In[57]:=
```
ClearAll["Global`*"];
Remove["Global`*"];
f[x_, r_] := (1 / 5) + (7 / 10) * (1 / (1 + Exp[80 * (1 - x)])) - r * x^4
f1[x_] = (1 / 5) + (7 / 10) * (1 / (1 + Exp[80 * (1 - x)]))
f2[x_, r_] = r * x^4

Manipulate[Show[Plot[{f1[x], f2[x, r]}, {x, -2, 2},
    PlotRange → {Full, {0, 1}}],
   ListPlot[{x, f2[x, r]} /. NSolve[f2[x, r] == f1[x], x, Reals],
    PlotStyle → Red]], {r, 0, 1}]
```

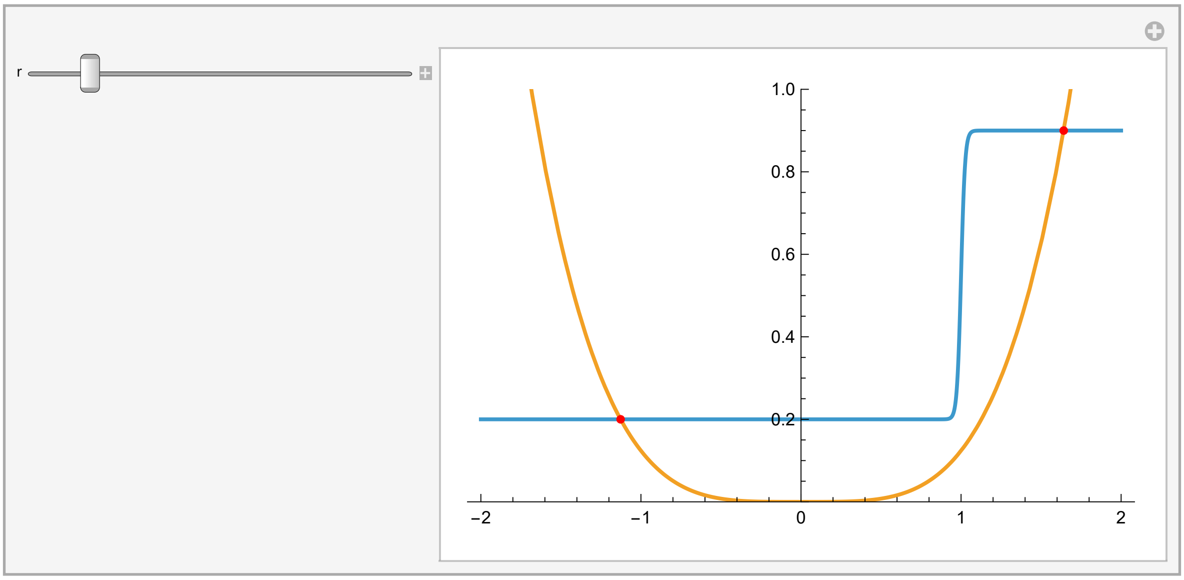
Out[60]=

$$\frac{1}{5}+\frac{7}{10\left(1+e^{80\,(1-x)}\right)}$$

Out[61]=

$$r\,x^4$$

Out[62]=



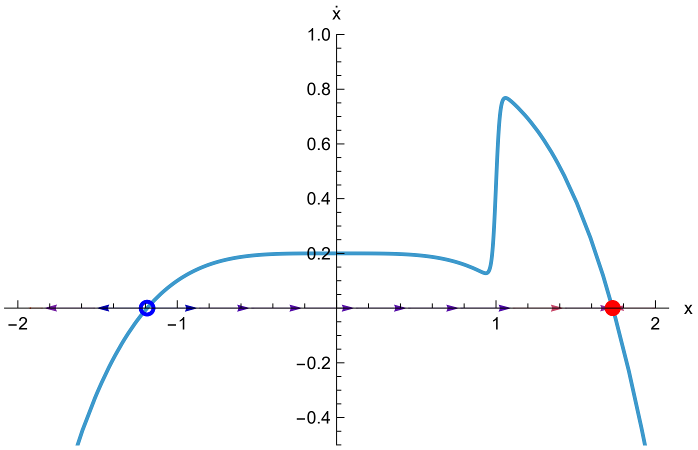
In[7]:=

Out[7]=

In[8]:=

```
(*rc =0.261*)
r = 0.1;
sol[rvar_] := NSolve[f[x, rvar] == 0, x, Reals];
p1 = Plot[f[x, r], {x, -2, 2},
   PlotRange → {Full, {-0.5, 1}}, AxesLabel → {"x", OverDot["x"]}];
p2 = StreamPlot[{f[x, r], 0}, {x, -2, 2}, {y, -0.1, 0.1}];
p3 = ListPlot[{{x /. sol[r]〚2〛, 0}}, PlotMarkers → {Graphics[{Red, Disk[]}], 10}];
