Exercise 2.4

In[230]:=
```
ClearAll["Global`*"]
ϕdot[ω_] := ω
ωdot[ϕ_, τ_] := Sin[ϕ] * (Cos[ϕ] - τ - 1)

H[ϕ_, ω_, τ_, C_] = Integrate[ϕdot[ω], ω] - Integrate[ωdot[ϕ, τ], ϕ] + C
```

Out[233]=
$$C + \frac{\omega^2}{2} - \text{Cos}[\phi] - \tau\,\text{Cos}[\phi] + \frac{\text{Cos}[\phi]^2}{2}$$

Set prefactor of ω to +1:

In[234]:=
```
Hnormalized[ϕ_, ω_, τ_, C_] = 2 * H[ϕ, ω, τ, C ] // Simplify
```

Out[234]=
$$2\,C + \omega^2 - 2\,(1 + \tau)\,\text{Cos}[\phi] + \text{Cos}[\phi]^2$$

Find C such that the system is -1.

In[235]:=
```
solC = Solve[Hnormalized[π / 2, 0, τ, C] == -1, C]
```

Out[235]=
$$\left\{\left\{C \to -\frac{1}{2}\right\}\right\}$$

Double check solution.

In[236]:=
```
Hnormalized[π / 2, 0, τ, C /. solC[[1]]]
```

Out[236]=
$$-1$$

Equal to -1, OK!

Insert C in the conserved quantity.

In[237]:=
```
Hnormalized[ϕ, ω, τ, C /. solC[[1]]]
```

Out[237]=
$$-1 + \omega^2 - 2\,(1 + \tau)\,\text{Cos}[\phi] + \text{Cos}[\phi]^2$$