Q1

In [1]:
```
import numpy as np

k_B = 1.380*10**-23
T= 300
eta = 10**-3
R=10**-6
k_x = 10**-6
k_y = 9*10**-6

gamma = 6*np.pi*eta*R

D = (k_B*T)/gamma

##Q1
tau_trap_x = gamma/k_x
tau_trap_y = gamma/k_y

print("tau_trap_x:", tau_trap_x,"\ntau_trap_y:", tau_trap_y)
```

```
tau_trap_x: 0.018849555921538756
tau_trap_y: 0.0020943951023931952
```

- τ_{trap} is the relaxation time, and choosing dt much smaller than the smallest relaxation time for x and y ensures that the simulation captures the fastest dynamics in both directions.

  To be on the safe side, i also divide the smallest τ_{trap} by 10, such that it is "much smaller".

- Since $k_y > k_x$ the restoring force in the y-direction is stronger, meaning the particle relaxes faster (smaller τ_{trap}) in the y-direction.

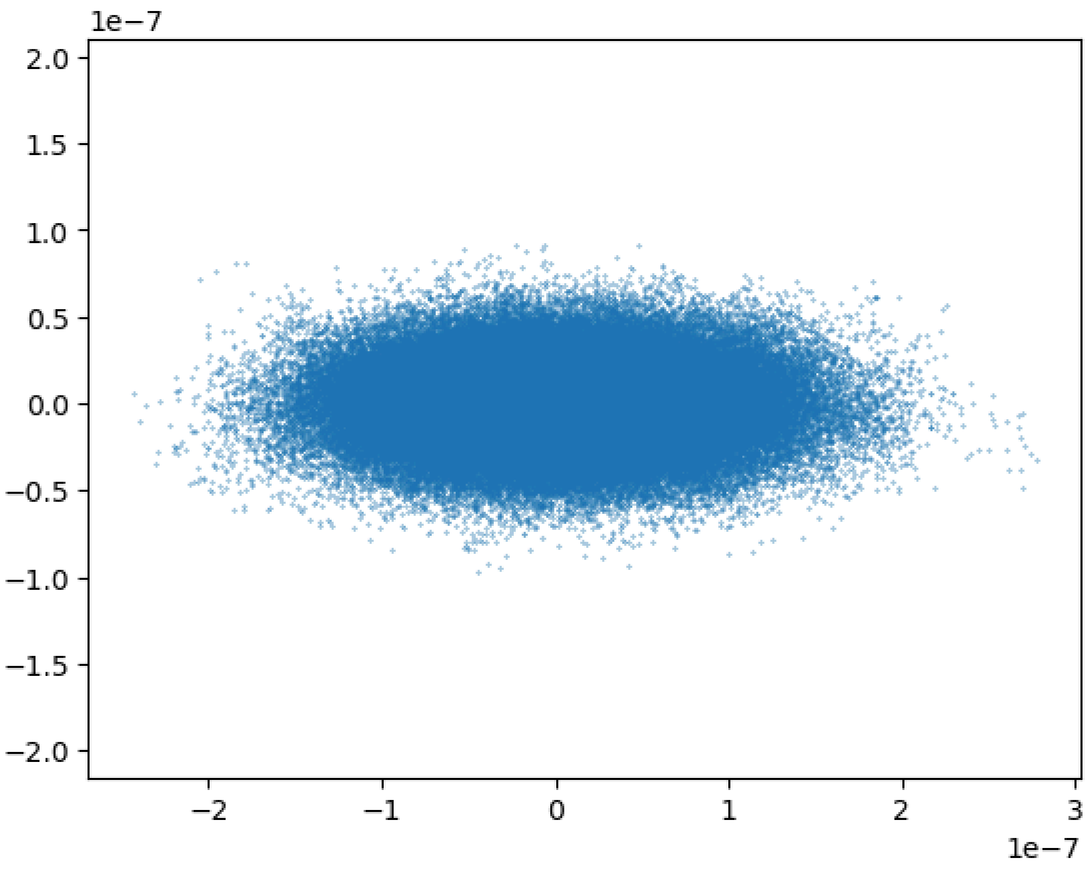
In [2]:
```
min_tau = np.min((tau_trap_x,tau_trap_y))
dt = min_tau/10
print("dt=",dt)
```

```
dt= 0.00020943951023931953
```

P1

In [3]:
```
import matplotlib.pyplot as plt
t_tot = 30
N= int(t_tot/dt)

x = np.zeros(N)
y = np.zeros(N)
w_x=np.random.normal(0,1,N)
w_y=np.random.normal(0,1,N)
for i in range(N-1):
    x[i+1] = x[i] - k_x*x[i]*dt/gamma + np.sqrt(2*k_B*T*dt/gamma)*w_x[i]
    y[i+1] = y[i] - k_y*y[i]*dt/gamma + np.sqrt(2*k_B*T*dt/gamma)*w_y[i]
```

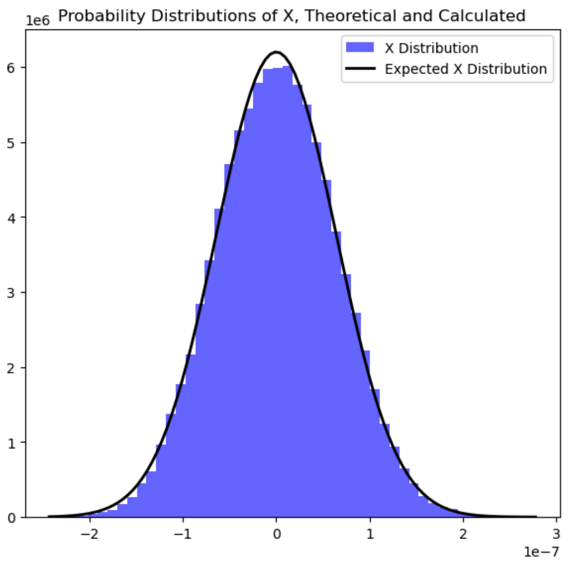
```
### P1
##scale to nm
plt.plot(x,y,'.',markersize=0.6)
plt.axis('equal')
plt.show()
```

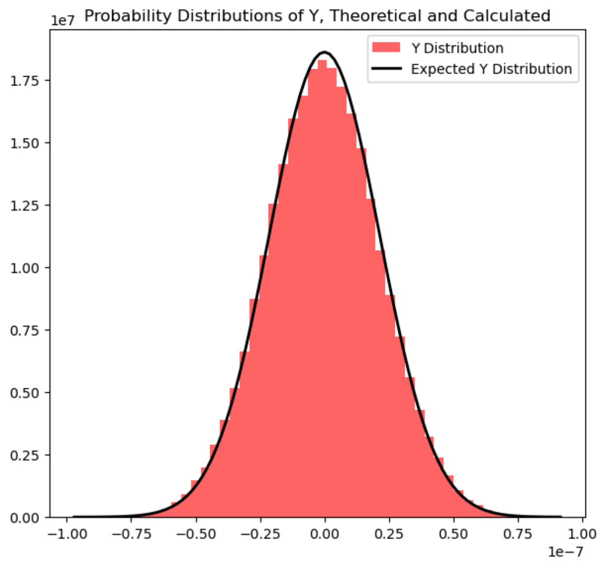


P2

In [4]:
```
#### P2
fig, axes = plt.subplots(1, 2, figsize=(12, 6))

# Plot for X distribution and theoretical distribution
axes[0].hist(x, bins=50, density=True, alpha=0.6, color='blue', label='X Distrib
x_generated = np.linspace(np.min(x), np.max(x), 100)
U_x = 0.5 * k_x * x_generated**2
p_x = np.exp(-U_x / (k_B * T))
p_x /= np.sum(p_x) * (x_generated[1] - x_generated[0])  # Normalize
axes[0].plot(x_generated, p_x, color='black', label='Expected X Distribution', l
axes[0].set_title('Probability Distributions of X, Theoretical and Calculated')
axes[0].legend()

# Plot for Y distribution and theoretical distribution
axes[1].hist(y, bins=50, density=True, alpha=0.6, color='red', label='Y Distribu
y_generated = np.linspace(np.min(y), np.max(y), 100)
U_y = 0.5 * k_y * y_generated**2
p_y = np.exp(-U_y / (k_B * T))
p_y /= np.sum(p_y) * (y_generated[1] - y_generated[0])  # Normalize
axes[1].plot(y_generated, p_y, color='black', label='Expected Y Distribution', l
axes[1].set_title('Probability Distributions of Y, Theoretical and Calculated')
```

```
axes[1].legend()

plt.tight_layout()
plt.show()
```





Q2

In [5]:
```
sigma_x = np.var(x)
sigma_y = np.var(y)

harmonic_trap_x = k_B*T/k_x
harmonic_trap_y = k_B*T/k_y

print("sigma_x:",sigma_x, "vs Harmonic_trap_x:", harmonic_trap_x)
print("sigma_y:",sigma_y, "vs Harmonic_trap_y:", harmonic_trap_y)
```

```
sigma_x: 4.076958049174085e-15 vs Harmonic_trap_x: 4.1399999999999994e-15
sigma_y: 4.794308002801998e-16 vs Harmonic_trap_y: 4.599999999999999e-16
```

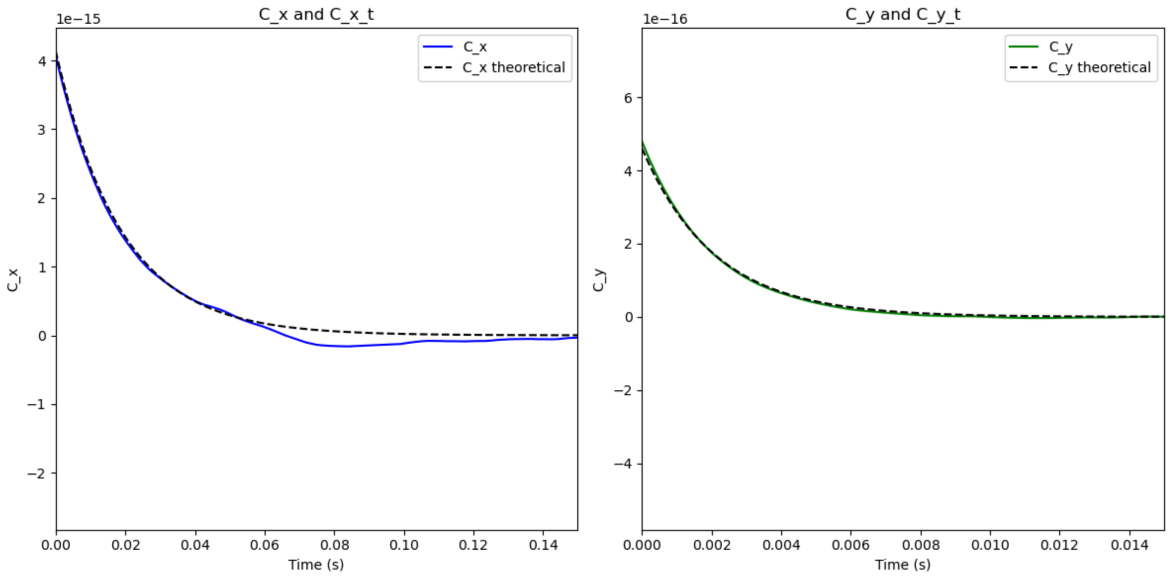
x has the larger variance because y has higher stiffness and therefore wont move as much as x, thus the variance is smaller for y than x.

The real and theoretical values seem very similar.