p4 = ListPlot[{{x /. sol[r]〚1〛, 0}}, PlotMarkers → {Graphics[{Blue, Circle[]}], 10}];
Show[p1, p2, p3, p4]
```

Out[14]=



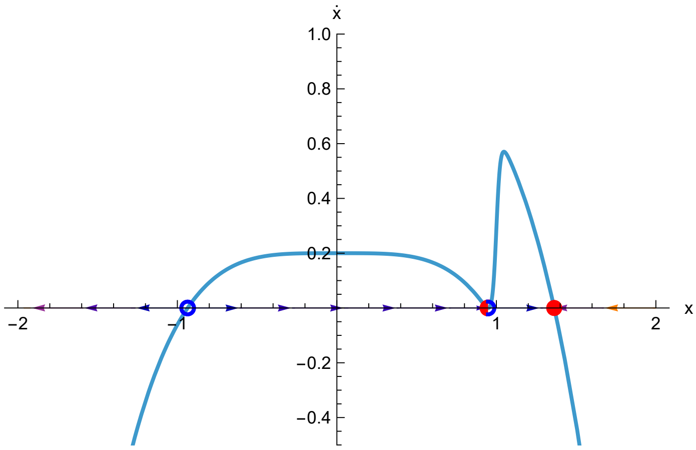
First: "{} has zero length and no first element."

Out[∘]=

```
First[{}]
```

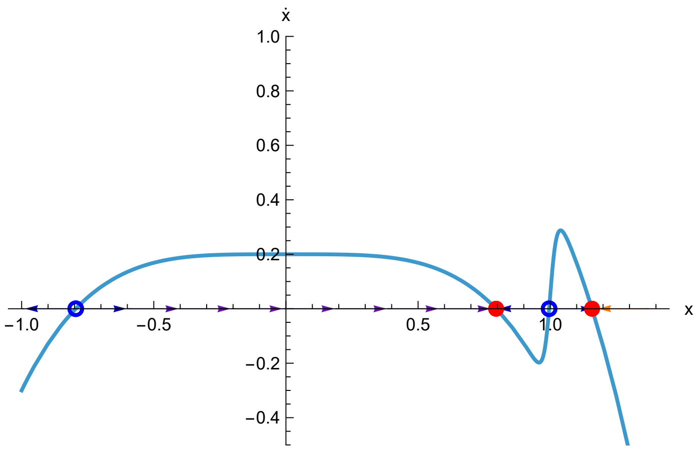
In[15]:=
```
r = 0.261;
sol[rvar_] := NSolve[f[x, rvar] == 0, x, Reals];
p1 = Plot[f[x, r], {x, -2, 2},
   PlotRange → {Full, {-0.5, 1}}, AxesLabel → {"x", OverDot["x"]}];
p2 = StreamPlot[{f[x, r], 0}, {x, -2, 2}, {y, -0.1, 0.1}];
p3 =
  ListPlot[{{x /. sol[r]⟦1⟧, 0}}, PlotMarkers → {Graphics[{Blue, Circle[]}], 10}];
p4 = ListPlot[{{x /. sol[r]⟦2⟧, 0}}, PlotMarkers →
    {Graphics[{Red, Disk[{0, 0}, 1, {Pi / 2, 3 Pi / 2}], Red, Circle[{0, 0}, 1,
        {Pi / 2, 3 Pi / 2}], Blue, Circle[{0, 0}, 1, {-Pi / 2, Pi / 2}]}], 10}];
p5 = ListPlot[{{x /. sol[r]⟦4⟧, 0}}, PlotMarkers → {Graphics[{Red, Disk[]}], 10}];
Show[p1, p2, p3, p4, p5]
```

