Exercise 4.1

task a)

In[61]:=
```
ClearAll["Global`*"]
σ = 10;
b = 8 / 3;
r = 28;

xdot[x_, y_, z_] := σ * (y - x)
ydot[x_, y_, z_] := r * x - y - x * z
zdot[x_, y_, z_] := x * y - b * z

sol = Solve[{xdot[x, y, z] == 0, ydot[x, y, z] == 0, zdot[x, y, z] == 0}, {x, y, z}]
```

Out[68]=

$$\{\{x \to 0, y \to 0, z \to 0\}, \{x \to -6\sqrt{2}, y \to -6\sqrt{2}, z \to 27\}, \{x \to 6\sqrt{2}, y \to 6\sqrt{2}, z \to 27\}\}$$

```
Clear[eig1, eig2, eig3]
jacobi = D[{xdot[x, y, z], ydot[x, y, z], zdot[x, y, z]}, {{x, y, z}}] ;
eig1 = Eigenvalues[jacobi /. sol[[1]]]
eig2 = Eigenvalues[jacobi /. sol[[2]]]
eig3 = Eigenvalues[jacobi /. sol[[3]]]
```

Out[90]=

$$\left\{\frac{1}{2}\left(-11-\sqrt{1201}\right), \frac{1}{2}\left(-11+\sqrt{1201}\right), -\frac{8}{3}\right\}$$

Out[91]=

$\{-13.9\ldots, 0.0940\ldots + 10.2\ldots\, i, 0.0940\ldots - 10.2\ldots\, i\}$

Out[92]=

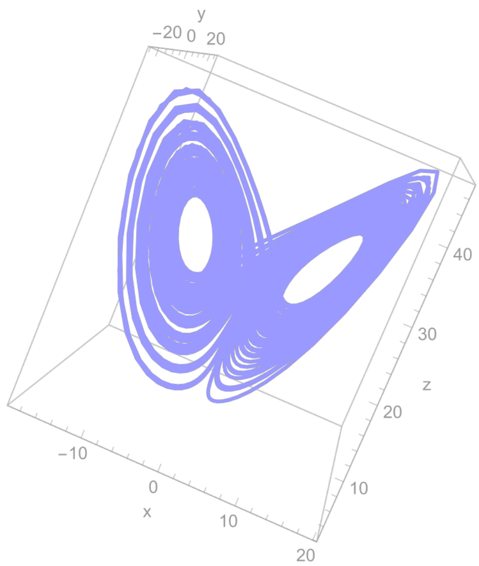
$\{-13.9\ldots, 0.0940\ldots + 10.2\ldots\, i, 0.0940\ldots - 10.2\ldots\, i\}$

For a fixed point to be stable, we need ALL Re(λ) < 0, thus this system only has unstable fixed points.

task b)

```
ClearAll["Global`*"]

σ = 10;
b = 8 / 3;
r = 28;

xdot[x_, y_, z_] := σ (y - x)
ydot[x_, y_, z_] := r x - y - x z
zdot[x_, y_, z_] := x y - b z

initialConditions = {x[0] == 0.1, y[0] == 0.1, z[0] == 0.1};
timeRange = {t, 0, 100};

lorenzEquations = {
   x'[t] == xdot[x[t], y[t], z[t]],
   y'[t] == ydot[x[t], y[t], z[t]],
   z'[t] == zdot[x[t], y[t], z[t]],
   initialConditions};

sol = NDSolve[lorenzEquations, {x, y, z}, timeRange];

ParametricPlot3D[
 Evaluate[{x[t], y[t], z[t]} /. sol], {t, 20, 100},
 PlotRange → All,
 AxesLabel → {"x", "y", "z"},
 PlotStyle → {Thick, Blue}]
```

Out[166]=



task c)

```
ClearAll["Global`*"]
xdot[x_, y_, z_] := σ * (y - x);
ydot[x_, y_, z_] := r * x - y - x * z;
zdot[x_, y_, z_] := x * y - b * z;
```

```
jacobi = D[{xdot[x, y, z], ydot[x, y, z], zdot[x, y, z]}, {{x, y, z}}]
```

Out[194]=

```
{{-σ, σ, 0}, {r - z, -1, -x}, {y, x, -b}}
```

task d)

In[195]:=

```
Tr[jacobi]
```

Out[195]=

```
-1 - b - σ
```