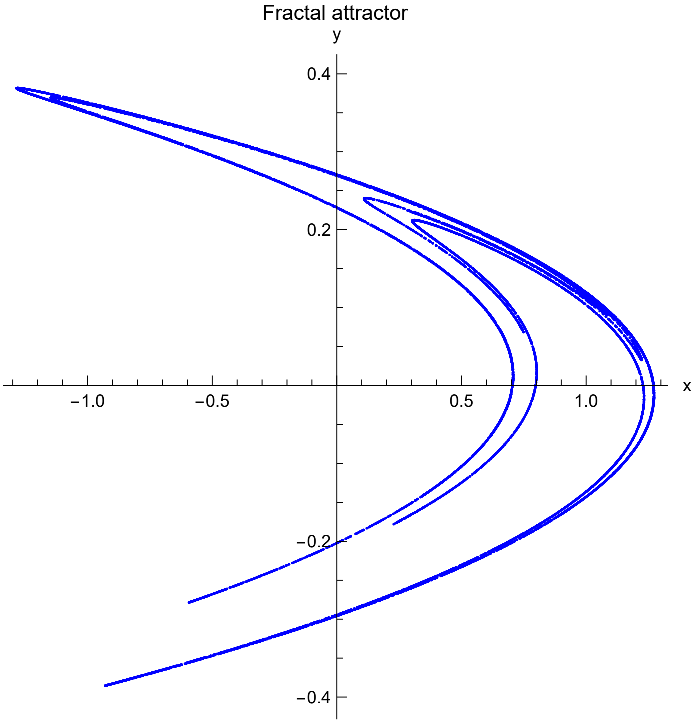
Exercise 5.2

task a)

```
In[47]:= ClearAll["Global`*"]
         a = 1.4;
         b = 0.3;
         iterations = 100000;
         transient = 5000;

         initialConditions =
           {{0, 0}, {0.1, 0}, {0, 0.1}, {-0.1, 0}, {0, -0.1}, {0.2, 0.2}, {-0.2, -0.2}};

         HenonMap[{x_, y_}] := {y + 1 - a x^2, b x};

         allPoints = Flatten[Table[NestList[HenonMap, init, iterations]⟦transient ;;⟧,
             {init, initialConditions}], 1];

         subsetPoints = RandomSample[allPoints, 15000];

         (*Due to large file, dont plot all points!*)
         ListPlot[subsetPoints,
          AspectRatio → 1,
          AxesLabel → {"x", "y"},
          PlotStyle → {Blue, PointSize[0.001]},
          PlotRange → All,
          ImageSize → Medium,
          PlotLabel → "Fractal attractor"]
```

Out[56]=

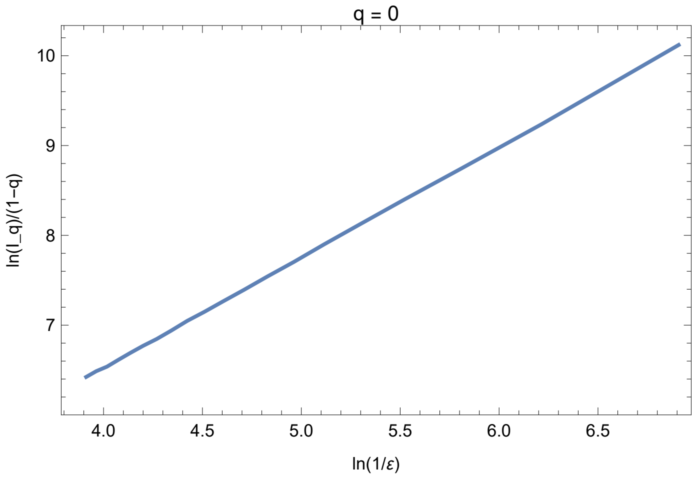


task b)