Out[22]=



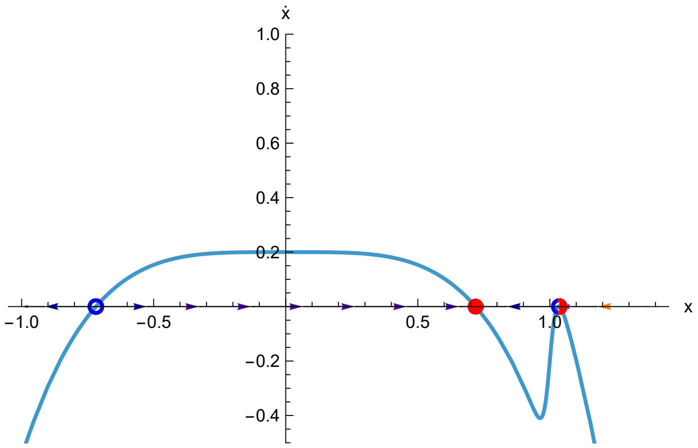
In[23]:=
```
r = 0.5;
sol[rvar_] := NSolve[f[x, rvar] == 0, x, Reals];
p1 = Plot[f[x, r], {x, -1, 1.4},
   PlotRange → {Full, {-0.5, 1}}, AxesLabel → {"x", OverDot["x"]}];
p2 = StreamPlot[{f[x, r], 0}, {x, -1, 1.4}, {y, -0.05, 0.05}];
p3 =
  ListPlot[{{x /. sol[r]⟦1⟧, 0}}, PlotMarkers → {Graphics[{Blue, Circle[]}], 10}];
p4 = ListPlot[{{x /. sol[r]⟦2⟧, 0}}, PlotMarkers → {Graphics[{Red, Disk[]}], 10}];
p5 = ListPlot[{{x /. sol[r]⟦3⟧, 0}}, PlotMarkers → {Graphics[{Blue, Circle[]}], 10}];
p6 = ListPlot[{{x /. sol[r]⟦4⟧, 0}}, PlotMarkers → {Graphics[{Red, Disk[]}], 10}];
Show[p1, p2, p3, p4, p5, p6]
```

Out[31]=



In[32]:=
```
(*This is r_c since it is the largest r that
  causes a change in the number of fixed points. For r > 0.749,
we will always have 2 fixed points. Moreover,
for r > 0.749 the temperature will always decrease
  indicating that the dynamics bifurcates to a snowball earth. *)
r = 0.749;
sol[rvar_] := NSolve[f[x, rvar] == 0, x, Reals];
p1 = Plot[f[x, r], {x, -1, 1.4},
   PlotRange → {Full, {-0.5, 1}}, AxesLabel → {"x", OverDot["x"]}];
p2 = StreamPlot[{f[x, r], 0}, {x, -1, 1.4}, {y, -0.05, 0.05}];
p3 =
  ListPlot[{{x /. sol[r][[1]], 0}}, PlotMarkers → {Graphics[{Blue, Circle[]}], 10}];
p4 = ListPlot[{{x /. sol[r][[2]], 0}}, PlotMarkers → {Graphics[{Red, Disk[]}], 10}];
p5 = ListPlot[{{x /. sol[r][[4]], 0}}, PlotMarkers →
    {Graphics[{Red, Disk[{0, 0}, 1, {-Pi / 2, Pi / 2}], Blue, Circle[{0, 0}, 1,
        {Pi / 2, 3 Pi / 2}], Red, Circle[{0, 0}, 1, {-Pi / 2, Pi / 2}]}], 10}];
Show[p1, p2, p3, p4, p5]
```

