



Logic

introduction to symbol logic

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Preface

This is an example of a math document written in L^AT_EX using `amsthm`, supporting unified numbering for theorems, definitions, and examples, with `cleveref` for auto-referencing.

— Eric

Contents

Chapter 1

Foundation

1.1 Fundamentals of Logic

Statements can be meaningfully claimed to be true or false.

If $E(x)$ is an expression which becomes a statement when x is replaced by an object or class, then E is a **property**.

We write $\exists!x \in X : E(x)$ when exactly one object $\in X$ has property E exists.

Let A and B be statements. Then we can define a new **statement** implication :

$$(A \rightarrow B) := (\neg A) \vee B$$

1.2 Sets

If $X \subseteq Y$ and $X \neq Y$, then X is called **proper subset** of Y . We denote this relationship by $X \subset Y$.

If X is a set and E is a property then $\{x \in X; E(x)\}$ is the subset of X consisting of all elements x of X such that $E(x)$ is true. Then the set:

$$\phi_X := \{x \in X; x \neq x\}$$

is called the **empty subset** of X .

Remark 1.2.1. Let E be a property, then

$$x \in \phi_X \rightarrow E(x)$$

is true for each $x \in X$

1.3 The Power Set

The power set of X is the set consists of all the subsets of X . Sometimes it's written 2^X .

1.4 Families of Sets

Let A be a nonempty set and for each $\alpha \in A$, let A_α be a set. Then $\{A_\alpha; \alpha \in A\}$ is called a **family of sets** and A is an **index set** for this family.

1.5 function

A **function** from X to Y is a rule which, for each element of X , specifies exactly one element of Y . The set X is called the **domain** of f and Y is called **codomain** of f . And

$$im(f) := \{y \in Y; \exists x \in X : y = f(x)\}$$

is called the **image** of f .

Remark 1.5.1. we should notice that the image of f is not equal to the codomain of f .