**Using the Poisson Process to Simulate Ice Hockey Games**

**Introduction**

Ice hockey is a contact sport played on ice, where two teams of skaters use their sticks to shoot a puck into the opponents net to score points. The teams usually consist of six players each. Each team has a goaltender and five who skate on the ice take the puck and score against the opposing team. Ice hockey is typically popular in countries with cooler climates such as Canada, Russia, the United States, the Baltic states, etc. The sport is typically played during the fall, winter and spring. Teams can only score one point at a time. Each team can also play defense to thwart the opponent from scoring points. This is typically done through defendants matching their backward speed to the opponent’s forward speed. This makes it less convoluted for the defendants to obtain the puck and go on offense.

The Poisson process is a stochastic counting process that models the times at which arrivals enter a occur. Arrivals may occur at arbitrary positive times. This means that the Poisson process is a non-decreasing, non-negative process. It is derived from the fact that if a collection of random points in some space follows forms a Poisson process, then the number of points in a region of finite size follows a Poisson distribution. The Poisson process can be useful for modeling radioactive decay, telephone calls, number of customers entering a shop, scoring in sports such as basketball, football soccer etc.

The primary focus of this project will be to use the Poisson process to simulate scoring in ice hockey games. For events to follow Poisson process, the events must be random, rare and memoryless. For hockey, this is mostly to be the case except in the case when teams pull their goalies. The scoring is random, rare and memoryless. If the scoring was not memoryless, then the goals would occur in bunches analogous to baseball. Pulling the goalie occurs when teams need an extra attacker. Teams typically pull their goalies towards the end of games or during a delayed penalty call. Therefore, because pulling the goalie does not satisfy the conditions of a Poisson process, this project will not include it.

The goal will be to determine the probability of one team winning a game against another team. To determine this probability, results by (DeJardine 2013) are used to generate a function that considers the difference in score using the Skellam distribution. Probabilities will be calculated for two National Hockey League (NHL) teams, the St. Louis Blues and the and the Boston Bruins. A discussion will follow.

**Methods**

Two teams are compared during a hockey game. The goal is to determine which team wins a game given that its regular season game or a playoff game. This can be calculated using the results from (DeJardine 2013). It is known that scoring for each team follows a Poisson process with parameter . Thus, scoring for teams one and two follow Poisson processes and , with parameters and . Therefore, the objective is to determine and . To find these probabilities, the goal differential between teams is considered. To account for this, the Skellam distribution must be introduced.

Definition: If and are two independent random variables with rates and , then the difference follows a Skellam distribution with probability density function

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where is the modified Bessel function of the first kind of order . This function is defined as

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Here, is the goal differential at time . If , team one is winning by goals and if , team two is winning by goals. Analogous to (DeJardine 2013), each scenario will be covered briefly.

At the end of the third quarter or at sixty minutes, the probability of team one winning is

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and the probability of team two winning is

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In the case that there is a tie, there are two overtime scenarios considered. The first scenario is when overtime occurs in a playoff game. Each team plays for twenty minutes quarters until one team scores. The first team to score, wins the game. The probability that team one wins a playoff game in overtime is

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and the probability of team two winning a playoff game in overtime is

,

where is the probability that the teams will be tied at the end of regulation (i.e. .

In the case of a regular season game, each team plays for a five-minute quarter until the first team scores. Since and represent the average number of goals over sixty minutes, is used to represent the five-minute quarter. Thus, the probability that team one will win within the five-minute regular season overtime is

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and the probability of team two winning is

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Shootouts are also considered during overtime in regular season games. If no team scores within the five-minute overtime, then they move on to a shootout. Shootouts are like penalty shots, where the goalie must defend against a scorer on the other team who attempts to shoot the puck into the goal from a specified distance. This is analogous penalty shots in soccer. The probability of a shootout occurring is , and each team has an equal chance of winning. Thus, the probability of a team winning after a shootout is

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Now to determine who wins a game, the probability for each scenario is summed up for each team. Thus, the probability of team one winning a regular game is

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The probability of team two winning a regular season game is

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The probability of team one winning a playoff game is

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The probability of team two winning a playoff game is

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**Results**

The code is written in R and is provided in the Appendix, along with the call functions below. It suffices to provide some examples of the Skellam distribution using two NHL hockey teams. In the 2019 playoffs, the St. Louis Blues averaged 3 goals per game and the Boston Bruins averaged 3.3125 goals per game before the start of the championship series. Given that the Blues are up by two goals, the probability that the Blues win is

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The probability of the Bruins winning, given that they are up by two goals is

Now consider the playoffs, when each team wins given that they are tied. The probability of the Blues winning is

and the probability of the bruins winning is

Now Consider the case of a regular season game when the teams are tied. The Blues averaged 3.0121 goals per game during the 2018-2019 season, while the Bruins averaged 3.1585 goals in the same season. The probability of the Blues winning is

and the probability of the Bruins winning is

**Discussion**

From the results above, the Blues have a higher chance of winning a game against the Bruins when they are ahead opposed to when they are tied. These probabilities can fluctuate with changes in the and (number of goals that the selected team is leading by) parameters. Given that the teams are tied, the probability of a Blues winning a playoff game against the Bruins are analogous to the probability of the Blues winning against the Bruins a regular season game. In both scenarios, this probability can fluctuate, depending on the values of and .