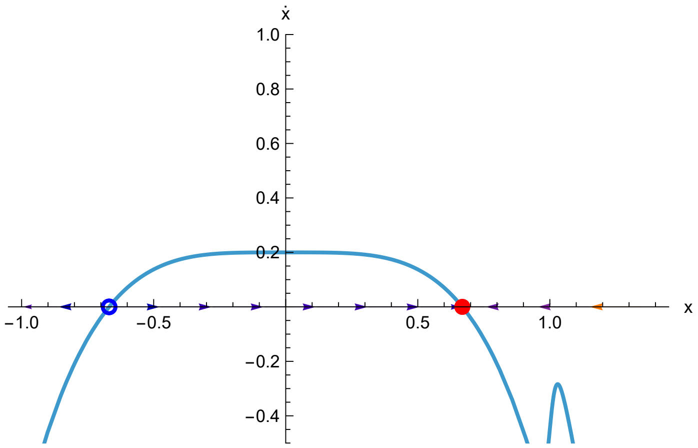
Out[39]=



In[40]:=
```
(*Not necessary to keep.*)
r = 1;
sol[rvar_] := NSolve[f[x, rvar] == 0, x, Reals];
p1 = Plot[f[x, r], {x, -1, 1.4},
   PlotRange → {Full, {-0.5, 1}}, AxesLabel → {"x", OverDot["x"]}];
p2 = StreamPlot[{f[x, r], 0}, {x, -1, 1.4}, {y, -0.05, 0.05}];
p3 =
  ListPlot[{{x /. sol[r][[1]], 0}}, PlotMarkers → {Graphics[{Blue, Circle[]}], 10}];
p4 = ListPlot[{{x /. sol[r][[2]], 0}}, PlotMarkers → {Graphics[{Red, Disk[]}], 10}];

Show[p1, p2, p3, p4]
```

Out[46]=



In[47]:=
```
ClearAll["Global`*"];
Remove["Global`*"];
f[x_, r_] := (1/5) + (7/10) * (1/(1 + Exp[80 * (1 - x)])) - r * x^4

(*Range for r values*)
rValues = N[Range[0, 1, 1/1000]];

interval1Points = {}; (*(-∞,0)*)
interval2Points = {}; (*(0,0.95)*)
interval3Points = {}; (*(0.95,1.04)*)
interval4Points = {}; (*(1.04,∞)*)

Do[solutions = NSolve[f[x, r] == 0, x, Reals];
  Do[point = {r, x} /. solution;
   If[point[[2]] < 0, AppendTo[interval1Points, point],
    If[0 ≤ point[[2]] < 0.95, AppendTo[interval2Points, point],
     If[0.95 < point[[2]] < 1.04, AppendTo[interval3Points, point],
      (*"Else""*)AppendTo[interval4Points, point]]]],
   {solution, solutions}], {r, rValues}];


Show[
 ListPlot[interval1Points, PlotRange → {{0, 1}, {-2, 2}},
  PlotStyle → {Red, Dashed}, Joined → True],
 ListPlot[interval2Points, PlotRange → {{0, 1}, {-2, 2}},
  PlotStyle → {Red}, Joined → True], ListPlot[interval3Points,
  PlotRange → {{0, 1}, {-2, 2}}, PlotStyle → {Red, Dashed}, Joined → True],
 ListPlot[interval4Points,
  PlotRange → {{0, 1}, {-2, 2}}, PlotStyle → {Red}, Joined → True],
 Graphics[{
   {Black, Arrow[{{0.8, 1.5}, {0.749, 1.04}}],
    Text[Style["Saddle-Node", Black], {0.8, 1.6}]}, {Black,
    Arrow[{{0.2, 0.5}, {0.261, 0.95}}], Text[Style["Saddle Node", Black], {0.2, 0.4}]}
  }],
 AxesLabel → {"r", "x"}, PlotLabel → "Bifurcation Diagram"
]
```

Out[56]=

Bifurcation Diagram