Exercise 2.1 a)

$\sigma = -1$

In[351]:=

```
ClearAll["Global`*"]
f1[x_, y_, sigma_] := (sigma + 3) * x + 4 * y
f2[x_, y_, sigma_] := -(9 / 4) * x + (sigma - 3) * y

sol = Solve[
  {f1[x, y, -1] == 0,
   f2[x, y, -1] == 0},
  {x, y}]

p1 = StreamPlot[
    {f1[x, y, -1], f2[x, y, -1]},
    {x, -10, 10},
    {y, -10, 10},
    AxesLabel → {"x", "y"},
    PlotRange → All,
    StreamStyle → Blue,
    Frame → True,
    FrameLabel → {"x", "y"}
   ];

fp = {x, y} /. sol[[1]]
p2 = Graphics[{Red, PointSize[Large], Point[fp] }];

Show[p1, p2,
 PlotLabel → "Fixed Point: " <> ToString[fp],
 FontSize → 14,
 FontWeight → Bold,
 Epilog → {Text[Style["Stable Fixed Point", Red, Bold, 10], fp + {1, 1}]}
]
```

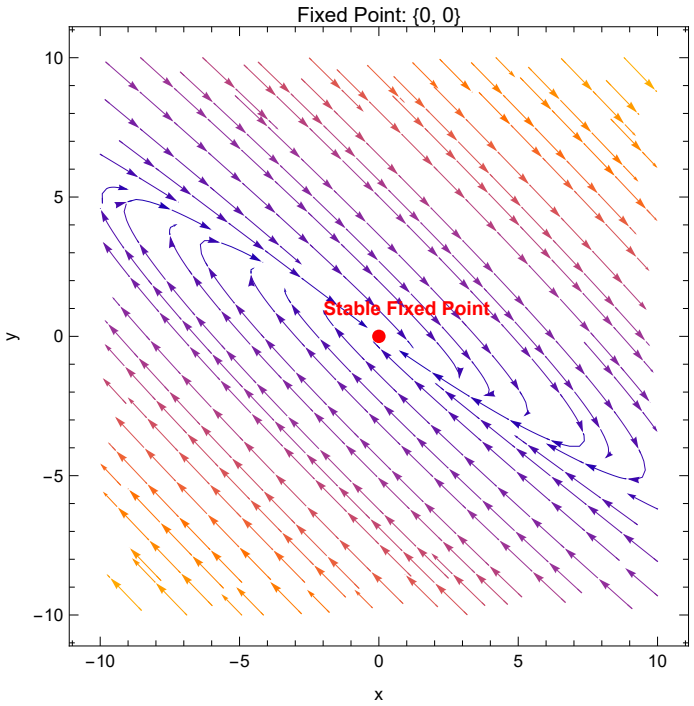
Out[354]=

```
{{x → 0, y → 0}}
```

Out[356]=

```
{0, 0}
```

Out[358]=



$\sigma = 0$

In[399]:=

```
ClearAll["Global`*"]
f1[x_, y_, sigma_] := (sigma + 3) * x + 4 * y;
f2[x_, y_, sigma_] := -(9 / 4) * x + (sigma - 3) * y;

sol = Solve[
  {f1[x, y, 0] == 0,
   f2[x, y, 0] == 0},
  {x, y}]

J[x_, y_] = D[{f1[x, y /. sol[[1]], 0], f2[x, y /. sol[[1]], 0]}, {{x, y}}]

Eigenvalues[J[x, y]]
(*Both eigenvalues = 0, thus line of fixed points neither stable or unstable.*)

p1 = StreamPlot[
    {f1[x, y, 0], f2[x, y, 0]},
    {x, -10, 10},
    {y, -10, 10},
    AxesLabel → {"x", "y"},
    PlotRange → All,
    StreamStyle → Blue, Frame → True, FrameLabel → {"x", "y"}
  ];

fp[x_] = y /. sol[[1]]

fpPlot = Plot[fp[x], {x, -10, 10},
    PlotStyle → {Red, Thick},
    PlotRange → Full
  ];

Show[p1, fpPlot,
 PlotLabel → "Fixed Point line: y = -3x/4",
 PlotRange → All,
 FontSize → 14,
 FontWeight → Bold]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[402]=

$$\left\{\left\{y \to -\frac{3\,x}{4}\right\}\right\}$$

Out[403]=

```
{{0, 0}, {0, 0}}
```

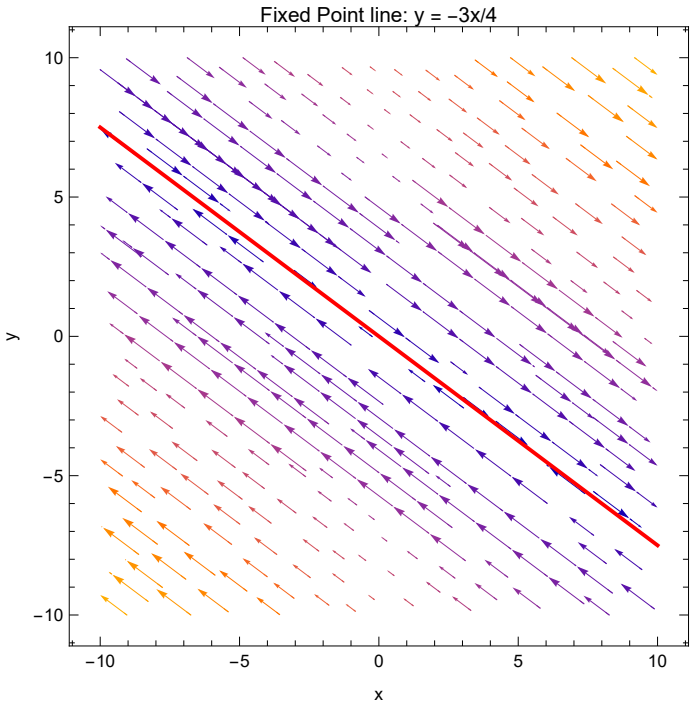
Out[404]=

```
{0, 0}
```

Out[406]=

$$-\frac{3\,x}{4}$$

Out[408]=



For sigma = 0, we get a line of fixed points, and since the stability matrix J and both eigenvalues are 0 we get $\tau = \Delta = 0$, thus the line of fixed points are neither stable or unstable.

$\sigma = -1$

In[409]:=

```
ClearAll["Global`*"]
f1[x_, y_, sigma_] := (sigma + 3) * x + 4 * y
f2[x_, y_, sigma_] := -(9 / 4) * x + (sigma - 3) * y

sol = Solve[
  {f1[x, y, 1] == 0,
   f2[x, y, 1] == 0} ,
  {x, y}]

p1 = StreamPlot[
   {f1[x, y, 1], f2[x, y, 1]},
   {x, -10, 10},
   {y, -10, 10},
   AxesLabel → {"x", "y"},
   PlotRange → All,
   StreamStyle → Blue, Frame → True, FrameLabel → {"x", "y"}
  ];
fp = {x, y} /. sol[[1]]

p2 = Graphics[
   {Red, PointSize[Large],
    Point[fp] }];

Show[p1, p2,
 PlotLabel → "Fixed Point: " <> ToString[fp],
 FontSize → 14,
 FontWeight → Bold,
 Epilog → {Text[Style["Unstable Fixed Point", Red, Bold, 10], fp + {1, 1}]}
]
```

Out[412]=

```
{{x → 0, y → 0}}
```

Out[414]=

```
{0, 0}
```

Out[416]=