P3

In [6]:
```
C_x = np.zeros(N)
for n in range(N):
    frac = 1/(N-n)
    C_x[n] = frac* np.sum(x[n:]*x[:N-n])

C_y = np.zeros(N)
for n in range(0,N):
    frac = 1/(N-n)
    C_y[n] = frac* np.sum(y[n:]*y[:N-n])


t = np.arange(0,N)*dt
C_x_t = (k_B * T / k_x) * np.exp(-k_x * t / gamma)
```

```
C_y_t = (k_B * T / k_y) * np.exp(-k_y * t / gamma)


# Create the figure and subplots
fig, axes = plt.subplots(1, 2, figsize=(12, 6))

# Plot C_x and C_x_t on the first subplot
axes[0].plot(t, C_x, color="blue", label="C_x")
axes[0].plot(t, C_x_t, "k--", label="C_x theoretical")  # Black dashed line for
axes[0].set_xlabel("Time (s)")
axes[0].set_ylabel("C_x")
axes[0].legend()
axes[0].set_title("C_x and C_x_t")
axes[0].set_xlim([0, 0.15])

# Plot C_y and C_y_t on the second subplot
axes[1].plot(t, C_y, color="green", label="C_y")
axes[1].plot(t, C_y_t, "k--", label="C_y theoretical")  # Black dashed line for
axes[1].set_xlabel("Time (s)")
axes[1].set_ylabel("C_y")
axes[1].legend()
axes[1].set_title("C_y and C_y_t")
axes[1].set_xlim([0, 0.015])

# Show the plot
plt.tight_layout()
plt.show()
```



In [18]:
```python
import numpy as np
def regularize(x_nu, t_nu, t):
    """
    Function to regularize a time non-uniformly sampled trajectory.

    Parameters
    ==========
    x_nu : Trajectory (x component) non-uniformly sampled in time.
    t_nu : Time (non-uniform sampling).
    t : Time (wanted sampling).
    """
    x = np.zeros(np.size(t))
    m = np.diff(x_nu) / np.diff(t_nu)  # Slopes of the different increments.

    s = 0  # Position in the wanted trajectory.
    for i in range(np.size(t_nu) - 1):

        # Select the spots in x (wanted trajectory) to set.
        s_end = np.where(t < t_nu[i+1])[0][-1]

        # Assign the values of the segment.
        x[s:s_end + 1] = x_nu[i] + m[i] * (t[s:s_end + 1]-t_nu[i])

        # Update the position in the wanted trajectory.
        s = s_end + 1

    return x
```

P1

In [19]:
```python
def lw_1D(T,alpha,v):
    import numpy as np
    x = []
    t = []
    x.append(0)
    t.append(0)
    #Continue until the previous time exceeds the duration
    while t[-1]<T:
        dt = np.random.rand()**(-1/(3-alpha))
        #Cumulative sum
        t.append(t[-1] + dt)
        w = np.random.choice([-1,1])
        x.append(x[-1] + v*w*dt)
    return x,t
```

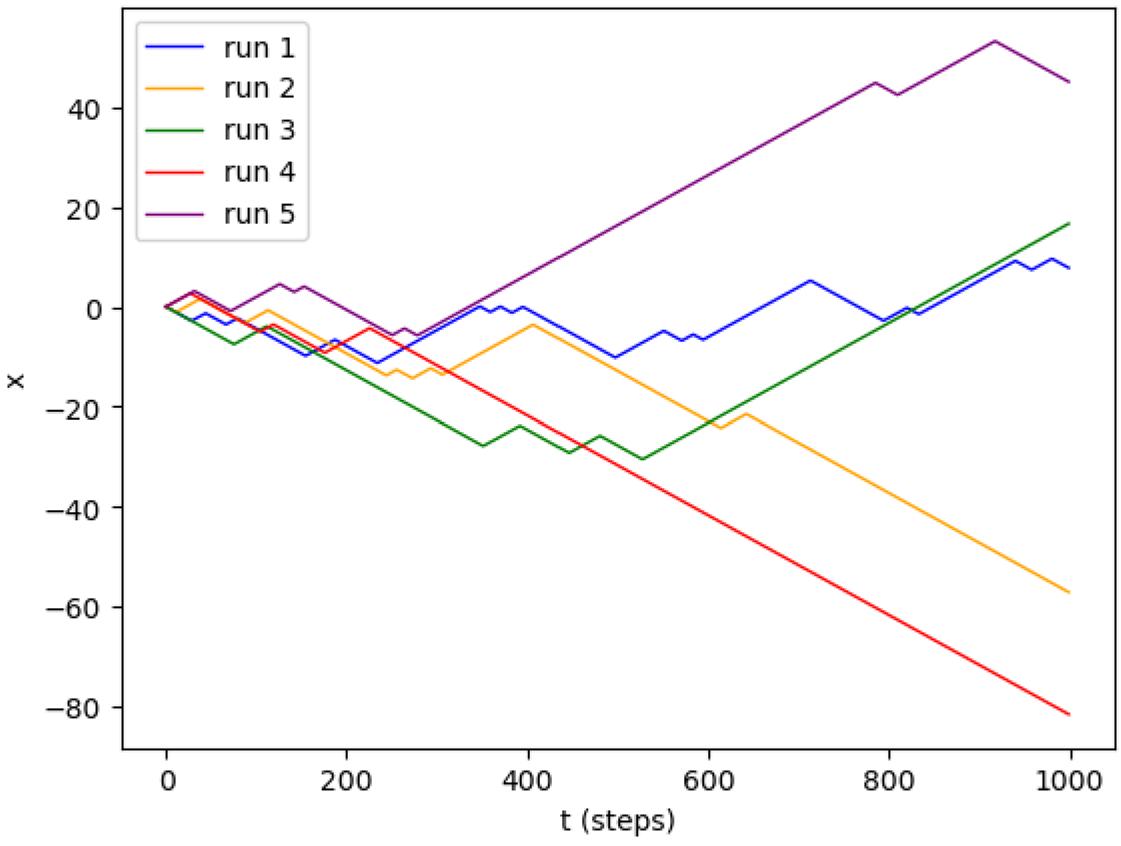
Uncomment to visualize what the regularization does.

In [ ]:
```python
duration = 100
dt = 0.1
## round up to cover all steps, also scale by dt. Used for regularization.
t = np.arange(int(np.ceil(duration / dt))) * dt
v = 1 # constant velocity
alpha = 2
runs = 5
trajectories = np.zeros((runs,len(t)))
for run in range(runs):
    x, t_sum = lw_1D(duration,alpha,v)
    x_r = regularize(x,t_sum,t)
    trajectories[run] = x_r

"""
x, t_sum = lw_1D(duration,alpha,v)
x_r = regularize(x,t_sum,t)
"""
```

```
Out[ ]: '\nx, t_sum = lw_1D(duration,alpha,v)\nx_r = regularize(x,t_sum,t)\n'
```

Uncomment to visualize what the regularization does.

```
In [21]: import matplotlib.pyplot as plt
colors = ['blue', 'orange', 'green', 'red', 'purple']
for run in range(runs):
    plt.plot(trajectories[run], '-', color=colors[run], linewidth=1, label=f'run


"""plt.plot(t_sum,x, 'o-', color='g', linewidth=1, label='Levi walk alpha=2')
plt.plot(t, x_r, '.', color='k', label='Regularized')
plt.xlim([0, 3])
plt.ylim([0, 3])"""

plt.legend()
plt.xlabel('t (steps)')
plt.ylabel('x')
plt.show()
```

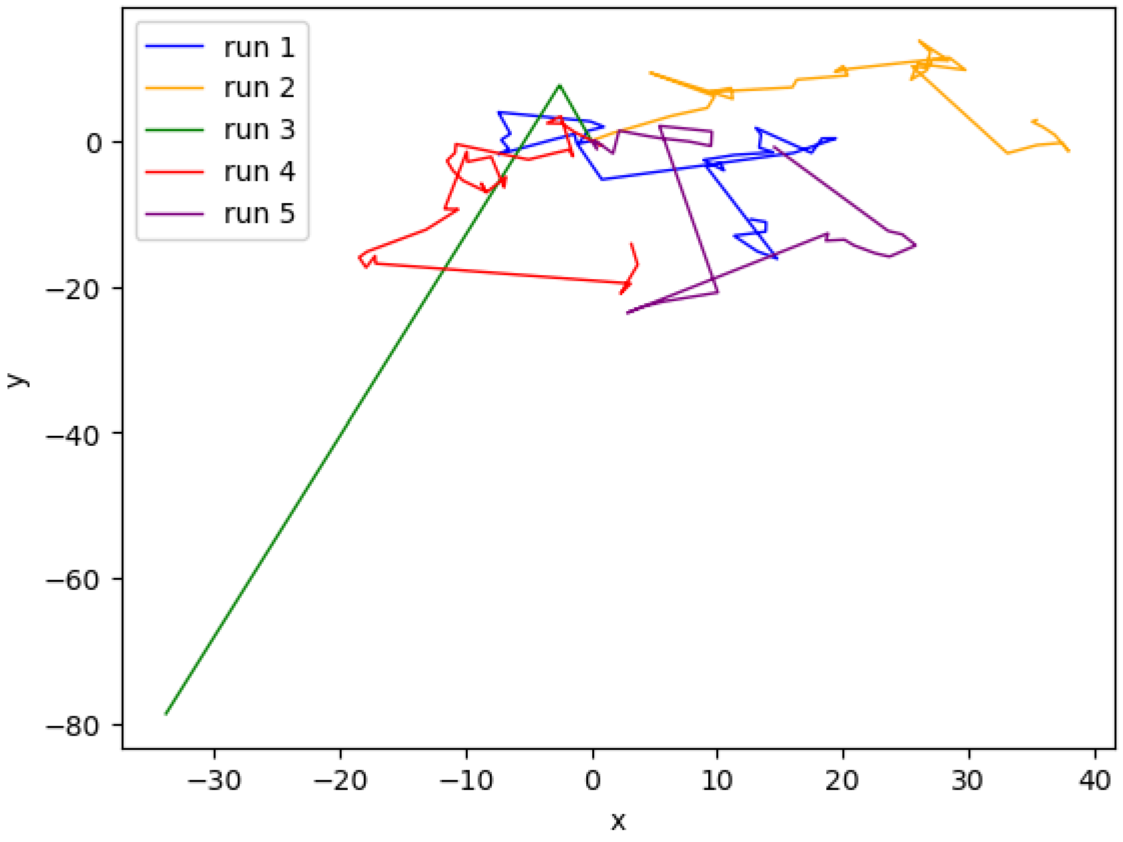


P2

In [22]:
```
def lw_2D(T,alpha,v):
    import numpy as np
    t = []
    x = []
    y = []
    x.append(0)
    y.append(0)
    t.append(0)
    while t[-1]<T:
        dt = np.random.rand()**(-1/(3-alpha))
        #Cumulative sum
        t.append(t[-1] + dt)
        phi = np.random.rand()*2*np.pi
        x.append(x[-1] + v*np.cos(phi)*dt)
        y.append(y[-1] + v*np.sin(phi)*dt)
    return x,y,t
```

In [23]:
```
duration = 100
dt = 0.1
t = np.arange(int(np.ceil(duration / dt))) * dt
v = 1 # constant velocity
alpha = 2
runs = 5
trajectories = np.zeros((2,runs,len(t))) #row 0 is x, row 1 y.
for run in range(runs):
    x,y,t_sum = lw_2D(duration, alpha, v)
    x = regularize(x,t_sum,t)
    y = regularize(y,t_sum,t)
    trajectories[0][run] = x
    trajectories[1][run] = y
```

In [24]:
```
import matplotlib.pyplot as plt
colors = ['blue', 'orange', 'green', 'red', 'purple']
for run in range(runs):
    plt.plot(trajectories[0][run],trajectories[1][run], '-', color=colors[run],
plt.legend()
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



P3

- Time MSD: Measures how far a single particle moves over time.
- Ensemble MSD: Measures how far many particles move on average at a specific time.

In [25]:
```
def tMSD_1d(x, L):
    """
    Function to calculate the tMSD.

    Parameters
    ==========
    x : Trajectory (x component).
    L : Indicates the maximum delay (L * dt) considered.
    """

    tmsd = np.zeros(L)

    nelem = np.size(x)

    for n in range(L):
        Nmax = nelem - n
        dx = x[n:nelem] -  x[0: Nmax]
        tmsd[n] += np.mean(dx ** 2)

    return tmsd
```

In [26]:
```
def eMSD_1d(x):
    """
    Function to calculate the eMSD.

