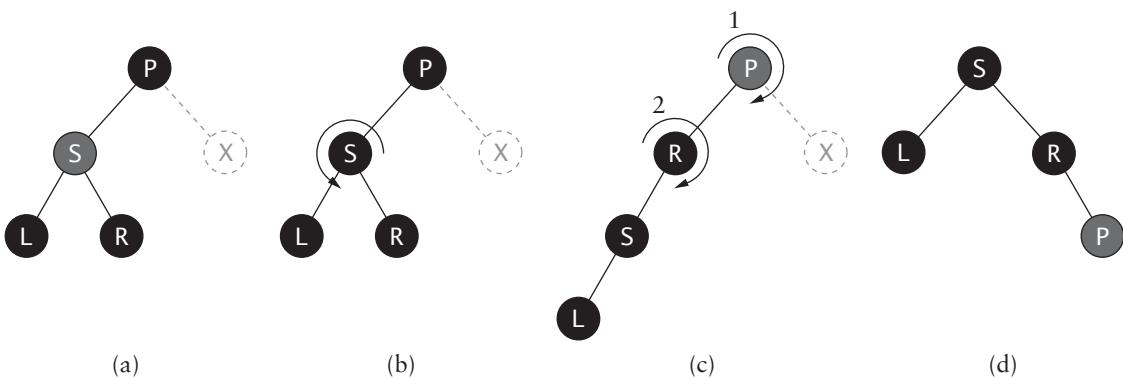


**FIGURE 9.77**  
Red–Black Removal Case 3



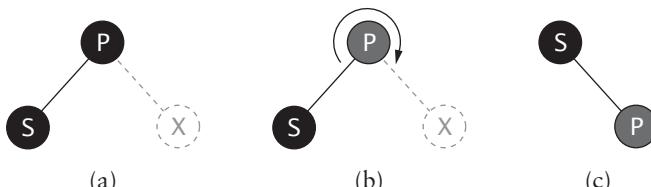
- Case 4: Parent is black and left child is red. Figure 9.78(a) shows the case where the parent P is black and the left child S is red. Since the black heights of S and X were equal before removing X, S must have two black children. We first change the color of the left child to black as shown in Figure 9.78(b). We rotate the child S left and change the color of P to red as shown in Figure 9.78(c). Then we rotate right twice, so that S is now where P was, thus restoring the black balance.

**FIGURE 9.78**  
Red–Black Removal Case 4



- Case 5: Parent is black and left child is black. Figure 9.79(a) shows the case where P is black and S is black. We then change the color of the parent to red and rotate. The black height of P has been reduced. Thus, we repeat the process at the next level (P's parent).

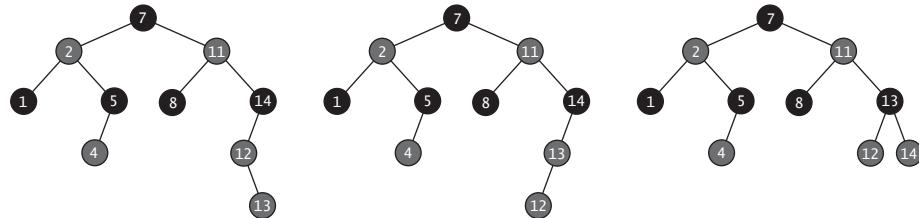
**FIGURE 9.79**  
Red–Black Removal  
Case 5



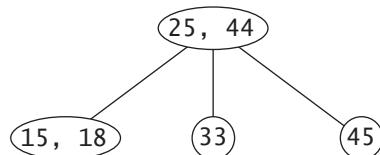
## Answers to Quick-Check Exercises

- When *mouse* is inserted (to the right of *morn*), node *morn* has a balance of +1, and node *priest* has a balance of -2. This is a case of Left–Right imbalance. Rotate left around *morn* and right around *priest*. Node *mouse* will have *morn* (*priest*) as its left (right) subtree.

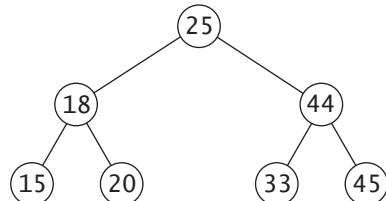
2. When we insert 12 as a red node, it has a black parent, so we are done. When we insert 13, we have the situation shown in the first of the following figures. This is the mirror image of Case 3 in Figure 9.25. We correct it by first rotating left around 12, giving the second of the following figures. Then we change 14 to red and 13 to black and rotate right around 13, giving the tree in the third figure.



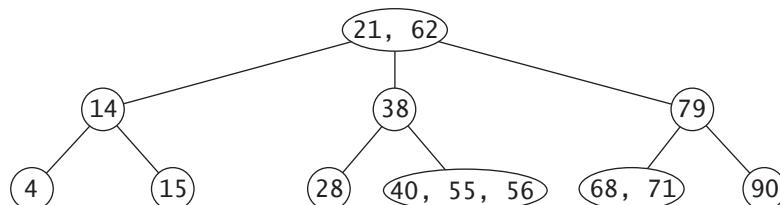
3. The 2–3 tree after inserting 45 is as follows.



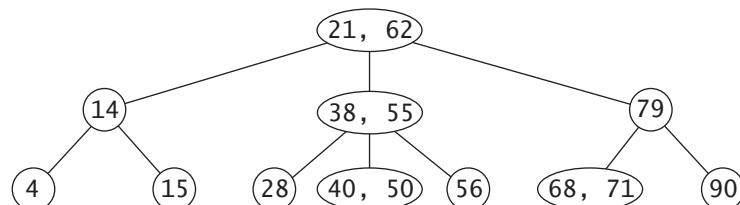
The 2–3 tree after inserting 20 is as follows.



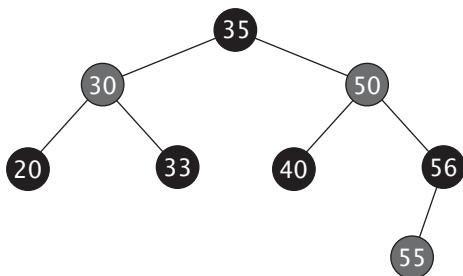
4. When 40 is inserted, the 4-node 14, 21, 38 is split and 21 is inserted into the root, 62. The node-14 has the children 4 and 15, and the node-38 has the children 28 and the 3-node 55, 56. We then insert 40 into the 3-node, making it a 4-node. The result follows.



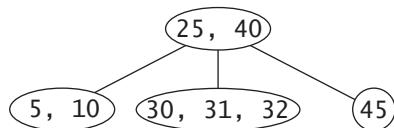
When we insert 50, the 4-node 40, 55, 56 is split and the 55 is inserted into the 2-node 38. Then 50 is inserted into the resulting 2-node, 40, making it a 3-node, as follows.



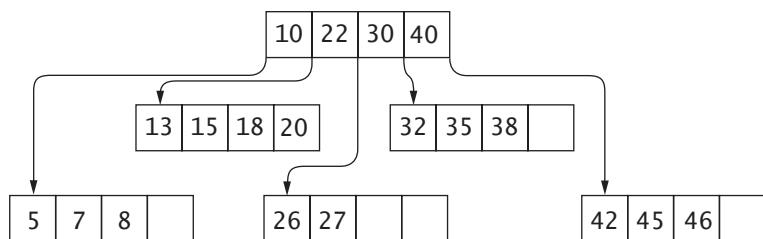
5. The equivalent Red–Black tree follows.



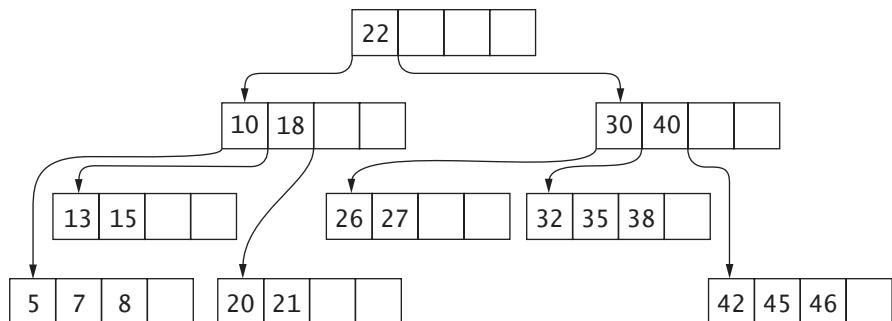
6. The equivalent 2–3–4 tree follows.



7. Insert 45 in a leaf.



To insert 21, we need to split node {13, 15, 18, 20} and pass 18 up. Then we split the root and pass 22 up to the new root.



# *Graphs*

## Chapter Objectives

- ◆ To become familiar with graph terminology and the different types of graphs
- ◆ To study a Graph ADT (abstract data type) and different implementations of the Graph ADT
- ◆ To learn the breadth-first and depth-first search traversal algorithms
- ◆ To learn some algorithms involving weighted graphs
- ◆ To study some applications of graphs and graph algorithms

One of the limitations of trees is that they cannot represent information structures in which a data item has more than one parent. In this chapter, we introduce a data structure known as a *graph* that will allow us to overcome this limitation.

Graphs and graph algorithms were being studied long before computers were invented. The advent of the computer made the application of graph algorithms to real-world problems possible. Graphs are especially useful in analyzing networks. Thus, it is not surprising that much of modern graph theory and application was developed at Bell Laboratories, which needed to analyze the very large communications network that is the telephone system. Graph algorithms are also incorporated into the software that makes the Internet function. You can also use graphs to describe a road map, airline routes, or course prerequisites. Computer chip designers use graph algorithms to determine the optimal placement of components on a silicon chip.

You will learn how to represent a graph, determine the shortest path through a graph, and find the minimum subset of a graph.

## Graphs

- 10.1** Graph Terminology
- 10.2** The Graph ADT and Edge Class
- 10.3** Implementing the Graph ADT
- 10.4** Traversals of Graphs
- 10.5** Applications of Graph Traversals
  - Case Study:* Shortest Path through a Maze
  - Case Study:* Topological Sort of a Graph
- 10.6** Algorithms Using Weighted Graphs

## 10.1 Graph Terminology

A graph is a data structure that consists of a set of *vertices* (or nodes) and a set of *edges* (relations) between the pairs of vertices. The edges represent paths or connections between the vertices. Both the set of vertices and the set of edges must be finite, and either set may be empty. If the set of vertices is empty, naturally the set of edges must also be empty. We restrict our discussion to simple graphs in which there is at most one edge from a given vertex to another vertex.

---

**EXAMPLE 10.1** The following set of vertices,  $V$ , and set of edges,  $E$ , define a graph that has five vertices, with labels A through E, and four edges.

$$\begin{aligned}V &= \{A, B, C, D, E\} \\E &= \{\{A, B\}, \{A, D\}, \{C, E\}, \{D, E\}\}\end{aligned}$$

Each edge is a set of two vertices. There is an edge between A and B (the edge  $\{A, B\}$ ), between A and D, between C and E, and between D and E. If there is an edge between any pair of vertices  $x, y$ , this means there is a path from vertex  $x$  to vertex  $y$  and vice versa. We discuss the significance of this shortly.

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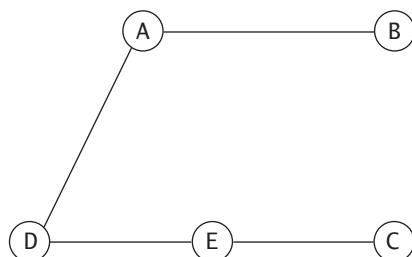
## Visual Representation of Graphs

Visually we represent vertices as points or labeled circles and the edges as lines joining the vertices. Figure 10.1 shows the graph from Example 10.1.

There are many ways to draw any given graph. The physical layout of the vertices, and even their labeling, are not relevant. Figure 10.2 shows two ways to draw the same graph.

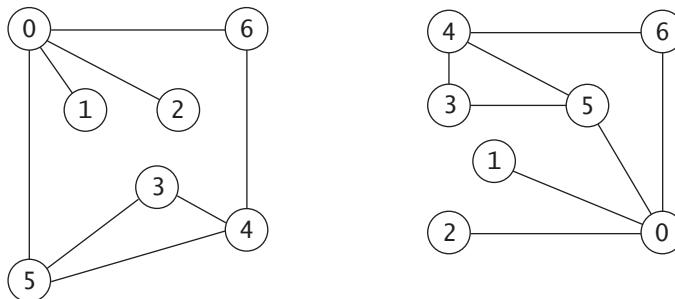
**FIGURE 10.1**

Graph Given in Example 10.1



**FIGURE 10.2**

Two Representations of the Same Graph



## Directed and Undirected Graphs

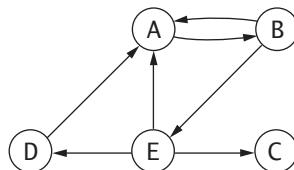
The edges of a graph are *directed* if the existence of an edge from A to B does not necessarily guarantee that there is a path in both directions. A graph that contains directed edges is known as a *directed graph* or *digraph*, and a graph that contains undirected edges is known as an *undirected graph* or simply a graph. A directed edge is like a one-way street; you can travel on it in only one direction. Directed edges are represented as lines with an arrow on one end, whereas undirected edges are represented as single lines. The graph in Figure 10.1 is undirected; Figure 10.3 shows a directed graph. The set of edges for the directed graph follows:

$$E = \{(A, B), (B, A), (B, E), (D, A), (E, A), (E, C), (E, D)\}$$

Each edge above is an ordered pair of vertices instead of a set as in an undirected graph. The edge (A, B) means there is a path from A to B. Observe that there is a path from both A to B and from B to A, but these are the only two vertices in which there is an edge in both directions. Our convention will be to denote an edge for a directed graph as an ordered pair  $(u, v)$  where this notation means that  $v$  (the destination) is adjacent to  $u$  (the source). We denote an edge in an undirected graph as the set  $\{u, v\}$ , which means that  $u$  is adjacent to  $v$  and  $v$  is adjacent to  $u$ . Therefore, you can create a directed graph that is equivalent to an undirected graph by substituting for each edge  $\{u, v\}$  the ordered pairs  $(u, v)$  and  $(v, u)$ . In general, when we describe graph algorithms in this chapter, we will use the ordered pair notation  $(u, v)$  for an edge.

**FIGURE 10.3**

Example of a Directed Graph



The edges in a graph may have values associated with them known as their *weights*. A graph with weighted edges is known as a *weighted graph*. In an illustration of a weighted graph, the weights are shown next to the edges. Figure 10.4 shows an example of a weighted graph. Each weight is the distance between the two cities (vertices) connected by the edge. Generally, the weights are nonnegative, but there are graph problems and graph algorithms that deal with negative weighted edges.

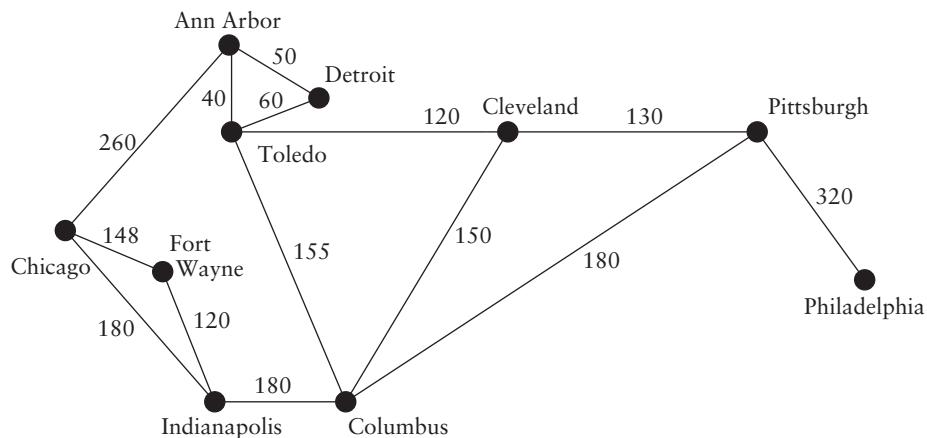
## Paths and Cycles

One reason we study graphs is to find pathways between vertices. We use the following definitions to describe pathways between vertices.

- A vertex is *adjacent* to another vertex if there is an edge to it from that other vertex. In Figure 10.4, Philadelphia is adjacent to Pittsburgh. In Figure 10.3, A is adjacent to D, but since this is a directed graph, D is not adjacent to A.

**FIGURE 10.4**

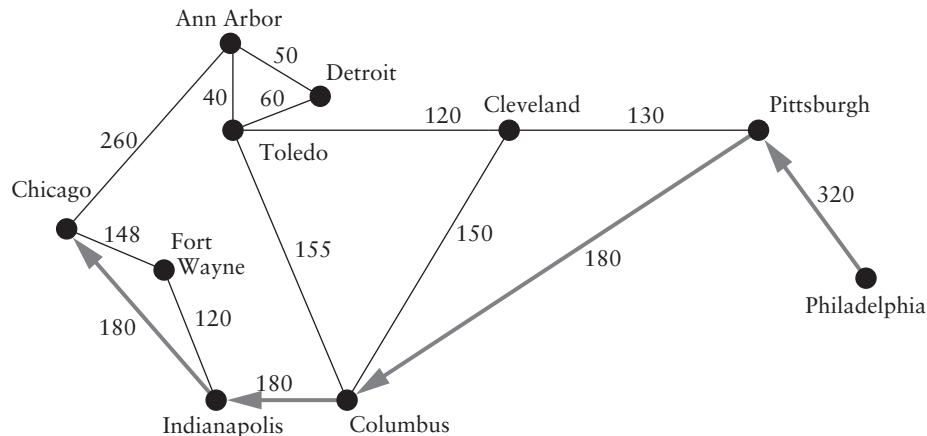
Example of a Weighted Graph



- A *path* is a sequence of vertices in which each successive vertex is adjacent to its predecessor. In Figure 10.5, the following sequence of vertices is a path: Philadelphia → Pittsburgh → Columbus → Indianapolis → Chicago.

**FIGURE 10.5**

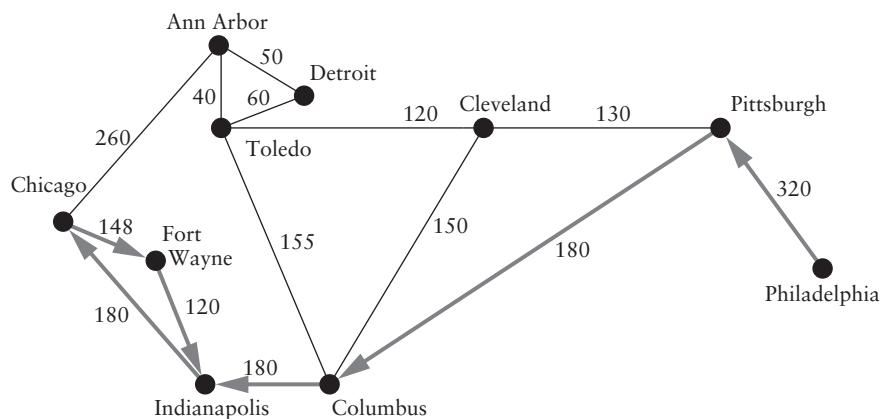
A Simple Path



- In a *simple path*, the vertices and edges are distinct, except that the first and last vertices may be the same. In Figure 10.5, the path Philadelphia → Pittsburgh → Columbus → Indianapolis → Chicago is a simple path. The path Philadelphia → Pittsburgh → Columbus → Indianapolis → Chicago → Fort Wayne → Indianapolis is a path but not a simple path (see Figure 10.6).

**FIGURE 10.6**

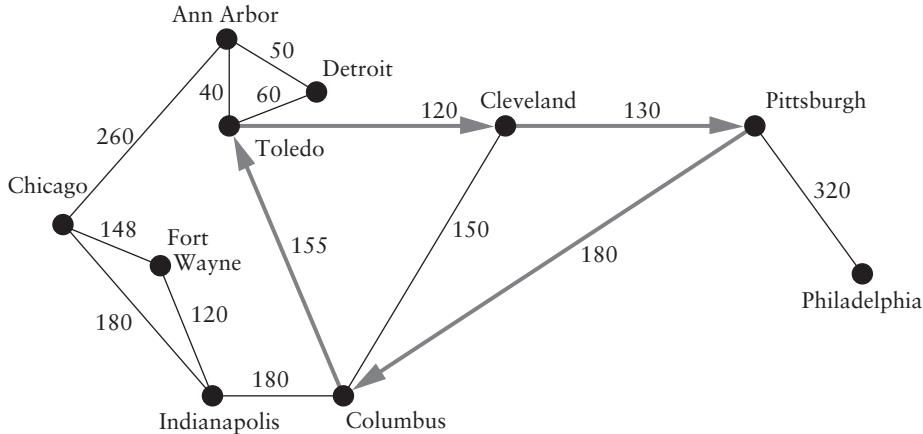
Not a Simple Path



- A *cycle* is a simple path in which only the first and final vertices are the same. In Figure 10.7, the path Pittsburgh → Columbus → Toledo → Cleveland → Pittsburgh is a cycle. For an undirected graph, a cycle must contain at least three distinct vertices. Thus, Pittsburgh → Columbus → Pittsburgh is not considered a cycle.

**FIGURE 10.7**

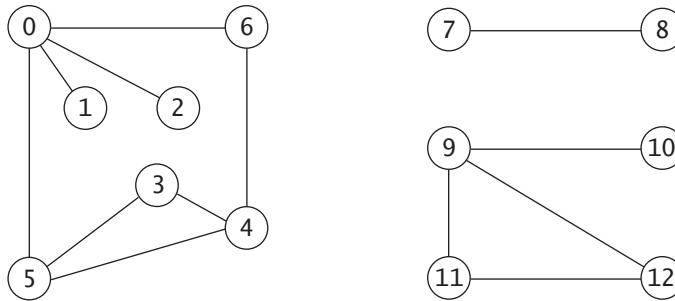
A Cycle



- An undirected graph is called a *connected graph* if there is a path from every vertex to every other vertex. Figure 10.7 is a connected graph, whereas Figure 10.8 is not.

**FIGURE 10.8**

Example of an Unconnected Graph



- If a graph is not connected, it is considered *unconnected*, but it will still consist of *connected components*. A connected component is a subset of the vertices and the edges connected to those vertices in which there is a path between every pair of vertices in the component. A single vertex with no edges is also considered a connected component. Figure 10.8 consists of the connected components  $\{0, 1, 2, 3, 4, 5, 6\}$ ,  $\{7, 8\}$ , and  $\{9, 10, 11, 12\}$ .

## Relationship between Graphs and Trees

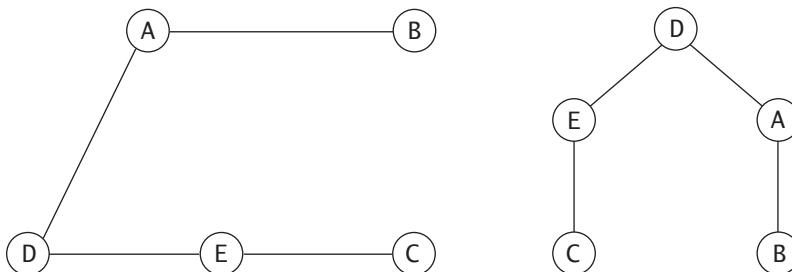
The graph is the most general of the data structures we have studied. It allows for any conceivable relationship among the data elements (the vertices). A tree is actually a special case of a graph. Any graph that is connected and contains no cycles can be viewed as a tree by picking one of its vertices (nodes) as the root. For example, the graph shown in Figure 10.1 can be viewed as a tree if we consider the node labeled D to be the root (see Figure 10.9).

## Graph Applications

We can use graphs to help solve a number of different kinds of problems. For example, we might want to know whether there is a connection from one node in a network to all others. If we can show that the graph is connected, then a path must exist from one node to every other node.

**FIGURE 10.9**

A Graph Viewed  
as a Tree



In college you must take some courses before you take others. These are called prerequisites. Some courses have multiple prerequisites, and some prerequisites have prerequisites of their own. It can be quite confusing. You may even feel that there is a loop in the maze of prerequisites and that it is impossible to schedule your classes to meet the prerequisites. We can represent the set of prerequisites by a directed graph. If the graph has no cycles, then we can find a solution. We can also find the cycles.

Another application would be finding the least-cost path or shortest path from each vertex to all other vertices in a weighted graph. For example, in Figure 10.4, we might want to find the shortest path from Philadelphia to Chicago. Or we might want to create a table showing the distance (miles in the shortest route) between each pair of cities.

## EXERCISES FOR SECTION 10.1

### SELF-CHECK

1. In the graph shown in Figure 10.1, what vertices are adjacent to D? Also check in Figure 10.3.
2. In Figure 10.3, is it possible to get from A to all other vertices? How about from C?
3. In Figure 10.4, what is the shortest path from Philadelphia to Chicago?



## 10.2 The Graph ADT and Edge Class

Java does not provide a Graph ADT, so we have the freedom to design our own. To write programs for the applications mentioned at the end of the previous section, we need to be able to navigate through a graph or traverse it (visit all its vertices). To accomplish this, we need to be able to advance from one vertex in a graph to all its adjacent vertices. Therefore, we need to be able to do the following:

1. Create a graph with the specified number of vertices.
2. Iterate through all of the vertices in the graph.
3. Iterate through the vertices that are adjacent to a specified vertex.
4. Determine whether an edge exists between two vertices.
5. Determine the weight of an edge between two vertices.
6. Insert an edge into the graph.

With the exception of item 1, we can specify these requirements in a Java interface. Since a Java interface cannot include a constructor, the requirements for item 1 can only be specified in the comment at the beginning of the interface.

Listing 10.1 gives the declaration of the Graph interface.

**LISTING 10.1**

Graph.java

```

.....
import java.util.*;

/** Interface to specify a Graph ADT. A graph is a set of vertices and
 * a set of edges. Vertices are represented by integers
 * from 0 to n - 1. Edges are ordered pairs of vertices.
 * Each implementation of the Graph interface should
 * provide a constructor that specifies the number of
 * vertices and whether or not the graph is directed.
 */
public interface Graph {

    // Accessor Methods
    /** Return the number of vertices.
     * @return The number of vertices
     */
    int getNumV();

    /** Determine whether this is a directed graph.
     * @return true if this is a directed graph
     */
    boolean isDirected();

    /** Insert a new edge into the graph.
     * @param edge The new edge
     */
    void insert(Edge edge);

    /** Determine whether an edge exists.
     * @param source The source vertex
     * @param dest The destination vertex
     * @return true if there is an edge from source to dest
     */
    boolean isEdge(int source, int dest);

    /** Get the edge between two vertices.
     * @param source The source vertex
     * @param dest The destination vertex
     * @return The Edge between these two vertices
     * or null if there is no edge
     */
    Edge getEdge(int source, int dest);

    /** Return an iterator to the edges connected to a given vertex.
     * @param source The source vertex
     * @return An Iterator<Edge> to the vertices connected to source
     */
    Iterator<Edge> edgeIterator(int source);
}

```

**Representing Vertices and Edges**

Before we can implement this interface, we must decide how to represent the vertices and edges of a graph. We can represent the vertices by integers from 0 up to, but not including,  $|V|$ . ( $|V|$  means the *cardinality of V*, or the number of vertices in set  $V$ .) For edges we will define the class *Edge* that will contain the source vertex, the destination vertex, and the

**TABLE 10.1**

The Edge Class

Data Field	Attribute
private int dest	The destination vertex for an edge
private int source	The source vertex for an edge
private double weight	The weight
Constructor	Purpose
public Edge(int source, int dest)	Constructs an Edge from source to dest. Sets the weight to 1.0
public Edge(int source, int dest, double w)	Constructs an Edge from source to dest. Sets the weight to w
Method	Behavior
public boolean equals(Object o)	Compares two edges for equality. Edges are equal if their source and destination vertices are the same. The weight is not considered
public int getDest()	Returns the destination vertex
public int getSource()	Returns the source vertex
public double getWeight()	Returns the weight
public int hashCode()	Returns the hash code for an edge. The hash code depends only on the source and destination
public String toString()	Returns a string representation of the edge

weight. For unweighted edges we will use the default value of 1.0. Table 10.1 shows the Edge class. Observe that an Edge is directed. For undirected graphs, we will always have two Edge objects: one in each direction for each pair of vertices that has an edge between them. A vertex is represented by a type `int` variable.

## EXERCISES FOR SECTION 10.2

### SELF-CHECK

1. Use the constructors in Table 10.1 to create the Edge objects connecting vertices 9 through 12 for the graph in Figure 10.8.

### PROGRAMMING

1. Implement the Edge class.



## 10.3 Implementing the Graph ADT

Because graph algorithms have been studied and implemented throughout the history of computer science, many of the original publications of graph algorithms and their implementations did not use an object-oriented approach and did not even use ADTs. The implementation of the graph was done in terms of fundamental data structures that were used directly in the algorithm. Different algorithms would use different representations.

Two representations of graphs are most common:

- Edges are represented by an array of lists called *adjacency lists*, where each list stores the vertices adjacent to a particular vertex.
- Edges are represented by a two-dimensional array, called an *adjacency matrix*, with  $|V|$  rows and  $|V|$  columns.

## Adjacency List

An adjacency list representation of a graph uses an array of lists. There is one list for each vertex. Figure 10.10 shows an adjacency list representation of a directed graph. The list referenced by array element 0 shows the vertices (1 and 3) that are adjacent to vertex 0. The vertices are in no particular order. For simplicity, we are showing just the destination vertex as the value field in each node of the adjacency list, but in the actual implementation the entire Edge will be stored. Instead of storing value = 1 (the destination vertex) in the first vertex adjacent to 0, we will store a reference to the Edge (0, 1, 1.0) where 0 is the source, 1 is the destination, and 1.0 is the weight. The Edge must be stored (not just the destination) because weighted graphs can have different values for weights.

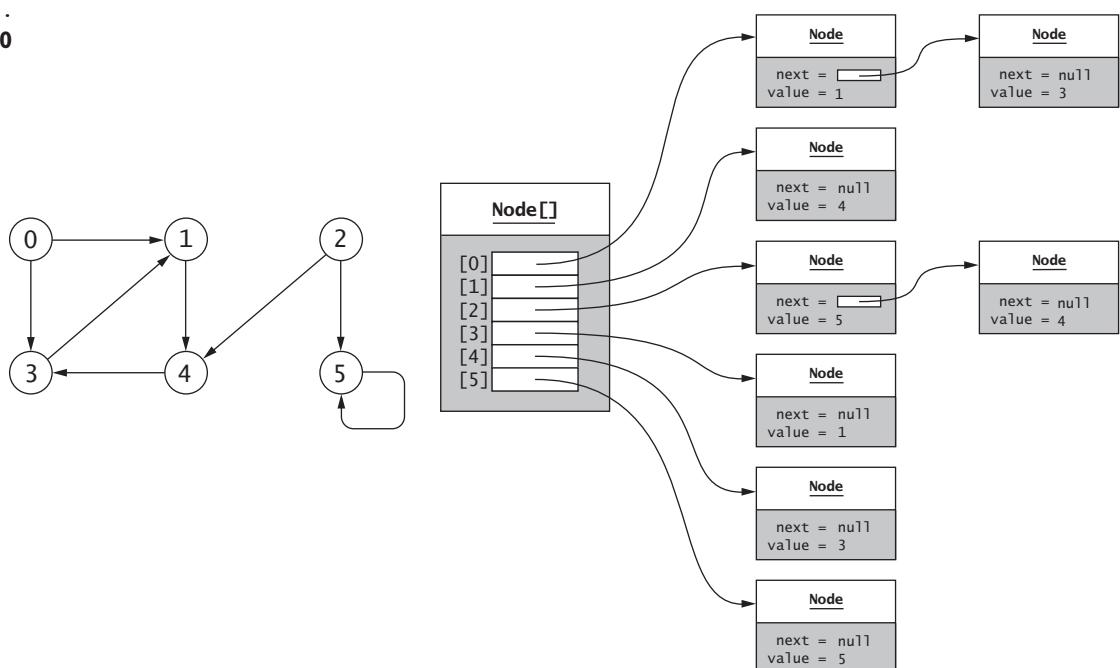
For an undirected graph (or simply a “graph”), symmetric entries are required. Thus, if  $\{u, v\}$  is an edge, then  $v$  will appear on the adjacency list for  $u$  and  $u$  will appear on the adjacency list for  $v$ . Figure 10.11 shows the adjacency list representation for an undirected graph. The actual lists will store references to Edges.

## Adjacency Matrix

The adjacency matrix uses a two-dimensional array to represent the graph. For an unweighted graph, the entries in this matrix can be **boolean** values, where **true** represents the presence of an edge and **false** its absence. Another popular method is to use the value 1 for an edge and 0 for no edge. The integer coding has benefits over the **boolean** approach for some graph algorithms that use matrix multiplication.

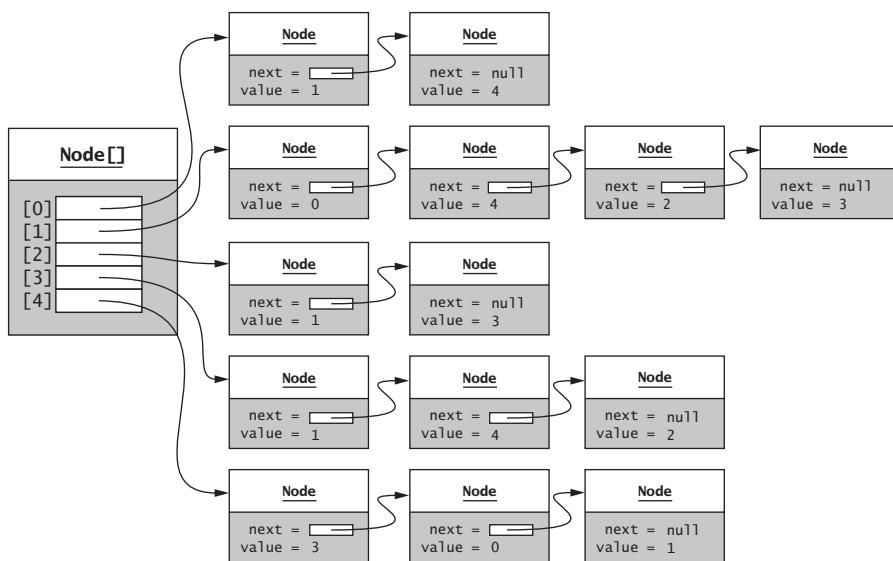
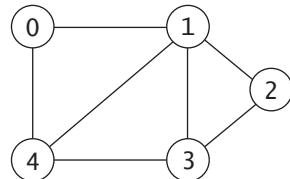
**FIGURE 10.10**

Adjacency List  
Representation of a  
Directed Graph



**FIGURE 10.11**

Adjacency List Representation of an Undirected Graph



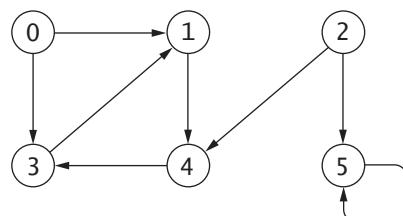
For a weighted graph, the matrix would contain the weights. Since 0 is a valid weight, we will use `Double.POSITIVE_INFINITY` (a special `double` value in Java that approximates the mathematical behavior of infinity) to indicate the absence of an edge, and in an unweighted graph we will use a weight of 1.0 to indicate the presence of an edge.

Figure 10.12 shows a directed graph and the corresponding adjacency matrix. Instead of using Edge objects, an edge is indicated by the value 1.0, and the lack of an edge is indicated by a blank space.

If the graph is undirected, then the matrix is symmetric, and only the lower diagonal of the matrix needs be saved (the colored squares in Figure 10.13).

**FIGURE 10.12**

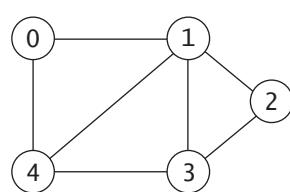
Directed Graph and Corresponding Adjacency Matrix



	Column	[0]	[1]	[2]	[3]	[4]	[5]
Row							
[0]		1.0		1.0			
[1]						1.0	
[2]						1.0	1.0
[3]		1.0					
[4]					1.0		
[5]							1.0

**FIGURE 10.13**

Undirected Graph and Adjacency Matrix Representation



	Column	[0]	[1]	[2]	[3]	[4]
Row						
[0]		1.0				1.0
[1]	1.0		1.0	1.0	1.0	
[2]		1.0		1.0		
[3]			1.0			1.0
[4]	1.0	1.0		1.0		

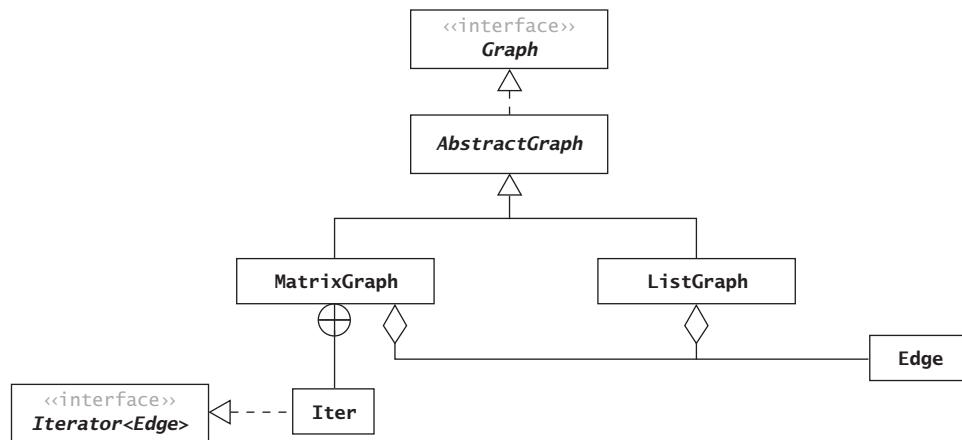
## Overview of the Hierarchy

We will describe Java classes that use each representation. Each class will extend a common abstract superclass. The interface `Graph` was introduced in Section 10.2. The class `Edge` was also described in that section.

We will define the class `AbstractGraph` to represent a graph in general. The classes `ListGraph` and `MatrixGraph` will provide concrete representations of graphs using an adjacency list and adjacency matrix, respectively (see Figure 10.14). The `MatrixGraph` class contains an inner class (indicated by the  $\oplus$  symbol) that we call `Iter`, which implements the `Iterator<Edge>` interface.

**FIGURE 10.14**

UML Class Diagram of Graph Class Hierarchy



## Class `AbstractGraph`

We will use an abstract class, `AbstractGraph`, as the common superclass for graph implementations. This will enable us to implement some of the methods for the `Graph` interface in the abstract superclass and leave other methods that are implementation specific to its subclasses. Graph algorithms will be designed to work on objects that meet the requirements defined by this abstract class. This class is summarized in Table 10.2. Note that the methods

**TABLE 10.2**

The Abstract Class `AbstractGraph`

Data Field	Attribute
<code>private boolean directed</code>	<code>true</code> if this is a directed graph
<code>private int numV</code>	The number of vertices
Constructor	Purpose
<code>public AbstractGraph(int numV, boolean directed)</code>	Constructs an empty graph with the specified number of vertices and with the specified <code>directed</code> flag. If <code>directed</code> is <code>true</code> , this is a directed graph
Method	Behavior
<code>public int getNumV()</code>	Gets the number of vertices
<code>public boolean isDirected()</code>	Returns <code>true</code> if the graph is a directed graph
<code>public void loadEdgesFromFile(Scanner scan)</code>	Loads edges from a data file
<code>public static Graph createGraph (Scanner scan, boolean isDirected, String type)</code>	Factory method to create a graph and load the data from an input file

`edgeIterator`, `getEdge`, `insert`, and `isEdge`, which are required by the `Graph` interface (see Listing 10.1), are implicitly declared abstract and must be declared in the concrete subclasses.

## Implementation

The implementation is shown in Listing 10.2. Method `loadEdgesFromFile` reads edges from individual lines of a data file (see Programming Exercise 1).

.....

**LISTING 10.2**  
`AbstractGraph.java`

```

import java.util.*;
import java.io.*;

/** Abstract base class for graphs. A graph is a set of vertices and
 *  a set of edges. Vertices are represented by integers
 *  from 0 to n - 1. Edges are ordered pairs of vertices.
 */
public abstract class AbstractGraph implements Graph {

    // Data Fields
    /** The number of vertices */
    private int numV;
    /** Flag to indicate whether this is a directed graph */
    private boolean directed;

    // Constructor
    /** Construct a graph with the specified number of vertices and the directed
     *  flag. If the directed flag is true, this is a directed graph.
     *  @param numV The number of vertices
     *  @param directed The directed flag
     */
    public AbstractGraph(int numV, boolean directed) {
        this.numV = numV;
        this.directed = directed;
    }

    // Accessor Methods
    /** Return the number of vertices.
     *  @return The number of vertices
     */
    public int getNumV() {
        return numV;
    }

    /** Return whether this is a directed graph.
     *  @return true if this is a directed graph
     */
    public boolean isDirected() {
        return directed;
    }

    // Other Methods
    /** Load the edges of a graph from the data in an input file. The file
     *  should contain a series of lines, each line with two or
     *  three data values. The first is the source, the second is
     *  the destination, and the optional third is the weight.
     *  @param scan The Scanner connected to the data file
     */
}

```

```

public void loadEdgesFromFile(Scanner scan) {
    // Programming Exercise 1
}

/** Factory method to create a graph and load the data from an input
 * file. The first line of the input file should contain the number
 * of vertices. The remaining lines should contain the edge data as
 * described under loadEdgesFromFile.
@param scan The Scanner connected to the data file
@param isDirected true if this is a directed graph,
    false otherwise
@param type The string "Matrix" if an adjacency matrix is to be
    created, and the string "List" if an adjacency list
    is to be created
@throws IllegalArgumentException if type is neither "Matrix"
    nor "List"
*/
public static Graph createGraph(Scanner scan, boolean isDirected,
    String type) {
    int numV = scan.nextInt();
    AbstractGraph returnValue;
    type = type.toLowerCase();
    switch (type) {
        case "matrix":
            returnValue = new MatrixGraph(numV, isDirected);
            break;
        case "list":
            returnValue = new ListGraph(numV, isDirected);
            break;
        default:
            throw new IllegalArgumentException();
    }
    returnValue.loadEdgesFromFile(scan);
    return returnValue;
}
}

```

## The ListGraph Class

The ListGraph class extends the AbstractGraph class by providing an internal representation using an array of lists. Table 10.3 describes the ListGraph class.

### The Data Fields

The class begins as follows:

```

import java.util.*;

/** A ListGraph is an extension of the AbstractGraph abstract class
 * that uses an array of lists to represent the edges.
 */
public class ListGraph extends AbstractGraph {

    // Data Field
    /** An array of Lists to contain the edges that
        originate with each vertex.
    */
    private List<Edge>[] edges;
    ...
}

```

**TABLE 10.3**

The ListGraph Class

Data Field	Attribute
private List<Edge>[] edges	An array of Lists to contain the edges that originate with each vertex
Constructor	Purpose
public ListGraph(int numV, boolean directed)	Constructs a graph with the specified number of vertices and directionality
Method	Behavior
public Iterator<Edge> edgeIterator(int source)	Returns an iterator to the edges that originate from a given vertex
public Edge getEdge(int source, int dest)	Gets the edge between two vertices
public void insert(Edge e)	Inserts a new edge into the graph
public boolean isEdge(int source, int dest)	Determines whether an edge exists from vertex source to dest

## The Constructor

The constructor allocates an array of `LinkedLists`, one for each vertex.

```
/** Construct a graph with the specified number of vertices and directionality.
 * @param numV The number of vertices
 * @param directed The directionality flag
 */
public ListGraph(int numV, boolean directed) {
    super(numV, directed);
    edges = new List[numV];
    for (int i = 0; i < numV; i++) {
        edges[i] = new LinkedList<Edge>();
    }
}
```

## The isEdge Method

Method `isEdge` determines whether an edge exists by searching the list associated with the source vertex for an entry. This is done by calling the `contains` method for the `List`.

```
/** Determine whether an edge exists.
 * @param source The source vertex
 * @param dest The destination vertex
 * @return true if there is an edge from source to dest
 */
public boolean isEdge(int source, int dest) {
    return edges[source].contains(new Edge(source, dest));
}
```

Observe that we had to create a dummy `Edge` object for the `contains` method to search for. The `Edge.equals` method does not check the edge weights, so the `weight` parameter is not needed.

## The insert Method

The `insert` method inserts a new edge (`source, destination, weight`) into the graph by adding that edge's data to the list of adjacent vertices for that edge's source. If the graph is not

directed, it adds a new edge in the opposite direction (`destination, source, weight`) to the list of adjacent vertices for that edge's destination.

```
/** Insert a new edge into the graph.
 * @param edge The new edge
 */
public void insert(Edge edge) {
    edges[edge.getSource()].add(edge);
    if (!isDirected()) {
        edges[edge.getDest()].add(new Edge(edge.getDest(), edge.getSource(),
                                             edge.getWeight()));
    }
}
```

### The `edgeIterator` Method

The `edgeIterator` method will return an `Iterator<Edge>` object that can be used to iterate through the edges adjacent to a given vertex. Because each `LinkedList` entry in the array `edges` is a `List<Edge>`, its `iterator` method will provide the desired object. Thus, the `edgeIterator` merely calls the corresponding `iterator` method for the specified vertex.

```
public Iterator<Edge> edgeIterator(int source) {
    return edges[source].iterator();
}
```

### The `getEdge` Method

Similar to the `isEdge` method, the `getEdge` method also requires a search. However, we need to program the search directly. We will use the enhanced `for` statement to access all edges in the list for vertex `source`. We compare each edge to a target object with source and destination set to the method arguments. The `equals` method does not compare edge weights, only the vertices.

```
/** Get the edge between two vertices.
 * @param source The source
 * @param dest The destination
 * @return the edge between these two vertices
 *         or null if an edge does not exist.
 */
public Edge getEdge(int source, int dest) {
    Edge target = new Edge(source, dest, Double.POSITIVE_INFINITY);
    for (Edge edge: edges[source]) {
        if (edge.equals(target))
            return edge; // Desired edge found, return it.
    }
    // Assert: All edges for source checked.
    return null; // Desired edge not found.
}
```

### The `MatrixGraph` Class

The `MatrixGraph` class extends the `AbstractGraph` class by providing an internal representation using a two-dimensional array for storing the edge weights

```
double[][] edges;
```

When a new `MatrixGraph` object is created, the constructor sets the number of rows (vertices) in this array. It implements the same methods as class `ListGraph` and also has an inner iterator class `Iter`. It needs its own iterator class because there is no `Iterator` class associated with an array. The implementation is left as a project (Programming Project 1).