The probabilities were calculated for only two NHL teams and the data for the average scores were collected from the NHL website. It is important to note that the two values and be calculated for each pair of teams. They should not be used as a national average. It is also important to remember that there is a difference between regular season games and playoff games. Playoff games tend to be more competitive than regular season games, because teams are competing for the championship. Therefore, the and values must be calculated to for both scenarios to reflect this. These values will ultimately affect the probability of one team winning over another.

Examining the probabilities calculated and the questions asked, it can be seen that (DeJardine 2013) only calculated the probabilities for one team to win against the other given a lead or a tie. The probabilities for a team to lead given a deficit have not been considered but should be considered in a future project. Also, the probability of one team loosing against another given a lead, a tie or a deficit has not been considered either and should be considered for a future study or project. This limitation is reflected in the code below, where negatives values of the parameter will return a null value. This way the function will not return false probabilities.

**Appendix**

**Code for Skellam Distribution**

# Skellam Dist Pr(D = X-Y = d)

# Skellam\_win(L1,L2,x,team, type)

# L1 = average number of goals for team 1

# L2 = average number of goals for team 2

# x = number of goals that the selected team is winning by (Team is selected with 'team' parameter)

# team = team we are calculating probability of win for {1 if team 1, 2 if team 2}

# type = Type of game{ 'r' for regular season, 'p' for playoffs}

Skellam\_Win <- function(L1, L2, x, team, type) {

S <- 0

bess <- function(theta) {

exp(2\*sqrt(L1\*L2)\*cos(theta))\*cos(theta)

}

I <- integrate(bess, lower = 0, upper = pi)

I <- (1/pi)\*I$value

S0 = exp(-(L1+L2))\*I

# For Team 1 Winning a regular season Game

if (x >= 0 &type == 'r' & team == 1) {

for (i in 0:x) {

S = S + exp(-(L1+L2))\*((L1/L2)^(i/2))\*I

}

S = S +S0\*((L1/(L1+L2))\*(1-exp(-(L1+L2)/12)) + 0.5\*exp(-(L1+L2)/12))

if (S > 1) {

S = 1

}

return(S)

}

#Probability of Team 2 Winning a regular season game.

else if (x >= 0 & type == 'r' & team == 2){

for (i in 0:x) {

S = S + exp(-(L1+L2))\*((L2/L1)^(i/2))\*I

}

S = S +S0\*((L2/(L1+L2))\*(1-exp(-(L1+L2)/12))+0.5\*exp(-(L1+L2)/12))

if (S > 1) {

S = 1

}

return(S)

}

#Probability of Team 1 winning a playoff game

else if (x >= 0 & type == 'p' & team == 1) {

for (i in 0:x) {

S = S + exp(-(L1+L2))\*((L1/L2)^(i/2))\*I

}

S = S + S0\*(L1/(L1+L2))

if (S > 1) {

S = 1

}

return(S)

}

#Probabiltiy of Team 2 winning a playoff game

else if (x >= 0 & type == 'p' & team == 2) {

for (i in 0:x) {

S = S + exp(-(L1+L2))\*((L2/L1)^(i/2))\*I

}

S = S + S0\*(L2/(L1+L2))

if (S > 1) {

S = 1

}

return(S)

}

else {

return(NULL)

}

}

**Probability of Blues winning playoff game against Bruins while up by 2**Skellam\_Win(3, 3.3125, 2, 1, type = "p")

**Probability of Bruins winning playoff game against Blues while up by 2**

Skellam\_Win(3, 3.3125, 2, 2, type = "p")

**Probability of Blues winning playoff game against Bruins while tied**

Skellam\_Win(3, 3.3125, 0, 1, type = "p")

**Probability of Bruins winning playoff game against Blues while tied**

Skellam\_Win(3, 3.3125, 0, 2, type = "p")

**Probability of Blues winning regular season game against Bruins while tied**

Skellam\_Win(3.0121, 3.1585, 0, 1, type = "r")

**Probability of Bruins winning regular season game against Blues while tied**

Skellam\_Win(3.0121, 3.1585, 0, 2, type = "r")

**References**

Dejardine, Zachary. “Poisson Processes and Applications in Hockey.” *Https://Scholar.google.com/Scholar?Cluster=8994603499844015940&Hl=En&as\_sdt=0,14&Sciodt=0,14*, 2013, www.lakeheadu.ca/sites/default/files/uploads/77/docs/DejardineFinal.pdf.