    Parameters
    ==========
    x : Trajectories: x[n_traj, i], bidimensional array.
    """

    N_traj, N_steps = x.shape

    # emsd = np.zeros(N_steps)

    emsd = np.mean(
        (x - np.repeat(x[:, 0].reshape(N_traj, 1), N_steps, axis=1)) ** 2,
        axis=0
    )

    return emsd
```

Calculate x_t for tMSD

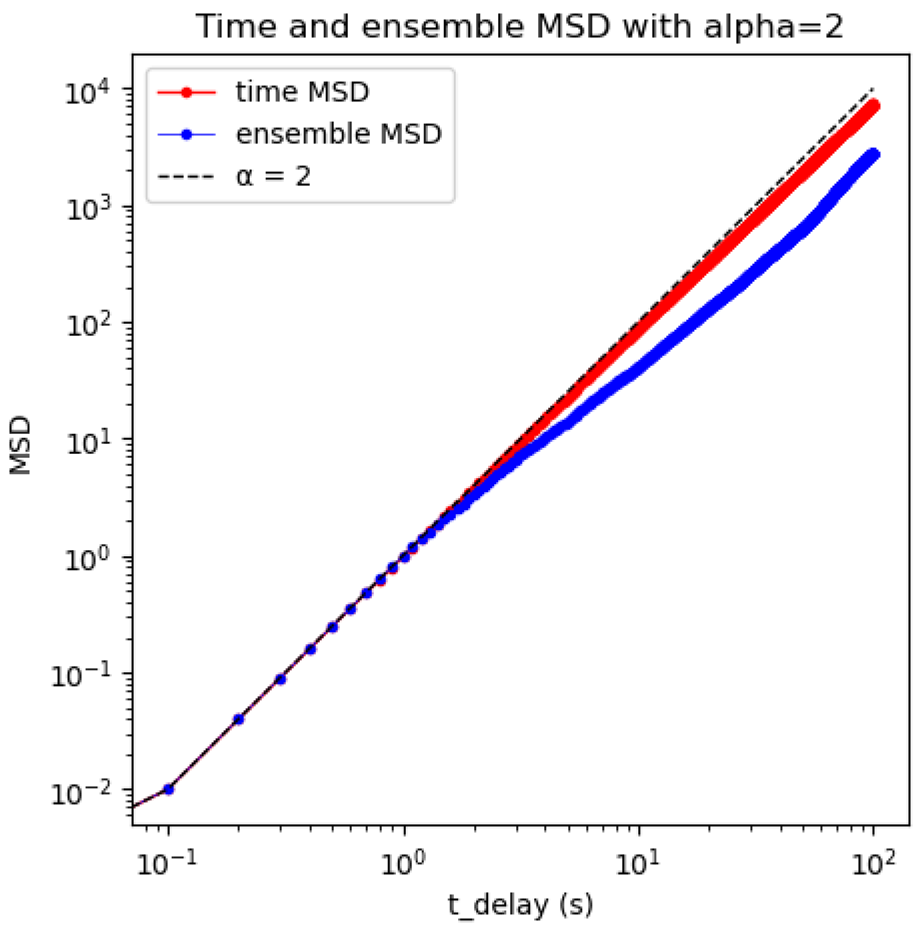
In [27]:
```
t_tot = 10000
N = 10000
dt = 0.1
v = 1

t_t = np.arange(int(np.ceil(t_tot / dt))) * dt
N_steps_t = np.size(t_t)

x_t, t_sum= lw_1D(t_tot, alpha, v)
x_t = regularize(x_t, t_sum, t_t)
```

Calculate x_e for eMSD

In [28]:
```
t_tot = 100
N = 100
dt = 0.1

# Regular sampling with dt.
t_e = np.arange(int(np.ceil(t_tot / dt))) * dt
N_steps_e = np.size(t_e)

N_traj = 100

x_e = np.zeros([N_traj, N_steps_e])

for i in range(N_traj):
    x, t_sum = lw_1D(t_tot, alpha, v)
    x_r = regularize(x,t_sum, t_e)
    x_e[i, :] = x_r
```

In [29]:
```
# Calculate eMSD
emsd = eMSD_1d(x_e)  # eMSD from ensemble trajectories.

# Calculate tMSD
tmsd = tMSD_1d(x_t, N_steps_e)  # tMSD from long trajectory.
```

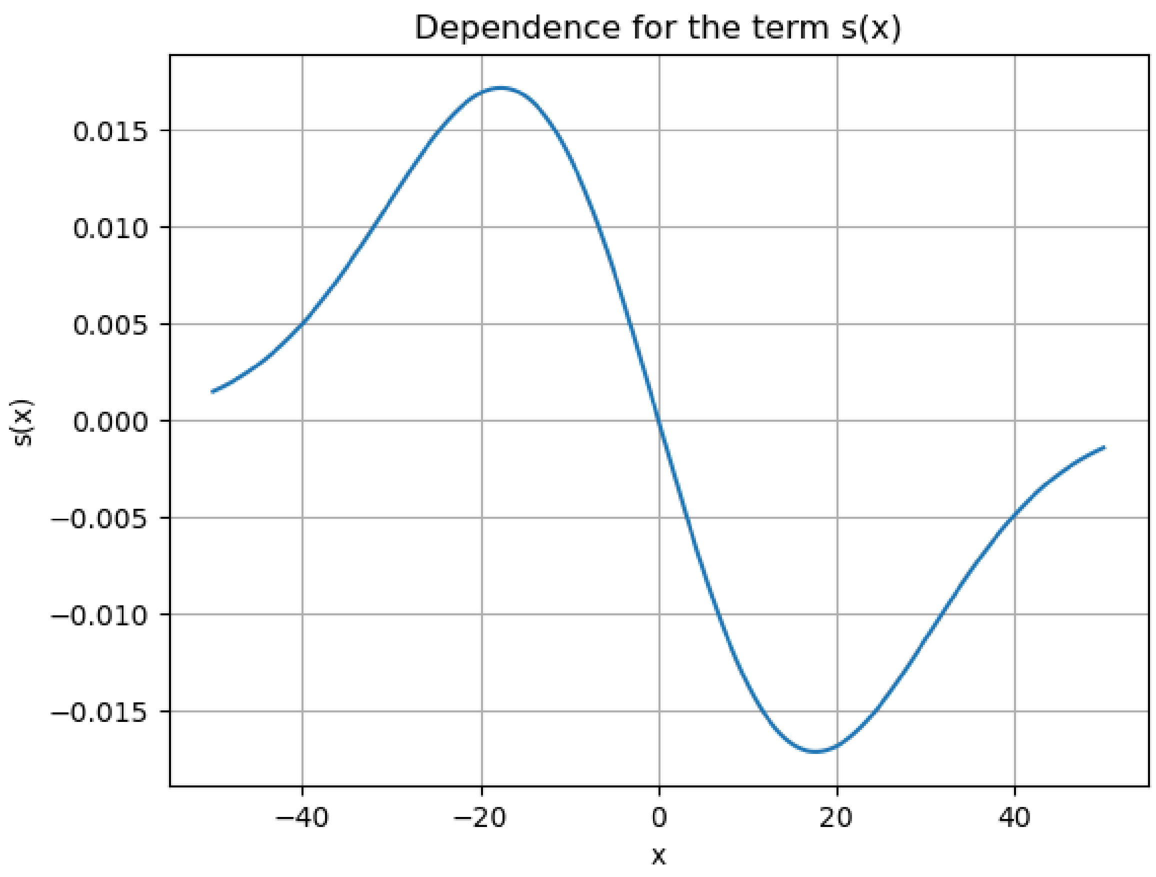
In [30]:
```
plt.figure(figsize=(5,5))
plt.loglog(t_e, tmsd, '.-', color='r', linewidth=1,
         label='time MSD')
plt.loglog(t_e, emsd, '.-', color='b', linewidth=0.5,
         label='ensemble MSD')
plt.loglog(t_e, t_e**alpha, 'k--', linewidth=1, label=f'α = {alpha}')
plt.title(f"Time and ensemble MSD with alpha={alpha}")
plt.legend()
plt.xlabel('t_delay (s)')
plt.ylabel('MSD')
plt.show()
```

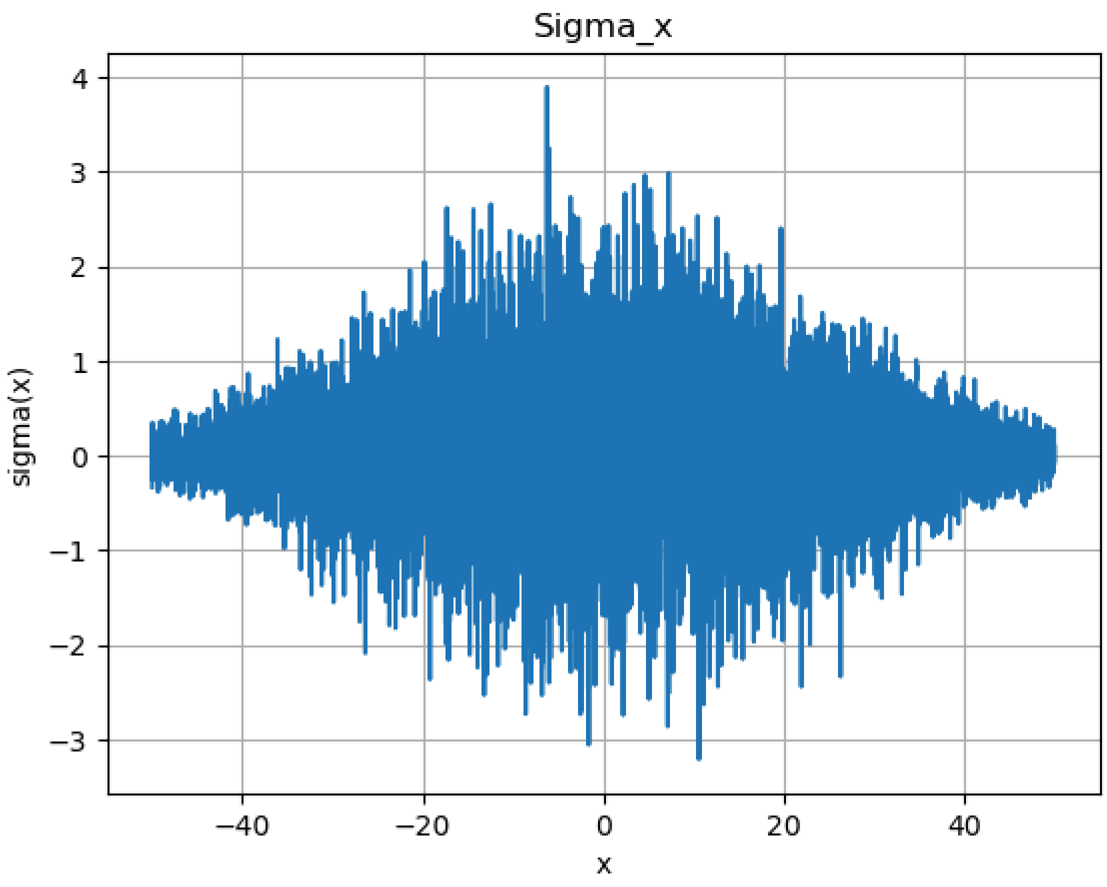


Seems to be ergodic since the time and ensemble MSD aligns closely.

P1

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
sigma0 = 1
w0 = 25
L = 100

xmin = -L/2
xmax = L/2

x = np.linspace(xmin,xmax,10000)
s_x = np.zeros(len(x))

for i in range(len(x)):
    tmp = -x[i]*(sigma0/w0)**2
    s_x[i] = tmp*np.exp(-(x[i]**2)/(w0**2))

plt.plot(x, s_x)
plt.xlabel('x')
plt.ylabel('s(x)')
plt.title('Dependence for the term s(x)')
plt.grid(True)
plt.show()


### Used to understand the equation.
rn = np.random.normal(0,1,len(x))
sigma = sigma0*np.exp(-(x**2)/(2*w0**2))*rn

plt.plot(x, sigma)
plt.xlabel('x')
plt.ylabel('sigma(x)')
plt.title('Sigma_x')
plt.grid(True)
plt.show()
```





P2

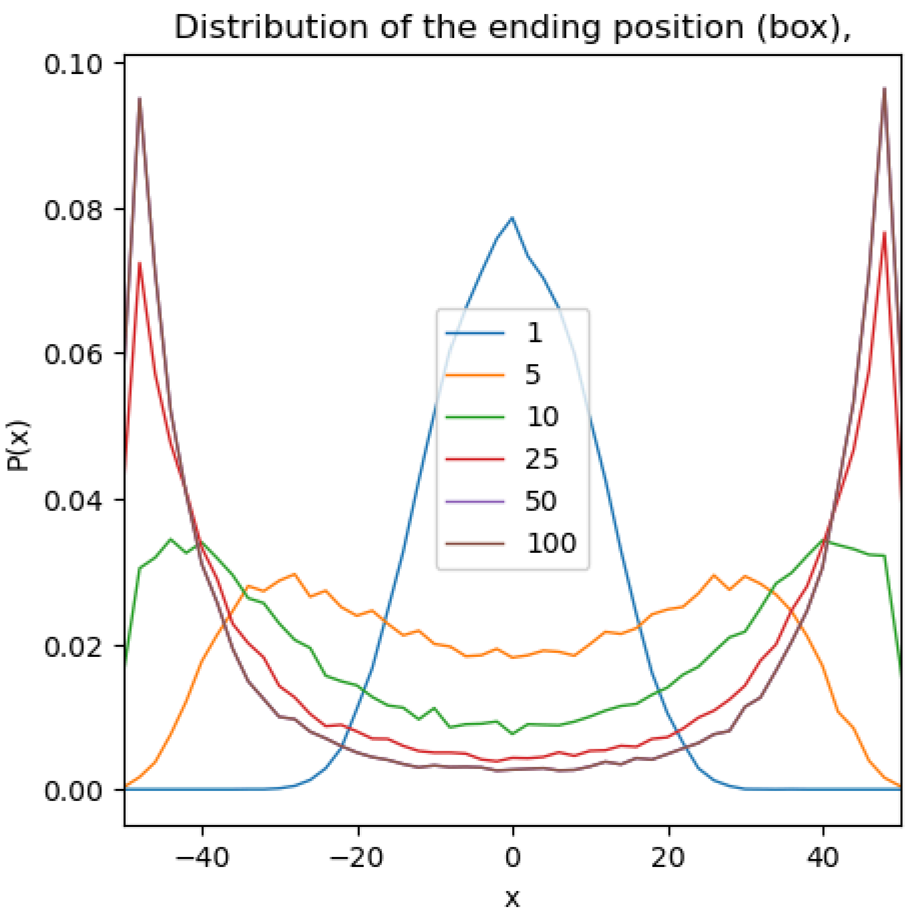
```
In [49]: alphas = np.array([0,0.5,1])
         sigma0 = 1
```

```
dt = 1  # Time step.
N_traj = 50000 # Number of independent trajectories.
t0 = 100  # Base value of the duration.
j_mult = np.array([1, 5, 10,25, 50, 100])

x0 = 0  # Initial position [m].

L = 100
x_min = - L / 2
x_max = L / 2

w0 = 25

x_finals = []

for idx,alpha in enumerate(alphas):

    x_fin = np.zeros([N_traj, np.size(j_mult)])  # Final positions.

    for j in range(np.size(j_mult)):

        # Set the number of steps to calculate further.
        if j > 1:
            N_steps = int(np.ceil((j_mult[j] - j_mult[j - 1]) * t0 / dt))
        else:
            N_steps = int(np.ceil(j_mult[j] * t0 / dt))

        rn = np.random.normal(0, 1, size=(N_traj, N_steps))

        if j > 1:
            x = x_fin[:, j - 1]
        else:
            x = np.zeros(N_traj)

        for step in range(N_steps):

            sigma_x = sigma0*np.exp((-x**2)/(2*w0**2))

            tmp = -x*(sigma0/w0)**2
            s_x = tmp*np.exp(-(x**2)/(w0**2))
            x += alpha*s_x*dt + sigma_x*np.sqrt(dt)* rn[:, step]


            # reflecting boundary conditions
            bounce_left = np.where(x < x_min)[0]  # Hitting box left end.
            x[bounce_left] = 2 * x_min - x[bounce_left]
            bounce_right = np.where(x > x_max)[0]  # Hitting box right end.
            x[bounce_right] = 2 * x_max - x[bounce_right]

        x_fin[:, j] = x

    x_finals.append(x_fin)
```