## Comparing Implementations

### Time Efficiency

The two implementations present a tradeoff. Which is best depends on the algorithm and the density of the graph. The density of a graph is the ratio of  $|E|$  to  $|V|^2$ . A *dense graph* is one in which  $|E|$  is close to but less than  $|V|^2$ , and a *sparse graph* is one in which  $|E|$  is much less than  $|V|^2$ . Therefore, for a dense graph we can assume that  $|E|$  is  $O(|V|^2)$ , and for a sparse graph we can assume that  $|E|$  is  $O(|V|)$ .

Many graph algorithms are of the form:

1. **for** each vertex  $u$  in the graph
2.     **for** each vertex  $v$  adjacent to  $u$
3.         Do something with edge  $(u, v)$ .

For an adjacency list representation, Step 1 is  $O(|V|)$  and Step 2 is  $O(|E_u|)$ , where  $|E_u|$  is the number of edges that originate at vertex  $u$ . Thus, the combination of Steps 1 and 2 will represent examining each edge in the graph, giving  $O(|E|)$ . For an adjacency matrix representation, Step 2 is also  $O(|V|)$ , and thus the overall algorithm is  $O(|V|^2)$ . Thus, for a sparse graph, the adjacency list gives better performance for this type of algorithm, whereas for a dense graph, the performance is the same for either representation.

Some graph algorithms are of the form

1. **for** each vertex  $u$  in some subset of the vertices
2.     **for** each vertex  $v$  in some subset of the vertices
3.         **if**  $(u, v)$  is an edge
4.             Do something with edge  $(u, v)$ .

For an adjacency matrix representation, Step 3 tests a matrix value and is  $O(1)$ , so the overall algorithm is  $O(|V|^2)$ . However, for an adjacency list representation, Step 3 searches a list and is  $O(|E_u|)$ , so the combination of Steps 2 and 3 is  $O(|E|)$  and the overall algorithm is  $O(|V||E|)$ . For a dense graph, the adjacency matrix gives the best performance for this type of algorithm, and for a sparse graph, the performance is the same for both representations.

Thus, if a graph is dense, the adjacency matrix representation is best, and if a graph is sparse, the adjacency list representation is best. Intuitively, this makes sense because a sparse graph will lead to a sparse matrix, or one in which most entries are `POSITIVE_INFINITY`. These entries are not included in a list representation, so they will have no effect on processing time. However, they are included in a matrix representation and will have an undesirable impact on processing time.

### Storage Efficiency

Note that storage is allocated for all vertex combinations (or at least half of them) in an adjacency matrix. So the storage required is proportional to  $|V|^2$ . If the graph is sparse (not many edges), there will be a lot of wasted space in the adjacency matrix. In an adjacency list, only the adjacent edges are stored.

On the other hand, in an adjacency list, each edge is represented by a reference to an `Edge` object containing data about the source, destination, and weight. There is also a reference to the next edge in the list. In a matrix representation, only the weight associated with an edge is stored. So each element in an adjacency list requires approximately four times the storage of an element in an adjacency matrix.

Based on this, we can conclude that the break-even point in terms of storage efficiency occurs when approximately 25 percent of the adjacency matrix is filled with meaningful data. That is, the adjacency list uses less (more) storage when less than (more than) 25 percent of the adjacency matrix would be filled.

## The MapGraph Class

We can achieve the performance benefits of both the ListGraph and MatrixGraph by making a slight modification to the ListGraph. Replacing the array of `List<Edge>` with an array of `Map<Integer, Edge>` allows us to query the existence of an edge in  $O(1)$  time, and using the `LinkedHashMap` allows iterating through the edges adjacent to a given vertex in  $O(|E_u|)$ . The constructor is changed to

```
public MapGraph(int numV, boolean isDirected) {
    super(numV, isDirected);
    outgoingEdges = new Map[numV];
    for (int i = 0; i < numV; i++) {
        outgoingEdges[i] = new LinkedHashMap<>();
    }
}
```

The `insertEdge` method is changed to

```
public void insertEdge(Edge edge) {
    int source = edge.getSource();
    int dest = edge.getDest();
    double weight = edge.getWeight();
    outgoingEdges[source].put(dest, edge);
    if (!isDirected()) {
        Edge reverseEdge = new Edge(dest, source, weight);
        outgoingEdges[dest].put(source, reverseEdge);
    }
}
```

The `isEdge` and `getEdge` methods are simplified to

```
public boolean isEdge(int source, int dest) {
    return outgoingEdges[source].containsKey(dest);
}

public Edge getEdge(int source, int dest) {
    return outgoingEdges[source].get(dest);
}
```

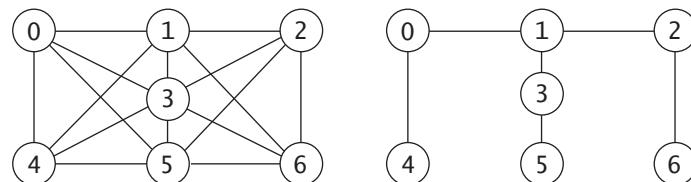
And the `edgeIterator` becomes

```
public Iterator<Edge> edgeIterator(int source) {
    return outgoingEdges[source].values().iterator();
}
```

## EXERCISES FOR SECTION 10.3

### SELF-CHECK

1. Represent the following graphs using adjacency lists.



2. Represent the graphs in Exercise 1 above using an adjacency matrix.
3. For each graph in Exercise 1, what are the  $|V|$ , the  $|E|$ , and the density? Which representation is best for each graph? Explain your answers.

**PROGRAMMING**

1. Implement the `loadEdgesFromFile` method for class `AbstractGraph`. If there are two values on a line, an edge with the default weight of 1.0 is inserted; if there are three values, the third value is the weight.

## 10.4 Traversals of Graphs

Most graph algorithms involve visiting each vertex in a systematic order. Just as with trees, there are different ways to do this. The two most common traversal algorithms are breadth first and depth first. Although these are graph traversals, they are more commonly called *breadth-first* and *depth-first search*.

### Breadth-First Search

In a breadth-first search, we visit the start node first, then all nodes that are adjacent to it next, then all nodes that can be reached by a path from the start node containing two edges, three edges, and so on. The requirement for a breadth-first search is that we must visit all nodes for which the shortest path from the start node is length  $k$  before we visit any node for which the shortest path from the start node is length  $k + 1$ . You can visualize a breadth-first traversal by “picking up” the graph at the vertex that is the start node, so the start node will be the highest node and the rest of the nodes will be suspended underneath it, connected by their edges. In a breadth-first search, the nodes that are higher up in the picked-up graph are visited before nodes that are lower in the graph.

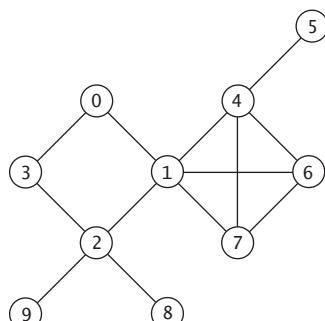
Breadth-first search starts at some vertex. Unlike the case of a tree, there is no special start vertex, so we will arbitrarily pick the vertex with label 0. We then visit it by identifying all vertices that are adjacent to the start vertex. Then we visit each of these vertices, identifying all of the vertices adjacent to them. This process continues until all vertices are visited. If the graph is not a connected graph, then the process is repeated with one of the unidentified vertices. In the discussion that follows, we use color to distinguish among three states for a node: identified (light gray), visited (dark gray), and not identified (white). Initially, all nodes are not identified. If a node is in the identified state, that node was encountered while visiting another, but it has not yet been visited.

### Example of Breadth-First Search

Consider the graph shown in Figure 10.15. We start at vertex 0 and color it light gray (see Figure 10.16(a)). We visit 0 and see that 1 and 3 are adjacent, so we color them light gray (to show that they have been identified). We are finished visiting 0 and now color it dark gray (see Figure 10.16(b)). So far we have visited node 0.

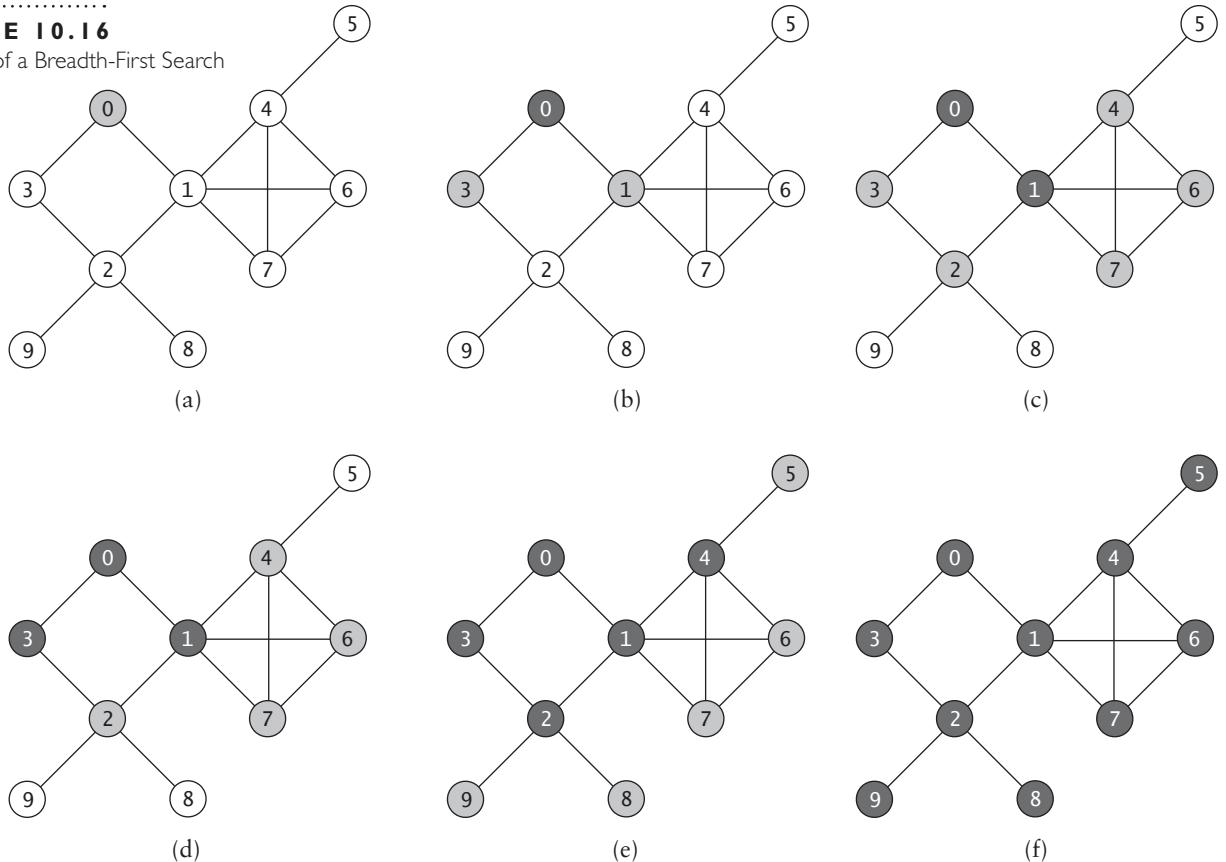
**FIGURE 10.15**

Graph to Be Traversed  
Breadth First



**FIGURE 10.16**

Example of a Breadth-First Search



We always select the first node that was identified (light gray) but not yet visited and visit it next. Therefore, we visit 1 and look at its adjacent vertices: 0, 2, 4, 6, and 7. We skip 0 because it is not colored white, and we color the others light gray. Then we color 1 dark gray (see Figure 10.16(c)). Now we have visited nodes 0 and 1.

Then we look at 3 (the first of the light gray vertices in Figure 10.16(c) to have been identified) and see that its adjacent vertex, 2, has already been identified and 0 has been visited, so we are finished with 3 (see Figure 10.16(d)). Now we have visited nodes 0, 1, and 3, which are the starting vertex and all vertices adjacent to it.

Now we visit 2 and see that 8 and 9 are adjacent. Then we visit 4 and see that 5 is the only adjacent vertex not identified or visited (Figure 10.16(e)). Finally, we visit 6 and 7 (the last vertices that are two edges away from the starting vertex), then 8, 9, and 5, and see that there are no unidentified vertices (Figure 10.16(f)). The vertices have been visited in the sequence 0, 1, 3, 2, 4, 6, 7, 8, 9, 5.

### Algorithm for Breadth-First Search

To implement breadth-first search, we need to be able to determine the first identified vertex that has not been visited so that we can visit it. To ensure that the identified vertices are visited in the correct sequence, we will store them in a queue (first-in, first-out). When we need a new node to visit, we remove it from the queue. We summarize the process in the following algorithm.

## Algorithm for Breadth-First Search

1. Take an arbitrary start vertex, mark it identified (color it light gray), and place it in a queue.
2. **while** the queue is not empty
3. Take a vertex,  $u$ , out of the queue and visit  $u$ .
4. **for** all vertices,  $v$ , adjacent to this vertex,  $u$
5.     **if**  $v$  has not been identified or visited
6.         Mark it identified (color it light gray).
7.         Insert vertex  $v$  into the queue.
8. We are now finished visiting  $u$  (color it dark gray).

Table 10.4 traces this algorithm on the graph shown earlier in Figure 10.15. The initial queue contents is the start node, 0. The first line shows that after we finish visiting vertex 0, the queue contains nodes 1 and 3, which are adjacent to node 0 and are colored light gray in Figure 10.16(b). The second line shows that after removing 1 from the queue and visiting 1, we insert its neighbors that have not yet been identified or visited: nodes 2, 4, 6, and 7.

Table 10.4 shows that the nodes were visited in the sequence 0, 1, 3, 2, 4, 6, 7, 8, 9, 5. There are other sequences that would also be valid breadth-first traversals.

We can also build a tree that represents the order in which vertices would be visited in a breadth-first traversal, by attaching the vertices as they are identified to the vertex from which they are identified. Such a tree is shown in Figure 10.17. Observe that this tree contains all of the vertices

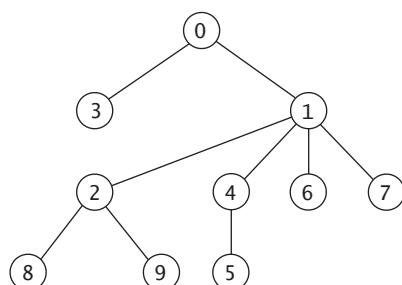
**TABLE 10.4**

Trace of Breadth-First Search of Graph in Figure 10.15

Vertex Being Visited	Queue Contents after Visit	Visit Sequence
0	1 3	0
1	3 2 4 6 7	0 1
3	2 4 6 7	0 1 3
2	4 6 7 8 9	0 1 3 2
4	6 7 8 9 5	0 1 3 2 4
6	7 8 9 5	0 1 3 2 4 6
7	8 9 5	0 1 3 2 4 6 7
8	9 5	0 1 3 2 4 6 7 8
9	5	0 1 3 2 4 6 7 8 9
5	Empty	0 1 3 2 4 6 7 8 9 5

**FIGURE 10.17**

Breadth-First Search  
Tree of Graph in  
Figure 10.15



and some of the edges of the original graph. A path starting at the root to any vertex in the tree is the shortest path in the original graph from the start vertex to that vertex, where we consider all edges to have the same weight. Therefore, the *shortest path* is the one that goes through the smallest number of vertices. We can save the information we need to represent this tree by storing the parent of each vertex when we identify it (Step 7 of the breadth-first algorithm).

### Refinement of Step 7 of Breadth-First Search Algorithm

7.1 Insert vertex  $v$  into the queue.

7.2 Set the parent of  $v$  to  $u$ .

### Performance Analysis of Breadth-First Search

The loop at Step 2 will be performed for each vertex. The inner loop at Step 4 is performed for  $|E_v|$  (the number of edges that originate at that vertex). The total number of steps is the sum of the edges that originate at each vertex, which is the total number of edges. Thus, the algorithm is  $O(|E|)$ .

### Implementing Breadth-First Search

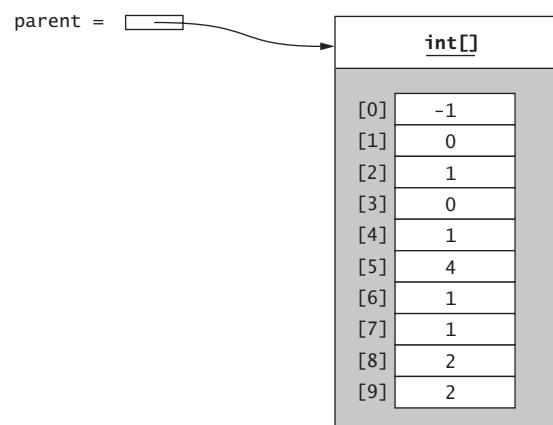
Listing 10.3 shows method `breadthFirstSearch`. Note that nothing is done when we have finished visiting a vertex (algorithm Step 8).

This method declares three data structures: `int[] parent`, `boolean[] identified`, and `Queue theQueue`. The array `identified` is used to keep track of the nodes that have been previously encountered, and `theQueue` is used to store nodes that are waiting to be visited.

The method returns array `parent`, which could be used to construct the breadth-first search tree. The element `parent[v]` contains the parent of vertex  $v$  in the tree. The statement

```
parent[neighbor] = current;
```

is used to “insert an edge into the breadth-first search tree.” It does this by setting the parent of a newly identified node (`neighbor`) as the node being visited (`current`). If we run the `breadthFirstSearch` method on the graph shown in Figure 10.15, then the array `parent` will be defined as follows:



If you compare array `parent` to Figure 10.17, you can see that `parent[i]` is the parent of vertex  $i$ . For example, the parent of vertex 4 is vertex 1. The entry `parent[0]` is  $-1$  because node 0 is the start vertex.

Although array `parent` could be used to construct the breadth-first search tree, we are generally not interested in the complete tree but rather in the path from the root to a given vertex.

Using array parent to trace the path from that vertex back to the root would give you the reverse of the desired path. For example, the path derived from parent for vertex 4 to the root would be 4 to 1 to 0. If you place these vertices in a stack and then pop the stack until it is empty, you will get the path from the root: 0 to 1 to 4.

.....  
**LISTING 10.3**

Class BreadthFirstSearch.java

```
/** Class to implement the breadth-first search algorithm. */
public class BreadthFirstSearch {

    /** Perform a breadth-first search of a graph.
     * @post The array parent will contain the predecessor
     *       of each vertex in the breadth-first search tree.
     * @param graph The graph to be searched
     * @param start The start vertex
     * @return The array of parents
    */
    public static int[] breadthFirstSearch(Graph graph, int start) {
        Queue<Integer> theQueue = new LinkedList<Integer>();
        // Declare array parent and initialize its elements to -1.
        int[] parent = new int[graph.getNumV()];
        for (int i = 0; i < graph.getNumV(); i++) {
            parent[i] = -1;
        }

        // Declare array identified and
        // initialize its elements to false.
        boolean[] identified = new boolean[graph.getNumV()];
        /* Mark the start vertex as identified and insert it into the queue */
        identified[start] = true;
        theQueue.offer(start);

        /* Perform breadth-first search until done */
        while (!theQueue.isEmpty()) {
            /* Take a vertex, current, out of the queue. (Begin visiting current). */
            int current = theQueue.remove();
            /* Examine each vertex, neighbor, adjacent to current. */
            Iterator<Edge> itr = graph.edgeIterator(current);
            while (itr.hasNext()) {
                Edge edge = itr.next();
                int neighbor = edge.getDest();
                // If neighbor has not been identified
                if (!identified[neighbor]) {
                    // Mark it identified.
                    identified[neighbor] = true;
                    // Place it into the queue.
                    theQueue.offer(neighbor);
                    /* Insert the edge (current, neighbor) into the tree. */
                    parent[neighbor] = current;
                }
            }
            // Finished visiting current.
        }
        return parent;
    }
}
```

## Depth-First Search

Another way to traverse a graph is depth-first search. In depth-first search you start at a vertex, visit it, and choose one adjacent vertex to visit. Then choose a vertex adjacent to that vertex to visit, and so on until you go no further. Then back up and see whether a new vertex (one not previously visited) can be found. In the discussion that follows, we use color to distinguish among three states for a node: being visited (light gray), finished visiting (dark gray), and not yet visited (white). Initially, of course, all nodes are not yet visited. Note that the color light gray is used in depth-first search to indicate that a vertex is in the process of being visited, whereas it was used in our discussion of breadth-first search to indicate that the vertex was identified.

### Example of Depth-First Search

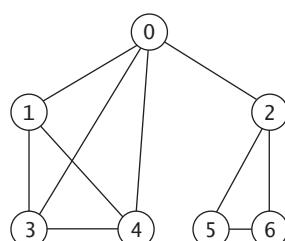
Consider the graph shown in Figure 10.18. We can start at any vertex, but for simplicity we will start at 0. The vertices adjacent to 0 are 1, 2, 3, and 4. We mark 0 as being visited (color it light gray; see Figure 10.19(a)). Next we consider 1. We mark 1 as being visited (see Figure 10.19(b)). The vertices adjacent to 1 are 0, 3, and 4. But 0 is being visited, so we recursively apply the algorithm with 3 as the start vertex. We mark 3 as being visited (see Figure 10.19(c)). The vertices adjacent to 3 are 0, 1, and 4. Because 0 and 1 are already being visited, we recursively apply the algorithm with 4 as the start vertex. We mark 4 as being visited (see Figure 10.19(d)). The vertices adjacent to 4 are 0, 1, and 3. All of these are being visited, so we mark 4 as finished (see Figure 10.19(e)) and return from the recursion. Now all of the vertices adjacent to 3 have been visited, so we mark 3 as finished and return from the recursion. Now all of the vertices adjacent to 1 have been visited, so we mark 1 as finished and return from the recursion to the original start vertex, 0. The order in which we started to visit vertices is 0, 1, 3, 4; the order in which vertices have become finished so far is 4, 3, 1.

We now consider vertex 2, which is adjacent to 0 but has not been visited. We mark 2 as being visited (see Figure 10.19(f)) and consider the vertices adjacent to it: 5 and 6. We mark 5 as being visited (see Figure 10.19(g)) and consider the vertices adjacent to it: 2 and 6. Because 2 is already being visited, we next visit 6. We mark 6 as being visited (see Figure 10.19(h)). The vertices adjacent to 6 (2 and 5) are already being visited. Thus, we mark 6 as finished and recursively return. The vertices adjacent to 5 have all been visited, so we mark 5 as finished and return from the recursion. All of the vertices adjacent to 2 have been visited, so we mark 2 as finished and return from the recursion.

Finally, we come back to 0. Because all of the vertices adjacent to it have also been visited, we mark 0 as finished and we are done (see Figure 10.19(i)). The order in which we started to visit all vertices is 0, 1, 3, 4, 2, 5, 6; the order in which we finished visiting all vertices is 4, 3, 1, 6, 5, 2, 0. The *discovery order* is the order in which the vertices are discovered. The *finish order* is the order in which the vertices are finished. We consider a vertex to be finished when we return to it after finishing all its successors.

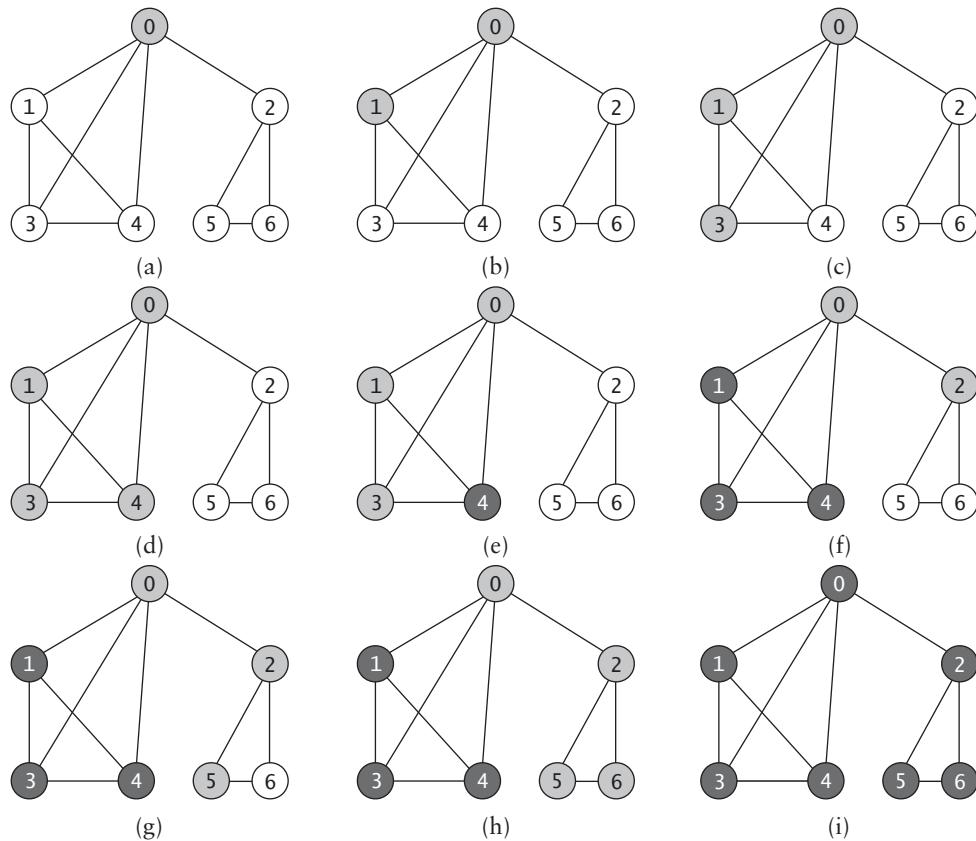
Figure 10.20 shows the depth-first search tree for the graph in Figure 10.18. A preorder traversal of this tree yields the sequence in which the vertices were visited: 0, 1, 3, 4, 2, 5, 6. The

**FIGURE 10.18**  
Graph to Be Traversed  
Depth First

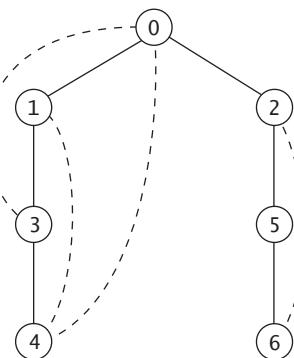


**FIGURE 10.19**

Example of Depth-First Search

**FIGURE 10.20**

Depth-First Search Tree  
of Figure 10.18



dashed lines are the other edges in the graph that are not part of the depth-first search tree. These edges are called *back edges* because they connect a vertex with its ancestors in the depth-first search tree. Observe that vertex 4 has two ancestors in addition to its parent 3: 1 and 0. Vertex 1 is a grandparent, and vertex 0 is a great-grandparent.

### Algorithm for Depth-First Search

Depth-first search is used as the basis of other graph algorithms. However, rather than embedding the depth-first search algorithm into these other algorithms, we will implement the depth-first search algorithm to collect information about the vertices, which we can then use in these other algorithms. The information we will collect is the discovery order (or the visit order) and the finish order.

The depth-first search algorithm follows. Step 5 recursively applies this algorithm to each vertex as it is discovered.

### Algorithm for Depth-First Search

1. Mark the current vertex,  $u$ , visited (color it light gray), and enter it in the discovery order list.
2. **for** each vertex,  $v$ , adjacent to the current vertex,  $u$
3.     **if**  $v$  has not been visited
4.         Set parent of  $v$  to  $u$ .
5.         Recursively apply this algorithm starting at  $v$ .
6. Mark  $u$  finished (color it dark gray) and enter  $u$  into the finish order list.

Observe that Step 6 is executed after the loop in Step 2 has examined all vertices adjacent to vertex  $u$ . Also, the loop at Step 2 does not select the vertices in any particular order.

Table 10.5 shows a trace of the algorithm as applied to the graph shown in Figure 10.19. We list each visit or finish step in column 1. Column 2 lists the vertices adjacent to each vertex when it begins to be visited. The discovery order (the order in which the vertices are visited) is 0, 1, 3, 4, 2, 5, 6. The finish order is 4, 3, 1, 6, 5, 2, and 0.

### Performance Analysis of Depth-First Search

The loop at Step 2 is executed  $|E_u|$  (the number of edges that originate at that vertex) times. The recursive call results in this loop being applied to each vertex. The total number of steps is the sum of the edges that originate at each vertex, which is the total number of edges  $|E|$ . Thus, the algorithm is  $O(|E|)$ .

There is an implicit Step 0 to the algorithm that colors all of the vertices white. This is  $O(|V|)$ ; thus, the total running time of the algorithm is  $O(|V|+|E|)$ .

.....  
**TABLE 10.5**

Trace of Depth-First Search of Figure 10.19

Operation	Adjacent Vertices	Discovery (Visit) Order	Finish Order
Visit 0	1, 2, 3, 4	0	
Visit 1	0, 3, 4	0, 1	
Visit 3	0, 1, 4	0, 1, 3	
Visit 4	0, 1, 3	0, 1, 3, 4	
Finish 4			4
Finish 3			4, 3
Finish 1			4, 3, 1
Visit 2	0, 5, 6	0, 1, 3, 4, 2	
Visit 5	2, 6	0, 1, 3, 4, 2, 5	
Visit 6	2, 5	0, 1, 3, 4, 2, 5, 6	
Finish 6			4, 3, 1, 6
Finish 5			4, 3, 1, 6, 5
Finish 2			4, 3, 1, 6, 5, 2
Finish 0			4, 3, 1, 6, 5, 2, 0

## Implementing Depth-First Search

The class `DepthFirstSearch` is designed to be used as a building block for other algorithms. When constructed, this class performs a depth-first search on a graph and records the start time, finish time, start order, and finish order. For an unconnected graph or for a directed graph (whether connected or not), a depth-first search may not visit each vertex in the graph. Thus, once the recursive method returns, the vertices need to be examined to see whether they all have been visited; if not, the recursive process repeats, starting with the next unvisited vertex. Thus, the depth-first search can generate more than one tree. We will call this collection of trees a *forest*. Also, it may be important that we control the order in which the vertices are examined to form the forest. Thus, one of the constructors for the `DepthFirstSearch` class enables its caller to specify the order in which vertices are examined to select a new start vertex. The default is normal ascending order. The class is described in Table 10.6, and part of the code is shown in Listing 10.4.

Each constructor allocates storage for the arrays `parent`, `visited`, `discoveryOrder`, and `finishOrder` and initializes all elements of `parent` to -1 (no parent). In the constructor in Listing 10.4, the `for` statement

```
for (int i = 0; i < n; i++) {
    if (!visited[i])
        depthFirstSearch(i);
}
```

**TABLE 10.6**Class `DepthFirstSearch`

Data Field	Attribute
<code>private int discoverIndex</code>	The index that indicates the discovery order
<code>private int[] discoveryOrder</code>	The array that contains the vertices in discovery order
<code>private int finishIndex</code>	The index that indicates the finish order
<code>private int[] finishOrder</code>	The array that contains the vertices in finish order
<code>private Graph graph</code>	A reference to the graph being searched
<code>private int[] parent</code>	The array of predecessors in the depth-first search tree
<code>private boolean[] visited</code>	An array of <code>boolean</code> values to indicate whether or not a vertex has been visited
Constructor	Purpose
<code>public DepthFirstSearch(Graph graph)</code>	Constructs the depth-first search of the specified graph selecting the start vertices in ascending vertex order
<code>public DepthFirstSearch(Graph graph, int[] order)</code>	Constructs the depth-first search of the specified graph selecting the start vertices in the specified order. The first vertex visited is <code>order[0]</code>
Method	Behavior
<code>public void depthFirstSearch(int s)</code>	Recursively searches the graph starting at vertex <code>s</code>
<code>public int[] getDiscoveryOrder()</code>	Gets the discovery order
<code>public int[] getFinishOrder()</code>	Gets the finish order
<code>public int[] getParent()</code>	Gets the parents in the depth-first search tree

calls the recursive depth-first search method. Method `depthFirstSearch` follows the algorithm shown earlier. If the graph is connected, all vertices will be visited after the return from the initial call to `depthFirstSearch`. If the graph is not connected, additional calls will be made using a start vertex that has not been visited.

In the constructor (not shown) that allows the client to control the order of selection for start vertices, the parameter `int[] order` specifies this sequence. To code this constructor, change the `if` statement in the `for` loop of the first constructor to

```
if (!visited[order[i]])
    depthFirstSearch(order[i]);
```

The rest of the code is the same.

.....

#### **LISTING 10.4**

`DepthFirstSearch.java`

```
/** Class to implement the depth-first search algorithm. */
public class DepthFirstSearch {

    // Data Fields
    /** A reference to the graph being searched. */
    private Graph graph;
    /** Array of parents in the depth-first search tree. */
    private int[] parent;
    /** Flag to indicate whether this vertex has been visited. */
    private boolean[] visited;
    /** The array that contains each vertex in discovery order. */
    private int[] discoveryOrder;
    /** The array that contains each vertex in finish order. */
    private int[] finishOrder;
    /** The index that indicates the discovery order. */
    private int discoverIndex = 0;
    /** The index that indicates the finish order. */
    private int finishIndex = 0;

    // Constructors
    /** Construct the depth-first search of a Graph starting at
        vertex 0 and visiting the start vertices in ascending order.
        @param graph The graph
    */
    public DepthFirstSearch(Graph graph) {
        this.graph = graph;
        int n = graph.getNumV();
        parent = new int[n];
        visited = new boolean[n];
        discoveryOrder = new int[n];
        finishOrder = new int[n];
        for (int i = 0; i < n; i++) {
            parent[i] = -1;
        }
        for (int i = 0; i < n; i++) {
            if (!visited[i])
                depthFirstSearch(i);
        }
    }

    /** Construct the depth-first search of a Graph
        selecting the start vertices in the specified order.
        The first vertex visited is order[0].
    */
}
```

```

@param graph The graph
@param order The array giving the order
in which the start vertices should be selected
*/
public DepthFirstSearch(Graph graph, int[] order) {
    // Same as constructor above except for the if statement.
}

/** Recursively depth-first search the graph starting at vertex current.
 * @param current The start vertex
 */
public void depthFirstSearch(int current) {
    /* Mark the current vertex visited. */
    visited[current] = true;
    discoveryOrder[discoverIndex++] = current;
    /* Examine each vertex adjacent to the current vertex */
    Iterator<Edge> itr = graph.edgeIterator(current);
    while (itr.hasNext()) {
        int neighbor = itr.next().getDest();
        /* Process a neighbor that has not been visited */
        if (!visited[neighbor]) {
            /* Insert (current, neighbor) into the depth-first search tree. */
            parent[neighbor] = current;
            /* Recursively apply the algorithm starting at neighbor. */
            depthFirstSearch(neighbor);
        }
    }
    /* Mark current finished. */
    finishOrder[finishIndex++] = current;
}
}

```

## Testing Method `depthFirstSearch`

Next, we show a `main` method that tests the class. It is a simple driver program that can be used to read a graph and then initiate a depth-first traversal. After the traversal, the driver program displays the arrays that represent the search results.

```

/** Main method to test depth-first search method
 * @pre args[0] is the name of the input file.
 * @param args The command line arguments
 */
public static void main(String[] args) {
    Graph g = null;
    int n = 0;
    try {
        Scanner scan = new Scanner(new File(args[0]));
        g = AbstractGraph.createGraph(scan, true, "List");
        n = g.getNumV();
    } catch (IOException ex) {
        ex.printStackTrace();
        System.exit(1);
        // Error
    }

    // Perform depth-first search.
    DepthFirstSearch dfs = new DepthFirstSearch(g);
    int[] dOrder = dfs.getDiscoveryOrder();

```

```

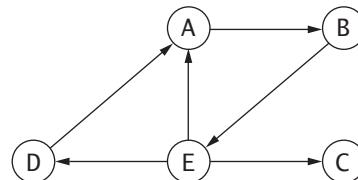
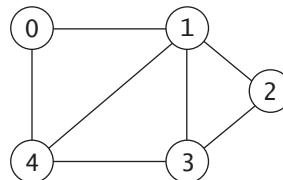
int[] fOrder = dfs.getFinishOrder();
System.out.println("Discovery and finish order");
for (int i = 0; i < n; i++) {
    System.out.println(dOrder[i] + " " + fOrder[i]);
}
}

```

## EXERCISES FOR SECTION 10.4

### SELF-CHECK

1. Show the breadth-first search trees for the following graphs.



2. Show the depth-first search trees for the graphs in Exercise 1 above.

### PROGRAMMING

1. Provide all accessor methods for class `DepthFirstSearch` and the constructor that specifies the order of start vertices.
2. Implement method `depthFirstSearch` without using recursion. *Hint:* Use a stack to save the parent of the current vertex when you start to search one of its adjacent vertices.



## 10.5 Applications of Graph Traversals

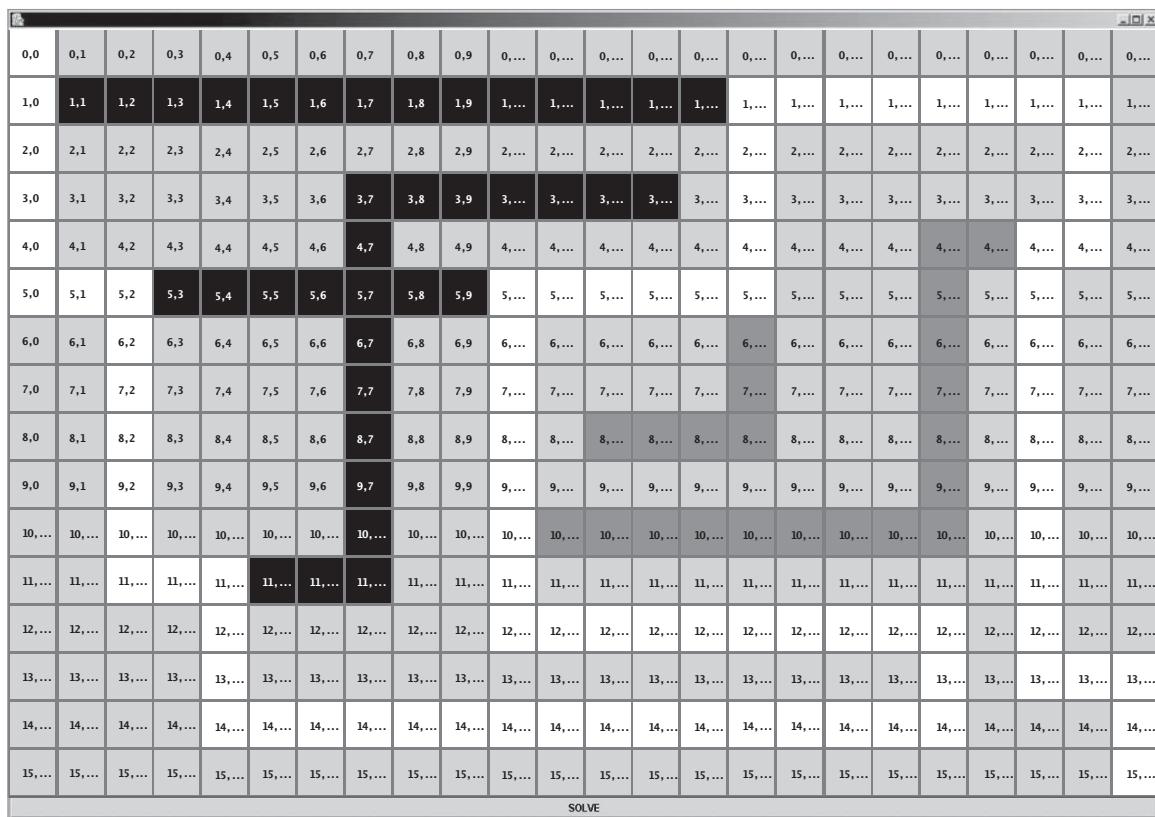
### CASE STUDY Shortest Path through a Maze

**Problem** We want to design a program that will find the shortest path through a maze. In Chapter 5, we showed how to write a recursive program that found a solution to a maze. This program used a backtracking algorithm that visited alternate paths. When it found a dead end, it backed up and tried another path, and eventually it found a solution.

Figure 10.21 shows a maze solution generated by this recursive program. The light-gray cells are barriers in the maze. The white squares show the solution path, the black squares show the squares that were visited but rejected, and the dark-gray squares were not visited. As you can see, the program did not find an optimal solution. (This is a consequence of the program advancing the solution path to the south before attempting to advance it to the east.) We want to find the shortest path, defined as the one with the fewest decision points in it.

**FIGURE 10.21**

## Recursive Solution to a Maze



**Analysis** We can represent the maze shown in Figure 10.21 by a graph, where we place a node at each decision point and at each dead end, as shown in Figure 10.22.

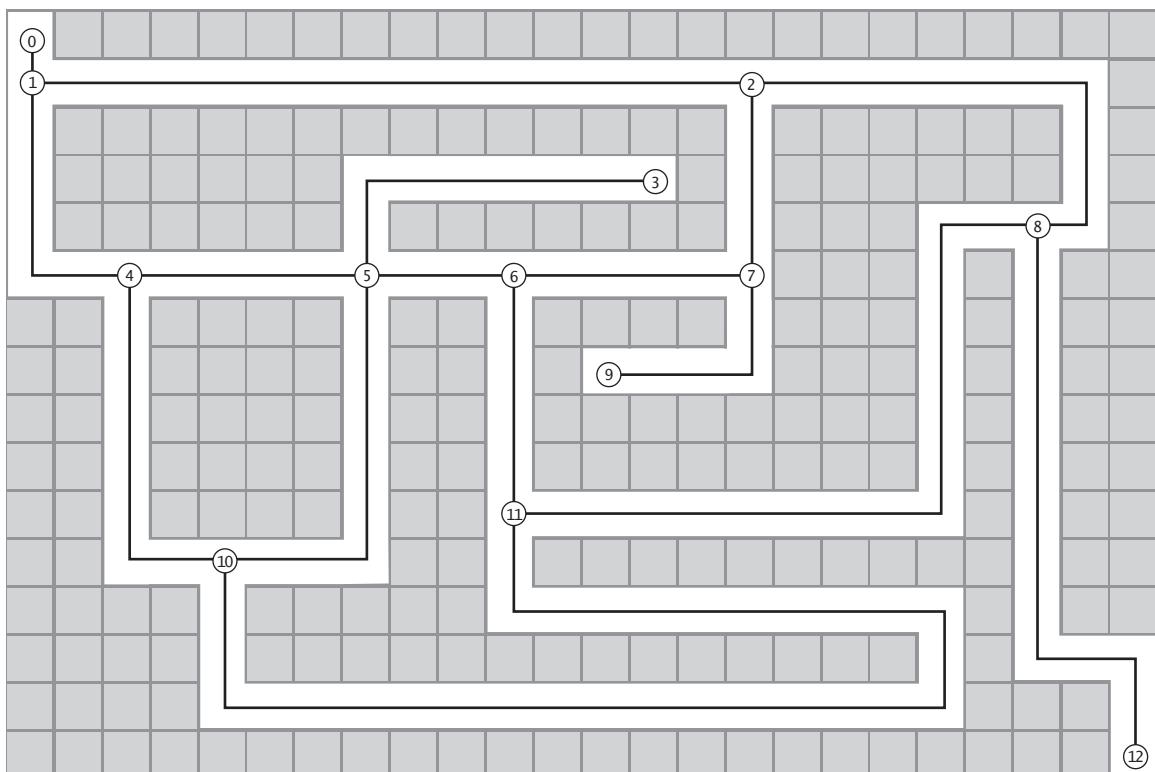
Now that we have the maze represented as a graph, we need to find the shortest path from the start point (vertex 0) to the end point (vertex 12). The breadth-first search method will return the shortest path from each vertex to its parent (the array of parent vertices), and we can use this array to find the shortest path to the end point. Recall that our shortest path will contain the smallest number of vertices, but not necessarily the smallest number of cells, in the path.

**Design** Your program will need the following data structures:

- An external representation of the maze, consisting of the number of vertices and the edges.
  - An object of a class that implements the Graph interface.
  - An array to hold the predecessors returned from the breadthFirstSearch method.
  - A stack to reverse the path.

**FIGURE 10.22**

Graph Representation of the Maze in Figure 10.21



The algorithm is as follows:

1. Read in the number of vertices and create the graph object.
2. Read in the edges and insert the edges into the graph.
3. Call the `breadthFirstSearch` method with this graph and the starting vertex as its argument. The method returns the array `parent`.
4. Start at  $v$ , the end vertex.
5. **while**  $v$  is not  $-1$
6.     Push  $v$  onto the stack.
7.     Set  $v$  to  $\text{parent}[v]$ .
8. **while** the stack is not empty
9.     Pop a vertex off the stack and output it.

### Implementation

Listing 10.5 shows the program. We assume that the graph that represents the maze is stored in a text file. The first line of this file contains the number of vertices. The edges are on subsequent lines. The method `loadEdgesFromFile` reads the source and destination vertices and inserts the edge into the graph. The rest of the code follows the algorithm.

