CS 630 - Fall 2020
Homework 2

Due: Wednesday, October 7

Reading : The reading on probabilistic algorithms found at the URL
http://www.cs.bu.edu/fac/homer/630/rand-alg1.pdf
Note: Please first read the notes from page 161 to 167. The earlier pages (pages 7-9) will be required too but only in about a week. This reading is from Randomized Algorithms by Motwani and Raghavan.

Problems:

1. LU decomposition for 2×2 singular matrices.

(i). **T** or **F**: The 2×2 matrix of all 0's has an LU decomposition. Show your LU decomposition or explain why there is none.

(ii) **T** or **F**: The 2×2 matrix of all -1's has an LU decomposition. Show your LU decomposition or explain why there is none.

(iii) **T** or **F**: Every singular 2×2 Boolean matrix has an LU decomposition. Prove or give a counterexample. Note: Entries of a Boolean matrix are 0's or 1's.

2. Find the LUP decomposition of the following 4 by 4 matrix M. Show one or two steps of your work, enough to show you are following the LUP algorithm.

```
2  -1  2  0
2  -1  0  3
4   2 -1  1
0   1  1  0
```

3. We saw in class that you can reduce "finding the inverse of M" to "solving the LUP decomposition of M," by using the LUP results from problem 2. You do this by setting up a system of 4 equations in four unknowns using the LUP decomposition. You do this once for each of the columns b of the I_4 matrix as the right hand side of Mx = b.

i. Write down the four Mx=b's that you have to solve for x in order to compute the inverse. However, you need not solve them for x, (except for what is asked in part ii. below).

ii. Find the solution of x in part i. for the b = the first column of I_4.

4. 1. You are given an algorithm A which can compute the square B $\times$ B of an n by n matrix in $O(n\sqrt{n})$ steps.

i Show how to use A to compute the product of any two n by n matrices M1 and M2, also in $O(n\sqrt{n})$.

ii. Show that your algorithm in part i. also takes $O(n\sqrt{n})$

5. (i). In a betting game you play against the house by picking 3 numbers from 1 to30. You do this with replacement, that is you can pick some number more than once. Then the lottery person (or machine) chooses three numbers at random, also from 1 to 30.

If you pick all three numbers correctly you win \$3000. Otherwise, you win \$400 if you picked any 2 out of the 3 numbers correctly. The game costs \$20 to play.

What is the expected value you earn when playing the game ? To do this define the relevant random variable and compute its expectation.

(ii). Assume you have 3 balls each a different color, red, blue and green. You put the balls in a box and without looking into the box you pick one of the balls at random and remember its color. Then the ball is put back into the box and do this 2 more times. So you might have seen only 1 color or 2 colors or 3 colors in the 3 picks.

What is the expected number of the different colors of the balls you see in your 3 picks ?
Explain how you found your answer,