In [50]:

```
# Histogram of the final positions.
bin_width = 2
bins_edges = np.arange(- L - bin_width / 2, L + bin_width / 2 + .1, bin_width)
bins = np.arange(- L, L + .1, bin_width)

p_distr = np.zeros([np.size(bins), np.size(j_mult)])  # Distributions.
```

```
for j in range(np.size(j_mult)):
    distribution = np.histogram(x_finals[0][:, j], bins=bins_edges)
    p_distr[:, j] = distribution[0] / np.sum(distribution[0])

plt.figure(figsize=(5, 5))
for j in range(np.size(j_mult)):
    plt.plot(bins, p_distr[:, j], '-', linewidth=1, label=str(j_mult[j]))
plt.title('Distribution of the ending position (box),')
plt.legend()
plt.xlabel('x')
plt.ylabel('P(x)')
plt.xlim([x_min, x_max])
plt.show()
```

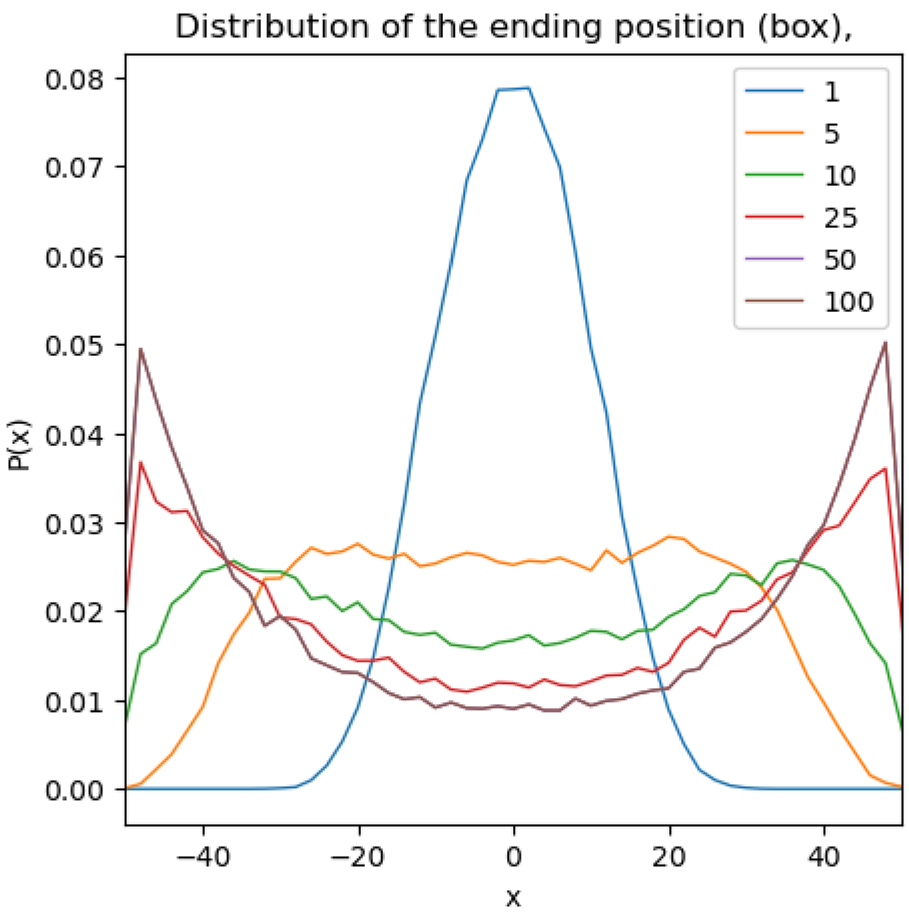


P3

In [51]:

```
# Histogram of the final positions.
bin_width = 2
bins_edges = np.arange(- L - bin_width / 2, L + bin_width / 2 + .1, bin_width)
bins = np.arange(- L, L + .1, bin_width)

p_distr = np.zeros([np.size(bins), np.size(j_mult)])  # Distributions.

for j in range(np.size(j_mult)):
    distribution = np.histogram(x_finals[1][:, j], bins=bins_edges)
    p_distr[:, j] = distribution[0] / np.sum(distribution[0])

plt.figure(figsize=(5, 5))
```

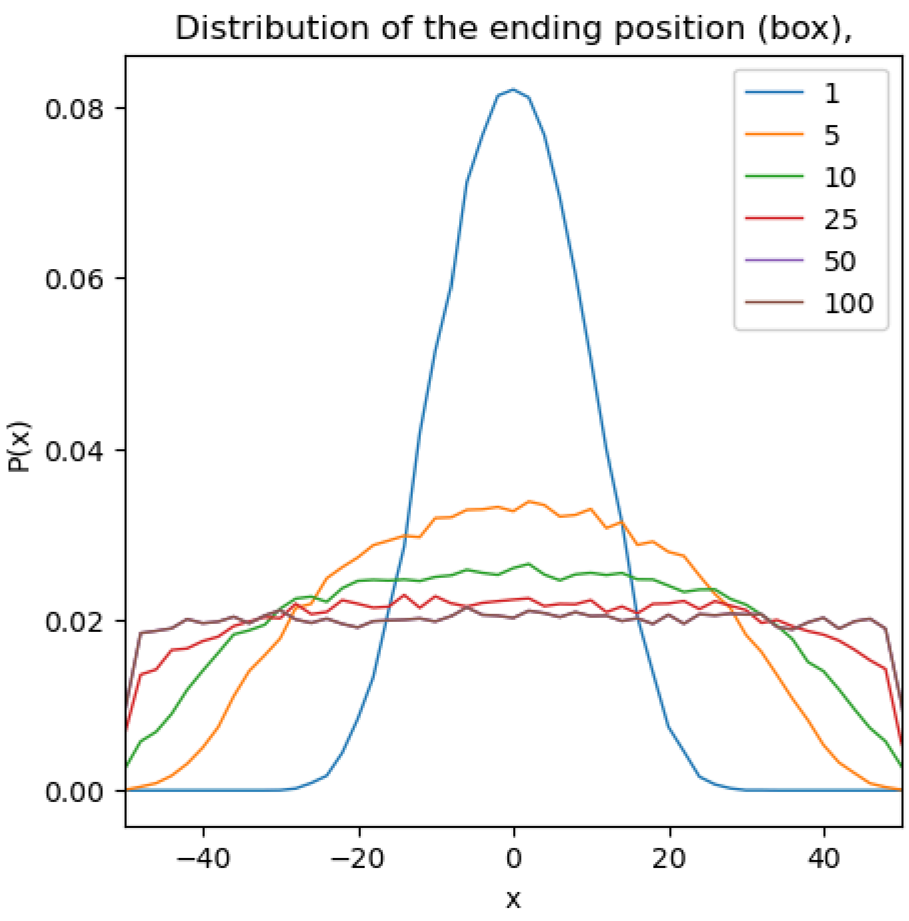
```
for j in range(np.size(j_mult)):
    plt.plot(bins, p_distr[:, j], '-', linewidth=1, label=str(j_mult[j]))
plt.title('Distribution of the ending position (box),')
plt.legend()
plt.xlabel('x')
plt.ylabel('P(x)')
plt.xlim([x_min, x_max])
plt.show()
```



P4

In [52]:

```
# Histogram of the final positions.
bin_width = 2
bins_edges = np.arange(- L - bin_width / 2, L + bin_width / 2 + .1, bin_width)
bins = np.arange(- L, L + .1, bin_width)

p_distr = np.zeros([np.size(bins), np.size(j_mult)])  # Distributions.

for j in range(np.size(j_mult)):
    distribution = np.histogram(x_finals[2][:, j], bins=bins_edges)
    p_distr[:, j] = distribution[0] / np.sum(distribution[0])

plt.figure(figsize=(5, 5))
for j in range(np.size(j_mult)):
    plt.plot(bins, p_distr[:, j], '-', linewidth=1, label=str(j_mult[j]))
plt.title('Distribution of the ending position (box),')
plt.legend()
plt.xlabel('x')
plt.ylabel('P(x)')
plt.xlim([x_min, x_max])
plt.show()
```



Q1

It seems as the multiplicative noise $\sigma(x)$ follows a gaussian distribution with mean 0 and standard deviation w_0, thus the standard deviation is symmetric around x=0, causing the system to be symmetric around 0. Moreover, we have reflective boundary conditions which further contributes to the symmetry of the system.

Note: From plotting the noise-induced drift in P1, and also plotting the right-most term in equation 2. We see that they seem to counteract eachother based on the position of x. Moreover, the noise-induced drift term increase when alpha increases, so when $\alpha = 1$ we fully incorporate the drift. That is why we see higher probabilities toward the boundaries for $\alpha < 1$, since the right-most term in eq 2 is larger than the noise-induced drift.

Task 1

P1

In [4]:
```
import math
import numpy as np

def replicas(x, y, L):
    """
    Function to generate replicas of a single particle.

    Parameters
    ==========
    x, y : Position.
    L : Side of the squared arena.
    """
    xr = np.zeros(9)
    yr = np.zeros(9)

    for i in range(3):
        for j in range(3):
            xr[3 * i + j] = x + (j - 1) * L
            yr[3 * i + j] = y + (i - 1) * L

    return xr, yr
```

In [5]:
```
def pbc(x, y, L):
    """
    Function to enforce periodic boundary conditions on the positions.

    Parameters
    ==========
    x, y : Position.
    L : Side of the squared arena.
    """

    outside_left = np.where(x < - L / 2)[0]
    x[outside_left] = x[outside_left] + L

    outside_right = np.where(x > L / 2)[0]
    x[outside_right] = x[outside_right] - L

    outside_up = np.where(y > L / 2)[0]
    y[outside_up] = y[outside_up] - L

    outside_down = np.where(y < - L / 2)[0]
    y[outside_down] = y[outside_down] + L

    return x, y
```

In [6]:
```
from functools import reduce

def interaction(x, y, theta, Rf, L):
    """
    Function to calculate the orientation at the next time step.

    Parameters
    ==========
    x, y : Positions.
    theta : Orientations.
    Rf : Flocking radius.
    L : Dimension of the squared arena.
    s : Discrete steps.
    """

    N = np.size(x)

    theta_next = np.zeros(N)

    # Preselect what particles are closer than Rf to the boundaries.
    replicas_needed = reduce(
        np.union1d, (
            np.where(y + Rf > L / 2)[0],
            np.where(y - Rf < - L / 2)[0],
            np.where(x + Rf > L / 2)[0],
            np.where(x - Rf > - L / 2)[0]
        )
    )

    for j in range(N):
        # Check if replicas are needed to find the nearest neighbours.
        if np.size(np.where(replicas_needed == j)[0]):
            # Use replicas.
            xr, yr = replicas(x[j], y[j], L)
            nn = []
            for nr in range(9):
                dist2 = (x - xr[nr]) ** 2 + (y - yr[nr]) ** 2
                nn = np.union1d(nn, np.where(dist2 <= Rf ** 2)[0])
        else:
            dist2 = (x - x[j]) ** 2 + (y - y[j]) ** 2
            nn = np.where(dist2 <= Rf ** 2)[0]

        # The list of nearest neighbours is set.
        nn = nn.astype(int)

        # Circular average.
        av_sin_theta = np.mean(np.sin(theta[nn]))
        av_cos_theta = np.mean(np.cos(theta[nn]))

        theta_next[j] = np.arctan2(av_sin_theta, av_cos_theta)

    return theta_next
```