**LISTING 10.5**

Program to Solve a Maze Using a Breadth-First Search

```

.....
import java.io.*;
import java.util.*;

/** Program to solve a maze represented as a graph.
    This program performs a breadth-first search of the graph
    to find the "shortest" path from the start vertex to the
    end. It is assumed that the start vertex is 0, and the
    end vertex is numV-1.
*/
public class Maze {

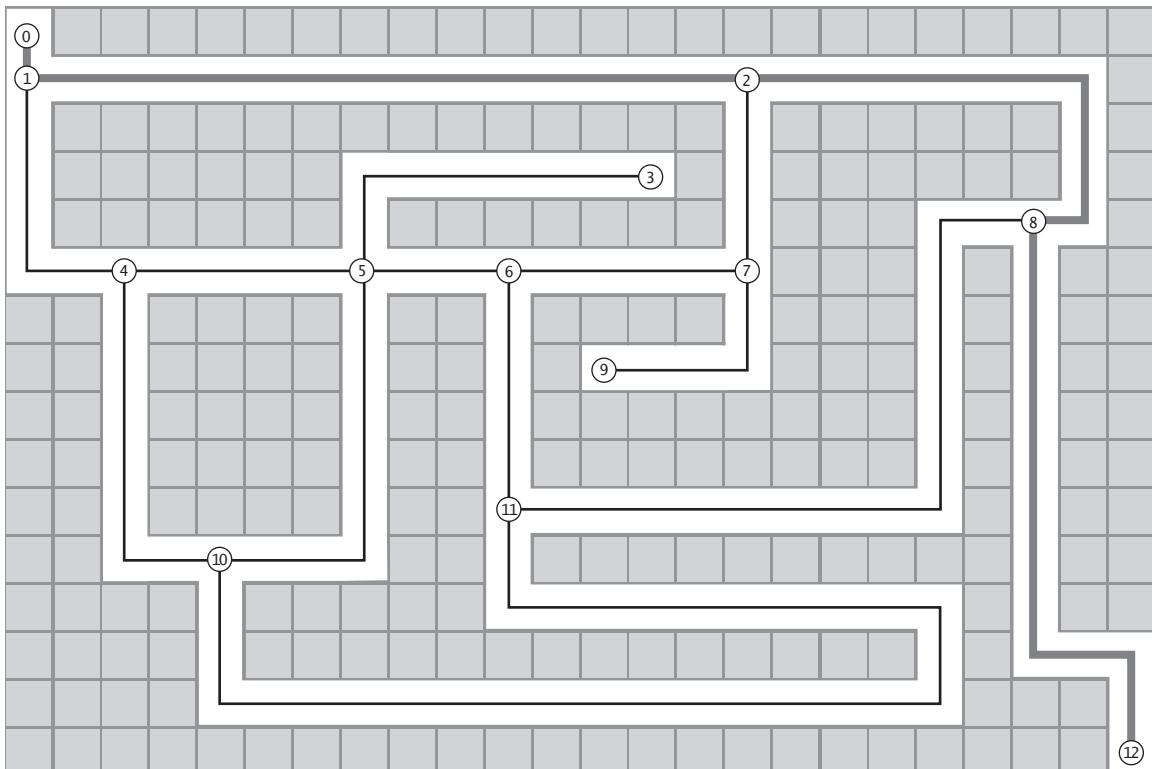
    /** Main method to solve the maze.
        @pre args[0] contains the name of the input file.
        @param args Command line argument
    */
    public static void main(String[] args) {
        int numV = 0;
        // The number of vertices.
        Graph theMaze = null;
        // Load the graph data from a file.
        try {
            Scanner scan = new Scanner(new File(args[0]));
            theMaze = AbstractGraph.createGraph(scan, false, "List");
            numV = theMaze.getNumV();
        } catch (IOException ex) {
            System.err.println("IO Error while reading graph");
            System.err.println(ex.toString());
            System.exit(1);
        }
        // Perform breadth-first search.
        int parent[] = BreadthFirstSearch.breadthFirstSearch(theMaze, 0);
        // Construct the path.
        Deque<Integer> thePath = new ArrayDeque<>();
        int v = numV - 1;
        while (parent[v] != -1) {
            thePath.push(v);
            v = parent[v];
        }
        // Output the path.
        System.out.println("The Shortest path is:");
        while (!thePath.isEmpty()) {
            System.out.println(thePath.pop());
        }
    }
}

```

**Testing** Test this program with a variety of mazes. Use mazes for which the original program finds the shortest path and mazes for which it does not. For the graph shown in Figure 10.23, the shortest path from 0 to 12 is  $0 \rightarrow 1 \rightarrow 2 \rightarrow 8 \rightarrow 12$ .

**FIGURE 10.23**

Solution to Maze in Figure 10.21



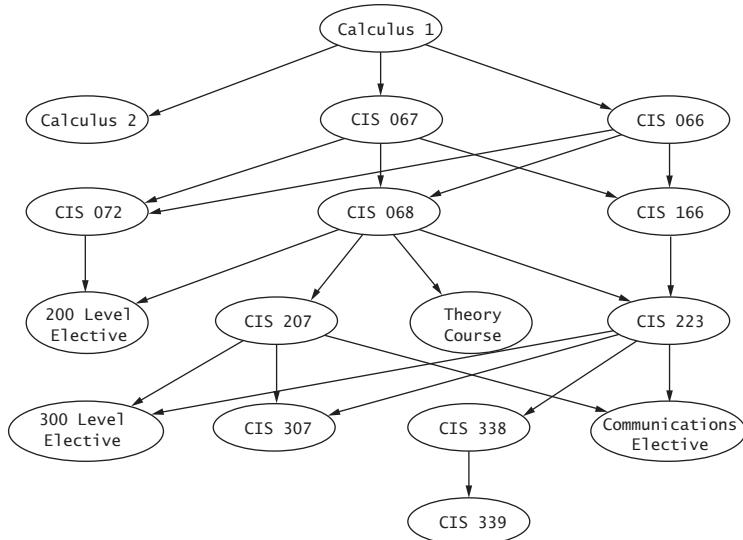
## CASE STUDY Topological Sort of a Graph

**Problem** There are many problems in which one activity cannot be started before another one has been completed. One that you may have already encountered is determining the order in which you can take courses. Some courses have prerequisites. Some have more than one prerequisite. Furthermore, the prerequisites may have prerequisites. Figure 10.24 shows the courses and prerequisites of a Computer Science program at the authors' university.

Graphs such as the one shown in Figure 10.24 are known as *directed acyclic graphs* (*DAGs*). They are directed graphs that contain no cycles; that is, there are no loops, so

**FIGURE 10.24**

Prerequisites for a Computer Science Program

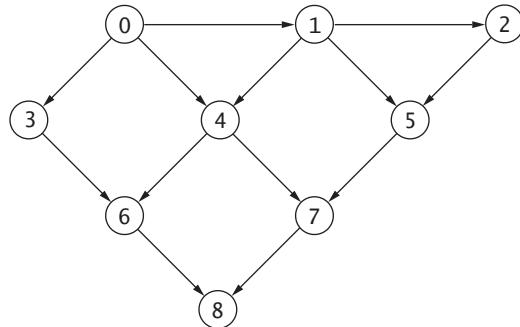


once you pass through a vertex, there is no path back to that vertex. Figure 10.25 shows another example of a DAG.

A *topological sort* of the vertices of a DAG is an ordering of the vertices such that if  $(u, v)$  is an edge, then  $u$  appears before  $v$ . This must be true for all edges. For example, 0, 1, 2, 3, 4, 5, 6, 7, 8 is a valid topological sort of the graph in Figure 10.25, but 0, 1, 5, 3, 4, 2, 6, 7, 8 is not because  $2 \rightarrow 5$  is an edge, but 5 appears before 2. There are many valid paths through the prerequisite graph and many valid topological sorts. Another valid topological sort is 0, 3, 1, 4, 6, 2, 5, 7, 8.

**FIGURE 10.25**

Example of a DAG



### Analysis

If there is an edge from  $u$  to  $v$  in a DAG, then if we perform a depth-first search of this graph, the finish time of  $u$  must be after the finish time of  $v$ . When we return to  $u$ , either  $v$  has not been visited or it has finished. It is not possible that  $v$  would be visited but not finished, because if it were possible, we would discover  $u$  on a path that had passed through  $v$ . That would mean that there is a loop or cycle in the graph.

For example, in Figure 10.25, we could start the depth-first search at 0, then visit 4, followed by 6, followed by 8. Then, returning to 4, we would have to visit 7 before returning to 0. Then we would visit 1, and from 1 we would see that 4 has finished. Alternatively, we could

start at 0 and then go to 1, and we would see that 4 has not been visited. What we cannot have happen is that we start at 0, then visit 4, and eventually get to 1 before finishing 4.

**Design** If we perform a depth-first search of a graph and then order the vertices by the inverse of their finish order, we will have one topological sort of a DAG. The topological sort produced by listing the vertices in the inverse of their finish order after a depth-first search of the graph in Figure 10.25 is 0, 3, 1, 4, 6, 2, 5, 7, 8.

### Algorithm for Topological Sort

1. Read the graph from a data file.
2. Perform a depth-first search of the graph.
3. List the vertices in reverse of their finish order.

### Implementation

We can use our `DepthFirstSearch` class to implement this algorithm. Listing 10.6 shows a program that does this. It begins by reading the graph from an input file. It then creates a `DepthFirstSearch` object `dfs`. The constructor of the `DepthFirstSearch` class performs the depth-first search and saves information about the graph. We then call the `getFinishOrder` method to get the vertices in the order in which they finished. If we output this array starting at `numVertices - 1`, we will obtain the topological sort of the graph.

.....

#### LISTING 10.6

`TopologicalSort.java`

```
import java.util.*;

/** This program outputs the topological sort of a directed graph
 *  that contains no cycles.
 */
public class TopologicalSort {

    /** The main method that performs the topological sort.
     * @param args[0] contains the name of the file
     *           that contains the graph. It has no cycles.
     * @param args The command line arguments
     */
    public static void main(String[] args) {
        Graph theGraph = null;
        int numVertices = 0;
        try {
            // Connect Scanner to input file.
            Scanner scan = new Scanner(new File(args[0]));
            // Load the graph data from a file.
            theGraph = AbstractGraph.createGraph(scan, true, "List");
            numVertices = theGraph.getNumV();
        } catch (Exception ex) {
            ex.printStackTrace();
            System.exit(1);
            // Error exit.
        }
        // Perform the depth-first search.
        DepthFirstSearch dfs = new DepthFirstSearch(theGraph);
        // Obtain the finish order.
        int[] finishOrder = dfs.getFinishOrder();
```

```
// Print the vertices in reverse finish order.  
System.out.println("The Topological Sort is");  
for (int i = numVertices - 1; i >= 0; i--) {  
    System.out.println(finishOrder[i]);  
}  
}  
}
```

**Testing** Test this program using several different graphs. Use sparse graphs and dense graphs. Make sure that each graph you try has no loops or cycles. If it does, the algorithm may display an invalid output.

## **EXERCISES FOR SECTION 10.5**

### SELF-CHECK

1. Draw the depth-first search tree of the graph in Figure 10.24 and then list the vertices in reverse finish order.
  2. List some alternative topological sorts for the graph in Figure 10.24.



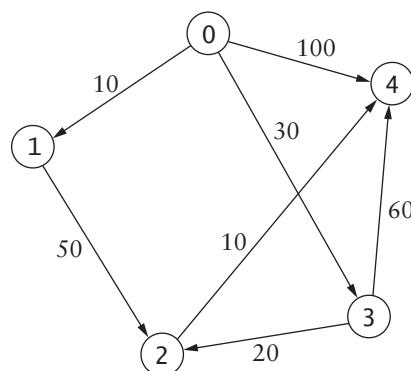
## 10.6 Algorithms Using Weighted Graphs

## Finding the Shortest Path from a Vertex to All Other Vertices

The breadth-first search discussed in Section 10.4 found the shortest path from the start vertex to all other vertices, assuming that the length of each edge was the same. We now consider the problem of finding the shortest path where the length of each edge may be different—that is, in a weighted directed graph such as that shown in Figure 10.26. The computer scientist Edsger W. Dijkstra developed an algorithm, now called Dijkstra's algorithm (“A Note on Two Problems in Connection with Graphs,” *Numerische Mathematik*, Vol. 1 [1959], pp. 269–271), to solve this problem. This algorithm makes the assumption that all of the edge values are positive.

## FIGURE 10.26

## Weighted Directed Graph



For Dijkstra's algorithm, we need two sets,  $S$  and  $V-S$ , and two arrays,  $d$  and  $p$ .  $S$  will contain the vertices for which we have computed the shortest distance, and  $V-S$  will contain the vertices that we still need to process. The entry  $d[v]$  will contain the shortest distance from  $s$  to  $v$ , and  $p[v]$  will contain the predecessor of  $v$  in the path from  $s$  to  $v$ .

We initialize  $S$  by placing the start vertex,  $s$ , into it. We initialize  $V-S$  by placing the remaining vertices into it. For each  $v$  in  $V-S$ , we initialize  $d$  by setting  $d[v]$  equal to the weight of the edge  $w(s, v)$  for each vertex,  $v$ , adjacent to  $s$  and to  $\infty$  for each vertex that is not adjacent to  $s$ . We initialize  $p[v]$  to  $s$  for each  $v$  in  $V-S$ .

For example, given the graph shown in Figure 10.26, the set  $S$  would initially be  $\{0\}$ , and  $V-S$  would be  $\{1, 2, 3, 4\}$ . The arrays  $d$  and  $p$  would be defined as follows.

$v$	$d[v]$	$p[v]$
1	10	0
2	$\infty$	0
3	30	0
4	100	0

The first row shows that the distance from vertex 0 to vertex 1 is 10 and that vertex 0 is the predecessor of vertex 1. The second row shows that vertex 2 is not adjacent to vertex 0.

We now find the vertex  $u$  in  $V-S$  that has the smallest value of  $d[u]$ . Using our example, this is 1. We now consider the vertices  $v$  that are adjacent to  $u$ . If the distance from  $s$  to  $u$  ( $d[u]$ ) plus the distance from  $u$  to  $v$  (i.e.,  $w(u, v)$ ) is smaller than the known distance from  $s$  to  $v$ ,  $d[v]$ , then we update  $d[v]$  to be  $d[u] + w(u, v)$ , and we set  $p[v]$  to  $u$ . In our example, the value of  $d[1]$  is 10, and  $w(1, 2)$  is 50. Since  $10 + 50 = 60$  is less than  $\infty$ , we set  $d[2]$  to 60 and  $p[2]$  to 1. We remove 1 from  $V-S$  and place it into  $S$ . We repeat this until  $V-S$  is empty.

After the first pass through this loop,  $S$  is  $\{0, 1\}$ ,  $V-S$  is  $\{2, 3, 4\}$ , and  $d$  and  $p$  are as follows:

$v$	$d[v]$	$p[v]$
1	10	0
2	60	1
3	30	0
4	100	0

We again select  $u$  from  $V-S$  with the smallest  $d[u]$ . This is now 3. The adjacent vertices to 3 are 2 and 4. The distance from 0 to 3,  $d[3]$ , is 30. The distance from 3 to 2 is 20. Because  $30 + 20 = 50$  is less than the current value of  $d[2]$ , 60, we update  $d[2]$  to 50 and change  $p[2]$  to 3. Also, because  $30 + 60 = 90$  is less than 100, we update  $d[4]$  to 90 and set  $p[4]$  to 3.

Now  $S$  is  $\{0, 1, 3\}$ , and  $V-S$  is  $\{2, 4\}$ . The arrays  $d$  and  $p$  are as follows:

$v$	$d[v]$	$p[v]$
1	10	0
2	50	3
3	30	0
4	90	3

Next, we select vertex 2 from  $V-S$ . The only vertex adjacent to 2 is 4. Since  $d[2] + w(2, 4) = 50 + 10 = 60$  is less than  $d[4]$ , 90, we update  $d[4]$  to 60 and  $p[4]$  to 2. Now  $S$  is  $\{0, 1, 2, 3\}$ ,  $V-S$  is  $\{4\}$ , and  $d$  and  $p$  are as follows:

v	d[v]	p[v]
1	10	0
2	50	3
3	30	0
4	60	2

Finally, we remove 4 from  $V-S$  and find that it has no adjacent vertices. We are now done. The array  $d$  shows the shortest distances from the start vertex to all other vertices, and the array  $p$  can be used to determine the corresponding paths. For example, the path from vertex 0 to vertex 4 has a length of 60, and it is the reverse of 4, 2, 3, 0; therefore, the shortest path is  $0 \rightarrow 3 \rightarrow 2 \rightarrow 4$ .

### Dijkstra's Algorithm

1. Initialize  $S$  with the start vertex,  $s$ , and  $V-S$  with the remaining vertices.
2. **for** all  $v$  in  $V-S$ 
  3. Set  $p[v]$  to  $s$ .
  4. **if** there is an edge  $(s, v)$ 
    5. Set  $d[v]$  to  $w(s, v)$ .
  6. **else**
    6. Set  $d[v]$  to  $\infty$ .
7. **while**  $V-S$  is not empty
  8. **for** all  $u$  in  $V-S$ , find the smallest  $d[u]$ .
  9. Remove  $u$  from  $V-S$  and add  $u$  to  $S$ .
  10. **for** all  $v$  adjacent to  $u$  in  $V-S$ 
    11. **if**  $d[u] + w(u, v)$  is less than  $d[v]$ 
      12. Set  $d[v]$  to  $d[u] + w(u, v)$ .
      13. Set  $p[v]$  to  $u$ .

### Analysis of Dijkstra's Algorithm

Step 1 requires  $|V|$  steps.

The loop at Step 2 will be executed  $|V - 1|$  times.

The loop at Step 7 will also be executed  $|V - 1|$  times.

Within the loop at Step 7, we have to consider Steps 8 and 9. For these steps, we will have to search each value in  $V-S$ . This decreases each time through the loop at Step 7, so we will have  $|V| - 1 + |V| - 2 + \dots + 1$ . This is  $O(|V|^2)$ . Therefore, Dijkstra's algorithm as stated is  $O(|V|^2)$ . We will look at possible improvements to this for sparse graphs when we discuss a similar algorithm in the next subsection.

### Implementation

Listing 10.7 provides a straightforward implementation of Dijkstra's algorithm using `HashSet vMinusS` to represent set  $V-S$ . We chose to implement the algorithm as a `static` method with the inputs (the graph and starting point) and outputs (predecessor and distance

array) passed through parameters. An alternative approach would be to make them data fields in a class that contained this method. We use iterators to traverse `vMinusS`.

If we used an adjacency list representation for the graph (i.e., class `ListGraph`, described earlier), then we would code Step 10 (update the distances) to iterate through the edges adjacent to vertex  $u$ , and then update the distance if the destination vertex was in `vMinusS`. The modified code follows:

```
// Update the distances.
Iterator<Edge> edgeIter = graph.edgeIterator(u);
while (edgeIter.hasNext()) {
    Edge edge = edgeIter.next();
    int v = edge.getDest();
    if (vMinusS.contains(new Integer(v))) {
        double weight = edge.getWeight();
        if (dist[u] + weight < dist[v]) {
            dist[v] = dist[u] + weight;
            pred[v] = u;
        }
    }
}
```

#### LISTING 10.7

Dijkstra's Shortest-Path Algorithm

```
/** Dijkstra's Shortest-Path algorithm.
 * @param graph The weighted graph to be searched
 * @param start The start vertex
 * @param pred Output array to contain the predecessors in the shortest path
 * @param dist Output array to contain the distance in the shortest path
 */
public static void dijkstrasAlgorithm(Graph graph, int start, int[] pred,
                                      double[] dist) {
    int numV = graph.getNumV();
    HashSet<Integer> vMinusS = new HashSet<>(numV);
    // Initialize V-S.
    for (int i = 0; i < numV; i++) {
        if (i != start) {
            vMinusS.add(i);
        }
    }
    // Initialize pred and dist.
    for (int v : vMinusS) {
        pred[v] = start;
        dist[v] = graph.getEdge(start, v).getWeight();
    }

    // Main loop
    while (vMinusS.size() != 0) {
        // Find the value u in V-S with the smallest dist[u].
        double minDist = Double.POSITIVE_INFINITY;
        int u = -1;
        for (int v : vMinusS) {
            if (dist[v] < minDist) {
                minDist = dist[v];
                u = v;
            }
        }
        // Remove u from vMinusS.
        vMinusS.remove(u);
    }
}
```

```

// Update the distances.
for (int v : vMinusS) {
    if (graph.isEdge(u, v)) {
        double weight = graph.getEdge(u, v).getWeight();
        if (dist[u] + weight < dist[v]) {
            dist[v] = dist[u] + weight;
            pred[v] = u;
        }
    }
}
}

```

## Minimum Spanning Trees

A *spanning tree* is a subset of the edges of a graph such that there is only one edge between each vertex, and all of the vertices are connected. If we have a spanning tree for a graph, then we can access all the vertices of the graph from the start node. The *cost of a spanning tree* is the sum of the weights of the edges. We want to find the *minimum spanning tree* or the spanning tree with the smallest cost. For example, if we want to start up our own long-distance phone company and need to connect the cities shown in Figure 10.4, finding the minimum spanning tree would allow us to build the cheapest network.

We will discuss the algorithm published by R. C. Prim (“Shortest Connection Networks and Some Generalizations,” *Bell System Technical Journal*, Vol. 36 [1957], pp. 1389–1401) for finding the minimum spanning tree of a graph. It is very similar to Dijkstra’s algorithm, but Prim published his algorithm in 1957, 2 years before Dijkstra’s paper that contains an algorithm for finding the minimum spanning tree that is essentially the same as Prim’s as well as the previously discussed algorithm for finding the shortest paths.

# Overview of Prim's Algorithm

The vertices are divided into two sets:  $S$ , the set of vertices in the spanning tree, and  $V-S$ , the remaining vertices. As in Dijkstra's algorithm, we maintain two arrays:  $d[v]$  will contain the length of the shortest edge from a vertex in  $S$  to the vertex  $v$  that is in  $V-S$ , and  $p[v]$  will contain the source vertex for that edge. The only difference between the algorithm to find the shortest path and the algorithm to find the minimum spanning tree is the contents of  $d[v]$ . In the algorithm to find the shortest path,  $d[v]$  contains the total length of the path from the starting vertex. In the algorithm to find the minimum spanning tree,  $d[v]$  contains only the length of the final edge. We show the essentials of Prim's algorithm next.

## Prim's Algorithm for Finding the Minimum Spanning Tree

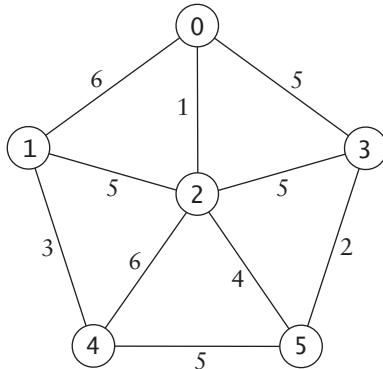
1. Initialize  $S$  with the start vertex,  $s$ , and  $V-S$  with the remaining vertices.
  2. **for** all  $v$  in  $V-S$ 
    - 3.     Set  $p[v]$  to  $s$ .
    - 4.     **if** there is an edge  $(s, v)$ 
      - 5.         Set  $d[v]$  to  $w(s, v)$ .
    - 6.     **else**
      - 6.         Set  $d[v]$  to  $\infty$ .
  7. **while**  $V-S$  is not empty

8.     **for** all  $u$  in  $V-S$ , find the smallest  $d[u]$ .
9.     Remove  $u$  from  $V-S$  and add it to  $S$ .
10.   Insert the edge  $(u, p[u])$  into the spanning tree.
11.   **for** all  $v$  in  $V-S$
12.      **if**  $w(u, v) < d[v]$
13.         Set  $d[v]$  to  $w(u, v)$ .
14.         Set  $p[v]$  to  $u$ .

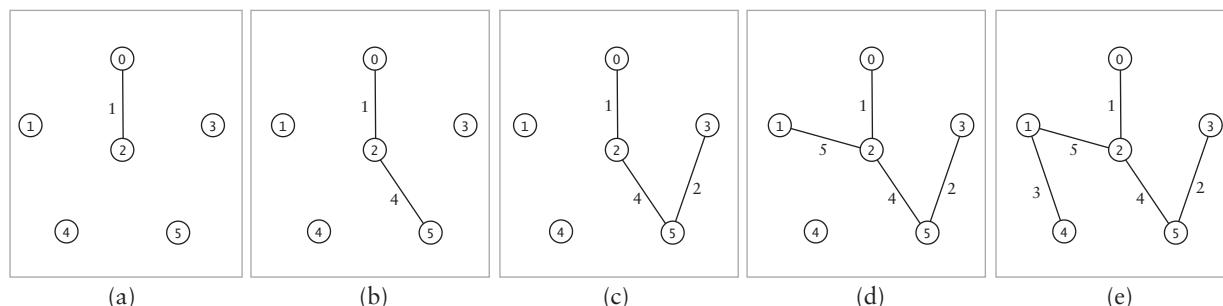
In the array  $d$ ,  $d[v]$  contains the length of the shortest known (previously examined) edge from a vertex in  $S$  to the vertex  $v$ , while  $v$  is a member of  $V-S$ . In the array  $p$ , the value  $p[v]$  is the source vertex of this shortest edge. When  $v$  is removed from  $V-S$ , we no longer update these entries in  $d$  and  $p$ .

**EXAMPLE 10.2** Consider the graph shown in Figure 10.27. We initialize  $S$  to  $\{0\}$  and  $V-S$  to  $\{1, 2, 3, 4, 5\}$ . The smallest edge from  $u$  to  $v$ , where  $u$  is in  $S$  and  $v$  is in  $V-S$ , is the edge  $(0, 2)$ . We add this edge to the spanning tree and add 2 to  $S$  (see Figure 10.28(a)). The set  $S$  is now  $\{0, 2\}$  and  $V-S$  is  $\{1, 3, 4, 5\}$ . We now have to consider all of the edges  $(u, v)$ , where  $u$  is either 0 or 2, and  $v$  is 1, 3, 4, or 5 (there are eight possible edges). The smallest one is  $(2, 5)$ . We add this to the spanning tree, and  $S$  now is  $\{0, 2, 5\}$  and  $V-S$  is  $\{1, 3, 4\}$  (see Figure 10.28(b)). The next smallest edge is  $(5, 3)$ . We insert that into the tree and add 3 to  $S$  (see Figure 10.28(c)). Now  $V-S$  is  $\{1, 4\}$ . The smallest edge is  $(2, 1)$ . After adding this edge (see Figure 10.28(d)), we are left with  $V-S$  being  $\{4\}$ . The smallest edge to 4 is  $(1, 4)$ . This is added to the tree, and the spanning tree is complete (see Figure 10.28(e)).

**FIGURE 10.27**  
Graph for Example 10.2



**FIGURE 10.28**  
Building a Minimum Spanning Tree Using Prim's Algorithm



## Analysis of Prim's Algorithm

Step 8 is  $O(|V|)$ . Because this is within the loop at Step 7, it will be executed  $O(|V|)$  times for a total time of  $O(|V|^2)$ . Step 11 is  $O(|E_u|)$ , the number of edges that originate at  $u$ . Because Step 11 is inside the loop of Step 7, it will be executed for all vertices; thus, the total is  $O(|E|)$ . Because  $|V|^2$  is greater than  $|E|$ , the overall cost of the algorithm is  $O(|V|^2)$ .

By using a priority queue to hold the edges from  $S$  to  $V-S$ , we can improve on this algorithm. Then Step 8 is  $O(\log n)$ , where  $n$  is the size of the priority queue. In the worst case, all of the edges are inserted into the priority queue, and the overall cost of the algorithm is then  $O(|E| \log |V|)$ . We say that the algorithm is  $O(|E| \log |V|)$  instead of saying that it is  $O(|E| \log |E|)$ , even though the maximum size of the priority queue is  $|E|$ , because  $|E|$  is bounded by  $|V|^2$  and  $\log |V|^2$  is  $2 \times \log |V|$ .

For a dense graph, where  $|E|$  is approximately  $|V|^2$ , this is not an improvement; however, for a sparse graph, where  $|E|$  is significantly less than  $|V|^2$ , it is. Furthermore, computer science researchers have developed improved priority queue implementations that give  $O(|E| + |V| \log |V|)$  or better performance.

## Implementation

Listing 10.8 shows an implementation of Prim's algorithm using a priority queue to hold the edges from  $S$  to  $V-S$ . The arrays  $p$  and  $d$  given in the algorithm description above are not needed because the priority queue contains complete edges. For a given vertex  $d$ , if a shorter edge is discovered, we do not remove the entry containing the longer edge from the priority queue. We merely insert new edges as they are discovered. Therefore, when the next shortest edge is removed from the priority queue, it may have a destination that is no longer in  $V-S$ . In that case, we continue to remove edges from the priority queue until we find one with a destination that is still in  $V-S$ . This is done with the following loop:

```
do {
    edge = pQ.remove();
    dest = edge.getDest();
} while(!vMinusS.contains(dest));
```

---

### LISTING 10.8

Prim's Minimum Spanning Tree Algorithm

```
/** Prim's Minimum Spanning Tree algorithm.
 * @param graph The weighted graph to be searched
 * @param start The start vertex
 * @return An ArrayList of edges that forms the MST
 */
public static ArrayList<Edge> primsAlgorithm(Graph graph, int start) {
    ArrayList<Edge> result = new ArrayList<>();
    int numV = graph.getNumV();
    // Use a HashSet to represent V-S.
    Set<Integer> vMinusS = new HashSet<>(numV);
    // Declare the priority queue.
    Queue<Edge> pQ = new PriorityQueue<>(numV,
        (e1, e2) -> Double.compare(e1.getWeight(), e2.getWeight()));
    // Initialize V-S.
    for (int i = 0; i < numV; i++) {
        if (i != start) {
            vMinusS.add(i);
        }
    }
}
```

```

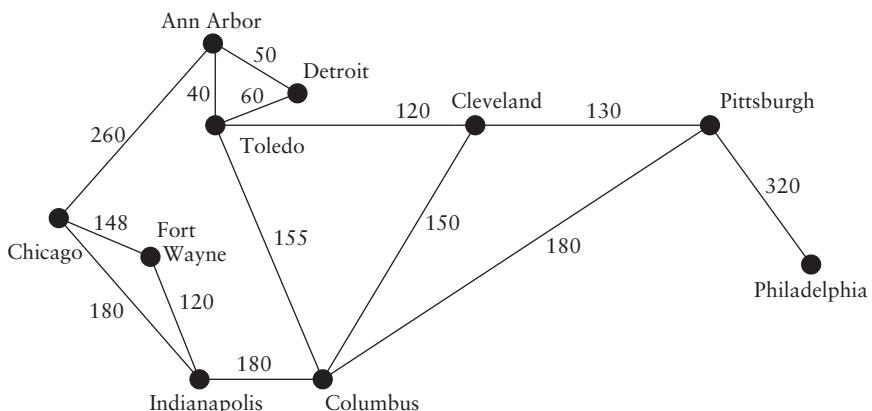
int current = start;
// Main loop
while (vMinusS.size() != 0) {
    // Update priority queue.
    Iterator<Edge> iter = graph.edgeIterator(current);
    while (iter.hasNext()) {
        Edge edge = iter.next();
        int dest = edge.getDest();
        if (vMinusS.contains(dest)) {
            pQ.add(edge);
        }
    }
    // Find the shortest edge whose source is in S and
    // destination is in V-S.
    int dest = -1;
    Edge edge = null;
    do {
        edge = pQ.remove();
        dest = edge.getDest();
    } while(!vMinusS.contains(dest));
    // Take dest out of vMinusS.
    vMinusS.remove(dest);
    // Add edge to result.
    result.add(edge);
    // Make this the current vertex.
    current = dest;
}
return result;
}

```

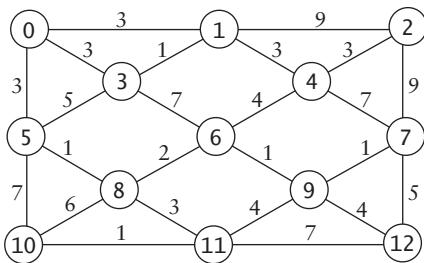
## EXERCISES FOR SECTION 10.6

### SELF-CHECK

1. Trace the execution of Dijkstra's algorithm to find the shortest path from Philadelphia to the other cities shown in the following graph.



2. Trace the execution of Dijkstra's algorithm to find the shortest paths from vertex 0 to the other vertices in the following graph.



3. Trace the execution of Prim's algorithm to find the minimum spanning tree for the graph shown in Exercise 2.
4. Trace the execution of Prim's algorithm to find the minimum spanning tree for the graph shown in Exercise 1.

## Chapter Review

- ◆ A graph consists of a set of vertices and a set of edges. An edge is a pair of vertices. Graphs may be either undirected or directed. Edges may have a value associated with them known as the weight.
- ◆ In an undirected graph, if  $\{u, v\}$  is an edge, then there is a path from vertex  $u$  to vertex  $v$ , and vice versa.
- ◆ In a directed graph, if  $(u, v)$  is an edge, then  $(v, u)$  is not necessarily an edge.
- ◆ If there is an edge from one vertex to another, then the second vertex is adjacent to the first. A path is a sequence of adjacent vertices. A path is simple if the vertices in the path are distinct except, perhaps, for the first and last vertex, which may be the same. A cycle is a path in which the first and last vertexes are the same.
- ◆ A graph is considered connected if there is a path from each vertex to every other vertex.
- ◆ A tree is a special case of a graph. Specifically, a tree is a connected graph that contains no cycles.
- ◆ Graphs may be represented by an array of adjacency lists. There is one list for each vertex, and the list contains the edges that originate at this vertex.
- ◆ Graphs may be represented by a two-dimensional square array called an adjacency matrix. The entry  $[u][v]$  will contain a value to indicate that an edge from  $u$  to  $v$  is present or absent.
- ◆ A breadth-first search of a graph finds all vertices reachable from a given vertex via the shortest path, where the length of the path is based on the number of vertices in the path.
- ◆ A depth-first search of a graph starts at a given vertex and then follows a path of unvisited vertices until it reaches a point where there are no unvisited vertices that are reachable. It then backtracks until it finds an unvisited vertex, and then continues along the path to that vertex.

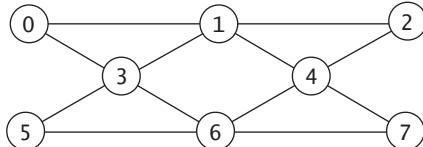
- ◆ A topological sort determines an order for starting activities that are dependent on the completion of other activities (prerequisites). The finish order derived from a depth-first traversal represents a topological sort.
- ◆ Dijkstra's algorithm finds the shortest path from a start vertex to all other vertices, where the distance from one vertex to another is determined by the weight of the edge between them.
- ◆ Prim's algorithm finds the minimum spanning tree for a graph. This consists of the subset of the edges of a connected graph whose sum of weights is the minimum and the graph consisting of only the edges in the subset is still connected.

## User-Defined Classes and Interfaces in This Chapter

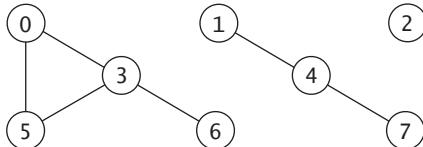
AbstractGraph	ListGraph
BreadthFirstSearch	MatrixGraph
DepthFirstSearch	MatrixGraph.Iter
Edge	Maze
Graph	TopologicalSort

## Quick-Check Exercises

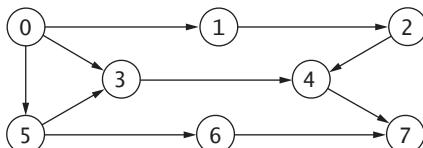
1. For the following graph:
  - a. List the vertices and edges.
  - b. True or false: The path  $0, 1, 4, 6, 3$  is a simple path.
  - c. True or false: The path  $0, 3, 1, 4, 6, 3, 2$  is a simple path.
  - d. True or false: The path  $3, 1, 2, 4, 7, 6, 3$  is a cycle.



2. Identify the connected components in the following graph.

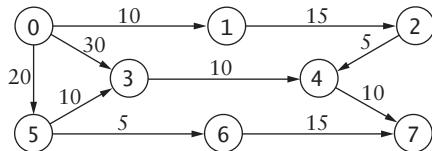


3. For the following graph:
  - a. List the vertices and edges.
  - b. Does this graph contain any cycles?

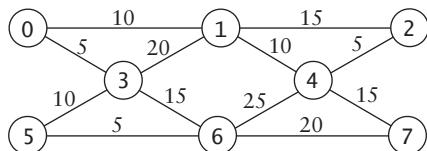


4. Show the adjacency matrices for the graphs shown in Questions 1, 2, and 3.
5. Show the adjacency lists for the graphs shown in Questions 1, 2, and 3.
6. Show the breadth-first search tree for the graph shown in Question 1, starting at vertex 0.
7. Show the depth-first search tree for the graph shown in Question 3, starting at vertex 0.
8. Show a topological sort of the vertices in the graph shown in Question 3.

9. In the following graph, find the shortest path from 0 to all other vertices.



10. In the following graph, find the minimum spanning tree.



## Review Questions

1. What are the different types of graphs?
2. What are the different types of paths?
3. What are two common methods for representing graphs? Can you think of other methods?
4. What is a breadth-first search? What can it be used for?
5. What is a depth-first search? What can it be used for?
6. Under what circumstances are the paths found by Dijkstra's algorithm not unique?
7. Under what circumstances is the minimum spanning tree unique?
8. What is a topological sort?

## Programming Projects

1. Design and implement the `MatrixGraph` class.
2. Rewrite method `dijkstrasAlgorithm` to use a priority queue as we did for method `primsAlgorithm`. When inserting edges into the priority queue, the weight is replaced by the total distance from the source vertex to the destination vertex. The source vertex, however, remains unchanged as it is the predecessor in the shortest path.
3. In both Prim's algorithm and Dijkstra's algorithm, edges are retained in the priority queue, even though a shorter edge to a given destination vertex has been found. This can be avoided, and thus performance improved, by using a `ModifiablePriorityQueue`. Extend the `PriorityQueue` class described in Chapter 6 as follows:

```

    /**
     * A ModifiablePriorityQueue stores Comparable objects. Items
     * may be inserted in any order. They are removed in priority
     * order, with the smallest being removed first, based on the
     * compareTo method. The insert method will return a value
     * known as a locator. The locator may be used to replace a
     * value in the priority queue.
    */
    public class ModifiablePriorityQueue<E> extends Comparable<E>
        extends PriorityQueue<E> {
        /**
         * Insert an item into the priority queue.
         * @param obj The item to be inserted
         * @return A locator to the item
        */
        int insert(E obj);

        /**
         * Remove the smallest item in the priority queue.
         * @return The smallest item in the priority queue
        */
    }
}

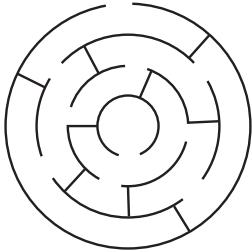
```

```

*/
E poll();
/** Replace the item at the specified location.
 * @param loc The locator value of the current item
 * @param newValue The new value
*/
void replaceItem(int loc, E newValue);
. . .

```

4. Implement Dijkstra's algorithm using the `ModifiablePriorityQueue`.
5. Implement Prim's algorithm using the `ModifiablePriorityQueue`.
6. A maze can be constructed from a series of concentric circles. Between the circles there are walls placed, and around the circles there are doors. The walls divide the areas between the circles into chambers, and the doors permit movement between chambers. The positions of the doors and walls are given in degrees measured counterclockwise from the horizontal. For example, the maze shown on the left can be described as follows:



Number of circles	4	
Position of doors	Outer circle	85–90
	Next inner circle	26–40, 135–146, 198–215, 305–319
	Next inner circle	67–90, 161–180, 243–256, 342–360
	Innermost circle	251–288
Position of walls	Outer ring	45, 135, 300
	Middle ring	0, 100, 225, 270
	Inner ring	65, 180

Write a program that inputs a description of a maze in this format and finds the shortest path from the outside to the innermost circle. The shortest path is the one that goes through the smallest number of chambers.

7. In Chapter 5 we discussed the class `MazeTest`, which reads a rectangular maze as a sequence of lines consisting of 0s and 1s, where a 0 represents an open square and a 1 represents a closed one. For example, the maze shown in Figure 10.21 and reproduced here has the following input file:

```

0111111111111111111111111111
000000000000000000000000001
011111111111111011111101
011111100000001011111101
011111101111111011000001
000000000000000011011011
110111101101111011011011
11011110110111011011011
11011110110111111011011
110111101100000000011011
110000001101111111111011
111101111100000000001011
111101111111111111101000
111100000000000000001110
111111111111111111111110

```

0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	0,...	0,...	0,...	0,...	0,...	0,...	0,...	0,...	0,...	0,...	0,...	0,...
1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,...	1,...	1,...	1,...	1,...	1,...	1,...	1,...	1,...	1,...	1,...	1,...
2,0	2,1	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	2,...	2,...	2,...	2,...	2,...	2,...	2,...	2,...	2,...	2,...	2,...	2,...
3,0	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,9	3,...	3,...	3,...	3,...	3,...	3,...	3,...	3,...	3,...	3,...	3,...	3,...
4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	4,...	4,...	4,...	4,...	4,...	4,...	4,...	4,...	4,...	4,...	4,...	4,...
5,0	5,1	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9	5,...	5,...	5,...	5,...	5,...	5,...	5,...	5,...	5,...	5,...	5,...	5,...
6,0	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9	6,...	6,...	6,...	6,...	6,...	6,...	6,...	6,...	6,...	6,...	6,...	6,...
7,0	7,1	7,2	7,3	7,4	7,5	7,6	7,7	7,8	7,9	7,...	7,...	7,...	7,...	7,...	7,...	7,...	7,...	7,...	7,...	7,...	7,...
8,0	8,1	8,2	8,3	8,4	8,5	8,6	8,7	8,8	8,9	8,...	8,...	8,...	8,...	8,...	8,...	8,...	8,...	8,...	8,...	8,...	8,...
9,0	9,1	9,2	9,3	9,4	9,5	9,6	9,7	9,8	9,9	9,...	9,...	9,...	9,...	9,...	9,...	9,...	9,...	9,...	9,...	9,...	9,...
10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...	10,...
11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...	11,...
12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...	12,...
13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...	13,...
14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...	14,...
15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...	15,...

Write a program that reads input in this format and finds the shortest path, where the distance along a path is defined by the number of squares covered.

8. A third possible representation of a graph is to use the `TreeSet` class to contain the edges. By defining a comparator that compares first on the source vertex and then the destination vertex, we can use the `subSet` method to create a view that contains only edges originating at a specified vertex and then use the iterator of that view to iterate through edges. Design and implement a class that meets the requirements of the `Graph` interface and uses a `TreeSet` to hold the edges.

## Answers to Quick-Check Exercises

1. a. Vertices: {0, 1, 2, 3, 4, 5, 6, 7}  
Edges: {{0, 1}, {0, 3}, {1, 2}, {1, 3}, {1, 4}, {2, 4}, {3, 5}, {3, 6}, {4, 6}, {4, 7}, {5, 6}, {6, 7}}  
b. True  
c. False  
d. True
2. The connected components are {0, 3, 5, 6}, {1, 4, 7}, and {2}.
3. a. Vertices: {0, 1, 2, 3, 4, 5, 6, 7}  
Edges: {(0, 1), (0, 3), (0, 5), (1, 2), (2, 4), (3, 4), (4, 7), (5, 3), (5, 6), (6, 7)}  
b. The graph contains no cycles.

4. For the graph shown in Question 1:

	Column	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Row	[0]		1		1				
	[1]	1		1	1	1			
	[2]		1			1			
	[3]	1	1				1	1	
	[4]		1	1				1	1
	[5]				1			1	
	[6]				1	1	1		1
	[7]					1		1	

For Question 2:

	Column	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Row	[0]				1		1		
	[1]					1			
	[2]								
	[3]	1					1	1	
	[4]		1						1
	[5]	1			1				
	[6]				1				
	[7]					1			

For Question 3:

	Column	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Row	[0]		1		1		1		
	[1]			1					
	[2]				1				
	[3]					1			
	[4]							1	
	[5]				1			1	
	[6]								1
	[7]								

For Question 1:

- [0] → 1 → 3
- [1] → 0 → 2 → 3 → 4
- [2] → 1 → 4
- [3] → 0 → 1 → 5 → 6
- [4] → 1 → 2 → 6 → 7
- [5] → 3 → 6
- [6] → 3 → 4 → 5 → 7
- [7] → 4 → 6

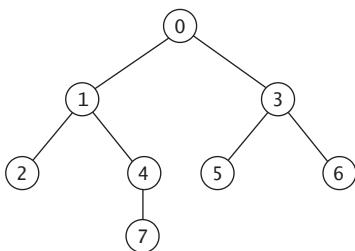
For Question 2:

- [0] → 3 → 5
- [1] → 4
- [2] →
- [3] → 0 → 5 → 6
- [4] → 1 → 7
- [5] → 0 → 3
- [6] → 3
- [7] → 4

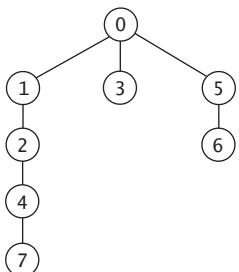
For Question 3:

- [0] → 1 → 3 → 5
- [1] → 2
- [2] → 4
- [3] → 4
- [4] → 7
- [5] → 3 → 6
- [6] → 7
- [7] →

6.



7.

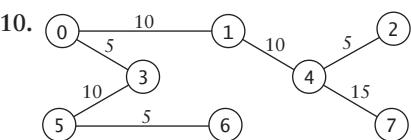


8. 0, 5, 6, 3, 1, 2, 4, 7

9.

Vertex	Distance	Path
1	10	$0 \rightarrow 1$
2	25	$0 \rightarrow 1 \rightarrow 2$
3	30	$0 \rightarrow 3$ (or $0 \rightarrow 5 \rightarrow 3$ )
4	30	$0 \rightarrow 1 \rightarrow 2 \rightarrow 4$
5	20	$0 \rightarrow 5$
6	25	$0 \rightarrow 5 \rightarrow 6$
7	40	$0 \rightarrow 5 \rightarrow 6 \rightarrow 7$ (or $0 \rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 7$ )

10.





# *Introduction to Java*

## Appendix Objectives

- ◆ To understand the essentials of object-oriented programming in Java
- ◆ To learn about the primitive data types of Java
- ◆ To understand how to use the control structures of Java
- ◆ To learn how to use predefined classes such as Math, String, StringBuilder, StringBuffer, and StringJoiner
- ◆ To introduce regular expressions and Pattern and Matcher classes
- ◆ To learn how to write and document your own Java classes
- ◆ To understand how to use arrays in Java
- ◆ To understand how to use enumerators in Java
- ◆ To learn how to perform input/output (I/O) in Java using simple dialog windows
- ◆ To introduce the Scanner class for input and the Formatter class for output
- ◆ To learn how to perform I/O in Java using streams and readers
- ◆ To learn how to use the **try-catch-finally** sequence to catch and process exceptions
- ◆ To understand what it means to throw an exception and how to throw an exception in a method

This appendix reviews object-oriented programming in Java. It is oriented to a student who has had a first course in programming in Java or another language and who, therefore, is familiar with control statements for selection and repetition, basic data types, arrays, and methods or functions. If your first course was in Java, you can skim this appendix for review or just use it as a reference as needed. However, you should read it more carefully if your Java course did not emphasize object-oriented design.

If your first course was not in Java, you should read this appendix carefully. If your first course followed an object-oriented approach but was in another language, you should concentrate on the differences between Java syntax and the language that you know. If you have programmed only in a language that was not object-oriented, you will need to concentrate on aspects of object-oriented programming and classes as well as Java syntax.

The appendix begins with an introduction to the Java environment and the Java Virtual Machine (JVM). Next, it covers the basic data types of Java, called primitive data types, and provides an introduction to objects and classes. Control structures and methods are then discussed.

The Java Application Programming Interface (API) provides a rich collection of classes that simplify programming in Java. The first Java classes that we cover are the `String`, `StringBuilder`, `StringBuffer`, `StringJoiner`, and `Math` classes. The `String` class provides several methods and an operator `+` (concatenation) that process sequences of characters (strings). The `Math` class provides many methods for performing standard mathematical computations.

