Exercise 5

task a)

Make a table similar to the lecture:

n	N(ϵ)	ϵ
0	1	1
1	4	1/3
2	16	1/9

Box-Counting dimension is given by:

$$D = \lim_{\epsilon \to \infty} \frac{ln(N(\epsilon))}{ln(1/\epsilon)} = \frac{ln(4^n)}{ln(3^n)} = \frac{n * ln(4)}{n * ln(3)} = \frac{ln(4)}{ln(3)}$$

task b)

We have $\lambda_a = 1/4$, $\lambda_b = 1/2$, and for each ϵ we have $N_{box}(\epsilon) = 4 * N_a(\epsilon) + 1 * N_b(\epsilon)$

From self similarity we have:
$N_{box}(\epsilon / \lambda_a) = N_a(\epsilon)$
$N_{box}(\epsilon / \lambda_b) = N_b(\epsilon)$

D0 is defined from: $N_{box}(\epsilon) = A\,\epsilon^{-D_0}$

Putting everything together we get:

$A\,\epsilon^{-D_0} = 4 * A\,(\epsilon / \lambda_a)^{-D_0} + A\,(\epsilon / \lambda_b)^{-D_0}$

=>

$\epsilon^{-D_0} = 4 * (\epsilon / \lambda_a)^{-D_0} + (\epsilon / \lambda_b)^{-D_0}$

=>

$1 = 4 * (1 / \lambda_a)^{-D_0} + (1 / \lambda_b)^{-D_0}$

=>

$1 = 4 * \lambda_a^{D_0} + \lambda_b^{D_0}$

In[6]:=
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λa = 1 / 4;
λb = 1 / 2;
sol = Solve[4 λa^D0 + λb^D0 == 1, D0, Reals]
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Out[8]= $\left\{\left\{D0 \to \frac{-\mathrm{Log}[2] + \mathrm{Log}\left[1 + \sqrt{17}\right]}{\mathrm{Log}[2]}\right\}\right\}$