In [7]:
```
from scipy.spatial import Voronoi, voronoi_plot_2d

def area_polygon(vertices):
    """
    Function to calculate the area of a Voronoi region given its vertices.

    Parameters
    ==========
    vertices : Coordinates (array, 2 dimensional).
    """
    N, dim = vertices.shape

    # dim is 2.
    # Vertices are listed consecutively.

    A = 0

    for i in range(N-1):
        # Below is the formula of the area of a triangle given the vertices.
        A += np.abs(
            vertices[- 1, 0] * (vertices[i, 1] - vertices[i + 1, 1]) +
            vertices[i, 0] * (vertices[i + 1, 1] - vertices[- 1, 1]) +
            vertices[i + 1, 0] * (vertices[- 1, 1] - vertices[i, 1])
        )

    A *= 0.5

    return A


def global_clustering(x, y, Rf, L):
    """
    Function to calculate the global alignment coefficient.

    Parameters
    ==========
    x, y : Positions.
    Rf : Flocking radius.
    L : Dimension of the squared arena.
    """

    N = np.size(x)

    # Use the replicas of all points to calculate Voronoi for
    # a more precise estimate.
    points = np.zeros([9 * N, 2])

    for i in range(3):
        for j in range(3):
            s = 3 * i + j
            points[s * N:(s + 1) * N, 0] = x + (j - 1) * L
            points[s * N:(s + 1) * N, 1] = y + (i - 1) * L

    # The format of points is the one needed by Voronoi.
    # points[:, 0] contains the x coordinates
    # points[:, 1] contains the y coordinates

    vor = Voronoi(points)
    '''
    vertices = vor.vertices  # Voronoi vertices.
    regions = vor.regions  # Region list.
    # regions[i]: list of the vertices indices for region i.
    # If -1 is listed: the region is open (includes point at infinity).
    point_region = vor.point_region  # Region associated to input point.
    '''

    # Consider only regions of original set of points (no replicas).
    list_regions = vor.point_region[4 * N:5 * N]

    c = 0
    for i in list_regions:
        indices = vor.regions[i]
        # print(f'indices = {indices}')
        if len(indices) > 0:
            if np.size(np.where(np.array(indices) == -1)[0]) == 0:
                # Region is finite.
                # Calculate area.
                A = area_polygon(vor.vertices[indices,:])
                if A < np.pi * Rf ** 2:
                    c += 1

    c = c / N

    return c
```

In [8]:
```
def global_alignment(theta):
    """
    Function to calculate the global alignment coefficient.

    Parameters
    ==========
    theta : Orientations.
    """

    N = np.size(theta)

    global_direction_x = np.sum(np.sin(theta))
    global_direction_y = np.sum(np.cos(theta))

    psi = np.sqrt(global_direction_x ** 2 + global_direction_y ** 2) / N


    return psi
```

In [72]:
```
N_part = 200
L = 100
v = 1
Rf = 2
eta = 0.01
dt = 1

x = (np.random.rand(N_part) - 0.5) * L  # in [-L/2, L/2]
y = (np.random.rand(N_part) - 0.5) * L  # in [-L/2, L/2]

theta = 2 * (np.random.rand(N_part) - 0.5) * np.pi  # in [-pi, pi]
```

In [ ]:
```
import time
from scipy.constants import Boltzmann as kB
from tkinter import *


step = 0
t_tot = 6000

config_x1 = []
config_y1 = []
config_theta1 = []
```

```
psi = np.zeros(t_tot+1)
c = np.zeros(t_tot+1)

while step <= t_tot:
    psi[step] = global_alignment(theta)
    c[step] = global_clustering(x, y, Rf, L)

    # Calculate next theta from the rule.
    dtheta = eta * (np.random.rand(N_part) - 0.5) * dt
    ntheta = interaction(x, y, theta, Rf, L) + dtheta
    nx = x + v * np.cos(ntheta)
    ny = y + v * np.sin(ntheta)

    # Reflecting boundary conditions.
    nx, ny = pbc(nx, ny, L)

    if(step == 0 or step == 2000 or step == 4000 or step == 6000):
        config_x1.append(nx[:])
        config_y1.append(ny[:])
        config_theta1.append(ntheta[:])

    step += 1
    if step % 100 == 0:
        print(step)

    x[:] = nx[:]
    y[:] = ny[:]
    theta[:] = ntheta[:]
```

```
100
200
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4600
4700
4800
4900
5000
5100
5200
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5400
5500
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5700
5800
5900
6000
```
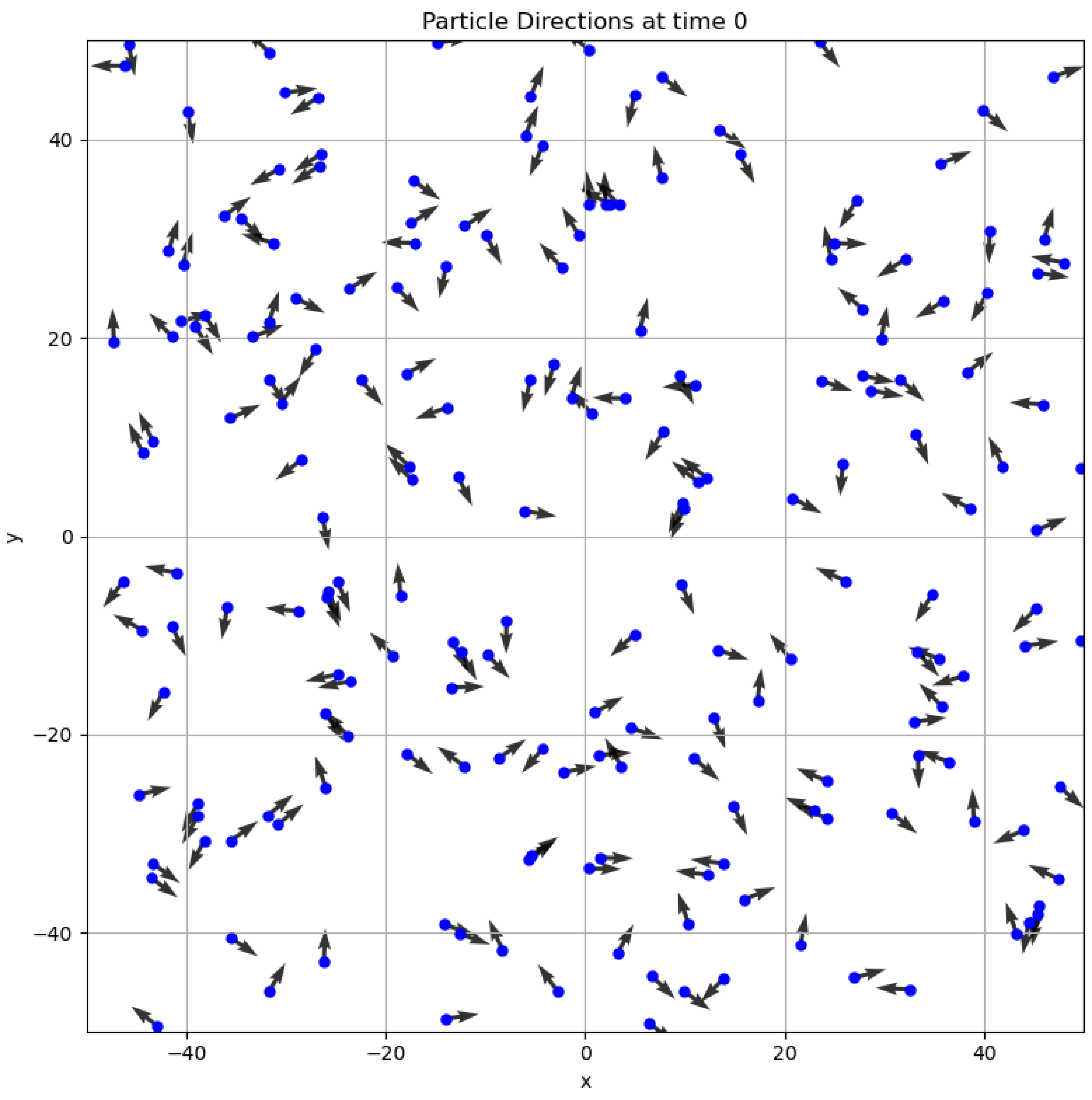
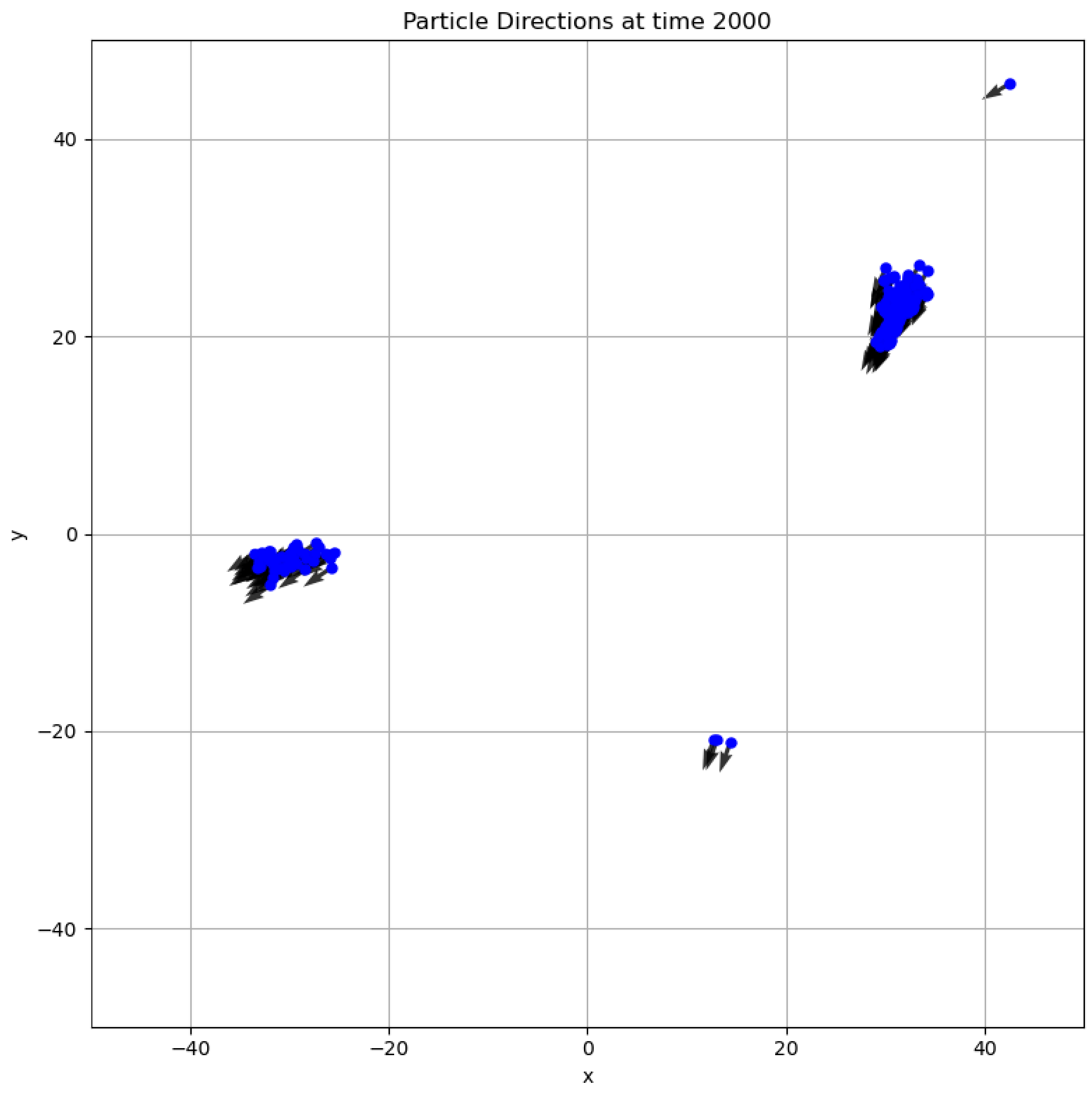
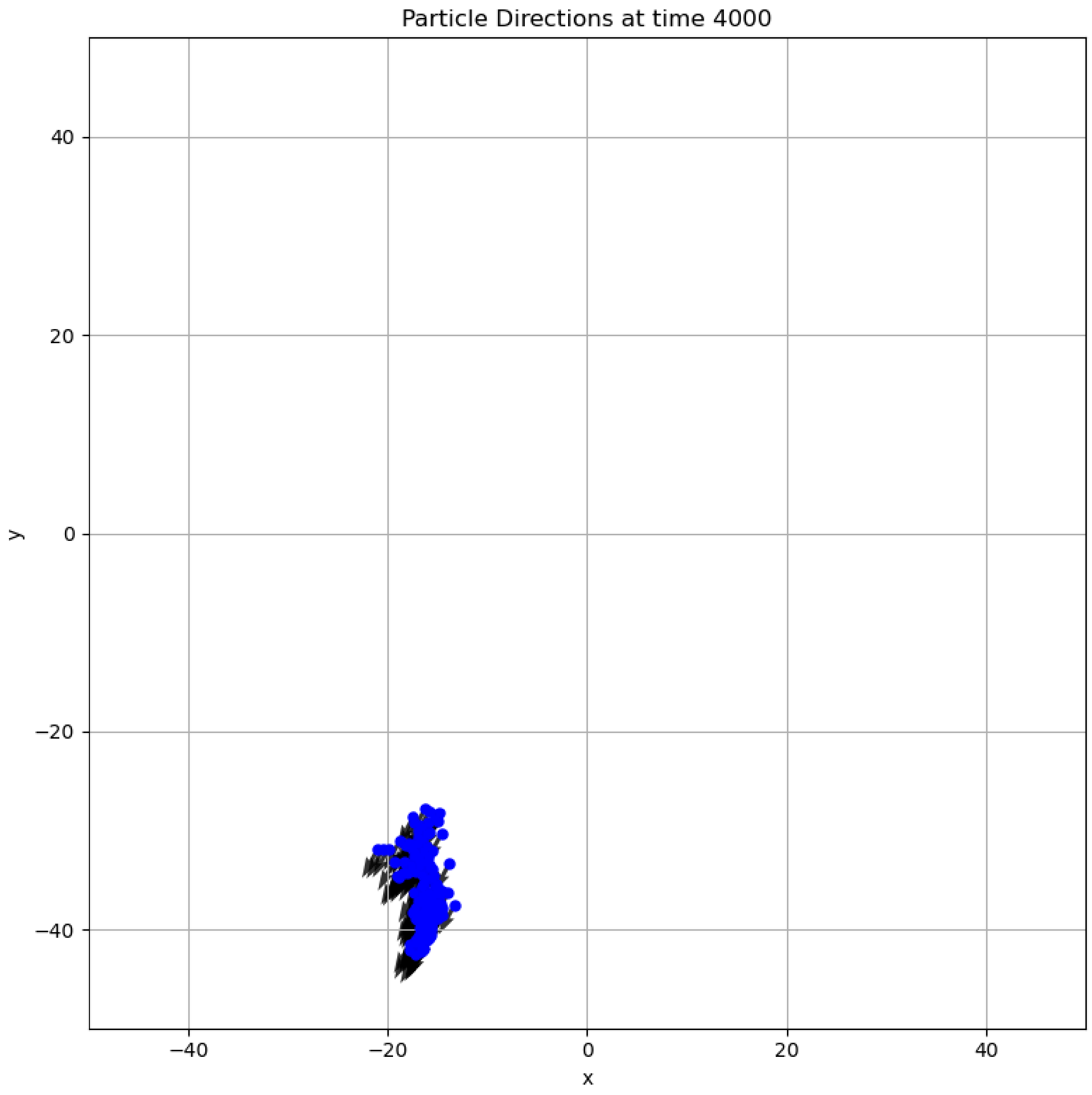
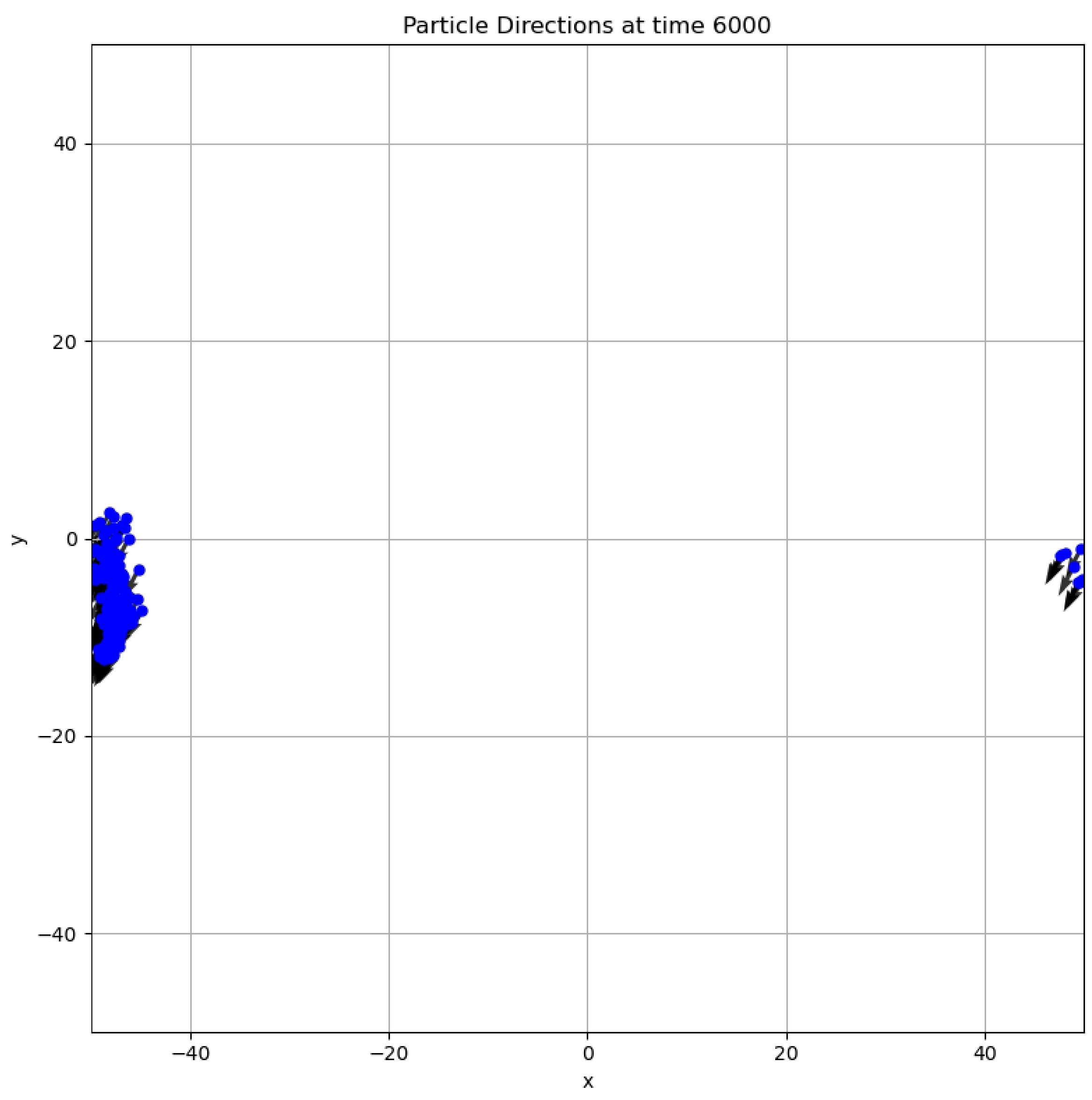

```
In [ ]: import numpy as np
        import matplotlib.pyplot as plt


        # Prepare time steps corresponding to snapshots
        time_steps = [0, 2000, 4000, 6000]

        # Plot arrows representing particle directions at each snapshot
        for i, time in enumerate(time_steps):
            plt.figure(figsize=(8, 8))
            x_positions_p1 = config_x1[i]
            y_positions_p1 = config_y1[i]
            angles = config_theta1[i]

            u_p1 = np.cos(angles)
            v_p1 = np.sin(angles)

            plt.plot(x_positions_p1, y_positions_p1, '.', color='blue', markersize=10)
            plt.quiver(x_positions_p1, y_positions_p1, u_p1, v_p1, color='black', angles
            """plt.scatter(x_positions_p1, y_positions_p1 )
            for j in range(len(x_positions_p1)):
                plt.arrow(x_positions_p1[j], y_positions_p1[j], u[j], v[j])"""
            plt.title(f'Particle Directions at time {time}')
            plt.xlabel('x')
            plt.ylabel('y')
            plt.grid()
            plt.xlim([-L/2,L/2])
            plt.ylim([-L/2,L/2])
            plt.tight_layout()
            plt.show()
```