Next, we show you how to design and write your own classes consisting of data fields and methods. We also discuss the Java wrapper classes, which enable a programmer to create and process objects that contain primitive-type values.

We describe a specific format for comments in classes. Using this commenting style enables you to generate HTML pages with clear and complete documentation for classes in the same form as the Java documentation provided on the Sun Web site.

We also review array objects in Java. We cover both one- and two-dimensional arrays.

Next, we discuss I/O. We show how to use the `JOptionPane` class (part of package `javax.swing`) to create dialog windows for data entry and for output. We also show how to use streams, readers, and the console for I/O.

Finally, we discuss how to handle exceptions and to throw exceptions.

---

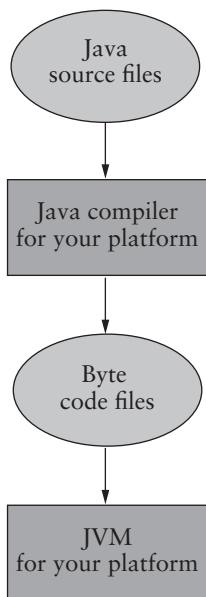
## Introduction to Java

- A.1** The Java Environment and Classes
- A.2** Primitive Data Types and Reference Variables
- A.3** Java Control Statements
- A.4** Methods and Class Math
- A.5** The `String`, `StringBuilder`, `StringBuffer`, and `StringJoiner` Classes
- A.6** Wrapper Classes for Primitive Types
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- A.8** Arrays
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- A.10** I/O Using Streams, Class Scanner, and Class `JOptionPane`
- A.11** Catching Exceptions
- A.12** Throwing Exceptions

## A.1 The Java Environment and Classes

Before we talk about the Java language, we will briefly discuss the Java environment and how Java programs are executed. Java, developed by Sun Microsystems Corporation, enjoys its popularity because it is a platform-independent, object-oriented language and because certain kinds of Java programs, called *applets*, can be embedded in Web pages. Being platform independent means that a Java program will run on any kind of computer. Although platform independence is a goal for all high-level language programs, it is not always achieved. Java

**FIGURE A.1**  
Compiling and Executing a Java Program



comes closer to achieving this goal than most by providing implementations of the JVM (discussed next) for many platforms.

## The Java Virtual Machine

Java is platform independent because the Java designers utilize the concept of a Java Virtual Machine (JVM), which is a software “computer” that runs inside an actual computer. Before you can execute a Java program, the classes in the Java program must first be translated from the Java language in which they were written into an executable form in the traditional way by a compiler program. Instead of a file of platform-dependent machine-language instructions, however, which is the normal output from a compiler, the Java compiler generates a file of platform-independent Java *byte code* instructions. When you execute the program, your computer’s JVM *interprets* each byte code instruction and carries it out. The JVM for machines running Microsoft Windows is different from the JVM for UNIX or Apple machines, but they all process byte code instructions in the same way (see Figure A.1).

## The Java Compiler

The Java compiler is also platform specific, even though it produces the same byte code file for a given Java source program on all platforms. It must be platform specific because it executes machine-language instructions for a particular platform, and these instructions are not the same for all platforms.

## Classes and Objects

In Java and object-oriented programming in general, the class is the fundamental programming unit. Every program is written as a collection of classes, and all code that you write must be part of a class. In Java, class definitions are stored in separate files with the extension `.java`; the file name must be the same as the class name defined within.

A *class* is a named description for a group of entities (called *objects* or *instances* of the class) that have the same characteristics. These characteristics are the attributes (*data fields*) for each object and the operations (*methods*) that can be performed on these objects.

If you are new to object-oriented design, you may be confused about the differences between a class and an object. A class is a general description of a group of entities that all have the same characteristics—that is, they can all perform the same kinds of actions, and the same pieces of information are meaningful for all of them. The individual entities are objects. For example, the class `House` would describe a collection of entities that each have a number of bedrooms, a number of bathrooms, a kind of roof, and so on (but not a horsepower rating or mileage); they can all be built, remodeled, assessed for property tax, and so on (but not have their transmission fluid changed). The house where you live and the house where your best friend lives can be represented by two objects of class `House`.

Classes extend Java by providing additional data types. For example, the class `String` is a predefined class that enables the programmer to process sequences of characters easily. We will discuss the `String` class in detail in Section A.5.

## The Java API

The Java programming language consists of a relatively small core language augmented by an extensive collection of *packages* (called *libraries* in other languages), which constitute the Java API and give Java additional capabilities. Each package contains a collection of related Java classes. We will use several of these packages in this textbook.

Among them are the `javax.swing` package and the `java.util` package. You can find out about these packages by accessing the Java Web site maintained by Oracle corporation at <http://docs.oracle.com/javase/8/docs/api/index.html>.

Java documentation is provided as a linked collection of Web pages. In Section A.7, we will discuss how you can write your own Java documentation that follows this style.

## The import Statement

Next, we show a sample Java source file (`HelloWorld.java`) that contains an application program (class `HelloWorld`). Our goal in the rest of this section is to give you an overview of the process of creating and executing an application program. The statements in this program will be covered in more detail later in this chapter.

```
import java.util.Scanner;

/** A HelloWorld class.
 *  @author Koffman and Wolfgang
 */
public class HelloWorld {

    public static void main(String[] args) {
        Scanner scanner = new Scanner(System.in);
        System.out.print("Enter your name: ");
        String name = scanner.nextLine();
        System.out.println("Hello " + name + ", welcome to Java!");
    }
}
```

The Java source file begins with the statement

```
import java.util.Scanner;
```

This statement tells the Java compiler to make the class `Scanner` defined in the `java.util` package accessible to this file. The semicolon at the end of the line is used to terminate a Java statement.

Class `HelloWorld` begins with the line

```
public class HelloWorld {
```

which identifies `HelloWorld` as a public class and makes it visible to other classes (or the JVM).

## Method main

The line

```
public static void main(String[] args) {
```

identifies the start of the definition for method `main`. This is the place where the JVM begins the execution of an application program. The words `public static void` tell the compiler that `main` is accessible outside of the class (`public`), it is a `static` method (explained in Section A.4), and it does not return a value (`void`). The part in parentheses after `main` describes the method's parameters, an array of `Strings`. We always write the heading for method `main` in this way.

The first statement declares a `Scanner` object that will be used to read input from the console (`System.in`). The second statement displays the message:

```
Enter your name:
```

on the console. The statement

```
String name = scanner.nextLine();
```

reads the characters that are typed on the console until the enter key is pressed. These characters are converted to a String and placed in the variable name. Assuming that the string “Katherine” was typed on the console, the statement:

```
System.out.println("Hello " + name + ", welcome to Java!");
```

will display the output:

```
Hello Katherine, welcome to Java!
```

## Execution of a Java Program

You can compile and run class `HelloWorld` using an Integrated Development Environment (IDE) or the Java Development Kit (JDK). If you are using an IDE, type this class into the edit window for class `HelloWorld.java` and select **Run**. If you are not using an IDE, you must create this file using an editor program and save it as file `HelloWorld.java`. Then you can use the command

```
javac HelloWorld.java
```

to get the Java compiler to compile it. This will create the Java byte code file called `HelloWorld.class`.

The command

```
java HelloWorld
```

starts the JVM and causes it to execute the byte code instructions in file `HelloWorld.class`. It begins execution with the byte code instructions for method `main`.

---

## EXERCISES FOR SECTION A.1

### SELF-CHECK

1. What is the Java Virtual Machine? Is it hardware or software? How does its role differ from that of the Java compiler?
2. Explain the statement: You can write a Java program once and run it anywhere.
3. Explain the relationship between a class and an object. Which is general and which is specific?




---

## A.2 Primitive Data Types and Reference Variables

Java distinguishes between two kinds of entities: primitive types (numbers, characters) and objects. Values associated with primitive-type data are stored in primitive-type variables. Objects, however, are associated with reference variables, which store an object’s address. We will discuss primitive types and introduce objects in this section; we describe objects in more detail throughout the chapter.

### Primitive Data Types

The primitive data types for Java represent numbers, characters, and boolean values (`true`, `false`) (see Table A.1). Integers are represented by data types `byte`, `short`, `int`, and `long`; real numbers are represented by `float` and `double`. The range of values for the data types is in increasing order in Table A.1.

**TABLE A.1**

Java Primitive Data Types in Increasing Order of Range

Data Type	Range of Values
<b>byte</b>	-128 through 127
<b>short</b>	-32,768 through 32,767
<b>int</b>	-2,147,483,648 through 2,147,483,647
<b>long</b>	-9,223,372,036,854,775,808 through 9,223,372,036,854,775,807
<b>float</b>	Approximately $\pm 10^{-38}$ through $\pm 10^{38}$ and 0 with 6 digits precision
<b>double</b>	Approximately $\pm 10^{-308}$ through $\pm 10^{308}$ and 0 with 15 digits precision
<b>char</b>	The Unicode character set
<b>boolean</b>	<b>true, false</b>

**TABLE A.2**

The First 128 Unicode Symbols

000	001	002	003	004	005	006	007
0	Null	Space	0	@	P	`	p
1		!	1	A	Q	a	q
2		"	2	B	R	b	r
3		#	3	C	S	c	s
4		\$	4	D	T	d	t
5		%	5	E	U	e	u
6		&	6	F	V	f	v
7	Bell	'	7	G	W	g	w
8	Backspace	(	8	H	X	h	x
9	Tab	)	9	I	Y	i	y
A	Line feed	*	:	J	Z	j	z
B	Escape	+	;	K	[	k	{
C	Form feed	,	<	L	\	l	
D	Return	-	=	M	]	m	}
E		.	>	N	^	n	~
F		/	?	O	_	o	delete

Type **char** is used in Java to represent characters. Java uses the Unicode character set (two bytes per character), which provides a much richer set of characters than the ASCII character set (one byte per character) used by many earlier languages. Table A.2 shows the first 128 Unicode characters, which correspond to the ASCII characters. These include the control characters and the Basic Latin alphabet. The Unicode for each character, expressed as a

hexadecimal number, consists of the three-digit column number (000 through 007) followed by the row number (0 through F). For example, the Unicode for the letter Q is 0051, and the Unicode for the letter q is 0071. The characters in the first two columns of Table A.2 and the Unicode character 007F (delete) are control characters. The hexadecimal digits A through F are equivalent to the decimal values 10 through 15. The hexadecimal number 007F is equivalent to the decimal number  $7 \times 16 + 15$ .

Java uses type **boolean** to represent logical data. The **boolean** data type has only two values: **true** and **false**. Some languages allow you to represent type **boolean** values using the integers 0 and 1, but Java does not allow you to do this.

## Primitive-Type Variables

Java uses declaration statements to declare and initialize primitive-type variables.

```
int countItems;
double sum = 0.0;
char star = '*';
boolean moreData;
```

The second and third of the preceding statements initialize variables **sum** and **star** to the values after the operator **=**. As shown, you can use primitive-type values (such as 0.0 and **'\*'**) as *literals* in Java statements. A literal is a constant value that appears directly in a statement.

Identifiers, such as variable names in Java, must consist of some combination of letters, digits, the underscore character, and the \$ character, beginning with a letter. Identifiers can't begin with a digit.



### PROGRAM STYLE

#### Java Convention for Identifiers

Many Java programmers use “camel notation” for variable names. All letters are in lowercase except for identifiers that are made up of more than one word. The first letter of each word, starting with the second word, is in uppercase (e.g., **thisLongIdentifier**). Camel notation gets its name from the appearance of the identifier, with the uppercase letters in the interior forming “humps.”

## Primitive-Type Constants

Java programmers usually use all uppercase letters for constant identifiers, with an underscore symbol between words. The keywords **static final** identify a constant value that is **static** (more on this later) and **final**—that is, can't be changed.

```
static final int MAX_SCORE = 999;
static final double G = 3.82;
```

## Operators

Table A.3 shows the Java operators in decreasing precedence. We will not use any of the bitwise operators, shifting operators, or conditional operator. The arithmetic operators (\*, /, +, -) can be used with any of the primitive numeric types or type **char**, but not with type **boolean**. This is also the case for the Java remainder operator (%) and the increment (++) and decrement (--) operators.

**TABLE A.3**  
Operator Precedence

Rank	Operator	Operation	Associativity
1	[]	Array subscript	Left
	()	Method call	
	.	Member access	
	++	Postfix increment	
	--	Postfix decrement	
2	++	Prefix increment	Right
	--	Prefix decrement	
	+ -	Unary plus or minus	
	!	Complement	
	~	Bitwise complement	
	( <i>type</i> )	Type cast	
	<b>new</b>	Object creation	
3	*, /, %	Multiply, divide, remainder	Left
4	+	Addition or string concatenation	Left
	-	Subtraction	
5	<<	Signed bit shift left	Left
	>>	Signed bit shift right	
	>>>	Unsigned bit shift right	
6	<, <=	Less than, less than or equal	Left
	>, >=	Greater than, greater than or equal	
	<b>instanceof</b>	Reference test	
7	==	Equal to	Left
	!=	Not equal to	
8	&	Bitwise and	Left
9	^	Bitwise exclusive or	Left
10		Bitwise or	Left
11	&&	Logical and	Left
12		Logical or	Left
13	?:	Conditional	Left
14	=	Assignment	Right
	*=, /=, %=, +=, -=, <<=, >>=, >>>=, &=,  =	Compound assignment	

## Postfix and Prefix Increment

In Java you can write statements such as

```
i = i + 1;
```

using the *increment operator*:

```
i++;
```

This form is the *postfix increment*. You can also use the *prefix increment*

```
++i;
```

but the postfix increment (or decrement) is more common.

When the postfix form is used in an expression (e.g., `x * i++`), the variable `i` is evaluated and then incremented. When the prefix form is used in an expression (e.g., `x * ++i`), the variable `i` is incremented before it is evaluated.

### EXAMPLE A.1

In the assignment

```
z = i++;
```

`i` is incremented, but `z` gets the value `i` had before it was incremented. So if `i` is 3 before the assignment statement, `z` would be 3 and `i` would be 4 after the assignment. In the assignment statement

```
z = ++i;
```

`i` is incremented and `z` gets its new value, so if `i` is 3 before the assignment, `z` and `i` would both be 4 after the assignment statement.



### PITFALL

#### Using Increment and Decrement in Expressions with Other Operators

In the preceding example, the increment operator is used with the assignment operator in the same statement. Similarly, the expression `x * i++` uses the multiplication and postfix increment operators. In this expression, the variable `i` is evaluated and then incremented. When the prefix form is used in an expression (e.g., `x * ++i`), the variable `i` is incremented before it is evaluated. However, you should avoid writing expressions like these, which could easily be interpreted incorrectly by the human reader.

## Type Compatibility and Conversion

In operations involving mixed-type operands, the numeric type of the smaller range is converted to the numeric type of the larger range. This means that if an operation involves a type `int` and a type `double` operand, the type `int` operand is automatically converted to type `double`. This is called a *widening conversion*.

In an assignment operation, a numeric type of a smaller range can be assigned to a numeric type of a larger range; for example, a type `int` expression can be assigned to a type `float` or `double` variable. Java performs the widening conversion automatically.

```
int item = . . . ;
double realItem = item;    // Valid - automatic widening
```

However, the converse is not true.

```
double y = . . . ;
int x = y; // Invalid assignment
```

This statement is invalid because it attempts to store a real value in an integer variable. It would cause the syntax error *possible loss of precision; double, required: int*. This means that a type **int** expression is required for the assignment. You can use explicit *type cast* operations to perform a *narrowing conversion* and ensure that the assignment statement will be valid. In the following statement, the expression `(int)` instructs the compiler to cast the value of `y` to type **int** before assigning the integer value to `x`.

```
int x = (int) y; // Cast to int before assignment
```

## Referencing Objects

In Java, you can declare reference variables that can reference objects of specified types. For example, the statement

```
String greeting;
```

declares a reference variable named `greeting` that can reference a `String` object. The statement

```
greeting = "hello";
```

specifies the particular `String` object to be referenced by `greeting`: the one that contains the characters in the string literal `"hello"`. What is actually stored in the memory cell allocated to `greeting` is the *address* of the area in memory where this particular object of type `String` is stored. We illustrate this in Figure A.2 by drawing an arrow from variable `greeting` to the object that it references (type `String`, value is `"hello"`). In contrast, the memory cell allocated to a primitive-type variable stores a value, not an address. Just as with the primitive variable declarations shown earlier, these two statements can be combined into one.

```
String greeting = "hello";
```

`String` objects are the only ones that can be created by assignment operations such as this one. We describe how to create other kinds of objects in the next section.

Two reference variables can reference the same object. The statement

```
String welcome = greeting;
```

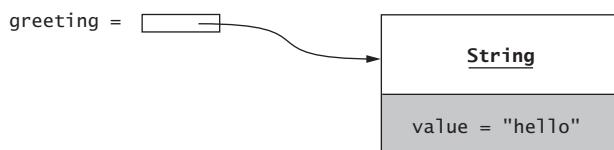
copies the address in `greeting` to `welcome`, so `String` variable `welcome` also references the object shown in Figure A.2.

## Creating Objects

The Java `new` operator can be used to create an instance of a class. The expression

```
new String("qwerty")
```

creates a new `String` instance (object) that stores the character sequence consisting of the first six characters of the top row of letters on the standard keyboard (called a “qwerty” keyboard). The expression `new String("qwerty")` invokes a special method for the `String` class called a *constructor*. A constructor executes whenever a new object of any type is created; in this case, it initializes the contents of a `String` object to the character sequence `"qwerty"`.



**FIGURE A.2**  
Variable `greeting`  
References a `String`  
Object

The object created by the expression `new String("qwerty")` is an *anonymous* or unnamed object. Normally we want to be able to refer to objects that we create. We can declare a reference variable of type `String` and assign this object to the reference variable:

```
String keyboard = new String("qwerty");
```

## EXERCISES FOR SECTION A.2

### SELF-CHECK

- For the following assignment statement, assume that `x`, `y` are type `double` and `m`, `n` are type `int`. List the order in which the operations would be performed. Include any widening and narrowing conversions that would occur.  
`m = (int) (x * y + m / n / y * (m + x));`
- What is the value assigned to `m` in Exercise 1 when `m` is 5, `n` is 3, `x` is 2.5, and `y` is 2.0?
- What is the difference between a reference variable and a primitive-type variable?
- Draw a diagram similar to Figure A.2 that shows the effect of the following statements.

```
String y = new String("abc");
String z = "def";
String w = z;
```

## A.3 Java Control Statements

The control statements of a programming language determine the flow of execution through a program. They fall into three categories: sequence, selection, and repetition.

### Sequence and Compound Statements

A group of statements that is executed in sequence is written as a *compound statement* delimited (enclosed) by braces. The statements execute in the order in which they are listed.

---

**EXAMPLE A.2** The following statements constitute a compound statement:

```
{
    double x = 3.45;
    double y = 2 * x;
    int i = (int) y;
    i++;
}
```

---

### Selection and Repetition Control

Table A.4 shows the Java control statements for selection and repetition. (Java uses the same syntax for control structures as do C and C++) We assume that you are familiar with basic programming control structures from your first course, so we won't dwell on them here. We will discuss the enhanced `for` statement in Chapter 2.

**TABLE A.4**

Java Control Statements

Control Structure	Purpose	Syntax
<b>if ... else</b>	Used to write a decision with <i>conditions</i> that select the alternative to be executed. Executes the first (second) alternative if the <i>condition</i> is true (false)	<pre>if (condition) {     . . . } else {     . . . }</pre>
<b>switch</b>	Used to write a decision with scalar values (integers, characters, enumerators) or strings that select the alternative to be executed. Executes the <i>statements</i> following the <i>label</i> that is the <i>selector</i> value. Execution falls through to the next <b>case</b> if there is no <b>return</b> or <b>break</b> . Executes the statements following <b>default</b> if the <i>selector</i> value does not match any <i>label</i>	<pre>switch (selector) {     case label : statements; break;     case label : statements; break;     . . .     default : statements; }</pre>
<b>while</b>	Used to write a loop that specifies the repetition <i>condition</i> in the loop header. The <i>condition</i> is tested before each iteration of the loop, and, if it is true, the loop body executes; otherwise, the loop is exited	<pre>while (condition) {     . . . }</pre>
<b>do . . . while</b>	Used to write a loop that executes at least once. The repetition <i>condition</i> is at the end of the loop. The <i>condition</i> is tested after each iteration of the loop, and, if it is true, the loop body executes again; otherwise, the loop is exited	<pre>do {     . . . } while (condition);</pre>
<b>for</b>	Used to write a loop that specifies the <i>initialization</i> , repetition <i>condition</i> , and <i>update</i> steps in the loop header. The <i>initialization</i> statements execute before loop repetition begins; the <i>condition</i> is tested before each iteration of the loop and, if it is true, the loop body executes; otherwise, the loop is exited. The <i>update</i> statements execute after each iteration	<pre>for (initialization; condition; update) {     . . . }</pre>
<b>for</b>	Used to write a loop that operates on each item in an array (see Section A.8). Each time through the loop body, the <i>variable</i> is assigned to the next item in the <i>array</i> . This is known as the enhanced for statement	<pre>for (type variable : array) {     . . . }</pre>

In Table A.3, each *condition* is a **boolean** expression in parentheses. Type **boolean** expressions often involve comparisons written using equality (`==`, `!=`) and relational operators (`<`, `<=`, `>`, `>=`). For example, the condition `(x + y > x - y)` is true if the sum of the two variables shown is larger than their difference. The logical operators `!` (not or complement), `&&` (and), and `||` (or) are used to combine **boolean** expressions. For example, the condition `(n >= 0 && n <= 10)` is true if `n` has a value between 0 and 10, inclusive.

Java uses short-circuit evaluation, which means that evaluation of a **boolean** expression terminates when its value can be determined. For example, if in the expression `bool1 || bool2`, `bool1` is true, the expression must be true, so `bool2` is not evaluated. Similarly, in the expression `bool3 && bool4`, if `bool3` is false, the expression must be false, so `bool4` is not evaluated.

---

**EXAMPLE A.3** In the condition

```
(num != 0 && sum / num)
```

if num is 0, the expression following `&&` is not evaluated. This prevents a division by zero.

---

**EXAMPLE A.4** The operator `%` in the condition (`nextInt % 2 == 0`) gives the remainder after an integer division, so the condition is true if `nextInt` is an even number. If `maxVal` has been defined, the following loops (`for` loop on the left, `while` loop on the right) store the sum of the even integers from 1 to `maxVal` in variable `sum` (initial value 0), and they store the product of the odd integers in variable `prod` (initial value 1).

```
for (int nextInt = 1;
      nextInt <= maxVal;
      nextInt++) {
    if (nextInt % 2 == 0) {
        sum += nextInt;
    } else {
        prod *= nextInt;
    }
}
```

```
int nextInt = 1;
while (nextInt <= maxVal) {
    if (nextInt % 2 == 0) {
        sum += nextInt;
    } else {
        prod *= nextInt;
    }
    nextInt++;
}
```

---

## Nested if Statements

You can write `if` statements that select among more than two alternatives by nesting one `if` statement inside another. Often each inner `if` statement will follow the keyword `else` of its corresponding outer `if` statement.

---

**EXAMPLE A.5** The following nested `if` statement has four alternatives. The conditions are evaluated in sequence until one evaluates to `true`. The compound statement following the first true condition then executes.

```
if (operator == '+') {
    result = x + y;
    addOp++;
}
else
    if (operator == '-') {
        result = x - y;
        subtractOp++;
    }
    else
        if (operator == '*') {
            result = x * y;
            multiplyOp++;
        }
        else
            if (operator == '/') {
                result = x / y;
                divideOp++;
            }
}
```

---



## PROGRAM STYLE

### Braces and Indentation in Control Statements

Java programmers often place the opening brace { on the same line as the control statement header. The closing brace } aligns with the first word in the control statement header. We will always indent the statements inside a control structure to clarify the meaning of the control statement.

Although we write the symbols } **else** { on one line, another popular style convention is to place the word **else** under the symbol } and aligned with **if**:

```
if (nextInt % 2 == 0) {
    sum += nextInt;
}
else {
    prod *= nextInt;
}
```

Some programmers omit the braces when a true task or false task or a loop body consists of a single statement. Others prefer to include them always, both for clarity and because having the braces will permit them to insert additional statements later if needed.



## PITFALL

### Omitting Braces around a Compound Statement

The braces in the preceding example delimit compound statements. Each compound statement consists of two statements. If you omit a brace, you will get the syntax error 'else' without 'if'.



## PROGRAM STYLE

### Writing if Statements with Multiple Alternatives

Java programmers often write nested **if** statements like those in the preceding example without indenting each nested **if**. The following multiple-alternative decision has the same meaning but is easier to write and read.

```
if (operator == '+') {
    result = x + y;
    addOp++;
} else if (operator == '-') {
    result = x - y;
    subtractOp++;
} else if (operator == '*') {
    result = x * y;
    multiplyOp++;
} else if (operator == '/') {
    result = x / y;
    divideOp++;
}
```

## The switch Statement

The **if** statement in Example A.5 could also be written as the following **switch** statement. Each **case** label (e.g., '+') indicates a possible value of the selector expression operator. The statements that follow a particular label execute if the selector has that value. The **break** statements cause an exit from the **switch** statement. Without them, execution would continue on to the statements in the next case. The last case, with label **default**, executes if the selector value doesn't match any case label. (Note that the compound statements for each case are not surrounded by braces.)

```
switch (operator) {
    case '+':
        result = x + y;
        addOp++;
        break;
    case '-':
        result = x - y;
        subtractOp++;
        break;
    case '*':
        result = x * y;
        multiplyOp++;
        break;
    case '/':
        result = x / y;
        divideOp++;
        break;
    default:
        // Do nothing
}
```

---

## EXERCISES FOR SECTION A.3

### SELF-CHECK

- What is the purpose of the **break** statement in the preceding **switch** statement? List the statements that would execute when operator is '-' with the **break** statements in place and if they were removed.
- What is the difference between a **while** loop and a **do ... while** loop? What is the minimum number of repetitions of the loop body with each kind of loop?

### PROGRAMMING

- Rewrite the **for** statement in Example A.4 using a **do ... while** loop.




---

## A.4 Methods and Class Math

Java programmers can use methods to define a group of statements that perform a particular operation. Methods are very similar to functions in other programming languages such as C and C++. The Java method `minChar` that follows returns the character with the smaller

Unicode value. The statements beginning with keyword `return` cause an exit from the method; the expression following `return` is the method result.

```
static char minChar(char ch1, char ch2) {
    if (ch1 <= ch2)
        return ch1;
    else
        return ch2;
}
```

The modifier `static` indicates that `minChar` is a *static method* or *class method*. A static method must be called by listing the name of the class in which it is defined, followed by a dot, then by the method name and any arguments. This is called *dot notation*. For example, the statement

```
char ch = ClassName.minChar('a', 'A');
```

would store the letter A in `ch` because uppercase letters have smaller codes than lowercase letters. (If method `minChar` is called within the class that defines it, the prefix `ClassName.` is not needed.) If the modifier `static` does not appear in a method header, the method is an *instance method*. We describe how to invoke instance methods next and show how to define them afterward.

## The Instance Methods `println` and `print`

Methods that are not preceded by the modifier `static` are instance methods. To call or invoke an instance method, you need to apply it to an object using dot notation:

```
object.method(arguments)
```

One instance method that is useful for output operations is the method `println` (defined in class `PrintStream`). It can be applied to the `PrintStream` object `System.out` (the console window), which is defined in the `System` class. It has a single argument of any data type. If `x` is a type `double` variable, the statement

```
System.out.println(x);
```

displays the value of `x` in the console window. The statement

```
System.out.println("Value of x is " + x);
```

has a `String` expression as its argument (+ means concatenate, or join, strings). The string consists of the character sequence `Value of x is` followed by the characters that represent the value of variable `x`. If `x` is 123.45, the output line will be

```
Value of x is 123.45
```

You would get the same effect using the statement pair

```
System.out.print("Value of x is ");
System.out.println(x);
```

The method `print` also displays its argument in the console window. However, it does not follow this information with the *newline* character, so the next execution of `print` or `println` will display information on the same output line.



### PITFALL

#### Static Methods Can't Call Instance Methods

A static method can call other static methods directly. Also, an instance method can call a static method. However, a static method, including method `main`, can't call an instance method without first creating an object and applying the instance method to that object.

## Call-by-Value Arguments

In Java, all method arguments are call-by-value. This means that if the argument is a primitive type, its value (not its address) is passed to the method, so the method can't modify the argument value and have the modification remain after return from the method. Some other programming languages provide a call-by-reference or call-by-address mechanism so that a method can modify a primitive-type argument.

If the argument is of a class type, the value that is passed to the method is the value of the reference variable, not the value of the object itself (see Section A.2). The reference variable value points to the object, allowing the method to access the object itself using the methods of the object's own class. Any modification to the object will remain after the return from the method. This will be discussed in Section A.7.

## The Class Math

Class Math is part of the Java language, and it provides a collection of methods that are useful for performing common mathematical operations. These are all **static** methods, so the prefix `Math.` is required in order to invoke a method of this class.

Table A.5 shows some of these methods. The first column shows the result type for each method followed by its *signature* (the method name and the argument types). For example, for method `ceil`, the first column shows that the method returns a type **double** result and has a type **double** argument. The data type *numeric* means that any of the numeric types can be used.

**TABLE A.5**  
Class Math Methods

Method	Behavior
<code>static numeric abs(numeric)</code>	Returns the absolute value of its <i>numeric</i> argument (the result type is the same as the argument type)
<code>static double ceil(double)</code>	Returns the smallest whole number that is not less than its argument
<code>static double cos(double)</code>	Returns the trigonometric cosine of its argument (an angle in radians)
<code>static double exp(double)</code>	Returns the exponential number $e$ (i.e., 2.718 ...) raised to the power of its argument
<code>static double floor(double)</code>	Returns the largest whole number that is not greater than its argument
<code>static double log(double)</code>	Returns the natural logarithm of its argument
<code>static numeric max(numeric, numeric)</code>	Returns the larger of its <i>numeric</i> arguments (the result type is the same as the argument types)
<code>static numeric min(numeric, numeric)</code>	Returns the smaller of its <i>numeric</i> arguments (the result type is the same as the argument type)
<code>static double pow(double, double)</code>	Returns the value of the first argument raised to the power of the second argument
<code>static double random()</code>	Returns a random number greater than or equal to 0.0 and less than 1.0
<code>static double rint(double)</code>	Returns the closest whole number to its argument
<code>static long round(double)</code>	Returns the closest <b>long</b> to its argument

(Continued)

**TABLE A.5**

Continued

Method	Behavior
<code>static int round(float)</code>	Returns the closest <code>int</code> to its argument
<code>static double sin(double)</code>	Returns the trigonometric sine of its argument (an angle in radians)
<code>static double sqrt(double)</code>	Returns the square root of its argument
<code>static double tan(double)</code>	Returns the trigonometric tangent of its argument (an angle in radians)
<code>static double toDegrees(double)</code>	Converts its argument (in radians) to degrees
<code>static double toRadians(double)</code>	Converts its argument (in degrees) to radians

## Escape Sequences

The `main` method in the following `SquareRoots` class contains a loop that displays the first 10 integers and their square roots (see Figure A.3).

```
public class SquareRoots {
    public static void main(String[] args) {
        System.out.println("n \tsquare root");
        for (int n = 1; n <= 10; n++) {
            System.out.println(n + "\t" + Math.sqrt(n));
        }
    }
}
```

The `println` statements use the *escape sequence* `\t`, the tab character, to align the column label “square root” with the numbers in the second output column (see Figure A.3). An escape sequence is a sequence of two characters beginning with the character `\`. Some escape sequences are used for special output control characters. Others are used to represent characters or symbols that have a special meaning in Java. For example, a double quote character by itself is a string delimiter, so we need to use the sequence `\"` to represent the double quote character in a string. Table A.6 lists some common escape sequences and their meaning.

The escape sequence that starts with `\u` represents a Unicode character. The character code uses four hexadecimal digits, where a hexadecimal digit is formed using four binary bits and ranges from 0 (all bits 0) to F (all four bits 1). The hexadecimal digit A corresponds to a decimal value of 10, and the hexadecimal digit F corresponds to a decimal value of 15.

**FIGURE A.3**

Sample Run of Class  
`SquareRoots`

n	square root
1	1.0
2	1.4142135623730951
3	1.7320508075688772
4	2.0
5	2.23606797749979
6	2.449489742783178
7	2.6457513110645907
8	2.8284271247461903
9	3.0
10	3.1622776601683795

**TABLE A.6**

Escape Sequences

Sequence	Meaning
\n	Start a new output line
\t	Tab character
\\\	Backslash character
\"	Double quote
\'	Single quote or apostrophe
\udddd	The Unicode character whose code is <i>dddd</i> where each digit <i>d</i> is a hexadecimal digit in the range 0 to F (0–9, A–F)

## EXERCISES FOR SECTION A.4

### SELF-CHECK

1. Identify the escape sequences in the following string. Show how this line would be displayed. Which of the escape sequences could be replaced by the second character of the pair without changing the effect?

```
System.out.println("Jane\'s motto is \n\"semper fi\"\n, according to Jim");
```

### PROGRAMMING

1. Write a Java program that displays all odd powers of 2 between 1 and 29. Display the power that 2 is being raised to, as well as the result, on each line. Use tab characters between numbers.
2. Write a Java program that displays *n* and the natural log of *n* for values of *n* of 1000, 2000, 4000, 8000, and so on. Display the first 20 lines for this sequence. Use tab characters between numbers.



## A.5 The String, StringBuilder, StringBuffer, and StringJoiner Classes

In this section, we discuss four Java classes that are used to process sequences of characters. We begin with the `String` class.

### The String Class

The `String` class defines a data type that is used to store a sequence of characters. Table A.7 describes some `String` class methods. The first column shows the result type for each method followed by its signature. For example, for method `charAt`, the first column shows that the method returns a type `char` result and has a type `int` argument. The second column describes

**TABLE A.7**String Methods in `java.lang.String`

Method	Behavior
<code>char charAt(int pos)</code>	Returns the character at position <code>pos</code>
<code>int compareTo(String)</code>	Returns a negative integer if this string's contents precede the argument string's contents in the dictionary; returns 0 if this string and the argument string have the same contents; returns a positive integer if this string's contents follow those of the argument string. This comparison is case-sensitive
<code>int compareToIgnoreCase(String)</code>	Returns a negative, zero, or positive integer according to whether this string's contents precede, match, or follow the argument string's contents in the dictionary, ignoring case
<code>boolean equals(Object)</code>	Returns <code>true</code> if this string's contents are the same as its argument string's contents
<code>boolean equalsIgnoreCase(String)</code>	Returns <code>true</code> if this string's contents are the same as the argument string's contents, ignoring case
<code>int indexOf(char)</code> <code>int indexOf(String)</code>	Returns the index within this string of the first occurrence of its character or string argument, or -1 if the argument is not found
<code>int indexOf(char, int index)</code> <code>int indexOf(String, int index)</code>	Returns the index within this string of the first occurrence of its first character or string argument, starting at the specified <code>index</code>
<code>int lastIndexOf(char)</code> <code>int lastIndexOf(String)</code>	Returns the index within this string of the rightmost occurrence of its character or string argument
<code>int lastIndexOf(char, int index)</code> <code>int lastIndexOf(String, int index)</code>	Returns the index within this string of the last occurrence of its first character or string argument, searching backward and stopping at the specified <code>index</code>
<code>int length()</code>	Returns the length of this string
<code>String replace(char oldChar, char newChar)</code>	Returns a new string resulting from replacing all occurrences of <code>oldChar</code> in this string with <code>newChar</code>
<code>String substring(int start)</code>	Returns a new string that is a substring of this string, starting at position <code>start</code> and going to the end of the string
<code>String substring(int start, int end)</code>	Returns a new string that is a substring of this string, starting with the character at position <code>start</code> and ending with the character at position <code>end - 1</code>
<code>String toLowerCase()</code>	Returns a new string in which all of the letters in this string are converted to lowercase
<code>String toUpperCase()</code>	Returns a new string in which all of the letters in this string are converted to uppercase
<code>String trim()</code>	Returns a new string in which all the white space is removed from both ends of this string
<code>static String format (String format, Object... args)</code>	Returns a new string with the arguments <code>args</code> formatted as prescribed by the string <code>format</code>
<code>String[] split(String pattern)</code>	Separates the string into an array of tokens, where each token is delimited by a string that matches the regular expression <code>pattern</code>

what the method does. The phrase “this string” means the string to which the method is applied by the dot notation. If type `Object` is listed as an argument type in column 1, any kind of object can be an argument. (We discuss type `Object` in Chapter 1.)

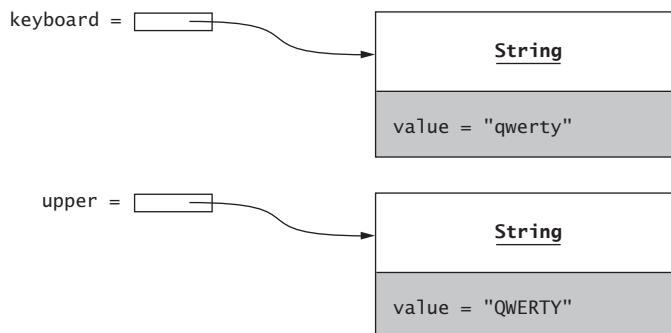
**EXAMPLE A.6** Assume that `keyboard` (type `String`) contains “`qwerty`”. We evaluate several expressions:

- `keyboard.charAt(0)` is 'q'.
- `keyboard.length()` is 6.
- `keyboard.indexOf('o')` is -1.
- `keyboard.indexOf('y')` is 5.

The statement

```
String upper = keyboard.toUpperCase();
```

creates a new `String` object, referenced by the variable `upper`, that stores the character sequence “`QWERTY`”, but the `String` object referenced by `keyboard` is unchanged, as shown here.



Finally, the expression

```
keyboard.charAt(keyboard.length() - 1)
```

applies two instance methods to `keyboard`. The inner call, to method `length`, returns the value 6; the outer call, to method `charAt`, returns `y`, the last character in the string (at position 5).

The method `substring` returns a new string containing a portion of the `String` object to which it is applied. If it is called with just one argument, the contents of the string returned will be all characters from its argument position to the end of the string. If it is called with two arguments, the contents of the string returned will be all characters from its first argument position up to, but excluding, the character at its second argument position. However, the string to which method `substring` is applied is not changed.

**EXAMPLE A.7** The expression

```
keyboard.substring(0, keyboard.length() - 1)
```

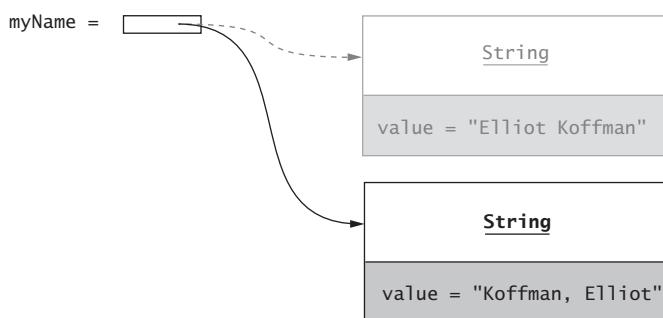
returns a new string “`qwert`” consisting of all characters except for the last character in the string referenced by `keyboard`. The contents of `keyboard` are unchanged.

## Strings Are Immutable

Strings are different from most other Java objects in that they are *immutable*. What this means is that you cannot modify a String object. If you attempt to do so, Java will create a new object that contains the modified character sequence. The following statements create a new String object storing the character sequence "Koffman, Elliot" that is referenced by myName (indicated by the black arrow in Figure A.4). The original String object still exists (at least temporarily) and contains the character sequence "Elliot Koffman", but it is no longer referenced by myName (indicated by the dashed arrow in Figure A.4).

```
String myName = "Elliot Koffman";
myName = myName.substring(7) + ", " + myName.substring(0, 6);
```

**FIGURE A.4**  
Old and New Strings  
Referenced by  
myName



### PITFALL

#### Attempting to Change a Character in a String

You might try to change the first character in `myName` using either of the following statements:

```
myName.charAt(0) = 'X'; // Invalid attempt to change character at
                      // position 0
myName[0] = 'X';      // Invalid attempt to treat string as array
```

Both statements cause syntax errors. The first statement will not work because method `charAt` returns a value, but a variable must precede the assignment operator. The second statement attempts to change the first character in a string by treating it as an array of characters. You can do this in some programming languages, but not in Java.

## The Garbage Collector

Storage space for objects that are no longer referenced is automatically reclaimed by the Java *garbage collector* so that the storage space can be reallocated and reused. The storage space occupied by the first String object in Figure A.4 will be reclaimed by the garbage collector. In other programming languages, the programmer is responsible for reclaiming any storage space that is no longer needed.

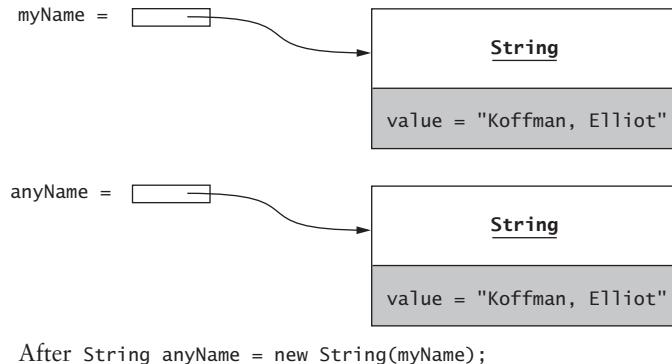
## Comparing Objects

You can't use the relational (`<`, `<=`, `>`, `<=`) or equality operators (`==`, `!=`) to compare the values stored in strings or other objects. After the assignment

```
String anyName = new String(myName);
```

the condition (`anyName == myName`) would be **false**, even though these variables have the same contents. The reason is that the `==` operator compares the *addresses* stored in `anyName` and `myName`, and the `String` objects that are referenced by these variables have different addresses (see Figure A.5).

**FIGURE A.5**  
Two `String` Objects  
at Different Addresses  
with the Same  
Contents

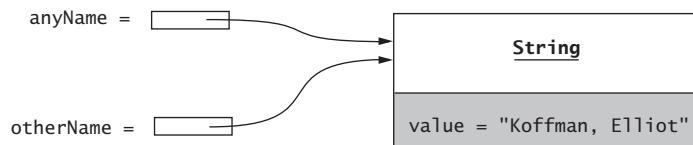


To compare the character sequences stored in two `String` objects, you need to use one of the Java `String` comparison methods: `equals`, `equalsIgnoreCase`, `compareTo`, or `compareToIgnoreCase`. In general, if you want to compare instances of classes that you write, you will need to write at least an `equals` and a `compareTo` method for that class.

### EXAMPLE A.8

If you execute the statement

`String otherName = anyName;`  
the variables `anyName` and `otherName` reference the same `String` object:



All of the following conditions are then **true**.

```

(anyName == otherName)
(anyName.equals(otherName))
(anyName.compareTo(otherName) == 0)
(anyName.equalsIgnoreCase(otherName))
  
```

If they had referenced different strings with the same contents, only the first condition would be **false** because of the address comparison. If they referenced different strings that contained the same words but one's contents were in uppercase and the other's contents were in lowercase, only the last condition would be **true**.

The `compareTo` and `compareToIgnoreCase` operators return negative or positive values according to whether the argument, in dictionary order, follows or precedes the string to which the method is applied. If `keyboard` contains the string "qwerty", the expression

```
keyboard.compareTo("rest")
```

is negative because "r" follows "q". For the same reason, the expression

```
"rest".compareTo(keyboard)
```

is positive.

The `compareTo` method performs a case-sensitive comparison, in which all of the uppercase letters precede all of the lowercase letters. If `keyboard` (containing "qwerty") is compared with "Rest", the results are the opposite of what we have just shown for comparing `keyboard` with "rest": `keyboard.compareTo("Rest")` is positive because "R" precedes "q", and "Rest". `compareTo(keyboard)` is negative.

To compare the contents of two strings in alphabetical order regardless of case, use the `compareToIgnoreCase` method: the expressions

```
keyboard.compareToIgnoreCase("rest")
keyboard.compareToIgnoreCase("Rest")
```

are both negative because "r" follows "q".

## The String.format Method

Java uses a default format for converting the primitive types to `Strings`. This default formatting is applied when you output a primitive value using `System.out.print` or `System.out.println`. For example, the output lines displayed by the statement

```
System.out.println(n + "\t" + Math.sqrt(n));
```

for `n = 1` and `n = 2` are as follows:

1	1.0
2	1.4142135623730951

Note that the numbers in each column are left-justified and that a large number of significant digits is shown for the square root of 2, but only one zero is shown for the square root of 1. Java 5.0 introduced the `Formatter` class and the `format` method to the `String` class that give us better control over the formatting of numeric values.

Using the `String format` method, we can rewrite the earlier `println` statement as

```
System.out.println(String.format("%2d%10.2f", n, Math.sqrt(n)));
```

Now the `format` method is called to build a formatted output string before `println` executes. This statement displays the following output lines for `n = 1` and `n = 2`:

1	1.00
2	1.41

The `format` method is unusual in that it takes a variable number of arguments. The first argument is a format string that specifies how the output string should be formed. The format string above contains a sequence of two format codes, `%2d` and `%10.2f`. Each format code describes how its corresponding argument should be formatted.

A format code begins with a % character and is optionally followed by an integer for width, a decimal point and an integer for precision (optional), and a type conversion specification (e.g., `d` for integer, `f` for real number, and `s` for string). The format code `%2d` means use two characters to represent the integer value of its corresponding argument (`n`); the format code `%10.2f` means use a total of 10 characters and 2 decimal places of precision to represent the real value of its corresponding argument (`Math.sqrt(n)`).

The width specifier gives the minimum number of characters that are used to represent a value. If more are required, then they will be inserted, but if fewer are required, then leading spaces are used to fill the character count (to achieve right-justification). If the width specifier is omitted, then the exact number of characters required to represent the value with the prescribed precision will be used.

The precision specifier (e.g., `.2`) is optional and applies only to the `f`-type conversion specification. It indicates the number of digits following the decimal point. If omitted, six digits are displayed following the decimal point.

You can also have characters other than format codes inside a format string. The arguments to be formatted are inserted in the formatted string exactly where their format specifiers appear. For example, when n is 2, the statement

```
String.format("Value of square root of %d is %.3f", n, Math.sqrt(n))  
creates the String
```

Value of square root of 2 is 1.414

The value 2 and the square root of 2 replace their format specifiers in the formatted string. Because the width specifiers are omitted, the exact number of characters required to represent the values to the desired precision is used.

