Jiun-Yan (Eric) Chen
CS 630 Homework #1
*Collaborators*: None

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**Please limit your answer to the following problems to at most 1/2 a page each.**

**Problem 1.** You are given an integer $t$ and a polynomial $p(x)$ of degree $n$ where all of the $n+1$ coefficient $a_i$ of $p$ are non-zero integers. You want to compute $p(t)$.

**i)** How many integer additions and integer multiplications does it take to compute $p(t)$ the normal way, i.e. plugging in $t$ for $x$ wherever $x$ occurs in $p(x)$ and then computing each term $a_i t^i$ from left to right and summing the values as you go?

To resolve each $a_i t^i$, there is a multiplication step $a_i \times t \times t \times ... \times t$ where there are $i$ $t$s
So for each term, there is $i+1$ multiplication steps. As there is $i+1$ terms, the total multiplications count is:

$$\sum_{n=1}^{i+1} n = \frac{i+1(i+2)}{2}$$

To sum up all terms in $p(t)$, there is $a_i t^i + a_{i-1} t^{i-1} ... + a_0$, where there are $i$ additions, as there are total of $i+1$ terms.

**ii)** Now see if you can find a better method to compute $p(t)$, using fewer than $\mathcal{O}(n^2)$ adds and multiplies. Describe your method and explain clearly how many arithmetic operations it uses.

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**Problem 2.** A *permutation matrix* $P$ is an $n \times n$ Boolean matrix with exactly one 1 in each row and one 1 in each column. It is called a permutation matrix because if you multiply $P$ by any $n \times 1$ column vector $v$ then the result $Pv$ is a permutation of $v$.

**i)** Permutation matrices are invertible. Explain how to construct the inverse of a permutation $P$ from $P$.

**ii)** What are the possible values of the determinant of $P$? Explain why your answer is true.

**iii)** Let $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. Construct the inverse of $P$ and compute the determinant of $P$.

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**Problem 3.**

**i)** Prove that the product of 2 lower triangular matrices is also lower triangular.

**ii)** Prove that the determinant of an upper triangular matrix is the product of its diagonal elements.

**iii)** Give an example of two $3 \times 3$ triangular matrices whose product is not triangular.

**iv)** Give an example of a non-singular $3 \times 3$ matrix $M$ which has no $LU$ decomposition, i.e. $M$ is not equal to $LU$ for an $L$ and $U$ where $L$ is unit triangular and $U$ is upper triangular.