Particle Directions at time 0
y
x
40
20
0
−20
−40
−40
−20
0
20
40

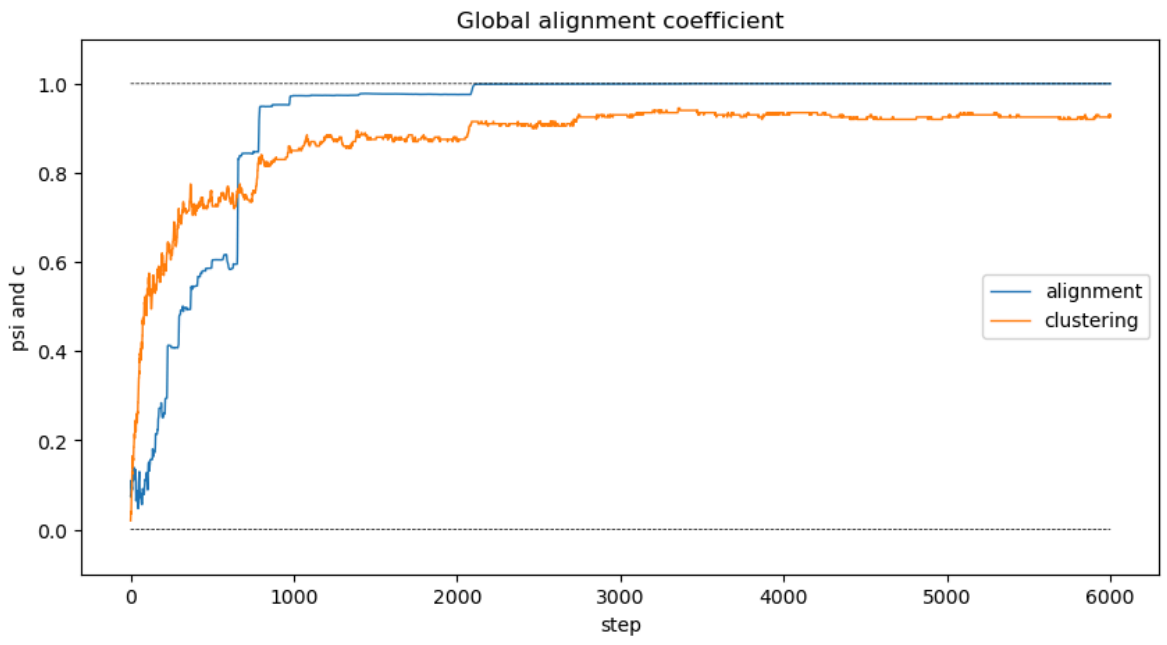

Particle Directions at time 2000
40
20
0
−20
−40
y
−40
−20
0
20
40
x


Particle Directions at time 4000
y
x
40
20
0
−20
−40
−40
−20
0
20
40



P2

```
In [75]: from matplotlib import pyplot as plt
         plt.figure(figsize=(10, 5))
         plt.plot(psi, '-', linewidth=1, label='alignment')
         plt.plot(c, '-', linewidth=1, label='clustering')
         plt.plot(0 * psi, '--', color='k', linewidth=0.5)
         plt.plot(0 * psi + 1, '--', color='k', linewidth=0.5)
         plt.title('Global alignment coefficient')
         plt.legend()
         plt.xlabel('step')
         plt.ylabel('psi and c')
         plt.ylim([-0.1, 1.1])
         plt.show()
```



Task 2

P3

In [9]:
```
import numpy as np
N_part = 200
L = 100
v = 1
Rf = 2
eta1 = 0.01
eta2 = 0.3
dt = 1

x = (np.random.rand(N_part) - 0.5) * L
y = (np.random.rand(N_part) - 0.5) * L

# Random orientation.
theta = 2 * (np.random.rand(N_part) - 0.5) * np.pi
```