Other format conversion characters used in this text are s (for string) and n for newline. The format code %10s causes its corresponding string argument to be formatted using 10 characters. Like numbers, strings are displayed right-justified. The format code %-10s will cause the string to be displayed left-justified. The format code %n will cause an operating system-specific newline sequence to be generated. On some operating systems, the newline is indicated by the \n character, or by the sequence \r\n, and on others by \r. The println method always terminates its output with the correct sequence for the operating system on which the program is executing. Using %n achieves the same result when using method format.

## The Formatter Class

You can also use the java.util.Formatter class to create Formatter objects for writing formatted output to the console (or elsewhere). The statement

```
Formatter fOut = new Formatter(System.out);  
creates a Formatter object fOut associated with the console. The statements  
fOut.format("%2d%10.2f\n", 1, Math.sqrt(1));  
fOut.format("%2d%10.2f\n", 2, Math.sqrt(2));
```

write to object fOut the pair of formatted strings shown earlier, each ending with a newline character ('\n'). Each string written to fOut will be displayed in the console window. You can actually apply the format method or the new printf method directly to System.out without wrapping System.out in a Formatter object.

```
System.out.printf("%2d%10.2f\n", 1, Math.sqrt(1));  
System.out.format("%2d%10.2f\n", 2, Math.sqrt(2));
```

## The String.split Method

Often we want to process individual pieces, or tokens, in a string. For example, in the string "Doe, John 5/15/65", we are likely to be interested in one or more of the particular pieces "Doe", "John", "5", "15", and "65". These pieces would have to be extracted from the string as *tokens*. You can retrieve tokens from a String object using the String.split method.

## Introduction to Regular Expressions

The argument to the split method is a special kind of string known as a *regular expression*. A regular expression is a string that describes another string or family of strings.

The simplest regular expression is a string that does not include any special characters, which matches itself. For example, the string " " represents a single space, and the string ", " represents a comma followed by a space. For the statements

```
String personData = "Doe, John 5/15/65";  
String[] tokens = personData.split(", ");
```

the string ", " matches the two characters following the letter e, so `tokens[0]` is "Doe" and `tokens[1]` is "John 5/15/65". This is not quite what we desire, as `tokens[1]` needs to be split further. The string `personData` is not changed by this operation. The character sequence comma followed by space is often called a *delimiter* because it separates the tokens.

## Matching One of a Group of Characters

A string enclosed in brackets ([ and ]) matches any one of the characters in the string, unless the first character in the string is ^, in which case the match is to any character not in the string that follows the ^. For example, the string "[, /]" will match a comma, a space, or a slash, and the string "[^abc]" will match any character that is not a, b, or c. To match a range of characters, separate the start and end character of the range with a '-'. For example, "[a-z]" will match any lowercase letter. For the statement

```
tokens = personData.split("[, /]");
```

`tokens[0]` is "Doe", `tokens[1]` is an empty string because the space character in `personData` is matched immediately by the space in the delimiter string, `tokens[2]` is "John", `tokens[3]` is "5", `tokens[4]` is "15", and `tokens[5]` is "65", so we are closer to what we desire.

## Qualifiers

Character groups match a single character. Qualifiers are applied to regular expressions to define a new regular expression that conditionally performs a match. These qualifiers are shown in Table A.8.

In the statement

```
String[] tokens = personData.split("[, /]+");
```

the argument "[, /]+" will match a string of one or more space, comma, and slash characters. Therefore, `tokens[0]` is "Doe", `tokens[1]` is "John", `tokens[2]` is "5", and so on. This is what we desire.

.....  
**TABLE A.8**

Regular Expression Qualifiers

Qualifier	Meaning
$X?$	Optionally matches the regular expression X
$X^*$	Matches zero or more occurrences of regular expression X
$X^+$	Matches one or more occurrences of regular expression X



### PITFALL

#### Not Using the + Qualifier to Define a Delimiter Regular Expression

As we explained above, if we had omitted the + qualifier from the delimiter string "[, /]", there would have been an empty string in the array of tokens. The reason for this is that the comma after Doe would match the comma in the delimiter. The `split` method would then save Doe in `tokens[0]` and search for another match to the delimiter. It would immediately find the space, and since there were no characters between the previously found delimiter and this one, an empty string would be stored. Using the + qualifier ensures that the comma and space in `personData` are treated as a single delimiter, not as two separate delimiters.

## Defined Character Groups

Several character groups are defined and are indicated by a letter preceded by two backslash characters. The defined groups are shown in Table A.9.

**TABLE A.9**

Defined Character Groups

Regular Expression	Equivalent Regular Expression	Description
\d	[0-9]	A digit
\D	[^0-9]	Not a digit
\s	[ \t\n\f\r]	A whitespace character (space, tab, newline, formfeed, or return)
\S	[^\s]	Not a whitespace character
\w	[a-zA-Z_0-9]	A word character (letter, underscore, or digit)
\W	[^\w]	Not a word character

---

**EXAMPLE A.9** We want to extract the symbols from an expression. We define a symbol as a string of letter or digit characters. The symbols are separated by one or more whitespace characters. The statement

```
String[] symbols = expression.split("\\s+");
```

will split the string `expression` into an array of symbols separated by whitespace characters.

---

**EXAMPLE A.10** We want to extract the words from a text. We define a word as a string of letter or digit characters. The characters can be in any language. Thus, the delimiters are any string that consists of one or more characters that are not letters or digits (e.g., whitespace, punctuation symbols, and parentheses). The regular expression "[^\P{L}\P{N}]+"<sup>1</sup> represents a string consisting of one or more characters that are not letters or digits. The statement

```
String[] words = line.split("[^\p{L}\p{N}]+");
```

will split the string `line` consisting of letters, digits, special characters, and punctuation symbols into an array of words. The meaning of `\p{L}` and `\p{N}` is discussed next.

---

## Unicode Character Class Support

The groups shown in Table A.9 apply only to the first 128 Unicode characters, which is adequate for processing English text. However, Java uses the Unicode characters that can be used to represent languages other than English. In these other languages, `a-z` do not represent all of the letters or may not be letters at all. For example, French includes the letters `à`, `á`, and `â`, which are distinct from `a`. Greek uses characters such as `α`, `β`, and `γ`. Selected character groups based on the Unicode character category are shown in Table A.10.

## The StringBuilder and StringBuffer Classes

Java provides a class called `StringBuilder` that, like `String`, also stores character sequences. However, unlike a `String` object, the contents of a `StringBuilder` object can be changed. Use a `StringBuilder` object to store a string that you plan to change; otherwise, use a `String`

**TABLE A.10**

Regular Expressions for Selected Unicode Character Categories

Regular Expression	Description
\p{L}	Letter
\p{Lu}	Uppercase letter
\p{Ll}	Lowercase letter
\p{Lt}	Titlecase letter
\p{N}	Numbers
\p{P}	Punctuation
\p{S}	Symbols
\p{Zs}	Spaces

**TABLE A.11**StringBuilder Methods in `java.lang.StringBuilder`

Method	Behavior
<code>StringBuilder append(anyType)</code>	Appends the string representation of the argument to this <code>StringBuilder</code> . The argument can be of any data type
<code>int capacity()</code>	Returns the current capacity of this <code>StringBuilder</code>
<code>StringBuilder delete(int start, int end)</code>	Removes the characters in a substring of this <code>StringBuilder</code> , starting at position <code>start</code> and ending with the character at position <code>end - 1</code>
<code>StringBuilder insert(int offset, anyType data)</code>	Inserts the argument <code>data</code> (any data type) into this <code>StringBuilder</code> at position <code>offset</code> , shifting the characters that started at <code>offset</code> to the right
<code>int length()</code>	Returns the length (character count) of this <code>StringBuilder</code>
<code>StringBuilder replace(int start, int end, String str)</code>	Replaces the characters in a substring of this <code>StringBuilder</code> (from position <code>start</code> through position <code>end - 1</code> ) with characters in the argument <code>str</code> . Returns this <code>StringBuilder</code>
<code>String substring(int start)</code>	Returns a new string containing the substring that begins at the specified index <code>start</code> and extends to the end of this <code>StringBuilder</code>
<code>String substring(int start, int end)</code>	Return a new string containing the substring in this <code>StringBuilder</code> from position <code>start</code> through position <code>end - 1</code>
<code>String toString()</code>	Returns a new string that contains the same characters as this <code>StringBuilder</code> object

object to store that string. Table A.11 describes the methods of class `StringBuilder`. In Table A.11, “this `StringBuilder`” means the `StringBuilder` object to which the method is applied through the dot notation. Methods `append`, `delete`, `insert`, and `replace` modify this `StringBuilder` object.

The `StringBuilder` class was introduced in Java 5.0 as a replacement for the `StringBuffer`. The `StringBuffer` has the same methods as the `StringBuilder`, but is designed for programs that have multiple threads of execution. All programs presented in this text are single-thread.

**EXAMPLE A.11** The following statements declare three `StringBuilder` objects using three different constructors. The default capacity of an empty `StringBuilder` object is 16 characters. The capacity of a `StringBuilder` object is automatically doubled as required to accommodate the character sequence that is stored.

```
StringBuilder sb1 = new StringBuilder();           // Capacity is 16
StringBuilder sb2 = new StringBuilder(30);         // Capacity is 30
StringBuilder sb3 = new StringBuilder("happy");    // Stores "happy"
                                                       // Capacity 16
```

The following statements result in the character sequence "happy birthday to you" being stored in `sb3`.

```
sb3.append("day me");      // "happyday me"
sb3.insert(9, "to ");      // "happyday to me"
sb3.insert(5, " birth");   // "happy birthday to me"
sb3.replace(18, 20, "you"); // "happy birthday to you"
```



## PITFALL

### String Index Out of Bounds

If an index supplied to any `String`, `StringBuilder`, or `StringBuffer` method is outside the valid range of character positions for the string object (i.e., if the index is less than 0 or greater than or equal to the string length), a `StringIndexOutOfBoundsException` will occur. This is a run-time error and will terminate program execution. We will discuss exceptions in more detail in Section A.11.

## Java 8 StringJoiner Class

Assume you have the array of `Strings`, `names`, as shown below:

[0]	Tom
[1]	Dick
[2]	Harry

You can format this into the String "Tom, Dick, Harry" using a `StringBuilder`:

```
StringBuilder stb = new StringBuilder();
stb.append(names[0]);
for (int i = 1; i < names.length; i++) {
    stb.append(", ");
    stb.append(names[i]);
}
String result = stb.toString();
```

Note that you needed to apply special handling for the first element in the array, and then start the loop at 1. The `StringJoiner` class was introduced in Java 8 to construct a sequence of characters separated by a delimiter. It also provides for an optional prefix and suffix. You can construct the String "Tom, Dick, Harry" with less effort using a `StringJoiner`:

```
StringJoiner sj = new StringJoiner(", ");
for (int i = 0; i < names.length; i++) {
    sj.add(names[i]);
}
String result = sj.toString();
```

The first line creates a new `StringJoiner sj` and specifies that the string `", "` will be used as the delimiter. In the loop, each element of array `names` is appended to `sj`.

Table A.12 describes the methods of class `StringJoiner`. The interface `CharSequence` is implemented by the `String`, `StringBuffer`, and `StringBuilder` classes, so objects of these classes may be used as arguments that are of type `CharSequence`.

**TABLE A.12**StringJoiner Methods in `java.util.StringJoiner`

Method	Behavior
<code>StringJoiner(CharSequence delimiter)</code>	Constructs an empty <code>StringJoiner</code> with the provided <code>delimiter</code> and no prefix or suffix
<code>StringJoiner(CharSequence delimiter, CharSequence prefix, CharSequence suffix)</code>	Constructs an empty <code>StringJoiner</code> with the provided <code>delimiter</code> , <code>prefix</code> , and <code>suffix</code>
<code>StringJoiner add(CharSequence newElement)</code>	Adds a copy of the given input to the <code>StringJoiner</code>
<code>int length()</code>	Returns the length of the resulting <code>String</code>
<code>StringJoiner merge(StringJoiner other)</code>	Adds the contents of the other <code>StringJoiner</code> to this <code>StringJoiner</code> . The other <code>StringJoiner</code> 's <code>delimiter</code> , <code>prefix</code> , and <code>suffix</code> are not copied
<code>StringJoiner setEmptyValue(CharSequence emptyValue)</code>	Sets the <code>emptyValue</code> to be returned by the <code>toString</code> method if no elements have been added via the <code>add</code> method
<code>String toString()</code>	Constructs a <code>String</code> consisting of the <code>prefix</code> , followed by the contents with each element separated by the <code>delimiter</code> and followed by the <code>suffix</code> . If no contents were added via a call to <code>add</code> , then a <code>String</code> consisting of the <code>prefix</code> followed by the <code>suffix</code> is returned, unless an empty value has been set

## EXERCISES FOR SECTION A.5

### SELF-CHECK

1. Evaluate each of these expressions.

```
"happy".equals("Happy")
"happy".compareTo("Happy")
"happy".equalsIgnoreCase("Happy")
"happy".equals("happy".charAt(0) + "Happy".substring(1))
"happy" == "happy".charAt(0) + "Happy".substring(1)
```

2. You want to extract the words in the string `"Nancy* has thirty-three*** fine!! teeth."` using the `split` method. What are the delimiter characters, and what should you use as the argument string?
3. Rewrite the following statements using `StringBuilder` objects:
- ```
String myName = "Elliot Koffman";
String myNameFirstLast = myName;
myName = myName.substring(7) + ", " + myName.substring(0, 6);
```
4. Rewrite the statements of Exercise 3 using a `StringJoiner` object.

5. What is stored in `result` after the following statements execute?

```
StringBuilder result = new StringBuilder();
String sentence = "Let's all learn how to program in Java";
String[] tokens = sentence.split("\\s+");
for (String token : tokens) {
    result.append(token);
}
```

6. Revise Exercise 5 to insert a newline character between the words in `result`.

### PRO GRAMMING

1. Write statements to extract the individual tokens in a string of the form "Doe, John 5/15/65". Use the `indexOf` method to find the string ", " and the symbol / and use the `substring` method to extract the substrings between these delimiters.
2. Write statements to extract the words in Self-Check Exercise 2 and then create a new `String` object with all the words separated by commas. Use `StringBuilder` to build the new string.
3. For Self-Check Exercise 4, write a loop to display all the tokens that are extracted.
4. Use a `StringJoiner` to create a string with a prefix of "(", a suffix of ")" and a delimiter of " + ". Store the elements of `String` array `symbols` in the `StringJoiner`.
5. Redo Programming Exercise 4 using a `StringBuilder`.



## A.6 Wrapper Classes for Primitive Types

We have seen that the primitive numeric types are not objects, but sometimes we need to process primitive-type data as objects. For example, we may want to pass a numeric value to a method that requires an object as its argument. Java provides a set of classes called *wrapper classes* whose objects contain primitive-type values: `Float`, `Double`, `Integer`, `Boolean`, `Character`, and so on. These classes provide constructor methods to create new objects that “wrap” a specified value. They also provide methods to “unwrap,” or extract, an object’s value and methods to compare two objects. Table A.13 shows some methods for wrapper class `Integer` (part of `java.lang`). The other numeric wrapper classes also provide these methods, except that method `parseInt` is replaced by a method `parseClassType`, where `ClassType` is the data type wrapped by that class.

In earlier versions of Java, a programmer could not mix type `int` values and type `Integer` objects in an expression. If you wanted to increment the value stored in `Integer` object `nInt`, you would have to unwrap the value, increment it, and then wrap the value in a new `Integer` object:

```
int n = nInt.intValue();
nInt = new Integer(n++);
```

Java 5.0 introduced a feature known as autoboxing/unboxing for primitive types. This enables programmers to use a primitive type in contexts where an `Object` is needed or to use a wrapper object in contexts where a primitive type is needed. Using autoboxing/unboxing, you can rewrite the statements above as

```
int n = nInt;
nInt = n++;
```

or even as the single statement:

```
nInt++;
```

**TABLE A.13**Methods for Class `Integer`

| Method                                     | Behavior                                                                                                                                   |
|--------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| <code>int compareTo(Integer anInt)</code>  | Compares two <code>Integer</code> s numerically                                                                                            |
| <code>double doubleValue()</code>          | Returns the value of this <code>Integer</code> as a <code>double</code>                                                                    |
| <code>boolean equals(Object obj)</code>    | Returns <code>true</code> if the value of this <code>Integer</code> is equal to its argument's value; returns <code>false</code> otherwise |
| <code>int intValue()</code>                | Returns the value of this <code>Integer</code> as an <code>int</code>                                                                      |
| <code>static int parseInt(String s)</code> | Parses the string argument as a signed integer                                                                                             |
| <code>String toString()</code>             | Returns a <code>String</code> object representing this <code>Integer</code> 's value                                                       |

**EXAMPLE A.12** The first pair of the following statements creates two `Integer` objects. The next pair unboxes the `int` value contained in each object. The next-to-last statement calls the static method `parseInt` to parse its string argument to an `int` (not `Integer`) value. The last statement displays the value (35) wrapped in `Integer` object `i1`.

```

Integer i1 = 35;           // Autoboxes 35.
Integer i2 = 1234;         // Autoboxes 1234.
Integer i3 = i1 + i2;      // Unboxes i1 and i2, autoboxes
                          // their sum 1269, and assigns
                          // it to i3.
int i2Val = i2++;          // Unboxes i2, increments it to
                          // 1235 and autoboxes it, and
                          // assigns 1234 to i2Val.
int i3Val = Integer.parseInt("-357"); // Parses "-357" to -357 and
                                      // assigns it to i3Val.
Integer i4 = new Integer("753");   // Autoboxes 753 and assigns
                                      // it to i4.
System.out.println(i1);        // Automatically calls
                          // toString() and displays 35.

```

## EXERCISES FOR SECTION A.6

### SELF-CHECK

- Do you think objects of a wrapper type are immutable or not? Explain your answer.
- For objects `i1`, `i2` in Example A.12, what do the following two statements display?

```

System.out.println(i1 + i2);
System.out.println(i1.toString() + i2.toString());

```

### PROGRAMMING

- Write statements that double the value stored in the `Integer` object referenced by `i1`. Draw a diagram showing the objects referenced by `i1` before and after these statements execute.
- There is no `*` (multiply) operator for type `Integer` objects. Suppose you have `Integer` objects `i1`, `i2`, `i3`. Write a statement to multiply the three type `int` values in these objects and store the product in an `Integer` object `i4`. Show how you would do this without using autoboxing/unboxing.

## A.7 Defining Your Own Classes

We mentioned earlier that a Java program is a collection of classes; consequently, when you write a Java program, you will develop one or more classes. We will show you how to write a Java class next.

A class `Person` might describe a group of objects, each of which is a particular human being. For example, instances of class `Person` would be yourself, your mother, and your father. A `Person` object could store the following data:

- Given name
- Family name
- ID number
- Year of birth

The following are a few of the operations that can be performed on a `Person` object:

- Calculate the person's age
- Test whether two `Person` objects refer to the same person
- Determine whether the person is old enough to vote
- Determine whether the person is a senior citizen
- Get one or more of the data fields for the `Person` object
- Set one or more of the data fields for the `Person` object

Figure A.6 shows a diagram of class `Person`. This figure uses the *Unified Modeling Language*<sup>TM</sup> (UML) to represent the class. UML diagrams are a standard means of documenting class relationships that is widely used in industry. The class is represented by a box. The top compartment of the box contains the class name. The data fields are shown in the middle compartment, and some of the methods are shown in the bottom compartment. Data fields are also called *instance variables* because each class instance (object) has its own storage for them. We discuss UML further in Appendix B.

**FIGURE A.6**  
Class Diagram for  
`Person`

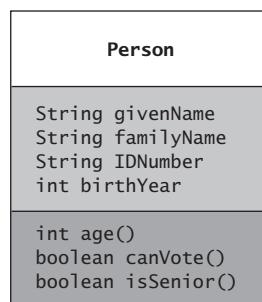
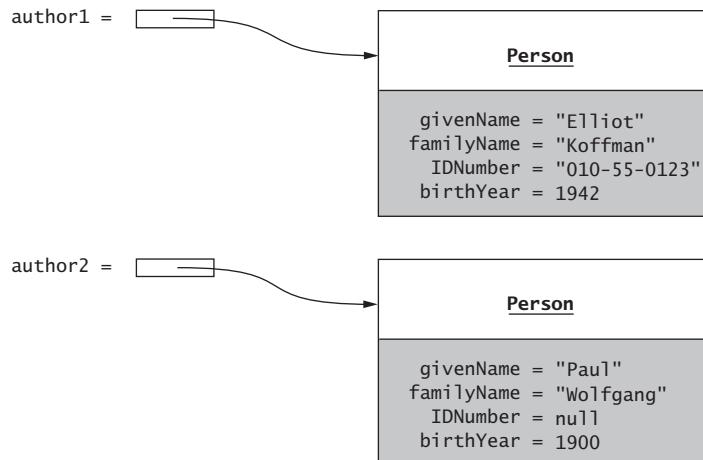


Figure A.7 shows how two objects or instances of the class `Person` (`author1` and `author2`) are represented in UML. A curved arrow from the reference variable for each object (`author1`, `author2`) points to the object, as we have shown in previous figures. Each object is represented by a box in which the top compartment contains the class name (`Person`), underlined, and the bottom compartment contains the data fields and their values. (For simplicity, we show the value of each `String` data field instead of a reference to a `String` object.)

Listing A.1 shows class `Person` and the instance methods for this class. The lines that are delimited by `/**` and `*/` are comments. They are program documentation, extremely important for

**FIGURE A.7**

Object Diagrams of Two Instances of Class Person



human programmers but ignored by the compiler. We discuss the form of the comments used in class Person at the end of this section.

We declare four data fields and two constants (all uppercase letters) before the methods (although many Java programmers prefer to declare methods before data fields). In the constant declarations, the modifier **final** indicates that the constant value may not be changed. The modifier **static** indicates that the constant is being defined for the class and does not have to be replicated in each instance. In other words, storage for the constant VOTE\_AGE is allocated once, regardless of how many instances of Person are created.

**LISTING A.1**

Class Person

```

/** Person is a class that represents a human being. */
public class Person {
    // Data Fields
    /** The given name */
    private String givenName;
    /** The family name */
    private String familyName;
    /** The ID number */
    private String IDNumber;
    /** The birth year */
    private int birthYear = 1900;

    // Constants
    /** The age at which a person can vote */
    private static final int VOTE_AGE = 18;
    /** The age at which a person is considered a senior citizen */
    private static final int SENIOR_AGE = 65;

    // Constructors
    /** Construct a person with given values
        @param first The given name
        @param family The family name
        @param ID The ID number
        @param birth The birth year
    */
  
```

```

public Person(String first, String family, String ID, int birth) {
    givenName = first;
    familyName = family;
    IDNumber = ID;
    birthYear = birth;
}

/** Construct a person with only an IDNumber specified.
 * @param ID The ID number
 */
public Person(String ID) {
    IDNumber = ID;
}

// Modifier Methods
/** Sets the givenName field.
 * @param given The given name
 */
public void setGivenName(String given) {
    givenName = given;
}

/** Sets the familyName field.
 * @param family The family name
 */
public void setFamilyName(String family) {
    familyName = family;
}

/** Sets the birthYear field.
 * @param birthYear The year of birth
 */
public void setBirthYear(int birthYear) {
    this.birthYear = birthYear;
}

// Accessor Methods
/** Gets the person's given name.
 * @return the given name as a String
 */
public String getGivenName() { return givenName; }

/** Gets the person's family name.
 * @return the family name as a String
 */
public String getFamilyName() { return familyName; }

/** Gets the person's ID number.
 * @return the ID number as a String
 */
public String getIDNumber() { return IDNumber; }

/** Gets the person's year of birth.
 * @return the year of birth as an int value
 */
public int getBirthYear() { return birthYear; }

// Other Methods
/** Calculates a person's age at this year's birthday.
 * @param year The current year
 * @return the year minus the birth year
 */

```

```

        public int age(int year) {
            return year - birthYear;
        }

        /**
         * Determines whether a person can vote.
         * @param year The current year
         * @return true if the person's age is greater than or
         *         equal to the voting age
        */
        public boolean canVote(int year) {
            int theAge = age(year);
            return theAge >= VOTE_AGE;
        }

        /**
         * Determines whether a person is a senior citizen.
         * @param year the current year
         * @return true if person's age is greater than or
         *         equal to the age at which a person is
         *         considered to be a senior citizen
        */
        public boolean isSenior(int year) {
            return age(year) >= SENIOR_AGE;
        }

        /**
         * Retrieves the information in a Person object.
         * @return the object state as a string
        */
        public String toString() {
            return "Given name: " + givenName + "\n"
                + "Family name: " + familyName + "\n"
                + "ID number: " + IDNumber + "\n"
                + "Year of birth: " + birthYear + "\n";
        }

        /**
         * Compares two Person objects for equality.
         * @param per The second Person object
         * @return true if the Person objects have same
         *         ID number; false if they don't
        */
        public boolean equals(Person per) {
            if (per == null)
                return false;
            else
                return IDNumber.equals(per.IDNumber);
        }
    }
}

```

## Private Data Fields, Public Methods

The modifier **private** sets the visibility of each variable or constant to *private visibility*. This means that these data fields can be accessed only within the class definition. Only class members with *public visibility* can be accessed outside of the class.

The reason for having private visibility for data fields is to control access to an object's data and to prevent improper use and processing of an object's data. If a data field is private, it can be processed outside of the class only by invoking one of the public methods that are part of the class. Therefore, the programmer who writes the public methods controls how the data field is processed. Also, the details of how the private data are represented and stored can be

changed at a later time by the programmer who implements the class, and the other programs that use the class (called the class's *clients*) will not need to be changed.

## Constructors

In Listing A.1, the two methods that begin with `public Person` are constructors. One of these methods is invoked when a new class instance is created. The constructor with four parameters is called if the values of all data fields are known before the object is created. For example, the statement

```
Person author1 = new Person("Elliot", "Koffman", "010-055-0123", 1942);
```

creates the first object shown in Figure A.7, initializing its data fields to the values passed as arguments.

The second constructor is called when only the value of data field `IDNumber` is known at the time the object is created.

```
Person author2 = new Person("030-555-5555");
```

In this case, data field `IDNumber` is set to "030-555-5555", but all the other data fields are initialized to the default values for their data type (see Table A.14) unless a different initial value is specified (1900 for `birthYear`). The `String` data fields are initialized to `null`, which means that no `String` object is referenced. You can use the modifier methods at a later time to set the values of the other data fields. The statement

```
author2.setGivenName("Paul");
```

**TABLE A.14**  
Default Values for Data Fields

| Data Field Type                          | Default Value                                               |
|------------------------------------------|-------------------------------------------------------------|
| <code>int</code> (or other integer type) | 0                                                           |
| <code>double</code> (or other real type) | 0.0                                                         |
| <code>boolean</code>                     | <code>false</code>                                          |
| <code>char</code>                        | \u0000 (the smallest Unicode character: the null character) |
| Any reference type                       | <code>null</code>                                           |

sets the data field `givenName` to reference the `String` object "Paul". Note that there is no `setIDNumber` method, so this data field value can't be assigned or changed at a later time.

## The No-Parameter Constructor

A constructor with no parameters is called the *no-parameter constructor* (or *no-argument constructor*). This constructor is sometimes called the default constructor because Java automatically defines this constructor with an empty body for a class that has no constructor definitions. However, if you define one or more constructors for a class, you must also explicitly define the no-parameter constructor, or it will be undefined for that class. Because two constructors are defined for class `Person`, but the no-parameter constructor is not, the statement

```
Person p = new Person(); // Invalid call to no-parameter constructor.
```

will not compile.

## Modifier and Accessor Methods

Because the data fields have private visibility, we need to provide public methods to access them. Normally, we want to be able to get or retrieve the value of a data field, so each data field in class Person has an accessor method (also called getter) that begins with the word get and ends with the name of the data field (e.g., `getFamilyName`). If we want to allow a class user to update or modify the value of a data field, we provide a modifier method (also called mutator or setter) beginning with the word set and ending with the name of the data field (e.g., `setGivenName`). Currently, there is an accessor for each data field in this example and a modifier for all but the `IDNumber` data field. The reason for this is to deny a client the ability to change a person's ID number.

The modifier methods are type `void` because they are executed for their effect (to update a data field), not to return a value. In the method `setBirthYear`,

```
public void setBirthYear(int birthYear) {
    this.birthYear = birthYear;
}
```

the assignment statement stores the integer value passed as an argument in data field `birthYear`. (We explain the reason for `this.` in the next subsection.)

The accessor method for data field `givenName`,

```
public String getGivenName() { return givenName; }
```

is type `String` because it returns the `String` object referenced by `givenName`. If the class designer does not want other users (clients) of the class to be able to access or change the data field values, these methods can be given private visibility.

## Use of `this.` in a Method

Method `setBirthYear` uses the statement

```
this.birthYear = birthYear;
```

to store a value in data field `birthYear`. We can use `this.aDataField` in a method to access a data field of the current object. Because we used `birthYear` as a parameter in method `setBirthYear`, the Java compiler will translate `birthYear` without the prefix `this.` as referring to the parameter `birthYear`, not to the data field. The reason is the declaration of `birthYear` as a parameter is local to the method and, therefore, hides the data field declaration.

## The Method `toString`

The last two methods, `toString` and `equals`, are found in most Java classes. The method `toString` creates a `String` object that represents the information stored in an object (the *state* of an object). The escape sequence `\n` is the newline character, and it terminates an output line when the string is displayed. A client of class `Person` could use the statement

```
System.out.println(author1.toString());
```

to display the state of `author1`. In fact, the statement

```
System.out.println(author1);
```

would also display the state of `author1` because `System.out.println` and `System.out.print` automatically apply method `toString` to an object that appears in their argument list. The following lines would be displayed by this statement.

```
Given name: Elliot
Family name: Koffman
ID number: 010-055-0123
Year of birth: 1942
```

## The Method `equals`

The method `equals` compares the object to which it is applied (*this* object) to the object that is passed as an argument. It returns `true` if the objects are determined to be the same based on the data they store. It returns `false` if the argument is `null` or if the objects are not the same. We will assume that two `Persons` are the same if they have the same ID number.



### PROGRAM STYLE

#### Using `toString` Instead of Displaying Data Fields

Java programmers use method `toString` to build a string that represents the object state. This string can then be displayed at the console, written to a file, displayed in a dialog window, or displayed in a Graphical User Interface (GUI). This is more flexible than the approach taken in many programming languages, in which each data field is displayed or written to a file.

```
public boolean equals(Person per) {
    if (per == null)
        return false;
    else
        return IDNumber.equals(per.IDNumber);
}
```

The second return statement returns the result of the method call  
`IDNumber.equals(per.IDNumber)`

Note that we can look at parameter `per`'s private `IDNumber` because `per` references an object of this class (type `Person`). Because `IDNumber` is type `String`, the `equals` method of class `String` is invoked with the `IDNumber` of the second object as an argument. If the two `IDNumber` data fields have the same contents, the `String equals` method will return `true`; otherwise, it will return `false`. The `Person equals` method returns the result of the `String equals` method. In Section 3.5, we discuss the `equals` method in more detail and show you a better way to write this method.



### PROGRAM STYLE

#### Returning a Boolean Value

Some programmers unnecessarily write `if` statements to return a `boolean` value. For example, instead of writing

```
return IDNumber.equals(per.IDNumber);
```

they write

```
if (IDNumber.equals(per.IDNumber))
    return true;
else
    return false;
```

Resist this temptation. The `return` statement by itself returns the value of the `if` statement condition, which must be `true` or `false`. It does this in a clear and succinct manner using one line instead of four.

## Declaring Local Variables in Class Person

There are three other methods declared in class `Person`. Methods `age`, `canVote`, and `isSenior` are all passed the current year as an argument. Method `canVote` calls method `age` to determine the person's age. The result is stored in local variable `theAge`. The result of calling method `canVote` is the value of the Boolean expression following the keyword `return`.

```
public boolean canVote(int year) {
    int theAge = age(year); // Local variable
    return theAge >= VOTE_AGE;
}
```

It really was not necessary to introduce local variable `theAge`; the call to method `age` could have been placed directly in the `return` statement (as it is in method `isSenior`). We wanted, however, to show you how to declare local variables in a Java method. The scope of the local variable `theAge` and the parameter `year` is the body of method `canVote`.



### PITFALL

#### Referencing a Data Field or Parameter Hidden by a Local Declaration

If you happen to declare a local variable (or parameter) with the same name as a data field, the Java compiler will translate the use of that name in a method as meaning the local variable (or parameter), not the data field. So if `theAge` was also declared as a data field in class `Person`, the statement

```
theAge++;
```

would increment the local variable, but the data field value would not change. To access the data field instead of the local variable, use the prefix `this.`, just as we did earlier when a parameter had the same name as a data field.



### PITFALL

#### Using Visibility Modifiers with Local Variables

Using a visibility modifier with a local variable would cause a syntax error because a local variable is visible only within the method that declares it. Therefore, it makes no sense to give it public or private visibility.

## An Application that Uses Class Person

To test class `Person`, we need to write a Java application program that contains a `main` method. The `main` method should create one or more instances of class `Person` and display the results of applying the class methods. Listing A.2 shows a class `TestPerson` that does this. To execute the `main` method, you must compile and run class `TestPerson`. As long as `Person` and `TestPerson` are in the same folder (directory), the application program will run. Figure A.8 shows a sample run.

**LISTING A.2**

Class TestPerson

```
.....  

LISTING A.2  

Class TestPerson  

/** TestPerson is an application that tests class Person. */  

public class TestPerson {  

    public static void main(String[] args) {  

        Person p1 = new Person("Sam", "Jones", "1234", 1940);  

        Person p2 = new Person("Jane", "Jones", "5678", 2000);  

        System.out.println("Age of " + p1.getGivenName() +  

                           " is " + p1.age(2014));  

        if (p1.isSenior(2014))  

            System.out.println(p1.getGivenName() +  

                               " can ride the subway for free");  

        else  

            System.out.println(p1.getGivenName() +  

                               " must pay to ride the subway");  

        System.out.println("Age of " + p2.getGivenName() +  

                           " is " + p2.age(2014));  

        if (p2.canVote(2014))  

            System.out.println(p2.getGivenName() + " can vote");  

        else  

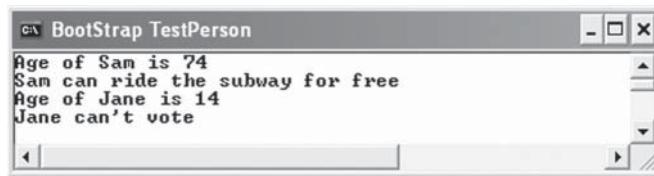
            System.out.println(p2.getGivenName() + " can't vote");  

    }  

}
```

**FIGURE A.8**

Sample Run of Class TestPerson



Although we will generally write separate application classes such as `TestPerson`, you could also insert the `main` method directly in class `Person` and then compile and run class `Person`. Program execution will start at the `main` method, and the result will be the same. If you use separate classes, make sure that you put them in the same folder (directory).

## Objects as Arguments

We stated earlier that Java arguments are passed by value. For primitive-type arguments, this protects the value of a method's argument and ensures that its value can't be changed by the method. However, this is not the case for arguments that are objects. If an argument is an object, its address is passed to the method, so the method parameter will reference the same object as the method argument. If the method happens to change a data field of its object parameter, that change will be made to the object argument. We illustrate this next.

---

**EXAMPLE A.13** Suppose method `changeGivenName` is defined as follows:

```
public void changeGivenName(Person per) {  

    per.givenName = this.givenName;  

}
```

Also suppose a client program declares `firstMan` and `firstWoman` as reference variables of type `Person`. After the method call

```
firstMan.changeGivenName(firstWoman)
```

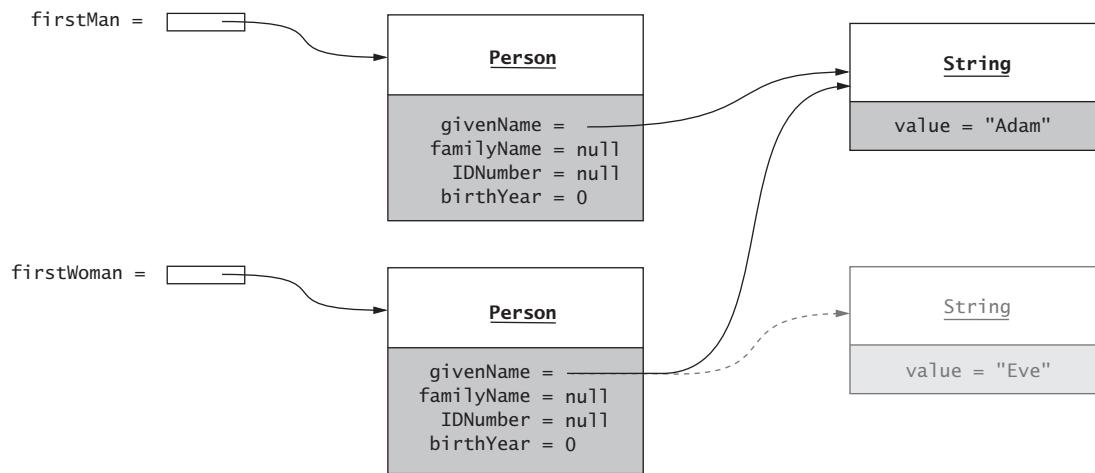
parameter `per` (declared in method `changeGivenName`) and reference variable `firstWoman` (declared in the client) will reference the same object. The statement

```
per.givenName = this.givenName;
```

will set the `givenName` data field of the object referenced by `per` (and `firstWoman`) to reference the same string as the `givenName` field of `this` object (the object referenced by `firstMan`). Figure A.9 shows the `givenName` data field of the objects referenced by `firstMan` and `firstWoman` after the foregoing statement executes.

**FIGURE A.9**

Reference Variables `firstMan` and `firstWoman`



## Classes as Components of Other Classes

Class `Person` has three data fields that are type `String`, so `String` objects are *components* of a `Person` object. In Figure A.10, this component relationship is indicated by the solid diamond symbol at the end of the line drawn from the box representing class `String` to the box representing class `Person`. Like the class diagram in Figure A.6 and the object diagrams in Figure A.7, Figure A.10 is a UML diagram, showing the relationships between classes. We will follow UML's set of conventions for documenting class relationships in this book.

**FIGURE A.10**

UML Diagram Showing that `String` Objects Are Components of Class `Person`



## Java Documentation Style for Classes and Methods

Java provides a standard form for writing comments and documenting classes, which we will use in this book. If you use this form, you can run a program called *Javadoc* (part of the JDK) to generate a set of HTML pages describing each class and its data fields and methods. These pages will look just like the ones that document the Java API classes on the Oracle Corporation Java Web site (<https://docs.oracle.com/javase/8/docs/api/>).

The Javadoc program focuses on text that is enclosed within the delimiters `/**` and `*/`. The introductory comment that describes the class is displayed on the HTML page exactly as it is written, so you should write that carefully. The lines that begin with the symbol `@` are Javadoc tags. They are described in Table A.15. In this book, we will use one `@param` tag for each method parameter. We will not use a `@return` tag for `void` methods. The first line of the comment for each method appears in the method summary part of the HTML page. The information provided in the tags will appear in the method detail part. Figures A.11 through A.13 show part of the documentation generated by running Javadoc for a class Person similar to the Person class in this chapter.

To run the Javadoc program, change to the directory that contains the source files that you would like to process. Then, to create the HTML documentation files, enter the command

```
pathName\javadoc className1.java className2.java
```

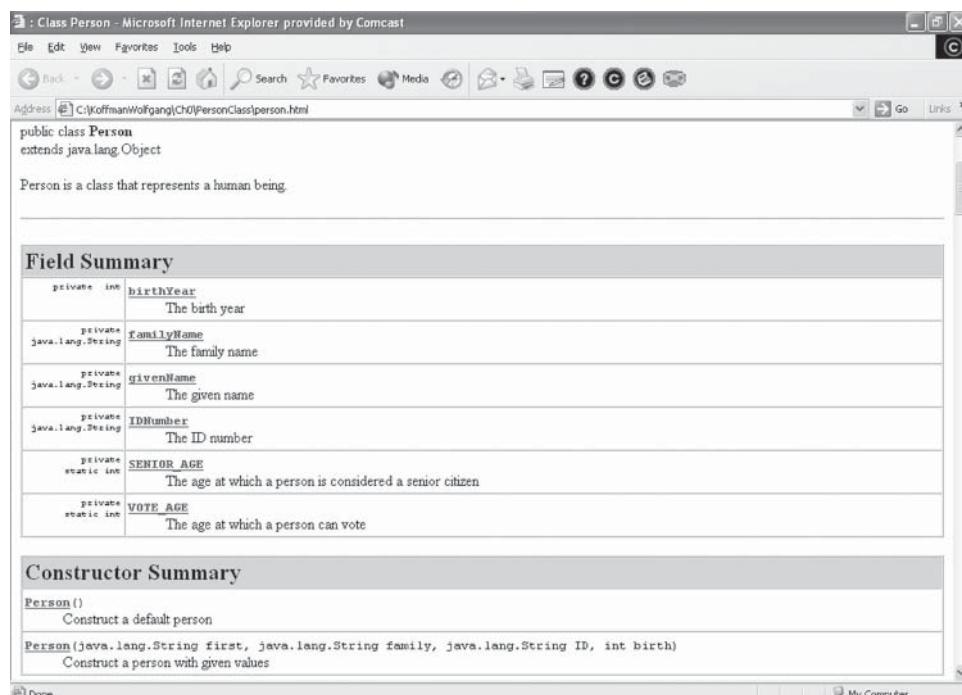
where `pathName` is the directory that contains the Javadoc program, and the Java source file names (`className1.java`, `className2.java`, etc.) follow the `javadoc` command. If you want to show the private data fields and methods, add the command line argument `-private`. If

.....  
**TABLE A.15**

Javadoc Tags

| Javadoc Tag and Example of Use            | Purpose                          |
|-------------------------------------------|----------------------------------|
| <code>@author Koffman and Wolfgang</code> | Identifies the class author      |
| <code>@param first The given name</code>  | Identifies a method parameter    |
| <code>@return The person's age</code>     | Identifies a method return value |

.....  
**FIGURE A.11**  
Field Summary for  
Class Person



**FIGURE A.12**

Method Summary for  
Class Person

| Method Summary                                           |                                                    |
|----------------------------------------------------------|----------------------------------------------------|
| <code>int age(int year)</code>                           | Calculates a person's age at this year's birthday. |
| <code>boolean canVote(int year)</code>                   | Determines whether a person can vote               |
| <code>boolean equals(Person per)</code>                  | Compares two Person objects for equality           |
| <code>int getBirthYear()</code>                          | Gets the person's year of birth                    |
| <code>java.lang.String getFamilyName()</code>            | Gets the person's family name                      |
| <code>java.lang.String getGivenName()</code>             | Gets the person's given name                       |
| <code>java.lang.String getIDNumber()</code>              | Gets the person's ID number                        |
| <code>boolean isSenior(int year)</code>                  | Determines whether a person is a senior citizen    |
| <code>void setBirthYear(int birthYear)</code>            | Sets the birthYear field                           |
| <code>void setFamilyName(java.lang.String family)</code> | Sets the familyName field                          |
| <code>void setGivenName(java.lang.String first)</code>   | Sets the givenName field                           |
| <code>void setIDNumber(java.lang.String ID)</code>       | Sets the IDNumber field                            |

**FIGURE A.13**

Method Detail for  
Class Person

**Method Detail**

**setGivenName**

```
public void setGivenName(java.lang.String first)
```

Sets the givenName field  
**Parameters:**  
 first - The given name

---

**setFamilyName**

```
public void setFamilyName(java.lang.String family)
```

Sets the familyName field  
**Parameters:**  
 family - The family name

---

**setIDNumber**

```
public void setIDNumber(java.lang.String ID)
```

Sets the IDNumber field  
**Parameters:**  
 ID - The ID number

you want to create documentation files for all the .java files in the directory, use the wildcard \* for the class name.

```
pathName\javadoc -private *.java
```

Another useful command line argument is -d *destinationFolder*, which allows you to specify a folder or directory other than the source folder for the Javadoc HTML files.

## EXERCISES FOR SECTION A.7

### SELF-CHECK

1. Explain why methods have public visibility but data fields have private visibility.
2. Download file Person.java from the textbook Web site and run javadoc on it.
3. Trace the execution of the following statements.

```
Person p1 = new Person("Adam", "Jones", "wxyz", 0);
p1.setBirthYear(1990);
Person p2 = new Person();
p2.setGivenName("Eve");
p2.setFamilyName(p1.getFamilyName());
p2.setBirthYear(p1.getBirthYear() + 10);
if (p1.equals(p2))
    System.out.println(p1 + "\nis same person as\n\n" + p2);
else
    System.out.println(p1 + "\nis not the same person as\n\n" + p2);
```

### PROGRAMMING

1. Write a method `getInitials` that returns a string representing a `Person` object's initials. There should be a period after each initial. Write Javadoc tags for the method.
2. Add a data field `motherMaidenName` to `Person`. Write an accessor and a modifier method for this data field. Modify class `toString` and class `equals` to include this data field. Assume two `Person` objects are equal if they have the same ID number and mother's maiden name. Write Javadoc tags for the method.
3. Write a method `compareTo` that compares two `Person` objects and returns an appropriate result based on a comparison of the ID numbers. That is, if the ID number of the object that `compareTo` is applied to is less than (is greater than) the ID number of the argument object, the result should be negative (positive). The result should be 0 if they have the same ID numbers. Write Javadoc tags for the method.
4. Write a method `switchNames` that exchanges a `Person` object's given and family names. Write Javadoc tags for the method.



## A.8 Arrays

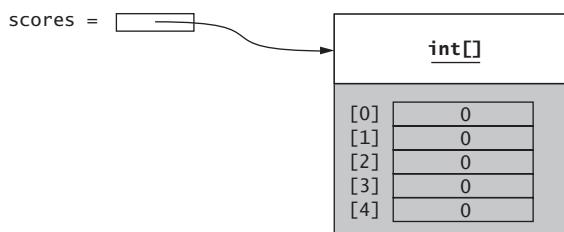
In Java, an array is also an object. The elements of an array are indexed and are referenced using a subscripted variable of the form:

```
arrayName[subscript]
```

Next, we show some different ways to declare arrays and allocate storage for arrays.

