Exercises 1 - 5: Probability in R. v. Clark (Sally)

> "In 1999 in England Sally Clark was found guilty of the murder of two of her sons. Both infants were found dead in the morning, one in 1996 and another in 1998, and she claimed the cause of death was sudden infant death syndrome (SIDS). No evidence of physical harm was found on the two infants so the main piece of evidence against her was the testimony of Professor Sir Roy Meadow, who testified that the chances of two infants dying of SIDS was 1 in 73 million. He arrived at this figure by finding that the rate of SIDS was 1 in 8,500 and then calculating that the chance of two SIDS cases was 8,500 $\times$ 8,500 $\approx$ 73 million."

Independent probabilities: likelihood of two deaths

Suppose the rate of sudden infant death syndrome (SIDS) is 1 in 8,500. Define:

- A to be 'Sally Clark's **first** born infant dies of SIDS'
- B to be 'Sally Clark's **second** born infant dies of SIDS'

Then the probabilities for either infanty dieing of SIDS (given no additional information) are equal $\mathbb{P}(A) = \mathbb{P}(B)$.

```
P_A <- 1/8500; P_A
```

```
## [1] 0.0001176471
```

Which is a probability of approximately 0.012%.

When we know Sally's first infant dies this changes the probability of the second infant. Since we beilve SIDS has a gentic link, A and B are not independent. Therefore $\mathbb{P}(A \wedge B) \neq \mathbb{P}(A) \cdot \mathbb{P}(B)$ and we must use conditional probability.

$$P(B|A) = \frac{\mathbb{P}(B \wedge A)}{\mathbb{P}(A)}$$

Assume the probibility that a child will die of SIDS given a sibling died of SIDS is

$$\mathbb{P}(B|A) = \frac{1}{100}$$

```
P_B_given_A <- 1/100
```

Then the probability of both infants dieing of SIDS is

$$\mathbb{P}(A \wedge B) = \mathbb{P}(A) \cdot \mathbb{P}(B|A)$$

```
P_A_and_B <- P_A * P_B_given_A
P_A_and_B
```

```
## [1] 1.176471e-06
```

Bayes' Theorem: Sally Clark's Murder Probability

Define these propositions to these symbols:

- M to be 'Mother is a murder'
- D to be 'Mother's two infants die with no evidence of harm'

To find the probability of the mother being a murder given her two childern died with no evidence of harm can be found by Bayes' Theorem.

$$\mathbb{P}(M|D) = \frac{\mathbb{P}(D|M) \cdot \mathbb{P}(M)}{\mathbb{P}(D)}$$

Suppose the probability of a murder finding a way to kill two of her infants without leaving evidence of physical harm is 50%. Then

$$\mathbb{P}(D|M) = 0.5$$

```
P_D_given_M <- 0.5
```

Assume the murder rate among mother is 1 in 1 million.

$$\mathbb{P}(M) = 10^{-6}$$

```
P_M <- 1/1000000
```

From the previous section proposition D is the same as $A \wedge B$ so $\mathbb{P}(D) = \mathbb{P}(A) \cdot \mathbb{P}(B|A)$.

```
P_D <- P_A_and_B
```

Therefore by Bayes' Theorem

```
P_M_given_D <- P_D_given_M * P_M / P_D
P_M_given_D
```

```
## [1] 0.425
```

Therefore given our assumptions:

- SIDS is genetic; $\mathbb{P}(B|A) = \frac{1}{100}$
- There is a 50% she could kill two infants with no evidence of harm; $\mathbb{P}(D|M) = 0.5$
- The murder rate for mothers is 1 in 1 million; $\mathbb{P}(M) = \frac{1}{1000000}$

The probability Sally Clark kill both her children is about 43%.