Exercises 1 - 5: Probability in R. v. Clark (Sally)

Independent probabilities: likelihood of two deaths

Suppose the rate of sudden infant death syndrome (SIDS) is 1 in 8,500. Define:

- A to be 'Sally Clark's **first** born infant dies of SIDS'
- B to be 'Sally Clark's **second** born infant dies of SIDS'

Then the probabilities for either infanty dieing of SIDS (given no additional information) are equal $\mathbb{P}(A) = \mathbb{P}(B)$.

```
P_A <- 1/8500; P_A
```

```
## [1] 0.0001176471
```

Which is a probability of approximately 0.012%.

When we know Sally's first infant dies this changes the probability of the second infant. Since we beilve SIDS has a gentic link, A and B are not independent. Therefore $\mathbb{P}(A \wedge B) \neq \mathbb{P}(A) \cdot \mathbb{P}(B)$ and we must use conditional probability.

$$P(B|A) = \frac{\mathbb{P}(B \wedge A)}{\mathbb{P}(A)}$$

Assume the probibility that a child will die of SIDS given a sibling died of SIDS is

$$\mathbb{P}(B|A) = \frac{1}{100}$$

```
P_B_given_A <- 1/100
```

Then the probability of both infants dieing of SIDS is

$$\mathbb{P}(A \wedge B) = \mathbb{P}(A) \cdot \mathbb{P}(B|A)$$

```
P_A_and_B <- P_A * P_B_given_A
P_A_and_B
```

```
## [1] 1.176471e-06
```

Bayes' Theorem: Sally Clark's Murder Probability

Define these propositions to these symbols:

- M to be 'Mother is a murder'
- D to be 'Mother's two infants die with no evidence of harm'

To find the probability of the mother being a murder given her two childern died with no evidence of harm can be found by Bayes' Theorem.

$$\mathbb{P}(M|D) = \frac{\mathbb{P}(D|M) \cdot \mathbb{P}(M)}{\mathbb{P}(D)}$$

Suppose the probability of a murder finding a way to kill two of her infants without leaving evidence of physical harm is 50%. Then

$$\mathbb{P}(D|M) = 0.5$$

```
P_D_given_M <- 0.5
```

Assume the murder rate among mother is 1 in 1 million.

$$\mathbb{P}(M) = 10^{-6}$$

```
P_M <- 1/1000000
```

From the previous section proposition D is the same as $A \wedge B$ so $\mathbb{P}(D) = \mathbb{P}(A) \cdot \mathbb{P}(B|A)$.

```
P_D <- P_A_and_B
```

Therefore by Bayes' Theorem

```
P_M_given_D <- P_D_given_M * P_M / P_D
P_M_given_D
```

```
## [1] 0.425
```

Therefore given our assumptions:

- SIDS is genetic; $\mathbb{P}(B|A) = \frac{1}{100}$
- There is a 50% she could kill two infants with no evidence of harm; $\mathbb{P}(D|M) = 0.5$
- The murder rate for mothers is 1 in 1 million; $\mathbb{P}(M) = \frac{1}{1000000}$

The probability Sally Clark kill both her children is about 43%.