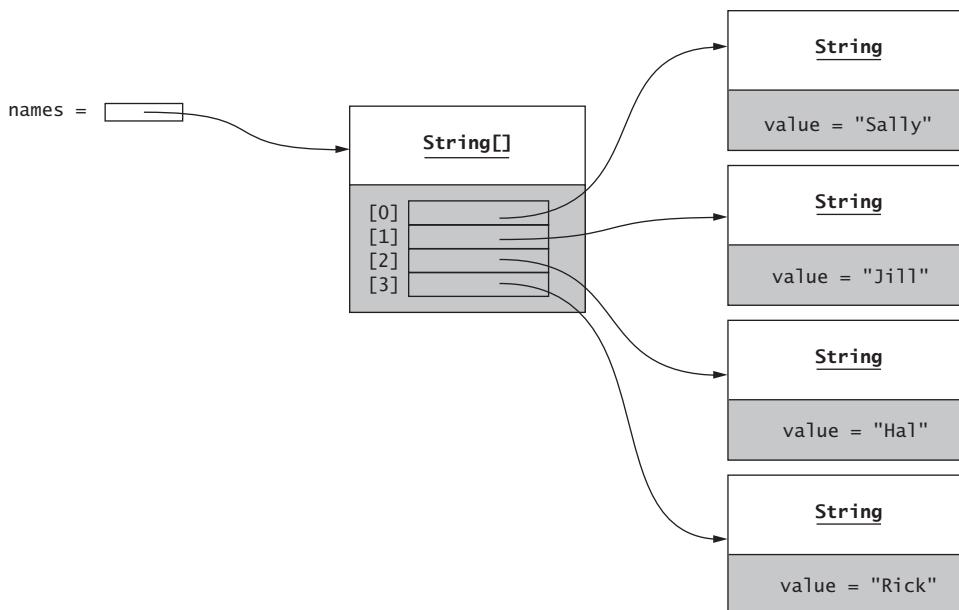
**EXAMPLE A.14** The following statement declares a variable `scores` that references a new array object that can store five type `int` values (subscripts 0 through 4) as shown. Each element is initialized to 0.

```
int[] scores = new int[5]; // An array with 5 type int values
```



**EXAMPLE A.15** The following statement declares a variable `names` that references a new array object that can store four type `String` objects. The values stored are specified in the *initializer list*.

```
String[] names = {"Sally", "Jill", "Hal", "Rick"};
```



### PITFALL

#### Out-of-Bounds Subscripts

Some programming languages allow you to use an array subscript that is outside of the array bounds. For example, if you attempt to reference `scores[5]`, a C or C++ compiler would access the first memory cell following the array `scores`. This is considered an error, but it is not detected by the run-time system and will probably lead to another error that will be detected farther down the road (before it does too much damage, you hope). Java, however, verifies that the current value of each array subscript is within the array bounds. If it isn't, you will get an `ArrayIndexOutOfBoundsException` error.

**EXAMPLE A.16** The first of the following statements declares a variable `people` that can reference an array object for storing type `Person` objects. Storage has not yet been allocated for the array object (or for the `Person` objects). The second statement assumes that `n` is defined, possibly through an input operation. The last statement allocates storage for an array object with `n` elements. Each array element can reference a type `Person` object, but initially each element has the value `null` (no object referenced).

```
// Declare people as type Person[].
Person[] people;
// Define n in some way.
int n = . . .
// Allocate storage for the array.
people = new Person[n];
```

We can create some `Person` objects and store them in the array. The following statements store two `Person` objects in array `people`.

```
people[0] = new Person("Elliot", "Koffman", "010-055-0123", 1942);
people[1] = new Person("Paul", "Wolfgang", "015-023-4567", 1945);
```



## PITFALL

### Forgetting to Declare Storage for an Array

As just shown, you can separate the declaration of variable `people` (the array reference variable) from the step that actually allocates storage (`people = new ...`). However, you can't reference the array elements before you allocate storage for the array. Similarly, if the array elements reference objects, you must separately allocate storage for each object.

## Data Field `length`

A Java array has a `length` data field that can be used to determine the array's size. The value of `names.length` is 4; the value of `people.length` is the same as the value of `n` when storage was allocated for the array. The subscripted variable `people[people.length - 1]` references the last element in array `people`. The following `for` statement can be used to display all the `Person` objects stored in array `people`, regardless of the array size.

```
for (int i = 0; i < people.length; i++ )
    if (people[i] != null)
        System.out.println(people[i] + "\n");
```



## PITFALL

### Using `length` Incorrectly

The value of data field `length` is set when storage is allocated for the array, and it is `final`. Therefore, it can't be changed by the programmer. A statement such as

```
people.length++; // invalid attempt to increment length
```

would cause a syntax error.

Another common error is using parentheses with `length`. The expression `people.length()` causes a syntax error because `length` is a data field, not a method, of an array.

## Method Arrays.copyOf

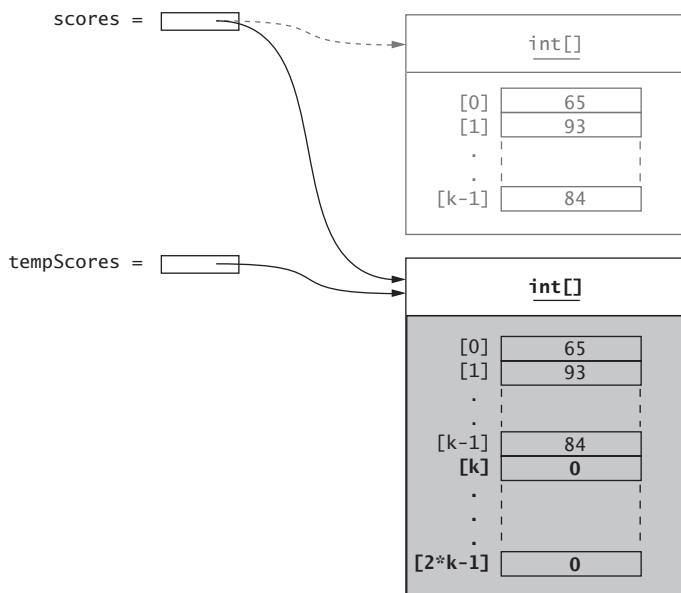
Although you can't change the length of a particular array object, you can copy the values stored in one array object to another array object using method `Arrays.copyOf`. This method returns a copy of a given array and either truncates or pads the copy to a new length. The method is overloaded for arrays of each primitive type, and there is a generic form for copying arrays of class types.

**EXAMPLE A.17** The following statements create a new array `tempScores` that is twice the size of array `scores` and contains a copy of elements in array `scores` to the first half of array `tempScores`. The remaining entries of `tempScores` are set to zero. Finally, we reset variable `scores` to reference the same array as `tempScores` (see Figure A.14). The storage originally allocated to store the elements of array `scores` can now be reclaimed by the garbage collector.

```
int[] tempScores = Arrays.copyOf(scores, 2 * scores.length);
scores = tempScores;
```

**FIGURE A.14**

Doubling the Size of the Array Referenced by Scores



## Method System.arraycopy

The method `Arrays.copyOf` makes a copy of the whole array. There is also the method `Arrays.copyOfRange`, which makes a copy of a part of an array returning the selected part as a new array. A general method that will copy a selected portion of an array into another array is `System.arraycopy`, which has the general form

```
System.arraycopy(source, sourcePos, destination, destPos, numElements);
```

The parameters `sourcePos` and `destPos` specify the starting positions in the `source` and `destination` arrays, respectively. The parameter `numElements` specifies the number of elements to copy. If this number is too large, an `ArrayIndexOutOfBoundsException` error occurs.

`System.arraycopy` is effectively the following:

```
for (int k = 0; k < numElements; k++) {
    destination[destPos + k] = source[sourcePos + k];
}
```

but is implemented within the JVM using native machine code instructions that efficiently copy a block of data from one location to another.

## Array Data Fields

It is very common in Java to encapsulate an array, together with the methods that process it, within a class. Rather than allocate storage for a fixed-size array, we would like the client to be able to specify the array size when an object is created. Therefore, we should define a constructor with the array size as a parameter and have the constructor allocate storage for the array. Class `Company` in Listing A.3 has a data field `employees` that references an array of `Person` objects. Both constructors allocate storage for a new array when a `Company` object is created. The client of this class can specify the size of the array by passing a type `int` value to the constructor parameter `size`. If no argument is passed, the no-parameter constructor sets the array size to `DEFAULT_SIZE`.

.....

### LISTING A.3

Class `Company`

```
/** Company is a class that represents a company.
   The data field employees provides storage for
   an array of Person objects.
*/
public class Company {
    // Data Fields
    /** The array of employees */
    private Person[] employees;

    /** The default size of the array */
    private static final int DEFAULT_SIZE = 100;

    // Methods
    /** Creates a new array of Person objects.
        @param size The size of array employees
    */
    public Company(int size) {
        employees = new Person[size];
    }

    public Company() {
        employees = new Person[DEFAULT_SIZE];
    }

    /** Sets field employees.
        @param emp The array of employees
    */
    public void setEmployees(Person[] emp) {
        employees = emp;
    }

    /** Gets field employees.
        @return employees array
    */
    public Person[] getEmployees() {
        return employees;
    }

    /** Sets an element of employees.
        @param index The position of the employee
        @param emp The employee
    */
    public void setEmployee(int index, Person emp) {
        if (index >= 0 && index < employees.length)
            employees[index] = emp;
    }
}
```

```

    /**
     * Gets an employee.
     * @param index The position of the employee
     * @return The employee object or null if not defined
     */
    public Person getEmployee(int index) {
        if (index >= 0 && index < employees.length)
            return employees[index];
        else
            return null;
    }

    /**
     * Builds a string consisting of all employee's
     * data, with newline characters between employees.
     * @return The object's state
     */
    public String toString() {
        StringBuilder result = new StringBuilder();
        for (int i = 0; i < employees.length; i++)
            result.append(employees[i] + "\n");
        return result.toString();
    }
}

```

There are modifier and accessor methods that process individual elements of array `Company` (`setEmployee` and `getEmployee`). Method `getEmployee` returns the type `Person` object at position `index`, or `null` if the value of `index` is out of bounds.

The `toString` method returns a string representing the contents of array `employees`. In the `for` loop, the argument in each call to method `append` is the string returned by applying method `Person.toString` to the current employee. This string is appended to the string representing the data for all employees with smaller subscripts.

The following `main` method illustrates the use of class `Company` and displays the state of object `comp`.

```

public static void main(String[] args) {
    Company comp = new Company(2);
    comp.setEmployee(0, new Person("Elliot", "K", "123", 1942));
    comp.setEmployee(1, new Person("Paul", "W", "234", 1945));
    System.out.println(comp);
}

```

## Array Results and Arguments

Method `setEmployees` in class `Company` takes a single argument `emp` that is type `Person[]`. The assignment statement

```
employees = emp;
```

resets array `employees` to reference the array argument. Storage allocated to the array previously referenced by `employees` can then be reclaimed by the garbage collector.

The return value of method `getEmployees` is type `Person[]`. The statement

```
return employees;
```

returns a reference to the array `employees`.

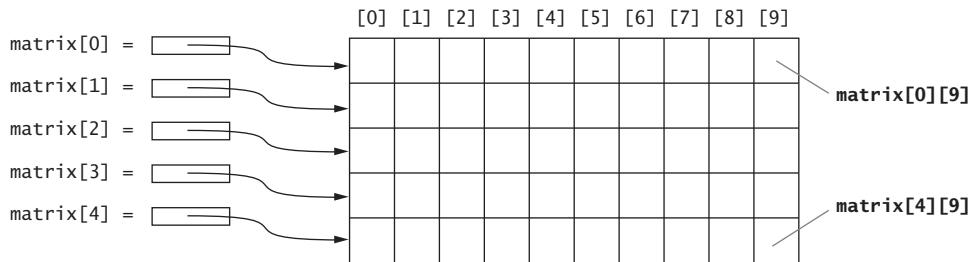
## Arrays of Arrays

A Java array can have other arrays as its elements. If all these arrays are of the same size, then the array of arrays is a two-dimensional array.

**EXAMPLE A.18** The declaration

```
double[][] matrix = new double[5][10];
```

allocates storage for a two-dimensional array, `matrix`, that stores 50 real numbers in 5 rows and 10 columns. The variable `matrix[i][j]` references the number with row subscript `i` and column subscript `j`. You can also declare arrays with more than two dimensions.



In Java you can have two-dimensional arrays with rows of different sizes. We illustrate this in the next two examples.

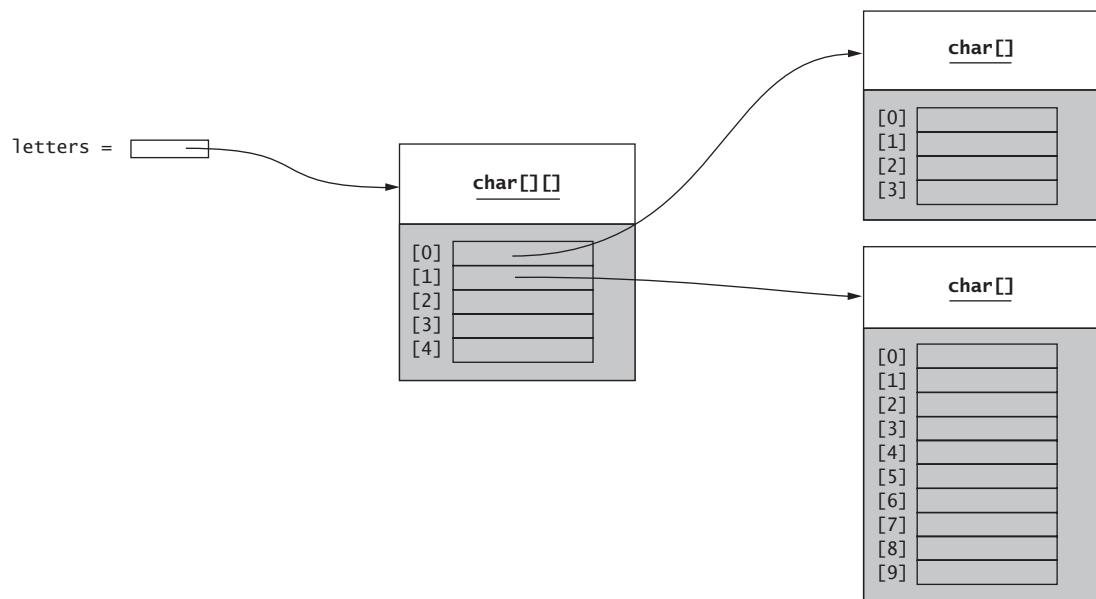
**EXAMPLE A.19** The declaration

```
char[][] letters = new char[5][];
```

allocates storage for a two-dimensional array of characters with five rows, but the number of columns in each row is not specified. The statements

```
letters[0] = new char[4];
letters[1] = new char[10];
```

define the size of the first two rows and allocate storage for them. The subscripted variable `letters[0]` references the first row; `letters.length` is 5, the number of rows in the array; `letters[1].length` is 10, the number of elements in the row with subscript 1.



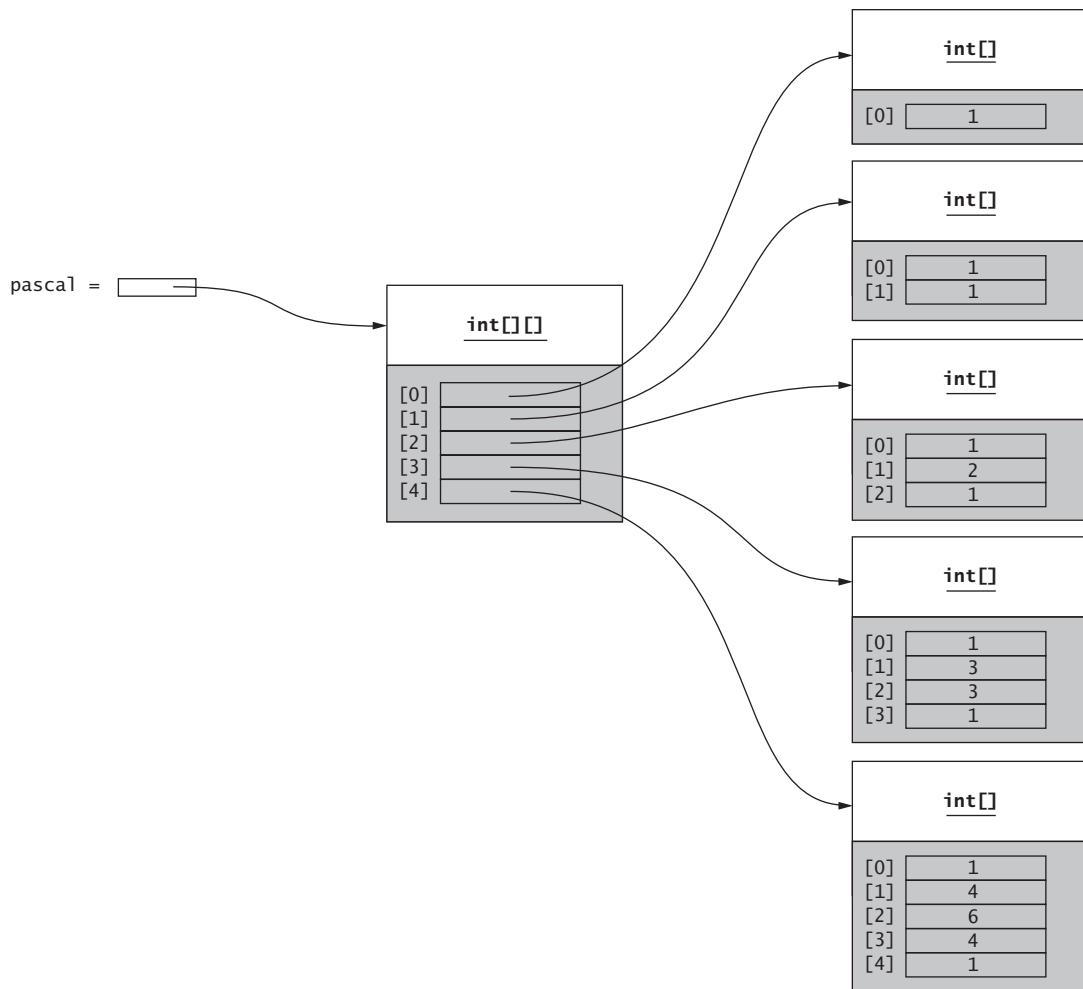
**EXAMPLE A.20** The declaration

```
int[][] pascal = {  
    {1},           // row 0  
    {1, 1},        // row 1  
    {1, 2, 1},  
    {1, 3, 3, 1},  
    {1, 4, 6, 4, 1},  
};
```

allocates storage for an array of arrays with five rows. The initializer list provides the values for each row, starting with row 0. The subscripted variable `pascal[0]` references the one-element array `{1}`, and `pascal[4]` references the array `{1, 4, 6, 4, 1}`. Each row has one more element than the previous one. The values shown above form a well-known mathematical entity called Pascal's triangle. Each element in a row, except for the first and last elements, is the sum of the two elements on either side of it in the previous row. For example, the number 6 in the last row is the sum of the numbers 3, 3 in the previous row. In mathematical notation,

$$\text{pascal}[i + 1][j] = \text{pascal}[i][j - 1] + \text{pascal}[i][j].$$

The first and last elements in each row are 1.



The following nested **for** statements sum all values in the Pascal triangle. In the outer **for** loop header, the expression `pascal.length` is the number of rows in the triangle. In the inner **for** loop header, the expression `pascal[row].length` is the number of columns in the array with subscript `row`.

```
int sum = 0;
for (int row = 0; row < pascal.length; row++)
    for (int col = 0; col < pascal[row].length; col++)
        sum += pascal[row][col];
```

---

## EXERCISES FOR SECTION A.8

### SELF-CHECK

1. Show the output that would be displayed by method `main` following Listing A.3.
2. Show that the formula for the interior elements of a Pascal triangle row is correct by evaluating it for each interior element of the last row.
3. What is the output of the following sample code fragment?

```
int[] x;
int[] y;
int[] z;
x = new int[20];
x[10] = 0;
y = x;
x[10] = 5;
System.out.println(x[10] + ", " + y[10]);
x[10] = 15;
z = new int[x.length];
System.arraycopy(x, 0, z, 0, 20);
x[10] = 25;
System.out.println(x[10] + ", " + y[10]+ ", " + z[10]);
```

4. What happens if you make a copy of an array of object references using method `System.arraycopy`? If the objects referenced by the new array are changed, how will this affect the original array?
5. Assume there is no initializer list for the Pascal triangle and you are trying to build up its rows. If row `i` has been defined, write statements to create row `i + 1`.

### PROGRAMMING

1. Write code for a method

```
public static boolean sameElements(int[] a, int[] b)
```

that checks whether two arrays have the same elements in some order, with the same multiplicities. For example, two arrays

121    144    19    161    19    144    19    11

and

11    121    144    19    161    19    144    19

would be considered to have the same elements because 19 appears three times in each array, 144 appears twice in each array, and all other elements appear once in each array.

2. Write an `equals` method for class `Company`. The result should be `true` if the employees of one company match element for element with the employees of a different company. Assume that the objects referenced by each array `employees` are in order by ID number.
3. For the two-dimensional array `letters` in Example A.19, assume `letters[i]` is going to be used to store an array that contains the individual characters in `String` object `next`. Allocate storage for `letters[i]` based on the length of `next` and write a loop that stores each character of `next` in the corresponding element of `letters[i]`. For example, the first character in `next` should be stored in `letters[i][0]`.



## A.9 Enumeration Types

In Java, we use classes and objects to represent things in the application domain. However, there are cases where the things we wish to represent are discrete objects with no attributes. For example, the colors of a traffic light: RED, YELLOW, and GREEN. One approach is to assign arbitrary integer values.

```
final static int RED = 0;
final static int YELLOW = 1;
final static int GREEN = 2;
```

This approach has limitations. For example, assume we also wanted to represent the colors of crayons:

```
final static int RED = 0;
final static int YELLOW = 1;
final static int BLUE = 2;
final static int GREEN = 3;
final static int ORANGE = 4;
final static int BROWN = 5;
final static int VIOLET = 6;
final static int BLACK = 7;
```

If these were both in the same program, there would be a conflict between the GREEN traffic light and the GREEN crayon.

Another limitation is input/output. We want our user to be able to enter "GREEN" instead of knowing that 2 represents green, and we want to output the string "GREEN" instead of the number 2.

The Java enumeration type allows us to declare a set of identifiers that can then be used to represent the concept in our application domain.



### SYNTAX Enumeration Type Declaration

#### FORM:

```
enum enumName {enumIdentifiers}
```

#### EXAMPLE:

```
enum Traffic {RED, YELLOW, GREEN}
```

#### INTERPRETATION:

An enumeration type `enumName` is defined with the listed identifiers as members. An enumeration type is a class and the `enumIdentifiers` are objects of the class. These objects are constants. You cannot create additional instances of the enumeration type.

## Using Enumeration Types

You can create variables of enumeration types, compare enumeration values for equality, and use them as case labels in switch statements. For example:

```
switch (lightColor) {
    case Traffic.GREEN:
        // keep going
        ...
        break;
    case Traffic.YELLOW:
        if /* able to stop at stop line */
            // apply brakes
        ...
    else
        // keep going
        ...
        break;
    case Traffic.RED:
        // apply brakes
        ...
        break;
}
```

Each enumeration type is a subtype of `Enum<E>` where the `<E>` is a generic parameter representing the enumeration type. (We explain generic parameters in more detail in Section 2.2.) Thus, our `Traffic` example is a subclass of `Enum<Traffic>`. Table A.16 shows the methods defined in `Enum<E>`.

For example, the following code

```
for (int i = 0; i < Traffic.values().length; i++) {
    System.out.println(Traffic.values()[i]);
}
```

will output

```
RED
YELLOW
GREEN
```

and the expression

```
Traffic.valueOf("RED")
```

will result in `Traffic.RED`.

.....  
**TABLE A.16**

Methods Defined in `Enum<E>`

| Method                                            | Behavior                                                                                                                                                                                                                                |
|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>public static E[] values()</code>           | Returns an array view of the enumeration. The indices are assigned in the order in which the enumerations were declared                                                                                                                 |
| <code>public static E valueOf(String name)</code> | Returns the enumeration constant with the specified name. The string must match exactly the identifier used to declare this enumeration value. If not, an <code>IllegalArgumentException</code> is thrown as described in Section A.12. |

## Assigning Values to Enumeration Types

An enumeration type declaration is a class declaration. Consequently, you can add additional members to the enumeration class. For example, you can assign arbitrary values by adding a field and providing a constructor. By default, the constructor is private and it is invoked when the enumeration constants are declared. For example, you could declare an enumeration Coin in which each enumeration identifier has a field value in parentheses which is the number of pennies it represents:

```
enum Coin {
    PENNY(1), NICKEL(2), DIME(10), QUARTER(25), HALF_DOLLAR(50);
    Coin(int value) {
        this.value = value;
    }
    private final int value;
    public int getValue() {return value;}
}
```

---

## EXERCISES FOR SECTION A.9

### SELF-CHECK

1. Define an enumeration Suit to represent the suits in a deck of cards: CLUBS, DIAMONDS, HEARTS, and SPADES.
2. Define an enumeration Rank to represent the cards in a suit: DEUCE, TREY, FOUR, FIVE, . . . NINE, TEN, JACK, QUEEN, KING, and ACE.

### PROGRAMMING

1. Define a class Card. This class should contain the Suit and Rank enumerations and two final fields to contain the Suit and Rank. Define a `toString` method that returns the string *rank* of *suit*, and a constructor that takes two Strings specifying the rank and suit.




---

## A.10 I/O Using Streams, Class Scanner, and Class JOptionPane

In this section, we will show you the basics of using streams for I/O in Java. An input stream is a sequence of bytes representing program data. An output stream is a sequence of bytes representing program output. You can store program data in the stream associated with the console, `System.in`. When you type data characters at the console keyboard, they are appended to `System.in`. The console window is associated with `System.out`, the standard output stream. We have used methods `print` and `println` to write information to this stream.

Besides using the console for I/O, you can create and save a text file (using a word processor or editor) and then use it as an input stream for a program. Similarly, a program can write characters to an output stream and save it as a disk file. Classes `BufferedReader` and `FileReader` are subclasses of `Reader`.

Internally Java works with characters. The conversion from bytes to characters is performed by a `Reader` for input, and the conversion from characters to bytes is performed by a `Writer` for output. We discuss the details of this conversion later in this section.

## The Scanner

The Scanner was introduced as part of Java 5.0. The Scanner greatly simplifies the process of reading data from the console or an input file because it breaks its input into *tokens* that are character sequences separated by *whitespace* (blanks and the *newline* character). For console input, the `next` and `hasNext` methods suspend execution until input is provided. Table A.17 summarizes selected methods of this class.

To use a Scanner (part of `java.util`) to read from the console, you need to create a new Scanner object and connect it to the console (`System.in`)

```
Scanner scanConsole = new Scanner(System.in);
```

We illustrate the use of a Scanner in the next example.

**TABLE A.17**

Selected Methods of the `java.util.Scanner` Class

| Constructor                                    | Behavior                                                                                                                                                                                                                       |
|------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>Scanner(File source)</code>              | Constructs a Scanner that reads from the specified file                                                                                                                                                                        |
| <code>Scanner(InputStream source)</code>       | Constructs a Scanner that reads from the specified InputStream                                                                                                                                                                 |
| <code>Scanner(Readable source)</code>          | Constructs a Scanner that reads from the specified Readable object. The interface <code>Readable</code> is the superclass for Readers                                                                                          |
| <code>Scanner(String source)</code>            | Constructs a Scanner that reads from the specified String object                                                                                                                                                               |
| Method                                         | Behavior                                                                                                                                                                                                                       |
| <code>boolean hasNext()</code>                 | Returns true if there is another token available for input                                                                                                                                                                     |
| <code>boolean hasNextDouble()</code>           | Returns true if the next token can be interpreted as a double value                                                                                                                                                            |
| <code>boolean hasNextInt()</code>              | Returns true if the next token can be interpreted as an int value                                                                                                                                                              |
| <code>boolean hasNextLine()</code>             | Returns true if there is another line available for input                                                                                                                                                                      |
| <code>IOException ioException()</code>         | Returns the IOException last thrown by the Readable object that is used to read the input. (The Scanner constructors create a Readable object to perform the actual input.)                                                    |
| <code>String next()</code>                     | Returns the next token                                                                                                                                                                                                         |
| <code>double nextDouble()</code>               | Returns the next token as a double value. Throws <code>InputMismatchException</code> if the input is not in the correct format                                                                                                 |
| <code>int nextInt()</code>                     | Returns the next token as an int value. Throws <code>InputMismatchException</code> if the input is not in the correct format                                                                                                   |
| <code>String nextLine()</code>                 | Returns the next line of input as a string. A line is a sequence of characters ending with the newline character (\n). It may contain several tokens. The newline character is processed, but it is not included in the string |
| <code>String findInLine(String pattern)</code> | Attempts to find the next occurrence of a substring that matches the regular expression defined by pattern. Returns the substring if found, or null if not found                                                               |

**EXAMPLE A.21** The program in Listing A.4 prompts the user for a name, an integer value, another name, and another integer value. It displays the sum of the two integer values read. A sample interaction follows:

```
For your blended family,
enter the wife's name: Teresa Heinz
Enter the number of her children: 3
Enter the husband's name: John Kerry
Enter the number of his children: 2
Teresa Heinz and John Kerry have 5 children.
```

#### LISTING A.4

A Program for Counting Children in a Blended Family

```
import java.util.*;

/** A class to count and display children in a blended family.
 */
public class BlendedFamily {
    public static void main(String[] args) {
        Scanner sc = new Scanner(System.in);
        System.out.print("For your blended family, \nenter the wife's name: ");
        String wife = sc.nextLine();
        System.out.print("Enter the number of her children: ");
        int herKids = sc.nextInt();

        System.out.print("Enter the husband's name: ");
        sc.nextLine(); // Skip over trailing newline character.
        String husband = sc.nextLine();
        System.out.print("Enter the number of his children: ");
        int hisKids = sc.nextInt();

        System.out.println(wife + " and " + husband + " have "
                           + (herKids + hisKids) + " children.");
    }
}
```



#### PITFALL

##### Not Skipping the Newline Character before Reading a String

In the preceding example, we used the statement pair

```
sc.nextLine(); // Skip over trailing newline character.
String husband = sc.nextLine();
```

to read a string into `husband`. The purpose of the first statement is to process the newline character at the end of the line containing the data value 3. If we did not include the first statement, the newline character following the data value 3 would terminate the scanning process immediately, storing an empty string in `husband`. Because data entry for `husband` was completed without the need for typing in additional data, the line

Enter the husband's name: Enter the number of his children:

would be displayed. At this point, if you enter the husband's name, you will get an `InputMismatchException`. If you enter an integer, you will get the incomplete output line:

Teresa Heinz and have 5 children.

## Using a Scanner to Read from a File

You can also use a Scanner to read from a file. To do this, you need to create a new Scanner object and connect it to the file.

```
Scanner scanFile = null;
String fileName = "dataFile.txt"; // data file name
try {
    scanFile = new Scanner(new File(fileName));
} catch (FileNotFoundException ex) {
    System.err.println(filename + " not found");
    System.exit(1);
}
```

We explain the try-catch statement in the next section. The try block connects Scanner `scanFile` to a data file; the name of the data file is stored in String `fileName`. The file name is passed as an argument to the `File` constructor. Class `File` is in `java.util.io`, which must be imported (as well as `java.util` for Scanner). The catch block executes if the specified file cannot be found.

## Exceptions

Exceptions are program errors that occur during the execution of a program. We will discuss exceptions in great detail in Section A.11. In this section, we will tell you just enough about them to enable you to use streams for I/O.

When you process streams, there is a reasonable chance that a system error will occur. For example, the system may not be able to locate your file, or an error could occur during a file read operation. For this reason, Java requires you to perform all file-processing operations within the try block of a try-catch sequence, as follows:

```
try {
    // Statements that perform file-processing operations
} catch (IOException ex) {
    ex.printStackTrace(System.err); // Display stack trace
    System.exit(1); // Exit with an error indication
}
```

If all operations in the try block execute without error, the catch block is skipped. If an `IOException` or error occurs, the try block is exited and the catch block executes. This catch block simply displays the sequence of method calls that led to the error (starting with the most recent one and working backward) in the console window (`System.err`—the standard error stream) and then exits with an error indication. If we did not exit the catch block after catching an error, the program would continue with the first statement following the catch block.

## Tokenized Input

Often a data line will consist of a group of data items separated by spaces. In Section A.5, we discussed how to extract the individual items (tokens) from each line in order to process them. You can also use a Scanner. The following loop adds all the numbers read from input stream `ins`.

```
double sum = 0.0;
Scanner sc = new Scanner(ins);
while (sc.hasNextDouble()) {
    nextNum = sc.nextDouble();
    sum += nextNum;
}
```

## Extracting Tokens Using Scanner.`findInLine`

You can use a Scanner to scan the characters in a string as well as the data in a file. The statement

```
Scanner scan = new Scanner(line);
```

creates a Scanner object to scan, or process the characters, in string `line`.

You can also extract substrings that match a specified pattern using Scanner method `findInLine`. If method `findInLine` is applied to `scan`, it will extract each sequence of characters in `line` matched by its regular expression argument. When there are no characters remaining that match the regular expression, `findInLine` will return `null`. The statements below store in `token` each sequence of digit and letter characters and display the tokens. Note that this regular expression is the same as the one used with method `split` with the `^` (not) symbol removed because we are extracting sequences of letters and digits instead of looking for delimiter characters that are not letters or digits.

```
String token;
while ((token = scan.findInLine("[\\p{L}\\p{N}]")) != null) {
    System.out.println(token);
}
```

## Using a BufferedReader to Read from an Input Stream

To use files as input streams, you must import `java.io`:

```
import java.io.*;
```

You also need to create a `BufferedReader` object:

```
String fileName = args[0]; // The first main parameter
BufferedReader ins = new BufferedReader(new FileReader(fileName));
```

Although this looks fairly complicated, you can think of it as “boilerplate” (or a template for creating a `BufferedReader`). The only part of this code that can change is the `String` argument passed to the `FileReader` constructor (`fileName` in this example). Variable `fileName` references the string passed as the first parameter (`args[0]`) to method `main`. This should be the name of a data file.

The `BufferedReader` constructor needs a parameter that is type `FileReader` (or type `InputStreamReader`). The `BufferedReader` class defines a method `readLine` that can be used to read the next data line in a file (or typed at the console); the method returns a `String` object that contains the characters in that data line.

## Output Streams

To write to an output file, you need to create an output stream. Use statements such as

```
String outFile = args[1]; // The second main parameter
PrintStream outs = new PrintStream(new FileOutputStream(outFile));
```

You can apply method `print` or `println` to the `PrintStream` object `outs`. Variable `outFile` references the same string as `args[1]`, the string passed as the second parameter to method `main`. This should be the external name of a file. When object `outs` is created, the stream it references is always empty. Any information previously stored in the file whose name is passed to `args[1]` will be lost.

## Passing Arguments to Method `main`

Earlier we set the variable `fileName` to reference the same string as `args[0]`, the first parameter for method `main`. You must specify the `main` method parameters before you run an application. When you are using the JDK and therefore running your applications from the console

command line (such as an “MS-DOS Prompt” window in Windows), you list the parameters after the name of the class you are executing. For example, if you are running application `FileTest.java` with parameters `indata.txt` and `output.txt`, use the command line

```
java FileTest indata.txt output.txt
```

When you are using an IDE, you can also specify parameters before running an application. Netbeans provides an option in the Run submenu under the Properties menu. Selecting this option brings up a window that has a text field with label Arguments. You can type the parameters into this text field.

## Closing Streams

After processing streams, you must disconnect them from the application. The statement

```
outs.close();
```

does this for stream `outs`. Data to be written to a file is stored in an *output buffer* in memory before it is written to the disk. The `close` statement ensures that any data in the output buffer is written to disk.



### PITFALL

#### Neglecting to Close an Output Stream

If you do not close a stream, it is not considered an error. However, you may find that not all the information written to the stream is actually stored in the corresponding disk file unless you close it.

## Try with Resources

Properly closing the I/O streams can be tricky, especially if more than one stream is involved. One or both may not have been created before an exception was thrown. Therefore, the code to close the streams must be in a finally block (discussed below) and needs to check whether the stream was opened and whether an exception was thrown. Java 7 introduced the try-with-resources syntax to automatically close any streams when the block is exited either normally or through an exception. The syntax is as follows:

```
try (// Declaration of input/output streams) {
    // statements that perform iI/O processing
} catch (IOException ex) {
    ex.printStackTrace();
    System.exit(1);
}
```

## A Complete File-Processing Application

We put all these pieces together in this example. In Listing A.5, the `main` method in class `FileTest` consists of a try-with-resources sequence. The `try` block creates two `BufferedReader` objects: `ins1` (associated with data `file1`) and `ins2` (associated with data `file2`). It also creates a `PrintStream` object `outs` (associated with an output file). The `while` loop invokes method `readLine` to read data lines from stream `ins1`, storing the information read from each line in the `String` object `first`. When the end of the data file is reached, `first` will contain `null`. If `first` is not `null` (the normal situation when a data line is read), a line is read into `String` object `second`. If `second` is `null`, the loop is exited via the `break` statement.

Otherwise, the contents of second are appended to first, and the new string is written to the output file.

```
outs.println(first + " " + second); // Append and write
```

This process continues until the end of the data file is reached, loop exit occurs, and the files are closed.

.....

### **LISTING A.5**

Class FileTest

```
public class FileTest {
    /**
     * Reads a line from an one input file and then from a second input file.
     * Concatenates the two lines and writes them to an output file.
     * Does this until all input lines have been read from one of the files.
     *
     * @param args The command line arguments
     *             args[0] The first input file name
     *             args[1] The second input file name
     *             args[2] The output file name
     */
    public static void main(String[] args) {
        if (args.length < 3) {
            System.err.println("Please provide three file names");
            System.exit(1);
        }
        try {
            BufferedReader ins1 = new BufferedReader(new FileReader(args[0]));
            BufferedReader ins2 = new BufferedReader(new FileReader(args[1]));
            PrintStream outs = new PrintStream(new FileOutputStream(args[2]));
        } {
            // Reads words and writes them to the output file until done.
            String first;
            while ((first = ins1.readLine()) != null) { // Read from file1
                String second = ins2.readLine(); // Read from file2
                if (second == null) {
                    break;
                }
                // Append and write
                outs.println(first + " " + second);
            }
        } catch (FileNotFoundException ex1) {
            System.err.println(ex1);
        } catch (IOException ex2) {
            System.err.println(ex2);
        }
    }
}
```

If file1.txt contains the three lines:

```
apple
cat
John
```

and if file2.txt contains the three lines:

```
butter
dog
Doe
```

then the output file will contain:

```
apple, butter
cat, dog
John, Doe
```

## Class InputStream and Character Codes (Optional)

Data is stored on external media or transmitted over the network as a sequence of bytes. The abstract class `InputStream` declares the abstract method `read` as follows:

```
/***
 * Reads the next byte of data. The value is returned as an
 * int in the range 0 to 255. If no data is available because
 * the end of stream has been reached, then -1 is returned.
 */
public abstract int read();
```

Class `FileInputStream` is a subclass of `InputStream` defined to read bytes from a file. Recall from Section A.2 that the Java `char` type is a 16-bit value used to represent the Unicode characters. The abstract class `Reader` defines the method `read` as follows:

```
/***
 * Reads a single character. The character read is returned as
 * an int in the range 0 to 65535, or -1 if the end of stream
 * has been reached.
 */
public int read();
```

Associated with the `Reader` is an `InputStream`. It is the Reader's responsibility to convert the bytes into characters. For bytes in the range of 0–127, this is straightforward (see Table A.2). The class `InputStreamReader` is the bridge between byte streams and character streams. The `InputStreamReader` uses the default character set to perform the conversion from byte values in the range of 128–255 into corresponding Unicode characters. We discuss character codes in the next section.

The class `FileReader`, which we used above, is a simple wrapper class that is effectively the following:

```
class FileReader extends Reader {
    private Reader r;
    public FileReader(File file) {
        r = new InputStreamReader(new FileInputStream(file));
    }
    public int read() throws IOException {
        return r.read();
    }
}
```

Similarly, when we construct a `Scanner` object from a file, the constructor creates a `FileInputStream` and then wraps it in an `InputStreamReader`.

## The Default Character Coding (Optional)

The default character set depends on the native operating system and the locale. On a Windows computer in the United States, the default character set is known as Cp1252 and is shown in Table A.18.

Using the default character encoding is fine if files are read and written by computers with the same operating system and at the same locale. However, if one wrote a file using the Cp1252 character encoding and then tried to read this file on a Macintosh computer that uses MacRoman as its default, some characters would have different interpretations. For example, a byte value of x80 representing the euro symbol € (\u20AC) in Cp1252 would be interpreted as the character

**TABLE A.18**

Code Table for the Cp1252 Character Set

|   | <b>8</b>  | <b>9</b>  | <b>A</b>   | <b>B</b>  | <b>C</b>  | <b>D</b>  | <b>E</b>  | <b>F</b>  |
|---|-----------|-----------|------------|-----------|-----------|-----------|-----------|-----------|
| 0 | €<br>20AC |           |            | °<br>00B0 | À<br>00C0 | Đ<br>00D0 | à<br>00E0 | ð<br>00F0 |
| 1 | ,         | i<br>2018 | í<br>00A1  | ±<br>00B1 | Á<br>00C1 | Ñ<br>00D1 | á<br>00E1 | ñ<br>00F1 |
| 2 | ,         | ,         | ¢<br>2014  | ¤<br>2019 | ž<br>00B2 | Â<br>00C2 | ò<br>00D2 | â<br>00E2 |
| 3 | f<br>0192 | ”<br>201C | £<br>00A3  | ³<br>00B3 | Ã<br>00C3 | Ó<br>00D3 | ã<br>00E3 | ó<br>00F3 |
| 4 | „<br>201E | ”<br>201D | ¤<br>00A4  | ‘<br>00B4 | Ä<br>00C4 | Ô<br>00D4 | ä<br>00E4 | ô<br>00F4 |
| 5 | …<br>2126 | •<br>2022 | ¥<br>00A5  | µ<br>00B5 | Å<br>00C5 | Õ<br>00D5 | å<br>00E5 | õ<br>00F5 |
| 6 | †<br>2020 | –<br>2013 | <br>00A6   | ¶<br>00B6 | Æ<br>00C6 | Ö<br>00D6 | æ<br>00E6 | ö<br>00F6 |
| 7 | ‡<br>2021 | –<br>2014 | §<br>00A7  | ·<br>00B8 | Ç<br>00C7 | ×<br>00D7 | ç<br>00E7 | ÷<br>00F7 |
| 8 | ^<br>02c6 | ~<br>02DC | ..<br>00A8 | ,<br>00B8 | È<br>00C8 | Ø<br>00D8 | è<br>00E8 | ø<br>00F8 |
| 9 | %<br>2030 | ™<br>2122 | ©<br>00A9  | ¹<br>00B9 | É<br>00C9 | Ù<br>00D9 | é<br>00E9 | ù<br>00F9 |
| A | Š<br>0160 | š<br>0161 | ¤<br>00AA  | º<br>00BA | Ê<br>00CA | Ú<br>00DA | ê<br>00EA | ú<br>00FA |
| B | <<br>2039 | ><br>203A | «<br>00AB  | »<br>00BB | Ë<br>00CB | Û<br>00DB | ë<br>00EB | û<br>00FB |
| C | Œ<br>0152 | œ<br>0153 | ¬<br>00AC  | ¼<br>00BC | Ì<br>00CC | Ü<br>00DC | ì<br>00EC | ü<br>00FC |
| D |           |           | –<br>00AD  | ½<br>00BD | Í<br>00CD | Ý<br>00DD | í<br>00ED | ý<br>00FD |
| E | Ž<br>017D | ž<br>017E | ®<br>00AE  | ¾<br>00BE | Î<br>00CE | Þ<br>00DE | î<br>00EE | þ<br>00FE |
| F |           | ÿ<br>0178 | -<br>00AF  | ¿<br>00BF | Ï<br>00CF | ß<br>00DF | ï<br>00EF | ÿ<br>00FF |

À (\U00C4) on the Mac. If the same file read on a Windows computer in Poland, which uses the default character coding Cp1250, the euro character would be read correctly, but a byte value of xf8 representing the ø character (\U00F8) would be interpreted as ſ (\U0159).

## UTF-8 (Optional)

The Universal Character Set Transform Format-8 bit (UTF-8) is a character encoding that can encode all possible Unicode characters. UTF-8 is used on most web pages and is the preferred coding scheme for file interchange. A variable number of bytes are used as shown in Table A.19. Unicode characters are called code points and are represented by U+xxxx, where xxxx is the

**TABLE A.19**

UTF-8 Coding

| Bits in Code Point | First Code Point | Last Code Point | Number of Bytes | First Byte | Second Byte | Third Byte | Fourth Byte |
|--------------------|------------------|-----------------|-----------------|------------|-------------|------------|-------------|
| 7                  | U+0000           | U+007F          | 1               | 0XXXXXX    |             |            |             |
| 11                 | U+0080           | U+07FF          | 2               | 110XXXX    | 10XXXXXX    |            |             |
| 16                 | U+0800           | U+FFFF          | 3               | 1110XXX    | 10XXXXXX    | 10XXXXXX   |             |
| 21                 | U+10000          | U+1FFFF         | 4               | 11110XXX   | 10XXXXXX    | 10XXXXXX   | 10XXXXXX    |

hexadecimal value. Note that Unicode contains more than 65,535 code points. Java internally uses UTF-16 that represents the code points U+0000 to U+D7FF and U+E000 to U+FFFF exactly, and represents the remaining code points as two adjacent Java characters.

Observe that the first 128 characters are represented by a single byte. The first byte of a multibyte sequence begins with a number of 1 bits that indicates how many bytes are in the sequence. The xs in the table represent the individual bits in the code point. For a code point that requires 11 bits, the high-order 5 bits follow the 110 in the first byte, and the remaining 6 bits follow the 10 in the second byte. For a code point that requires 16 bits, the first 4 bits follow the 1110 in the first byte and the remaining 12 bits are placed in the second and third bytes following the 10. For example, the euro character (U+20AC) requires 16 bits (0010 0000 1010 1100). Splitting these 16 bits into a 4-bit group followed by two 6-bit groups gives 0010 000010 101100. Thus, the UTF-8 encoding is the byte sequence 11100010 10000010 10101100 or in hexadecimal C2 82 AC.