In [10]:
```
from IPython.display import clear_output
from matplotlib import pyplot as plt
import time

step = 0
t_tot = 6000
half_N = N_part // 2

config_x = []
config_y = []
config_theta = []

psi2 = np.zeros(t_tot+1)
c2 = np.zeros(t_tot+1)
fig, ax = plt.subplots(figsize=(10, 10))

while step <= t_tot:
```

```python
    """ if step % 1 == 0:
        ax.clear()  # Clear previous plot.

        # Plot first subpopulation
        ax.plot(x[:half_N], y[:half_N], '.', markersize=10, color='blue', label=
        ax.quiver(x[:half_N], y[:half_N], np.cos(theta[:half_N]), np.sin(theta[:

        # Plot second subpopulation
        ax.plot(x[half_N:], y[half_N:], '.', markersize=10, color='red', label='
        ax.quiver(x[half_N:], y[half_N:], np.cos(theta[half_N:]), np.sin(theta[h

        # Plot boundary (if needed)
        ax.plot(Rf * np.cos(2 * np.pi * np.arange(360) / 360),
                Rf * np.sin(2 * np.pi * np.arange(360) / 360),
                '-', color='#FFA0FF', linewidth=3)

        ax.set_xlim([-L / 2, L / 2])
        ax.set_ylim([-L / 2, L / 2])
        ax.set_title(f'Step {step}')
        ax.legend(loc='upper right')  # Add legend to distinguish subpopulations

        display(fig)  # Display updated plot.
        clear_output(wait=True)  # Clear previous output."""

    psi2[step] = global_alignment(theta)
    c2[step] = global_clustering(x, y, Rf, L)

    dtheta = np.zeros(N_part)

    dtheta[:half_N] = eta1 * (np.random.rand(half_N) - 0.5) * dt
    dtheta[half_N:] = eta2 * (np.random.rand(N_part-half_N) - 0.5) * dt

    ntheta = interaction(x, y, theta, Rf, L) + dtheta
    nx = x + v * np.cos(ntheta)
    ny = y + v * np.sin(ntheta)

    # Reflecting boundary conditions.
    nx, ny = pbc(nx, ny, L)

    if(step == 0 or step == 2000 or step == 4000 or step == 6000):
        config_x.append(nx[:])
        config_y.append(ny[:])
        config_theta.append(ntheta[:])

    step += 1
    if step % 100 == 0:
        print(step)

    x[:] = nx[:]
    y[:] = ny[:]
    theta[:] = ntheta[:]
```

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5400
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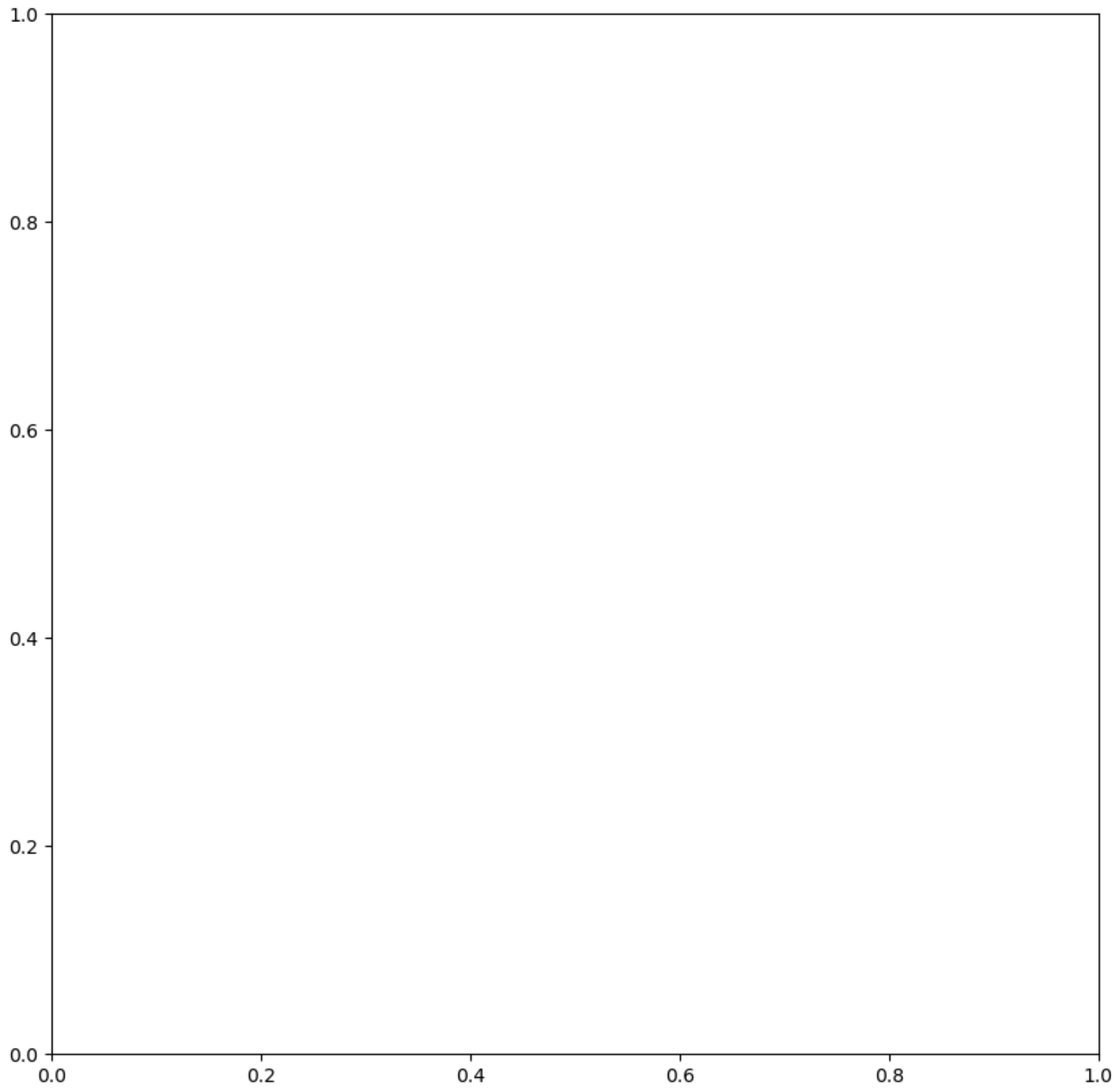
5600
5700
5800
5900
6000
```



In [11]:
```
time_steps = [0, 2000, 4000, 6000]
N_half = N_part // 2

for i, time in enumerate(time_steps):
    plt.figure(figsize=(8, 8))

    x_positions = config_x[i]
    y_positions = config_y[i]
    angles = config_theta[i]

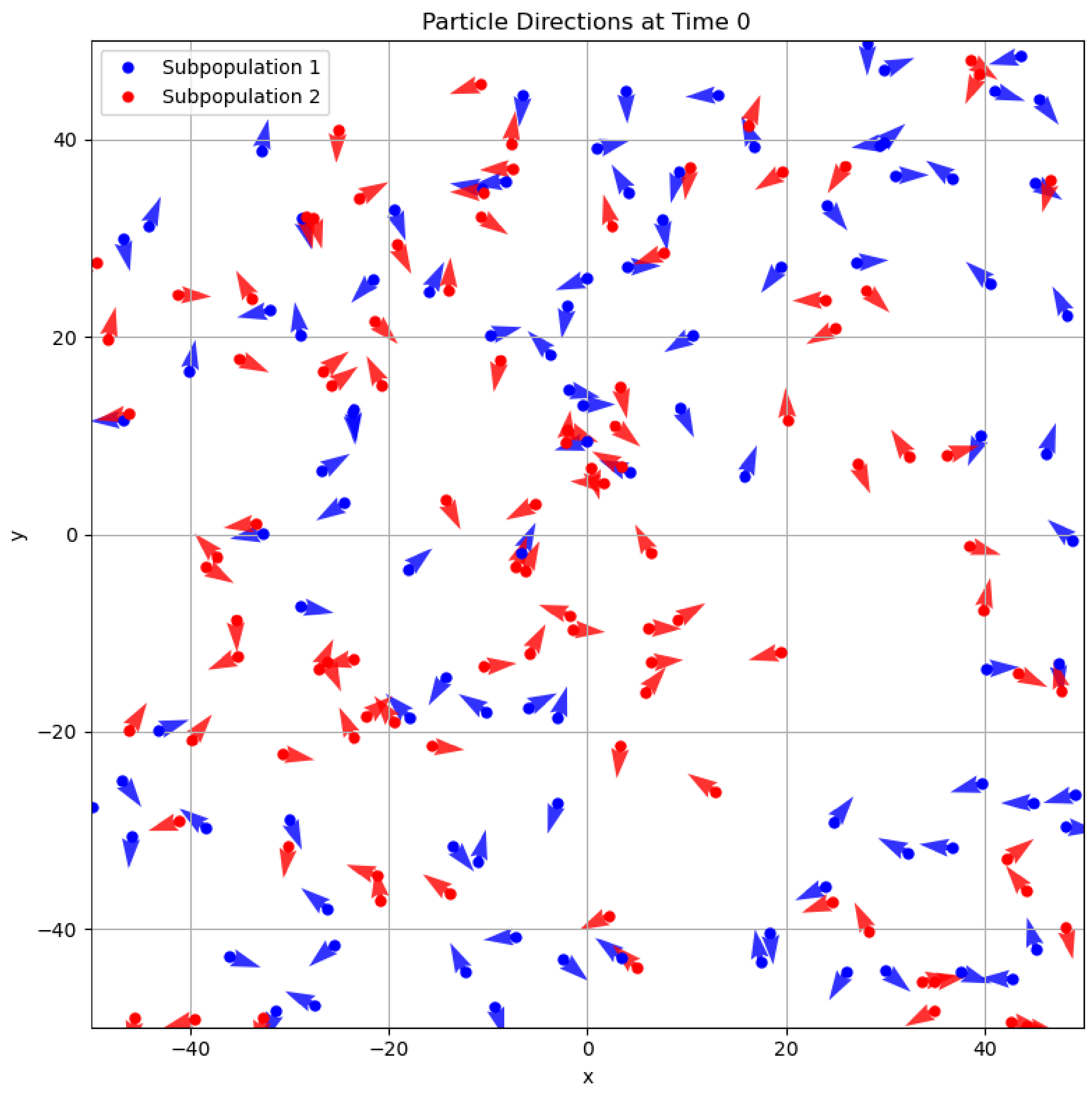
    x_positions_1, x_positions_2 = x_positions[:N_half], x_positions[N_half:]
    y_positions_1, y_positions_2 = y_positions[:N_half], y_positions[N_half:]
    angles_1, angles_2 = angles[:N_half], angles[N_half:]

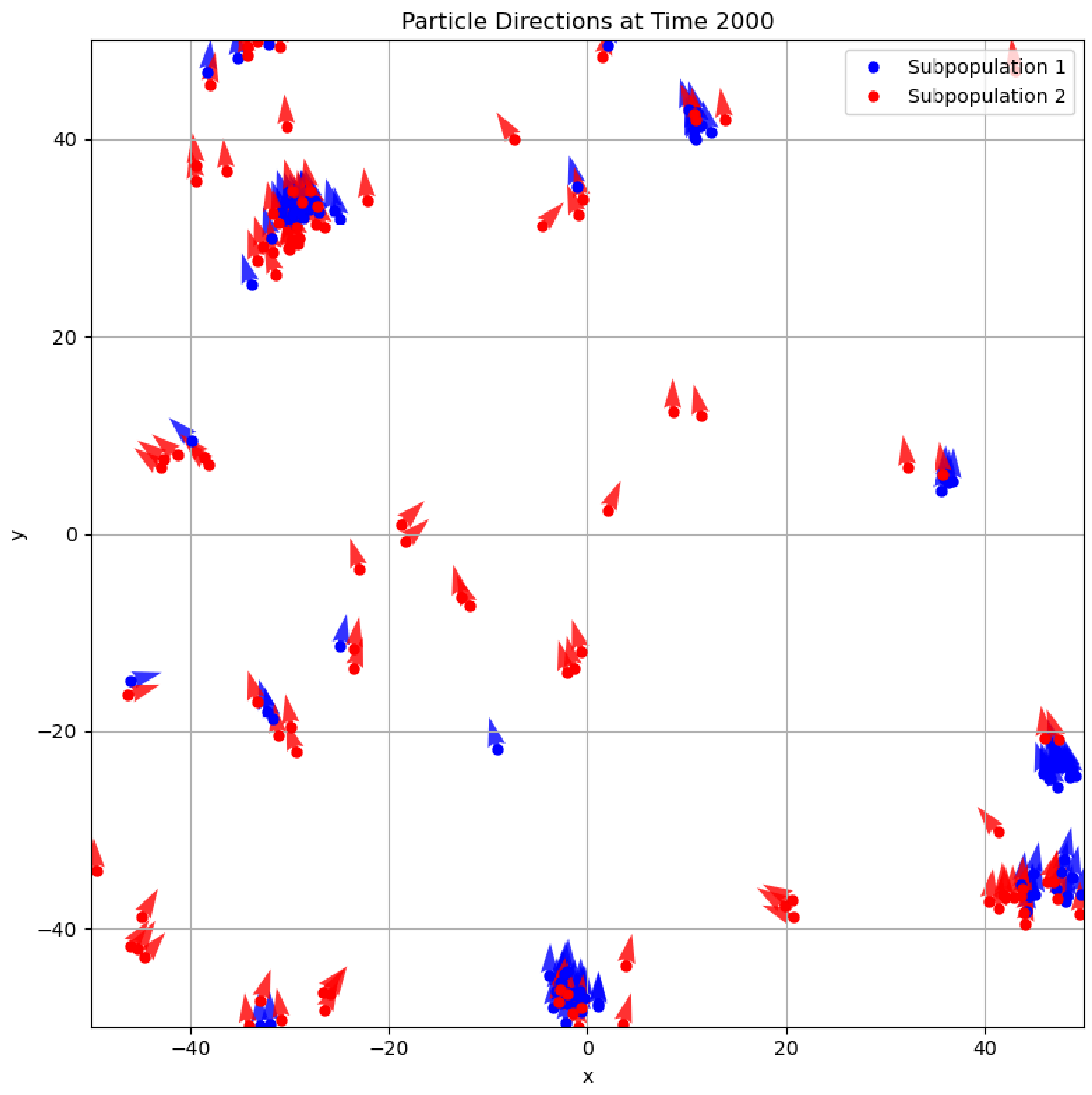
    u1, v1 = np.cos(angles_1), np.sin(angles_1)
    u2, v2 = np.cos(angles_2), np.sin(angles_2)

