CS 630 - Fall 2020
Homework 1

Due: Monday, September 21 by 12 noon - submit via Gradescope

Reading : Sections 28.1 and 28.2 pages 813-830 of the textbook Also, you read about matrices in appendix D of the textbook.

Problems: Please limit your answer to the following problems to at most 1/2 a pages each.

1. You are given a integer t and a polynomial p(x) of degree n where all of the n+1 coefficient a_i of p are non-zero integers. You want to compute p(t).
i. To compute p(t) in the normal way you plug in t for x wherever x occurs in p(x). Then you go through p from left to right and compute $a_i t^i$ for each term $a_i t^i$, i= 0,1,2,...,n, of p(t) and adding up the values of $a_i t^i$ as you go.
How many integer additions and integer multiplications does it take to do this computation of p(t)?

ii. Now see if you can find a better method to compute p(t). Your method should use fewer than than O(n $logn$) adds and multiplies.
Describe your method and then explain clearly what the number of arithmetic operations (+ and $\times$) is for your method.

2. A permutation matrix P is an n by n Boolean matrix with exactly one 1 in each row and one 1 in each column.

P is called a permutation matrix because if you multiply P by any n by 1 column vector V then the result PV is a permutation of V.

Given a permutation matrix P:
i. All permutation matrices are invertible. Explain how to construct the inverse of P from P.
ii. What are the possible values of the determinant of P ? Explain why your answer is correct.

iii. Let P =

$$\begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$$

Construct the inverse of P and compute P's determinant.

3. Look up the definitiond up lower triangular and upper triangular square matrices in the textbook.

i. Prove why the product of 2 lower triangular matrices is lower triangular

ii. Prove that the determinant of an upper triangular matrix is the product of its diagonal elements

iii. Give an example of two 3 by 3 triangular matrices whose product is not triangular

iv. Give and example of a non-singular 3 by 3 matrix M which has no LU decomposition. (So, M is not equal to LU for an L and U where L is unit lower triangular and U is upper triangular.)