## Specifying a Character Encoding (Optional)

The `FileReader` class shown earlier is a convenience class that creates a `Reader` that uses the default character encoding. To use a specified encoding, such as UTF-8, we need to use a `InputStream` to read the data and an `InputStreamReader` to perform the character coding:

```
BufferedReader ins = new BufferedReader(new InputStreamReader
    (new FileInputStream(args[0]), "UTF-8"));
```

## Input/Output Using Class JOptionPane

So far we have discussed console input and input from files. Many of the interactive programs you see use dialog windows for input and message windows for output (Table A.20). Class `JOptionPane` (part of the Swing package) enables this type of interaction. To use class `JOptionPane`, you should place the line

```
import javax.swing.JOptionPane; // Import class JOptionPane
```

before the class definition in your source file.

**TABLE A.20**Methods from Class `JOptionPane`

| Method                                                                    | Behavior                                                                                                                |
|---------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| <code>static String showInputDialog(String prompt)</code>                 | Displays a dialog window that shows the argument as a prompt and returns the character sequence typed by the user       |
| <code>static void showMessageDialog(Object parent, String message)</code> | Displays a window containing a message string (the second argument) inside the specified container (the first argument) |

**EXAMPLE A.22** The statement

```
String name = JOptionPane.showInputDialog("Enter your name");
```

displays the dialog window shown on the left in Figure A.15. After the OK button is clicked or the Enter key is pressed, variable `name` references a `String` object that stores the character sequence "Jane Doe". If Cancel is clicked, variable `name` stores `null`. The statement

```
JOptionPane.showMessageDialog(null, "Your name is " + name);
```

displays the message window shown on the right in Figure A.15. The first argument specifies the parent container in which this window will be placed. When the argument is `null`, the dialog window is placed in the middle of the screen (the window in which the program is executing).

**FIGURE A.15**

A Dialog Window (Left) and Message Window (Right)

**Converting Numeric Strings to Numbers**

A dialog window always returns a reference to a string. How can we convert numeric strings to numbers? Fortunately, as shown in Table A.21, class `Integer` provides a static method, `parseInt`, for converting strings consisting only of digit characters to numbers, and class `Double` provides a static method, `parseDouble`, for converting strings consisting of the characters for a real number (or integer) to a type `double` value.

**TABLE A.21**

Methods for Converting Strings to Numbers

| Method                                         | Behavior                                                                                                                                                                         |
|------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>static int parseInt(String)</code>       | Returns an <code>int</code> value corresponding to its argument string. A <code>NumberFormatException</code> occurs if its argument string contains characters other than digits |
| <code>static double parseDouble(String)</code> | Returns a <code>double</code> value corresponding to its argument string. A <code>NumberFormatException</code> occurs if its argument string does not represent a real number    |

**EXAMPLE A.23** The next pair of statements stores a type `int` value in `numStu` if `answer` references a `String` object that contains digit characters only.

```
String answer = JOptionPane.showInputDialog("Enter number of students");
int numStu = Integer.parseInt(answer);
```



## PITFALL

### Using Nonnumeric Strings with parseInt, parseDouble

If you pass to `parseInt` a String that contains characters that are not digit characters, you will get a `NumberFormatException` error. If you pass to `parseDouble` a String that contains characters that can't be in a number, you will also get a `NumberFormatException` error.

## GUI Menus Using Method `showOptionDialog`

Another useful method from class `JOptionPane` is method `showOptionDialog`. This method displays a menu of choices with a button for each choice (see Figure A.16). When a button is clicked, the method returns the index of the button pressed (0 for the first button, etc.). The index value can be used in a `switch` statement to select an alternative.

### EXAMPLE A.24

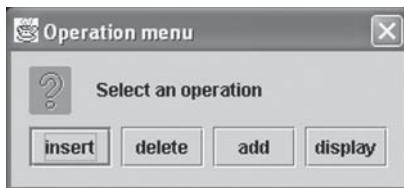
The statements

```
String[] choices = {"insert", "delete", "add", "display"};
int selection = JOptionPane.showOptionDialog(null,
    "Select an operation",
    "Operation menu",
    JOptionPane.YES_NO_CANCEL_OPTION,
    JOptionPane.QUESTION_MESSAGE, null,
    choices, choices[0]);
```

display the menu shown in Figure A.16. The array `choices` defines the button labels. After a button is clicked, the value stored in `selection` will be the index of that button, an integer from 0 to 3.

**FIGURE A.16**

Displaying a Menu



## EXERCISES FOR SECTION A.10

### SELF-CHECK

1. Show the statements that would be required, using the console for input, to read and store the data for a `Person` object prior to calling the constructor with four parameters.
2. Answer Exercise 1 above using a data file instead of the console.
3. What would happen if the output file name matched the name of a file already saved on disk? What could happen if the user forgets to close an output file?
4. When does the `catch` block in a `try-catch` sequence execute?

5. Show the statements that would be required, using `JOptionPane`, to read and store the data for a `Person` object prior to calling the constructor with four parameters.

### PROGRAMMING

1. Write a method for class `Person` that reads the data for a single employee from a `BufferedReader` object (the method argument). Assume there is one data item per line.
2. Write a method for class `Company` that reads the data for the `employees` array. This method should call the one needed for Programming Exercise 1.
3. Write a `main` method that reads the data for two `Person` objects, creates the objects, and displays the objects and a message indicating whether they represent the same `Person`.



## A.11 Catching Exceptions

When an exception is thrown, the normal sequence of execution is interrupted because the execution of subsequent statements would most likely be erroneous. The default behavior is for the JVM to halt program execution and to display an error message indicating which type of exception was thrown and where in the program it was thrown. The JVM also displays a stack trace that shows the sequence of method calls, starting at the method that threw the exception, then showing the method that called that method, and so on, all the way back to the `main` method.

The stack trace in Figure A.17 shows that an exception occurred during the execution of class `ExceptionDemo`. The exception was a `NullPointerException`. The exception was thrown in method `doSomethingElse` (at line 18 of class `ExceptionDemo`). Method `doSomethingElse` was called from method `doSomething` (at line 13). Method `doSomething` was called from method `main` (at line 7).

**FIGURE A.17**

Example of a Stack Trace for an Uncaught Exception

```
Command Prompt
Exception in thread "main" java.lang.NullPointerException
  at ExceptionDemo.doSomethingElse(ExceptionDemo.java:18)
  at ExceptionDemo.doSomething(ExceptionDemo.java:13)
  at ExceptionDemo.main(ExceptionDemo.java:7)
```

## Catching and Handling Exceptions

In the next few subsections, you will see how to avoid the default behavior when you write a method that may throw an exception. You will also see why it is advantageous to do this.

### The Try–Catch–Finally Sequence

One way to avoid uncaught exceptions is to write a try–catch sequence that actually “catches” an exception and “handles it” rather than relying on the JVM to do this.

```
try {
    // Statements that perform file-processing operations.
}
```

```

catch (IOException ex) {
    ex.printStackTrace(); // Display stack trace.
    System.exit(1); // Exit with an error indication.
}

```

If all statements in the `try` block execute without error, the `catch` block is skipped. If an `IOException` occurs, the `try` block is exited and the `catch` block executes. This particular `catch` block simply displays the sequence of method calls that led to the error (starting with the most recent one and working backward) in the console window (`System.err` – the standard error stream) and then exits with an error indication.

Although this handles the exception, it basically duplicates the default behavior for uncaught exceptions. Next, we show you how to use the `try-catch` sequence to recover from errors and continue the execution of your program.

### **Handling Exceptions to Recover from Errors**

In addition to reporting errors, exceptions provide us with the opportunity to recover from errors. One common source of exceptions is user input. For example, the method `JOptionPane.showInputDialog` displays a dialog window and allows the user to enter input. After the user enters input and presses the Enter key, the method will return a string containing the input characters. If we are expecting an integer value, we need to convert this string to an integer. The conversion is performed by method `parseInt`, which can cause a `NumberFormatException` to be thrown.

**EXAMPLE A.25** Method `readInt` (Listing A.6) returns the integer value that was typed into a dialog window by the program user. The method argument is the dialog window prompt.

The `while` loop repetition condition (`true`) ensures that the `try-catch` sequence will execute “forever” or until the user enters a correct data item. The statements

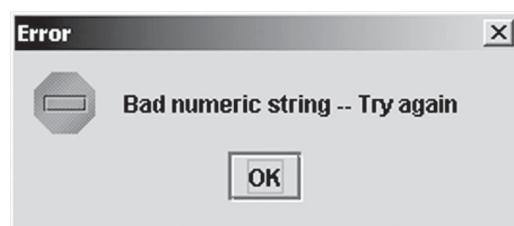
```

String numStr = JOptionPane.showInputDialog(prompt);
return Integer.parseInt(numStr);

```

display the dialog window and return an integer value if `numStr` contains only digit characters. If not, a `NumberFormatException` is thrown, which is handled by the `catch` clause. The `catch` block displays an error message window by calling `JOptionPane.showMessageDialog`. The last argument, `JOptionPane.ERROR_MESSAGE`, causes a window with a stop sign to appear (see Figure A.18). After closing this window, the user has another opportunity to enter a valid numeric string.

**FIGURE A.18**  
Bad Numeric String Error



### **LISTING A.6**

Method `readInt`

```

/** Method to return an integer data value.
@param prompt Message
@return The data value read as an int
*/

```

```

public static int readInt(String prompt) {
    while (true) { // Loop until valid number is read.
        try {
            String numStr = JOptionPane.showInputDialog(prompt);
            return Integer.parseInt(numStr);
        }
        catch (NumberFormatException ex) {
            JOptionPane.showMessageDialog(
                null,
                "Bad numeric string - Try again",
                "Error", JOptionPane.ERROR_MESSAGE);
        }
    }
}

```

---

## The try Block

The syntax for the try block is as follows:

```

try {
    Code that may throw an exception
}

```

## The catch Clauses and Blocks

Exceptions are caught by what is appropriately called a catch clause. A catch clause resembles a method and has the following syntax.

```

catch (ExceptionClass exceptionArgument) {
    Code to handle the exception
}

```

The code within the brackets is called the catch block. The catch clause(s) must follow a try block. There may be multiple catch clauses, one for each exception class that you wish to handle.

An exception matches a catch clause if the type of the exception is the same as the argument of the catch clause or is a subclass of the argument type. When an exception is thrown from within a try block, the associated catch clause(s) is (are) examined to see whether there is a match in the exception class for any catch clause. If so, that catch block executes. If not, a search is made back through the chain of method calls to see whether any of them occurs in a try block with an appropriate catch. If so, that catch block executes. We illustrate this next.

---

**EXAMPLE A.26** The body of method `readIntTwo` below contains just the statements in the try block of method `readInt` (Listing A.6), but the catch clause is omitted.

```

public static int readIntTwo(String prompt) {
    String numStr = JOptionPane.showInputDialog(prompt);
    return Integer.parseInt(numStr);
}

```

In this case, if a `NumberFormatException` is thrown by method `parseInt`, method `readIntTwo` will not be able to handle it, so `readIntTwo` is exited and a search is made for an appropriate catch clause in the caller of method `readIntTwo`. If method `readIntTwo` is called to read a value into `age` by the following try block:

```

try {
    // Enter a value for age.
    age = readIntTwo("Enter your age");
}

```

```

} catch (Exception ex) {
    System.err.println("Error occurred in call to readIntTwo");
    age = DEFAULT_AGE;
}

```

the catch clause will handle the `NumberFormatException` (because `NumberFormatException` is a subclass of `Exception`) and assign the value of `DEFAULT_AGE` to `age`. It will also display an error message on the console, and program execution will continue with the statement that follows this try-catch sequence.

---

Note that a `catch` clause is like a method, and the `catch` block is like a method body. The term `catch` clause refers to both the header and the body, whereas the term `catch` block refers to the body alone. This distinction is not generally that important, and you may see the terms used interchangeably in other texts and documentation.

---

**EXAMPLE A.27** `EOFException` is a subclass of `IOException`. The two `catch` clauses in the following code must appear in the sequence shown to avoid a `catch is unreachable` syntax error. The first `catch` block handles an `EOFException` that occurs when all the data in the file was processed (not an error). This `catch` block tells the program user so and then exits the program normally (`System.exit(0)`). The second `catch` clause processes any other I/O exception by calling method `printStackTrace` to display a stack trace like the one shown in Figure A.17. It exits the program with an error indication (`System.exit(1)`). (Note: The method `printStackTrace` is defined in the `Throwable` class and is inherited by all exception objects.)

```

catch (EOFException ex) {
    System.out.print("End of file reached ")
    System.out.println(" - processing complete");
    System.exit(0);
}
catch (IOException ex) {
    System.err.println("Input/Output Error:");
    ex.printStackTrace();
    System.exit(1);
}

```

---



## PITFALL

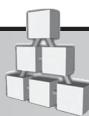
### Unreachable catch Block

Note that only the `catch` block within the first `catch` clause having an appropriate exception class executes. All other `catch` blocks are skipped. If a `catch` clause exception type is a subclass of an exception type in an earlier `catch` clause, the `catch` block in the later `catch` clause cannot execute, so the Java compiler will display a `catch is unreachable` syntax error. To correct this error, switch the order of the `catch` clauses so that the `catch` clause whose exception class is the subclass comes first.

### The `finally` Block

When an exception is thrown, the flow of execution is suspended and continues at the appropriate `catch` clause. There is no return to the `try` block. Instead, processing continues at the first statement after all of the `catch` clauses associated with the `try` from which the exception was thrown.

In some situations, allowing the program to continue after an exception without executing all the statements in the `try` block could cause problems. For example, if some calculations needed to be performed before returning from the method, these calculations would have to be duplicated in both the `try` block and every `catch` clause. To avoid such a duplication (which can be error-prone), the `finally` block can be used. The code in the `finally` block is executed either after the `try` block is exited or after a `catch` clause is exited (if one is executed). The `finally` block is optional. We show an example in the following syntax summary.



## SYNTAX try-catch-finally Sequence

### FORM:

```
try {
    Statements that may throw an exception
}
catch (ExceptionClass1 exceptionArgument1) {
    Statements to process ExceptionClass1
}
catch (ExceptionClass2 exceptionArgument2) {
    Statements to process ExceptionClass2
}
catch (ExceptionClassn exceptionArgumentn) {
    Statements to process ExceptionClassn
}
finally {
    Statements to be executed after the try block or the exception block exits
}
```

### EXAMPLE:

```
try {
    String sizeStr = JOptionPane.showInputDialog("Enter new size");
    size = Integer.parseInt(sizeStr);
}
catch (NumberFormatException ex) {
    size = DEFAULT_SIZE; // Use default value if input error.
}
finally {
    if (size > MAX_CAPACITY)
        size = MAX_CAPACITY;
}
```

### MEANING:

The statements in the `try` block execute through to completion unless an exception is thrown. If there is a `catch` clause to handle the exception, its `catch` block executes to completion. After the `try` block or `catch` block executes, the `finally` block executes to completion.

If there is no `catch` clause to handle the exception thrown in the `try` block, the `finally` block is executed, and then the exception is passed up the call chain until either it is caught by some other method in the call chain or it is processed by the JVM as an uncaught exception.

## Reporting the Error and Exiting

There are many cases in which an exception is thrown but there is no obvious way to recover. For example, reading from a file or the system console can result in an `IOException` being thrown. In this case, the catch clause should print the stack trace and exit, as follows:

```
catch (IOException ex) {
    ex.printStackTrace();
    System.exit(1);
}
```

The method call `System.exit(1)` causes a return to the operating system with an error indication.

## Checked and Unchecked Exceptions

There are two categories of exceptions: checked and unchecked. A *checked exception* is caused by an error that is beyond the programmer's control, such as an input/output error (`IOException`). An *unchecked exception* is caused by a program error. An example is an `IndexOutOfBoundsException`. Checked exceptions must always be handled in some way (discussed next). There is no requirement to handle unchecked exceptions.



### PITFALL

#### Ignoring Exceptions

Exceptions are designed to make the programmer aware of possible error conditions and to provide a way to handle them. Some programmers do not appreciate this feature and do the following:

```
catch (Exception e){}
```

Although this clause is syntactically correct and eliminates a lot of pesky error messages, it is almost always a bad idea. The program continues execution after the try–catch sequence with no indication that there was a problem. The statement that caused the exception to be thrown did not execute properly. The statements that follow it in the try block were not executed at all. The program will have hidden defects that will make its users very unhappy and could have even more serious consequences.



### PROGRAM STYLE

#### Using Exceptions to Enable Straightforward Code

In computer languages that did not provide exceptions, programmers had to incorporate error-checking logic throughout their code to check for many possibilities, some of which were of low probability. The result was sometimes messy, as follows:

```
Step A
if (Step A successful) {
    Step B
    if (Step B successful) {
        Step C
    } else {
        Report error in Step B
    }
}
```

```

        Cleanup after Step A
    }
} else {
    Report error in Step A
}

```

With exceptions this becomes much cleaner, as follows:

```

try {
    Step A
    Step B
    Step C
} catch (exception indicating Step B failed) {
    Report error in step B
    Cleanup after step A
} catch (exception indicating Step A failed) {
    Report error in step A
}

```

## EXERCISES FOR SECTION A.11

### SELF-CHECK

1. Assume that method `main` calls method `first` at line 10 of class `MyApp`, method `first` calls method `second` at line 10 of class `Others`, and method `second` calls method `parseInt` at line 20 of class `Other`. These calls result in a `NumberFormatException` at line 430 of class `Integer`. Show the stack trace.
2. Assume that you have `catch` clauses for exception classes `Exception`, `NumberFormatException`, and `RuntimeException` following a `try` block. Show the required sequence of `catch` clauses.

### PROGRAMMING

1. For the `try` block

```

try {
    numStr = in.readLine();
    num = Integer.parseInt(numStr);
    average = total / num;
}

```

write a `try-catch-finally` sequence with `catch` clauses for `ArithmetiException`, `NumberFormatException`, and `IOException`. For class `ArithmetiException`, set `average` to zero and display an error message indicating the kind of exception, display the stack trace, and exit with an error indication. After exiting the `try` block or the `catch` block for `ArithmetiException`, display the message "That's all folks" in the `finally` block.

## A.12 Throwing Exceptions

In the previous section, we showed how to catch and handle exceptions using the `try-catch` sequence. As an alternative to catching an exception in a lower-level method, you can allow it to be caught and handled by a higher-level method. You can do this in one of two ways:

1. You declare that the lower-level method may throw a checked exception by adding a `throws` clause to the method header.

2. You throw the exception in the lower-level method, using a `throw` statement, when the exception is detected.

## The throws Clause

The next example illustrates the use of the `throws` clause to declare that a method may throw a particular kind of checked exception. This is a useful approach if a higher-level module already contains a `catch` clause for this exception type. If you don't use the `throws` clause, you must duplicate the `catch` clause in the lower-level method to avoid an unreported exception syntax error.

**EXAMPLE A.28** Method `readData` reads two strings from the `BufferedReader` object `console` associated with `System.in` (the system console) and stores them in data fields `firstName` and `lastName`. Each call to method `readLine` may throw a checked `IOException`, so method `readData` cannot compile without the `throws` clause. If you omit it, you will get the syntax error `unreported exception: Java.io.IOException; must be caught or declared to be thrown.`

```
public void readData() throws IOException {
    BufferedReader console = new BufferedReader(
        new InputStreamReader(System.in));
    System.out.print("Enter first name: ");
    firstName = console.readLine();
    System.out.print("Enter last name: ");
    lastName = console.readLine();
}
```

If method `readData` is called by method `setNewPerson`, method `setNewPerson` must have a `catch` block that handles exceptions of type `IOException`.

```
public void setNewPerson() {
    try {
        readData();
        // Process the data read.
        .
        .
    } catch (IOException iOEx) {
        System.err.println("Call to readLine failed in readData");
        iOEx.printStackTrace();
        System.exit(1);
    }
}
```

If a method can throw more than one exception type, list them all after `throws` with comma delimiters. You will get an unreported exception syntax error if you omit any checked exception type. The compiler verifies that all class names listed are exception classes.



### PROGRAM STYLE

#### Using Javadoc @throws for Unchecked Exceptions

Listing unchecked exceptions in the `throws` clause is legal syntax but is considered poor programming practice. Instead you should use the Javadoc `@throws` tag to document any unchecked exceptions that may reasonably be expected to occur but are not caught in the method.

## The throw Statement

You can use a `throw` statement in a lower-level method to indicate that an error condition has been detected. When the `throw` statement executes, the lower-level method stops executing immediately, and the JVM begins the search for an exception handler as described earlier. This approach is usually taken if the exception is unchecked and is likely to be caught in a higher-level method. If the exception thrown is a checked exception, this exception must be declared in the `throws` clause of the method containing the `throw` statement.

**EXAMPLE A.29** The method `addOrChangeEntry` takes two `String` parameters: `name` and `number`. The `number` parameter is intended to represent a valid phone number. Therefore, we wish to validate its format to ensure that only validly formatted numbers are entered. Assuming that we have a method `isPhoneNumberFormat` that checks for a valid phone number, we could code the `addOrChangeEntry` method as follows:

```
public String addOrChangeEntry(String name, String number) {
    if (!isPhoneNumberFormat(number)) {
        throw new IllegalArgumentException ("Invalid phone number: " + number);
    }
    // Add/change the number.
    ...
}
```

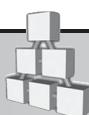
The `throw` statement creates and throws a new `IllegalArgumentException`, which can be handled farther back in the call chain or by the JVM if it is uncaught. The constructor argument ("Invalid phone number: " + `number`) for the new exception object is a message that describes the cause of the error.

If we call this method using the following try-catch sequence:

```
try {
    addOrChangeEntry(myName, myNumber);
} catch (IllegalArgumentException ex) {
    System.err.println(ex.getMessage());
}
```

and `myNumber` references the string "1xx1", which is not a valid phone number, the console output would be

```
Invalid phone number: 1xx1
```



## SYNTAX throw Statement

### FORM:

```
throw new ExceptionClass();
throw new ExceptionClass(detailMessage);
```

### EXAMPLE:

```
throw new FileNotFoundException("File " + fileSource + " not found");
```

### MEANING:

A new exception of type `ExceptionClass` is created and thrown. The optional `String` parameter `detailMessage` is used to specify an error message associated with this exception. If the higher-level method that catches this exception has the `catch` clause

```

        catch (ExceptionClass ex) {
            System.err.println(ex.getMessage());
            System.exit(1);
    }

```

the *detailMessage* will be written to the system error stream before system exit occurs.

---

**EXAMPLE A.30** Listing A.7 shows a second method `readInt` that has three arguments. As in method `readInt` in Listing A.6, the first argument is a prompt. The second and third arguments represent the end points for a range of integer numbers. The method returns the first integer value entered by the program user that is between the end points.

The `if` statement tests whether the end points define an empty range (`minN > maxN`). If so, the statement

```

throw new IllegalArgumentException("In readInt, minN " + minN
                                + " not <= maxN " + maxN);

```

throws an `IllegalArgumentException`, creating an instance of this class. The message passed to the constructor gives the cause of the exception. This message would be displayed by `printStackTrace` or returned by `getMessage` or `toString`.

If the range is not empty, the `while` loop executes. Its repetition condition (`!inRange`) is true as long as the user has not yet entered a value that is within the range defined by the end points. The `try` block displays a dialog window with a prompt that shows the valid range of values. The statement

```
inRange = (minN <= n && n <= maxN);
```

sets `inRange` to true when the value assigned to `n` is within this range. If so, the loop is exited and this value is returned. However, if the user enters a string that is not numeric, the `catch` block displays an error message. If the string is not numeric or its value is not in range, `inRange` remains false, so (`!inRange`) is true and the loop repeats, giving the user another opportunity to enter a valid number.

.....  
**LISTING A.7**

Method `readInt` (part of `MyInput.java`) with Three Parameters

```

/** Method to return an integer data value between two
 * specified end points.
 * @pre: minN <= maxN.
 * @param prompt Message
 * @param minN Smallest value in range
 * @param maxN Largest value in range
 * @throws IllegalArgumentException
 * @return The first data value that is in range
 */
public static int readInt(String prompt, int minN, int maxN) {
    if (minN > maxN) {
        throw new IllegalArgumentException("In readInt, minN " + minN
   + " not <= maxN " + maxN);
    }
    // Arguments are valid, read a number.
    boolean inRange = false; // Assume no valid number read.
    int n = 0;
    while (!inRange) { // Repeat until valid number read.

```

```

try {
    String line = JOptionPane.showInputDialog(
        prompt + "\nEnter an integer between "
        + minN + " and " + maxN);
    n = Integer.parseInt(line);
    inRange = (minN <= n && n <= maxN);
} catch (NumberFormatException ex) {
    JOptionPane.showMessageDialog(
        null,
        "Bad numeric string – Try again",
        "Error", JOptionPane.ERROR_MESSAGE);
}
} // End while
return n; // n is in range
}

```

---



## PROGRAM STYLE

### Reasons for Throwing Exceptions

You might wonder what is gained by intentionally throwing an exception. If it is not caught farther back in the call chain, it will go uncaught and will cause your program to terminate. However, in the examples in this section, it would not make any sense to continue with either an empty range (in `readInt`) or an invalid phone number (in `addOrChangeEntry`). In fact, the loop in method `readInt` would execute forever if the range of acceptable values was empty. Because the boundary parameters `minN` and `maxN` are defined in a higher-level method, it would also make no sense to try to get new values in `readInt`. However, if the exception is passed back and caught at the point where the boundary points are defined, the programmer can get new boundary values and call method `readInt` again instead of terminating the program.

### Catching versus Throwing Exceptions

You can always avoid handling exceptions where they occur by declaring that they are thrown or by throwing them and letting them be handled farther back in the call chain. In general, though, it is better to handle an exception where it occurs rather than to pass it back. This gives you the opportunity to recover from the error and to continue on with the execution of the current method. We did this, for example, for `NumberFormatExceptions` in both `readInt` methods (see Listings A.6 and A.7). If an error is a nonrecoverable error, however, and is also likely to occur farther back in the call chain, you might as well allow the exception to be handled at the farthest point back in the call chain rather than duplicate the error-handling code in several methods. We recommend the following guidelines:

- If an exception is recoverable in the current method, handle the exception in the current method.
- If a checked exception is likely to be caught in a higher-level method, declare that it can occur using a `throws` clause, and use a `@throws` tag to document this in the Javadoc comment for this method.
- If an unchecked exception is likely to be caught in a higher-level method, use a `@throws` tag to document this fact in the Javadoc comment for the method. However, it is not necessary to use a `throws` clause with unchecked exceptions.

## EXERCISES FOR SECTION A.12

### SELF-CHECK

1. Explain the difference between the `throws` clause and the `throw` statement.
2. When would it be better to declare an exception rather than catch it in a method?
3. When would it be better to throw an exception rather than catch it in a method?
4. What kind of exceptions should appear in a `throws` clause?
5. For the following situations, indicate whether it would be better to catch an exception, declare an exception, or throw an exception in the lower-level method. Explain your answer and show the code required for the lower-level method to do it.
  - a. A lower-level method contains a call to method `readLine`; the higher-level method that calls it contains a `catch` clause for class `IOException`.
  - b. A method contains a call to method `readLine` to enter a value that is passed as an argument to a lower-level method. The lower-level method's argument must be a positive number.
  - c. A lower-level method contains a call to method `readLine`, but the higher-level method that calls it does not have a `catch` clause for class `IOException`.
  - d. A lower-level method reads a data string and converts it to type `int`. The higher-level method contains a `catch` clause for class `NumberFormatException`.
  - e. A lower-level method detects an unrecoverable error that is an unchecked exception.

### PROGRAMMING

1. The syntax display for the `throw` statement had the following example:
 

```
throw new FileNotFoundException("File " + fileSource + " not found");
```

 Write a `catch` clause for a method farther back in the call chain that handles this exception.
2. Method `setElementOfX` shown below validates that the parameters `index` and `val` are in bounds before accessing array `x`. Rewrite this method so that it throws exceptions during array access if `val` is out of bounds if `index` is out of bounds. Pass an appropriate detail message to the new exception object. Your modified method should be type `void` because there is no longer a reason to return a `boolean` error indicator. Show catch blocks for a higher-level method that would handle these exceptions.

```
public boolean setElementOfX(int index, int val) {
    if (index >= 0 && index < x.length
        && val >= MIN_VAL && val <= MAX_VAL) {
        x[index] = val;
        return true;
    } else {
        return false;
    }
}
```



# Appendix Review

- ◆ A Java program is a collection of classes. A programmer can use classes defined in the Java API to simplify the task of writing new programs and can define new classes to use as building blocks in future programs. Use

```
import packageName.*;
```

or

```
import packageName.ClassName;
```

to make the public names defined in a package or class accessible to the current file.

- ◆ The JVM enables a Java program written for one machine to execute on any other machine that has a JVM. The JVM is able to execute instructions that are written in Java byte code. The byte code instructions are found in the .class file that is created when a Java source file is compiled.
- ◆ Java defines a set of primitive data types that are used to represent numbers (**int**, **double**, **float**, etc.), characters (**char**), and **boolean** data. Characters are represented using Unicode. Primitive-type variables are used to store primitive data. The Java programmer can use reference variables to reference objects. Wrapper classes can be used to encapsulate (wrap) a primitive-type value in an object.
- ◆ The control structures of Java are similar to those found in other languages: sequence (a compound statement), selection (**if** and **switch**), and repetition (**while**, **for**, **do ... while**).
- ◆ There are two kinds of methods: static (or class) methods and instance methods. Static methods are called using

```
ClassName.methodName(arguments)
```

but instance methods must be applied to objects:

```
objectReference.methodName(arguments)
```

- ◆ The Java **String**, **StringBuilder**, **StringJoiner**, and **StringBuffer** classes are used to reference objects that store character strings. **String** objects are immutable, which means they can't be changed, whereas **StringBuilder**, **StringJoiner** and **StringBuffer** objects can be modified.
- ◆ Make sure you use methods such as **equals** and **compareTo** to compare the contents of two **String** objects (or any objects). The operator **==** compares the addresses of two objects, not their contents.
- ◆ You can use the **String.format** method or **Formatter** objects to create and display formatted strings.
- ◆ You can declare your own Java classes and create objects (instances) of these classes using the **new** operator. A constructor call must follow the **new** operator. A constructor has the same name as its class, and a class can define multiple constructors. The no-parameter constructor is defined by default if no constructors are explicitly defined.
- ◆ A class has data fields (instance variables) and instance methods. The default values for data fields are 0 or 0.0 for numbers, \u0000 for characters, **false** for **boolean**, and **null** for reference variables. A constructor initializes data fields to values specified by its arguments. Generally, data fields have private visibility (accessible only within the class), whereas methods have public visibility (accessible outside the class).

- ◆ Array variables can reference array objects. You must use the `new` operator to allocate storage for the array object.

```
int[] anArray = new int[mySize];
```

The elements of an array can store primitive-type values or references to other objects. Arrays of arrays (multidimensional arrays) are permitted. The data field `length` represents the size of an array and is always accessible, but `length` can't be changed by the programmer. However, an array variable can be reset to reference a different array object with a different size.

- ◆ Class `JOptionPane` (part of Swing) can be used to display dialog windows for data entry (method `showInputDialog`) and message windows for output (method `showMessageDialog`).

- ◆ The `Scanner` class in `java.util` can be used to read numbers and strings from the console (`System.in`) using methods `nextInt`, `nextDouble`, and `nextLine`.

- ◆ The stream classes in `java.io` can enable you to read data from input files and write data to output files. Use statements like

```
Scanner ins = new Scanner (new File(inputFileName));
PrintWriter outs = new PrintWriter(new FileWriter(outFileName));
```

to associate the input stream `ins` and the output stream `outs` with specified files. Many file operations must be performed within a `try-catch` sequence that catches `IOException` exceptions. You must close an output file when you have finished writing all information to it.

- ◆ The default behavior for exceptions is for the JVM to catch them by printing an error message and a call stack trace and then terminating the program. You can use the `try-catch-finally` sequence to catch and handle exceptions, possibly to recover from the error and continue, thereby avoiding the default behavior.

- ◆ There are two categories of exceptions: checked and unchecked. Checked exceptions are generally due to an error condition external to the program. Unchecked exceptions are generally due to a programmer error or a dire event.

- ◆ A method that can throw a checked exception must either catch it or declare that it is thrown using the `throws` declaration. If you throw it, you must catch it further back in the call sequence. Methods do not have to catch unchecked exceptions, and they should not be declared in the `throws` clause.

- ◆ Use the `throw` statement to throw an unchecked exception when you detect one in a method. You should catch this exception farther back in the call sequence, or it will be processed by the JVM as an uncaught exception.

## Java Constructs Introduced in This Appendix

|                      |                      |
|----------------------|----------------------|
| <code>boolean</code> | <code>main</code>    |
| <code>catch</code>   | <code>new</code>     |
| <code>char</code>    | <code>private</code> |
| <code>class</code>   | <code>public</code>  |
| <code>double</code>  | <code>static</code>  |
| <code>final</code>   | <code>throw</code>   |
| <code>finally</code> | <code>throws</code>  |
| <code>for</code>     | <code>try</code>     |
| <code>int</code>     | <code>while</code>   |

## Java API Classes Introduced in This Appendix

|                            |                                  |
|----------------------------|----------------------------------|
| java.io.BufferedReader     | java.lang.Integer                |
| java.io.FileReader         | java.lang.NumberFormatException  |
| java.io.InputStreamReader  | java.lang.Object                 |
| java.io.IOException        | java.lang.String                 |
| java.io.OutputStreamWriter | java.lang.StringBuffer           |
| java.io.PrintWriter        | java.lang.StringBuilder          |
| java.lang.Boolean          | java.util.Formatter              |
| java.lang.Character        | java.util.InputMismatchException |
| java.lang.Double           | java.util.Scanner                |
| java.lang.Exception        | java.util.StringJoiner           |
| java.lang.FileWriter       | javax.swing.JOptionPane          |
| java.lang.Math             |                                  |

## User-Defined Interfaces and Classes in This Appendix

|          |            |             |
|----------|------------|-------------|
| Company  | HelloWorld | SquareRoots |
| FileTest | Person     | TestPerson  |

## Quick-Check Exercises

1. The Java compiler translates Java source code to \_\_\_\_\_, which are executed by the \_\_\_\_\_.
2. Java \_\_\_\_\_ are embedded in \_\_\_\_\_, whereas Java \_\_\_\_\_ are stand-alone programs.
3. A Java program is a collection of \_\_\_\_\_. Execution of a Java application begins at method \_\_\_\_\_.
4. Java classes declare \_\_\_\_\_ and \_\_\_\_\_. Generally, the \_\_\_\_\_ have public visibility and the \_\_\_\_\_ have private visibility.
5. An \_\_\_\_\_ method is invoked by applying it to an \_\_\_\_\_; a \_\_\_\_\_ method is not.
6. If you use the operator == with objects, you are comparing their \_\_\_\_\_, not their \_\_\_\_\_.
7. To associate an input stream with the console, you must wrap an \_\_\_\_\_ object in a \_\_\_\_\_ object.
8. To associate an output stream with the console, you must wrap a \_\_\_\_\_ object in a \_\_\_\_\_ object.
9. To associate an input stream with a text file, you must wrap a \_\_\_\_\_ object in a \_\_\_\_\_ object.
10. Method \_\_\_\_\_ of class JOptionPane normally has \_\_\_\_\_ as its first argument and a \_\_\_\_\_ as its second argument.

## Review Questions

1. Discuss how a Java source file is processed prior to execution and why this approach makes Java platform independent.
2. Declare storage for an array of arrays that will store a list of integers in its first row, the squares of all but the last integer in its second row, and the cubes of all but the last two integers in its third row. Assume that the size of the first row and its integer values are entered by the program user. Read this data into the array and store the required squares and cubes in the array.
3. Draw diagrams that illustrate the effect of each of the following statements.

```
String s1 = "woops";
String s2 = new String(s1);
String s3 = s1;
s1 = new String("Oops!");
```

What are the values of `s1 == s2`, `s1 == s3`, and `s2 == s3`? What are the values of `s1.equals(s2)`, `s1.equals(s3)`, `s2.equals(s3)`? What are the values of `s1.compareTo(s2)`, `s1.compareTo(s3)`, `s2.compareTo(s3)`?

4. Write a class `Fraction` with integer `numerator` and `denominator` data fields. The default value of `denominator` should be 1. Define a constructor with two arguments for this class and one with just one argument (the value of the numerator). Define a method `multiply` that multiplies this `Fraction` object with the one specified by its argument and returns a new `Fraction` object as its result. Define a method `toDecimal` that returns the value of the fraction as a decimal number (be careful about integer division). Define a `toString` method for this class that represents a `Fraction` object as a string of the form `numerator / denominator`.
5. Write a `main` method that reads two `Fraction` objects using class `JOptionPane`. Multiply them and display their result as a fraction and as a decimal number using the instance methods defined in Review Question 4. Use class `JOptionPane` to display the results.
6. Write a `main` method that reads two `Fraction` objects from the console. Multiply them and display their result as a fraction and as a decimal number using the instance methods defined in Review Question 4. Use the console to display the results.

## Programming Projects

1. Complete the definition of the `Fraction` class described in Review Question 4. Provide all the methods listed in that question and methods to add, subtract, and divide two fractions. Also, provide methods `equals` and `compareTo` to compare two `Fraction` objects.
2. Provide a class `MatrixOps` that has a two-dimensional array of `double` values as its data field. Provide the following methods:

```
MatrixOps() // Default constructor
MatrixOps(int numNows) // Sets the number of rows
MatrixOps(int numRows, int numCols) // Sets the number of rows and columns
MatrixOps(double[][] mat) // Stores the specified array
void setMatrix(double[][] mat) // Stores the specified array
double[][] getMatrix() // Gets the array
void setRow(int row, double[] rowVals) // Stores the array of rowVals in row
double[] getRow(int row) // Returns the specified row
void setElement(int row, int col) // Sets the specified element
double getElement(int row, int col) // Returns the specified element
double sum() // Returns sum of the values in the array
double findMax() // Returns the largest value in the array
double findMin() // Returns the smallest value in the array
double[][] transpose() // Returns the transpose of the matrix
double[] multiply(double[][] mat2) // Returns the product of two matrices
String toString() // Returns a string representing the array
```

3. Modify class `Person` to include a person's hours worked and hourly rate as data fields. Provide modifier and accessor methods for the new data fields and a method `calcSalary` that returns a person's salary. Also, modify method `toString`. Provide a method `calcPayroll` for class `Company` that returns the weekly payroll amount for a company (gross payroll only; don't be concerned about withholding, payroll taxes, etc.). Write a `main` method that reads the employee data for a `Company` object from a data file. Display the data stored and the calculated payroll in a message window using class `JOptionPane`. Also, write this information to an output file.
4. Write a class that stores a collection of exam scores in an array. Provide methods to find the average score, to assign a letter grade based on a standard scale, to sort the scores so they are in increasing order, and to display the scores. Test the methods of this class.
5. Write a class `Student` that stores a person's name, an array of scores for each person, an average exam score, and a letter grade. Write a class `Gradebook` that stores an instructor's name, a section ID, a course name, and an array of `Student` records. Write the following methods to process this array.
  - Load the array of `Student` records with data read from a text file.
  - Write all information stored to an output file.
  - Calculate and store each student's average exam score in that student's record.
  - Calculate and store the average score for each student in that student's record.

- Assign a letter grade to each student based on that student's average exam score.
- Sort the array of student records so that all information is in increasing order by student.
- Sort the array of student records so that all information is in decreasing order by exam score.

Write a client program that reads the data for a class and performs all the operations in the list above. Display the information in a `Gradebook` object after all the data is stored and again after all student information been calculated and stored. Also, display the information after sorting it by name and after sorting it by exam score.

## Answer to Quick-Check Exercises

1. The Java compiler translates Java source code to *byte code instructions*, which are executed by the *JVM*.
2. Java *applets* are embedded in Web pages, whereas Java *applications* are stand-alone programs.
3. A Java program is a collection of *classes*. Execution of a Java application begins at method *main*.
4. Java classes declare *data fields* and *methods*. Generally, the *methods* have public visibility and the *data fields* have private visibility.
5. An *instance* method is invoked by applying it to an *object*; a *static* (or *class*) method is not.
6. If you use the operator `==` with objects, you are comparing their *addresses*, not their *contents*.
7. To associate an input stream with the console, you must wrap an *InputStreamReader* object in a *BufferedReader* object.
8. To associate an output stream with the console, you must wrap a *FileWriter* object in a *PrintWriter* object.
9. To associate an input stream with a text file, you must wrap a *FileReader* object in a *BufferedReader* object.
10. Method *showMessageDialog* of class `JOptionPane` normally has `null` as its first argument and a *prompt string* as its second argument.

# *Overview of UML*

The Unified Modeling Language (UML) represents the unification of earlier object-oriented design modeling techniques. Specifically, notations developed by Grady Booch, Ivar Jacobson, and James Rumbaugh were adapted to form the initial version. This version was submitted to the Object Modeling Group for formal standardization. Since that initial submission, the UML standard has undergone several revisions and continues to be revised.

We call UML a modeling *language* much in the same way we call Java a programming language. There is a formal definition of the syntax and semantics. There are software tools that are used both to draw the diagrams and to capture the underlying design information. These tools can then be used to analyze the resulting model, verify the model's consistency, and generate code.

UML defines 12 types of diagrams. In this text, we use only two of them: the class diagram and the sequence diagram. Throughout the text, where we use these diagrams, we provide brief explanations of the diagram and the meaning of the notations used. The purpose of this appendix is to provide a more complete reference to the diagrams as they are used in this text.

In this text, we use a notation that has been adapted from the UML standard to match the syntax of Java more closely. Other books may use slightly different versions of these diagrams that follow the standard syntax, but the principles are the same.

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## Overview of UML

- B.1** The Class Diagram
- B.2** Sequence Diagrams

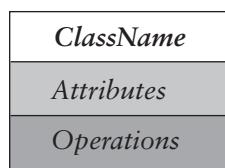
## B.1 The Class Diagram

The *class diagram* shows the classes and their relationships. It is a static diagram that represents the structure of the program. The classes (including interfaces) are represented by rectangles, and lines between the classes represent the relationships. The style of a line, symbols on the ends of the lines, and text placed near the line are used to indicate the kind of relationship being modeled.

A large amount of information about the structure of a program can be represented in a class diagram. If all of the possible information was presented, the diagram would become quite cluttered. Therefore, the practice is to show only the essential information. For example, in a class diagram, the complete method declaration can show the method's visibility, return type, name, and parameter types. Sometimes only the method's name is necessary, in which case you would elect to suppress the other information. Also, some methods may not be significant to the discussion, so those methods need not be shown. Sometimes only the class name is the essential item, and thus the methods and attributes are not shown.

**FIGURE B.1**

General Representation of a Class



### Representing Classes and Interfaces

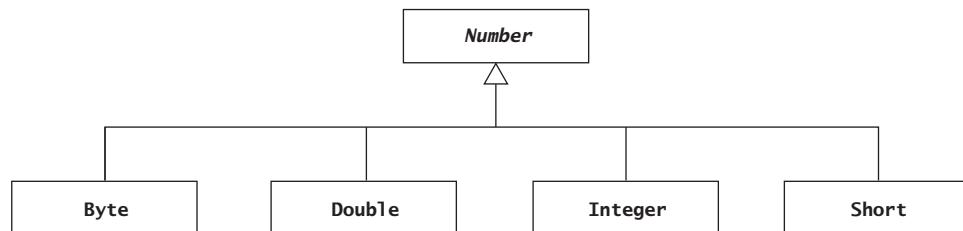
A class is represented by a rectangle divided into three segments as shown in Figure B.1.

#### The Class Name

Every class has a name that distinguishes it from other classes. In Java, a class may be (and usually is) a member of a package, in which case we may show the complete name including the package name (e.g., `java.util.Stack`). In other cases, we just show the class name (e.g., `Node`). Italics indicate abstract classes. The class name is centered in the box representing the class. For example, Figure B.2 shows the abstract class `Number` and the concrete classes derived from it.

**FIGURE B.2**

The Abstract Class `Number` and Concrete Subclasses



### Interfaces

The word *interface* enclosed in double angle brackets (`<<interface>>`, called guillemets) placed before the class name is used to indicate that this class is an interface. Because interfaces, like abstract classes, cannot be instantiated, the name is shown in italics (see Figure B.3).

**FIGURE B.3**

The Interface List

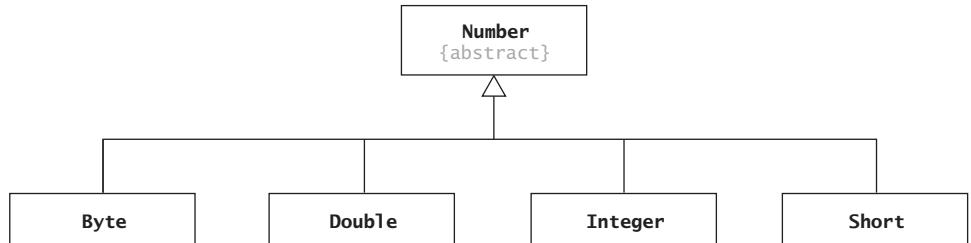


### Alternative UML Syntax for Class Names

In other texts, you may see the class name in a bold sans-serif font. Also, abstract classes may be indicated by `{abstract}`, as shown in Figure B.4.

**FIGURE B.4**

Alternative Syntax for Indicating an Abstract Class



### The Attributes

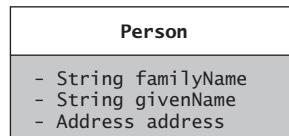
The *attributes* of a class are the data fields. As a minimum we show the name. Optionally we can also show the visibility and type. The visibility is indicated by the symbols shown in Table B.1.

**TABLE B.1**

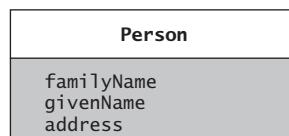
Visibility Specifiers

| Symbol | Visibility |
|--------|------------|
| +      | public     |
| -      | private    |
| #      | protected  |
| ~      | package    |

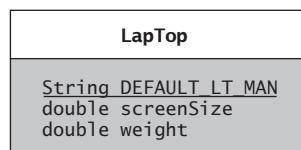
In this text, we use the Java language syntax to indicate the type of an attribute by placing the type name before the attribute name. For example, the class Person could have the attributes `familyName`, `givenName`, and `address`, as shown in the following figure:



Where they are not essential to the current discussion, we will omit the visibility indicator, the type, or both, as shown in the following figure:

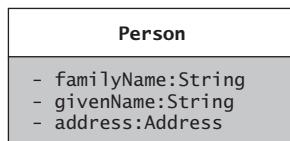


Static attributes are indicated by underlining their name. For example, the class LapTop has the static attribute `DEFAULT_LT_MAN`.



## Standard UML Syntax for Attribute Types

In other texts, you may see a different syntax for showing the attribute type. The UML standard specifies that the attribute type be specified following the name and separated by a colon.