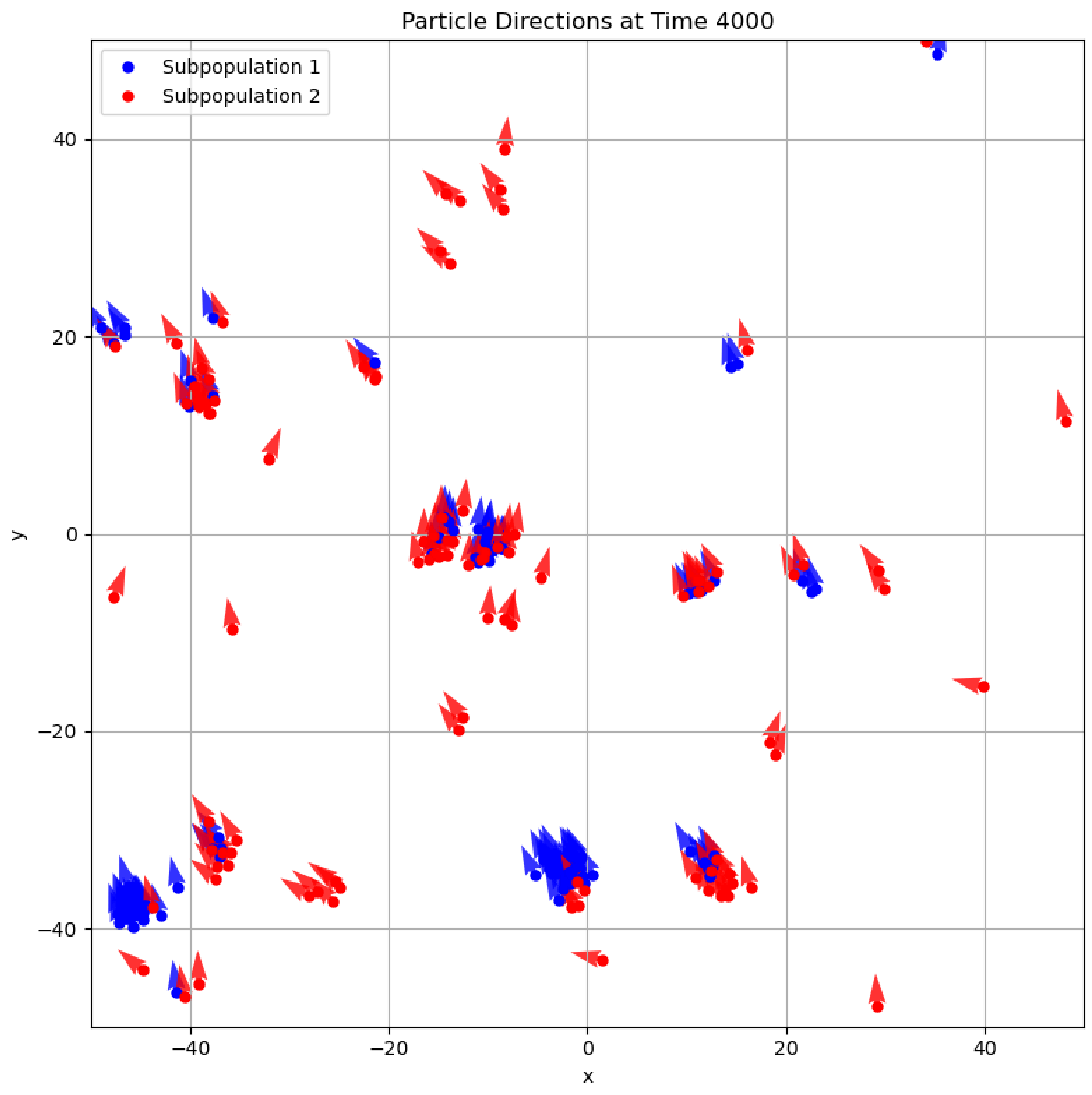
    plt.plot(x_positions_1, y_positions_1, '.', color='blue', markersize=10, lab
    plt.plot(x_positions_2, y_positions_2, '.', color='red', markersize=10, labe

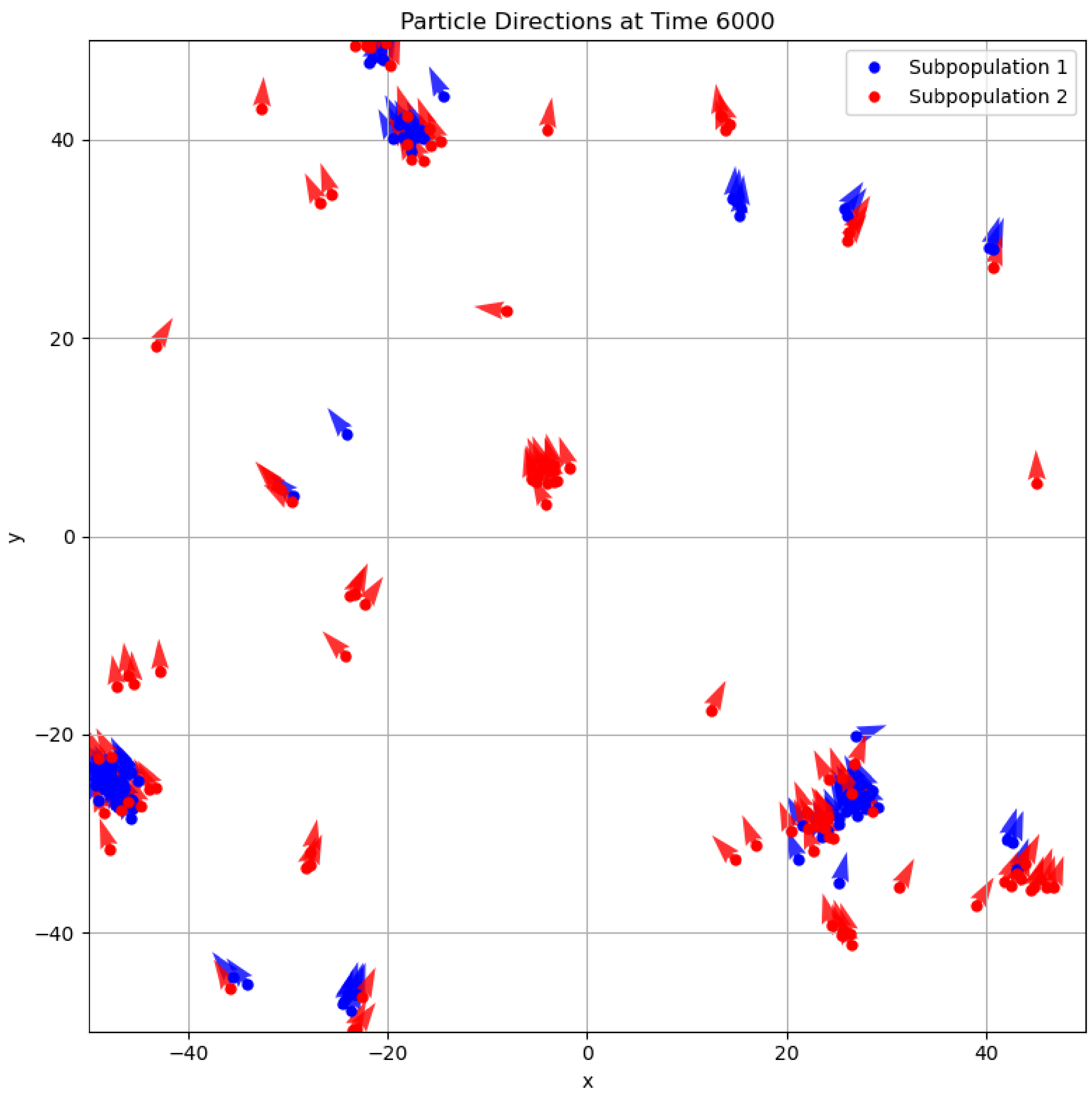
    plt.quiver(x_positions_1, y_positions_1, u1, v1, color='blue', angles='xy', 
    plt.quiver(x_positions_2, y_positions_2, u2, v2, color='red', angles='xy', s

    plt.title(f'Particle Directions at Time {time}')
    plt.xlabel('x')
    plt.ylabel('y')
    plt.grid()
    plt.xlim([-L / 2, L / 2])
    plt.ylim([-L / 2, L / 2])
    plt.legend()
    plt.tight_layout()
    plt.show()
```



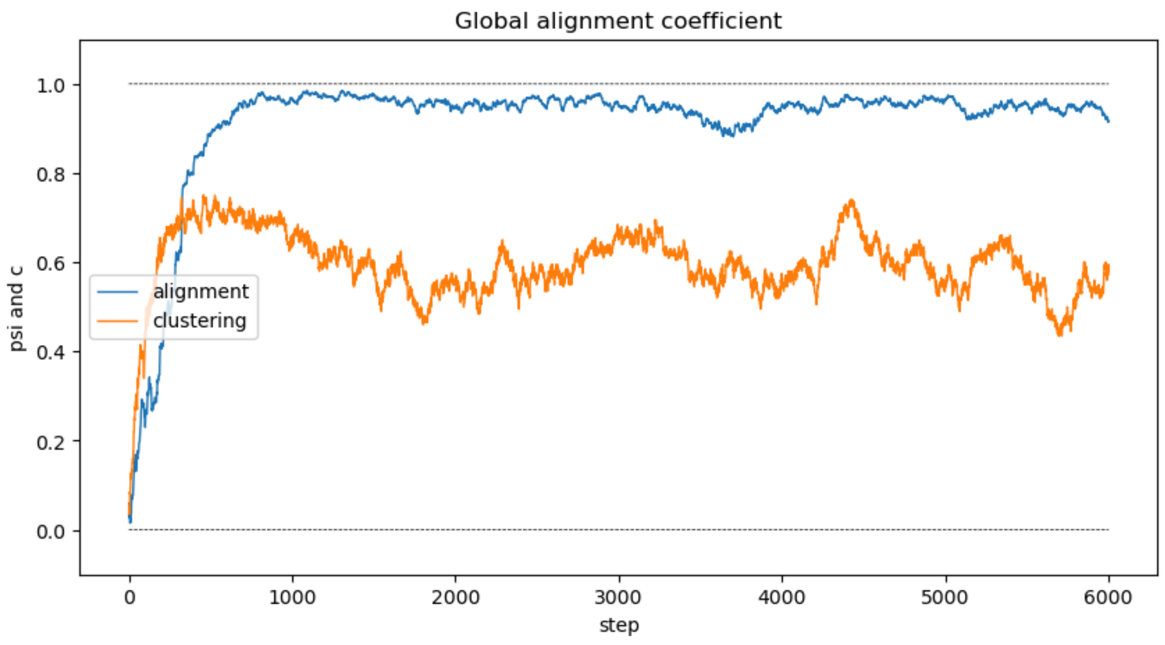

Particle Directions at Time 2000
Subpopulation 1
Subpopulation 2
x
y
−40
−20
0
20
40


Particle Directions at Time 4000
Subpopulation 1
Subpopulation 2
40
20
0
−20
−40
y
−40
−20
0
20
40
x



P4

In [12]:
```
from matplotlib import pyplot as plt
plt.figure(figsize=(10, 5))
plt.plot(psi2, '-', linewidth=1, label='alignment')
plt.plot(c2, '-', linewidth=1, label='clustering')
plt.plot(0 * psi2, '--', color='k', linewidth=0.5)
plt.plot(0 * psi2 + 1, '--', color='k', linewidth=0.5)
plt.title('Global alignment coefficient')
plt.legend()
plt.xlabel('step')
plt.ylabel('psi and c')
plt.ylim([-0.1, 1.1])
plt.show()
```



Q1

Studying the simulation, i can see that when clusters are formed, the low-noise particles remain aligned whereas the high-noise particles tend to defer from the alignment of the cluster. That is, the high-noise particles tend to move more randomly, disrupting the overall order and clustering of the system.

The low-noise particles try to form clusters, but the presence of the high-noise particles makes it harder for the entire system to achieve a coherent structure.

In summary: The presence of high-noise particles disrupts the alignment and clustering behavior demonstrated by low-noise particles, resulting in a less coherent system.