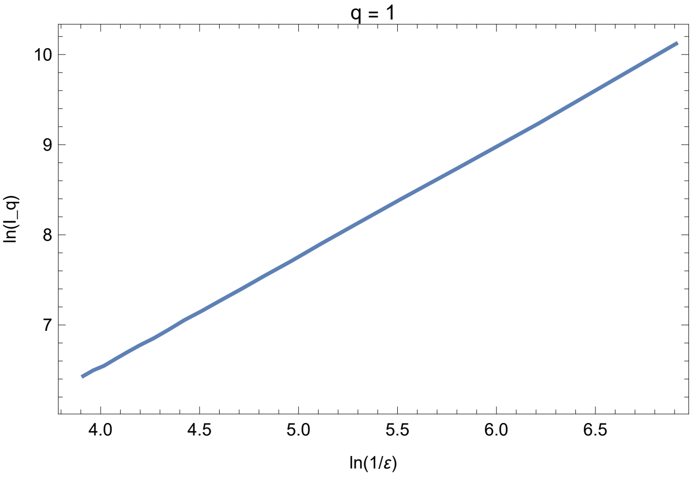
In[57]:=
```
ClearAll["Global`*"]

a = 1.4;
b = 0.3;
iterations = 2 * 10^6;
transient = 5000;
eps = Range[0.001, 0.02, 0.001];

HenonMap[{x_, y_}] := {y + 1 - a x^2, b x};

data = NestList[HenonMap, {0.1, 0.1}, iterations][[transient ;;]];
x = data[[All, 1]];
y = data[[All, 2]];

I0 = ConstantArray[0, Length[eps]];
I1 = ConstantArray[0, Length[eps]];
I2 = ConstantArray[0, Length[eps]];

xmin = Round[Min[x], 0.1];
xmax = Round[Max[x], 0.1];
ymin = Round[Min[y], 0.1];
ymax = Round[Max[y], 0.1];

Do[e = eps[[i]];
  bins = BinCounts[data, {xmin, xmax, e}, {ymin, ymax, e} ];
  P = Flatten[bins];
  P = Select[P, # > 0 &];
```

```
    P = P / Total[P];
    I0⟦i⟧ = Total[P^0];
    I1⟦i⟧ = Total[(1 / P)^P];
    I2⟦i⟧ = Total[P^2];
    ,
    {i, Length[eps]}];

logInvEpsilon = Log[1 / eps];

q0Plot = ListLinePlot[Transpose[{logInvEpsilon, Log[I0]}],
    PlotLabel → "q = 0",
    Frame → True,
    FrameLabel → {"ln(1/ε)", "ln(I_q)/(1-q)"},
    ImageSize → Medium]
q1Plot = ListLinePlot[Transpose[{logInvEpsilon, Log[I1]}],
    PlotLabel → "q = 1",
    Frame → True,
    FrameLabel → {"ln(1/ε)", "ln(I_q)"},
    ImageSize → Medium]
q2Plot = ListLinePlot[Transpose[{logInvEpsilon, -Log[I2]}],
    PlotLabel → "q = 2",
    Frame → True,
    FrameLabel → {"ln(1/ε)", "ln(I_q)/(1-q)"},
    ImageSize → Medium]
```

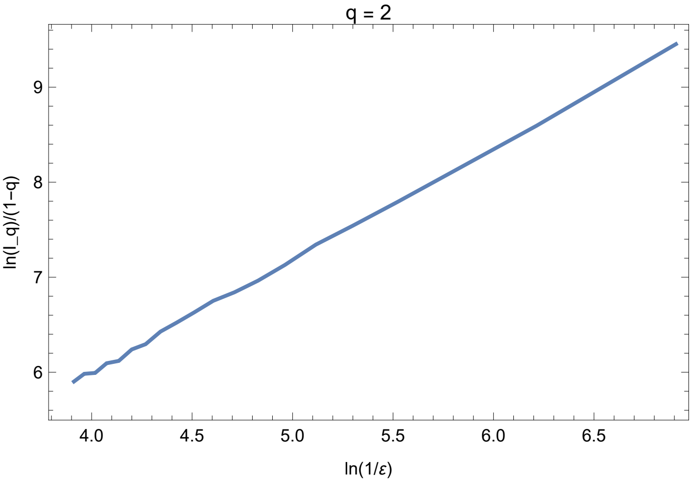
Out[76]=



Out[77]=



Out[78]=



task c)

```
In[79]:= ClearAll[modelD0, modelD1, modelD2, D0, D1, D2, x];

         modelD0 = LinearModelFit[Transpose[{logInvEpsilon, Log[I0]}], x, x];
         modelD1 = LinearModelFit[Transpose[{logInvEpsilon, Log[I1]}], x, x];
         modelD2 = LinearModelFit[Transpose[{logInvEpsilon, -Log[I2]}], x, x];

         D0 = modelD0["BestFit"][[2]]
         D1 = modelD1["BestFit"][[2]]
         D2 = modelD2["BestFit"][[2]]
```

```
Out[83]=
         1.23212 x
Out[84]=
         1.22867 x
Out[85]=
         1.18066 x
```

task d)

```
In[86]:= (*Could not make it work.*)
```

task e)

```
In[87]:= (*Based of my code from 4.3*)
         ClearAll["Global`*"]
         a = 1.4;
         b = 0.3;

         tmax = 100 000;

         x0 = 0.1;
         y0 = 0.1;

         HenonStep[{x_, y_}] := {y + 1 - a x^2, b x};

         jacobi[{x_, y_}] := {{-2 a x, 1}, {b, 0}};

         trajectory = Table[0, {tmax}];
         trajectory⟦1⟧ = {x0, y0};

         Q = IdentityMatrix[2];
         R = IdentityMatrix[2];

         λAccumulate = ConstantArray[0., 2];

         For[k = 2, k ≤ tmax, k++,
           trajectory⟦k⟧ = HenonStep[trajectory⟦k - 1⟧];
           J = jacobi[trajectory⟦k - 1⟧];
           M = J.Transpose[Q];
           {Qnew, R} = QRDecomposition[M];
           Q = Qnew;
           λAccumulate += Log[Abs[Diagonal[R]]]
          ];

         λ = λAccumulate / tmax
Out[101]=
         {0.417962, -1.62192}
```

task f)

In[102]:=

```
ClearAll[k, m, point1, point2]
point1 = {1, λ[[1]]};
point2 = {2, λ[[1]] + λ[[2]]};

(*y = kx+m => when x=0 => y=m, k=Δy/Δx*)
k = (point2[[2]] - point1[[2]]) / (point2[[1]] - point1[[1]]);

m = point1[[2]] - k * point1[[1]];

y[x_] := k * x + m;
DL = Solve[y[x] == 0, x]
(*Very close to my D_1*)
```

Out[108]=

```
{{x → 1.2577}}
```

In[109]:=

```
ClearAll;
```