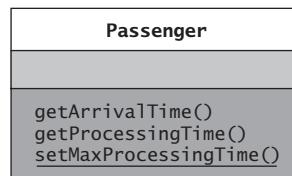


## The Operations

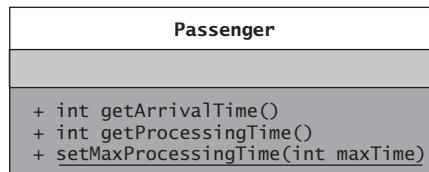
The *operations* are the methods of the class. At a minimum, we show the method name followed by a pair of parentheses. An empty set of parentheses does not necessarily indicate that this method takes no parameters. Italics are used to indicate an abstract method, and underlining is used to indicate a static method. For example, Figure B.5 shows the class Passenger with the static method `setMaxProcessingTime` and the nonstatic methods `getArrivalTime` and `getProcessingTime`. The attributes are not shown.

We may also show the visibility, the parameter types, and the return type. The visibility is shown using the same symbols as used for the attributes (see Table B.1). In this text, we use the Java method declaration syntax, as shown in Figure B.6, to show the parameter types and return type. A return type of `void`, however, will not be shown.

**FIGURE B.5**  
The Class Passenger

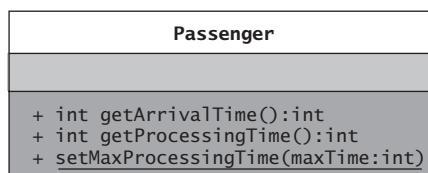


**FIGURE B.6**  
Class Passenger  
Showing the Return  
and Parameter Types  
of Its Operations



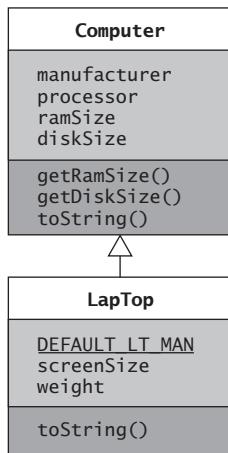
## Standard UML Syntax for Operations

In other texts, you may see a different syntax for showing the parameter types and return type. The UML standard specifies that the parameter type be preceded by a colon and shown following the parameter name and that the return type be shown following the operation name, also preceded by a colon. The class Passenger using this syntax is shown in the following figure:



**FIGURE B.7**

Class Laptop as a Subclass of Computer



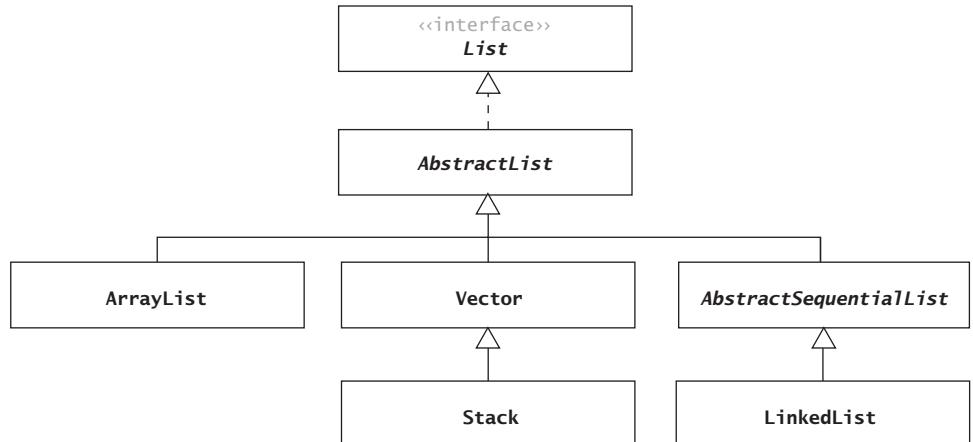
## Generalization

UML uses the term *generalization* to describe the relationship between a superclass and its subclasses. Drawing a solid line with a large open arrowhead pointing to the superclass shows generalization. Figure B.7 shows the class LapTop as a subclass of Computer.

A dashed line with a large open arrowhead is used to show that a class implements an interface. Figure B.8 shows that the abstract class `AbstractList` implements the `List` interface and that the classes `ArrayList`, `Vector`, and `AbstractSequentialList` are subclasses of `AbstractList`. `Stack` is a subclass of `Vector`, and `LinkedList` is a subclass of `AbstractSequentialList`.

**FIGURE B.8**

The List Interface and Classes that Implement It



## Inner or Nested Classes

A class that is declared within the body of another class is called an inner or nested class. In UML, this relationship is indicated by a solid line between the two classes, with what the UML standard calls an *anchor* on the end connected to the enclosing class. The anchor is a cross inside a circle. For example, in Figure B.9, the class `Node` is declared as an inner class of the class `KWLinkedList`.

**FIGURE B.9**

Node as an Inner Class



## Association

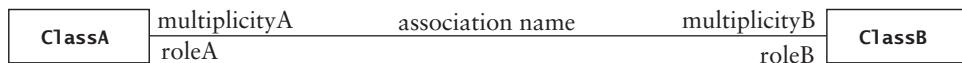
An *association* between classes represents a relationship between objects of those classes. In object-oriented terminology, we say that “object A sends a message to object B.” This statement implies two things:

1. There is a method in class B that will receive the message.
2. There must be a reference within class A that references object B.

An association indicates the presence of the reference required by condition 2. Thus, in the analysis process in which we examine a use case and determine the flow of information from one object to another, we identify the requirements for methods and associations. Note that the association may represent a data field or it may represent a parameter.

Figure B.10 shows the UML notation for an association. The *association name*, multiplicities, and roles are all optional. The association name is a name given to the association. The *multiplicity* represents the number of objects of that class that participate in the association. Where the association is implemented as a data field, the *role name* is generally used as the name of the data field. Thus, in ClassA, there would be a reference of type ClassB with the name roleB. The role name may have a visibility specifier (see Table B.1). The role and multiplicity may be either above or below the line.

**FIGURE B.10**  
UML Notation for an Association



Multiplicity represents the number of objects of the class that are related to the other class. Thus, *multiplicityB* represents the number of objects of ClassB that are associated with an object of ClassA, and *multiplicityA* represents the number of objects of ClassA that are associated with an object of ClassB. Multiplicity may be either a single number or a range of numbers. The symbol \* is used to indicate an indefinite number. A range of numbers is specified by a low bound followed by a high bound separated by two periods. Examples are shown in Table B.2.

In addition, an arrow can be placed at one or both ends of the line. The presence of an arrow indicates the navigation direction. Thus, if there is an arrow on the ClassB end, then objects of ClassA can send messages to objects of ClassB, but objects of ClassB cannot send messages to objects of ClassA. The absence of arrows generally represents that navigation in both directions is possible, but it may also mean that the navigation is not being shown.

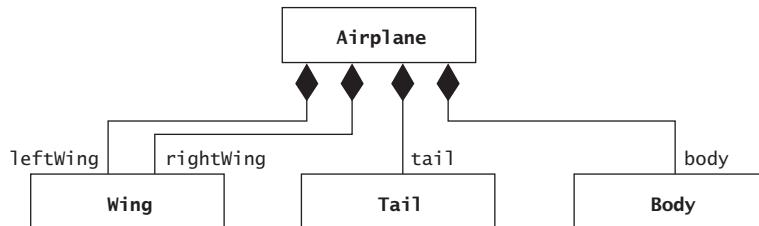
**TABLE B.2**  
Multiplicity Examples

| Multiplicity | Meaning                                            |
|--------------|----------------------------------------------------|
| 1            | There is only 1                                    |
| 1..5         | There is at least 1, and there may be as many as 5 |
| 3..*         | There are at least 3                               |
| *            | There could be any number, including 0             |

## Aggregation and Composition

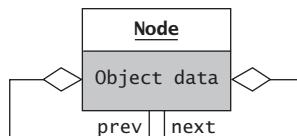
In those cases where we wish to show that an association definitely is represented by a data field, we place a diamond on the end of the line next to the class that will contain the data field. This represents the *has-a* relationship. If the diamond is open, this is called an *aggregation*, and if the diamond is filled, this is called a *composition*. The difference is that in a composition, the component objects are not considered to have an independent existence. For example, an Airplane is composed of two wings, a body, and a tail, none of which would exist unless it was a component of an Airplane. This would be modeled as shown in Figure B.11.

**FIGURE B.11**  
Airplane Composed of Wing, Tail, and Body

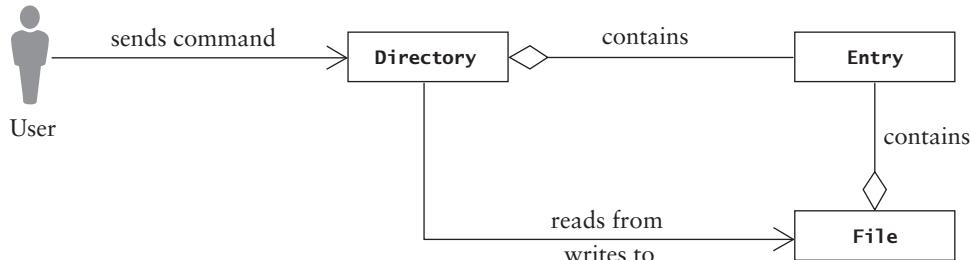


However, a Node in either a linked list or a tree has references to other Nodes, but these other nodes are independent entities, and the value of the reference can be changed. Thus, we use the open diamond as shown in Figure B.12. Observe that the references are to the same class (Node).

**FIGURE B.12**  
A Node in a Double-Linked List



**FIGURE B.13**  
The Directory and File Classes as Aggregations Of Entries



**FIGURE B.14**  
Generic Class Representation



We will indicate a generic class by placing the generic parameter(s) in a dotted rectangle in the upper right corner of the rectangle that models the class. Thus, the generic class `ArrayList<E>` is modeled by the diagram shown in Figure B.14. Alternatively, you can just write the class name as `ArrayList<E>`.

An invocation of a generic class is indicated by including the actual parameters inside a pair of less than and greater than symbols following the name. This is the same notation used by the Java language. Thus, an `ArrayList` of `Strings` would be written `ArrayList<String>`.

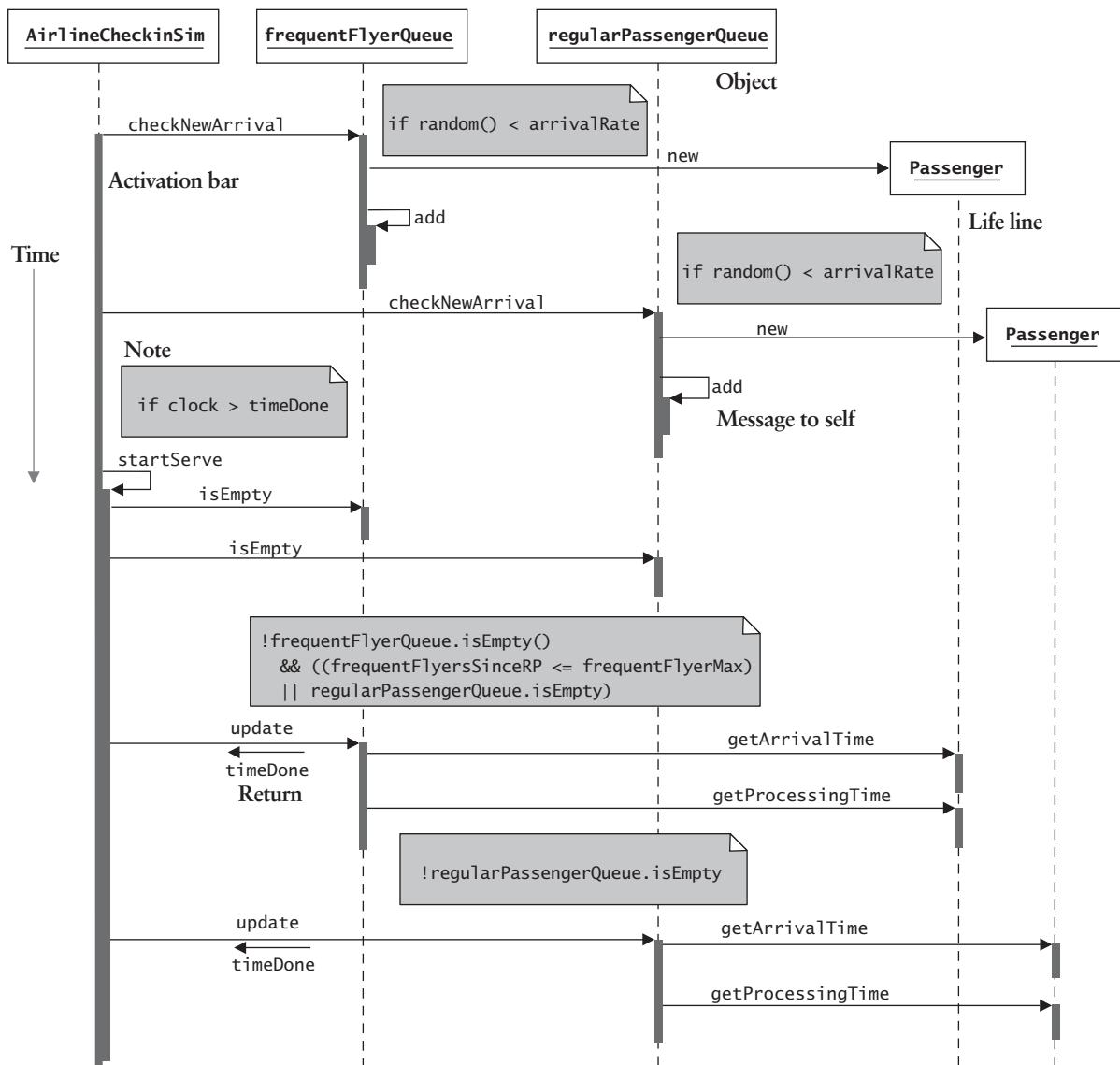
## B.2 Sequence Diagrams

*Sequence diagrams* are used to show the flow of information through the program. Sequence diagrams are generally developed on a use-case basis and show the message sequence associated with a particular use case. The purpose of developing a sequence diagram is to identify the messages that are passed from one object to another. This then identifies the requirements for the corresponding classes. Recall that if *objectA* sends a message to *objectB*, then

1. *ClassB* must have a method to process that message.
2. *ClassA* must have a reference to an *objectB*.

**FIGURE B.15**

Sequence Diagram Example



Thus, when you enter a message on a sequence diagram, you identify a requirement for a method and an association to be entered on the class diagram. Many UML modeling software tools automate the process of keeping the sequence diagrams and class diagram consistent.

Figure B.15 shows an example of a sequence diagram. This is a two-dimensional diagram with time running down the vertical axis and objects listed across the horizontal axis. The ordering across the horizontal axis is insignificant.

## Time Axis

Time flows down the vertical axis. Generally the scale is not significant, but for some applications, where timing is critical, a precise timing scale can be used. The sequence along the time axis is significant.

## Objects

Objects are listed across the horizontal axis. Their order is insignificant. An object is represented by a rectangle with the name of the object underlined. For anonymous objects, the name of the class is given.

Objects are listed across the top of the sequence diagram unless they are created during the time period represented by the sequence diagram. If an object is created, then it is shown lower in the diagram, at the point at which it is created. As shown in Figure B.15, two Passenger objects are created during the sequence of events depicted.

## Life Lines

Flowing down from each object is its *life line*. This is a dashed line that begins when the object is created and ends when the object is destroyed. There is no way to destroy an object explicitly in Java, so the life lines will continue to the bottom of the diagram.

## Activation Bars

The thin long rectangles along the life line are *activation bars*. These represent the time that the object is responding to a given message. Note that if a second message is received while a message is being processed, a second activation bar is drawn on top of and to the right of the first activation bar. This can be seen in Figure B.15, where the AirlineCheckinSim object sends itself the startServe message, or where the frequentFlyerQueue and regularPassengerQueue objects send themselves the insert message.

## Messages

Messages are indicated by a horizontal arrow from the sending object to the receiving object. The name of the message is shown above the arrow. Optionally, the parameters may be shown in parentheses following the message name. Also, a small reverse direction arrow may be used to indicate a return value with the value shown below it. An example of this is shown in Figure B.15, where timeDone is returned to the AirlineCheckinSim object in response to the update message sent to the frequentFlyerQueue.

## Use of Notes

Notes may be used on any UML diagram. They are free-form text enclosed in a rectangle with the upper right corner folded down.

The purpose of the sequence diagram is to identify the sequence of messages that occur during a use case. For a given instance of a use case, not all messages will be sent. For example, as shown in Figure B.15, the checkNewArrival message to the frequentFlyerQueue may or may not result in the creation of a new Passenger object. Notes can be used to document the conditions for sending a message. For example, the checkNewArrival message is sent when the result of the random number generator is less than arrivalRate.



# Glossary

**2-3 tree** A search tree in which each node may have two or three children.

**2-3-4 tree** A search tree in which each node may have two, three, or four children.

**2-node** A node in a 2-3 or 2-3-4 tree with two children.

**3-node** A node in a 2-3 or 2-3-4 tree with three children.

**abstract class** A class that contains at least one abstract method.

**abstract data type** An implementation-independent specification of a set of data items and the operations performed on those data items.

**abstraction** A model of a physical entity or activity.

**abstract method** The specification of the signature of a method without its implementation. Abstract methods are declared in interfaces and abstract classes. A concrete class that is a subclass of an abstract class or an implementation of an interface must implement each abstract method declared in the abstract superclass or interface.

**acceptance testing** A sequence of tests that demonstrate to the customer that a software product meets all of its requirements. Acceptance testing generally is observed by a customer representative.

**activation bar** The thick line along the lifeline in a sequence diagram that indicates the time that a method is executing in response to the receipt of a message.

**activation frame** An area of memory allocated to store the actual parameters and local variables for a particular call to a method. In Java, references to activation frames are stored on the run-time stack. When a method is called, a new activation frame is pushed onto the stack, and when a method exits, the activation frame is popped.

**actor** An entity that is external to a given software system. In many cases, an actor is a human user of the software system, but an actor may be another system.

**adapter class** A class that provides the same or very similar functionality as another class but with different method signatures. The actual work is performed by delegation to the methods in the other class.

**address** A number that represents an object's location in memory.

**adjacency lists** A representation of a graph in which the vertices (the destinations) adjacent to a given vertex (the source) are stored in a list associated with that vertex. The actual edge (source, destination, weight) from the source vertex to the destination may be stored.

**adjacency matrix** A representation of a graph in which the presence or absence of an edge is indicated by a value in a

matrix that is indexed by two vertices. The value stored is 0 for no edge, 1 for an edge in an unweighted graph, and the weight itself for a weighted graph.

**adjacent [vertex]** In a directed graph, a vertex,  $v$ , is adjacent to another vertex,  $u$ , if there is an edge,  $(u, v)$ , from vertex  $u$  to vertex  $v$ . In an undirected graph,  $v$  is adjacent to  $u$  if there is an edge,  $\{u, v\}$ , between them.

**aggregation** An association between two classes in which one class is composed of a collection of objects of the other class.

**analysis** In the waterfall model, the phase of the software life cycle (workflow in the Unified Model) during which the requirements are clarified and the overall architecture of the solution is determined.

**ancestor** A node in a tree that is at a higher level than a given node and from which there is a path to that node (the descendant).

**ancestor–descendant relationship** A generalization of the parent–child relationship. (See *ancestor* and *descendant*.)

**anchor** The symbol  $\oplus$  that is used in a UML class diagram to indicate that a class is an inner class of another class.

**annotations** Directions to the compiler and other language-processing tools; they do not affect the execution of the program.

**anonymous method** A method that does not have an explicit name or method declaration but is declared as a lambda expression.

**anonymous object** An object for which there is no named reference. The Java `new` operator returns a reference to an anonymous object.

**anonymous reference** A reference to an object that itself has no name. Anonymous references are the result of a cast operation.

**applet** A top-level Java GUI class that is intended to be displayed in a frame that is under the control of a web browser.

**assertion** A statement that is true about the current value of one or more variables state of a method.

**association** A relationship between two classes.

**attributes** The set of data values that determine the state of an object. Generally, the attributes of a class are represented by data fields within the class.

**auto-boxing** A new Java feature (in Java 1.5) that performs automatic conversion between the primitive types and their corresponding wrapper classes.

**AVL tree** A self-balancing binary search tree in which the difference between the heights of subtrees is stored in each tree node. The insertion and removal algorithms use rotations to maintain this difference within the range  $-1$  to  $+1$ .

**back edges** An edge that is discovered during a depth-first search that leads to an ancestor in the depth-first search tree.

**backtracking** An approach to implementing a systematic trial-and-error search for a solution. When a dead end is reached, the algorithm follows a path back to the decision point that leads to the dead end, and then moves forward along a different path.

**balanced binary search tree** A binary search tree in which the height of each pair of subtrees is approximately the same.

**base case** The case in a recursive algorithm that can be solved directly.

**big-O notation** The specification of a set of functions that represent the upper bound of a given function. Formally, the function  $f(n)$  is said to be  $O(g(n))$  if there are constants  $c > 0$  and  $n_0 > 0$  such that for all  $n > n_0$ ,  $cg(n) \geq f(n)$ .

**binary search** The process of searching a sorted sequence that begins by examining the middle element. If the middle element is greater than the target, then the search is applied recursively to the lower half; if it is less than the target, the search is applied recursively to the upper half.

**binary search tree** A binary tree in which the items in the left subtree of a node are all less than that node, and the items in the right subtree are all greater than that node.

**binary tree** A tree in which each node has 0, 1, or 2 children. The children are distinguished by the names left and right. If a node has one child, that child is distinguished as being a left child or a right child.

**black-box testing** A testing approach in which the internal structure of the item being tested is not known or taken into account in the design of test cases. The test cases are based only on the functional requirements for the item being tested.

**block** A compound statement that may contain local variables and class declarations.

**bottom-up design** A design process in which the lower level methods are designed first. A lowest-level method is one that does not depend on other methods to perform its function.

**boundary condition** A value of a variable that causes a different path to be taken. For example, in the statement `if (x > C) { ... } else { ... }`, the value of `C` is a boundary condition.

**branch** In a tree, the link between a parent node and one of its children.

**branch coverage** A measure of testing thoroughness. Each alternative from a decision point (`if`, `switch`, or `while` statement) is considered a branch. If a test exercises a branch, then that branch is considered covered. The ratio of the covered branches to the total number of branches is the branch coverage. See also *path coverage* and *statement coverage*.

**breadth-first search** A way of searching through a graph in which the vertices adjacent to a given vertex are all examined and placed into a queue. Once all the adjacent vertices are examined, the next vertex is removed from the queue. Thus, vertices are examined in increasing distance (as measured by the number of edges) from the starting vertex.

**breadth-first traversal** See *breadth-first search*.

**breakpoint** A point in a program at which the debugger is instructed to suspend execution when it is reached. This allows for examination of the value of variables at a given point before execution is resumed.

**B-tree** A balanced search tree in which each node is a leaf or may have up to  $n$  children and  $n-1$  data items. The leaves are all at the bottom level. Each node (except for the root) is kept at least half full. That is, each node has between  $(n-1)/2$  and  $n-1$  data items. The root is either a single node (leaf) or it has at least one data item and two children.

**bucket** The list of keys stored in a hash table entry that uses chaining. All the keys in the list map to the index of that table entry.

**bucket hashing** See *chaining*.

**byte code** The platform-independent representation of a Java program that is the output of the Java compiler and is the input to the Java Virtual Machine (JVM). The JVM then interprets this input to execute the program.

**casting** When applied to a reference to an object, casting reinterprets that reference to refer to an object of a different type. The object must be of the target type (or a subclass of the target type) for the cast to be valid. When applied to a primitive numeric value, a cast represents a conversion to an equivalent value of the target primitive numeric type.

**catch block** The sequence of statements that will be executed when an exception is caught by a `catch` clause.

**catch clause** The specification of an exception type and the statements to be executed when an exception of that type is caught. One or more `catch` clauses follow a `try` block and will catch the exceptions thrown from that `try` block.

**chaining** An approach to hashing in which all keys that are mapped to a given entry in the hash table are placed into a list. The list is called a bucket.

**checked exception** An exception that either must be declared in a `throws` declaration or caught by a `try-catch` sequence.

**child** A node in a tree that is the immediate descendant of another node.

**class** The fundamental programming unit in a Java program. A class consists of a collection of zero or more data fields (instance variables) and zero or more methods that operate on those data fields.

**class diagram** A UML diagram that shows a number of classes and the relationships between them.

**class method** See *static method*.

**client** A class or method that uses a given class.

**closed-box testing** See *black-box testing*.

**collection hierarchy** The hierarchy of classes in the Java API that consists of classes designed to represent collections of other objects.

**collision** The mapping of two or more keys into the same position in a hash table.

**complete binary tree** A binary tree in which each node is a leaf or has two children.

**component** In a GUI application, an object displayed on the screen that can interact with the user.

**component testing** The testing of an individual part of a program by itself. In a Java program, a component may be a method or a class.

**composition** The association between two classes in which objects of one class are part of another class. The parts generally do not have an independent existence but are created when the parent object is created. For example, an `Airplane` object is composed of a `Body` object, two `Wing` objects, and a `Tail` object.

**compound statement** Zero or more statements enclosed within braces `{ ... }`.

**concrete class** (actual class) A class for which objects can be instantiated.

**connected components** A set of vertices within a graph for which there is a path between every pair of vertices.

**connected graph** A graph that consists of a single connected component.

**constructor** A method that initializes an object when it is first created.

**container** In a GUI application, a component that contains other components.

**contract** The specification of the pre- and postconditions of a method.

**cost of a spanning tree** The sum of the weights of the edges.

**coverage testing** See *branch coverage*.

**cycle** A path in a graph in which the first and final vertices are the same.

**data abstraction** The specification of the data items of a problem and the operations to be performed on these data items that does not specify how the data items will be represented and stored in memory. See also *abstract data type*.

**data field** (instance variable) A variable that is part of a class.

**debugging** The process of finding and removing defects (bugs) from a program.

**deep copy** A copy of an object in which data field values and references to immutable objects are simply duplicated, but each reference to a mutable object references a copy of that object. If there are mutable references in any object that is copied, these also reference a copy of that object. The effect is that you can change any value in a deep copy of an object without modifying the original object.

**default constructor** The no-parameter constructor that is generated by the Java compiler if no constructors are defined.

**default visibility** The same as *package visibility*.

**defensive programming** An approach to designing a program that builds in statements to test the values of variables that might result in an exception or run-time error (to be sure that they are valid) before statements that use the variables are executed.

**delegation** The implementation of a method in one class that merely calls a method in another class.

**delimiter characters** Characters that are defined to separate a string into tokens.

**depth (level)** The number of nodes in a path from the root to a node.

**depth-first search** A method of searching a graph in which adjacent vertices are examined along a path until a dead end is reached. The search then backtracks until an unexamined vertex is found, and the search continues with that vertex.

**depth-first traversal** See *depth-first search*.

**deque** A data structure that combines the features of a stack and queue. Items may be inserted in one end and removed from either.

**descendant** In a tree, a lower node that can be reached by following a path from a given node.

**design** The process by which classes and methods are identified and defined to create a program that satisfies a given set of requirements.

**detail message** An optional string to be displayed when an exception is thrown that provides additional information about the conditions that led to the exception.

**dialog** In a GUI application, a window that provides information or asks for data entry.

**digraph** See *directed graph*.

**directed acyclic graph** A directed graph that contains no cycles.

**directed edge** An edge in a directed graph.

**directed graph** A graph in which every edge is considered to have a direction. If  $u$  and  $v$  are vertices in a graph, then the presence of the edge  $(u, v)$  indicates that  $v$  is adjacent to  $u$ , but  $u$  may not be adjacent to  $v$ . Contrast with *undirected graph*.

**discovery order** The order in which vertices are discovered in a depth-first search.

**downcast** A reinterpretation of a reference from a superclass to a subclass. In Java, downcasts are tested for validity. See also *casting*.

**driver** A method whose purpose is to call a method being tested and provide it with appropriate argument values. Usually, the result of executing the method is displayed immediately to the user.

**edges** In a graph, the links between pairs of vertices.

**escape sequence** A sequence of characters beginning with the backslash (\), which is used to indicate another character that cannot be directly entered. For example, the sequence \n represents the newline character.

**Euler tour** A path around a tree, starting and ending with the root. The tree is always kept to the left of the path when viewed from the direction of travel along the path.

**event** The occurrence of an external input or an internal state change.

**exclusive or (XOR)** A graphics drawing mode in which drawing a shape twice has the effect of erasing the original shape from the image.

**extending** The process of adding functionality by defining a new class that adds data fields and/or adds or overrides methods of an existing class.

**external node** See *leaf*.

**factory method** A method that is responsible for creating objects of a class. Generally, a factory method will be associated with an abstract class or interface and will choose an appropriate concrete class that extends the abstract class or implements the interface based on parameters passed to the factory method and/or system parameters. Returns a reference to a new object of this concrete class.

**finally block** A block preceded by the key word **finally**. Part of the **try-catch-finally** sequence.

**finish order** The order in which the vertices are finished in a depth-first search. A vertex is considered finished when all of the paths to adjacent vertices have been finished.

**forest** A collection of trees that may result from a depth-first search of a directed graph or an unconnected graph.

**frame** A top-level container in a GUI application. A frame consists of a window with a border around it.

**full binary tree** A binary tree in which the nodes at all but the deepest level contain two children. At the deepest level, all nodes that have two children are to the left of those that have no children, and there is at most one node with a left child that is between these two groups.

**functional interface** An interface that declares exactly one abstract method.

**functional programming** A language feature in which functions (methods) can be assigned to variables or passed as arguments to other functions.

**functional testing** Testing that concentrates on verifying that software meets its functional requirements.

**garbage collector** The process of reclaiming memory that no longer has a reference to it. This process generally runs in the background.

**generalization** The relationship between two classes in which one class is the superclass and the other is a subclass. The superclass is a generalization of the subclass.

**generic class** A class with type parameters that are specified when instances are created. These parameters specify the actual data type for the internal data fields of the object that is created.

**generic method** A method with type parameters that are used to represent the data type of its formal parameters. The type parameters are specified when the method is called and enable the method to process actual parameters of different data types.

**generic type** A type that is defined in terms of another type where that other type may be specified as a parameter. For example, the class `List<E>` is a `List` designed to hold objects of type `E`, where `E` may be any other class and is specified when the object is created.

**glass-box testing** Testing that takes the internal structure of the unit being tested into account.

**graph** A mathematical structure consisting of a set of vertices and edges. The edges represent a relationship between the vertices.

**hash code** A function that transforms an object into an integer value that may be used as an index into a hash table.

**heapsort** A sort algorithm in which the items being sorted are inserted into a heap, and then removed one at a time.

**height of a tree** The number of nodes in a path from the root to the deepest leaf.

**Huffman code** A varying-length binary code in which each symbol is assigned a code whose length is inversely proportional to the frequency with which that symbol appears (or is expected to appear) in a message. The resulting coded message is the minimum possible length.

**immutable** A class that is immutable has no methods to change the value of its data fields. An immutable object cannot be changed.

**implement (an interface)** To provide in a class an implementation of all of the methods specified by an interface.

**increment operator** The operator that has the side effect of adding 1 to its operand.

**index** A value that specifies a position within an array.

**infix notation** Mathematical notation in which the operators are between the operands.

**inherit** To receive from an ancestor. In an object-oriented language, a subclass inherits the visible methods and data fields from its superclass. These inherited methods and data fields appear to clients of the subclass as if they were members of that class.

**initializer list** A list of values, enclosed in braces, that initializes the values in an array.

**inner class** A class that is defined within another class. Methods of inner classes have access to the data fields and methods of the outer class in which they are defined and vice versa.

**inorder predecessor** For a binary search tree, the inorder predecessor of an item is the largest item that is less than this item. The node containing an item's inorder predecessor would be visited just prior to that item in an inorder traversal.

**insertion sort** A sorting algorithm in which each item is inserted into its proper place in the sorted region.

**instance** See `object`.

**instance method** A method that is associated with an object. Contrast with `static method`.

**instanceof operator** The Java operator that returns true if a reference variable references an instance of a specified class or interface.

**instance variables** A variable of a class that is associated with an object (i.e., a data field of an object). Contrast with `static variable`.

**integration testing** Testing in which the interaction of the components or units of a software program is validated.

**interface** The external view of a class. In Java, an interface is a class that defines nothing more than public abstract methods and constants.

**internal node** A node in a tree that has one or more children. Contrast with `leaf`.

**interpret** To translate or understand the meaning of. The Java Virtual Machine interprets the machine-independent byte code in terms of specific machine-language instructions for the computer on which it is executing.

**iteration** In a loop, a complete execution of the loop body. In the Unified model of the software life cycle, a sequence of activities that results in the release of a set of software artifacts.

**iterator** An object that accesses the objects contained in a collection one at a time.

**Javadoc** The commenting convention defined for Java programs. Also, the program that generates documentation

from the comments that follow this convention in a program.

**key** A value or reference that is unique to a particular object and thereby identifies that object (e.g., a social security number).

**lambda expression** A method without a name that is declared for a single use (see *anonymous method*).

**Last In, First Out (LIFO)** An organization of data such that the most recently inserted item is the one that is removed first.

**last-line recursion** A recursive algorithm or method in which the recursive call is the last executable statement.

**leaf (node)** A node in a tree that has no children. Contrast with *internal node*.

**left rotation** The transformation of a binary search tree in which the right child of the current root becomes the new root and the old root becomes the left child of the new root.

**level of a node** The number of nodes in a path from the root to this node.

**life line** The dotted vertical line in a UML sequence diagram that indicates the lifetime of an object.

**linear probing** A collision resolution method in which sequential locations in a hash table are searched to find the item sought or an empty location.

**linear search** A search algorithm in which items in a sequence are examined sequentially.

**link** A reference from one node to another.

**literal** A constant value that appears directly in a statement.

**logic error** An error in the design of an algorithm or program. Contrast with *syntax error*.

**logical view** A description of the data stored in an object that does not specify the physical layout of the data in memory.

**loop invariant** An assertion that is true before each execution of the loop body and is true when the loop exits.

**many-to-one mapping** An association among items in which more than one item (a key) is associated with a single item (a value).

**marker** An interface that is defined with no methods or constants. It is used to give a common name to a family of interfaces or classes.

**merge** The process of combining two sorted sequences into a single sorted sequence.

**merge sort** A sorting algorithm in which sorted sub-sequences are merged to form larger sorted sequences.

**message** In an object-oriented design, a message represents an occurrence of a method call.

**message to self** A message that is passed from an object to itself. It represents a method calling another method within the same class.

**method** A sequence of statements that can be invoked (or called) passing a fixed number of values as arguments and optionally returning a value.

**method declaration** The specification of the name, parameters, and return type of a method. See also *signature*.

**method overloading** The presence of multiple methods in a class with the same name but different signatures.

**method overriding** The replacement of an inherited method with a different implementation in a subclass.

**minimum spanning tree** A subset of the edges of a connected graph such that the graph remains connected and the sum of the weights of the edges is the minimum.

**multiplicity** An indication of the number of objects in an association.

**narrowing conversion** A conversion from a type that has a larger range of values to a type that has a smaller one.

**nested class** See *inner class*.

**network** A system consisting of interconnected entities.

**newline** The special character that indicates the end of a line of input or output.

**new operator** The Java operator that creates objects (or instances) of a class.

**node** An object to store data in a linked list or tree. This object will also contain references to other nodes.

**object** An example or instance of a class. Internally, it is an area of memory that is structured as defined by a class. The methods of that class operate on the values defined within this memory area.

**object-oriented design** A design approach that identifies the entities, or objects, that participate in a problem or system and then designs classes to model these objects within a program.

**onto mapping** A mapping in which each value in the value set is mapped to by at least one member of the key set.

**open-box testing** See *glass-box testing*.

**operations** The methods defined in a class.

**operator** For classes, operator is another name for *method*. For primitive types, it represents a predefined function on one or two values (e.g., addition).

**output buffer** A memory area in which information written to an output stream is stored prior to being written to disk.

**override** Replace a method inherited from a superclass by one defined in a subclass.

**package** A grouping of classes under a common package name.

**package visibility** A level of visibility whereby variables and methods are visible to methods defined in classes within the same package.

**panel** A general-purpose GUI component that can be used as a drawing surface or to contain other GUI components.

**parent** The node that is directly above a node within a tree.

**partitioning** The process of separating a sequence into two sequences; used in quicksort.

**path** In a graph, a sequence of vertices in which each vertex is adjacent to its predecessor.

**path coverage** A measure of testing thoroughness. If a test exercises a path, then that path is considered covered. The ratio of the covered paths to the total number of paths is

- the path coverage.** See also *branch coverage* and *statement coverage*.
- phase** In the Unified Model of the software life cycle, the span of time between two major milestones.
- physical view** A view of an object that considers its actual representation in computer memory.
- pivot** In the quicksort algorithm, a value in the sequence being sorted that is used to partition the sequence. The sequence is partitioned into values that are less than or equal to the pivot and values that are greater than the pivot.
- polymorphism** Many forms or many shapes. In a Java program, a method defined in a superclass (or interface) may be called through a reference to that superclass (or interface). The actual method executed is the one that overrides that method and is defined in the concrete subclass object that is referenced by the superclass (or interface) variable.
- pop** Remove the top element of a stack.
- postcondition** An assertion that will be true after a method is executed, assuming that the preconditions were true before the method is executed.
- postfix increment** The increment operator (e.g., `i++`) that has the side effect of incrementing the variable to which it is applied, but its current value is the value of the variable before the increment takes place (e.g., `i`).
- postfix notation** A mathematical notation in which the operators appear after their operands.
- precedence** The degree of binding of infix operators. Operators of higher precedence are evaluated before operators of lower precedence.
- precondition** An assertion that must be true before a method is executed for the method to perform as specified.
- prefix increment** The increment operator (e.g., `++i`) that has the side effect of incrementing the variable to which it is applied, and its current value is the value of the variable after the increment takes place (e.g., `i + 1`).
- private visibility** A level of visibility whereby variables and methods are visible only to methods defined in the same class.
- proof by induction** A proof method that demonstrates that a proposition is true for a base case (usually 0) and then demonstrates that if the proposition is true for an arbitrary value ( $k$ ), it is then true for the successor of that value ( $k + 1$ ).
- protected visibility** A level of visibility whereby variables and methods are visible to methods defined in the same class, subclasses of that class, or the same package.
- pseudorandom** A computer-generated sequence of values that appear to be random because they pass various statistical tests that are consistent with those that would be produced by a truly random sequence.
- public visibility** A level of visibility whereby variables and methods are visible to all methods regardless of which class or package they are defined in.
- quadratic probing** In a hash table, a collision resolution technique in which the sequence of locations that are examined increases as the square of the number of probes made.
- queuing theory** The branch of mathematics developed to solve problems associated with queues by developing mathematical models for these problems.

- quicksort** A sorting algorithm in which a sequence is partitioned into two subsequences, one that is less than or equal to a pivot value and the other that is greater than the pivot value. The process is then recursively applied to the subsequences until a subsequence with one item is reached.
- random access** The ability to access any object in a collection by means of an index.
- recursive case** A case in a recursive algorithm that is solved by applying the algorithm to a transformed version of its parameter.
- recursive data structure** A data structure that is defined in terms of itself.
- recursive method** A method that calls itself.
- Red-Black tree** A self-balancing binary search tree that maintains balance by distinguishing the nodes by one of two states: “red” or “black.” Algorithms for insertion and deletion maintain balance by ensuring that the number of black nodes in any path from the root to a leaf is the same.
- refactoring** The process of improving code without changing its functionality.
- reference variable** A variable that references an object.
- rehashing** The process of moving the items in one hash table to a larger hash table using hashing to find each item’s new location.
- requirements specification** A document that specifies what a program or system is to do without specifying how it is done.
- reusable code** Code written for one program that can be used in another.
- right rotation** The transformation of a binary search tree in which the left child of the current root becomes the new root and the old root becomes the right child of the new root.
- root** The node in a tree that has no parent and is at the top level.
- run-time error** An error that is detected when the program executes. In Java, run-time errors are detected by the Java Virtual Machine.
- starter method** See *wrapper method*.
- seed** The initial value in a pseudorandom number sequence. Changing the seed causes a different sequence to be generated by the pseudorandom number generator.
- selection sort** A sort algorithm in which the smallest item is selected from the unsorted portion of the sequence and placed into the next position in the sorted portion.
- self-balancing search tree** A search tree with insertion and removal algorithms that maintain the tree in balance. See *2-3 tree*, *2-3-4 tree*, *AVL tree*, *balanced binary search tree*, and *Red-Black tree*.
- sequence diagram** A UML diagram that shows the sequence of messages between objects that are required to perform a given function or realize a use case.
- set difference** For sets A and B,  $A - B$  is the subset of a set, A, that does not contain elements of some other set, B.
- set intersection** A set of the elements that are common to two sets.
- set union** A set of the elements that are in one set or the other.

**Shell sort** A variation on insertion sort in which elements separated by a value known as the gap are sorted using the insertion sort algorithm. This process repeats using a decreasing sequence of values for the gap.

**sibling** One of two or more nodes in a tree that have a common parent.

**signature** A method's name and the types of its parameters. The return type is not part of the signature because it is illegal to have two methods with the same signature and different return types.

**simple path** A path that contains no cycles.

**simulation** The process of modeling a physical system using a computer program.

**single-step execution** In debugging, the process of executing one statement at a time so that the user may examine the values of variables after each statement is executed.

**skip-list** A randomized variant of an ordered linked list with additional parallel lists. Parallel lists at higher levels skip geometrically more items. Searching begins at the highest level to quickly get to the right part of the list, then uses progressively lower level lists. A new item is added by randomly selecting a level, then inserting it in order in the lists for that and all lower levels. With enough levels, searching is  $O(\log n)$ .

**spanning tree** A minimum subset of vertices of a connected graph that still results in a connected graph.

**stack trace** A listing of the sequence of method calls that starts where an error is detected and ends at the program invocation.

**state** The current value of all of the data fields in an object.

**statement coverage** A measure of testing thoroughness. If a test exercises a statement, then the statement is considered covered. The ratio of the covered statements to the total number of statements is the statement coverage. See also *branch coverage* and *path coverage*.

**static method** A method defined within a class but not associated with any particular object of that class.

**static variable** A variable defined in a class that is not a member of any particular object but is shared by all objects of the class.

**step into** When debugging in single-step mode, setting the next statement to be executed to be the first statement of the method. Each individual statement in the method is executed in sequence.

**step over** When debugging in single-step mode, setting the method call to be treated as a single statement.

**stepwise refinement** The process of breaking a complicated problem into simpler problems. This process is repeated with the smaller problems until a problem of solvable size is reached.

**strongly typed language** A programming language in which the type of objects is verified when arguments are bound to parameters and when values are assigned to variables. A syntax error occurs if the types are not compatible.

**stub** A dummy method that is used to test another method. A stub takes the place of a method that the method being tested calls. A stub typically will return a known result.

**subclass** A class that is an extension of another class. A subclass inherits the members of its superclass.

**subset** A set that contains only elements that are in some other set. A subset may contain any or all of the elements of the other set, or it may be the empty set.

**subtree of a node** The tree that consists of this node as its root.

**superclass** A class that has a subclass. See *subclass*.

**syntax error** An error that violates the syntax rules of the language. Syntax errors are generally the result of a mistake in entering the program into the computer (typographical error) or a misunderstanding of the language syntax. Syntax errors are detected by the compiler.

**system testing** Testing of a complete program or solution to a problem.

**tail recursion** See *last-line recursion*.

**test case** An individual test.

**test-driven development** A software-development approach that involves writing the tests and the methods in parallel. The method is refined and adapted to satisfy each new test that is introduced.

**test framework** A set of classes and procedures used to design and conduct tests.

**test harness** A method that executes the individual test cases of a test suite and records the results.

**test runner** A main method in a testing program that initiates a series of tests.

**test suite** A collection of test cases.

**throw an exception** Indicate that the situation that causes an exception has been detected.

**Timsort** A sorting method based on merge sort that takes advantage of sorted subsets that may exist in the data being sorted. Timsort is used in Java 8 to sort lists of objects.

**token** A character or string extracted from a larger string. Tokens are separated by *delimiter characters*.

**topological sort** An ordering of a sequence of items for which a partial order is defined that does not violate the partial order. For example, if *a* is defined to be before *b* (*a* is a prerequisite of *b*) by the partial order, then *a* will not appear later in the sequence than *b*. A partial order is defined by a directed acyclic graph.

**tree traversal** The process of systematically visiting each node in a tree.

**try block** A block preceded by the reserved word **try**. Part of the **try-catch-finally** sequence.

**try-catch-finally sequence** A sequence consisting of a **try** block followed by one or more **catch** clauses and optionally followed by a **finally** block. Or a **try** block followed by a **finally** block. Exceptions that are thrown by the **try** block are handled by the **catch** clauses that follow it. Statements in the **finally** block are executed either after the **try** block exits normally or when a **catch** block that handles an exception exits.

**type cast** The process of converting from one type in to another.

**unchecked exception** An exception that does not have to be declared in a **throws** statement or have the statements that might throw it enclosed within a **try** block.

**undirected edge** An edge in an undirected graph.

**undirected graph** A graph in which no edge has a direction. If  $u$  and  $v$  are vertices in a graph, then the presence of the edge  $\{u, v\}$  indicates that  $v$  is adjacent to  $u$  and  $u$  is adjacent to  $v$ . Contrast with *directed graph*.

**Unified Modeling Language (UML)** A language to describe the modeling of an object-oriented design that is unification of several previous modeling systems. Specifically, the modeling techniques developed by Booch, Jacobson, and Rumbaugh were combined to form the initial version. UML has since evolved and is defined by a standard issued by the Object Modeling Group.

**unit testing** Testing of an individual unit of a software program. In Java, a unit is generally a method or class.

**unnamed reference** See *anonymous reference*.

**unwinding the recursion** The process of returning from a sequence of method calls and forming the result.

**upcast** Casting a reference to a superclass or interface type.

**user interface (UI)** The way in which the user and a program interact, or the class that provides this interaction.

**vertices** The set of items that are part of a graph. The vertices are related to one another by edges.

**weight** A value associated with an edge in a weighted graph.

**weighted graph** A graph in which each edge is assigned a value.

**widening conversion** A conversion from a type that has a smaller set of values to one that has a larger set of values.

**window** A top-level container in a GUI application. Generally, a window is a rectangular area on the display surface. See also *frame*.

**wrapper class** A class that encapsulates a primitive data type.

**wrapper method** A method whose only purpose is to call a recursive method, perhaps providing initial values for some parameters and returning the result. Also called a starter method.

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