Logic

introduction to symbol logic

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Preface

This is an example of a math document written in LaTeX using `amsthm`, supporting unified numbering for theorems, definitions, and examples, with `cleveref` for auto-referencing.

— Eric

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Chapter 1

Introduction

1.1 Abstraction

Example 1.1.1. Models are constructed by abstraction.

All men are mortal.
Socrates is a man.
Therefore, Socrates is mortal.

In this example, the conclusion is depended only on the **form** of the sentences. What Socrates and mortal means is not important, it does matter what **”All”** means. The logical correctness of these deductions is due to their form but is independent of their content. This criterion is vague, but it is just this sort of vagueness that prompts us to turn to mathematical models.

A major goal will be to give, within a model, a precise version of this criterion. The questions (about our model) we will initially be most concerned with are these:

1. What does it mean for one sentence to **“follow logically”** from certain others?
2. If a sentence does follow logically from certain others, what methods of proof might be necessary to establish this fact?
3. Is there a gap between what we can **proof** in an axiomatic system (say for the natural numbers) and what is **true** the natural numbers?
4. What is the connection between logic and computability?

Chapter 2

Sentential Logic

2.1 Language of Sentential Logic

Definition 2.1.1. The five symbols: $\neg$, $\vee$, $\wedge$, $\rightarrow$ and $\leftrightarrow$ are called **sentential connective symbols**. The sentential connective symbols, together with the parentheses, are the **logical symbols**. The sentence symbols are called **parameters**(nonlogical symbols). An **expression** is a finite sequences of symbols.

Definition 2.1.2. Well formed formula satisfies the following consequences:

a) every sentence symbol is a wff.

b) if α and β are wffs, the $\alpha \rightarrow \beta$, $\neg\alpha$, $\alpha \leftrightarrow \beta$, $\alpha \wedge \beta$, $\alpha \vee \beta$ are also wffs.

c) No expression is a wff unless it is compelled to be one by (a) and (b).

Proposition 2.1.3. *If* $\mathbf{S}$ *is a set of wffs containing all the sentence symbols and closed under all five formula-building operations, then* $\mathbf{S}$ *is the set of all wffs.*

Proof. $\square$

Chapter 3

More Results

3.1 Propositions and Corollaries

Proposition 3.1.1. *The set of rational numbers* $\mathbb{Q}$ *is dense.*

Corollary 3.1.2. *There is a rational number between any two real numbers.*